Optimising Compressor Stage Performance Using Particle Swarm Optimisation

AE310: Course Project

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Optimisation Problem

Problem Definition

Problem

Design last stage of low pressure axial compressor(LPC) in order to achieve pressure ratio of 1.2.

Design Requirements

- *Total Pressure Ratio* $= 1.2$
- *Operating absolute mach number* $= 0.5$
- *Tip diameter* $= 0.75m$
- *Hub tip radius ratio* $= 0.5$
- *Rotational Speed* $= 10{,}000$ *rpm*

Constraints

Hard Constraints

Design Constraints which a solution has to satisfy in order to be a potential solution.

Hard Constraints :

- Total Pressure Ratio = 1.2
- Operating absolute mach number = 0.5
- Geometry Constraints
- Relative Mach Number at tip of Blade is less 1
- Diffusion factor for rotor hub and stator should be less than 0.6, and same less than 0.4 for rotor tip
- **Air Out-let angle at stator = 0**

Design constraints which can be violated to generate potential solutions.

Soft Constraints :

- Design Rotational Speed
- deHaller Number

Flow profile in compressor is highly complex and 3-Dimensional in nature. Therefore a lot of empirical formulae and constants are involved in parameter calculations.
In designing stage of compressor first we assume reasonable stage efficiency and then calculate blade parameters, from the calculated data back calculate efficiency and try to match with assumed efficiency.

Assumptions :

- Enthalpy change is only function of axial direction
- Radial velocity is very small hence negligible
- Blades are able to sustain structural loading
- Rectangular Blades are designed, constant chord

Design Strategy AND Cost Function

Problem Simplifications

Generally stage efficiencies are 90% to 95 %, since this design is for last stage so flow profile is distorted and efficiencies are reduced. Therefore assume efficiency of 90%. Hence first generate solution on this efficiency and back calculate to match with this assumed one.

In order to have optimum design the tip leakage has to be minimum, which can be controlled by minimising camber at tip. Therefore along with optimising efficiency camber will be minimized.

Design Strategy AND Cost Function

Cost Functions

Blade row efficiency is given by:

$$\eta_b = 1 - \frac{loss\_coefficient}{theoretical\_pressure\_rise}$$

Theoretically, if reaction at mean diameter is not very far from 50% then rotor and stator has same blade efficiency. Also η_b value can be regarded same as isentropic efficiency(η_{st}) of whole stage. Theoretical pressure rise is :

$$P_{th} = 1 - \frac{cos^2\alpha 1}{cos^2\alpha 2}$$

Loss coefficient is :

$$C_D = solidity * loss\_coefficient\frac{cos^3\alpha_m}{cos^2\alpha 1}$$

Design Strategy AND Cost Function

Cost Functions

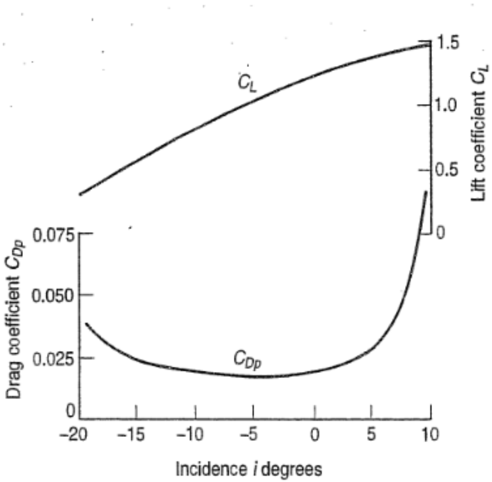

Figure: Drag Coefficient for Cascade of fixed geometrical form

Design Strategy AND Cost Function

Cost Functions

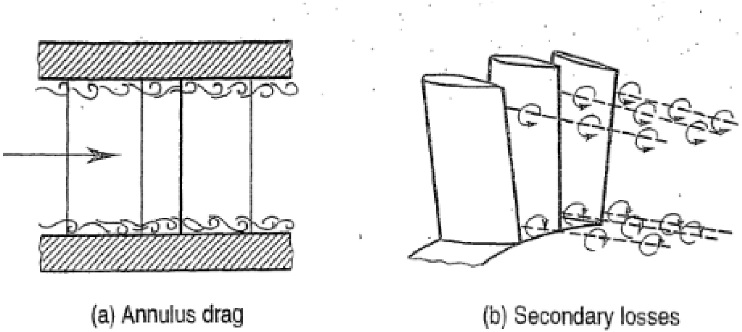

Figure: Secondary losses and Annulus losses

C_D is total drag loss given as $C_D = C_{Dp} + C_{DS} + C_{DA}$ C_{DS} and C_{DA} is given by:

$$C_{DS} = 0.018 C_L^2$$
$$C_{DA} = 0.020 \frac{blade\_spacing}{blade\_height}$$

From figure C_{Dp} at zero incidence is 0.018.
C_L is given as :

$$C_L = 2(s/c)(tan\beta 1 - tan\beta 2)Cos\alpha_m - C_{Dp} tan\alpha_m$$

Design Variables

Dependent Variables

Following are the dependent variables :

- Air inlet and outlet angles for rotor and stator
- Blade inlet and outlet angles for rotor and stator
- Absolute and Relative Velocities
- Rotational Speed
- Axial and tangential velocity components
- Reaction of rotor and stator
- Diffusion factor of Rotor and Stator
- Total initial and final temperature
- Deflection angle, Spacing and Chord of blades
- Stagger, Camber and deHaller Number

Design Variables

Independent Variables

Apart from design specifications we need 4 more variables to completely design blade, which are as follows.

- Aspect Ratio of Blade
- Tip Mach Number
- 3-D design constant value
- Temperature Difference across stage

Design Variable Constraints

Independent Variables

- From aerodynamic point, in order to have minimum losses and better efficiency, the following criterion have to be met.
  - Temperature difference across a stage should be in the range of 15 - 30 K.
  - 3-D design constant value in the range of 0-1.
- As stated tip mach number less than 1.
- Since the blades are larger, choose aspect ratio in the range of 3 - 5.

Generate Population

Initial Population

Satisfying the constraints and taking values for independent variable as mentioned first generate 50 potential candidates.
On calculating the efficiency of each candidate it is found that some have efficiencies near to 0.86, which is close to assumed efficiency.
Use **Particle Swarm Optimisation** method to tune any one of the four independent variable and generate next population.

Update Population

Velocity Function

Since we have 4 independent variables, each particle has 4 independent degrees of freedom to move in the design space and find the optimal solution.
Set maximum and minimum velocity of the particular dimension same as the extreme values of that dimension.
For first generation set velocities of particular dimension same as position value of that dimension.
Use the following formula to generate the next velocity:

$$v_{id} = K[v_{id} + c_1 rand(p_{id} - x_{id}) + c_2 rand(p_{gd} - x_{id})]$$

where K =0.729 and $c_1 = c_2 = 2.05$
rand is random number $\in (0, 1)$

Generate Population

Iteration termination Criterion

After generating the next stage of population, find out efficiency of each particle. If efficiency is better than old one, update particle best with new particle best else keep the old one.
The following criterion can break the loop :

- Maximum number of iterations
- If particle efficiency is in range of 0.895 to 0.905

Now to minimize camber, store 100 potential particles satisfying above loop criterion and check for minimum camber and efficiency closest to 0.9.

Solution

Because of the element of randomness every-time the programme runs, number of iteration to converge is different. With 50 particles the solution converged to loop criterion in less than 600 iterations almost every-time.

Solution

Sample Solutions

```
Iteration_number 254
Iteration_number 255
Iteration_number 256
Iteration_number 257
Iteration_number 258
Iteration_number 259
Iteration_number 260
Iteration_number 261
efficiency is 0.896538,pressure rise 1.199180,tip_camber 15.978209

Iteration_number 262
efficiency is 0.896538,pressure rise 1.199180,tip_camber 15.978209

Iteration_number 263
efficiency is 0.896538,pressure rise 1.199180,tip_camber 15.978209

Iteration_number 264
efficiency is 0.896538,pressure rise 1.199180,tip_camber 15.978209

Iteration_number 265
Done Enjoy!!
```

Figure: Sample Solution

Solution

Sample Solutions

```
Iteration_number 468
efficiency is 0.895492,pressure rise 1.198932,tip_camber 16.069420

efficiency is 0.896524,pressure rise 1.199177,tip_camber 15.959480

Iteration_number 469
efficiency is 0.895492,pressure rise 1.198932,tip_camber 16.069420

efficiency is 0.896524,pressure rise 1.199177,tip_camber 15.959480

Iteration_number 470
efficiency is 0.896524,pressure rise 1.199177,tip_camber 15.959480

Iteration_number 471
efficiency is 0.896524,pressure rise 1.199177,tip_camber 15.959480

Iteration_number 472
efficiency is 0.896524,pressure rise 1.199177,tip_camber 15.959480

Iteration_number 473
efficiency is 0.896524,pressure rise 1.199177,tip_camber 15.959480

Iteration_number 474
efficiency is 0.896524,pressure rise 1.199177,tip_camber 15.959480

Iteration_number 475
Done Enjoy!!
```

Solutions

Sample Solutions

total_effi	pressu	tip_ca	tip_mach	dT	n	AR
0.896812	1.199	16.15	0.70004	29	0.444	5
0.895172	1.199	15.66	0.74831	29	0.444	5
0.895204	1.199	15.97	0.71617	29	0.444	4.5
0.895701	1.199	15.86	0.73362	29	0.444	5
0.895204	1.199	15.97	0.71617	29	0.444	4.5
0.895701	1.199	15.86	0.73362	29	0.444	5
0.895204	1.199	15.97	0.71617	29	0.444	4.5

Figure: Optimised Solutions

Solution

Generate Blade Profile

From the calculated 100 potential particles find out which has minimum camber and efficiency close to 0.9.
The solution found is

1. Tip Mach Number = 0.703207425
2. Temperature Change = 29K
3. 3-D constant = 0.444444
4. Aspect Ratio = 5

Generally number of blades present in LPC compressor is around 40-60. In this optimisation, the value obtained is 44.

Conclusions

- It was found as number of starting particles are increased, number of iterations involved decreases at a large rate. It was also found that if starting particle is 10, the solution doesn't converge even in 10000 iterations.
- It was found if K value in velocity variable is set to 1 then number of iterations increases to 2000-3000.
- The flow is highly complex in a compressor. A lot of empirical formulae is involved in calculation, so above result is just a good starting point of design. Once starting point is known, CAD model is made and tested using CFD to check efficiency match.