ENGINEERING MECHANICS - STATICS

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DISTRIBUTED FORCES

Session- 8

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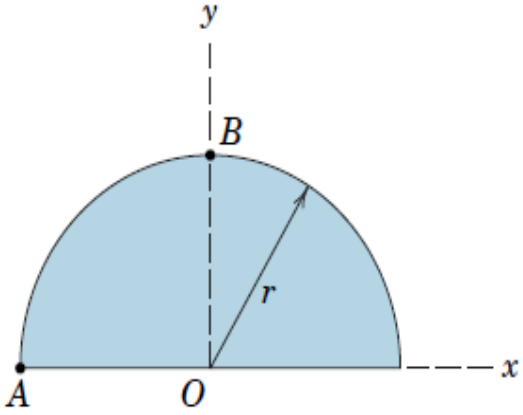
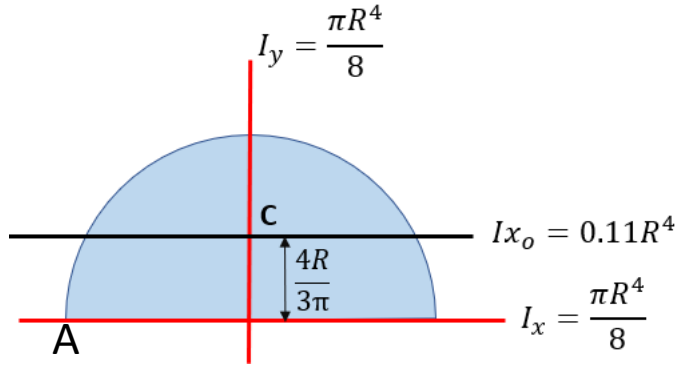
Problem A/6 Determine the polar moments of inertia of the semicircular area about points A and B.



Polar Moment of inertia of the Semicircular area about point "A":

$$MI\ about\ centroidal\ z-axis = \bar{I}_z = \bar{I}_x + \bar{I}_y$$

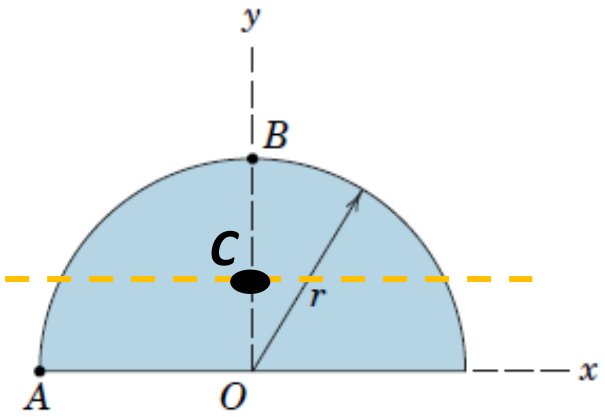
Here, $\bar{I}_x = 0.11r^4,\ \bar{I}_y = \frac{\pi r^4}{8}$



$$(\bar{I}_z) = \mathbf{0.503\ r^4}$$

$$PMI\ about\ Point"A" = \bar{I}_z + Ad^2 \text{ --------(1)}$$

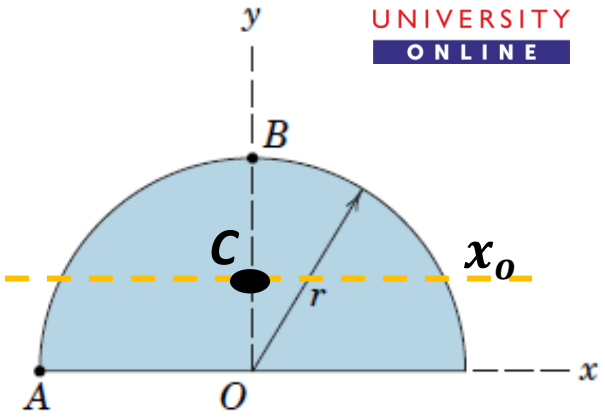
$$d = \sqrt{r^2 + \left(\frac{4r}{3\pi}\right)^2} = 1.086\ r \qquad A = \frac{\pi r^2}{2}$$



Substituting in equation (1)

$$PMI\ about\ Point\text{A} = 0.503\ r^4 + \frac{\pi r^2}{2}(1.086\ r)^2$$

$$\boxed{PMI\ about\ Point\text{A} = 2.355\ r^4}$$



Polar Moment of inertia of the Semicircular area about point "B":

$PMI\ about\ Point\mathrm{B} = \bar{I}_z + Ad^2$ --------(2)

Here,

$\bar{I}_z = 0.503\, r^4$

$A = \frac{\pi r^2}{2}$

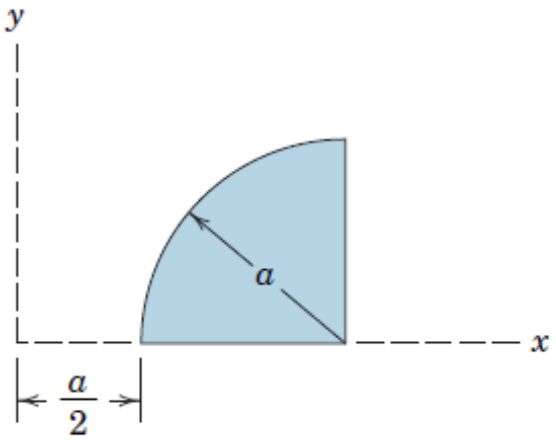
$d = r - \frac{4r}{3\pi} = 0.5755\ \mathrm{r}$

Substituting in equation (2)

$$PMI\ about\ Point\text{"B"} = 0.503\, r^4 + \frac{\pi r^2}{2}(0.5755\, r)^2$$

$$\boxed{PMI\ about\ Point\text{"B"} = 1.023\, r^4}$$

Problem A/7. Determine the moment of inertia of the quarter-circular area about the y-axis.



Solution:

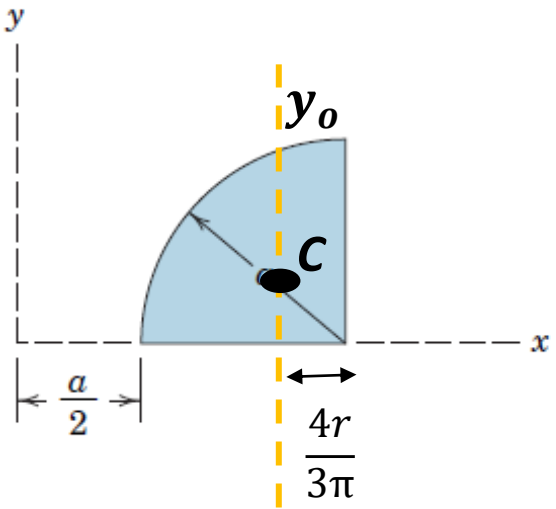
Moment of inertia of the quarter-circular area about the y-axis "I_y" $= \bar{I}_y + Ad^2$ ------(1)

Here

$$\bar{I}_y = 0.055\, r^4 = 0.055\, a^4$$

$$A = \frac{\pi r^2}{4} = \frac{\pi a^2}{4}$$

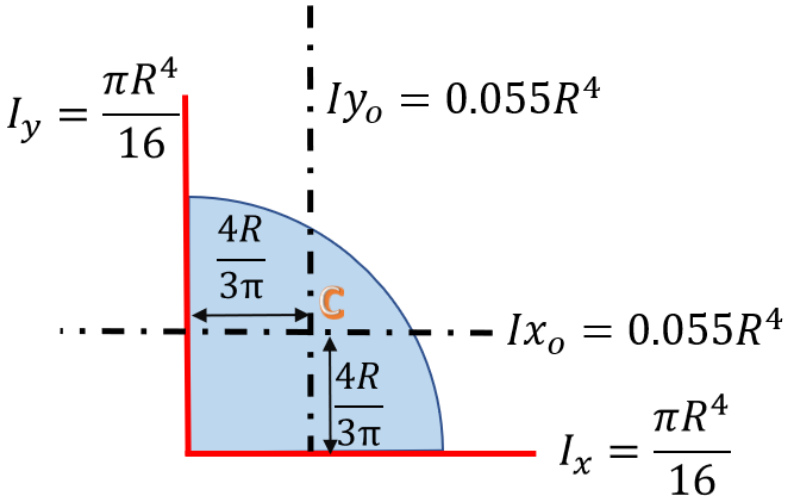
$$d = \frac{a}{2} + \left(a - \frac{4\,a}{3\,\pi}\right) = 1.0755\, a$$



Substituting in equation (1)

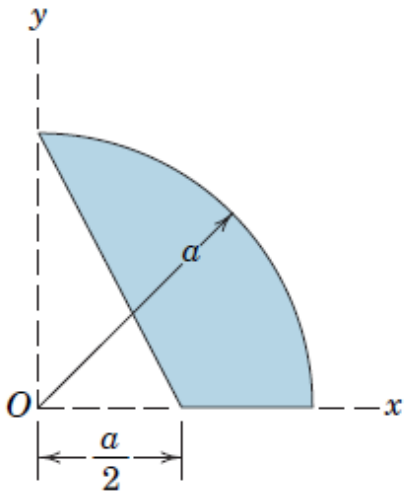
$$I_y = 0.055\, a^4 + \left(\frac{\pi a^2}{4}\right)(1.0755\, a)^2$$

$$\boxed{I_y = 0.9636\, a^4}$$



Moment of Inertia: Numerical

Problem A/19 Determine the moment of inertia of the shaded area about the x-axis.



Moment of inertia of the shaded area about the x-axis:

$$I_x = \begin{pmatrix} MI\ of\ a\ quarter\ circular \\ lamina\ about\ x-Axis \end{pmatrix} - \begin{pmatrix} MI\ of\ a\ triangle\ about \\ x-Axis \end{pmatrix}$$

$$I_x = \left(\frac{\pi r^4}{16}\right) - \left(\frac{bh^3}{12}\right)$$

Here,

r = a, b= a/2 & h = a

Substituting in the above equation

$$I_x = \left(\frac{\pi a^4}{16}\right) - \left(\frac{\left(\frac{a}{2}\right)(a)^3}{12}\right) = 0.154a^4$$

$$\boxed{I_x = 0.154a^4}$$

THANK YOU

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