Deep Learning Principles & Applications

Chapter 5 – Improving the way deep neural networks learn

Sudarsan N.S. Acharya (sudarsan.acharya@manipal.edu)

Topics

1. Why deep neural networks tend to overfit?
2. Loss-based Regularization Approaches
3. Why does loss-based regularization work?
4. Dropout regularization - idea
5. Dropout regularization - practical details
6. Why does dropout regularization work?
7. Why initializing weights is important?
8. Weight-initialization techniques
9. Normalizing activations in a deep neural network
10. Batch normalization - practical details

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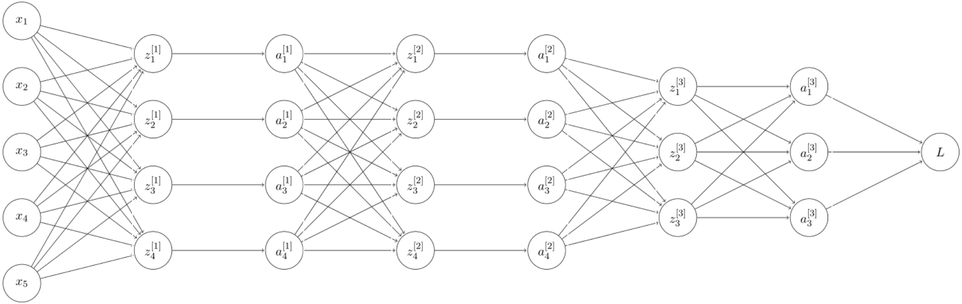

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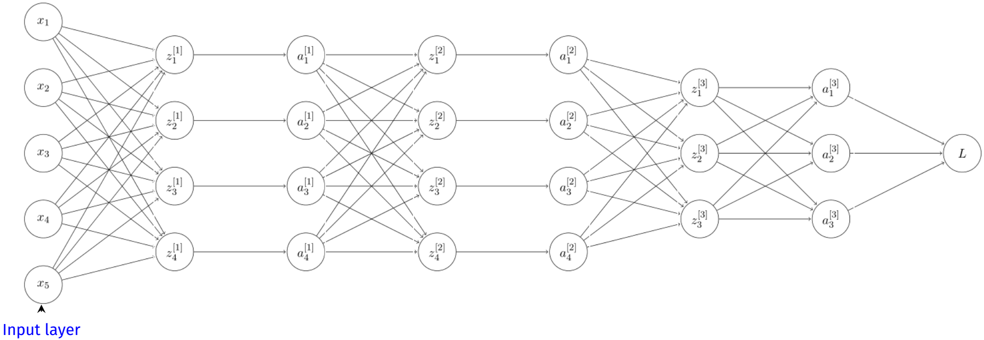

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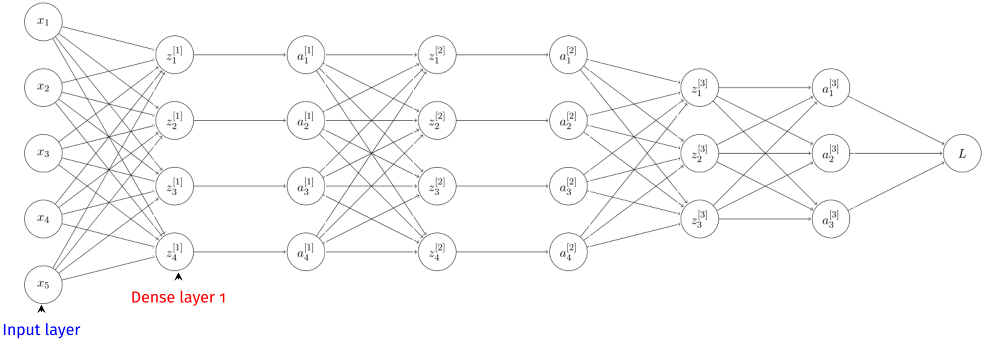

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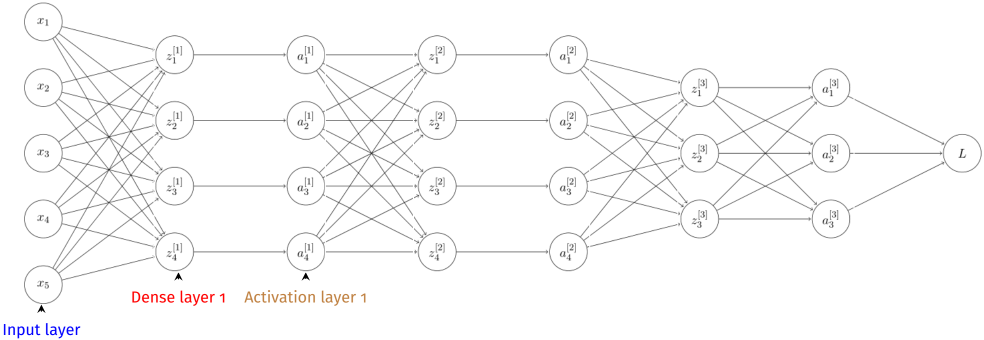

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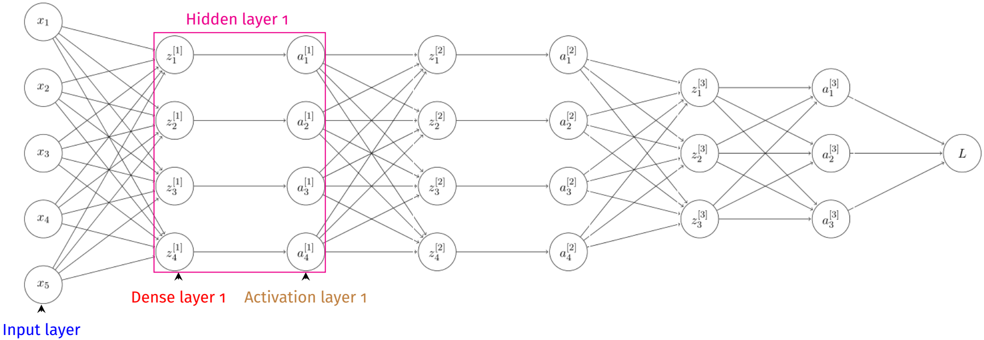

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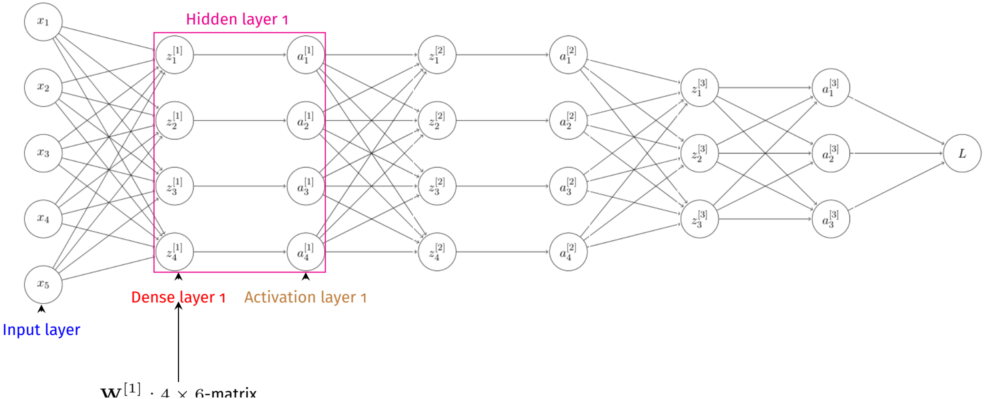

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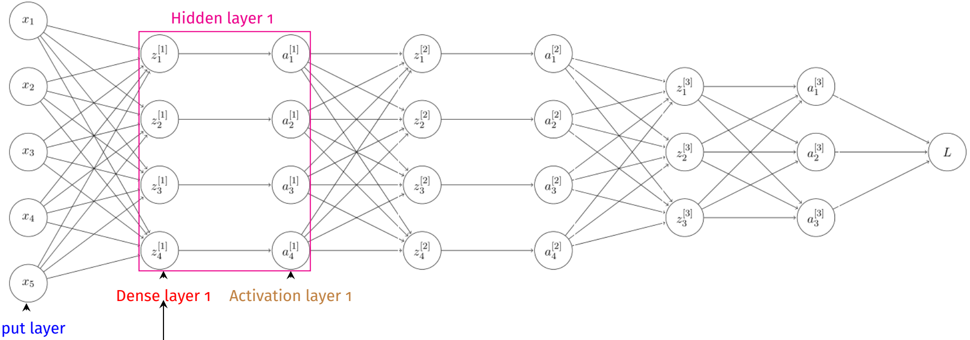

Loss = $L\left(\mathbf{W}^{[1]}\right) + \lambda\left(\|w_{1,1:5}^{[1]}\|^2 + \|w_{2,1:5}^{[1]}\|^2 + \|w_{3,1:5}^{[1]}\|^2 + \|w_{4,1:5}^{[1]}\|^2\right)$

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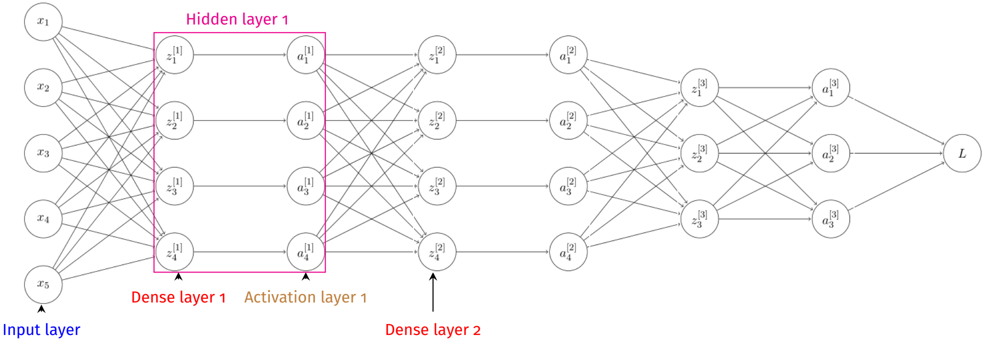

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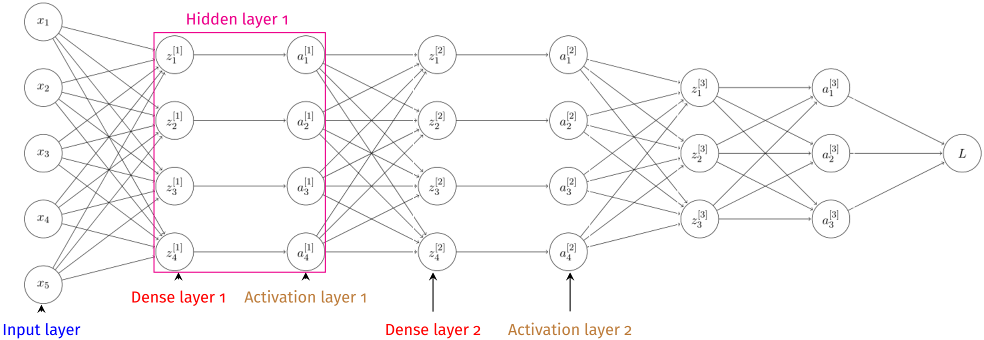

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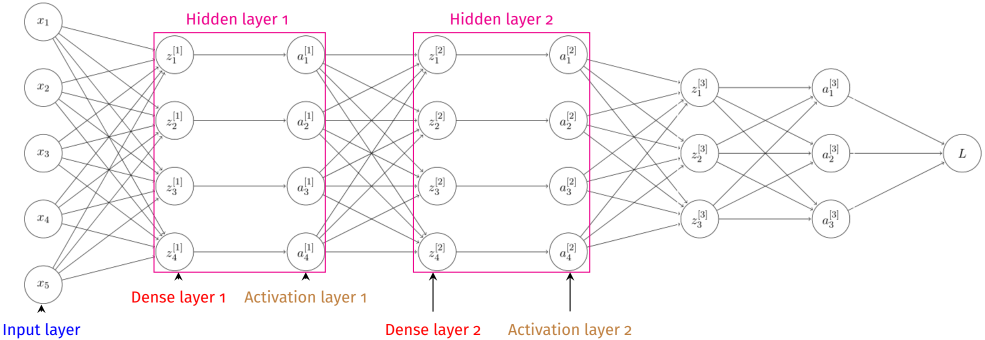

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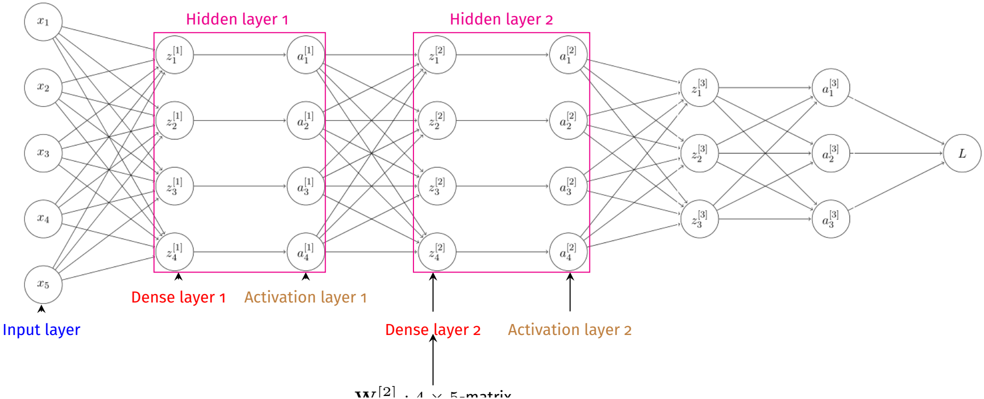

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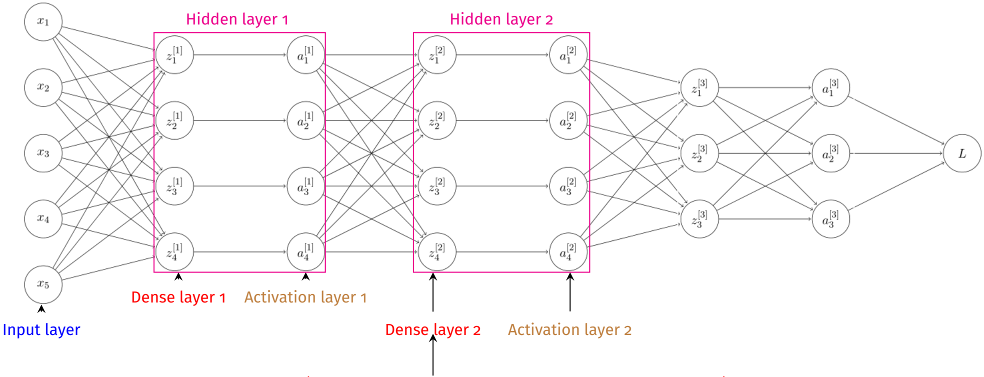

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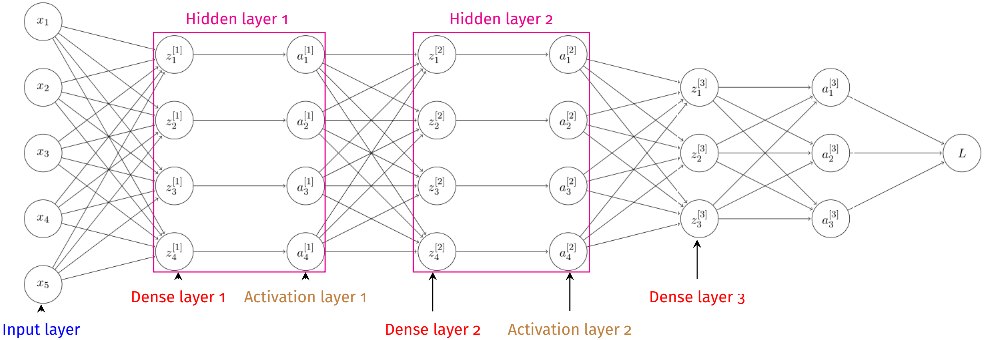

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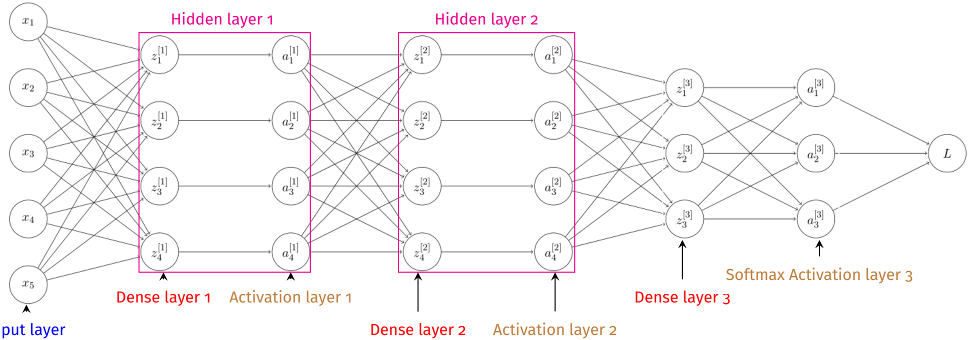

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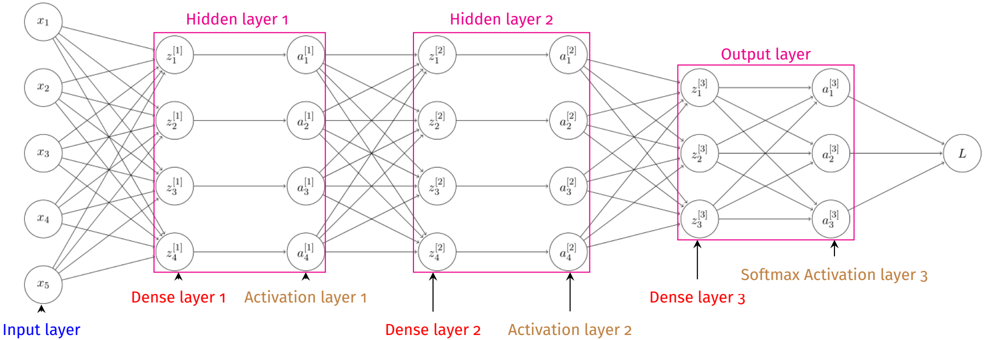

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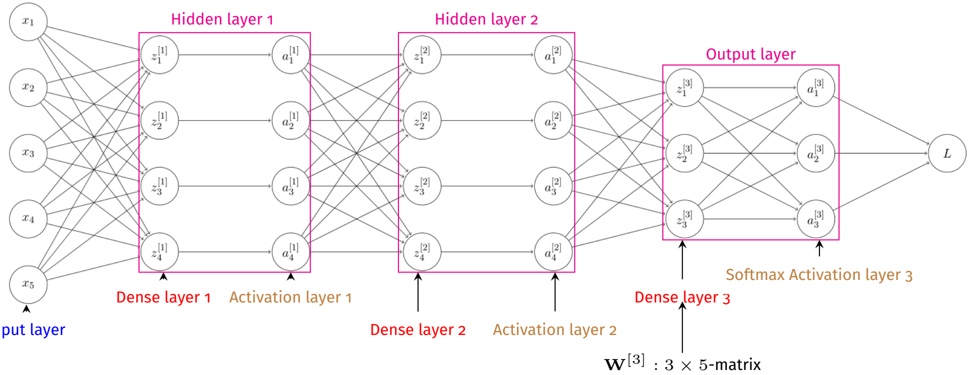

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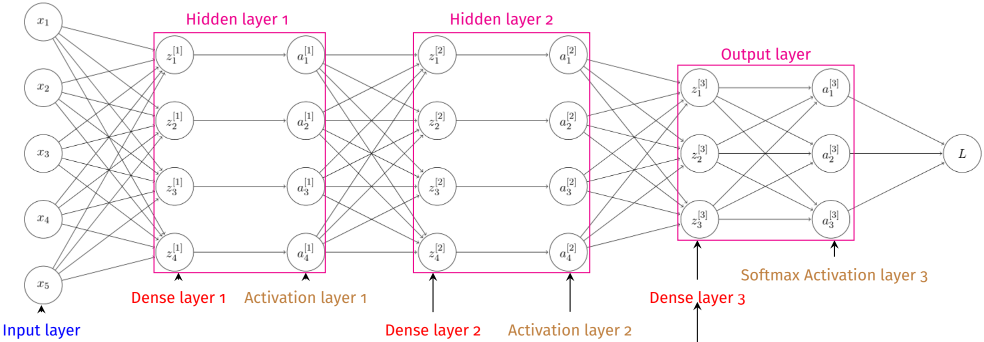

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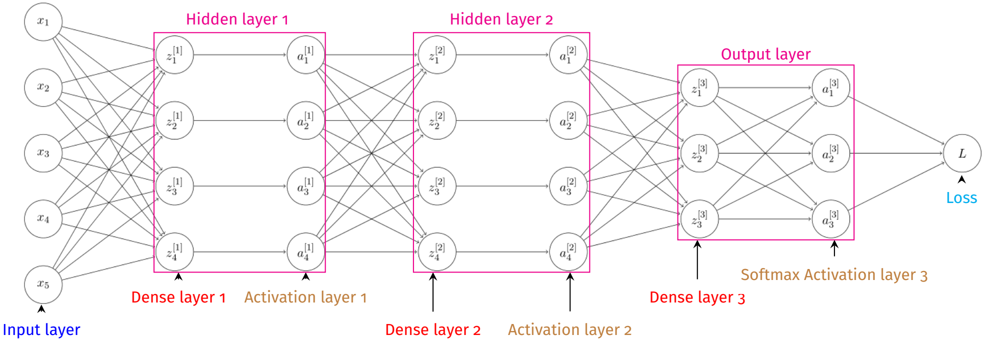

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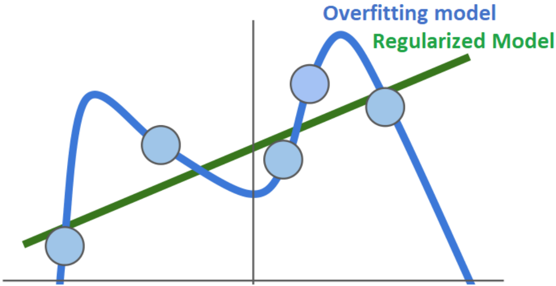

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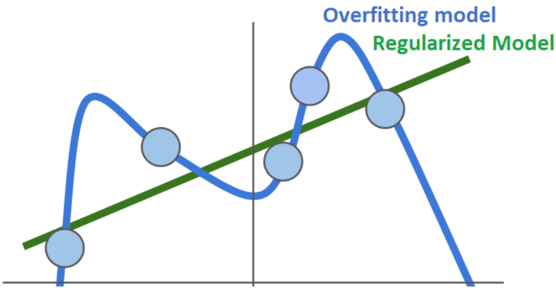

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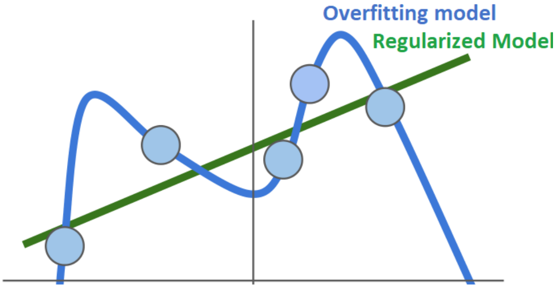

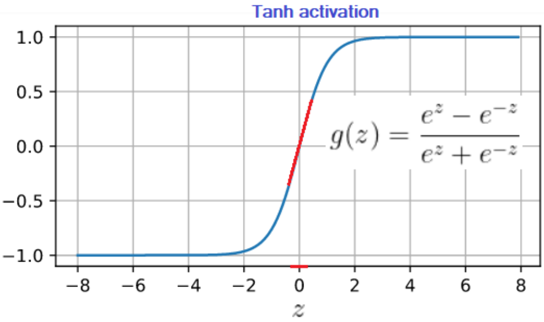

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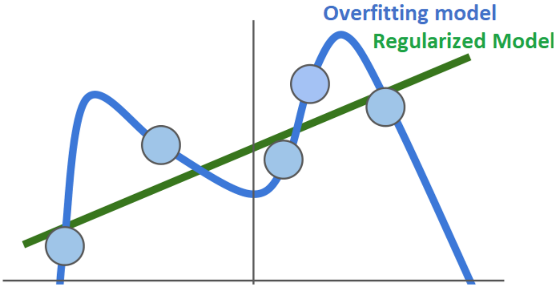

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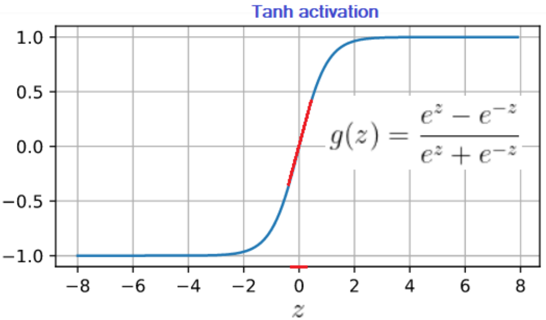

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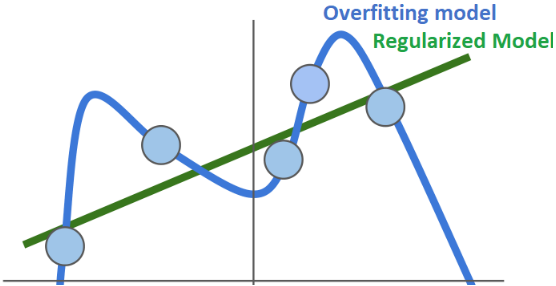

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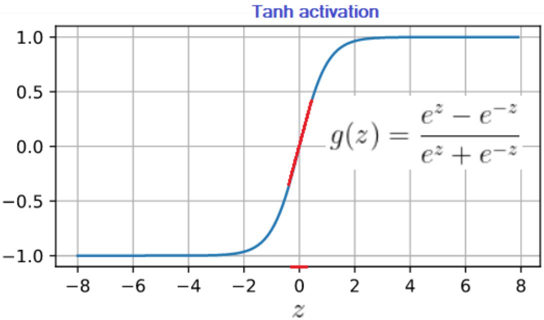

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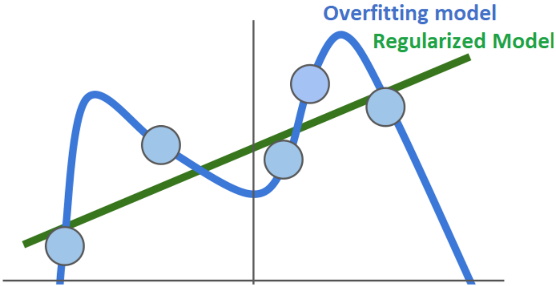

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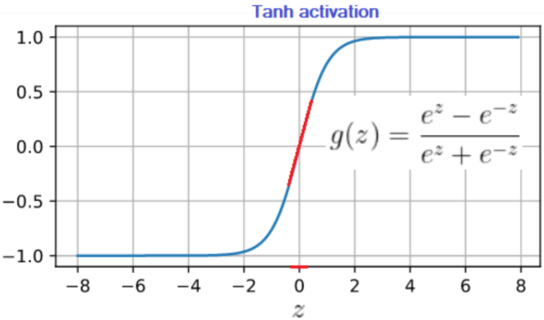

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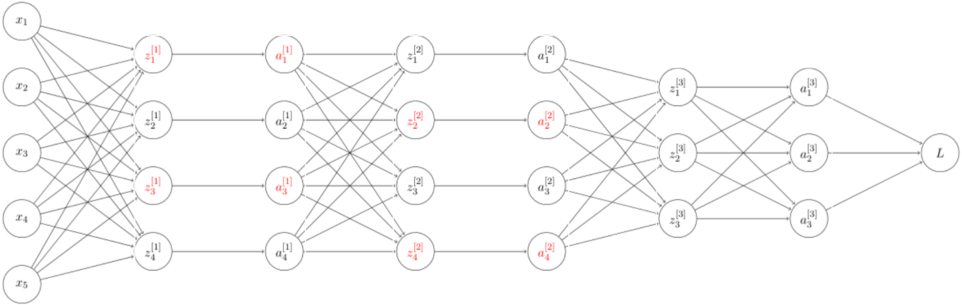

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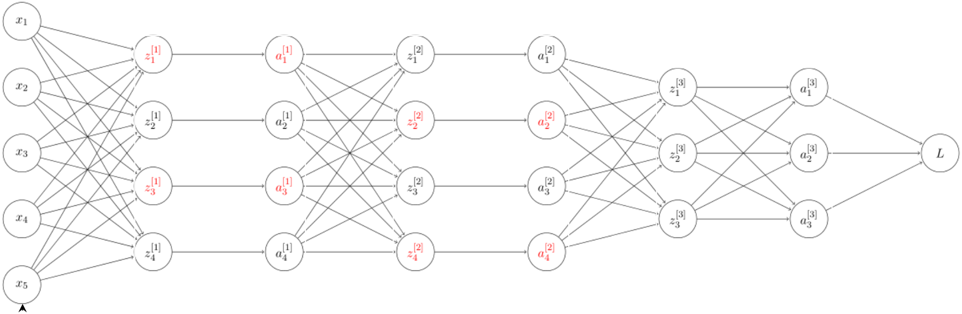

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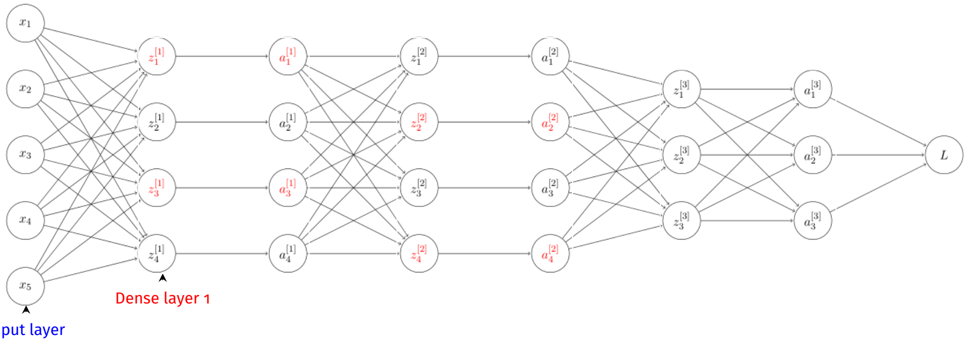

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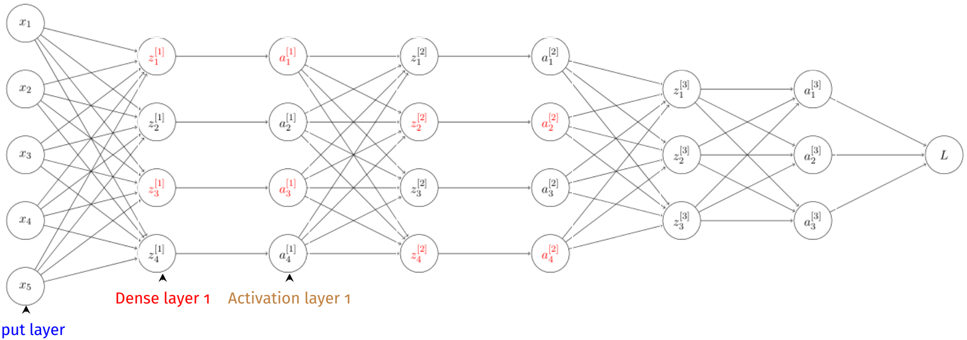

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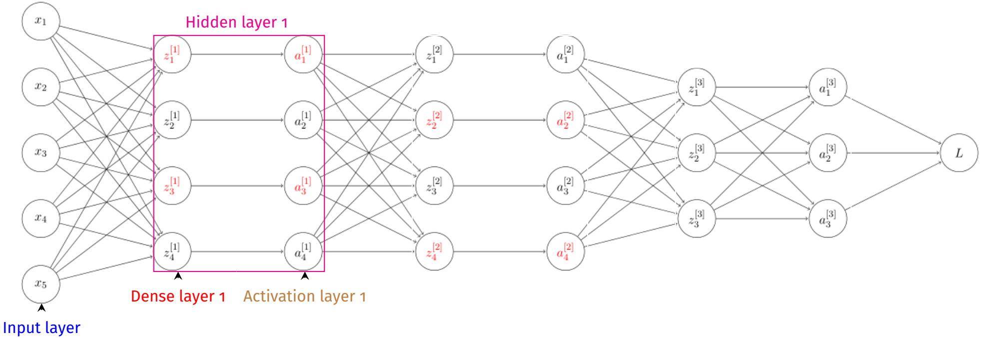

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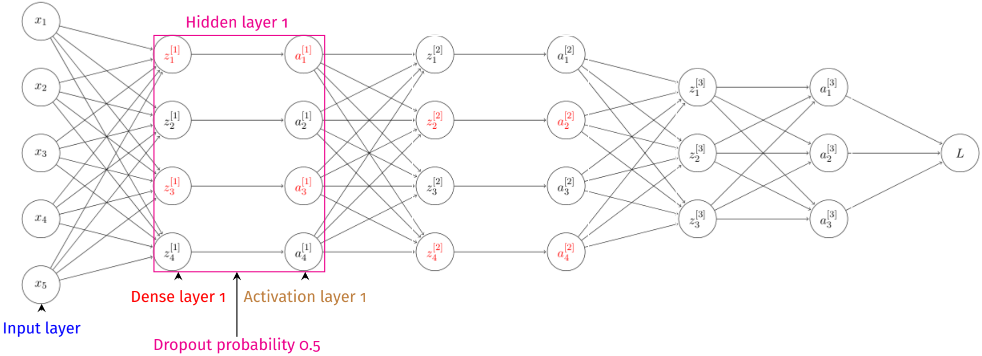

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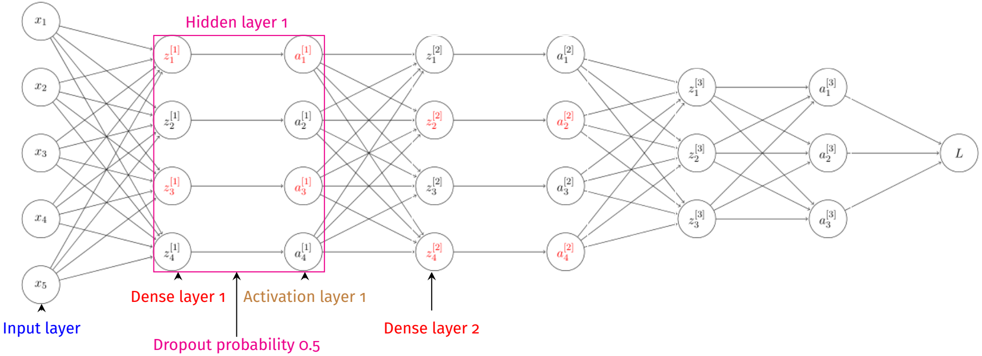

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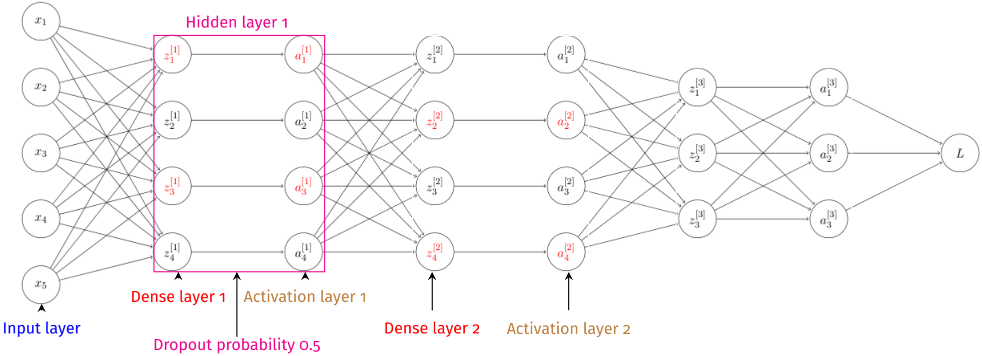

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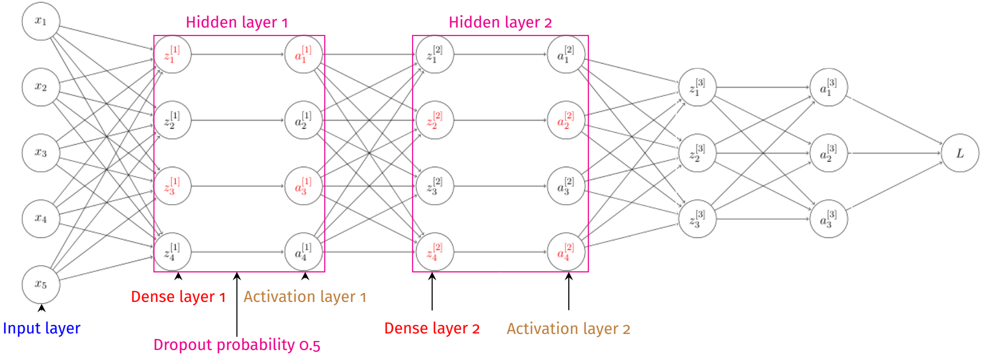

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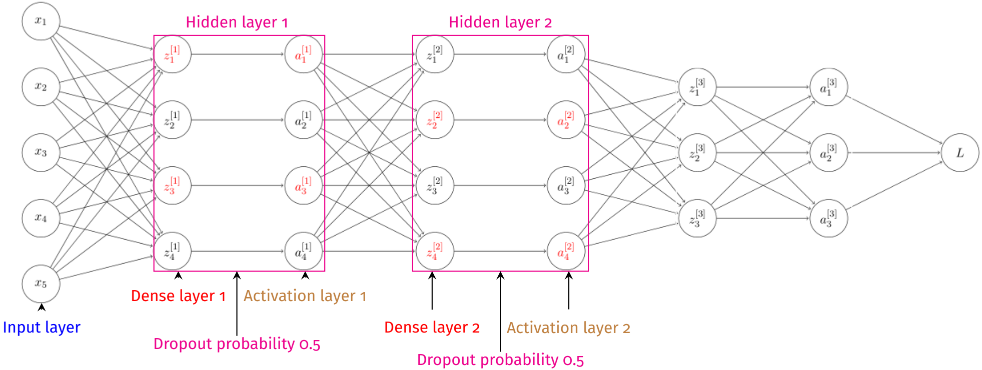

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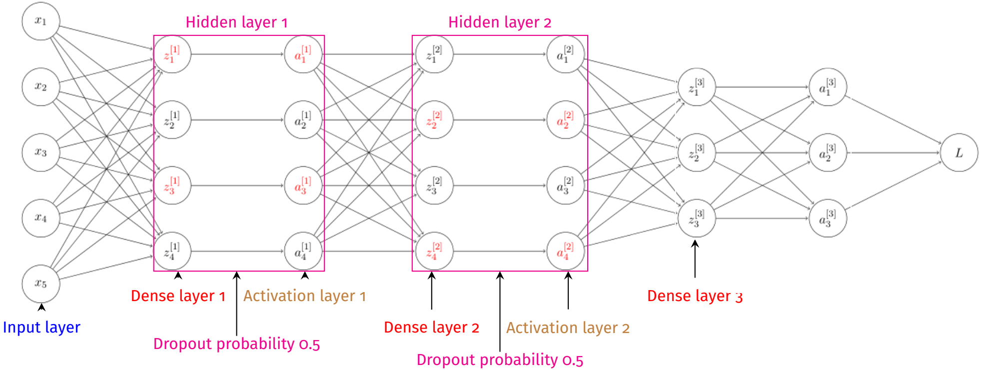

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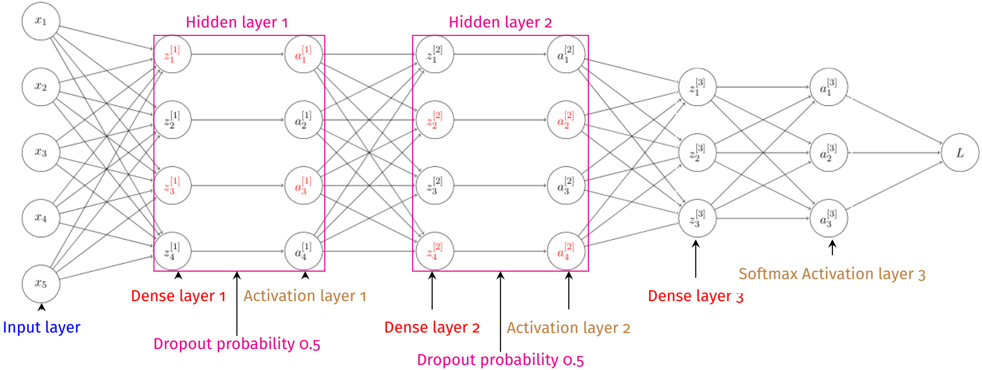

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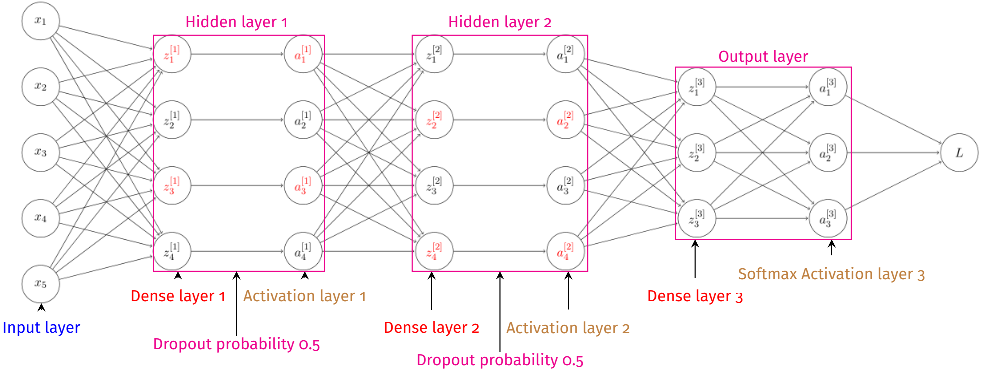

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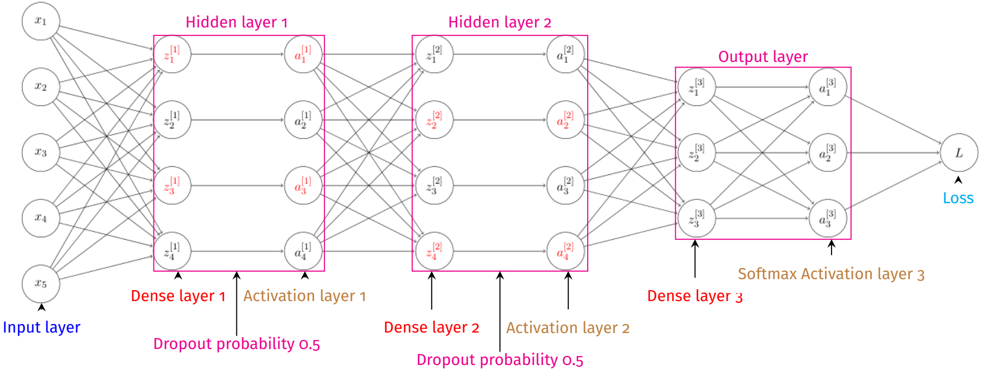

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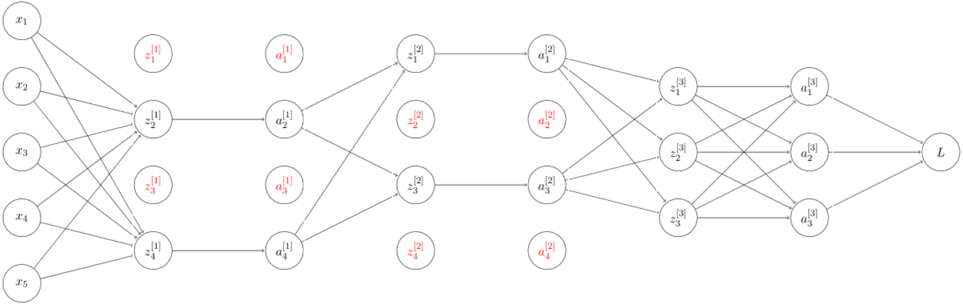

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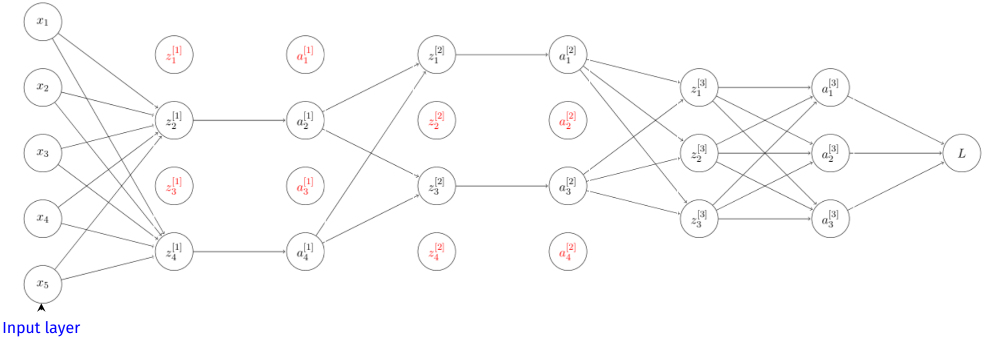

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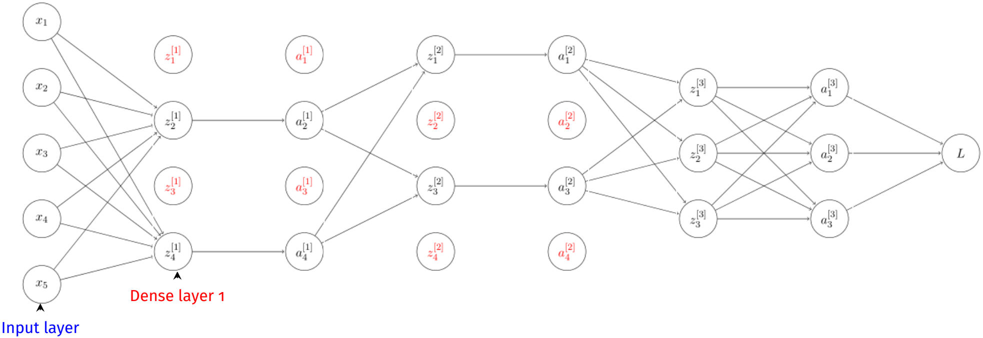

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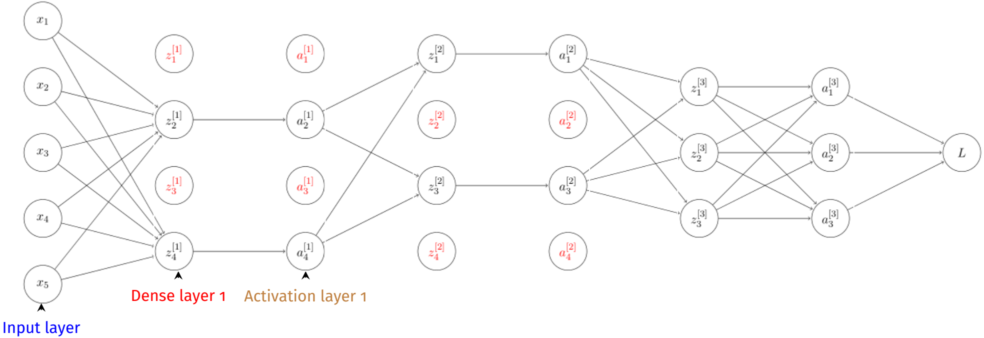

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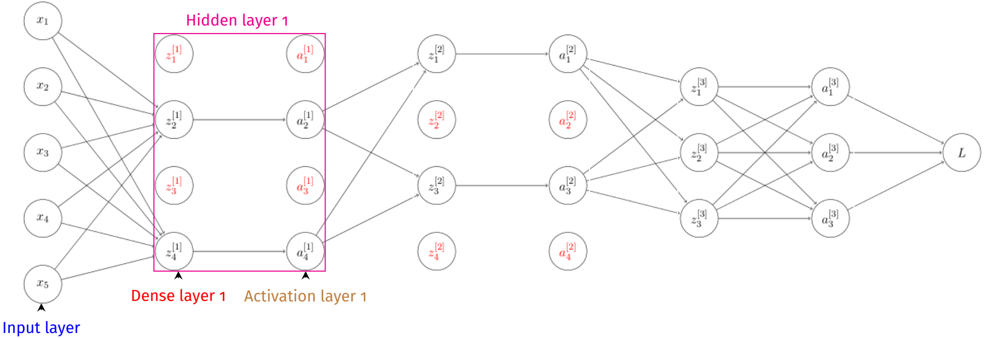

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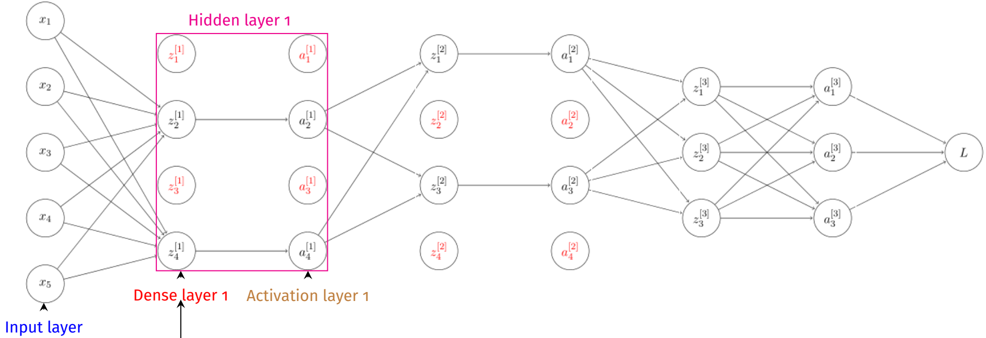

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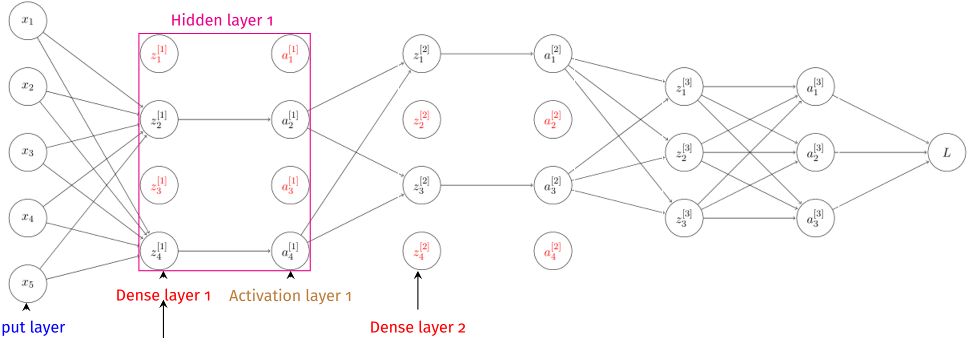

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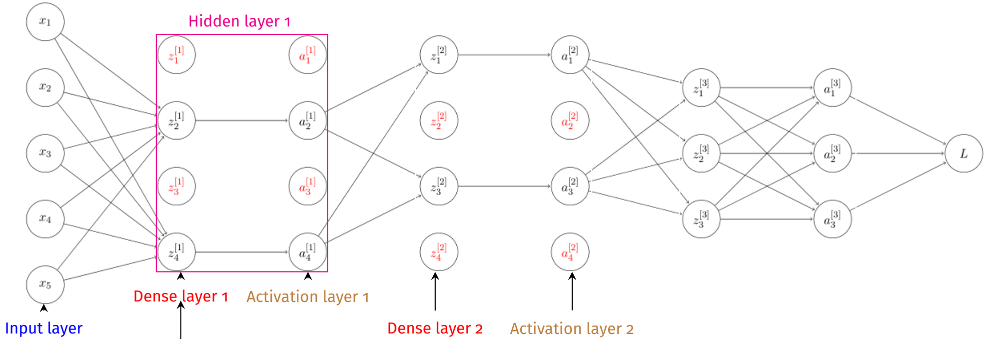

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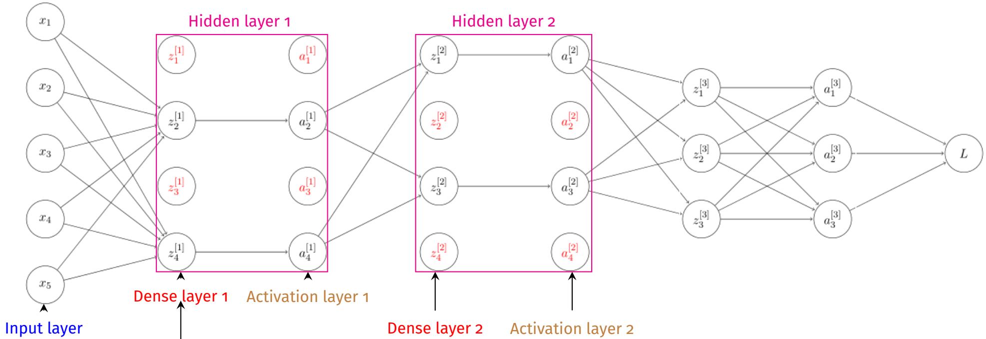

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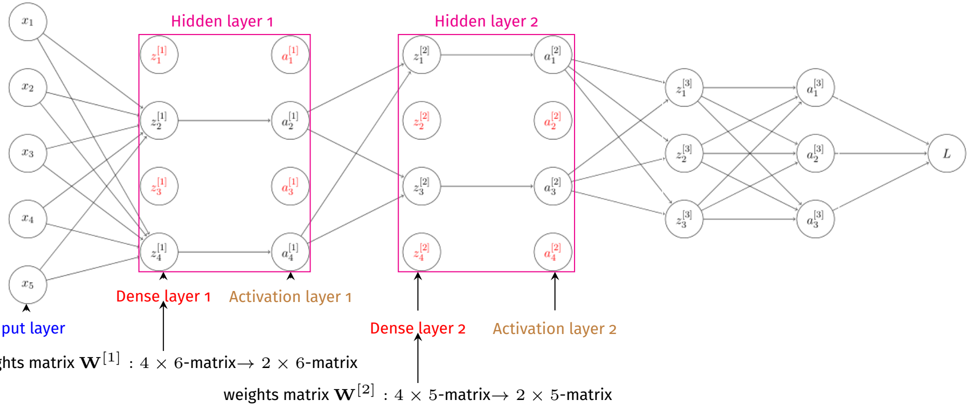

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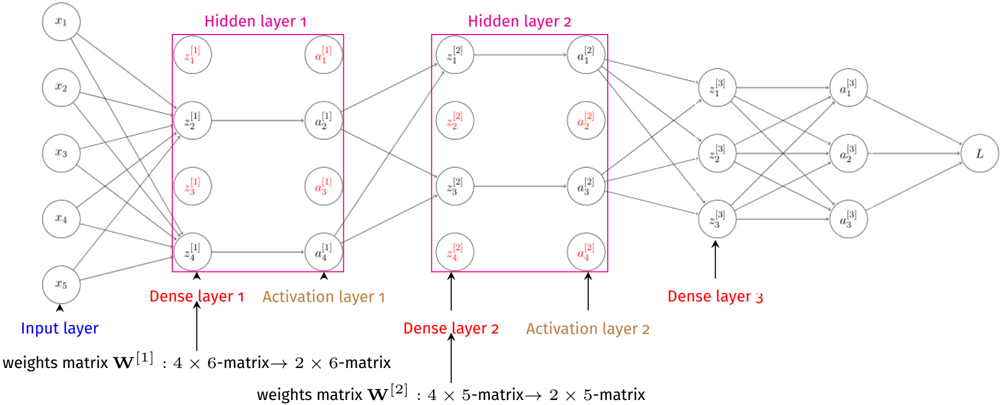

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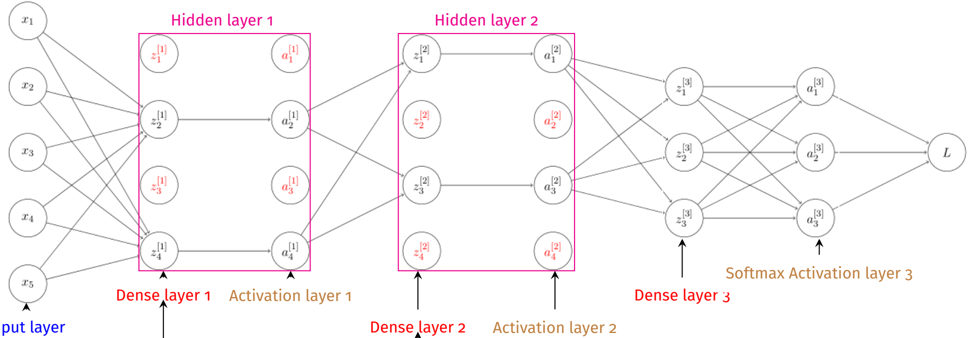

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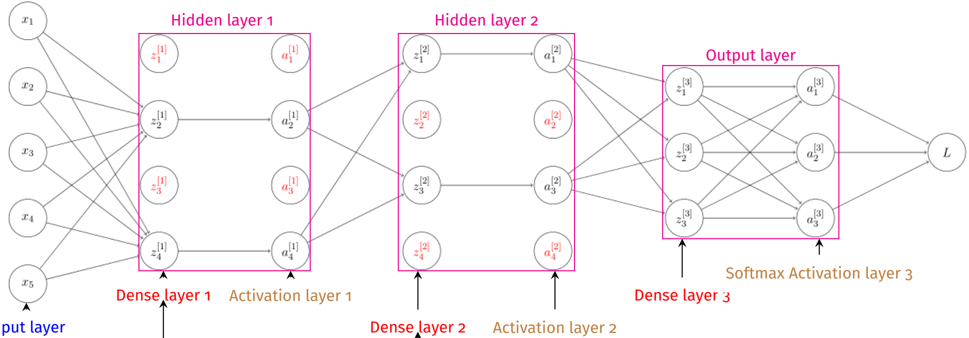

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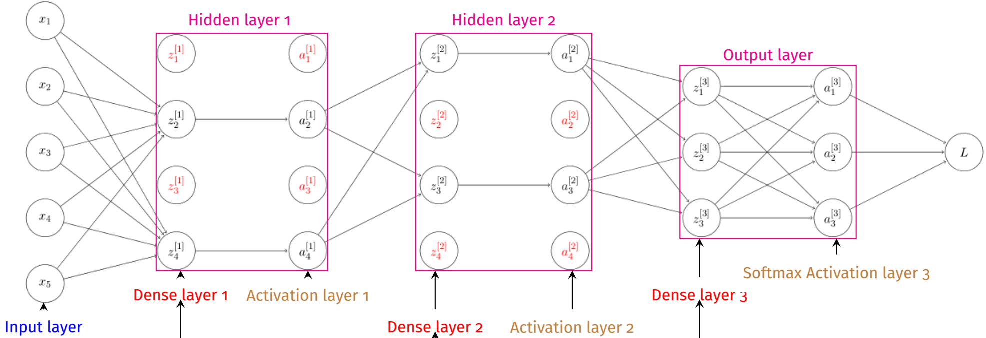

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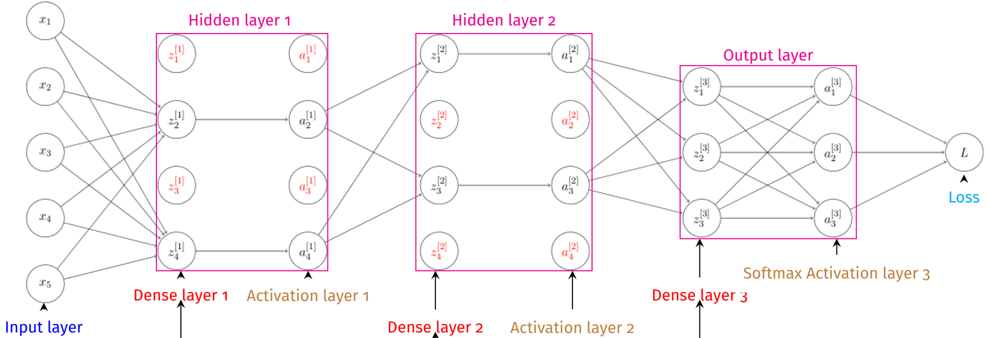

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- In a deep neural network with multiple layers, layers with higher number of nodes are typically associated with a higher dropout probability that ones with smaller number of nodes.

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- An excellent resource to visualize this: https://www.deeplearning.ai/ai-notes/initialization/index.html

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- The same idea can be applied to speed up the learning of the weights associated with deeper dense layers in a deep neural network.

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- Calculate batch norm-updated raw scores for each sample in the batch as $\tilde{\mathbf{z}}^{[l](i)} = \gamma \mathbf{z}_{\text{norm}}^{[l](i)} + \beta$, where γ are β are learnable parameters just like the weights which let the mean and variance of the batch norm-updated raw scores to not to be tied to 0 and 1, respectively, because of the normalization step