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MLPA ASSIGNMENT - 01

1 → $W = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $b = -4$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$W^T x + b = 0$

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + (-4) = 0$$

$$x_1 + 2x_2 + 3x_3 - 4 = 0$$

$$x = \begin{bmatrix} 4 - 2x_2 - 3x_3 \\ x_2 \\ x_3 \end{bmatrix}.$$

$$\Rightarrow x = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

2] Distance of sample $x^{(1)}$ from plane $w^T x + b = 0$ is

$$\frac{| w^T x^{(1)} + b |}{\|w\|}$$

Scalar Projection $= \dfrac{V.w}{\|w\|}$

$v \to v_1$, $w = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

$V.w \Rightarrow w^T . x^{(1)}$

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Scalar projection $\Rightarrow \dfrac{w^T x^{(i)}}{\|w\|}$

Distance of $x^{(i)}$ from the plane is magnitude of scalar projection

$$\text{Distance} = \left|\frac{w^T x^{(i)}}{\|w\|}\right| = \frac{|w^T x^{(i)}|}{\|w\|}$$

Distance of sample $x^{(i)}$ from plane is $w^T x^{(i)} + b = 0$

$$\Rightarrow \frac{|w^T x^{(i)} + b|}{\|w\|}$$

6] Samples $\Rightarrow x^{(1)}, x^{(2)}, \ldots, x^{(7)}$
Label $= y^{(1)}, y^{(2)}, \ldots, y^{(7)}$

$$\text{maximize}\left(\underbrace{\text{min of } \frac{|w^T x^{(i)} + b|}{\|w\|}}_{v}\right) =$$

$$\Rightarrow \text{maximize}\left[\text{minimize}\left(\frac{w^T x^{(i)} + b}{\|w\|}\right)\right] = \frac{v}{\|w\|}$$

$$y^{(i)}(w^T x^{(i)} + b) \geq 1$$

7] $x_2 = -2x_1 + 4$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x \\ -2x + 4 \end{bmatrix}$$

$= x_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix}$

It can be visualized as a straight line with the sum of vector $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and the specific point on the line $\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

8] $$X = \begin{array}{ccccc} x^{(1)} & x^{(2)} & x^{(3)} & x^{(4)} & x^{(5)} \end{array}$$
$$X = \begin{bmatrix} -1 & -1 & 0 & 2 & -2 \\ -1 & 1 & 4 & -3 & -2 \end{bmatrix}$$

$3x_1 - 4x_2 + 1 = 0$

Coeff of x_1 & x_2 are 3 & -4

$x = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$

Normal vector, $\mu = \frac{x}{\|x\|} = \frac{\begin{bmatrix} 3 \\ -4 \end{bmatrix}}{\sqrt{(3)^2 + (-4)^2}}$

$\mu = \frac{1}{5} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$

b] The full margin width

$\mu = \frac{2}{\|x\|} = \frac{2}{\sqrt{(3)^2 + (-4)^2}}$

$= \frac{2}{5}$

c] $3x_1 - 4x_2 + 1 = 0$

$$x^T = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & 4 \\ 2 & -3 \\ -2 & -2 \end{bmatrix}$$

Directed distance, $d = \frac{w^T x_i + b}{\|x\|}$, $b = 1$

$$d_1 = \frac{3 \times 1 + -4 (-1) + 1}{\sqrt{(3)^2 + (-4)^2}} = \frac{8}{5}$$

$$d_2 = \frac{3(-1) + (-4) \times 1 + 1}{\sqrt{25}} = \frac{-6}{5}$$

$$d_3 = \frac{3 \times 0 + -4 \times 4 + 1}{5} = \frac{-15}{5} = -3$$

$$d_4 = \frac{3 \times 2 + (-4) \times (-3) + 1}{5} = \frac{19}{5}$$

$$d_5 = \frac{3 \times -2 + (-4) \times (-2) + 1}{5} = \frac{3}{5}$$

The smallest ~~distance~~ margin is x_3 $d_3 = -3$

The largest margin x_4 $d_4 = \frac{19}{5}$

3] The dot product of w & the vector on plane is always zero.