

Problem Set - 1

07-01-24

3x5 weights matrix W , 5-vector x

- ① Shape of Wx = 3-vector
- ② Shape of gradient $\nabla_{W_1}(Wx)$
 $(3 \times 5) \times (3 \times 1) \rightarrow 3 \times 5 \times 3$
- ③ If $W = \begin{bmatrix} w_1^T \\ w_2^T \\ w_3^T \end{bmatrix}$, Shape of w_1 = 5 column vector
- ④ Shape of Gradient $\nabla_{w_1}(Wx)$
 $= (5 \times 1) \times (3 \times 1) \rightarrow \boxed{5 \times 3}$
- ⑤ Calculate $\nabla_{w_1}(Wx)$, given $Wx = [w_1^T x \cdot w_2^T x \cdot w_3^T x]$

$$Wx = \begin{bmatrix} w_1^T x \\ w_2^T x \\ w_3^T x \end{bmatrix}$$

$$\nabla_{w_1}(Wx) = \begin{bmatrix} w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \\ w_{15} \end{bmatrix} \quad \cancel{w_{12} \ w_{13} \ w_{14} \ w_{15}}$$

$$\begin{bmatrix} w_{11}x_1 + w_{12}x_2 + w_{13}x_3 + w_{14}x_4 + w_{15}x_5 \\ w_{21}x_1 + w_{22}x_2 + w_{23}x_3 + w_{24}x_4 + w_{25}x_5 \\ w_{31}x_1 + w_{32}x_2 + w_{33}x_3 + w_{34}x_4 + w_{35}x_5 \end{bmatrix}$$

$$\nabla_{w_1}(Wx) = \begin{bmatrix} x_1 & 0 & 0 \\ x_2 & 0 & 0 \\ x_3 & 0 & 0 \\ x_4 & 0 & 0 \\ x_5 & 0 & 0 \end{bmatrix}$$

⑥

Similar to ⑤

$$\nabla_{W_2}(Wx) =$$

$$\begin{bmatrix} 0 & x_1 & 0 \\ 0 & x_2 & 0 \\ 0 & x_3 & 0 \\ 0 & x_4 & 0 \\ 0 & x_5 & 0 \end{bmatrix}$$

$$\nabla_{W_3}(Wx) = \begin{bmatrix} 0 & 0 & x_1 \\ 0 & 0 & x_2 \\ 0 & 0 & x_3 \\ 0 & 0 & x_4 \\ 0 & 0 & x_5 \end{bmatrix}$$

⑦

$$\nabla_W(Wx)$$

- 3D - tensor

$$\begin{bmatrix} [x_1, 0, 0], \\ [x_2, 0, 0], \\ [x_3, 0, 0], \\ [x_4, 0, 0], \\ [x_5, 0, 0] \end{bmatrix},$$

$$\begin{bmatrix} [0, x_1, 0], \\ [0, x_2, 0], \\ [0, x_3, 0], \\ [0, x_4, 0], \\ [0, x_5, 0] \end{bmatrix},$$

$$\begin{bmatrix} [0, 0, x_1], \\ [0, 0, x_2], \\ [0, 0, x_3], \\ [0, 0, x_4], \\ [0, 0, x_5] \end{bmatrix} \end{bmatrix}$$