ENGINEERING MATHEMATICS I

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HIGHER ORDER LINEAR DIFFERENTIAL EQUATIONS

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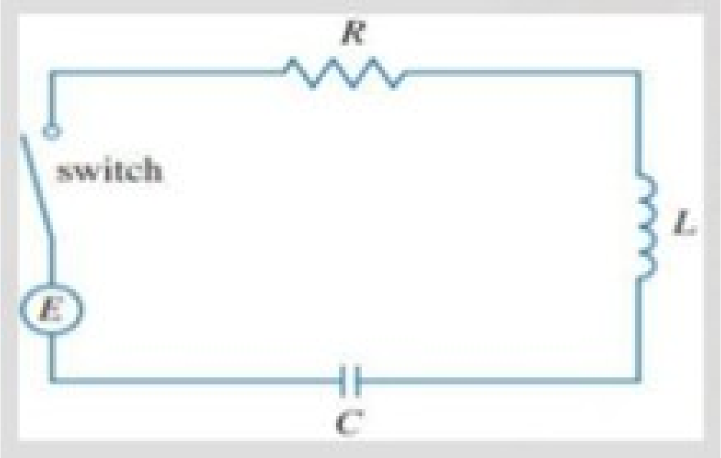
CLASS CONTENT:

- Applications of LDE to electric circuits

RLC Circuit

A simple RLC circuit consists of an inductor of inductance ‘L’ (in henries), a Capacitor of capacitance ‘C’(in farads) and a resistor of resistance ‘R’ (in ohms) and a source of electromotive force E (t) (in volts) are connected in series.

The current ‘I’ through the circuit is measured in amperes and the charge ‘Q’ on the capacitor is measured in coulombs.



The equation for the current I(t) in the RLC circuit is obtained by considering the voltage drops across three components.

The voltage drop across the inductor is given by $E_L = L\frac{dI}{dt}$

The voltage drop across the resistor is given by $E_R = R\,I$

The voltage drop across the capacitor is given by $E_C = \frac{Q}{C}$

From Kirchoff's Voltage Law,

The voltage supplied by the battery = The voltage drop across the circuit components

(the sum of these voltage drops is equal to the electromotive force)

i.e, $E(t) = V_L + V_R + V_c$

$$E(t) = L\frac{dI}{dt} + R\,I + \frac{Q}{C}$$

'I' is the current in the entire circuit and 'Q' is the charge on the capacitor.

We know that, $I = \frac{dQ}{dt}$

$$\boxed{L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E(t)}$$

This is the differential equation which describes LCR – Circuit.

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=E(t)$$

It is a second order linear differential equation with constant coefficients. If the charge and the current are known at zero time, we can get an expression for Q(t) and I(t) .

Example: Find an expression for I(t) in RLC - circuit with R = 8 ohms , L = 2 henry, C = 0.1 farad, which is connected to a source of voltage E(t) = 10 volts , assuming zero charge and zero current when t = 0.

Solution: The differential equation of RLC – circuit is

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=E(t)$$

Using the values of R, L, C and E from the given problem, we get

$$2\frac{d^2Q}{dt^2}+8\frac{dQ}{dt}+\frac{Q}{0.1}=10$$

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$$\Rightarrow \frac{d^2Q}{dt^2}+4\frac{dQ}{dt}+5Q=5$$

The above equation is a non homogeneous LDE of order 2 with constant coefficients.

To find CF :

Auxiliary Equation is

$$m^2+4m+5=0$$

$$\text{The roots are } m=-2\pm i$$

$$CF=e^{-2t}(c_1\cos t+c_2\sin t)$$

To find PI

PI is given by,

$$PI = \frac{5}{D^2 + 4D + 5} = 1$$

$\therefore$ The general solution is $Q(t) = CF + PI$

$$\therefore Q(t) = e^{-2t}(c_1 \cos t + c_2 \sin t) + 1 \quad --------(1)$$

To find c_1 and c_2

Using the given initial conditions, I(0) = 0 and Q(0) = 0 eqn (1)

$$0 = e^{-2.0}(c_1 \cos 0 + c_2 \sin 0) + 1$$

$$0 = 1.(c_1 .1 + c_2 .0) + 1$$

$$c_1 = -1 \text{ - - - - - - - -}(2)$$

To find c_2

differentiate eqn (1) wrt ‘t’ and then use the given initial conditions.

$$\therefore \frac{dQ}{dt} = e^{-2t}(c_1(-\sin t) + c_2(\cos t)) + (-2)e^{-2t}(c_1(\cos t) + c_2(\sin t))$$

Using the value of c_1, we get

$$\therefore \frac{dQ}{dt} = I(t) = e^{-2t}(\sin t + c_2(\cos t)) + (-2)e^{-2t}(-1(\cos t) + c_2(\sin t)) \quad -----(3)$$

Given, I(0) = 0

$$0 = 1.(\sin 0 + c_2(\cos 0)) + (-2)e^{0}(-1(\cos 0) + c_2(\sin 0))$$

$$\Rightarrow \quad c_2 = -2$$

Using the values of c_1 and c_2 in (1), we get

Thus, $Q(t) = e^{-2t}(-\cos t - 2.\sin t) + 1$ and

$$I(t) = 5e^{-2t}\sin t$$

THANK YOU

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