UE25MA141A: ENGINEERING MATHEMATHICS - I

Unit - 2: HIgher Order Ordinary Differential Equations

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Type 3: $X = x^m$

If $X = x^m$, then the particular integral is

$$y_p = \text{P.I} = \frac{1}{f(D)} x^m = \Big[f(D)\Big]^{-1} x^m$$

Expand $\Big[f(D)\Big]^{-1} x^m$ in ascending powers of D using binomial theorem and operate on x^m term by term. Since $(m+1)th$ term and higher derivatives are zero, it is not necessary to consider terms beyond D^m

Solve $(D^2 + 3D + 2)y = x^3 + x^2$

Complementary Function :
The auxillary equation is given by

$$m^2 + 3m + 2 = 0 \implies m - -1, -2$$

Thus C.F. is given by

$$y_c(x) = c_1 e^{-x} + c_2 e^{-2x}$$

Particular Integral :

$$\begin{aligned} y_p &= \frac{1}{D^2 + 3D + 2}(x^3 + x^2) \\ &= \frac{1}{2\left[1 + \frac{(D^2+3D)}{2}\right]}(x^3 + x^2) \end{aligned}$$

Contd.

Expanding the powers of $\left[1+\frac{(D^2+3D)}{2}\right]$, we get

$$y_p = \frac{1}{2}\left[1 - \frac{D^2+3D}{2} + \left(\frac{D^2+3D}{2}\right)^2 - \left(\frac{D^2+3D}{2}\right)^3 + ..\right](x^3+x^2)$$

$$= \frac{1}{2}\left[x^3 + x^2 - \frac{1}{2}(6x+2+9x^2+6x) + \frac{1}{4}(54x+54) - \frac{81}{4}\right]$$

$$= \frac{1}{2}\left[x^3 - \frac{7x^2}{2} + \frac{30x}{4} - \frac{31}{4}\right]$$

Therefore the general solution is

$$y = c_1e^{-x} + c_2e^{-2x} + \frac{1}{2}\left[x^3 - \frac{7x^2}{2} + \frac{30x}{4} - \frac{31}{4}\right]$$

Solve $(2D^2 + 2D + 3)y = x^2 + 2x - 1.$

The auxillary equation is

$$2m^2 + 2m + 3 = 0 \implies m = \frac{-2 \pm \sqrt{5}i}{2}$$

The complementary function is

$$y_c = e^{-\frac{x}{2}} \left(c_1 \cos \frac{\sqrt{5}}{2} x + \sin \frac{\sqrt{5}}{2} x \right)$$

The Particular Integral is

$$\begin{aligned} y_p &= \frac{1}{2D^2 + 2D + 3}(x^2 + 2x - 1) \\ &= \frac{1}{3\left(1 + \frac{2D^2+2D}{3}\right)}(x^2 + 2x - 1) \end{aligned}$$

Expanding the powers of $\left(1+\frac{2D^2+2D}{3}\right)^{-1}$, we get

$$y_p = \frac{1}{3}\Big[1-\left(\frac{2D^2+2D}{3}\right)+\left(\frac{2D^2+2D}{3}\right)^2\Big](x^2+2x-1)$$
$$= \frac{1}{3}\left(x^2+2x-1-\frac{4}{3}-\frac{2}{3}(2x+2)+\frac{8}{9}\right)$$
$$= \frac{1}{3}\left(x^2+\frac{2x}{3}-\frac{25}{9}\right)$$

The general solution is

$$y = e^{-\frac{x}{2}}\left(c_1\cos\frac{\sqrt{5}}{2}x+\sin\frac{\sqrt{5}}{2}x\right)+\frac{1}{3}\left(x^2+\frac{2x}{3}-\frac{25}{9}\right)$$

Type 4 : $X = e^{ax}V$, V being a function of x

When $X = e^{ax}V$, V being a function of x, the particular integral is given by

$$\begin{aligned} y_p = \textbf{P.I.} &= \frac{1}{f(D)} e^{ax} V(x) \\ &= e^{ax} \frac{1}{f(D+a)} V(x) \\ &= e^{ax} [f(D+a)]^{-1} V(x) \end{aligned}$$

where $V(x)$ is of some particular form.

Solution:

Complementary Function: The A.E. equation is given by

$$m^2+2=0 \implies m=\pm\sqrt{2}i$$

The C.F. is given by

$$y_c = c_1\cos\sqrt{2}x + c_2\sin\sqrt{2}x$$

Particular Integral

$$\begin{aligned} y_p &= \frac{1}{D^2+2}\,x^2e^{3x} \\ &= \frac{e^{3x}}{(D+3)^2+2}\,x^2 \\ &= \frac{e^{3x}}{(D^2+6D+11)}\,x^2 \\ &= \frac{e^{3x}}{11(1+\frac{D^2+6D}{11})}\,x^2 \end{aligned}$$

(contd.)

Expanding powers of $\frac{D^2+6D}{11}$, we get

$$y_p = \frac{e^{3x}}{11}\left[1 - \left(\frac{D^2+6D}{11}\right) + \left(\frac{D^2+6D}{11}\right)^2 - \left(\frac{D^2+6D}{11}\right)^3\right]x^2$$

Since x^2 is the highest power, terms containing D^3 and higher order vanishes. Therefore,

$$\begin{aligned} y_p &= \frac{e^{3x}}{11}\left[1 + \left(-\frac{D^2}{11} - \frac{6D}{11}\right) + \left(\frac{36}{11^2}D^2\right)\right]x^2 \\ &= \frac{e^{3x}}{11}\left[x^2 - \frac{12x}{11} + \frac{50}{121}\right] \end{aligned}$$

Hence the general solution is

$$y(x) = y_c + y_p \quad = c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x + \frac{e^{3x}}{11}\left[x^2 - \frac{12x}{11} + \frac{50}{121}\right]$$

Solve $(D^2 - 2D + 1)y = e^x sinx$

Solution:

Complementary Function:
The Auxillary equation is

$$m^2 - 2m + 1 = 0 \implies m = 1, 1$$

The C.F. is

$$y_c(x) = (c_1 + c_2 x)e^x$$

Contd.

The particular Integral is

$$
\begin{aligned}
y_p(x) &= \frac{1}{D^2 - 2D + 1} e^x \sin x \\
&= \frac{1}{(D-1)^2} e^x \sin x \\
&= e^x \frac{1}{(D+1-1)^2} \sin x \\
&= e^x \frac{1}{D^2} sinx \\
&= -e^x \sin x
\end{aligned}
$$

The general solution is

$$
y(x) = (c_1 + c_2 x)e^x - e^x \sin x
$$

THANK YOU