ELEMENTS OF ELECTRICAL ENGINEERING
Course Code : UE25EE141A/B

FACULTY CONTRIBUTED:

Department of EEE, RR Campus

Prof . Jyothi T N

Prof. Vadhiraj K P P

Prof. Kruthika N

Prof. Suma S

Prof. Pushpa K R

Prof. Sangeeta Modi

Department of ECE, EC Campus

Prof. Lokesh L

Prof. Dhanashree G Bhate

Dr. Renuka R Kajur

Prof. Rajesh Chandrashekar

Prof. Sangam Kumar G H

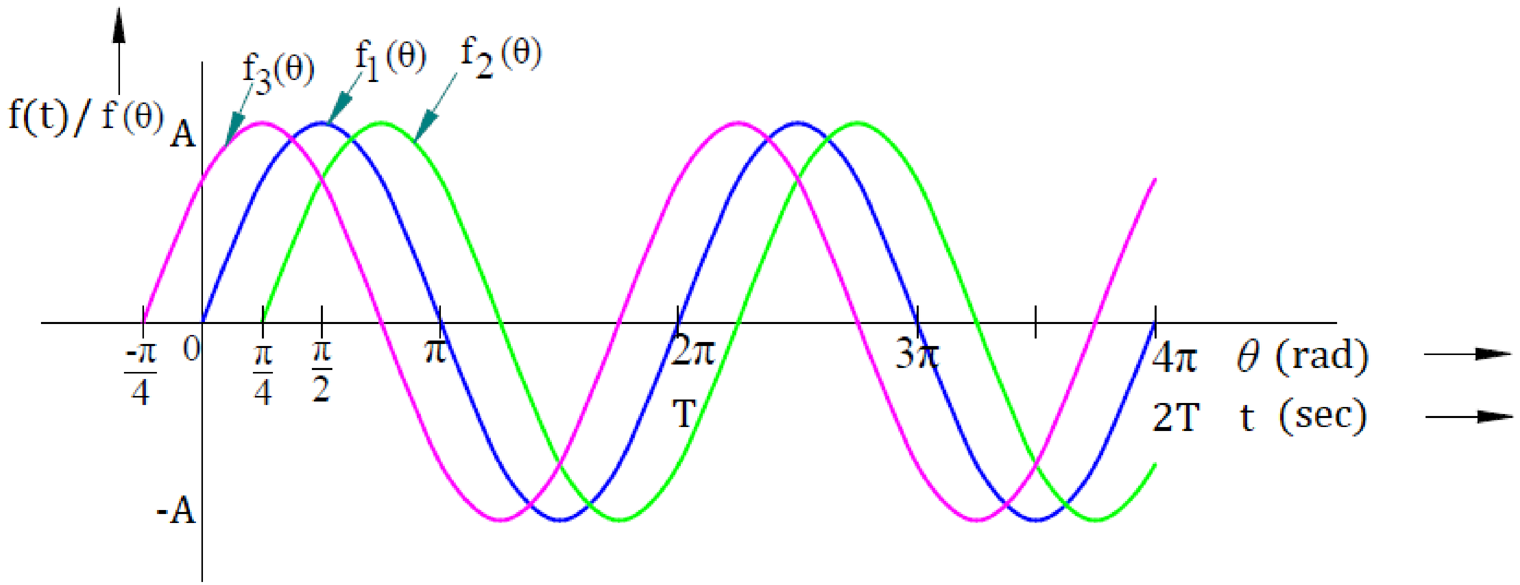
PES UNIVERSITY

ELEMENTS OF ELECTRICAL ENGINEERING (UE25EE141A/B)

Concept of phase lag and phase lead, Concept of phasor and phasor diagram; Mathematical representation of a phasor.

Jyothi T.N
Department of Electrical & Electronics Engineering

Concept of Phase Lag and Phase Lead



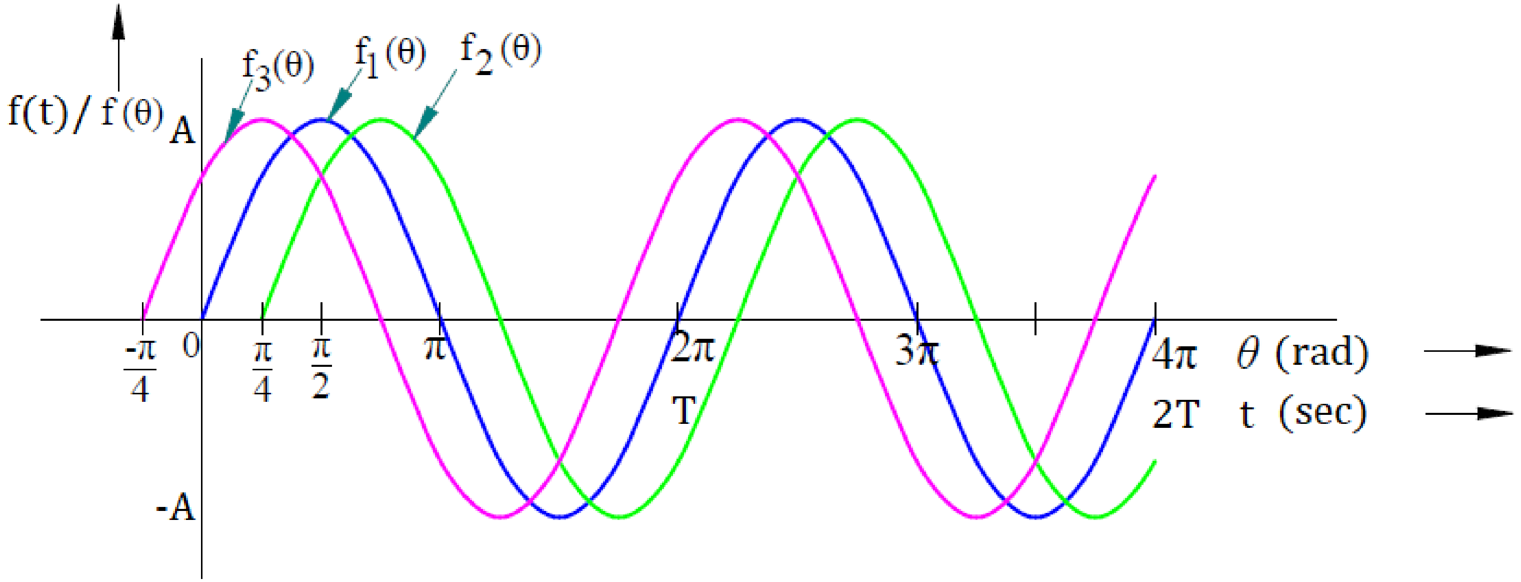
$f_1(t)=A\sin(\omega t)$ represents a reference sine wave.

$f_2(t)=A\sin(\omega t-\frac{\pi}{4})$ lags reference sine wave by $\frac{\pi}{4}$ rad.

$f_3(t)=A\sin(\omega t+\frac{\pi}{4})$ leads reference sine wave by $\frac{\pi}{4}$ rad.

Also, $f_2(t)$ lags $f_3(t)$ by $\frac{\pi}{2}$ rad.

Concept of Phase Lag and Phase Lead



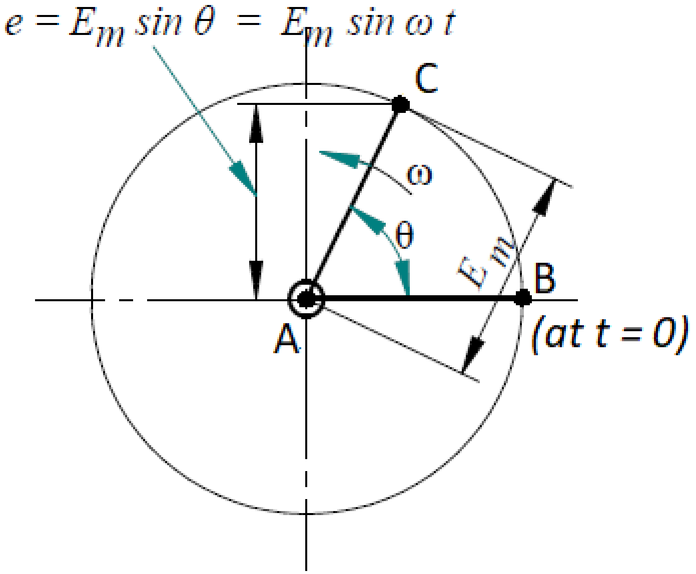
In general, sinusoidal function is represented as

A sin(ωt + ϕ) where ϕ represents the phase angle.

If ϕ is positive, it leads the reference sine wave and lags if ϕ is negative.

Concept of Phasor

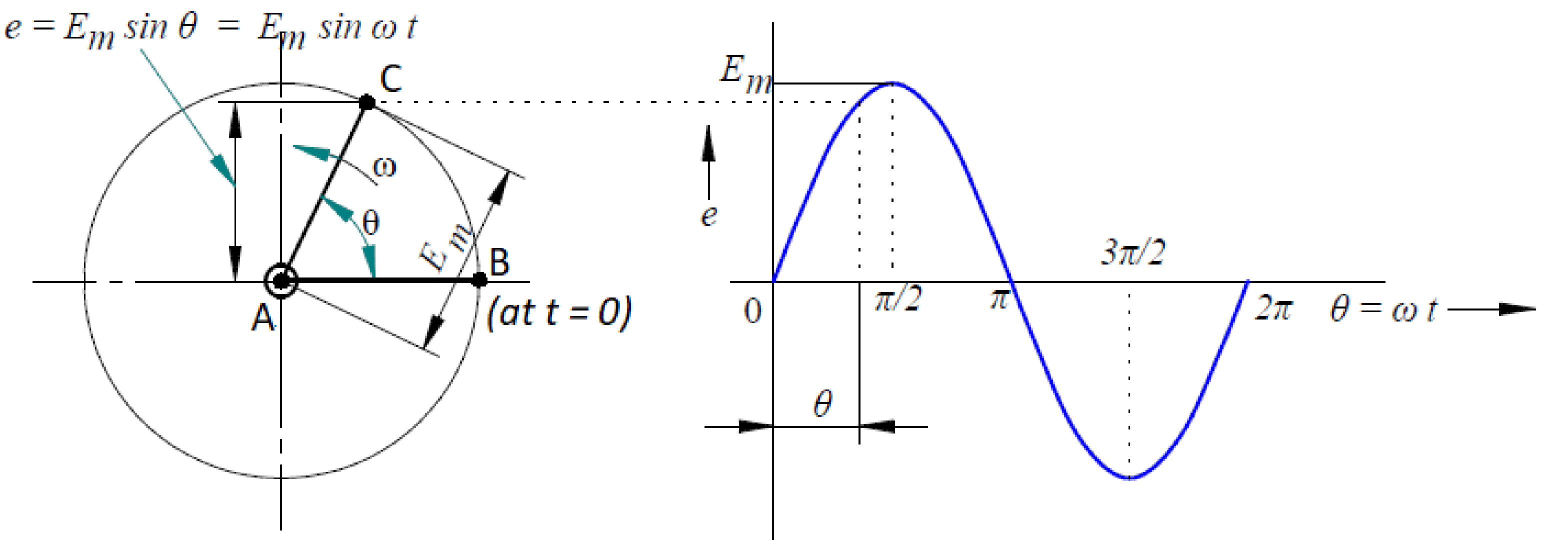
Let us consider a rotating crank of length E_m lying at 0° position at t = 0 and rotating anticlockwise at an angular speed of 'ω' rad/s.



At general time 't', it would be at an angle θ = ωt

Its vertical projection defines a sinusoidal function.

Concept of Phasor



(a) Crank

(b) Sinusoidal waveform

Thus the above rotating crank represents a sinusoidal function of the form $E_m\sin(\omega t)$

Concept of Phasor

Similarly, a sinusoidal function of the form $E_m \sin(\omega t + \phi)$ can be represented by another rotating crank of same length 'E_m' and rotating with same angular speed 'ω' rad/s anticlockwise but lying at an angle 'ϕ' at $t = 0$.

Thus, any sinusoidal function can be represented by a rotating crank and it is called '**Phasor representation**' of a sinusoidal function.

A **Phasor** is a rotating vector which effectively represents a sinusoidal function.

Phasor Diagram

When a number of sinusoidal functions are to be represented as phasors, it is represented using a diagram called **phasor diagram**.

While drawing a phasor diagram, all phasors must be represented corresponding to same point in time. It is usually preferred to represent them at a time t = 0. Then, angular position of each sinusoidal function corresponds to its phase angle.

Note: Only sinusoidal functions of same frequency can be represented together as a phasor diagram. Also, the length of the phasor is its RMS value.

Phasor Diagram – Example

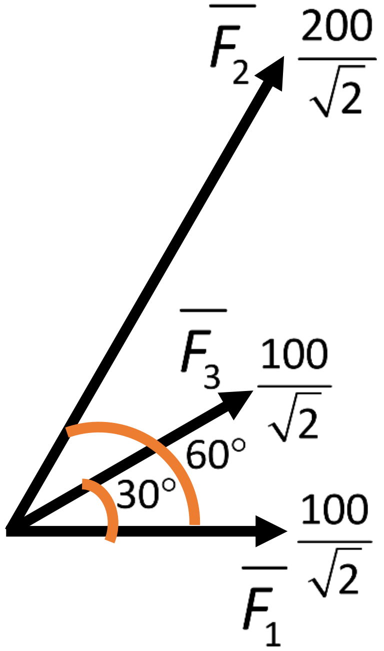
Consider the following sinusoidal functions

i) $f_1(t) = 100\sin(100\pi t)$

ii) $f_2(t) = 200\sin(100\pi t + 60°)$

iii) $f_3(t) = 100\cos(100\pi t - 60°)$

Let us represent them using a phasor diagram.



Note: Convert a cosine function to sine form before representing as a phasor.

For instance,

$f_3(t) = 100\cos(100\pi t - 60°)$

$= 100\sin(100\pi t - 60° + 90°)$

$= 100\sin(100\pi t + 30°)$

Mathematical Representation of a Phasor

A phasor is mathematically represented as

Phasor = Magnitude $\angle$Phase Angle

Where, magnitude is the RMS value.

For instance, Consider these sinusoidal functions

i) $f_1(t) = 100\sin(100\pi t)$ ii) $f_2(t) = 200\sin(100\pi t + 60°)$

iii) $f_3(t) = 100\cos(100\pi t - 60°)$

Let us represent them using phasor representation.

$$f_1(t) = 100\sin(100\pi t) \Rightarrow \overline{F_1} = \frac{100}{\sqrt{2}}\angle 0°$$

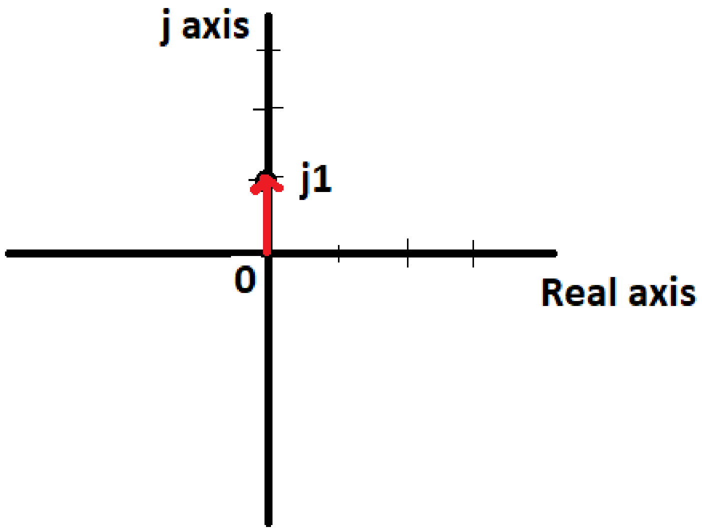
$$f_2(t) = 200\sin(100\pi t + 60°) \Rightarrow \overline{F_2} = \frac{200}{\sqrt{2}}\angle 60°$$

$$f_3(t) = 100\cos(100\pi t - 60°) = 100\sin(100\pi t + 30°)$$

$$\Rightarrow \overline{F_3} = \frac{100}{\sqrt{2}}\angle 30°$$

j operator

‘j’ operator in phasor representation is analogous to ‘i’ operator in complex mathematics.



In rectangular form, j = (0 + j1)

In polar form, $j = 1\angle 90°$

Conversion between the forms

Polar to Rectangular conversion :

Let us consider a polar number $r\angle\theta$

It can be converted to rectangular form (A + jB) using

$A = r\cos\theta$; $B = r\sin\theta$

Rectangular to Polar conversion :

Let us consider a rectangular number (A + jB)

It can be converted to polar form $r\angle\theta$ using

$r = \sqrt{A^2+B^2}$; $\theta = \mathrm{Tan}^{-1}(\frac{B}{A})$

θ will be positive if ‘B’ is positive and it is negative if ‘B’ is negative.

Addition, Subtraction, Multiplication & Division of Phasors

Addition & Subtraction of Phasors:

Addition & subtraction of phasors would be easier in rectangular form.

For instance, let $\overline{F_1}=(A_1+jB_1)$ & $\overline{F_2}=(A_2+jB_2)$

$$\overline{F_1}+\overline{F_2} = (A_1+A_2)+j(B_1+B_2)$$

$$\overline{F_1}-\overline{F_2} = (A_1-A_2)+j(B_1-B_2)$$

Multiplication & Division of Phasors:

Multiplication & Division of phasors would be easier in Polar form.

For instance, let $\overline{F_1} = r_1\angle\theta_1$ & $\overline{F_2} = r_2\angle\theta_2$

$$\overline{F_1} * \overline{F_2} = r_1 * r_2\angle(\theta_1+\theta_2)$$

$$\frac{\overline{F_1}}{\overline{F_2}} = \frac{r_1}{r_2}\angle(\theta_1-\theta_2)$$

j operator properties

'j' operator when multiplied to a phasor, does not change the magnitude of the phasor but rotates the phasor anticlockwise by 90°

For instance, if $\bar{F} = 3\angle 60°$, then $j\bar{F} = 1\angle 90° * 3\angle 60° = 3\angle 150°$

$$j^2 = 1\angle 90° * 1\angle 90° = 1\angle 180° = \cos(180°) + j\sin(180°) = -1$$

Similarly, $j^3 = j^2 * j = -j$

And $j^4 = j^2 * j^2 = 1$

Text Book & References

Text Book:

1. “Basic Electrical Engineering” S.K Bhattacharya, 1st Edition Pearson India Education Services Pvt. Ltd., 2017
2. “Basic Electrical Engineering”, D. C. Kulshreshta, 2nd Edition, McGraw-Hill. 2019
3. “Special Electrical Machines” E G Janardanan, PHI Learning Pvt. Ltd., 2014

Reference Books:

1. “Engineering Circuit Analysis” William Hayt, Jack Kemmerly, Jamie Phillips and Steven Durbin, 10th Edition McGraw Hill, 2023

2. “Electrical and Electronic Technology” E. Hughes (Revised by J. Hiley, K. Brown & I.M Smith), 12th Edition, Pearson Education, 2016.

THANK YOU

Jyothi T.N
Department of Electrical & Electronics Engineering
jyothitn@pes.edu