CL7_Q1. Distinguish between phase and group velocities.

Answer

Phase velocity: The phase velocity of the waves is defined as the velocity of an arbitrary point marked on the wave (the high frequency component) as the wave propagates and is given by

$$v_p = \frac{\omega}{k}$$

Phase velocity refers to the velocity of a monochromatic wave.

Group velocity: The velocity of the wave packet (wave group) is defined as the group velocity and given by $v_g = \frac{d\omega}{dk}$

The group velocity is the velocity with which the entire group of waves would travel.

CL7_Q2. Show that the phase velocity of the de-Broglie waves for a particle is a function of the wavelength?

Answer

If we consider a harmonic wave, the wave has a single wavelength and a single frequency. The velocity of propagation of the wave is given by

$$v_p = \nu\lambda$$

Using, $\nu = \frac{\omega}{2\pi}$ and $\lambda = \frac{2\pi}{k}$ into the above equation, we get

$$v_p = \frac{\omega}{2\pi} \times \frac{2\pi}{k} = \frac{\omega}{k}$$

v_p is called the phase velocity. The velocity with which the plane of equal phase travels through the medium is known as the phase velocity. It thus represents the velocity of propagation of the wave front.

As $E = h\nu$ and $p = \frac{h}{\lambda}$, we get

$$v_p = \frac{E}{h} \times \frac{h}{p} = \frac{E}{p}$$

CL7_Q3. Distinguish between phase velocity and group velocity. Give the relation between them and under what circumstances is the group velocity equal to phase velocity?

Answer

The phase velocity is the velocity of a travelling wave whereas group velocity is the velocity of the wave packet formed due to the superposition of two or more individual waves.

$$v_p = \frac{\omega}{k}$$

$$v_g = \frac{d\omega}{dk}$$

$$v_g = v_p - \lambda \frac{dv_p}{d\lambda}$$

In a non-dispersive medium, all the individual waves travel with the same velocity, therefore $\frac{dv_{ph}}{d\lambda} = 0$ or the phase velocity is a constant with respect to wavelength. Then $v_g = v_p$