



ENGINEERING PHYSICS

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ENGINEERING PHYSICS

Unit I : Review of concepts leading to Quantum Mechanics



Week #2 Class #8

- Analysis of the wave packet
- Heisenberg's Uncertainty Principle
- Gamma Ray microscope
- Other applications

ENGINEERING PHYSICS

Unit I : Review of concepts leading to Quantum Mechanics



➤ *Suggested Reading*

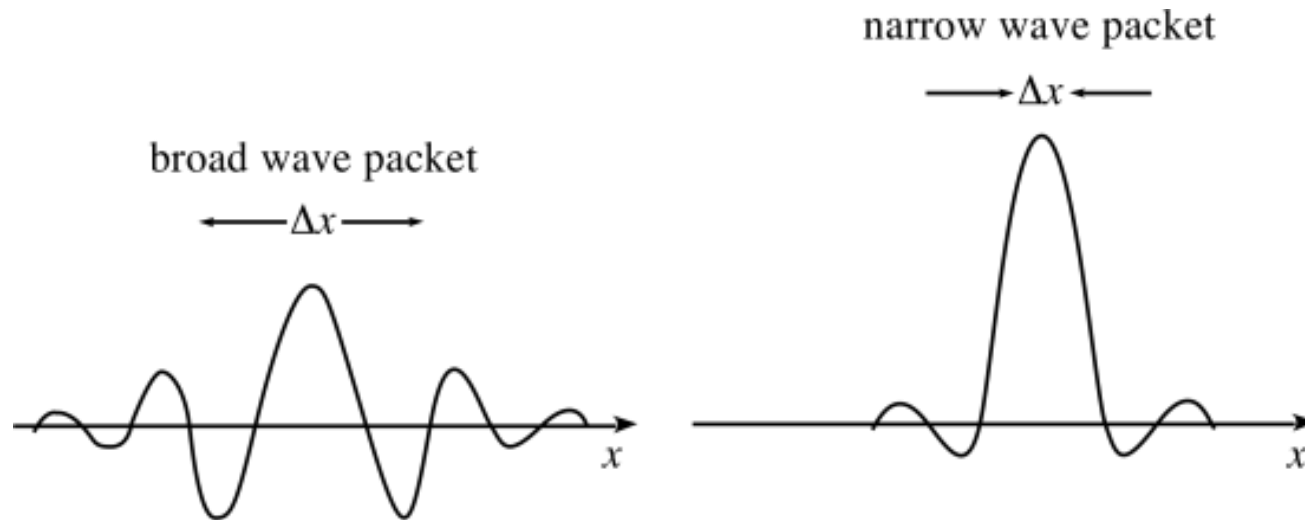
1. *Concepts of Modern Physics, Arthur Beiser, Chapter 3*
2. *Learning Material prepared by the Department of Physics*

➤ *Reference Videos*

1. *Video lectures : MIT 8.04 Quantum Physics I*

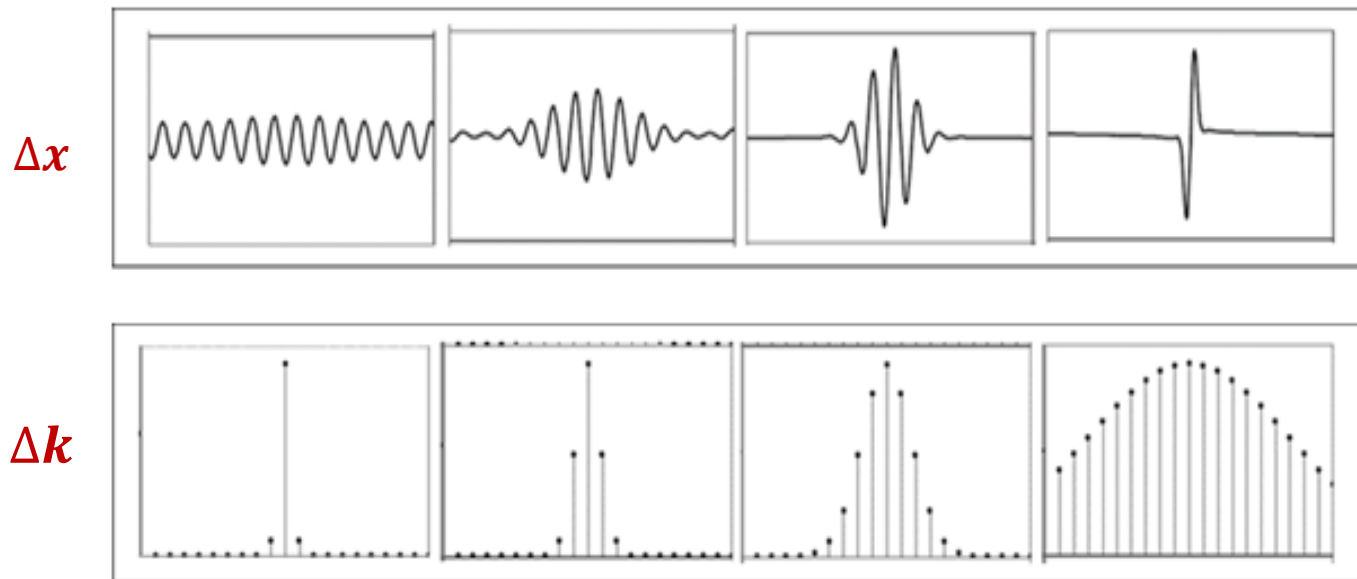
Analysis of Wave Packets

- *Wave packets are formed by the superposition of waves*
- *More number of components waves result in a narrow wave packet*
- *More number of component waves also result in a spread in the propagation constant*



Analysis of Wave Packets

- *Wave packets have an inherent component of uncertainties*
- *A broad wave packet has a high uncertainty in position, but a high accuracy in momentum*
- *A narrow wave packet results in a high accuracy in position and a high uncertainty in the momentum*



- The spread in x can be described by σ_x
- The spread in k can be described by σ_k
- The product of the standard deviations $\sigma_x \cdot \sigma_k \geq \frac{1}{2}$
- Replacing the standard deviation by the inaccuracies

$$\Delta x \cdot \Delta k \geq \frac{1}{2}$$

- Replacing $k = \frac{p}{\hbar}$
- The product of the uncertainties $\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$

Heisenberg's Uncertainty Principle

Position momentum uncertainty:

The position and momentum of a particle cannot be determined simultaneously with unlimited precision.

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

Δx - *the uncertainty in the position*

Δp - *the uncertainty in the momentum*

Energy time uncertainty:

The energy and life time of a particle in a state cannot be determined simultaneously with unlimited precision.

ΔE - the uncertainty in the energy of the particle

Δt - the uncertainty in the life time of the particle in the state

The product of the uncertainties $\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$

Uncertainty relation for circular motion:

The angular position and angular momentum of a particle in a circular motion cannot be determined simultaneously with unlimited precision.

$\Delta\theta$ - the uncertainty in the angular position

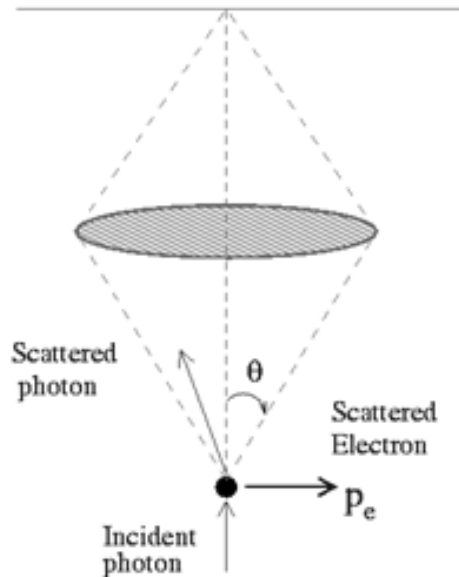
ΔL - the uncertainty in the angular momentum

The product of the uncertainties $\Delta\theta \cdot \Delta L \geq \frac{\hbar}{2}$

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Gamma ray microscope a thought experiment

- *Experiment to “measure” the position of an electron*
- *Construct a “microscope” capable of imaging the electron*



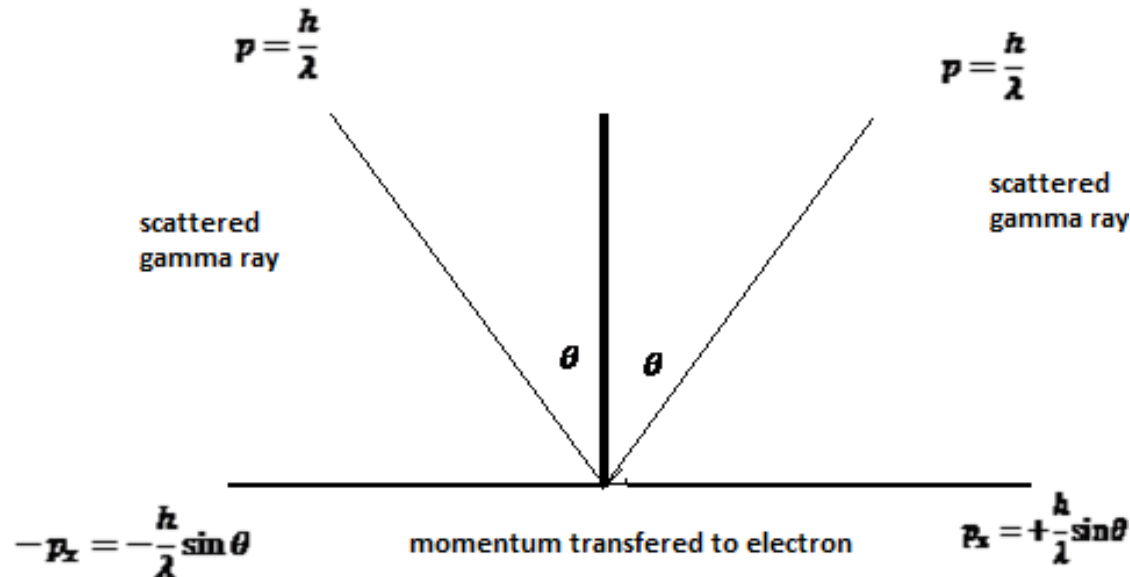
Gamma ray microscope a thought experiment

- *Wavelength of radiation to be comparable to size of electron*
- *Gamma rays wavelengths $\approx 10^{-12} \text{ m}$*
- *Resolution of the microscope $\Rightarrow \Delta x \approx \frac{\lambda}{\sin\theta}$*

Uncertainty in the position of the electron

Gamma ray microscope a thought experiment

- *High energy gamma rays impart momentum to the electrons*
– *Compton effect with gamma rays*
- *Electron momentum along x direction* $\pm \frac{h}{\lambda} \sin \theta$
- *Minimum electron momentum uncertainty* $\approx \Delta p = 2 \frac{h}{\lambda} \sin \theta$



- *Product of the uncertainties*
- $\Delta x \times \Delta p = 2 \frac{h \sin \theta}{\lambda} \cdot \frac{\lambda}{\sin \theta} = 2h$
- *Greater than $\frac{h}{4\pi}$!*
- *Confirms that the position and momentum cannot be simultaneously determined accurately.*

Other applications – electron confinement in a nucleus

- *In a radioactive β decay an electron is emitted from the nucleus with energies of the order of 8 MeV*
- *Estimate the energy of the electron from the uncertainty principle*
- *Electron if present in the nucleus has to be confined to the diameter of the nucleus*
- *The uncertainty in the position is $\Delta x \approx 10^{-14}m$*

- *Uncertainty in the momentum of electron confined to the nucleus*
- $\Delta p = \frac{h}{4\pi\Delta x} = \frac{h}{4\pi \times 10^{-14}} = 5.28 \times 10^{-21} \text{ kg ms}^{-1}$
- *Minimum momentum of the electron cannot be lesser than the uncertainty in momentum $p \approx \Delta p$*
- *A simple non relativistic estimation for the energy*
- $E = \frac{p^2}{2m} = \frac{\Delta p^2}{2m} = \frac{1}{2m} \left(\frac{h}{2\Delta x} \right)^2 \approx 96 \text{ MeV}$
- *Relativistic calculations yield energy >20 MeV*
- *Hence electrons cannot be part of the nucleus !*

- *De Broglie hypothesis and Heisenberg's Uncertainty principles ...*
 - *two fundamental ideas which cannot be violated in any theory of quantum systems*

The concepts which true of the uncertainty principle

1. Uncertainty principle is based on the measurement accuracies of equipments used.
2. The position of a particle cannot be determined accurately
3. The momentum of a particle always has an uncertainty which is related to the uncertainty in the position of the particle
4. Electrons cannot be confined to a nucleus as it is energetically not feasible
5. Electrons cannot be confined to a nucleus as it's size is bigger the nuclear diameter



THANK YOU

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