Deep Learning Principles & Applications

Chapter 4 – Deep Neural Networks

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Topics

1. Deep L-layer neural network: architecture
2. Deep L-layer neural network: notation
3. Deep L-layer neural network forward propagation
4. Accounting for bias in a deep L-layer neural network
5. Minibatch forward propagation in a deep L-layer neural network
6. Deep L-layer neural network gradient calculation using backward propagation
7. Minibatch backward propagation in a deep L-layer neural network

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Deep L-layer neural network: architecture

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Zero hidden layer neural network

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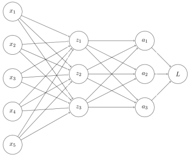

Deep L-layer neural network: architecture

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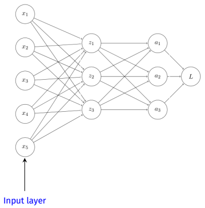

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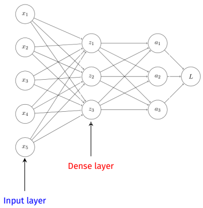

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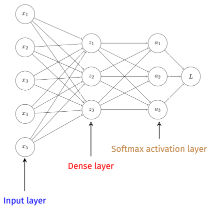

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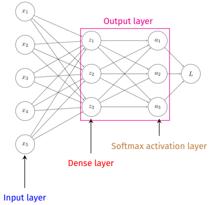

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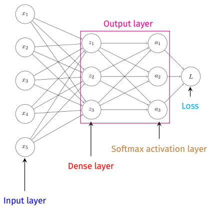

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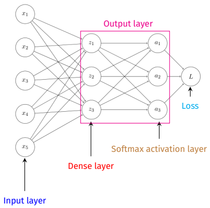

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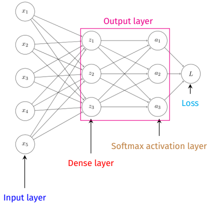

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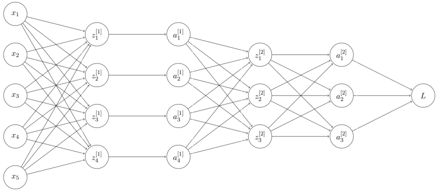

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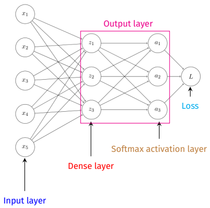

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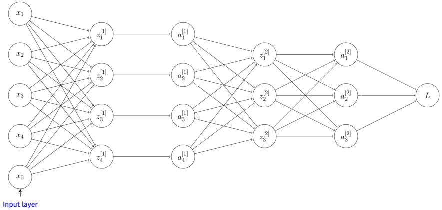

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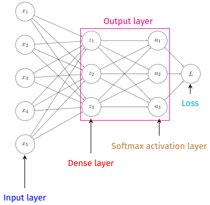

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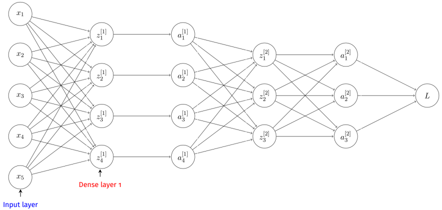

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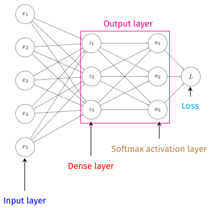

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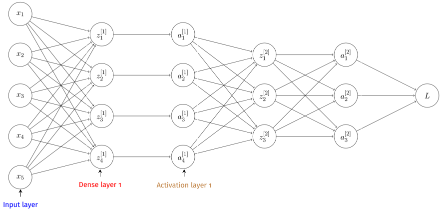

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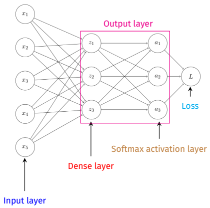

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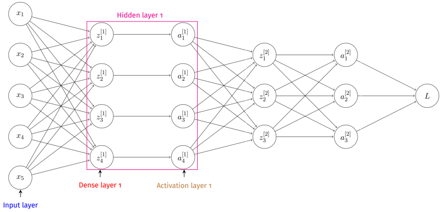

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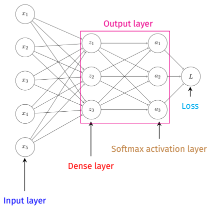

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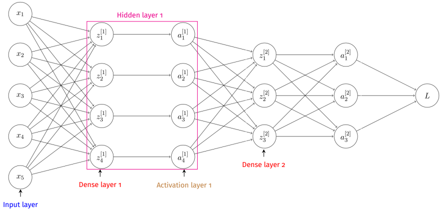

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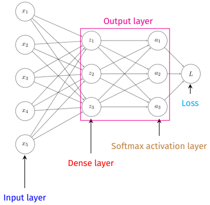

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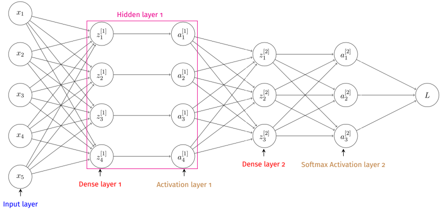

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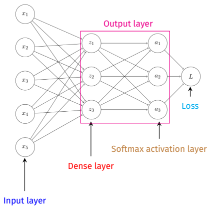

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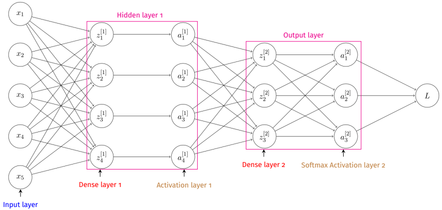

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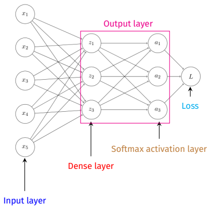

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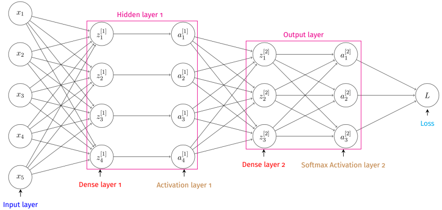

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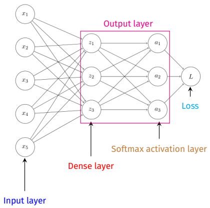

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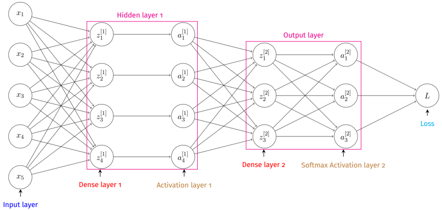

2 hidden layer neural network

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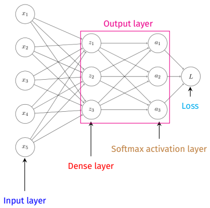

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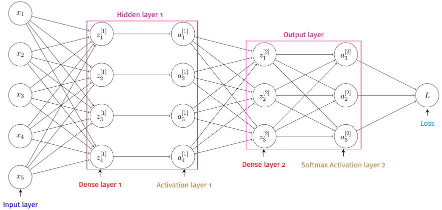

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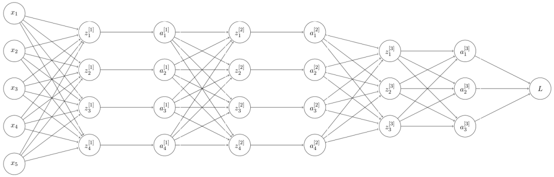

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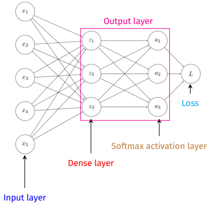

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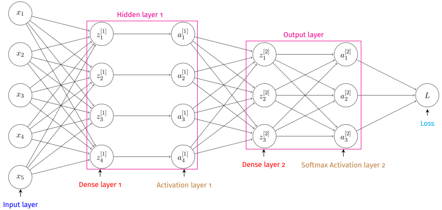

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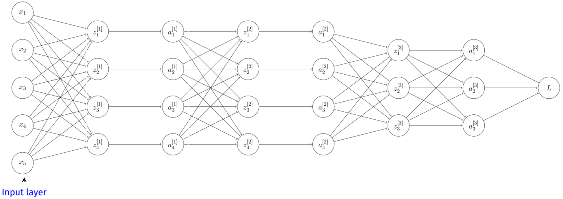

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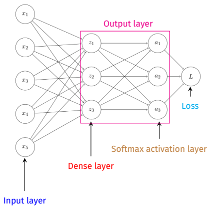

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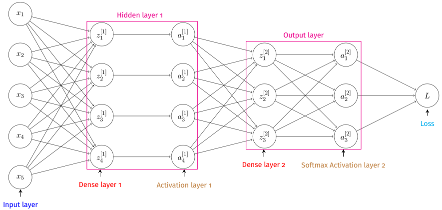

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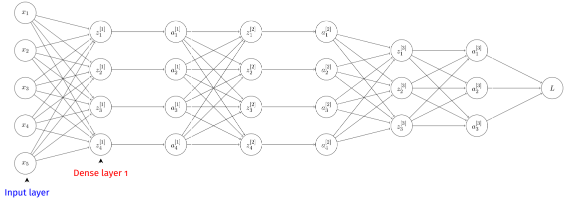

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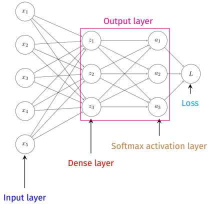

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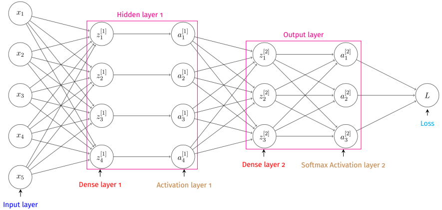

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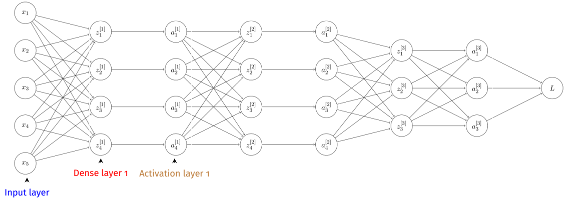

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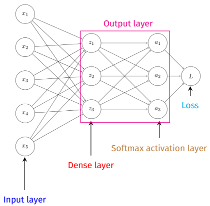

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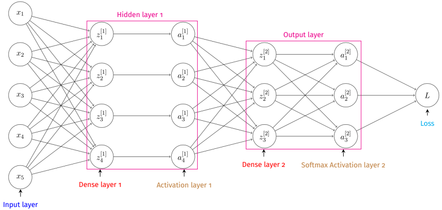

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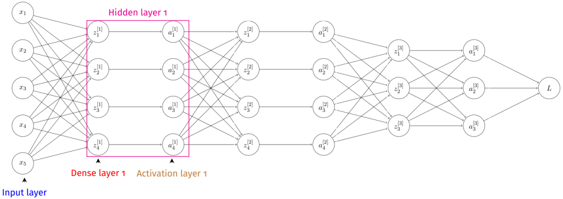

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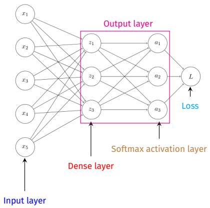

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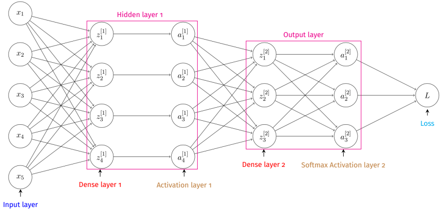

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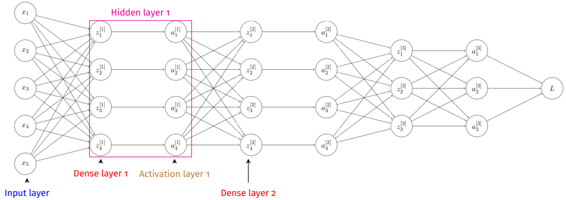

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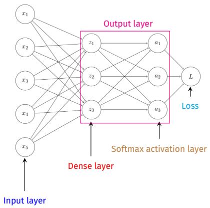

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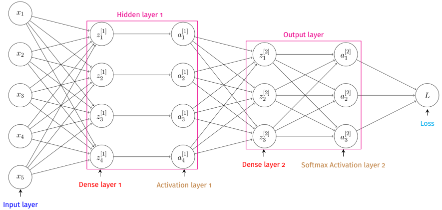

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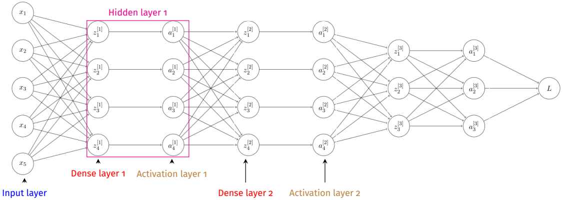

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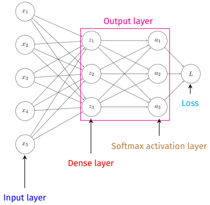

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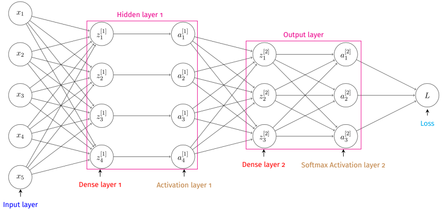

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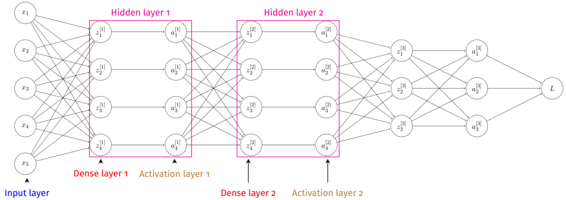

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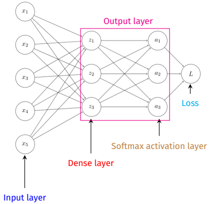

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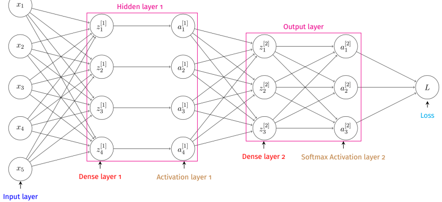

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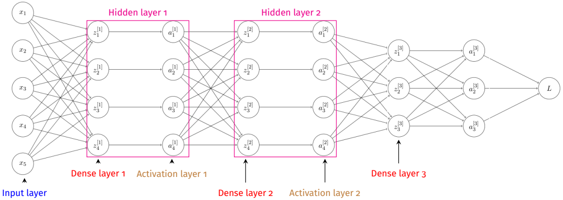

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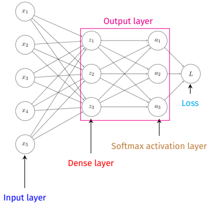

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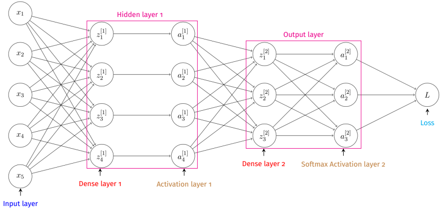

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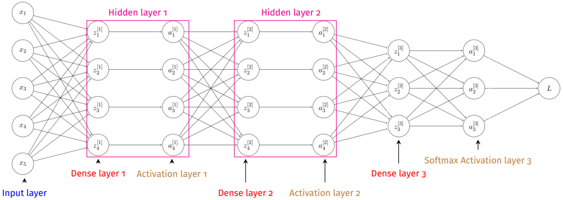

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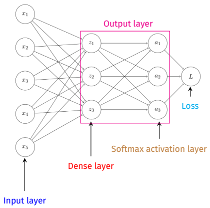

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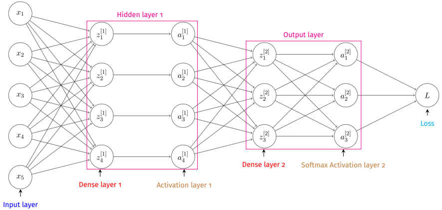

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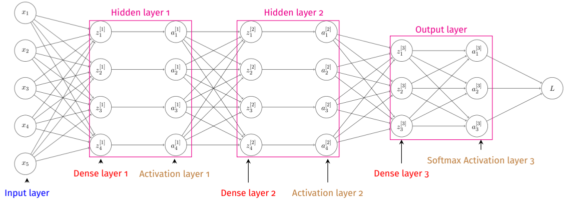

Deep L-layer neural network: architecture

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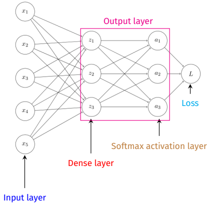
(Institution of Eminence Deemed to be University)

Zero hidden layer neural network



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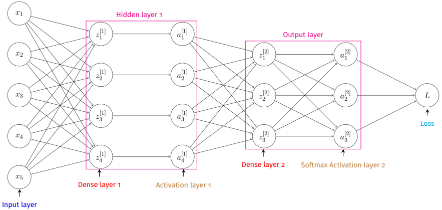

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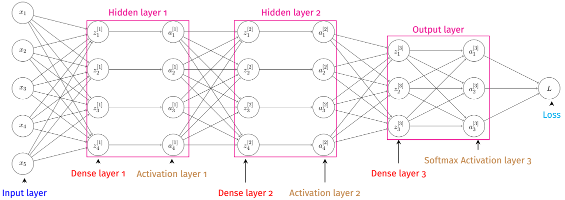

Deep L-layer neural network: notation and building blocks

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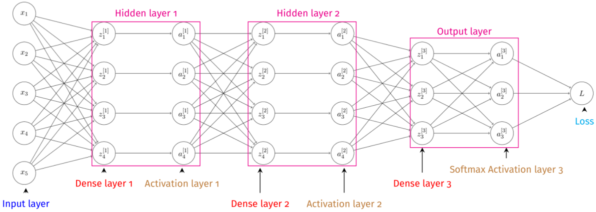

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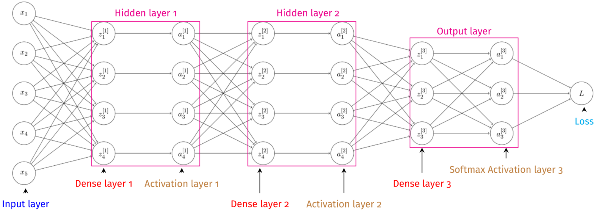

- Total number of layers $= L + 1$.

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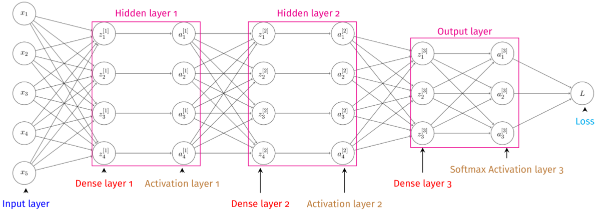
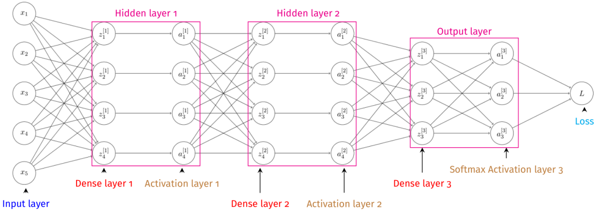


- Total number of layers $= L + 1$.
- Layer index: $l = \underbrace{0}_{\text{(input layer)}}, \underbrace{1, 2, \ldots L-1,}_{\text{hidden layers}} \underbrace{L}_{\text{(output layer)}}.$

Deep L-layer neural network: notation and building blocks



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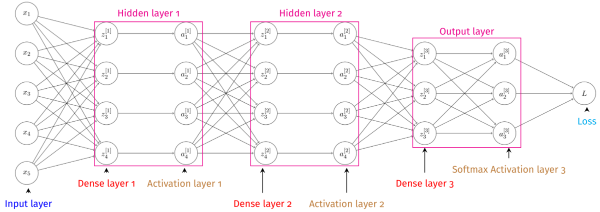

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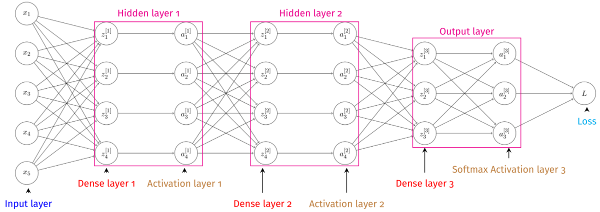

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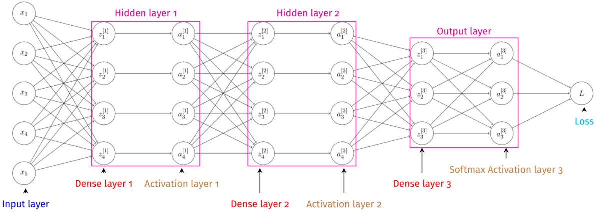

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Deep L-layer neural network: notation and building blocks

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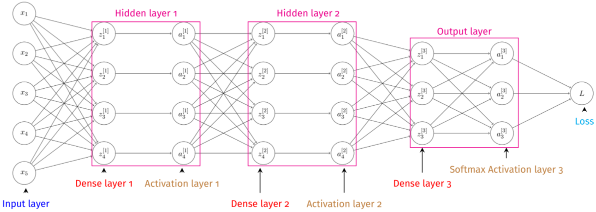

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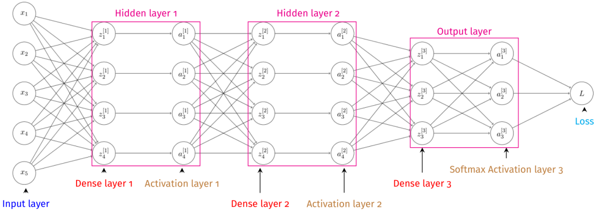

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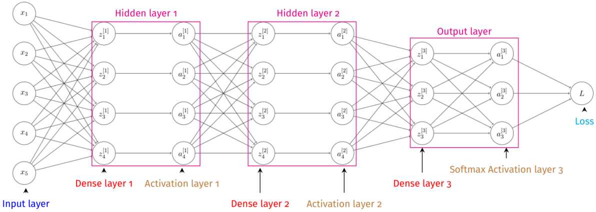

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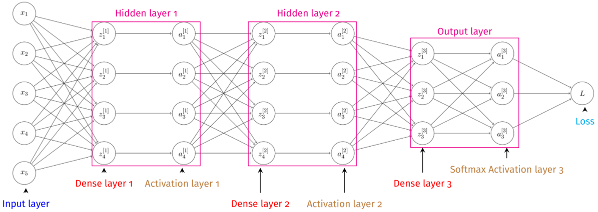

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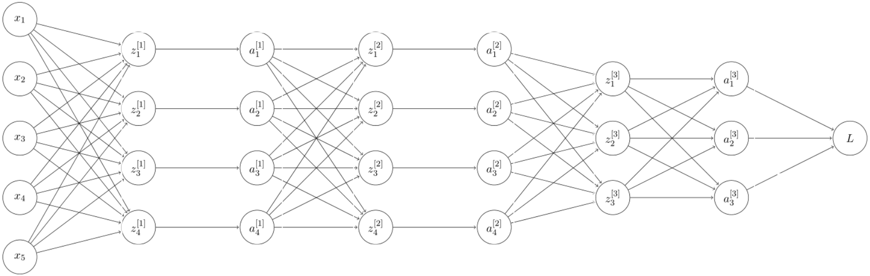

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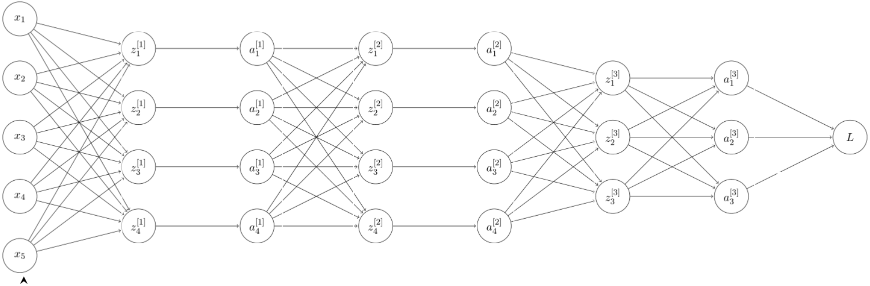

Input layer: $\mathbf{a}^{[0]}$

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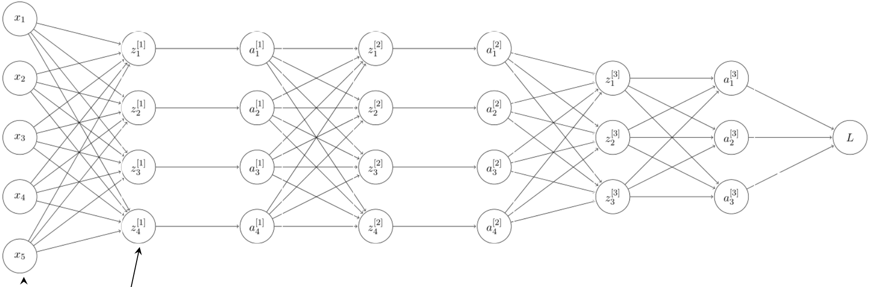

Input layer: $\mathbf{a}^{[0]}$

Dense layer 1: $\mathbf{z}^{[1]} = \mathbf{W}^{[1]}\mathbf{a}^{[0]}$

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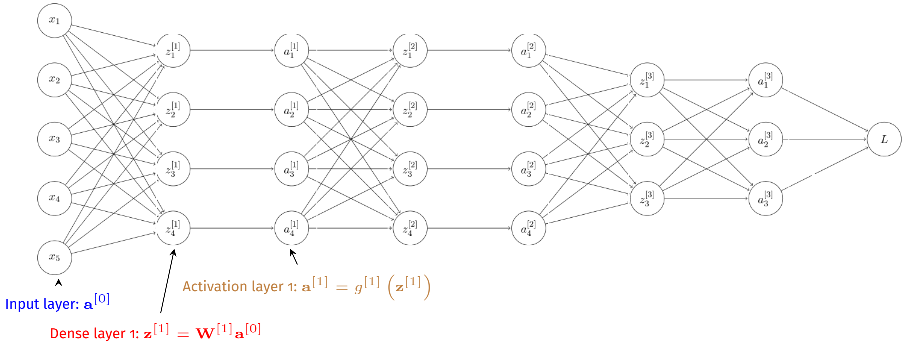

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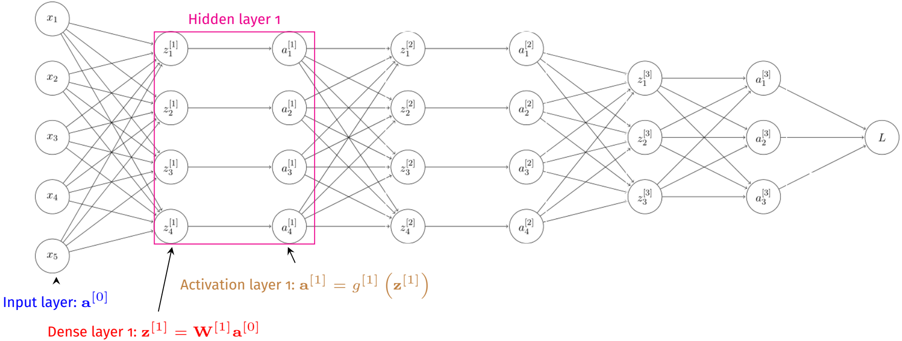

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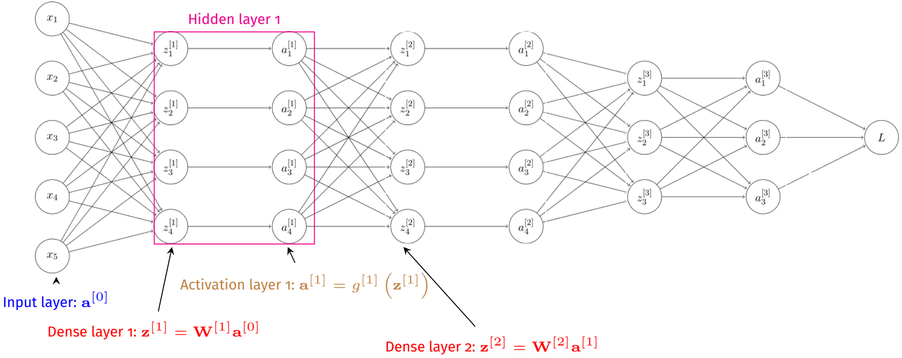

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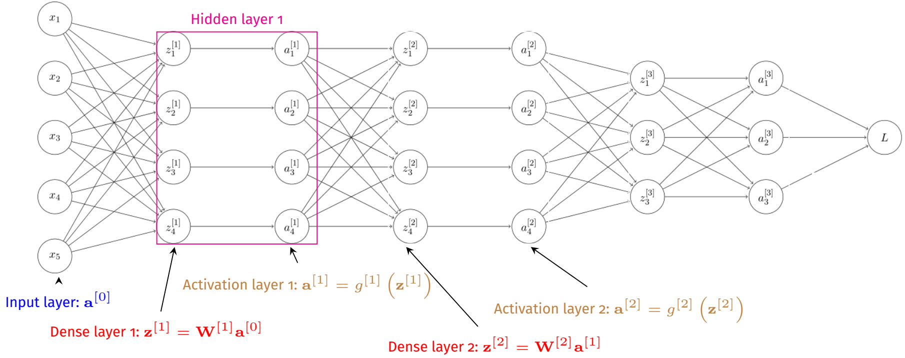

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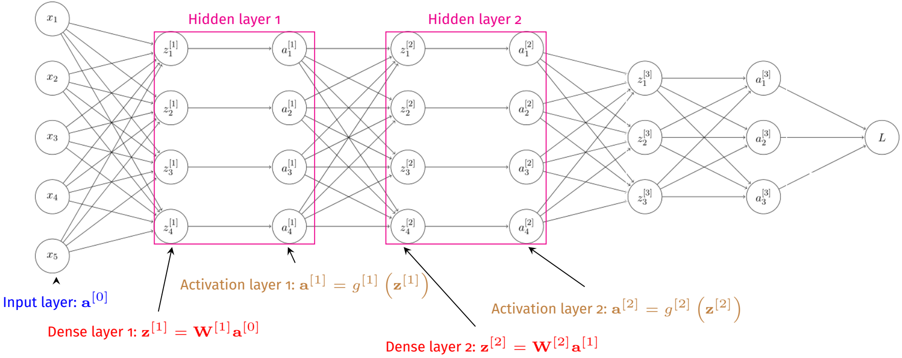

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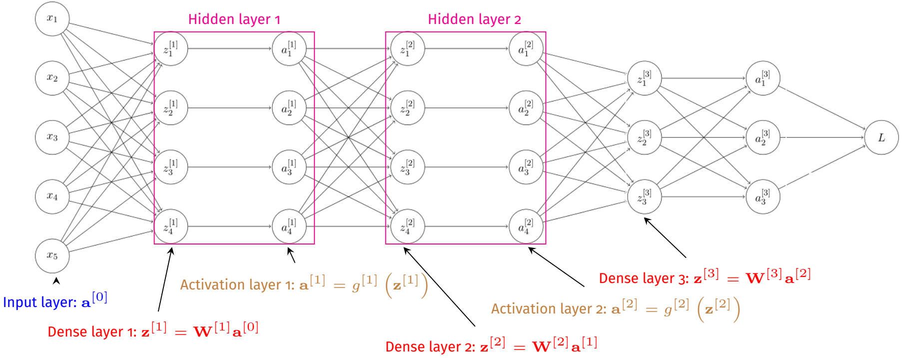

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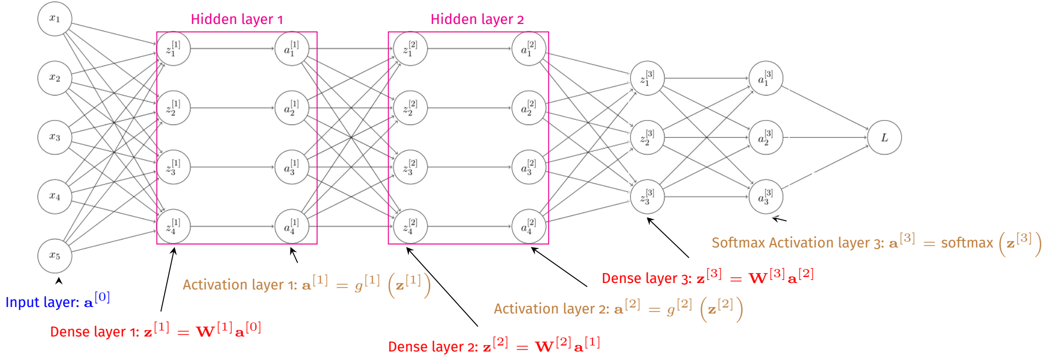

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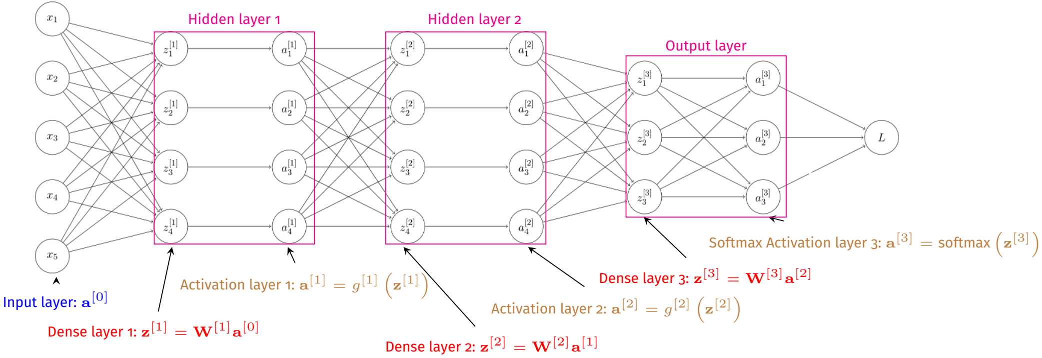

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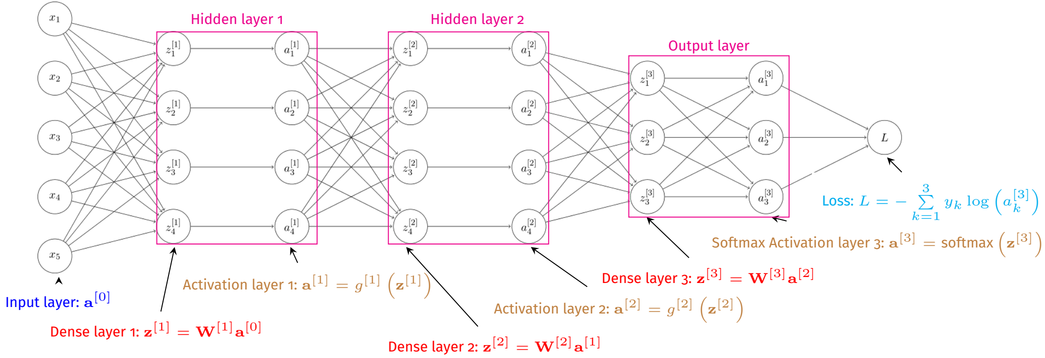

Accounting for bias in a deep L-layer neural network

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Minibatch forward propagation in a deep L-layer neural network

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Minibatch forward propagation in a deep L-layer neural network

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Minibatch forward propagation in a deep L-layer neural network

- Suppose we have minibatch comprising b samples represented as the $5 \times b$-matrix $\mathbf{X} = \begin{bmatrix} \mathbf{x}^{(1)} & \mathbf{x}^{(2)} & \dots & \mathbf{x}^{(b)} \end{bmatrix}$, with one-hot encoded true labels represented as the $3 \times b$-matrix (3 possible output categories) $\mathbf{Y} = \begin{bmatrix} \mathbf{y}^{(1)} & \dots & \mathbf{y}^{(b)} \end{bmatrix}$.
- For hidden layer 1, we calculate

$$\begin{cases} \underbrace{\begin{bmatrix} \mathbf{z}^{[1](1)} & \dots & \mathbf{z}^{[1](b)} \end{bmatrix}}_{\mathbf{Z}^{[1]}} = \begin{bmatrix} \mathbf{W}^{[1]}\mathbf{a}_B^{[0](1)} & \dots & \mathbf{W}^{[1]}\mathbf{a}_B^{[0](b)} \end{bmatrix} = \mathbf{W}^{[1]} \underbrace{\begin{bmatrix} \mathbf{a}_B^{[0](1)} & \dots & \mathbf{a}_B^{[0](b)} \end{bmatrix}}_{\mathbf{A}_B^{[0]}} = \mathbf{W}^{[1]}\mathbf{A}_B^{[0]}, \\ \underbrace{\begin{bmatrix} \mathbf{a}^{[1](1)} & \dots & \mathbf{a}^{[1](b)} \end{bmatrix}}_{\mathbf{A}^{[1]}} = \end{cases}$$

Minibatch forward propagation in a deep L-layer neural network

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Minibatch forward propagation in a deep L-layer neural network

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Minibatch forward propagation in a deep L-layer neural network

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Minibatch forward propagation in a deep L-layer neural network

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Minibatch forward propagation in a deep L-layer neural network

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- Similarly, we calculate for hidden layer 2, $\mathbf{Z}^{[2]} = \mathbf{W}^{[2]}\mathbf{A}_B^{[1]}$ and $\mathbf{A}^{[2]} = g^{[2]}\left(\mathbf{Z}^{[2]}\right)$, and for hidden layer 3, $\mathbf{Z}^{[3]} = \mathbf{W}^{[3]}\mathbf{A}_B^{[2]}$ and $\mathbf{A}^{[3]} = \text{softmax}\left(\mathbf{Z}^{[3]}\right)$.

Minibatch forward propagation in a deep L-layer neural network

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Minibatch forward propagation in a deep L-layer neural network

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- Setting $\hat{\mathbf{Y}} = \mathbf{A}^{[3]}$, the predicted probabilities matrix for the minibatch, the categorical crossentropy (CCE) loss for the ith sample is $L_i = \sum_{k=1}^{3} -y_k^{(i)} \log\left(\hat{y}_k^{(i)}\right)$,

Minibatch forward propagation in a deep L-layer neural network

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- Setting $\hat{\mathbf{Y}} = \mathbf{A}^{[3]}$, the predicted probabilities matrix for the minibatch, the categorical crossentropy (CCE) loss for the ith sample is $L_i = \sum_{k=1}^{3} -y_k^{(i)} \log\left(\hat{y}_k^{(i)}\right)$, which leads to the average categorical crossentropy (CCE) batch loss for the minibatch as

Minibatch forward propagation in a deep L-layer neural network

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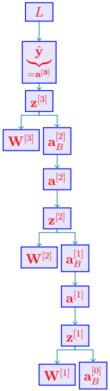
Deep L-layer neural network gradient calculation using backward propagation

Deep L-layer neural network gradient calculation using backward propagation

Computation graph:

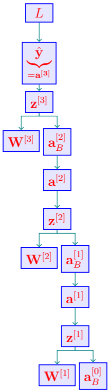
Deep L-layer neural network gradient calculation using backward propagation

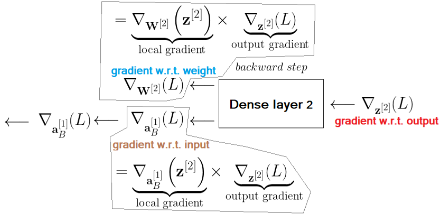
Computation graph:



Deep L-layer neural network gradient calculation using backward propagation

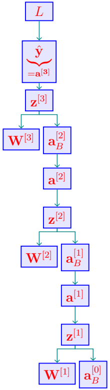
Computation graph:

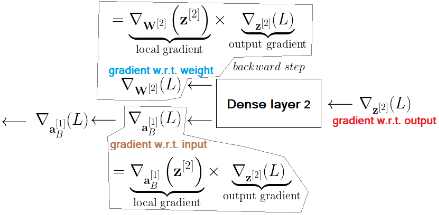




Deep L-layer neural network gradient calculation using backward propagation

Computation graph:

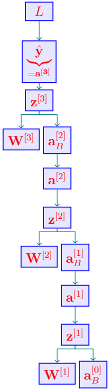


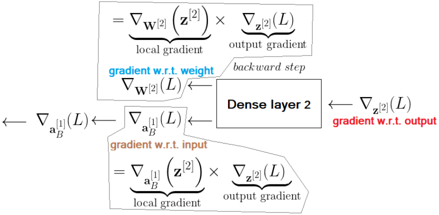


During backward propagation, a dense layer looks at the gradient flowing backward from its output side and does two things:

Deep L-layer neural network gradient calculation using backward propagation

Computation graph:





During backward propagation, a dense layer looks at the gradient flowing backward from its output side and does two things: (1) calculates the local gradient w.r.t. its weights and multiplies that with the output gradient so that the weights can be updated

Deep L-layer neural network gradient calculation using backward propagation

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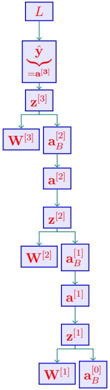
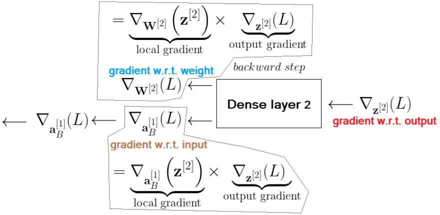
Computation graph:

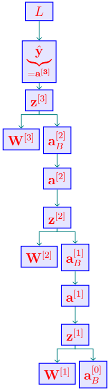


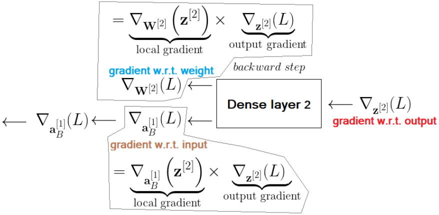


During backward propagation, a dense layer looks at the gradient flowing backward from its output side and does two things: (1) calculates the local gradient w.r.t. its weights and multiplies that with the output gradient so that the weights can be updated (2) calculates the local gradient w.r.t. its input and multiplies that with the output gradient so that the resulting gradient flowing backward from its input side is returned;

Deep L-layer neural network gradient calculation using backward propagation

Computation graph:





During backward propagation, a dense layer looks at the gradient flowing backward from its output side and does two things: (1) calculates the local gradient w.r.t. its weights and multiplies that with the output gradient so that the weights can be updated (2) calculates the local gradient w.r.t. its input and multiplies that with the output gradient so that the resulting gradient flowing backward from its input side is returned; this then becomes the gradient flowing backward from the output side of the previous activation layer.

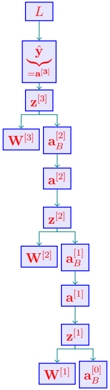
Deep L-layer neural network gradient calculation using backward propagation – continued

Deep L-layer neural network gradient calculation using backward propagation – continued

Computation graph:

Deep L-layer neural network gradient calculation using backward propagation – continued

Computation graph:



Deep L-layer neural network gradient calculation using backward propagation – continued

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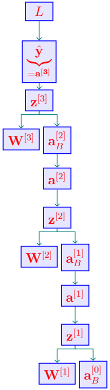
Computation graph:



gradient w.r.t input

$$\nabla_{\mathbf{z}^{[1]}}(L) = \underbrace{\nabla_{\mathbf{z}^{[1]}}\left(\mathbf{a}^{[1]}\right)}_{\text{local gradient}} \times \underbrace{\nabla_{\mathbf{a}^{[1]}}(L)}_{\text{output gradient}}$$

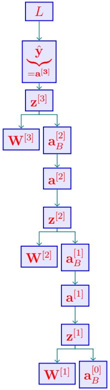
$\longleftarrow$ **Activation layer 1** $\longleftarrow$

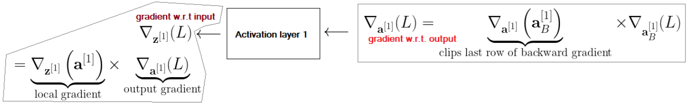
gradient w.r.t. output

$$\nabla_{\mathbf{a}^{[1]}}(L) = \underbrace{\nabla_{\mathbf{a}^{[1]}}\left(\mathbf{a}_B^{[1]}\right)}_{\text{clips last row of backward gradient}} \times \nabla_{\mathbf{a}_B^{[1]}}(L)$$

Deep L-layer neural network gradient calculation using backward propagation – continued

Computation graph:





During backward propagation, an activation layer does the following:

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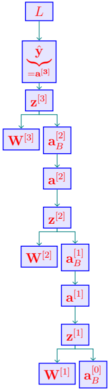

gradient w.r.t input

$$\nabla_{\mathbf{z}^{[1]}}(L) \longleftarrow \boxed{\text{Activation layer 1}} \longleftarrow \nabla_{\mathbf{a}^{[1]}}(L) = \underbrace{\nabla_{\mathbf{a}^{[1]}}\left(\mathbf{a}_B^{[1]}\right)}_{\text{clips last row of backward gradient}} \times \nabla_{\mathbf{a}_B^{[1]}}(L)$$

gradient w.r.t. output

$$= \underbrace{\nabla_{\mathbf{z}^{[1]}}\left(\mathbf{a}^{[1]}\right)}_{\text{local gradient}} \times \underbrace{\nabla_{\mathbf{a}^{[1]}}(L)}_{\text{output gradient}}$$

During backward propagation, an activation layer does the following: it clips the gradient flowing backward sent by the subsequent dense layer such that the last row of the gradient is removed;

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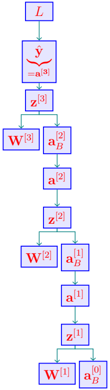

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$$\nabla_{\mathbf{a}^{[1]}}(L) = \underbrace{\nabla_{\mathbf{a}^{[1]}}\left(\mathbf{a}_B^{[1]}\right)}_{\text{clips last row of backward gradient}} \times \nabla_{\mathbf{a}_B^{[1]}}(L)$$

gradient w.r.t. output

During backward propagation, an activation layer does the following: it clips the gradient flowing backward sent by the subsequent dense layer such that the last row of the gradient is removed; the resulting gradient is then sent flowing backward on the output side for the activation layer;

Deep L-layer neural network gradient calculation using backward propagation – continued

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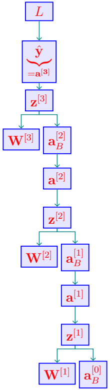

$$\underbrace{\nabla_{z^{[1]}}(L)}_{\text{gradient w.r.t input}} \longleftarrow \boxed{\text{Activation layer 1}} \longleftarrow \underbrace{\nabla_{a^{[1]}}(L)}_{\text{gradient w.r.t. output}} = \underbrace{\nabla_{a^{[1]}}\left(a_B^{[1]}\right)}_{\text{clips last row of backward gradient}} \times \nabla_{a_B^{[1]}}(L)$$

$$= \underbrace{\nabla_{z^{[1]}}\left(a^{[1]}\right)}_{\text{local gradient}} \times \underbrace{\nabla_{a^{[1]}}(L)}_{\text{output gradient}}$$

During backward propagation, an activation layer does the following: it clips the gradient flowing backward sent by the subsequent dense layer such that the last row of the gradient is removed; the resulting gradient is then sent flowing backward on the output side for the activation layer; after that, it calculates its local gradient which is multiplied by the clipped output gradient so that the resulting gradient flowing backward from its input side is returned;

Deep L-layer neural network gradient calculation using backward propagation – continued

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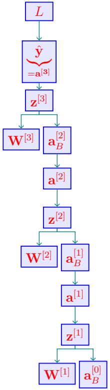

gradient w.r.t input

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Minibatch backward propagation in a deep L-layer neural network

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$$L = \frac{1}{b}\left[L_1 + \cdots + L_b\right]$$

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$$
\begin{aligned}
L &= \frac{1}{b}\left[L_1 + \cdots + L_b\right] \\
\Rightarrow \nabla_{\mathbf{W}^{[2]}}(L) &= \frac{1}{b}\left[\nabla_{\mathbf{W}^{[2]}}(L_1) + \cdots + \nabla_{\mathbf{W}^{[2]}}(L_b)\right] \\
&= \frac{1}{b}\left(\underbrace{\left[\nabla_{\mathbf{W}^{[2]}}\left(\mathbf{z}^{[2](1)}\right) \times \nabla_{\mathbf{z}^{[2](1)}}\left(\mathbf{a}^{[2](1)}\right) \times \nabla_{\mathbf{a}^{[2](1)}}(L_1)\right]}_{\text{sample 1}} + \cdots\right. \\
&\quad \left. + \underbrace{\left[\nabla_{\mathbf{W}^{[2]}}\left(\mathbf{z}^{[2](b)}\right) \times \nabla_{\mathbf{z}^{[2](b)}}\left(\mathbf{a}^{[2](b)}\right) \times \nabla_{\mathbf{a}^{[2](b)}}(L_b)\right]}_{\text{sample } b}\right).
\end{aligned}
$$