Deep Learning Principles & Applications

Chapter 3 – Shallow Neural Network

Sudarsan N.S. Acharya (sudarsan.acharya@manipal.edu)

Softmax classifier as a zero hidden layer neural network

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A *layered visualization* of applying the softmax classifier to a sample $\mathbf{x}$ with 5 features and correct output label y from 3 possible output labels:

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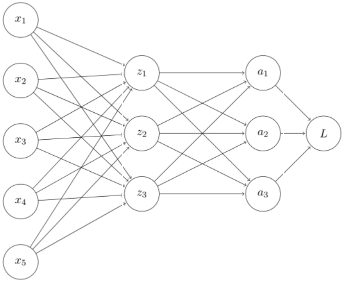

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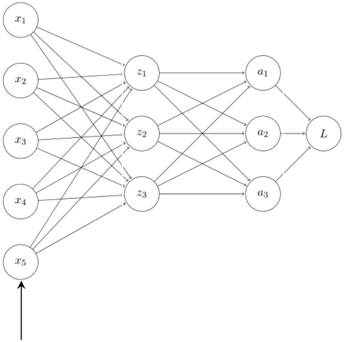

Input layer

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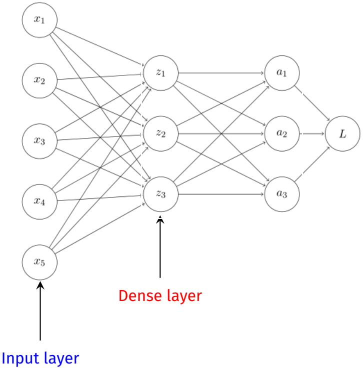

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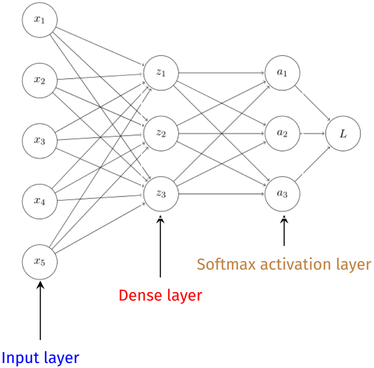

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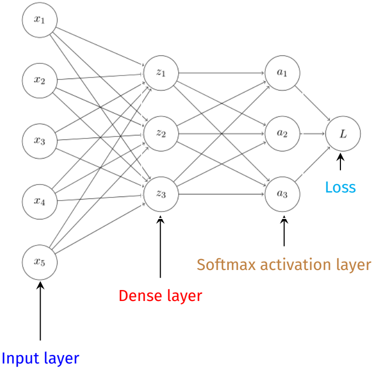

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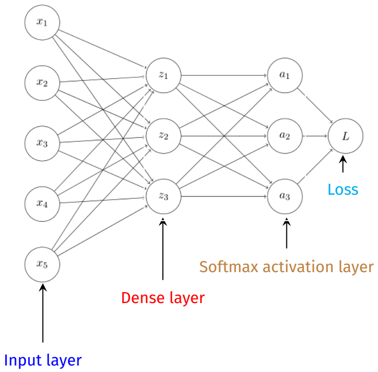

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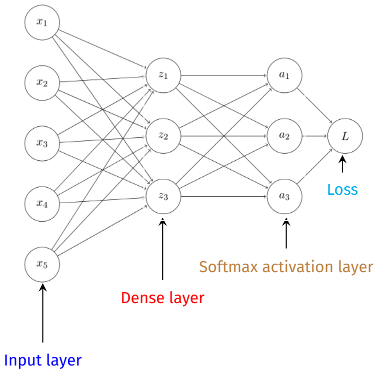

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A *layered visualization* of applying the softmax classifier to a sample x with 5 features and correct output label y from 3 possible output labels:

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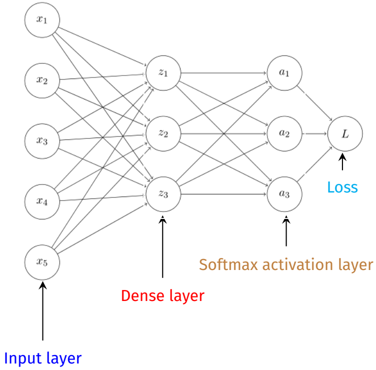

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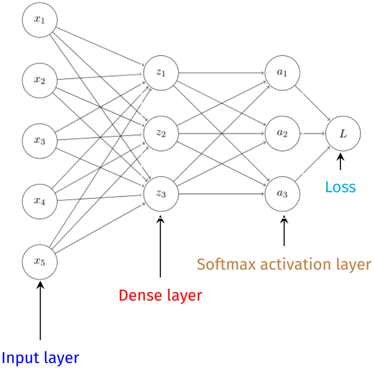

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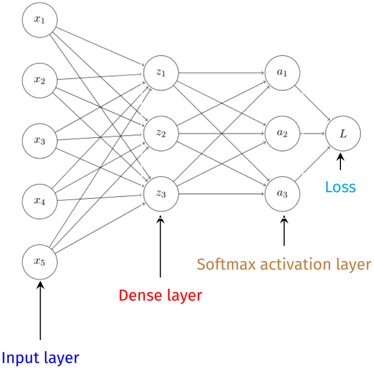

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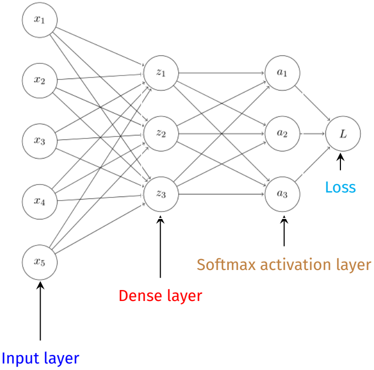

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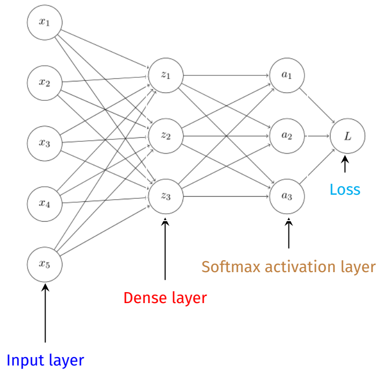

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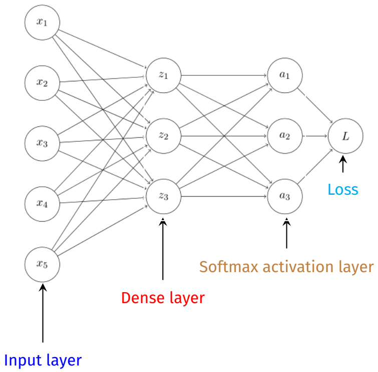

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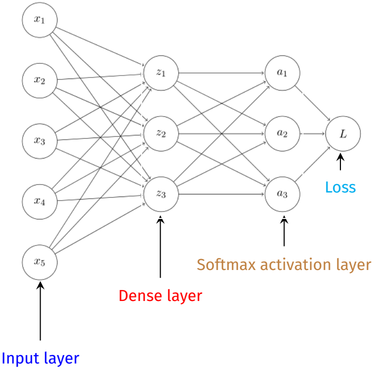

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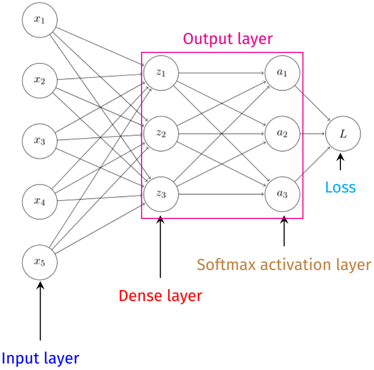

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- Dense + Softmax activation layers put together is called the output layer.

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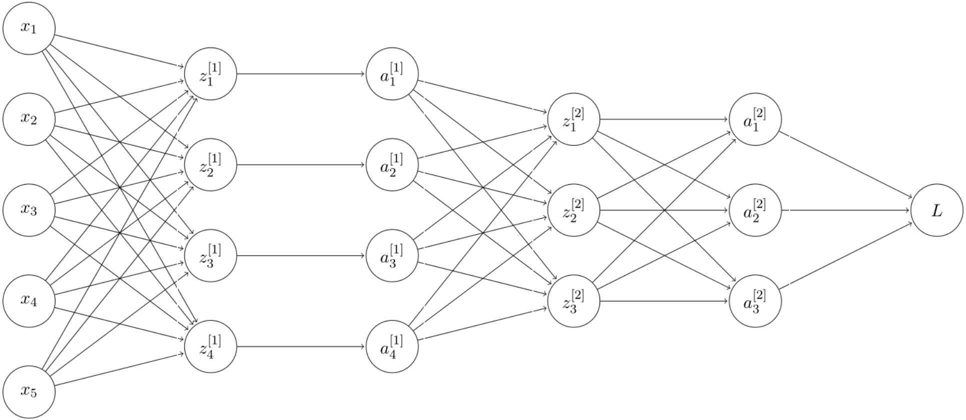

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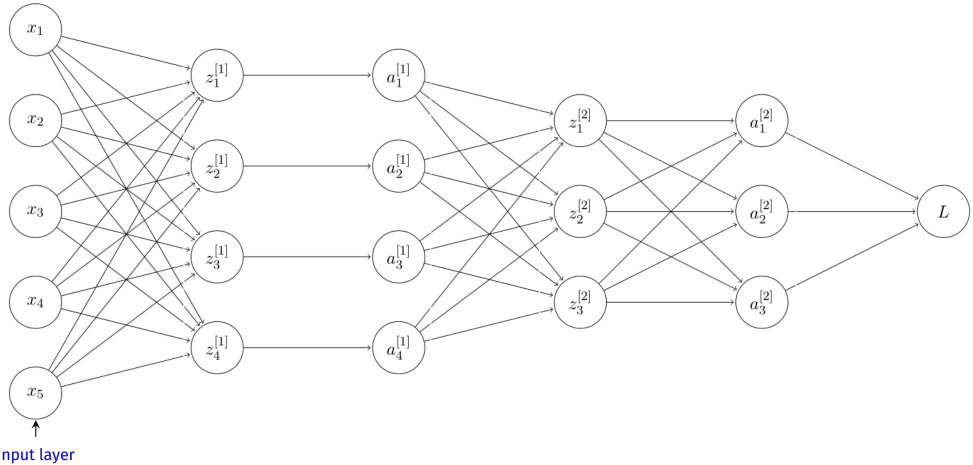

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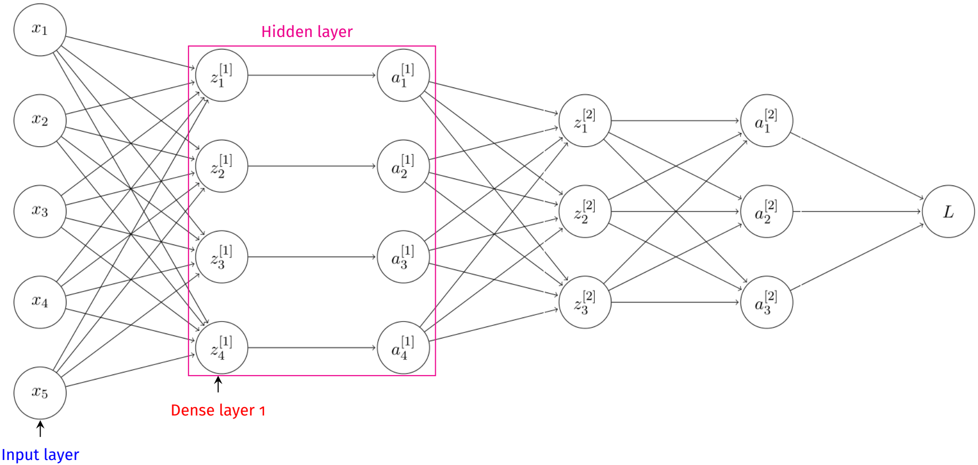

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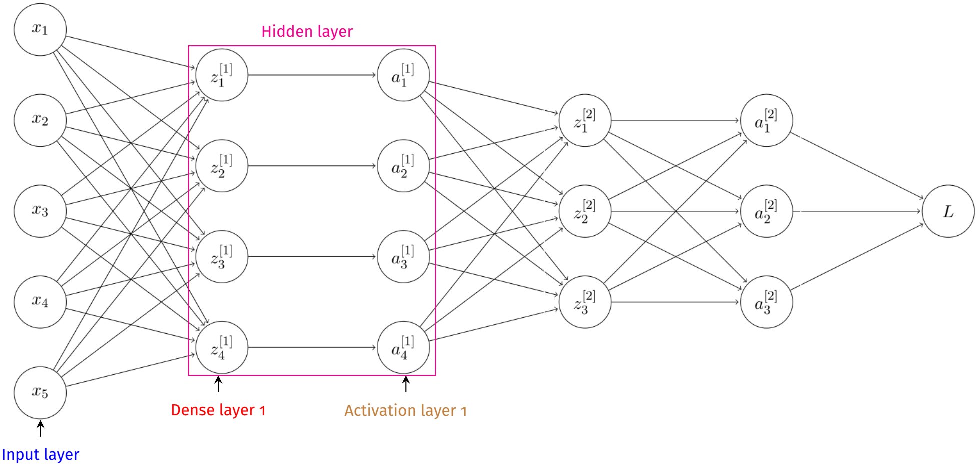

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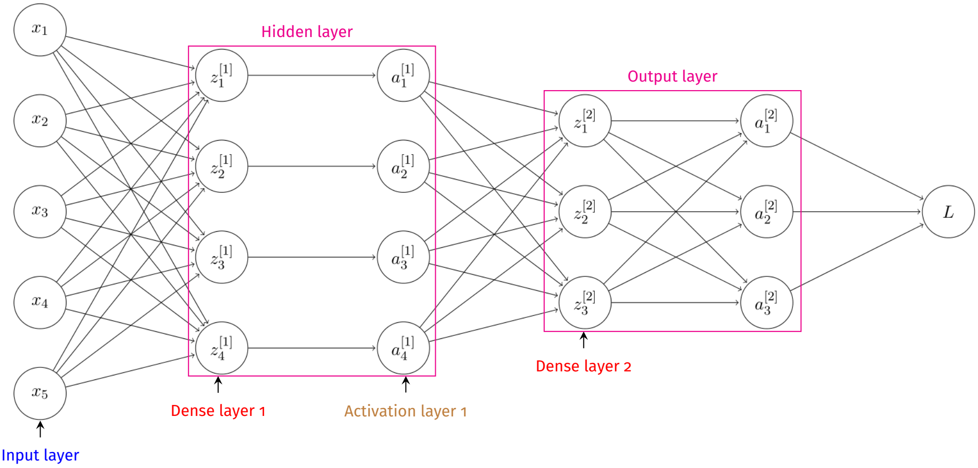

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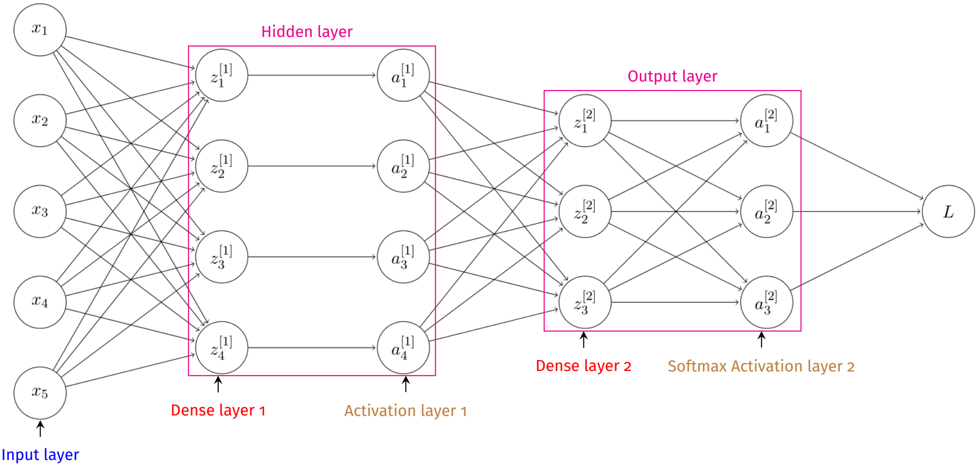

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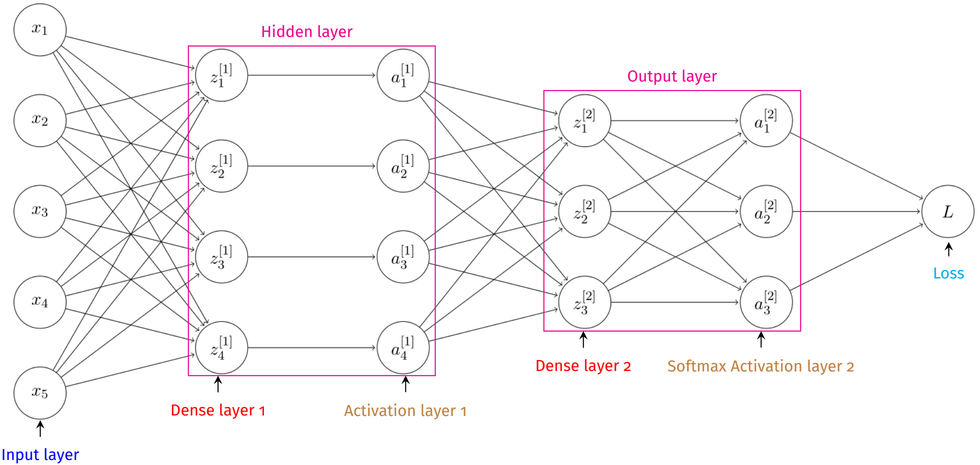

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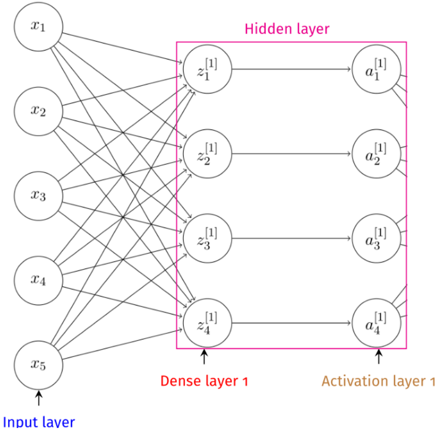

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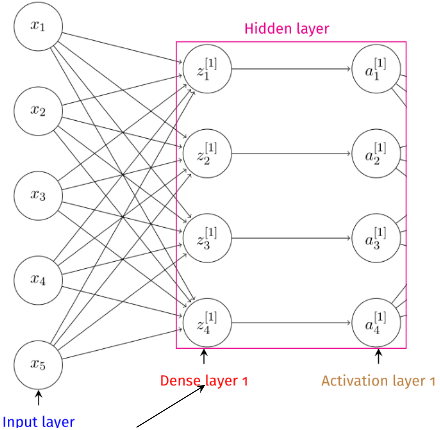

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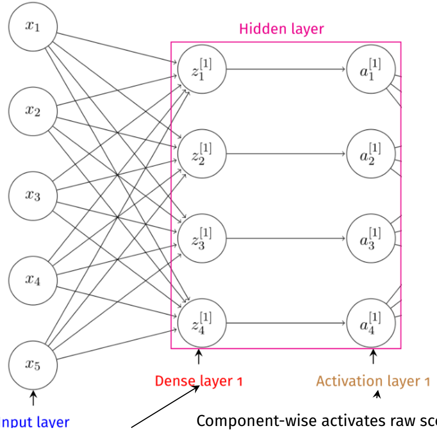

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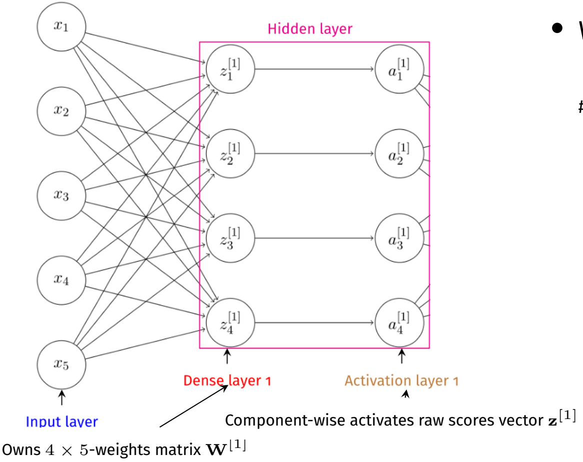

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$$\underbrace{4}_{\text{\# output raw scores in dense layer 1}} \times \underbrace{5}_{\text{\# features in input layer}}$$

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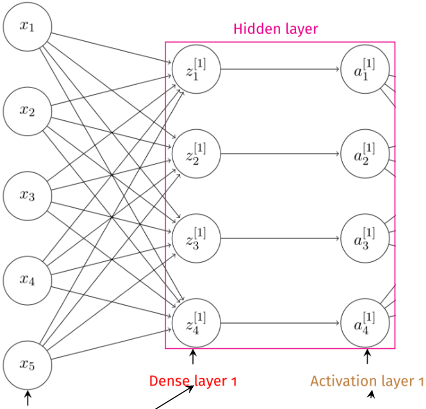

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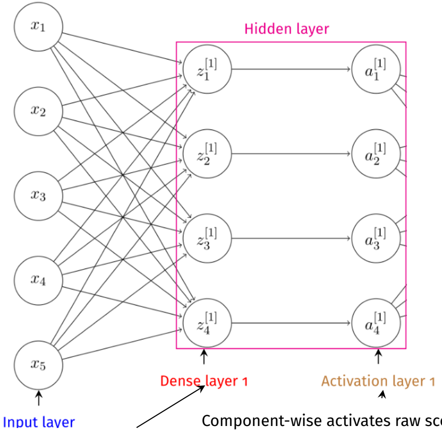

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- Activated scores for activation layer 1 is $\mathbf{a}^{[1]} = \begin{bmatrix} a_1^{[1]} \\ a_2^{[1]} \\ a_3^{[1]} \\ a_4^{[1]} \end{bmatrix} = \begin{bmatrix} g\left(z_1^{[1]}\right) \\ g\left(z_2^{[1]}\right) \\ g\left(z_3^{[1]}\right) \\ g\left(z_4^{[1]}\right) \end{bmatrix}$, where g is the *layer's activation function*.

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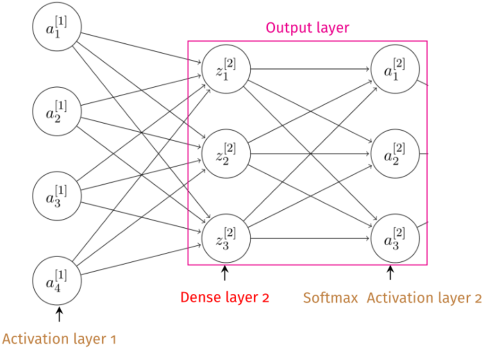

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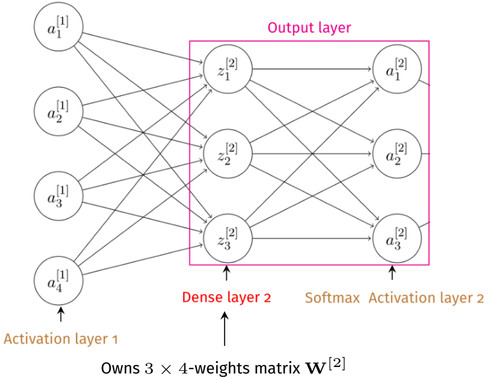

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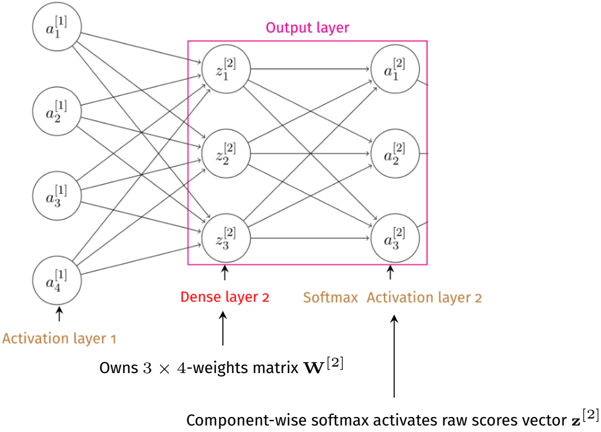

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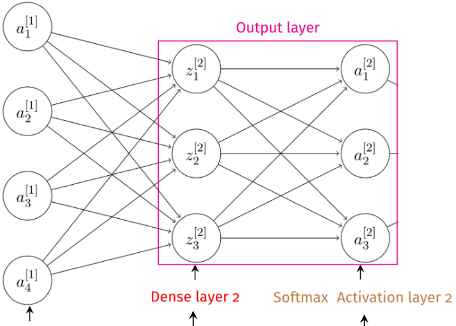

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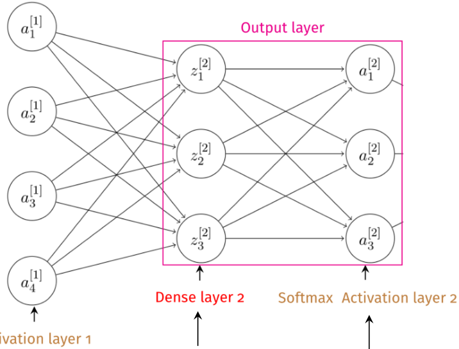

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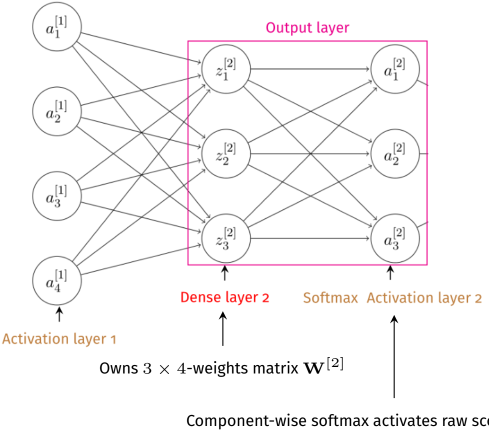

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- Activated scores for softmax activation layer 2 is

$$\mathbf{a}^{[2]} = \begin{bmatrix} a_1^{[2]} \\ a_2^{[2]} \\ a_3^{[2]} \end{bmatrix} = \text{softmax}\left(\mathbf{z}^{[2]}\right) = \text{softmax}\left(\begin{bmatrix} z_1^{[2]} \\ z_2^{[2]} \\ z_3^{[2]} \end{bmatrix}\right).$$

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- However, the identity activation function does not lead to *nonlinear* learning that can capture potential nonlinear relationship between the input and output.
- *Nonlinear learning* is achieved using nonlinear activation functions.

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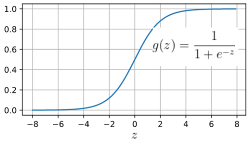

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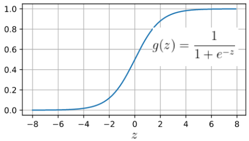

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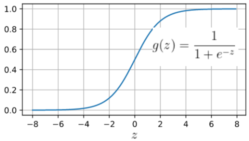

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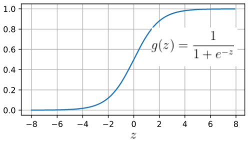

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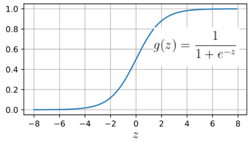

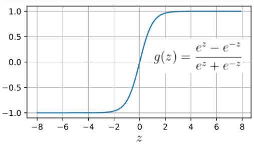

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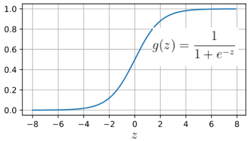

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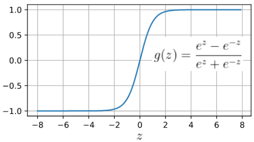
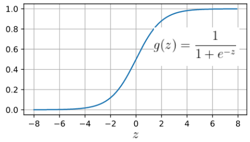
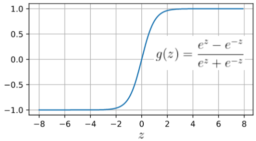


- tanh (hyperbolic tangent) activation function.

Activation functions: need and types – continued



- Sigmoid activation function.
- Activated output between 0 and 1.
- Almost linear behavior for small inputs around 0.



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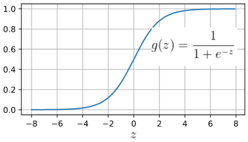
Activation functions: need and types – continued



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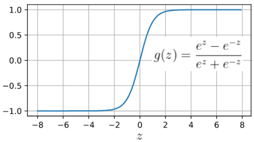

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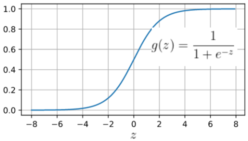
Activation functions: need and types – continued



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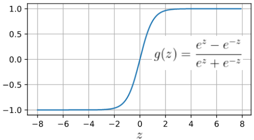

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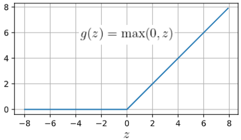
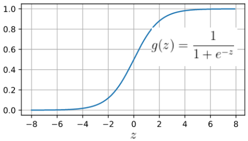


Activation functions: need and types – continued



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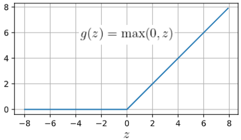
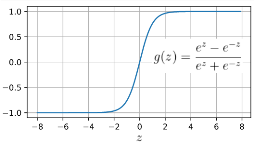


- ReLU (rectified linear unit) activation function.



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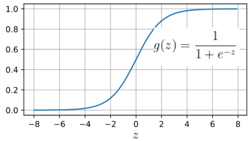
Activation functions: need and types – continued



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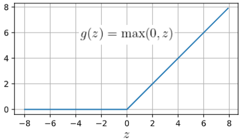

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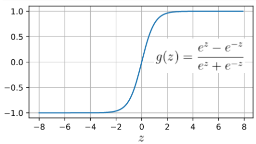

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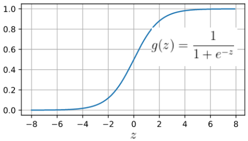
Activation functions: need and types – continued



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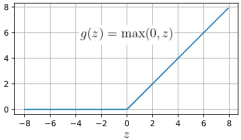
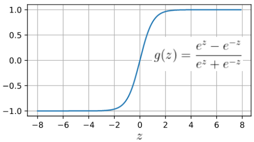


- ReLU (rectified linear unit) activation function.
- Negative inputs clipped to 0, non-negative ones transmitted as such.
- Simple and effective.



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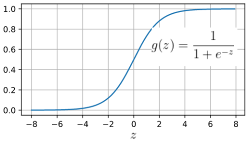

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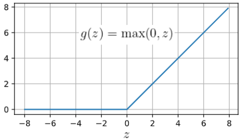

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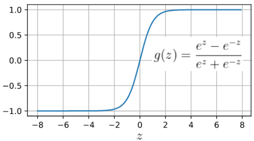

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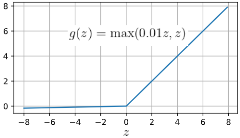
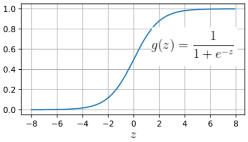


Activation functions: need and types – continued



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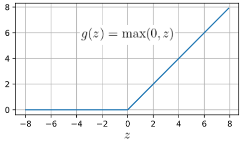

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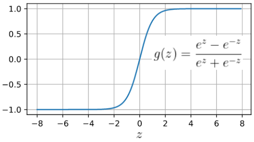

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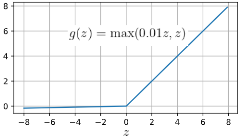
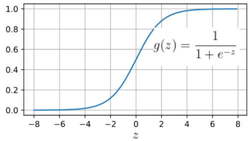


- Leaky ReLU (leaky rectified linear unit) activation function.

Activation functions: need and types – continued



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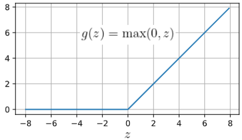

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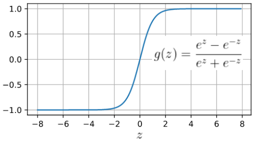

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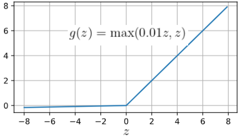
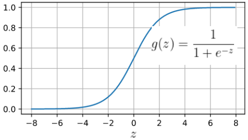


- Leaky ReLU (leaky rectified linear unit) activation function.
- Negative inputs transmitted with a small multiplicative factor while non-negative ones transmitted as such just like ReLU.

Activation functions: need and types – continued



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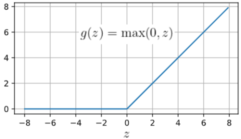

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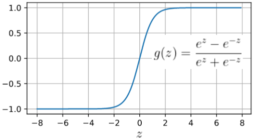

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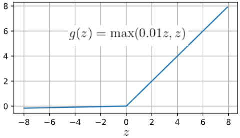

- Leaky ReLU (leaky rectified linear unit) activation function.
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- Just like ReLU, simple and effective.

Gradients of activation functions

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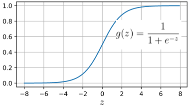

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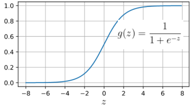
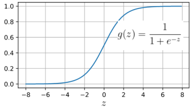


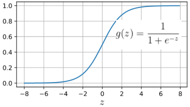
- Sigmoid activation function.

Gradients of activation functions



- Sigmoid activation function.
- $\nabla_z(g(z)) = \frac{1}{\left(1+e^{-z}\right)^2} = g(z)(1-g(z)).$

Gradients of activation functions



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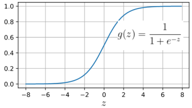

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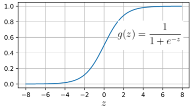

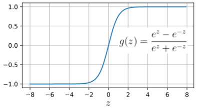

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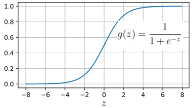

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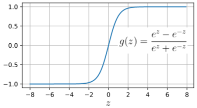

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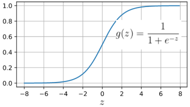
Gradients of activation functions



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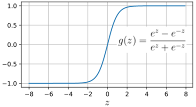

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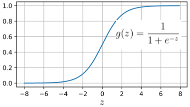
Gradients of activation functions



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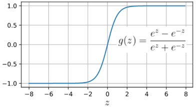

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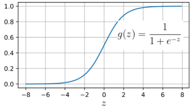

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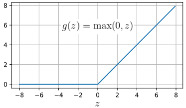

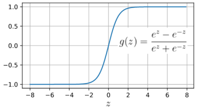

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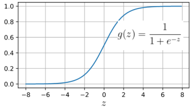

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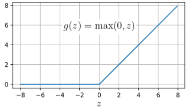

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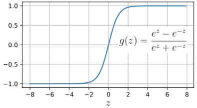

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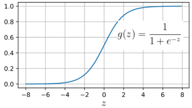

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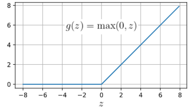

- ReLU (rectified linear unit) activation function.
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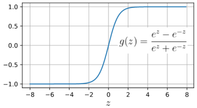

- tanh (hyperbolic tangent) activation function.
- $\nabla_z(g(z)) = 1 - \left(\frac{e^z - e^{-z}}{e^z + e^{-z}}\right)^2 = 1 - g(z)^2.$
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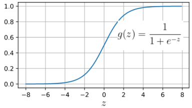
Gradients of activation functions



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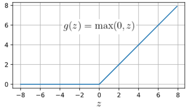

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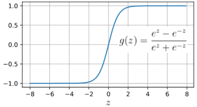

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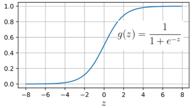

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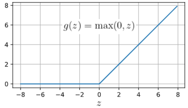

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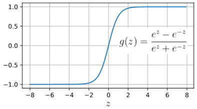

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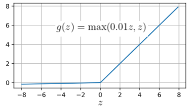

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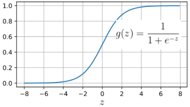

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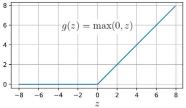

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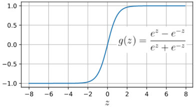

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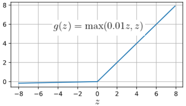

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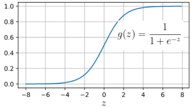

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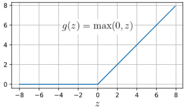

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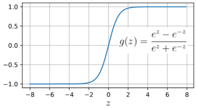

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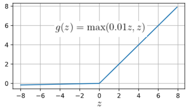

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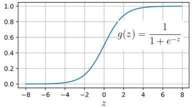

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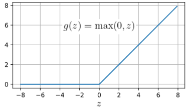

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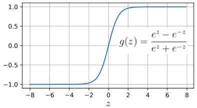

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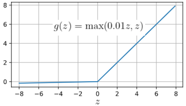

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Zero hidden layer neural network (softmax classifier) forward propagation revisited

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Softmax classifier to a sample x with 5 features and correct output label y from 3 possible output labels (bias feature 1 ignored for clarity):

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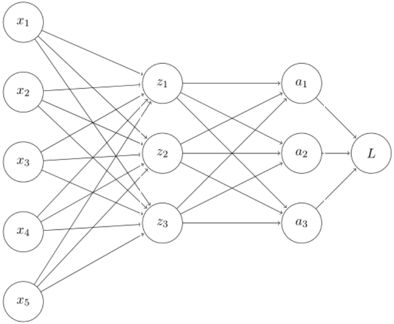

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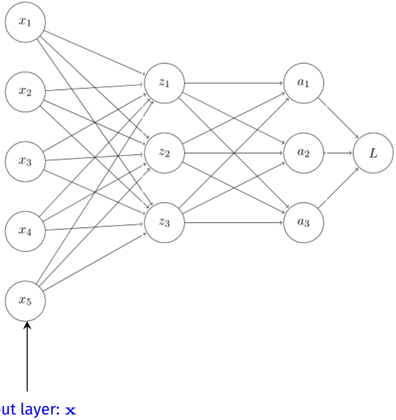

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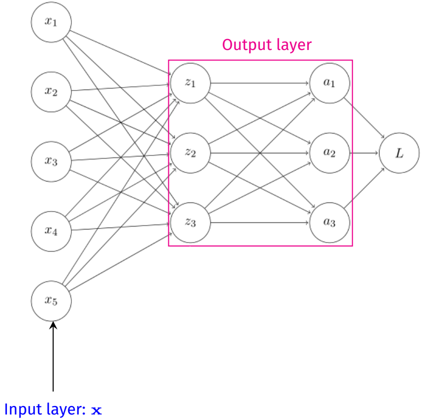

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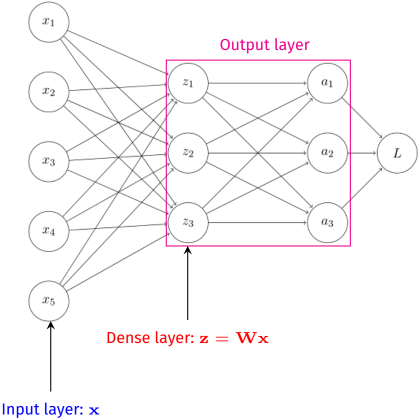

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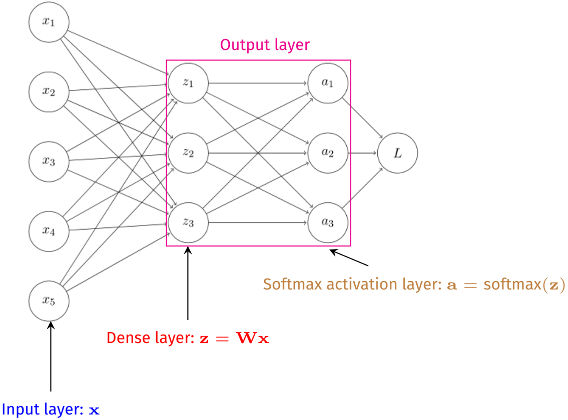

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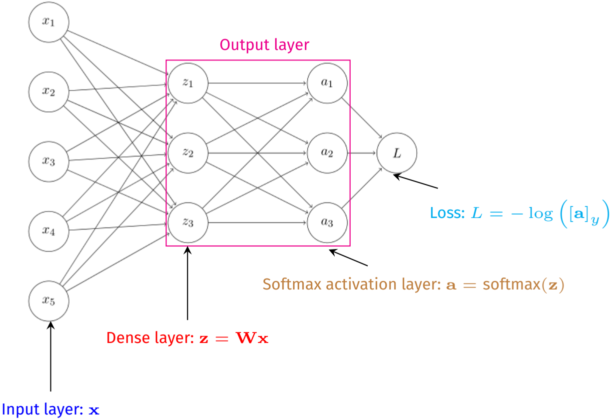

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\end{tikzcd}
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Gradient calculation using chain rule:

$$\nabla_{\mathbf{w}}(L) = \quad \nabla_{\mathbf{z}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

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Gradient calculation using chain rule:

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$$
\begin{aligned}
\nabla_{\mathbf{w}}(L) &= \nabla_{\mathbf{W}}(\mathbf{z}) \times \nabla_{\mathbf{z}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L) \\
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$$= \nabla_{\mathbf{W}}(\mathbf{z}) \times \nabla_{\mathbf{z}}(\mathbf{a}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

$$\nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right).$$

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$$

Gradient calculation using chain rule:

$$\nabla_{\mathbf{w}}(L) = \nabla_{\mathbf{W}}(\mathbf{z}) \times \nabla_{\mathbf{z}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

$$= \nabla_{\mathbf{W}}(\mathbf{z}) \times \nabla_{\mathbf{z}}(\mathbf{a}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

$$\nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right) \times \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right).$$

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&= \nabla_{\mathbf{W}}\left(\mathbf{W}\mathbf{x}\right) \times \nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right) \times \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right).
\end{aligned}$$

We will calculate the gradient term-by-term backwards.

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

We will now calculate the last gradient $\nabla_{\hat{\mathbf{y}}}(L) = \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right)$ in

$$\nabla_{\mathbf{W}}\left(\mathbf{W}\mathbf{x}\right) \times \nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right) \times \boxed{\nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right)}:$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

We will now calculate the last gradient $\nabla_{\hat{\mathbf{y}}}(L) = \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right)$ in

$$\nabla_{\mathbf{W}}\left(\mathbf{W}\mathbf{x}\right) \times \nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right) \times \boxed{\nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right)}:$$

$$\nabla_{\hat{\mathbf{y}}}(L) = \begin{bmatrix} \nabla_{\hat{y}_1}(L) \\ \nabla_{\hat{y}_2}(L) \\ \nabla_{\hat{y}_3}(L) \end{bmatrix}$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

We will now calculate the last gradient $\nabla_{\hat{\mathbf{y}}}(L) = \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right)$ in

$$\nabla_{\mathbf{W}}\left(\mathbf{W}\mathbf{x}\right) \times \nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right) \times \boxed{\nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right)}:$$

$$\nabla_{\hat{\mathbf{y}}}(L) = \begin{bmatrix} \nabla_{\hat{y}_1}(L) \\ \nabla_{\hat{y}_2}(L) \\ \nabla_{\hat{y}_3}(L) \end{bmatrix} = \begin{bmatrix} \nabla_{\hat{y}_1}\left(-y_1 \log\left(\hat{y}_1\right) - y_2 \log\left(\hat{y}_2\right) - y_3 \log\left(\hat{y}_3\right)\right) \\ \nabla_{\hat{y}_2}\left(-y_1 \log\left(\hat{y}_1\right) - y_2 \log\left(\hat{y}_2\right) - y_3 \log\left(\hat{y}_3\right)\right) \\ \nabla_{\hat{y}_3}\left(-y_1 \log\left(\hat{y}_1\right) - y_2 \log\left(\hat{y}_2\right) - y_3 \log\left(\hat{y}_3\right)\right) \end{bmatrix}$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

We will now calculate the last gradient $\nabla_{\hat{\mathbf{y}}}(L) = \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log(\hat{y}_k)\right)$ in

$$\nabla_{\mathbf{W}}(\mathbf{W}\mathbf{x}) \times \nabla_{\mathbf{z}}(\text{softmax}(\mathbf{z})) \times \boxed{\nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log(\hat{y}_k)\right)}:$$

$$\nabla_{\hat{\mathbf{y}}}(L) = \begin{bmatrix} \nabla_{\hat{y}_1}(L) \\ \nabla_{\hat{y}_2}(L) \\ \nabla_{\hat{y}_3}(L) \end{bmatrix} = \begin{bmatrix} \nabla_{\hat{y}_1}\left(-y_1 \log(\hat{y}_1) - y_2 \log(\hat{y}_2) - y_3 \log(\hat{y}_3)\right) \\ \nabla_{\hat{y}_2}\left(-y_1 \log(\hat{y}_1) - y_2 \log(\hat{y}_2) - y_3 \log(\hat{y}_3)\right) \\ \nabla_{\hat{y}_3}\left(-y_1 \log(\hat{y}_1) - y_2 \log(\hat{y}_2) - y_3 \log(\hat{y}_3)\right) \end{bmatrix} = \begin{bmatrix} -y_1/\hat{y}_1 \\ -y_2/\hat{y}_2 \\ -y_3/\hat{y}_3 \end{bmatrix}.$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

We will now calculate the middle gradient $\nabla_{\mathbf{z}}(\mathbf{a}) = \nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right)$ in $\nabla_{\mathbf{W}}\left(\mathbf{W}\mathbf{x}\right) \times \boxed{\nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right)} \times \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right)$:

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

We will now calculate the middle gradient $\nabla_{\mathbf{z}}(\mathbf{a}) = \nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right)$ in $\nabla_{\mathbf{W}}\left(\mathbf{W}\mathbf{x}\right) \times \boxed{\nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right)} \times \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right)$:

$$\nabla_{\mathbf{z}}(\mathbf{a}) = \nabla_{\mathbf{z}}\left(\begin{bmatrix} \frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}} \\ \frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}} \\ \frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}} \end{bmatrix}\right)$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

We will now calculate the middle gradient $\nabla_{\mathbf{z}}(\mathbf{a}) = \nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right)$ in $\nabla_{\mathbf{W}}\left(\mathbf{W}\mathbf{x}\right) \times \boxed{\nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right)} \times \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right)$:

$$\nabla_{\mathbf{z}}(\mathbf{a}) = \nabla_{\mathbf{z}}\left(\begin{bmatrix} \frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}} \\ \frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}} \\ \frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}} \end{bmatrix}\right) = \begin{bmatrix} \nabla_{\mathbf{z}}\left(\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{\mathbf{z}}\left(\frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{\mathbf{z}}\left(\frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) \end{bmatrix}$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

We will now calculate the middle gradient $\nabla_{\mathbf{z}}(\mathbf{a}) = \nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right)$ in $\nabla_{\mathbf{W}}\left(\mathbf{Wx}\right) \times \boxed{\nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right)} \times \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right)$:

$$\nabla_{\mathbf{z}}(\mathbf{a}) = \nabla_{\mathbf{z}}\left(\begin{bmatrix} \frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}} \\ \frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}} \\ \frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}} \end{bmatrix}\right) = \begin{bmatrix} \nabla_{\mathbf{z}}\left(\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{\mathbf{z}}\left(\frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{\mathbf{z}}\left(\frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) \end{bmatrix}$$

$$= \begin{bmatrix} \nabla_{z_1}\left(\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_1}\left(\frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_1}\left(\frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) \\ \nabla_{z_2}\left(\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_2}\left(\frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_2}\left(\frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) \\ \nabla_{z_3}\left(\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_3}\left(\frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_3}\left(\frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) \end{bmatrix}$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

We will now calculate the middle gradient $\nabla_{\mathbf{z}}(\mathbf{a}) = \nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right)$ in

$$\nabla_{\mathbf{W}}\left(\mathbf{W}\mathbf{x}\right) \times \boxed{\nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right)} \times \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right):$$

$$\nabla_{\mathbf{z}}(\mathbf{a}) = \nabla_{\mathbf{z}}\left(\begin{bmatrix} \frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}} \\ \frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}} \\ \frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}} \end{bmatrix}\right) = \begin{bmatrix} \nabla_{\mathbf{z}}\left(\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{\mathbf{z}}\left(\frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{\mathbf{z}}\left(\frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) \end{bmatrix}$$

$$= \begin{bmatrix} \nabla_{z_1}\left(\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_1}\left(\frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_1}\left(\frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) \\ \nabla_{z_2}\left(\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_2}\left(\frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_2}\left(\frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) \\ \nabla_{z_3}\left(\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_3}\left(\frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_3}\left(\frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) \end{bmatrix}$$

$$\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}} - \left(\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}\right)^2 = a_1 - a_1^2 = a_1(1-a_1)$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

We will now calculate the middle gradient $\nabla_{\mathbf{z}}(\mathbf{a}) = \nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right)$ in

$$\nabla_{\mathbf{W}}\left(\mathbf{W}\mathbf{x}\right) \times \boxed{\nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right)} \times \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right):$$

$$\nabla_{\mathbf{z}}(\mathbf{a}) = \nabla_{\mathbf{z}}\left(\begin{bmatrix} \frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}} \\ \frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}} \\ \frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}} \end{bmatrix}\right) = \begin{bmatrix} \nabla_{\mathbf{z}}\left(\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{\mathbf{z}}\left(\frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{\mathbf{z}}\left(\frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) \end{bmatrix}$$

$$= \begin{bmatrix} \nabla_{z_1}\left(\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_1}\left(\frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_1}\left(\frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) \\ \nabla_{z_2}\left(\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_2}\left(\frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_2}\left(\frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) \\ \nabla_{z_3}\left(\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_3}\left(\frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_3}\left(\frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) \end{bmatrix}$$

$$\frac{-e^{z_1}e^{z_2}}{(e^{z_1}+e^{z_2}+e^{z_3})^2} = -a_1a_2$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

We will now calculate the middle gradient $\nabla_{\mathbf{z}}(\mathbf{a}) = \nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right)$ in $\nabla_{\mathbf{W}}\left(\mathbf{W}\mathbf{x}\right) \times \boxed{\nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right)} \times \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right)$:

$$\nabla_{\mathbf{z}}(\mathbf{a}) = \nabla_{\mathbf{z}}\left(\begin{bmatrix} \frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}} \\ \frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}} \\ \frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}} \end{bmatrix}\right) = \begin{bmatrix} \nabla_{\mathbf{z}}\left(\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{\mathbf{z}}\left(\frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{\mathbf{z}}\left(\frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) \end{bmatrix}$$

$$= \begin{bmatrix} \nabla_{z_1}\left(\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_1}\left(\frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_1}\left(\frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) \\ \nabla_{z_2}\left(\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_2}\left(\frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_2}\left(\frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) \\ \nabla_{z_3}\left(\frac{e^{z_1}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_3}\left(\frac{e^{z_2}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) & \nabla_{z_3}\left(\frac{e^{z_3}}{e^{z_1}+e^{z_2}+e^{z_3}}\right) \end{bmatrix} = \begin{bmatrix} a_1(1-a_1) & -a_2 a_1 & -a_3 a_1 \\ -a_1 a_2 & a_2(1-a_2) & -a_3 a_2 \\ -a_1 a_3 & -a_2 a_3 & a_3(1-a_3) \end{bmatrix}.$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

We will now calculate the last gradient $\nabla_{\mathbf{W}}(\mathbf{W}\mathbf{x})$ in

$$\boxed{\nabla_{\mathbf{W}}\left(\mathbf{W}\mathbf{x}\right)} \times \nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right) \times \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right)$$ using the fact that $\mathbf{W} = \begin{bmatrix} \mathbf{w}_1^{\mathrm{T}} \\ \mathbf{w}_2^{\mathrm{T}} \\ \mathbf{w}_3^{\mathrm{T}} \end{bmatrix}$:

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

Gradient shape = shape of $\mathbf{W}$ $\times$ shape of $\mathbf{Wx}$ $= (3 \times 5) \times 3$ which is a 5×3-matrix repeating 3 times

We will now calculate the last gradient $\nabla_{\mathbf{W}}(\mathbf{Wx})$ in

$$\boxed{\nabla_{\mathbf{W}}(\mathbf{Wx})} \times \nabla_{\mathbf{z}}(\text{softmax}(\mathbf{z})) \times \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log(\hat{y}_k)\right)$$ using the fact that $\mathbf{W} = \begin{bmatrix} \mathbf{w}_1^{\mathrm{T}} \\ \mathbf{w}_2^{\mathrm{T}} \\ \mathbf{w}_3^{\mathrm{T}} \end{bmatrix}$:

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

Gradient shape = shape of $\mathbf{W}$ $\times$ shape of $\mathbf{Wx}$ $= (3 \times 5) \times 3$ which is a 5×3-matrix repeating 3 times

We will now calculate the last gradient $\nabla_{\mathbf{W}}(\mathbf{Wx})$ in

$$\boxed{\nabla_{\mathbf{W}}(\mathbf{Wx})} \times \nabla_{\mathbf{z}}(\text{softmax}(\mathbf{z})) \times \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log(\hat{y}_k)\right)$$ using the fact that $\mathbf{W} = \begin{bmatrix} \mathbf{w}_1^{\mathrm{T}} \\ \mathbf{w}_2^{\mathrm{T}} \\ \mathbf{w}_3^{\mathrm{T}} \end{bmatrix}$:

$$\nabla_{\mathbf{W}}(\mathbf{Wx}) \quad =$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

Gradient shape = shape of $\mathbf{W}$ $\times$ shape of $\mathbf{W}\mathbf{x} = (3 \times 5) \times 3$ which is a 5×3-matrix repeating 3 times

We will now calculate the last gradient $\nabla_{\mathbf{W}}(\mathbf{W}\mathbf{x})$ in

$$\boxed{\nabla_{\mathbf{W}}\left(\mathbf{W}\mathbf{x}\right)} \times \nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right) \times \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right)$$ using the fact that $\mathbf{W} = \begin{bmatrix} \mathbf{w}_1^{\mathrm{T}} \\ \mathbf{w}_2^{\mathrm{T}} \\ \mathbf{w}_3^{\mathrm{T}} \end{bmatrix}$:

$$\nabla_{\mathbf{W}}(\mathbf{W}\mathbf{x}) = \nabla_{\mathbf{W}}\left(\begin{bmatrix} \mathbf{w}_1^{\mathrm{T}} \\ \mathbf{w}_2^{\mathrm{T}} \\ \mathbf{w}_3^{\mathrm{T}} \end{bmatrix}\mathbf{x}\right) =$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

Gradient shape = shape of $\mathbf{W}$ $\times$ shape of $\mathbf{Wx} = (3 \times 5) \times 3$ which is a 5×3-matrix repeating 3 times

We will now calculate the last gradient $\nabla_{\mathbf{W}}(\mathbf{Wx})$ in

$$\boxed{\nabla_{\mathbf{W}}(\mathbf{Wx})} \times \nabla_{\mathbf{z}}(\text{softmax}(\mathbf{z})) \times \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log(\hat{y}_k)\right) \text{ using the fact that } \mathbf{W} = \begin{bmatrix}\mathbf{w}_1^{\mathrm{T}}\\ \mathbf{w}_2^{\mathrm{T}}\\ \mathbf{w}_3^{\mathrm{T}}\end{bmatrix}:$$

$$\nabla_{\mathbf{W}}(\mathbf{Wx}) = \nabla_{\mathbf{W}}\left(\begin{bmatrix}\mathbf{w}_1^{\mathrm{T}}\\ \mathbf{w}_2^{\mathrm{T}}\\ \mathbf{w}_3^{\mathrm{T}}\end{bmatrix}\mathbf{x}\right) = \nabla_{\mathbf{W}}\left(\begin{bmatrix}\mathbf{w}_1^{\mathrm{T}}\mathbf{x}\\ \mathbf{w}_2^{\mathrm{T}}\mathbf{x}\\ \mathbf{w}_3^{\mathrm{T}}\mathbf{x}\end{bmatrix}\right) =$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

Gradient shape = shape of $\mathbf{W}$ $\times$ shape of $\mathbf{Wx}$ $= (3 \times 5) \times 3$ which is a 5×3-matrix repeating 3 times

We will now calculate the last gradient $\nabla_{\mathbf{W}}(\mathbf{Wx})$ in

$$\boxed{\nabla_{\mathbf{W}}\left(\mathbf{Wx}\right)} \times \nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right) \times \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right) \text{ using the fact that } \mathbf{W} = \begin{bmatrix} \mathbf{w}_1^{\mathrm{T}} \\ \mathbf{w}_2^{\mathrm{T}} \\ \mathbf{w}_3^{\mathrm{T}} \end{bmatrix}:$$

$$\nabla_{\mathbf{W}}(\mathbf{Wx}) = \nabla_{\mathbf{W}}\left(\begin{bmatrix} \mathbf{w}_1^{\mathrm{T}} \\ \mathbf{w}_2^{\mathrm{T}} \\ \mathbf{w}_3^{\mathrm{T}} \end{bmatrix}\mathbf{x}\right) = \nabla_{\mathbf{W}}\left(\begin{bmatrix} \mathbf{w}_1^{\mathrm{T}}\mathbf{x} \\ \mathbf{w}_2^{\mathrm{T}}\mathbf{x} \\ \mathbf{w}_3^{\mathrm{T}}\mathbf{x} \end{bmatrix}\right) = \begin{array}{l} \begin{bmatrix} \nabla_{\mathbf{w}_1}\left(\mathbf{w}_1^{\mathrm{T}}\mathbf{x}\right) & \nabla_{\mathbf{w}_1}\left(\mathbf{w}_2^{\mathrm{T}}\mathbf{x}\right) & \nabla_{\mathbf{w}_1}\left(\mathbf{w}_3^{\mathrm{T}}\mathbf{x}\right) \end{bmatrix} \\ \begin{bmatrix} \nabla_{\mathbf{w}_2}\left(\mathbf{w}_1^{\mathrm{T}}\mathbf{x}\right) & \nabla_{\mathbf{w}_2}\left(\mathbf{w}_2^{\mathrm{T}}\mathbf{x}\right) & \nabla_{\mathbf{w}_2}\left(\mathbf{w}_3^{\mathrm{T}}\mathbf{x}\right) \end{bmatrix} \\ \begin{bmatrix} \nabla_{\mathbf{w}_3}\left(\mathbf{w}_1^{\mathrm{T}}\mathbf{x}\right) & \nabla_{\mathbf{w}_3}\left(\mathbf{w}_2^{\mathrm{T}}\mathbf{x}\right) & \nabla_{\mathbf{w}_3}\left(\mathbf{w}_3^{\mathrm{T}}\mathbf{x}\right) \end{bmatrix} \end{array}$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

Gradient shape = shape of $\mathbf{W}$ $\times$ shape of $\mathbf{Wx}$ $= (3 \times 5) \times 3$ which is a 5×3-matrix repeating 3 times

We will now calculate the last gradient $\nabla_{\mathbf{W}}(\mathbf{Wx})$ in

$$\boxed{\nabla_{\mathbf{W}}(\mathbf{Wx})} \times \nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right) \times \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right) \text{ using the fact that } \mathbf{W} = \begin{bmatrix} \mathbf{w}_1^{\mathrm{T}} \\ \mathbf{w}_2^{\mathrm{T}} \\ \mathbf{w}_3^{\mathrm{T}} \end{bmatrix}:$$

$$\nabla_{\mathbf{W}}(\mathbf{Wx}) = \nabla_{\mathbf{W}}\left(\begin{bmatrix} \mathbf{w}_1^{\mathrm{T}} \\ \mathbf{w}_2^{\mathrm{T}} \\ \mathbf{w}_3^{\mathrm{T}} \end{bmatrix}\mathbf{x}\right) = \nabla_{\mathbf{W}}\left(\begin{bmatrix} \mathbf{w}_1^{\mathrm{T}}\mathbf{x} \\ \mathbf{w}_2^{\mathrm{T}}\mathbf{x} \\ \mathbf{w}_3^{\mathrm{T}}\mathbf{x} \end{bmatrix}\right) = \begin{matrix} \nabla_{\mathbf{w}_1}\left(\mathbf{w}_1^{\mathrm{T}}\mathbf{x}\right) & \nabla_{\mathbf{w}_1}\left(\mathbf{w}_2^{\mathrm{T}}\mathbf{x}\right) & \nabla_{\mathbf{w}_1}\left(\mathbf{w}_3^{\mathrm{T}}\mathbf{x}\right) \\ \nabla_{\mathbf{w}_2}\left(\mathbf{w}_1^{\mathrm{T}}\mathbf{x}\right) & \nabla_{\mathbf{w}_2}\left(\mathbf{w}_2^{\mathrm{T}}\mathbf{x}\right) & \nabla_{\mathbf{w}_2}\left(\mathbf{w}_3^{\mathrm{T}}\mathbf{x}\right) \\ \nabla_{\mathbf{w}_3}\left(\mathbf{w}_1^{\mathrm{T}}\mathbf{x}\right) & \nabla_{\mathbf{w}_3}\left(\mathbf{w}_2^{\mathrm{T}}\mathbf{x}\right) & \nabla_{\mathbf{w}_3}\left(\mathbf{w}_3^{\mathrm{T}}\mathbf{x}\right) \end{matrix}$$

$$\nabla_{\mathbf{w}_i}\left(\mathbf{w}_j^{\mathrm{T}}\mathbf{x}\right) = \begin{cases} \mathbf{x} & \text{if } i = j, \\ \mathbf{0} & \text{if } i \neq j. \end{cases}$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

Gradient shape = shape of $\mathbf{W}$ $\times$ shape of $\mathbf{Wx}$ $= (3 \times 5) \times 3$ which is a 5×3-matrix repeating 3 times

We will now calculate the last gradient $\nabla_{\mathbf{W}}(\mathbf{Wx})$ in

$$\boxed{\nabla_{\mathbf{W}}\left(\mathbf{Wx}\right)} \times \nabla_{\mathbf{z}}\left(\text{softmax}(\mathbf{z})\right) \times \nabla_{\hat{\mathbf{y}}}\left(\sum_{k=1}^{3} -y_k \log\left(\hat{y}_k\right)\right) \text{ using the fact that } \mathbf{W} = \begin{bmatrix} \mathbf{w}_1^{\mathrm{T}} \\ \mathbf{w}_2^{\mathrm{T}} \\ \mathbf{w}_3^{\mathrm{T}} \end{bmatrix}:$$

$$\nabla_{\mathbf{W}}(\mathbf{Wx}) = \nabla_{\mathbf{W}}\left(\begin{bmatrix} \mathbf{w}_1^{\mathrm{T}} \\ \mathbf{w}_2^{\mathrm{T}} \\ \mathbf{w}_3^{\mathrm{T}} \end{bmatrix}\mathbf{x}\right) = \nabla_{\mathbf{W}}\left(\begin{bmatrix} \mathbf{w}_1^{\mathrm{T}}\mathbf{x} \\ \mathbf{w}_2^{\mathrm{T}}\mathbf{x} \\ \mathbf{w}_3^{\mathrm{T}}\mathbf{x} \end{bmatrix}\right) = \begin{array}{l} \begin{bmatrix} \nabla_{\mathbf{w}_1}(\mathbf{w}_1^{\mathrm{T}}\mathbf{x}) & \nabla_{\mathbf{w}_1}(\mathbf{w}_2^{\mathrm{T}}\mathbf{x}) & \nabla_{\mathbf{w}_1}(\mathbf{w}_3^{\mathrm{T}}\mathbf{x}) \end{bmatrix} \\ \begin{bmatrix} \nabla_{\mathbf{w}_2}(\mathbf{w}_1^{\mathrm{T}}\mathbf{x}) & \nabla_{\mathbf{w}_2}(\mathbf{w}_2^{\mathrm{T}}\mathbf{x}) & \nabla_{\mathbf{w}_2}(\mathbf{w}_3^{\mathrm{T}}\mathbf{x}) \end{bmatrix} \\ \begin{bmatrix} \nabla_{\mathbf{w}_3}(\mathbf{w}_1^{\mathrm{T}}\mathbf{x}) & \nabla_{\mathbf{w}_3}(\mathbf{w}_2^{\mathrm{T}}\mathbf{x}) & \nabla_{\mathbf{w}_3}(\mathbf{w}_3^{\mathrm{T}}\mathbf{x}) \end{bmatrix} \end{array} = \begin{array}{l} \begin{bmatrix} \mathbf{x} & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & \mathbf{x} & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & \mathbf{x} \end{bmatrix} \end{array}$$

$$\nabla_{\mathbf{w}_i}\left(\mathbf{w}_j^{\mathrm{T}}\mathbf{x}\right) = \begin{cases} \mathbf{x} & \text{if } i = j, \\ \mathbf{0} & \text{if } i \neq j. \end{cases}$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

The zero hidden layer (softmax classifier) gradient can be written as:

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

The zero hidden layer (softmax classifier) gradient can be written as:

Computation graph:

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

The zero hidden layer (softmax classifier) gradient can be written as:

Computation graph:

$$\begin{array}{c} L \\ \downarrow \\ \underbrace{\hat{\mathbf{y}}}_{=\mathbf{a}} \\ \downarrow \\ \mathbf{z} \\ \downarrow \\ \mathbf{W} \end{array}$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

The zero hidden layer (softmax classifier) gradient can be written as:

Computation graph:

$$\begin{tikzcd} \boxed{L} \arrow[d] \\ \boxed{\underbrace{\hat{\mathbf{y}}}_{=\mathbf{a}}} \arrow[d] \\ \boxed{\mathbf{z}} \arrow[d] \\ \boxed{\mathbf{W}} \end{tikzcd}$$

Gradient calculation using chain rule:

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

The zero hidden layer (softmax classifier) gradient can be written as:

Computation graph:

$$
\begin{tikzcd}
\boxed{L} \arrow[d] \\
\boxed{\underbrace{\hat{\mathbf{y}}}_{=\mathbf{a}}} \arrow[d] \\
\boxed{\mathbf{z}} \arrow[d] \\
\boxed{\mathbf{W}}
\end{tikzcd}
$$

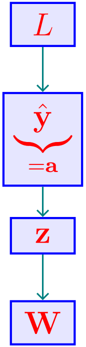
Gradient calculation using chain rule:

$$\nabla_{\mathbf{W}}(L) =$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

The zero hidden layer (softmax classifier) gradient can be written as:

Computation graph:



Gradient calculation using chain rule:

$$\nabla_{\mathbf{w}}(L) = \nabla_{\hat{\mathbf{y}}}(L)$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

The zero hidden layer (softmax classifier) gradient can be written as:

Computation graph:

$$\begin{tikzcd} \boxed{L} \arrow[d] \\ \boxed{\underbrace{\hat{\mathbf{y}}}_{=\mathbf{a}}} \arrow[d] \\ \boxed{\mathbf{z}} \arrow[d] \\ \boxed{\mathbf{W}} \end{tikzcd}$$

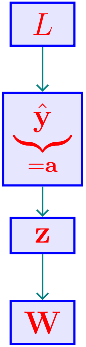
Gradient calculation using chain rule:

$$\nabla_{\mathbf{W}}(L) = \qquad \nabla_{\mathbf{z}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

The zero hidden layer (softmax classifier) gradient can be written as:

Computation graph:



Gradient calculation using chain rule:

$$\nabla_{\mathbf{W}}(L) = \nabla_{\mathbf{W}}(\mathbf{z}) \times \nabla_{\mathbf{z}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

The zero hidden layer (softmax classifier) gradient can be written as:

<u>Computation graph:</u>

$$\begin{tikzcd} \boxed{L} \arrow[d] \\ \boxed{\underbrace{\hat{\mathbf{y}}}_{=\mathbf{a}}} \arrow[d] \\ \boxed{\mathbf{z}} \arrow[d] \\ \boxed{\mathbf{W}} \end{tikzcd}$$

<u>Gradient calculation using chain rule:</u>

$$\begin{aligned} \nabla_{\mathbf{w}}(L) &= \nabla_{\mathbf{W}}(\mathbf{z}) \times \nabla_{\mathbf{z}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L) \\ &= \nabla_{\mathbf{W}}(\mathbf{z}) \times \nabla_{\mathbf{z}}(\mathbf{a}) \times \nabla_{\hat{\mathbf{y}}}(L) \end{aligned}$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

The zero hidden layer (softmax classifier) gradient can be written as:

Computation graph:

$$
\begin{tikzcd}
\boxed{L} \arrow[d] \\
\boxed{\underbrace{\hat{\mathbf{y}}}_{=\mathbf{a}}} \arrow[d] \\
\boxed{\mathbf{z}} \arrow[d] \\
\boxed{\mathbf{W}}
\end{tikzcd}
$$

Gradient calculation using chain rule:

$$
\begin{aligned}
\nabla_{\mathbf{w}}(L) &= \nabla_{\mathbf{W}}(\mathbf{z}) \times \nabla_{\mathbf{z}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L) \\
&= \nabla_{\mathbf{W}}(\mathbf{z}) \times \nabla_{\mathbf{z}}(\mathbf{a}) \times \nabla_{\hat{\mathbf{y}}}(L)
\end{aligned}
$$

$$
\begin{bmatrix} -y_1/\hat{y}_1 \\ -y_2/\hat{y}_2 \\ -y_3/\hat{y}_3 \end{bmatrix}
$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

The zero hidden layer (softmax classifier) gradient can be written as:

Computation graph:

$$
\begin{tikzcd}
\boxed{L} \arrow[d] \\
\boxed{\underbrace{\hat{\mathbf{y}}}_{=\mathbf{a}}} \arrow[d] \\
\boxed{\mathbf{z}} \arrow[d] \\
\boxed{\mathbf{W}}
\end{tikzcd}
$$

Gradient calculation using chain rule:

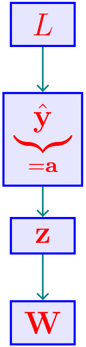
$$
\begin{aligned}
\nabla_{\mathbf{w}}(L) &= \nabla_{\mathbf{W}}(\mathbf{z}) \times \nabla_{\mathbf{z}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L) \\
&= \nabla_{\mathbf{W}}(\mathbf{z}) \times \nabla_{\mathbf{z}}(\mathbf{a}) \times \nabla_{\hat{\mathbf{y}}}(L)
\end{aligned}
$$

$$
\times \begin{bmatrix} a_1(1-a_1) & -a_2a_1 & -a_3a_1 \\ -a_1a_2 & a_2(1-a_2) & -a_3a_2 \\ -a_1a_3 & -a_2a_3 & a_3(1-a_3) \end{bmatrix} \times \begin{bmatrix} -y_1/\hat{y}_1 \\ -y_2/\hat{y}_2 \\ -y_3/\hat{y}_3 \end{bmatrix}
$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

The zero hidden layer (softmax classifier) gradient can be written as:

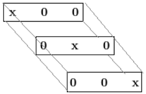
<u>Computation graph:</u>



<u>Gradient calculation using chain rule:</u>

$$
\begin{aligned}
\nabla_{\mathbf{w}}(L) &= \nabla_{\mathbf{W}}(\mathbf{z}) \times \nabla_{\mathbf{z}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L) \\
&= \nabla_{\mathbf{W}}(\mathbf{z}) \times \nabla_{\mathbf{z}}(\mathbf{a}) \times \nabla_{\hat{\mathbf{y}}}(L)
\end{aligned}
$$

=

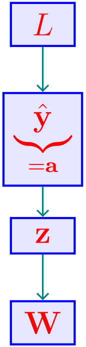


$$
\times \begin{bmatrix} a_1(1-a_1) & -a_2a_1 & -a_3a_1 \\ -a_1a_2 & a_2(1-a_2) & -a_3a_2 \\ -a_1a_3 & -a_2a_3 & a_3(1-a_3) \end{bmatrix} \times \begin{bmatrix} -y_1/\hat{y}_1 \\ -y_2/\hat{y}_2 \\ -y_3/\hat{y}_3 \end{bmatrix}
$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

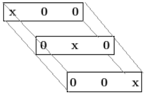
The zero hidden layer (softmax classifier) gradient can be written as:

Computation graph:



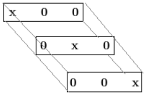
Gradient calculation using chain rule:

$$\nabla_{\mathbf{w}}(L) = \nabla_{\mathbf{W}}(\mathbf{z}) \times \nabla_{\mathbf{z}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L)$$
$$= \nabla_{\mathbf{W}}(\mathbf{z}) \times \nabla_{\mathbf{z}}(\mathbf{a}) \times \nabla_{\hat{\mathbf{y}}}(L)$$



$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} v_1\mathbf{x}^T \\ v_2\mathbf{x}^T \\ v_3\mathbf{x}^T \end{bmatrix} = \mathbf{v}\mathbf{x}^T$$

=

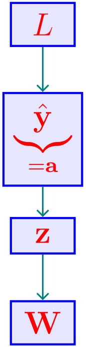


$$\times \begin{bmatrix} a_1(1-a_1) & -a_2a_1 & -a_3a_1 \\ -a_1a_2 & a_2(1-a_2) & -a_3a_2 \\ -a_1a_3 & -a_2a_3 & a_3(1-a_3) \end{bmatrix} \times \begin{bmatrix} -y_1/\hat{y}_1 \\ -y_2/\hat{y}_2 \\ -y_3/\hat{y}_3 \end{bmatrix}$$

Zero hidden layer neural network (softmax classifier) gradient calculation using backward propagation – continued

The zero hidden layer (softmax classifier) gradient can be written as:

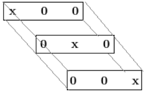
Computation graph:



Gradient calculation using chain rule:

$$\begin{aligned}\nabla_{\mathbf{w}}(L) &= \nabla_{\mathbf{W}}(\mathbf{z}) \times \nabla_{\mathbf{z}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L) \\ &= \nabla_{\mathbf{W}}(\mathbf{z}) \times \nabla_{\mathbf{z}}(\mathbf{a}) \times \nabla_{\hat{\mathbf{y}}}(L)\end{aligned}$$

=

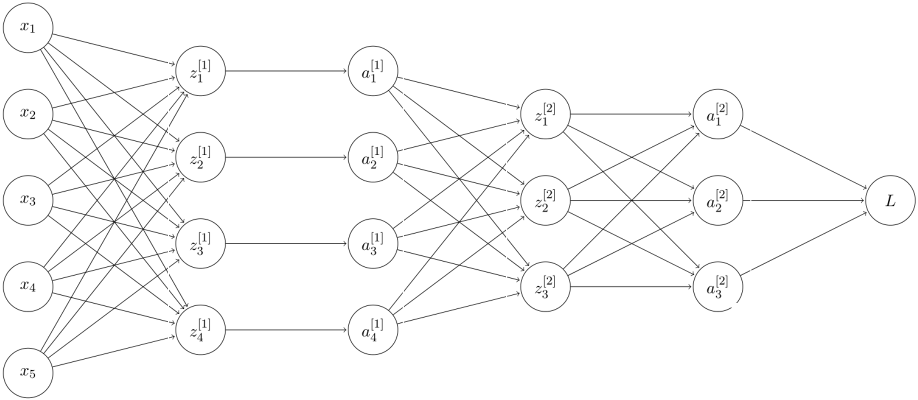


$$\times \begin{bmatrix} a_1(1-a_1) & -a_2a_1 & -a_3a_1 \\ -a_1a_2 & a_2(1-a_2) & -a_3a_2 \\ -a_1a_3 & -a_2a_3 & a_3(1-a_3) \end{bmatrix} \times \begin{bmatrix} -y_1/\hat{y}_1 \\ -y_2/\hat{y}_2 \\ -y_3/\hat{y}_3 \end{bmatrix}$$

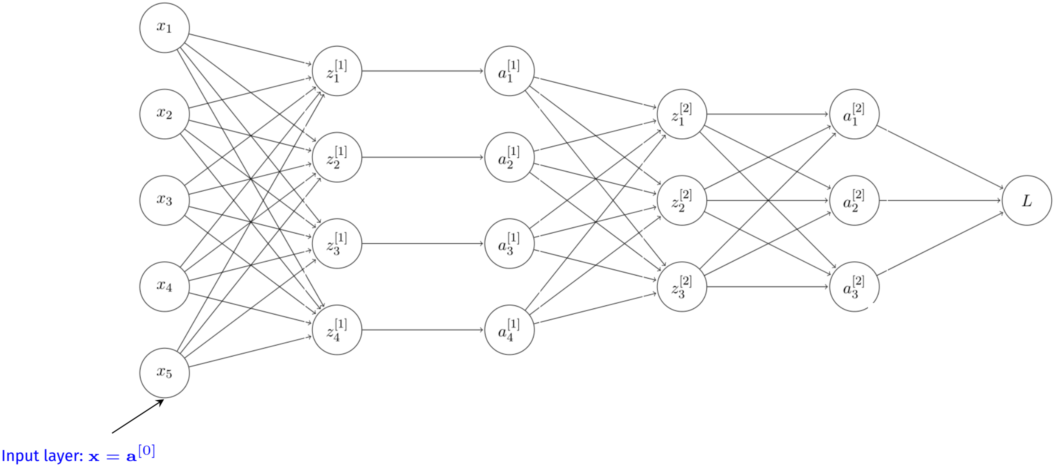
$$= \begin{bmatrix} a_1(1-a_1) & -a_2a_1 & -a_3a_1 \\ -a_1a_2 & a_2(1-a_2) & -a_3a_2 \\ -a_1a_3 & -a_2a_3 & a_3(1-a_3) \end{bmatrix} \times \begin{bmatrix} -y_1/\hat{y}_1 \\ -y_2/\hat{y}_2 \\ -y_3/\hat{y}_3 \end{bmatrix} \mathbf{x}^{\mathrm{T}}.$$

Single hidden layer neural network forward propagation

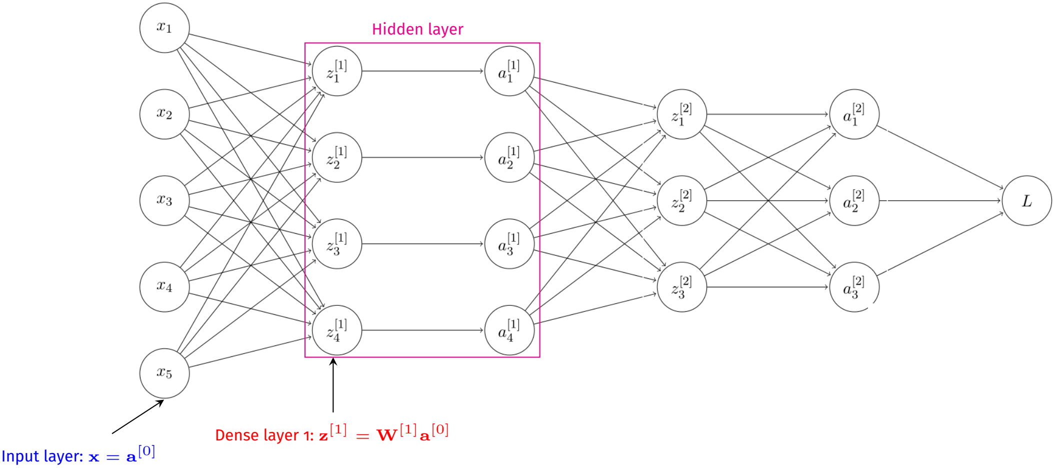
Single hidden layer neural network forward propagation



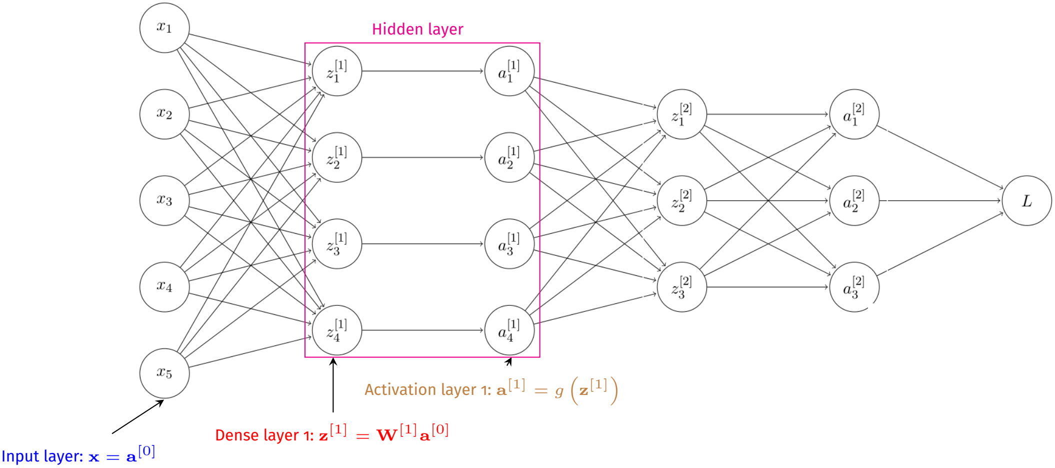
Single hidden layer neural network forward propagation



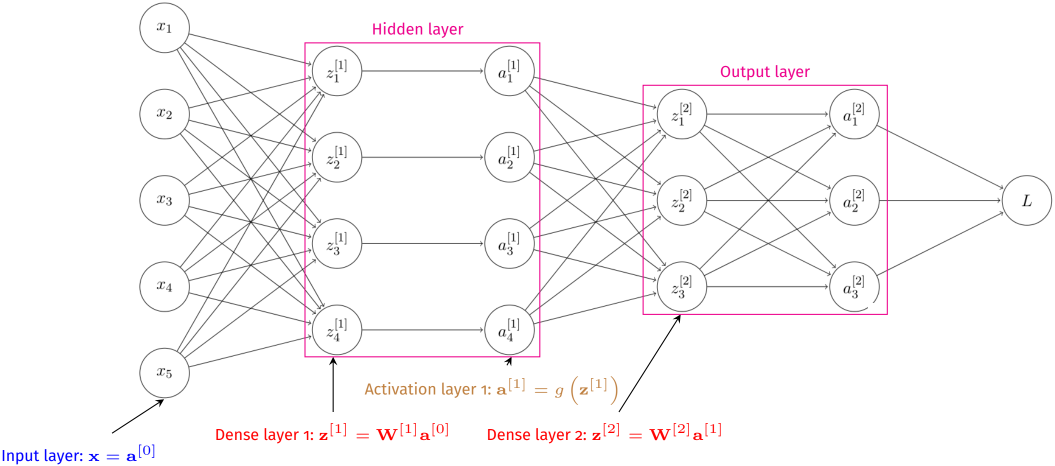
Single hidden layer neural network forward propagation



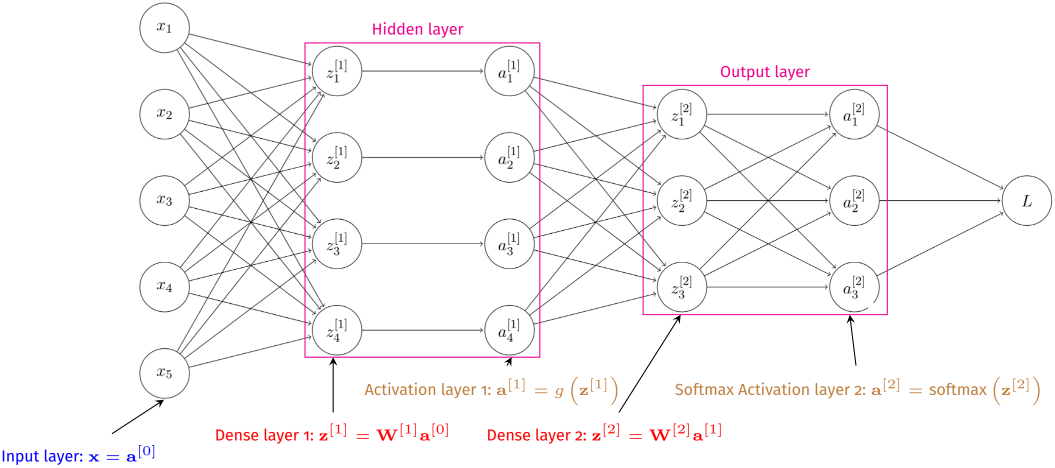
Single hidden layer neural network forward propagation



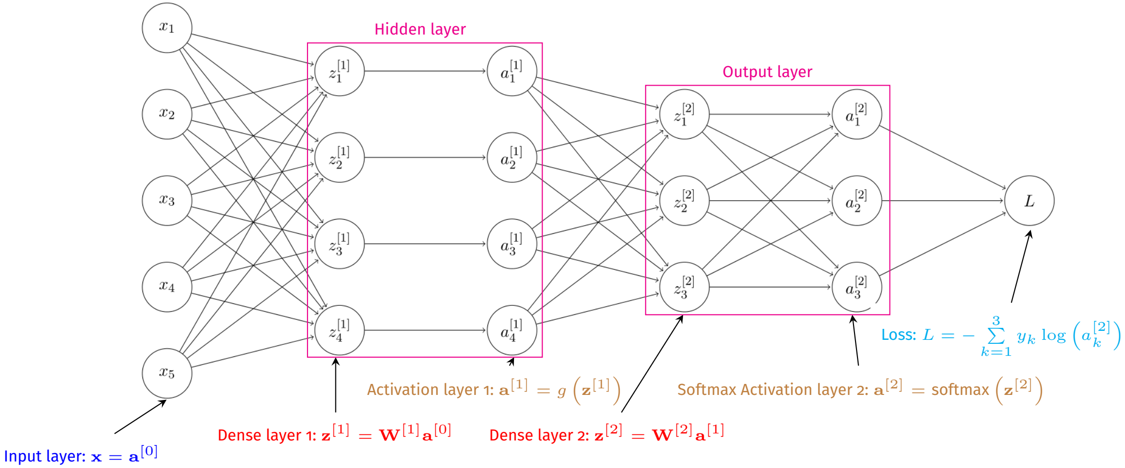
Single hidden layer neural network forward propagation



Single hidden layer neural network forward propagation



Single hidden layer neural network forward propagation



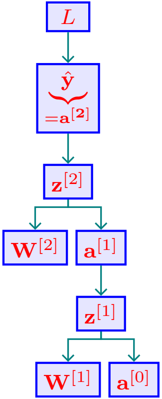
Single hidden layer neural network gradient calculation calculation using backward propagation

Single hidden layer neural network gradient calculation calculation using backward propagation

Computation graph:

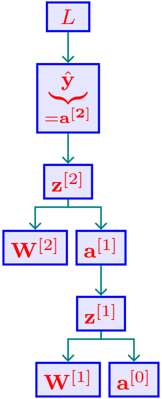
Single hidden layer neural network gradient calculation calculation using backward propagation

Computation graph:



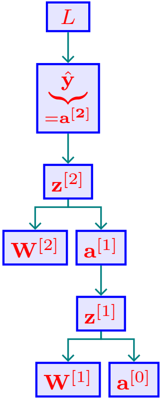
Single hidden layer neural network gradient calculation calculation using backward propagation

Computation graph: Gradient calculation using chain rule:



Single hidden layer neural network gradient calculation calculation using backward propagation

Computation graph:

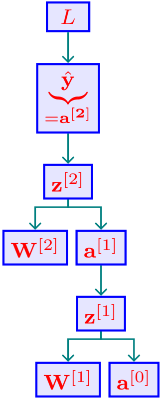


Gradient calculation using chain rule:

$$\nabla_{\mathbf{W}^{[2]}}(L) =$$

Single hidden layer neural network gradient calculation calculation using backward propagation

Computation graph:

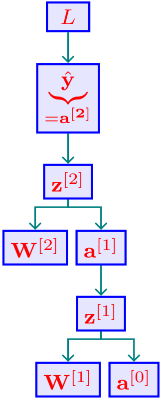


Gradient calculation using chain rule:

$$\nabla_{\mathbf{W}^{[2]}}(L) = \qquad \nabla_{\hat{\mathbf{y}}}(L)$$

Single hidden layer neural network gradient calculation calculation using backward propagation

Computation graph:

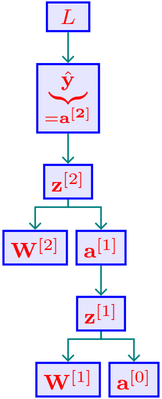


Gradient calculation using chain rule:

$$\nabla_{\mathbf{W}^{[2]}}(L) = \qquad \nabla_{\mathbf{z}^{[2]}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

Single hidden layer neural network gradient calculation calculation using backward propagation

Computation graph:

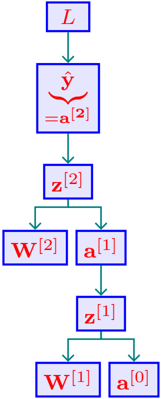


Gradient calculation using chain rule:

$$\nabla_{\mathbf{W}^{[2]}}(L) = \nabla_{\mathbf{W}^{[2]}}(\mathbf{z}^{[2]}) \times \nabla_{\mathbf{z}^{[2]}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

Single hidden layer neural network gradient calculation calculation using backward propagation

Computation graph:



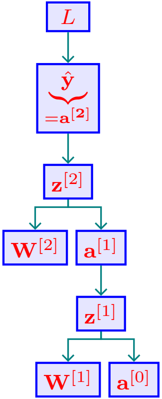
Gradient calculation using chain rule:

$$\nabla_{\mathbf{W}^{[2]}}(L) = \nabla_{\mathbf{W}^{[2]}}(\mathbf{z}^{[2]}) \times \nabla_{\mathbf{z}^{[2]}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

$$\nabla_{\mathbf{W}^{[1]}}(L) =$$

Single hidden layer neural network gradient calculation calculation using backward propagation

Computation graph:



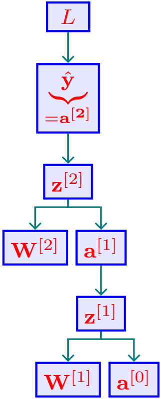
Gradient calculation using chain rule:

$$\nabla_{\mathbf{W}^{[2]}}(L) = \nabla_{\mathbf{W}^{[2]}}(\mathbf{z}^{[2]}) \times \nabla_{\mathbf{z}^{[2]}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

$$\nabla_{\mathbf{W}^{[1]}}(L) = \qquad \nabla_{\hat{\mathbf{y}}}(L)$$

Single hidden layer neural network gradient calculation calculation using backward propagation

Computation graph:



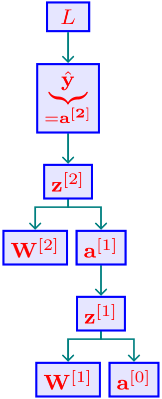
Gradient calculation using chain rule:

$$\nabla_{\mathbf{W}^{[2]}}(L) = \nabla_{\mathbf{W}^{[2]}}(\mathbf{z}^{[2]}) \times \nabla_{\mathbf{z}^{[2]}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

$$\nabla_{\mathbf{W}^{[1]}}(L) = \qquad \nabla_{\mathbf{z}^{[2]}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

Single hidden layer neural network gradient calculation calculation using backward propagation

Computation graph: **Gradient calculation using chain rule:**

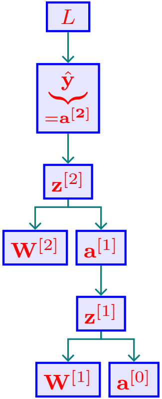


$$\nabla_{\mathbf{W}^{[2]}}(L) = \nabla_{\mathbf{W}^{[2]}}(\mathbf{z}^{[2]}) \times \nabla_{\mathbf{z}^{[2]}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

$$\nabla_{\mathbf{W}^{[1]}}(L) = \qquad \nabla_{\mathbf{a}^{[1]}}(\mathbf{z}^{[2]}) \times \nabla_{\mathbf{z}^{[2]}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

Single hidden layer neural network gradient calculation calculation using backward propagation

Computation graph:



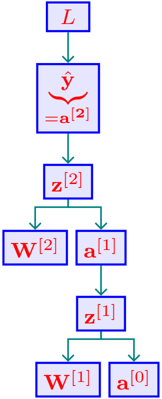
Gradient calculation using chain rule:

$$\nabla_{\mathbf{W}^{[2]}}(L) = \nabla_{\mathbf{W}^{[2]}}(\mathbf{z}^{[2]}) \times \nabla_{\mathbf{z}^{[2]}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

$$\nabla_{\mathbf{W}^{[1]}}(L) = \nabla_{\mathbf{z}^{[1]}}(\mathbf{a}^{[1]}) \times \nabla_{\mathbf{a}^{[1]}}(\mathbf{z}^{[2]}) \times \nabla_{\mathbf{z}^{[2]}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

Single hidden layer neural network gradient calculation calculation using backward propagation

Computation graph: Gradient calculation using chain rule:

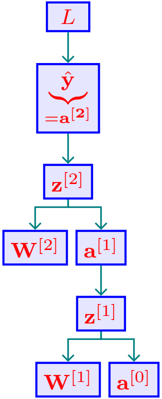


$$\nabla_{\mathbf{W}^{[2]}}(L) = \nabla_{\mathbf{W}^{[2]}}(\mathbf{z}^{[2]}) \times \nabla_{\mathbf{z}^{[2]}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

$$\nabla_{\mathbf{W}^{[1]}}(L) = \nabla_{\mathbf{W}^{[1]}}(\mathbf{z}^{[1]}) \times \nabla_{\mathbf{z}^{[1]}}(\mathbf{a}^{[1]}) \times \nabla_{\mathbf{a}^{[1]}}(\mathbf{z}^{[2]}) \times \nabla_{\mathbf{z}^{[2]}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

Single hidden layer neural network gradient calculation calculation using backward propagation

Computation graph:



Gradient calculation using chain rule:

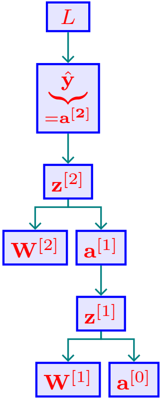
$$\nabla_{\mathbf{W}^{[2]}}(L) = \nabla_{\mathbf{W}^{[2]}}(\mathbf{z}^{[2]}) \times \nabla_{\mathbf{z}^{[2]}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

$$\nabla_{\mathbf{W}^{[1]}}(L) = \nabla_{\mathbf{W}^{[1]}}(\mathbf{z}^{[1]}) \times \nabla_{\mathbf{z}^{[1]}}(\mathbf{a}^{[1]}) \times \nabla_{\mathbf{a}^{[1]}}(\mathbf{z}^{[2]}) \times \nabla_{\mathbf{z}^{[2]}}(\hat{\mathbf{y}}) \times \nabla_{\hat{\mathbf{y}}}(L)$$

$$= \left[\nabla_{\mathbf{z}^{[1]}}\left(a_1^{[1]}\right) \quad \nabla_{\mathbf{z}^{[1]}}\left(a_2^{[1]}\right) \quad \nabla_{\mathbf{z}^{[1]}}\left(a_3^{[1]}\right) \quad \nabla_{\mathbf{z}^{[1]}}\left(a_4^{[1]}\right)\right]$$

Single hidden layer neural network gradient calculation calculation using backward propagation

Computation graph:



Gradient calculation using chain rule:

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$$= \left[\nabla_{\mathbf{z}^{[1]}}\left(g\left(z_1^{[1]}\right)\right) \quad \nabla_{\mathbf{z}^{[1]}}\left(g\left(z_2^{[1]}\right)\right) \quad \nabla_{\mathbf{z}^{[1]}}\left(g\left(z_3^{[1]}\right)\right) \quad \nabla_{\mathbf{z}^{[1]}}\left(g\left(z_4^{[1]}\right)\right)\right]$$

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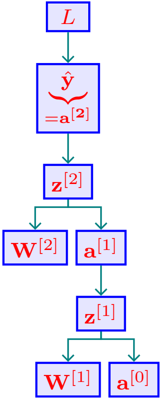

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$$= \begin{bmatrix} \nabla_{z_1^{[1]}} g\left(z_1^{[1]}\right) & \nabla_{z_1^{[1]}} g\left(z_2^{[1]}\right) & \nabla_{z_1^{[1]}} g\left(z_3^{[1]}\right) & \nabla_{z_1^{[1]}} g\left(z_4^{[1]}\right) \\ \nabla_{z_2^{[1]}} g\left(z_1^{[1]}\right) & \nabla_{z_2^{[1]}} g\left(z_2^{[1]}\right) & \nabla_{z_2^{[1]}} g\left(z_3^{[1]}\right) & \nabla_{z_2^{[1]}} g\left(z_4^{[1]}\right) \\ \nabla_{z_1^{[1]}} g\left(z_3^{[1]}\right) & \nabla_{z_3^{[1]}} g\left(z_2^{[1]}\right) & \nabla_{z_3^{[1]}} g\left(z_3^{[1]}\right) & \nabla_{z_3^{[1]}} g\left(z_4^{[1]}\right) \\ \nabla_{z_4^{[1]}} g\left(z_1^{[1]}\right) & \nabla_{z_4^{[1]}} g\left(z_2^{[1]}\right) & \nabla_{z_4^{[1]}} g\left(z_3^{[1]}\right) & \nabla_{z_4^{[1]}} g\left(z_4^{[1]}\right) \end{bmatrix}$$

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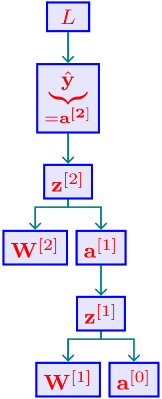

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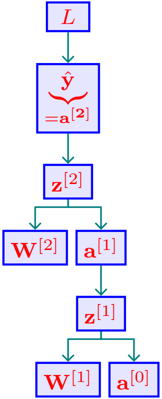

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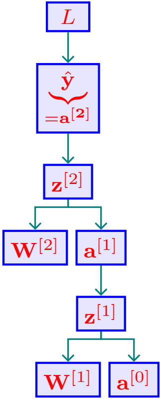

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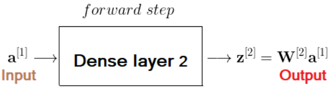

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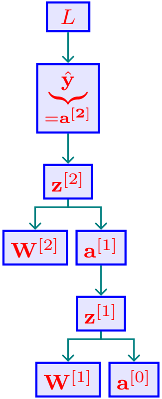

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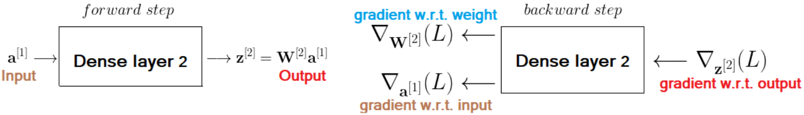

Parameters and hyperparameters

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