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ENGINEERING PHYSICS

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Week #3 Class #10

- **Concept of observables of the state of a system**
- **Operators in quantum mechanics**
- **Expectation values of observables**

ENGINEERING PHYSICS

Unit I : Review of concepts leading to Quantum Mechanics



➤ *Suggested Reading*

1. *Concepts of Modern Physics, Arthur Beiser, Chapter 5*
2. *Learning Material prepared by the Department of Physics*

➤ *Reference Videos*

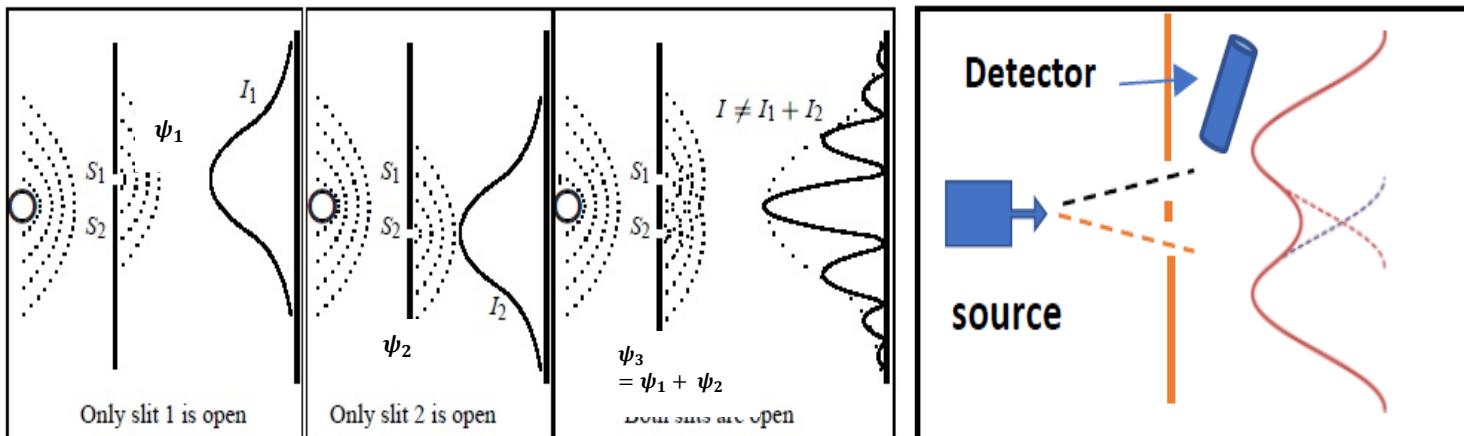
1. *Video lectures : MIT 8.04 Quantum Physics I*

Observables

- *All experimentally measurable parameters of a physical system are observables*
 - *Position*
 - *momentum*
 - *Energy of a state*
 - *life time of electrons*
 - *Spin of a system*
- *Multiple measurements yield average values of the parameters*

Double slit experiment revisited

- *Quantum mechanically observations on physical systems do not yield information about the system*
- *Observations lead to a collapse of the wave function - double slit experiment with an “observer”*
- *Observables in quantum mechanics have to be estimated probabilistically*



Operators

- *Wave functions contain information about the quantum system*
- *Mathematical operators can be used to extract information about the physical state in terms of the observables*

Operators

- A normalized wave function contain information about the quantum system $\psi = Ae^{\frac{i}{\hbar}(px-Et)}$ - eigen function
- A mathematical operator \hat{G} operating on the wavefunction can result in the eigen value G of the observable
- The eigen value equation $\hat{G}\psi = G\psi$

Momentum operator -

- *The partial derivative of ψ with respect to position yields*

$$\frac{\partial \psi}{\partial x} = \left(\frac{ip}{\hbar} \right) \psi$$

$$\left\{ -i\hbar \frac{\partial}{\partial x} \right\} \psi = p \psi$$

- *The momentum operator $\hat{p} = \left\{ -i\hbar \frac{\partial}{\partial x} \right\}$*

Operating on the eigen function yields the momentum eigen value

Kinetic energy operator -

- *The second derivative of ψ with respect to position yields*

$$\frac{\partial^2 \psi}{\partial x^2} = \left(\frac{ip}{\hbar} \right)^2 \psi$$
$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = \frac{p^2}{2m} \psi = KE\psi$$

The kinetic energy operator $\widehat{KE} = \left\{ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \right\}$

*operating on the eigen function yields the eigen value
of the kinetic energy of quantum system*

Total energy operator -

- **The derivative of ψ with respect to time yields**

$$\frac{\partial \psi}{\partial t} = \left(-\frac{iE}{\hbar} \right) \psi$$

$$\left\{ i\hbar \frac{\partial}{\partial t} \right\} \psi = E\psi$$

The total energy operator $\hat{E} = \left\{ i\hbar \frac{\partial}{\partial t} \right\}$

**Operating on the eigen function yields the eigen value
of the total energy of quantum system**

This is also called as the Hamiltonian operator \hat{H}

Operators

Position operator -

- ***The position operator has to be discussed in the momentum space***
- ***The position operator \hat{x} operating on ψ***

$$\hat{x}\psi = x\psi$$

- ***yields the eigen value of position of the quantum system***

Potential energy operator -

- *Potential energy operator is not explicitly described*
- *The eigen value of the potential energy can be inferred as the difference of the total energy and the kinetic energy*
- *The eigen value equation for the potential energy is*

$$\hat{V}\psi = V\psi$$

Expectation values of observables

Quantum mechanics predicts only the most probable values of the observables of a physical system

- the expectation values \equiv the average of repeated measurements on the system.

The eigen value equation for momentum

$$\hat{p}\psi = p\psi$$

The operation $\psi^*\hat{p}\psi = \psi^*p\psi = p\psi^*\psi$

$\psi^*\psi$ is the probability density

$p\psi^*\psi$ should be the probability of the eigen value

Expectation values of observables

The spread in the wave packet can yield a range of $p\psi^*\psi$

Integrated over the range of x for the extend of the wave packet

$$\int \psi^* \hat{p} \psi dx = \int \psi^* p \psi dx = \langle p \rangle \int \psi^* \psi dx$$

$\langle p \rangle$ is the most probable value of the momentum.

Thus the expectation value of the momentum is written as

$$\langle p \rangle = \frac{\int \psi^* \hat{p} \psi dx}{\int \psi^* \psi dx}$$

Expectation values of observables

In general an operator \hat{G} of the observable g

Gives the expectation value of the observable

$$\langle g \rangle = \frac{\int \psi^* \hat{G} \psi dx}{\int \psi^* \psi dx}$$

And in three dimensional space

$$\langle g \rangle = \frac{\int \Psi^* \hat{G} \Psi dV}{\int \Psi^* \Psi dV}$$

Understanding concepts of observables, operators and expectation values

- 1. Observables in quantum mechanics are obtained as results of experiments**
- 2. The total energy operator is $\left\{ i\hbar \frac{\partial}{\partial t} \right\}$**
- 3. The position operator as obtained from the momentum space would be $\left\{ -i\hbar \frac{\partial}{\partial p} \right\}$**
- 4. The total energy operator is Hamiltonian operator**
- 5. Expectation value of position in 1d is $\langle x \rangle = \frac{\int \psi^* \hat{x} \psi dx}{\int \psi^* \psi dx}$**



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THANK YOU

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