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## 1. Introduction to Magnetic Materials

Every material when placed in an external magnetic field  $H$  responds in different magnitudes and different ways. Some materials get magnetized either temporarily or permanently, others do not get magnetized but repulse the external magnetic fields. Materials are classified according to the magnetization state of the material.

The intensity of magnetization ( $M$ ) of a material is defined as the dipole moment per unit volume of the material.

$M = \frac{m \cdot 2l}{2l \cdot a} = \frac{m}{a}$  which is the dipole strength per unit area of cross section.

The magnetization of the material  $M = \chi_m H$

where the magnetic susceptibility  $\chi_m$  of the material is the ratio of the magnetization  $M$  to the magnetic field  $H$  i.e.,  $\chi_m = \frac{M}{H}$

Consider a long solenoid of length  $l$  with air as the core and having  $n$  turns. The magnetic field strength when a current  $I$  flows through the solenoid  $H = \frac{nI}{l}$  A/m.

The magnetic flux density  $B_0 = \mu_0 H$  Tesla ( $\text{Wb/m}^2$ )

Where  $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$  or  $(4\pi \times 10^{-7} \text{ TmA}^{-1})$

When a material of susceptibility  $\chi_m$  is introduced inside the solenoid the total magnetic flux of the system is the sum of the flux due to the coil and the flux due to the magnetization of the material.

$$B = \mu_0 H + \mu_0 M = \mu_0 (H + M) = \mu_0 (H + \chi_m H) = \mu_0 (1 + \chi_m) H$$

Defining the relative permeability of the material  $\mu_r = (1 + \chi_m)$

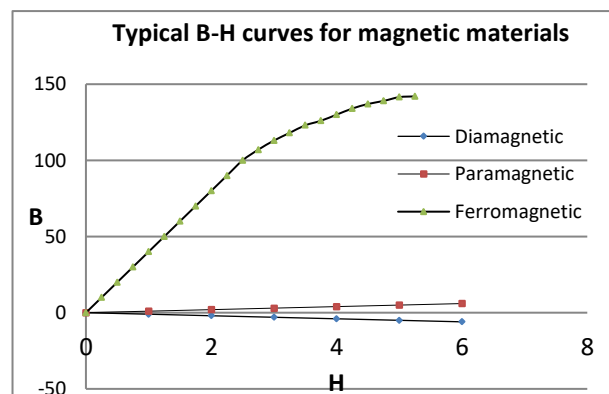
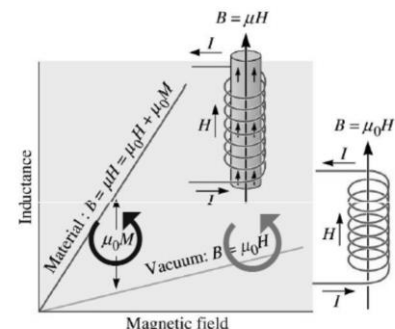
The magnetic flux density  $B = \mu_0 \mu_r H = \mu H$ , where  $\mu = \mu_0 \mu_r$  is the permeability of the material.

Magnetic materials can be classified on their susceptibility to be magnetized by an external magnetic field and on their relative permeability.

Materials with susceptibilities less than 0 ( $\chi_m$  negative) are classified as diamagnetic materials. They tend to repel the lines of force of the external magnetic field. The diamagnetic susceptibilities are in the range of  $-10^{-3}$  to  $-10^{-6}$ . Diamagnetic materials possess a relative permeability less than 1.

*The first truly scientific study of magnetism was made by the Englishman William Gilbert (1540–1603), who published his classic book *On the Magnet* in 1600. He experimented with lodestones (magnetite) and iron magnets, formed a clear picture of the Earth's magnetic field, and cleared away many superstitions that had clouded the subject. From his experiments, he concluded that the Earth was itself magnetic and that this was the reason compasses pointed north (previously, some believed that it was the pole star (Polaris) or a large magnetic island on the north pole that attracted the compass).*

*The existence of isolated magnetic poles, or monopoles, is not forbidden by any known law of nature, and serious efforts to find monopoles have been made [P. A. M. Dirac, *Proc. R. Soc. Lond.*, A133 (1931) p. 60; H. Jeon and M. J. Longo, *Phys. Rev. Lett.*, 75 (1995) pp. 1443–1446]. The search has not so far been successful.*



Materials with a small positive susceptibility  $>0$  ( $\chi_m$  positive) are classified as paramagnetic materials. Paramagnetic materials tend to show feeble magnetization in the presence of an external field. The susceptibility of the material is of the order of  $10^{-4}$  to  $10^{-5}$  and relative permeability is slightly more than 1.

Materials with a large positive susceptibility  $\gg 0$  ( $\chi_m$  positive and large) are classified as ferromagnetic materials. Ferromagnetic materials are easily magnetized in the presence of an external magnetic field and exhibit a large relative permeability of the order of  $10^5$ - $10^6$ .

## 2. Origin of Magnetism in materials

Any current loop creates a magnetic field with the direction of the current deciding the direction of the field. It is imperative that the atomic level currents due to the electrons in the orbits could result in magnetic fields. The spin of the electrons can also be one of the major contributors to the magnetization in materials. In addition, there is also a feeble contribution from the nuclei as well which could be neglected for all practical evaluations. The net contribution from an atom to the magnetization would depend on the magnitudes of the individual components and the statistical behavior of the collection of atoms would decide the magnetic susceptibility of the material.

The basic atomic model of orbiting electrons around the nucleus constitutes tiny current loops at atomic levels. If the electrons are assumed to move in an orbit of radius  $r$  with a constant speed  $v$ , the period of rotation  $T$ , then the current  $I = \frac{e}{T} = \frac{e\omega}{2\pi} = \frac{ev}{2\pi r}$

The magnetic moment associated with the electron's moment in the orbit

$$\mu_e = I \cdot A = I \cdot \pi r^2 = \frac{ev}{2\pi r} \pi r^2 = \frac{1}{2} evr = \frac{1}{2} e\omega r^2$$

The angular momentum of the orbiting electron  $L = m_e v r$  and the magnetic moment can be written as

$$\mu_{orb} = \left( \frac{e}{2m_e} \right) L$$

Thus the magnetic moment of the electron is proportional to its angular momentum. The ratio of the magnetic moment to the angular momentum  $\gamma = \frac{\mu_{orb}}{L} = \left( \frac{e}{2m_e} \right)$  is known as the gyro magnetic ratio. Due to the negative charge of the electron, the vectors  $\vec{\mu}$  and  $\vec{L}$  point in opposite directions and perpendicular to the plane of the orbit. The angular momentum of the electron is a multiple of  $\hbar$  (quantum mechanical concepts).

The magnetic effect of the orbital motion of the electron is either zero or very small because of the cancellation of the magnetic moments of electrons orbiting in opposite directions.

Apart from the orbital magnetic moments, electrons have the property of spin and a spin angular momentum  $\vec{S}$  associated with it.  $\vec{S}$  is of the same order as the angular momentum  $\vec{L}$ .

The magnetic moment due to the spin of the electrons is almost twice that of the orbital movement and the contribution from the spin to the magnetic moment can be written as  $\mu_{spin} = \sqrt{s(s+1)} \frac{e\hbar}{m}$ .

The total magnetic moment of an atom is the vector sum of orbital and spins magnetic moments.

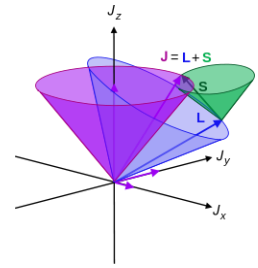
The net magnetic moment due to electrons

$\mu_e = g_e \frac{e\hbar}{2m}$  where  $g_e$  is the Lande  $g$  factor (spectroscopic splitting factor of the energy levels) given by  $g_e = 1 + \frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)}$

From this the smallest non zero value of the spin magnetic moment due to electrons (when  $s=j$  and  $l=0$ ) is obtained as  $\mu_s = \frac{e\hbar}{m}$

This is twice the orbital magnetic moment and hence determines the susceptibility of materials. The magnetic moment being quantized; the smallest unit of magnetic moment  $\mu_B = \frac{e\hbar}{2m} = 9.27 \times 10^{-24} \text{ J/T}$  is called the Bohr magneton.

The magnetic moments of protons and neutrons are much smaller than the electron magnetic moment and are usually neglected.

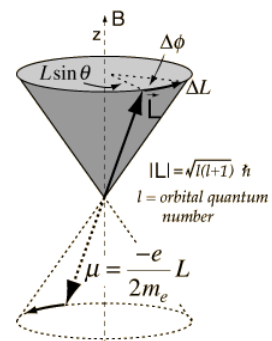


### 3. Larmor precession

A magnetic moment directed at some finite angle with respect to the applied magnetic field direction, experiences a torque  $\mu \times B$ . This causes  $\mu$  to precess (rotate) about the magnetic field direction. (This is analogous to the precession of a spinning top around the gravity field.)

For static magnetic moments or classical current loops, the torque tends to line up the magnetic moments with the magnetic field  $B$ , so this represents its lowest energy configuration.

In the case of an electron in orbit around a nucleus, the magnetic moment is proportional to the angular momentum of the electron. The torque exerted the external field produces a change in angular momentum which is perpendicular to that angular momentum, causing the magnetic moment to precess around the direction of the magnetic field rather than settle down in the direction of the magnetic field. This is called Larmor precession.



When a torque is exerted perpendicular to the angular momentum  $L$ , it produces a change in angular momentum  $\Delta L$  which is perpendicular to  $L$ , causing it to precess about the  $z$  axis. The effect of the torque as follows:

$$\tau = \mu \times B = |\mu B \sin \theta| = \frac{\Delta L}{\Delta t} = \frac{L \sin \theta \Delta \phi}{\Delta t} = L \sin \theta \omega_L$$

The precession angular velocity (Larmor frequency) is given by  $\frac{d\phi}{dt} = \omega_L = \frac{e}{2m_e} B$  since  $\frac{\mu}{L} = \left(\frac{e}{2m_e}\right)$ .

The Larmor frequency forms the basis for magnetic resonance.

The Larmor precession frequency for electrons is 14GHz/ T which forms the basis of the electron spin resonance.

The Larmor precession frequency for protons is 7.8 MHz/ T which forms the basis of the Nuclear magnetic resonance.

Thus, the magnetic moment due to a precessing charge can be evaluated as

$\mu_{\text{ind}} = \frac{1}{2} e \omega r^2 = \frac{Be^2 r^2}{4m}$  where  $r$  is the radius of the precessing orbit. The sign of this induced magnetic moment is negative by Lenz's law ( it opposes the magnetic field that induced the precession).

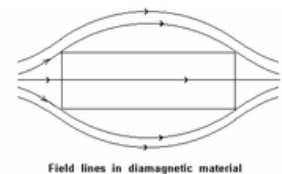
If there are  $N_a$  number of atoms per unit volume and each atom contains  $Z$  electrons we can estimate the total induced magnetisation as  $M = N_a Z \mu_{ind} = -\frac{N_a Z e^2 r^2 B}{4m} = -\frac{N_a Z e^2 r^2 \mu_0 H}{4m}$

Hence the induced susceptibility  $\chi_m = \frac{M}{H} = -\frac{N_a Z e^2 r^2 \mu_0}{4m}$ .

This will yield an extremely small negative value of the order of  $10^{-5}$  for most materials and is recognized as the diamagnetic response of all materials.

#### 4. Diamagnetic materials

Diamagnetism is a property due to which certain materials are repelled by both poles of a magnet. When a short rod of diamagnetic material is placed in a magnetic field, it aligns itself at right angles to or across the direction of the magnetic lines of force. The meaning of term “diamagnetic” in Greek is “across magnetic”.



The orbital motion of electrons is equivalent to tiny atomic current loops, producing magnetic fields. In most materials, the magnetic fields of the orbiting electrons balance each other and add up to zero in the absence of an external field. However, in the presence of an external magnetic field, the interaction of external field with the electrons induces an internal field which tends to align the current loops in a direction opposite to the applied field.

This effect on atomic scale is similar to that due to Lenz's law induced magnetic fields tend to oppose the change which created them. If magnetic response is only due to these phenomena then the material is classified as diamagnetic. Diamagnetism is exhibited by a substance only in the presence of an externally applied magnetic field.

Atoms with closed electronic shells and sub shells are generally diamagnetic ( $-ve \chi_m$  and  $\mu_r < 1$ ). The classical diamagnetic susceptibility is given by

$$\chi_{dia} = -\frac{Ne^2\mu_0}{6m}\langle r^2 \rangle$$

where  $N$  is the number of atoms per unit volume and  $\langle r^2 \rangle$  is the average value of the square of the radius of the current loops. From the above it is observed that diamagnetic susceptibility is temperature independent.

All materials have inherent diamagnetism. Diamagnetism is detectable and observable only with sensitive instruments and in the presence of powerful magnets. Examples of diamagnetic substances include water, wood, sodium chloride, most organic compounds such as petroleum, some plastics, and many metals including copper, particularly the heavy metals with many core electrons, such as mercury, gold, bismuth and antimony. Most substances usually considered as nonmagnetic may be strictly called as diamagnetic.

The volume magnetic susceptibility of water is  $= -9.05 \times 10^{-6}$

Bismuth the most strongly diamagnetic material has  $= -1.66 \times 10^{-4}$ .

However, superconductors show strong diamagnetic effect (and offer no resistance to the formation of the current loops and conduction of electricity). Lines of magnetic flux are deflected away from the material due to diamagnetism and are expelled completely. These materials can be used in creating strong repulsive magnetic fields and are used in levitation experiments / applications.

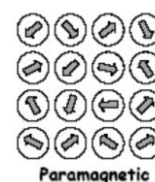
## 5. Para magnetism

Paramagnetic materials are weakly attracted by magnetic fields. When a small rod of paramagnetic material is placed in a magnetic field, it aligns with or alongside the lines of force. The term paramagnetic is derived from a Greek word meaning “alongside magnetic”.

Para magnetism is generally a characteristic of materials with partially filled 3d shells, where the Hund’s rule restricts the anti-parallel alignment resulting in the net spin magnetic moment for the atoms.

*Pierre Curie (1859–1906) was a French physicist who worked extensively on magnetic materials. He and his wife, Marie (Sklodowska) Curie (1867–1934), later became famous for their research on radioactivity.*

Materials with unpaired electron spins in the atomic / molecular orbitals have permanent magnetic dipoles even in the absence of an applied field. These dipoles are randomly oriented and do not interact with one another. The thermal energy of the system being greater than the magnetic energy  $k_B T > \mu \cdot B$ , the thermal agitation result in random ordering and a net zero magnetic moment.



In the presence of an external magnetic field these dipoles tend to align to the applied direction leading to a net magnetic moment in the direction of the applied field. The induced magnetization is weak and linearly dependent on the applied magnetic field. When the magnetic field is removed, the dipoles relax back to their normal random orientation and the magnetization becomes zero. Aluminium, Oxygen titanium, FeO are some examples of materials which are attracted by a magnetic field.

## 6. Quantum theory of Paramagnetic materials (solids).

Some of the features of the quantum theory of Para magnetism are:

The magnetic dipoles of a molecule / atom /ion in a material can be written as

$$\mu = g\mu_B m_j$$

where  $g_e$  is the Lande g factor and  $\mu_B$  is the Bohr magneton. In an external magnetic field B the dipoles are allowed only certain fixed orientations of the total angular momentum J and described by  $M_j$ .

For a given J (the total angular momentum) the allowed orientations are  $2j+1$  ( $j, j-1, j-2, \dots, 0, \dots, -j$ )

and the energy of interaction with an external magnetic field B is given by

$$E_j = \mu \cdot B = g\mu_B m_j \cdot B = g\mu_o \mu_B H m_j$$

For a material with N dipoles per unit volume, the net magnetization is the statistical average of the allowed  $\mu_j$  orientations and is given by

$$M = N \left[ \frac{\sum_{-j}^{+j} m_j g \mu_B \exp\left(\frac{m_j g \mu_o \mu_B H}{kT}\right)}{\sum \exp\left(\frac{m_j g \mu_o \mu_B H}{kT}\right)} \right] \quad \text{eqn. 1.0}$$

Clearly the magnetization will depend on the value of the exponent  $\left(\frac{m_j g \mu_o \mu_B H}{kT}\right)$ .

**Case 1:** If  $m_j g \mu_o \mu_B H \ll kT$  i.e. the thermal energy is significantly higher (high temperatures) than the magnetic interaction energy with field, the ratio will be much less than 1.

Expanding the exponential terms as a power series and simplifying the terms the net magnetization

$$M = Ng\mu_B \cdot \frac{g\mu_B H}{KT} \frac{j(j+1)}{3} = \frac{Ng^2\mu_o\mu_B^2 H}{KT} \frac{j(j+1)}{3}$$

The magnetic susceptibility  $\chi = \frac{M}{H} = \frac{Ng^2\mu_o\mu_B^2}{KT} \frac{j(j+1)}{3} = \frac{C}{T}$  which is the classical Curies law of paramagnetic

$$\left\{ \text{Simplification of the expression } \left[ \frac{\sum_{-j}^{+j} m_j g\mu_B \exp\left(\frac{m_j g\mu_o\mu_B H}{kT}\right)}{\sum \exp\left(\frac{m_j g\mu_o\mu_B H}{kT}\right)} \right] \right\}$$

Let  $\exp\left(\frac{m_j g\mu_o\mu_B H}{kT}\right) = \exp^{m_j x}$ , then the expression simplifies to  $\frac{\sum_{-j}^{+j} m_j \exp^{m_j x}}{\sum \exp^{m_j x}}$

Since the exponent is small we can expand the exponential term as a series (summation neglecting the higher order term

in  $x$ ,  $\frac{\sum_{-j}^{+j} m_j(1+m_j x)}{\sum(1+m_j x)} = \frac{\sum_{-j}^{+j} m_j + \sum_{-j}^{+j} m_j^2 x}{\sum_{-j}^{+j} 1 + \sum_{-j}^{+j} m_j x}$

$m_j$  can take values from  $-j$  to  $j$  including 0 and  $x$  is independent of  $j$  and hence

$$\sum_{-j}^{+j} m_j = j + (j-1) + \dots + 1 + 0 - 1 - 2 \dots - (j-1) - j = 0$$

$$\sum_{-j}^{+j} m_j^2 = 2 \times [1^2 + 2^2 + \dots + (j-1)^2 + (j)^2] = 2 \times \frac{j(j+1)(2j+1)}{6} = \frac{j(j+1)(2j+1)}{3}$$

$$\sum_{-j}^{+j} 1 = 2j + 1$$

Hence  $\frac{\sum_{-j}^{+j} m_j(1+m_j x)}{\sum(1+m_j x)} = \frac{0 + \frac{j(j+1)(2j+1)}{3} x}{(2j+1) + 0} = x \frac{j(j+1)}{3} \}$

materials where the Curie constant  $C = \frac{Ng^2\mu_o\mu_B^2}{K} \frac{j(j+1)}{3}$ .

The paramagnetic susceptibility is thus inversely proportional to the absolute temperature.

Curie's law is obeyed under generally prevailing conditions of low magnetization ( $\mu H < k_B T$ ), i.e., when only a relatively small fraction of atomic dipoles are aligned, with the magnetic field. Curie's law indicates that the Magnetization  $M$  depends on the applied field  $H$  (the stronger magnetic field aligns more dipoles) and the susceptibility  $\chi$ . However,  $\chi$  is inversely proportional to the temperature i.e., the magnetization decreases if the temperature is increased (increased thermal agitation prevents alignment of dipoles).

**Case 2:** If  $m_j g\mu_o\mu_B H \gg KT$  i.e. the magnetic interaction energy is much greater than the thermal energy the exponent will be greater than 1.

Equation 1 can be written as

$$M = Ng\mu_B \left[ \frac{\sum_{-j}^{+j} m_j \exp\left(\frac{m_j g\mu_o\mu_B H}{kT}\right)}{\exp\left(\frac{m_j g\mu_o\mu_B H}{kT}\right)} \right] = Ng\mu_B \frac{\sum_{-j}^{+j} m_j \exp^{m_j x}}{\sum \exp^{m_j x}} \quad \text{where } x = \frac{g\mu_o\mu_B H}{kT}$$

This can be simplified by taking  $a = xj$  to

$$M = Ng\mu_B j \left[ \frac{2j+1}{2j} \coth\left(\frac{2j+1}{2j} a\right) - \frac{1}{2j} \coth\left(\frac{a}{2j}\right) \right] = Ng\mu_B j \cdot B_j(a)$$

where  $B_j(a) = \left[ \frac{2j+1}{2j} \coth\left(\frac{2j+1}{2j}a\right) - \frac{1}{2j} \coth\left(\frac{a}{2j}\right) \right]$  is the Brillouin function which describes the probability of all the dipoles to orienting in the direction of the external field.

When the dipoles are aligned completely in the direction of the external field, the magnetization of the material is saturated, and the saturation magnetization is given by  $M_s = Ng\mu_B j$ .

Hence the magnetization can be written as  $M = M_s B_j(a)$ .

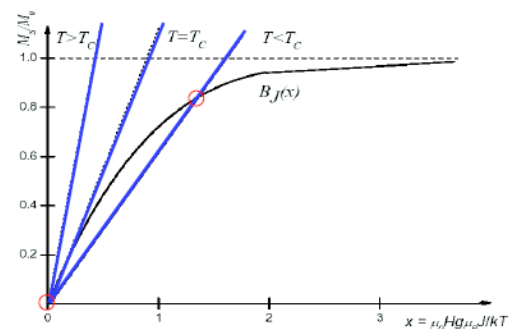
When  $j = \frac{1}{2}$  the Brillouin function reduces to  $\tanh(a)$ .

In the limit of large  $j$  the Brillouin function converges to the classical Langevin's function  $L(a) = \coth(a) - \frac{1}{a}$ .

[  $L(a)$  can be written as a series expansion in  $a$  as  $\frac{a}{3} - \frac{a^3}{45} + \frac{2a^5}{945} - \dots$ . For small values of  $a$ , the function then converges to  $\frac{a}{3}$ . ]

For small values of  $a$  (large values of  $T$ ) the magnetization reduces to  $M = M_s \frac{a}{3}$  and the susceptibility is given by  $\chi = \frac{Ng^2 \mu_B^2}{3KT}$  which is similar to the Curies theory of Paramagnetism.

The variation of  $\frac{M}{M_s}$  is represented in the figure for three different cases of the temperature (as compared to  $T_c$  the Curie temperature of the material)



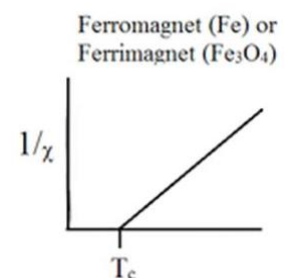
Examples of paramagnetic materials at room temperature include Aluminium (Al), Manganese (Mn), platinum (Pt), Oxygen (gas and liquid), and rare earth ions Ce, Nd.

## 7. Weiss Molecular field concept.

The variation of  $\frac{1}{\chi}$  vs  $T$  is expected to be a straight line passing through the origin. However, some materials show a variation in this behaviour with the straight line showing a positive intercept  $T_c$  on the temperature axis.

Weiss in 1907 showed that this could be due to a molecular field which is the field at any point due to the neighbouring dipoles. If  $M$  is the magnetization in the material, then the magnetic field at any point is proportional to  $M$  and hence the field at any point in the material could have an additional contribution  $= \lambda M$ . This field has the tendency to align the dipoles in the direction of the field and hence would be a case of spontaneous magnetisation. Therefore, the Curies law is modified as  $\frac{M}{H + \lambda M} = \frac{C}{T}$  which gives us  $\chi = \frac{C}{T - T_c}$  where  $T_c = \lambda C$  is the Curie temperature above which the materials show the paramagnetic behaviour. Below  $T_c$  the material behaves as a ferromagnetic material. However, most of the common materials have a low  $T_c$  and hence at normal temperatures does not show the spontaneous magnetization.

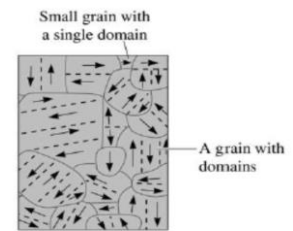
*Pierre Weiss (1865–1940), French physicist deserves to be called the “Father of Modern Magnetism” because almost the whole theory of ferromagnetism is due to him, and his ideas also permeate the theory of ferrimagnetism. Most of his work was done at the University of Strasbourg.*





## 8. Ferromagnetism

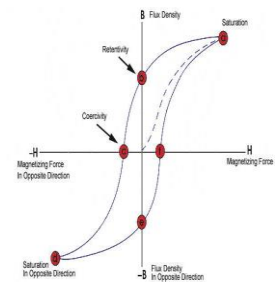
Ferromagnetism is the property by which certain materials are attracted by a magnet and form permanent magnets. Ferromagnetic materials have very high magnetic susceptibilities, ranging from 1000 up to 100,000. Ferromagnetic materials exhibit spontaneous magnetization and exhibit sharp hysteresis characteristics in the magnetisation versus magnetising field.



Ferromagnetism can be considered as a case of a Paramagnetic material with a high molecular field constant  $\lambda$  and a very high  $T_c$ .

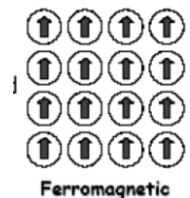
The local molecular field can result in a high ordering of spins in certain materials like Fe, Co, Ni etc, result in the materials being easily magnetised).

The presence of unpaired spins in the 3d shells of these materials gives a large dipole moment for the atom. (Cobalt has a  $T_c > 1000K$ , Fe has a  $T_c > 750K$  and Ni has a  $T_c > 350K$ )



In Ferromagnetic materials when an external magnetic field is applied the magnetic moments are aligned in the direction of the applied field and domains grow at the expense of their neighbours. Microscopic regions in which large numbers of dipoles are aligned are called magnetic domains. The unpaired electron spins to line up parallel with each other. This quantum mechanical interaction at the atomic level gives rise to long range order and creates magnetic domains. The alignment directions of the individual regions are random throughout the material. Hence the bulk of the material is usually unmagnetized or weakly magnetized in the absence of magnetic field.

The magnetisation of the material in the presence of an external magnetic field tends to exhibit the classic M-H hysteresis. When the external field is strong and all the spins are aligned the magnetisation shows a saturation value. When the external field is removed, sizable number of dipoles still maintains the alignment resulting in a net magnetization which is the retentivity of the material. Thus, the material gets permanently magnetized. This remnant magnetisation can be removed if a coercive field  $H_c$  is applied in the reverse direction. Since the material exhibits hysteresis, it is obvious that it has a memory of the previous experience of external fields and hence can be used as memory materials. The flipping of the magnetisation in opposite directions is achieved by applying a saturation magnetisation field  $H_s$  in the appropriate direction.



$$T < T_c, H = 0, M \neq 0$$

All ferromagnets have a maximum temperature at which the thermal motion of the dipoles becomes violent, the alignment of dipole moments and spontaneous magnetization cannot be maintained. Phase change from ferromagnetic to paramagnetic behaviour with the usual weak magnetization occurs. This critical temperature is called as the Curie temperature. The magnetic susceptibility above the Curie temperature is given by  $\chi = \frac{C}{T - \theta}$  where  $\theta$  is the transition temperature constant.

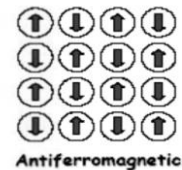
Ferromagnetic materials can be characterized by their Curie temperature in addition to their Permeability, Coercive field and Remnant Magnetization.

Apart from iron, other materials exhibiting ferromagnetic property are nickel, cobalt, some of the rare earths gadolinium, dysprosium, samarium and neodymium in alloys with cobalt. They have important

applications in fabrication of magnets, electromagnets, transformers; magnetic storage media e.g. tape recording, computer hard disks etc.

### 9. Anti-ferro magnetic materials

The magnetisation of a material also depends on the type of ordering that exist in some of the ordered materials. In the case of ferro magnetic materials it is seen that there is a long-range ordering of the magnetic dipoles that lead to strong magnetisation.

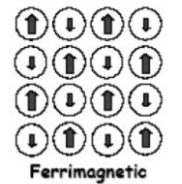


$$T < T_c, H = 0, M = 0$$

The class of anti-ferro magnetic materials such as MnO, NiO, CoO etc in which the electron spins associated with the atoms at different crystallographic sites are ordered such that the net magnetisation of the material is zero below a certain temperature called as the Neel temperature  $T_N$ . Above the Neel temperature the materials behave as paramagnetic with the magnetic susceptibility inversely proportional to temperature.

### 10. Ferri magnetic materials

Ferri magnetic materials are a class of ordered structures in which the magnetic moments at crystal sites are anti parallel and unequal. These materials are treated as two sub lattices of a crystal with different magnetic moments and anti parallel alignments. Generally these materials contain cations of two or more types with different magnetic moments and hence show a net magnetisation not equal to zero.



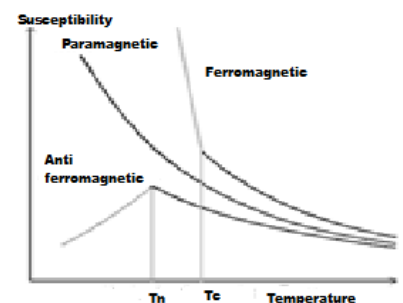
Examples of such materials are  $\text{NiFe}_2\text{O}_4$ ,  $\text{CoFe}_3\text{O}_4$ , and  $\text{BaFe}_{12}\text{O}_{19}$  etc.

$$T < T_c, H = 0, M \neq 0$$

Ferri magnetic materials like ferro magnetic materials show significant magnetisation below the transition temperatures. Above  $T_c$  the materials display paramagnetic behaviour.

### 11. Magnetic susceptibility vs temperature

The temperature dependence of the susceptibility for the Para, Ferro and anti-ferro magnetic materials are summarised as shown. For paramagnetic materials the susceptibility  $\chi_m$  varies monotonically as  $1/T$  following the Curies law.



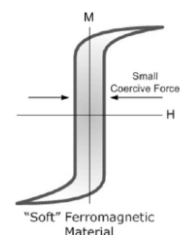
Ferromagnetic materials follow the Curie Weiss law and exhibit a paramagnetic behaviour above the Curie temperature.

Anti-ferromagnetic materials show an increase in susceptibility till the Neel temperature above which the material behaves as a paramagnetic material.

### 12. Soft and Hard Magnetic materials

Ferromagnetic materials can be classified as soft and hard materials depending on the nature of the hysteresis displayed by the material.

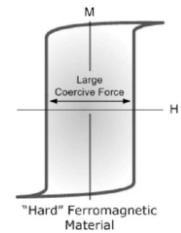
Soft magnetic materials are easily magnetisable and de-magnetisable. They are characterised by a small coercivity and large saturation magnetisation. The hysteresis loop is narrow and has low losses.



Soft magnetic materials find application in high frequency switching of the magnetisation and are used in transformers, motors and generators.

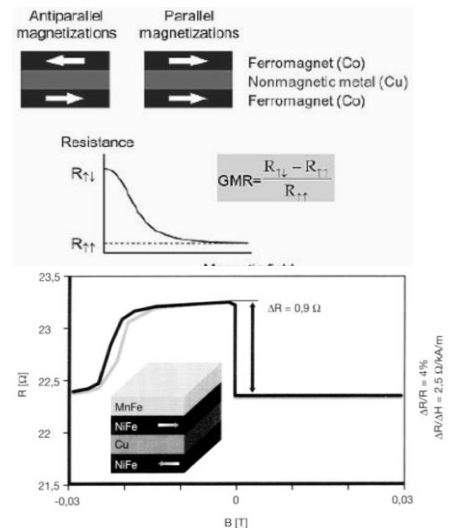
Hard magnetic materials on the other hand are difficult to magnetise and demagnetise. They exhibit a very high retentivity and require a large coercive field to demagnetise the material. The area under the hysteresis curve is large indicating the large amount of energy loss.

The high retentivity of the material makes it ideal for magnetic storage of information as in hard disk drives. The current capabilities are close to 1 Terra bits per square inch.



### 13. Giant Magneto Resistance device.

**scattering of electron is reduced when** Magneto resistance has been observed in layered magnetic materials where the resistance across the thickness of two magnetic layers (generally Cobalt) separated by a non magnetic layer (generally Copper) shows a dependence on the magnetisation states of the individual layers. The resistance to current flow depends on the direction of magnetisation of the two layers and can show large variations in the resistance. The effect has been attributed to the spin scattering of the electrons when they flow through the material. The scattering of electron is reduced when the magnetisation of the two layers is parallel. When the spin state of the two layers are anti parallel the scattering and hence the resistance increases.

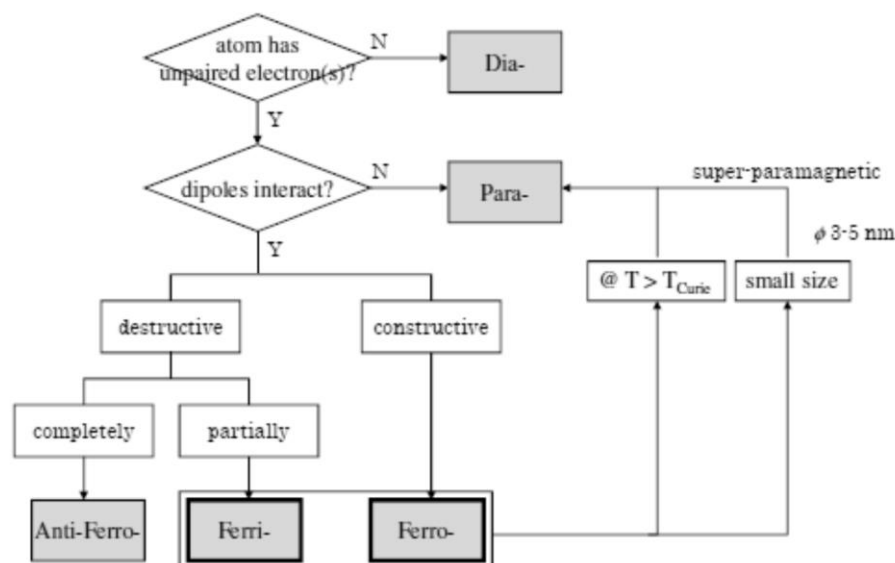


When the magnetic layers are in the range of 3 – 5nm with the separation layer about 25nm, it has been observed that the change in resistance can be as high as 50% of which is significant for any practical measurements and applications.

The Giant Magneto Resistor finds application in the read head of magnetic memories. One of the layers ferromagnetic (soft magnetic material) layer is pinned to an anti-ferro magnetic material and the other (soft magnetic material) free layer's spin orientation is flipped by the magnetisation of the domain (hard magnetic material) on the recording media.

### 14. General classification of magnetic materials

A general scheme for classifying materials according to the atomic characteristics is represented.



**Questions and Problem set**

1. Derive an expression for the Bohr Magneton and calculate the value.
2. Discuss the concepts of Larmor precession and gyromagnetic ratio.
3. Elaborate on two tests which can differentiate a paramagnetic material from a diamagnetic material.
4. Discuss the parameters that distinguish ferro, antiferro and ferri magnetic materials.
5. Classify magnetic materials on the basis of the magnetic susceptibility.
6. Bring out the differences between soft and hard magnetic materials.
7. Write a note on applications of magnetic materials.
8. Discuss the magnetic hysteresis of materials.
9. Discuss magnetic memory materials.
10. An electron in a hydrogen atom moves in a circular orbit of radius 0.065nm. The electron makes  $10^{16}$  revolutions per second. Calculate the orbital magnetic moment.
11. The saturation magnetic induction of nickel is  $0.65 \text{ Wb m}^{-2}$ . If the density of nickel is  $8906 \text{ kg m}^{-3}$  and atomic weight is 514.7. Calculate the magnetic moment of nickel in Bohr magneton.

***Magnetism tidbits***

- James Clerk Maxwell (1831–1879), Scottish physicist, who developed the classical theory of electromagnetic fields described by the set of equations known as Maxwell's equations.
- Pierre Curie (1859–1906) was a French physicist who worked extensively on magnetic materials. He and his wife, Marie (Sklodowska) Curie (1867–1934), later became famous for their research on radioactivity.

## **Contents**

- 1. Electric fields in dielectrics – concepts of polarization**
- 2. Electric field due to dipoles in the dielectric**
- 3. Polarization mechanisms**
- 4. Non-linear dielectrics – classification of non-centro symmetric systems**
  - a. Piezoelectric**
  - b. Pyroelectrics**
  - c. Ferroelectrics**
- 5. Solved Problems and practice problems**

## Unit V Dielectrics

### Polarisation in dielectrics

Dielectrics are a class of material which respond to external electric fields, and find use in the electronics and electrical industry.

From our knowledge of capacitors, the dielectric constant of a material is

- $\epsilon_r = \frac{\text{capacitance of a parallel plate capacitor with the material}}{\text{capacitance of the capacitor without the material}} = \frac{C}{C_0}$
- $\epsilon_r = \frac{\text{field across a parallel plate capacitor without the material}}{\text{field across the capacitor with the material}} = \frac{E_0}{E}$

where  $E_0$  is the applied electric field and  $E$  is the net electric field across the dielectric.

An external electric field can cause the center of positive and negative charges in the crystal to shift from their mean position. This separation of charges results in creation of dipoles in the system. The net dipole moment created per unit volume is called the polarization  $\mathbf{P}$ . The polarization in the material is dependent on the net electric field experienced by the dielectric.

If  $E_0$  is the applied electric field applied to a parallel plate capacitor the surface charge density on the plates of the capacitor  $\sigma = \epsilon_0 E_0$

If a dielectric material is placed between the plates of the capacitor, the surface density of charge due to polarization  $\sigma_p = \epsilon_0 E'$  where  $E'$  is called the depolarization field.

The net electric field  $E$  between the plates is reduced by a factor which is the dielectric constant  $\epsilon_r$  of the material i.e.,  $E = \frac{E_0}{\epsilon_r}$  or  $E_0 = \epsilon_r E$

The net electric field  $E = E_0 - E' = \epsilon_r E - \frac{\sigma_p}{\epsilon_0}$

Simplifying  $\sigma_p = \epsilon_0 \epsilon_r E - \epsilon_0 E = \epsilon_0 (\epsilon_r - 1) E$

Hence the polarization in the material due to a net electric field is given by

$$\mathbf{P} = \sigma_p = \epsilon_0 (\epsilon_r - 1) \mathbf{E}$$

The polarisation of the material can also be written as

$\mathbf{P} = \epsilon_0 \chi \mathbf{E}$  where  $\chi$  is the dielectric susceptibility of the material and is a measure of the ability to create dipoles in the material. The polarization is linearly proportional to the applied electric field for pure dielectrics.

This gives us the relation between susceptibility and dielectric constant

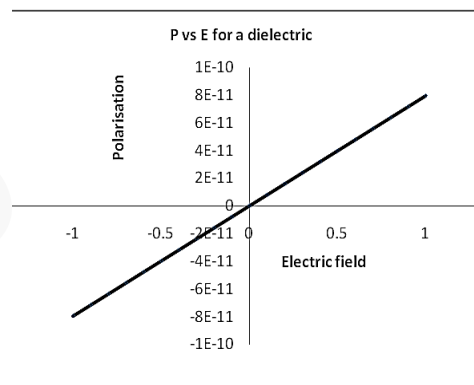
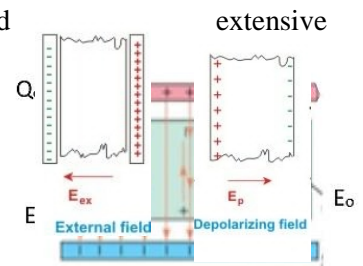
$$\chi = (\epsilon_r - 1) \text{ or } \epsilon_r = 1 + \chi.$$

If  $N$  is the number of atoms per unit volume and  $\alpha_e$  is the polarisability of the individual dipoles then the polarization can be written as  $\mathbf{P} = \epsilon_0 \chi \mathbf{E} = N \alpha_e \mathbf{E}$ .

$$\text{Or } \chi = \frac{N \alpha_e}{\epsilon_0}$$

Polarization in dielectrics is thus proportional to the net electric field.

### Electric fields in a dielectric material



To understand the polarization mechanism in totality, it is essential to understand the components of the electric fields, which exist in a real dielectric, apart from the applied electric field  $E$  and the depolarization field  $E'$ . In general the local electric field inside a dielectric material comprises of four components of electric field.

$$E_{loc} = E_o + E_1 + E_2 + E_3$$

$E_o$  is the external applied electric field ,

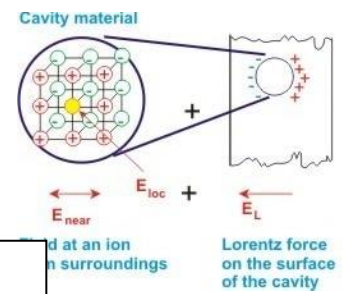
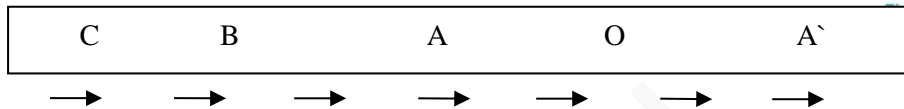
$E_1$  is the depolarization field,

$E_2$  is the Lorentz field on the surface of the spherical cavity (scooped out of the dielectric and

$E_3$  is the field due to other dipoles lying within the sphere.

{ In the case of cubic structures the net Lorentz field can be shown to be zero. The electric field due to the dipoles inside the spherical cavity can be estimated as follows.

Consider a linear array of dipoles at A, B, C A', B' and C' with dipole moment  $\mu$  at regular intervals of  $d$ .



The electric field at A and A' are equal in strength and direction and equal to

$$E_A = E_{A'} = \frac{2\mu}{4\pi\epsilon_0 d^3} \quad \text{and the net electric field } E_{AA'} = \frac{\mu}{\pi\epsilon_0 d^3}.$$

The electric field at O due to the pair of dipoles equidistant from O can be written as

$E = E_{AA'} + E_{BB'} + E_{CC'} \dots = \frac{\mu}{\pi\epsilon_0} \left( \frac{1}{d^3} + \frac{1}{(2d)^3} + \frac{1}{(3d)^3} \dots \right) = \frac{\mu}{\pi\epsilon_0 d^3} \sum \frac{1}{n^3} = \frac{1.2\mu}{\pi\epsilon_0 d^3} = \frac{\gamma P}{\epsilon_0}$  where  $\gamma$  is a constant that depends on the geometry of the material. From the series we can evaluate the contribution of the nearest neighbors and find that only first few pair of neighbors contributes significantly to the electric field at any site. ###

In the case of a dielectric with a cubic structure the effective field can be written  $E_{in} = \frac{P}{3\epsilon_0}$

Thus the net electric field across the dielectrics will be greater by this factor and given by

$$E_{loc} = E + E_{in} = E + \frac{P}{3\epsilon_0}$$

From the definition of polarization we have  $P = N\alpha_e E_{loc} = N\alpha_e \left( E + \frac{P}{3\epsilon_0} \right) = \epsilon_0(\epsilon_r - 1)E$

Solving this we get  $\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha_e}{3\epsilon_0}$ . This is the Clausius Mosotti relation which relates the macroscopic dielectric constant to the microscopic polarisability of the material.

It is obvious that the effective microscopic polarisability is greater when the internal fields are considered and should evolve in a more realistic dielectric constant of the material.

[ derivation of the Clausius Mosotti relation:

$$P = N\alpha_e \left( E + \frac{P}{3\epsilon_0} \right)$$

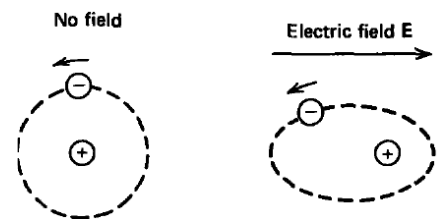
Rearranging terms we get  $P \left( 1 - \frac{N\alpha_e}{3\epsilon_0} \right) = N\alpha_e E$

$$P = \frac{N\alpha_e E}{\left( 1 - \frac{N\alpha_e}{3\epsilon_0} \right)} = \epsilon_0 (\epsilon_r - 1) E, \text{ simplifying } \frac{N\alpha_e/\epsilon_0}{\left( 1 - \frac{N\alpha_e}{3\epsilon_0} \right)} = (\epsilon_r - 1) \text{ Taking the inverse}$$

$$\frac{\left( 1 - \frac{N\alpha_e}{3\epsilon_0} \right)}{N\alpha_e/\epsilon_0} = \frac{1}{\frac{N\alpha_e}{\epsilon_0}} - \frac{1}{3} = \frac{1}{(\epsilon_r - 1)} \text{ leading to } \frac{1}{\frac{N\alpha_e}{\epsilon_0}} = \frac{1}{3} + \frac{1}{(\epsilon_r - 1)} = \frac{(\epsilon_r + 2)}{3(\epsilon_r - 1)} \text{ Hence } \frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha_e}{3\epsilon_0} \quad ]$$

## Polarisation mechanisms

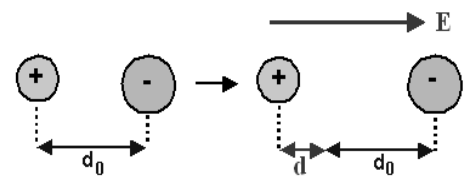
**Electronic polarization** : When an atom is subjected to an electric field  $E$ , the nucleus and the electron cloud will try to move in opposite directions. The nucleus moves in the direction of the applied field and the electron cloud in the direction opposite to that of the applied field. The centre of the negatively charged electron cloud no longer coincides with the positive nucleus and hence results in an induced dipole. The polarization produced due to this induced dipole is called “Electronic polarization”.



The electronic polarizability is  $\alpha_e = 4\pi\epsilon_0 R^3$  where  $R$  is the radius of the atom. The electronic polarization is temperature independent since the size of the atom does not depend on temperature.

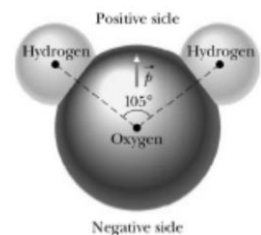
## Ionic Polarisation

Dielectric materials with ionic bonding there exists regular arrangement of positive and negative ions. Ions between any two bond is then a dipole. Because of the symmetric arrangement of the dipoles in the material, the net dipole moment in the material may be zero.



In the presence of an external electric field the elongation of the bond will result in an increased the dipole moment along the direction of the field. However when the ions are displaced from their equilibrium position, a restoring force in the form of the columbic force of attraction tends to constrain the bond elongation.

The displacement of the ions results in a net induced dipole moment per unit volume or the electric field induces polarization in the material. The ionic polarisability  $\alpha_i$  is independent of temperature and depends on the Young's modulus of the material.



## Orientational polarization:

Polar molecules have permanent dipole moments since the net charge centers are separated by a small distance. These dipoles can move and rotate freely (polar liquids - such as the water molecule). However due to the random motion of the molecules at normal temperatures, there is no net polarization in the material.

In the presence of an external field the dipoles tend to align in the direction of the field. This results in a net dipole moment for the material and is called orientational polarization.

The polarisation is given by  $P = \frac{N\mu^2 E}{3kT}$

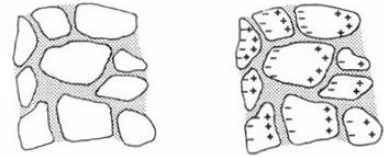


Where  $N$  is the no of dipoles per unit volume,  $\mu$  is the dipole moment of the individual molecules and  $T$  the temperature.

Increase in temperature increases the random motion of the dipoles and hence the orientational polarization is highly temperature dependent.

### Space charge limited polarisation

In general when two materials of different conducting phases meet at an interface, a potential barrier layer is formed resulting in an accumulation of charges of the opposite polarities on either sides of the interface. This can be modeled to be a polarization of charges in the bulk of the material. The width of the barrier is generally very small and can result in reasonably high capacitance. These are however very sensitive to voltages and frequencies and may not be effective at normal working conditions.

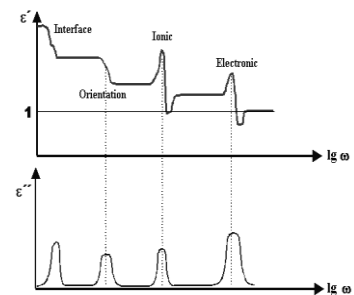


### Frequency dependence of dielectric constant.

The response of the dielectric constant to an applied ac electric field can be understood by considering the ability of the individual dipoles in the material to follow the external AC field. The dielectric constant of an ideal dielectric which has all mechanism contributing to the polarization can be expressed as

$$\epsilon_r = \epsilon_{sp} + \epsilon_{or} + \epsilon_{io} + \epsilon_e$$

The AC electric field induces the dipoles due to the individual polarization mechanisms have to align in the direction of the field. At low frequencies, the dipoles due to all mechanisms tend to align with the electric field easily. As the frequency of the applied electric field increases, the dipoles alignment tends to lag behind the field (depending on the inertia of the dipole). This results in a effective decrease in the polarization of the material as the frequency increases.



The AC response of the dielectric constant can be represented as  $\epsilon = \epsilon' + j\epsilon''$  where  $\epsilon'$  is the real part of the dielectric constant and  $\epsilon''$  is the imaginary part of the dielectric constant which describes the losses in the dielectric.

At low frequencies of about 100Hz, the contribution to the dielectric constant from the space charge polarization disappears and hence the dielectric constant decreases. Beyond this and up to about  $10^9$  Hz remains constant implying all the other three polarization mechanisms are active. At round  $10^9$  Hz (the microwave region) the orientation polarization stops responding to the electric field and the contribution from this mechanism to the dielectric constant become zero and the effective dielectric constant of the material decreases to a lower value.

At this point only two polarization mechanisms are contributing to the dielectric constant – the ionic and electronic polarization mechanisms.

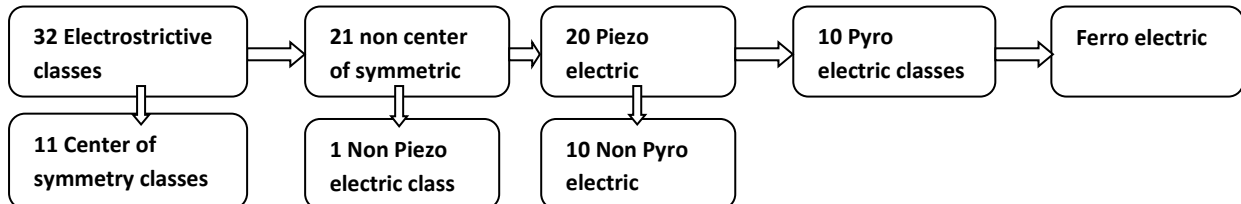
Around  $10^{13}$  Hz (far infra red) the frequency of the AC field is in close to the vibrational frequencies of the bond and resonance phenomena sets in. The dielectric constant shows an initial increase and exhibits a resonance type of behavior around the resonant frequency.

Beyond this range the dielectric constant is only due to the electronic polarization. As the frequency of the AC approach  $10^{15}$  Hz - the visible range which corresponds to the visible absorption frequency and a resonant condition sets in with a behavior similar to the ionic range.

The material may be treated as transparent to frequencies  $> 10^{16}$  Hz and the dielectric constant should be that of vacuum or free space.

The dielectric loss  $\epsilon''$  represents the energy loss in the dipoles following the electric fields. As the frequency increases generally the inertia of the dipoles becomes significant and at particular frequency ranges the dipoles become inert to the increasing ac frequency. } }###

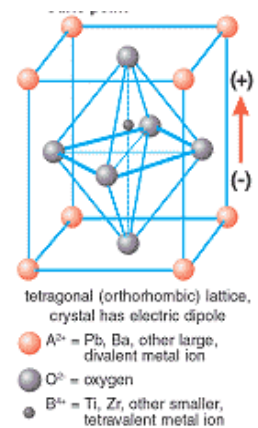
### Classification of non linear dielectric materials



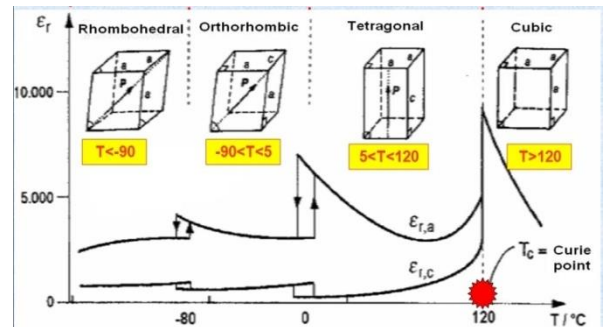
### Non Centro-symmetric system

Out of a total of 32 crystal point groups (see the course related to structure of materials), 21 are non-centrosymmetric *i.e.* crystals not having a center of symmetry.

BaTiO<sub>3</sub> is a good example of a non centro symmetric crystal. The unit cell is cubic above the curie temperature of about 120 C, when the material is centro symmetric and behaves as a paraelectric. Between 5C and 120C the material is in the tetragonal phase exhibiting ferroelectric behavior. Below 5C and -90C the material has a orthogonal phase and below -90C the material is in the orthorhombic phase. In all these phases also the material exhibit ferroelectric behavior in varying proportions.



These dipoles arise due to the fact that in the tetragonal unit cell of BaTiO<sub>3</sub>, the Ti<sup>4+</sup> cation is surrounded by six O<sup>2-</sup> anions in a slightly deformed octahedral configuration, and can occupy one of two asymmetrical sites. In either position, the Ti<sup>4+</sup> cation is not coincident with the negative charge center of the oxygen anions by a small fraction of an Angstrom, creating an electric dipole. The energy barrier between the two possible Ti atom positions is sufficiently low to permit motion of the atom between sites by the coercion of an electric field, and the material can thus be directionally polarized with ease.



Non centro symmetric crystals can respond to an external stimulus producing polarization or surface charges which show up as a potential across the element. However these properties are highly anisotropic and could be described as a tensor (which describes the direction dependent properties of materials). Piezoelectric behavior is the response of these crystals to external mechanical pressures; Pyroelectric behavior is the response to thermal changes and Ferroelectric behavior is a response of the material to external electric fields.

### Piezoelectric materials

Piezoelectric Effect is appearance of an electrical potential across some faces of some crystals when a pressure is applied to the crystal. Pierre Curie and his brother Jacques discovered the effect in 1880. Subsequently the inverse piezoelectric effect was observed as a distortion in the crystal when an electrical field is applied.

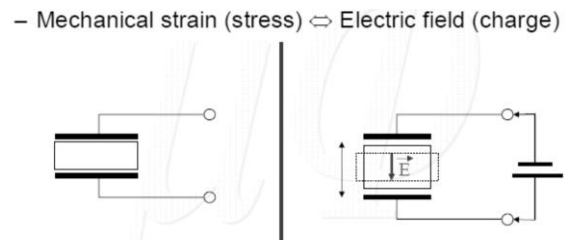
The term piezoelectricity refers to the fact that, when a crystal is strained, an electric field is produced within the substance. As a result of this field, a potential difference develops across the sample, and by measuring this potential one may determine the field. The inverse effect - that an applied field produces strain - has also been observed. It is explained by the displacement of ions, causing the electric polarization of the crystal's structural units. When an electrical field is applied, the ions are displaced by electrostatic forces, resulting in the mechanical deformation of the whole crystal.

The piezoelectric effect is very small. A field of  $10^3$  V/cm in quartz ( $\text{SiO}_2$ ) produces a strain of only  $10^{-7}$ . That is, a rod 1 cm long changes its length by  $10\text{\AA}$ . Conversely, even small strains can produce enormous electric fields.

The piezoelectric effect is often used to convert electrical energy into mechanical energy, and vice versa; i.e., these materials are used to make transducers. For instance, an electric signal applied to the end of a quartz rod generates a mechanical strain, which consequently leads to the propagation of a mechanical wave - a sound wave - down the rod.

Piezo electric materials like PZT ( $\text{Pb}(\text{Zr,Ti})\text{O}_3$ ), and Quartz find extensive application in electronics as sensors and actuators.

- Piezoelectric microbalances are used as very sensitive chemical and biological sensors.
- Atomic force and scanning tunneling microscopes for precision manipulation of sample at nano meter levels
- Transient pressure measurement to study explosives, internal combustion engines (knock sensors), and any other vibrations, accelerations, or impacts.
- Energy Harvesting from impact on the ground
- Electric igniters
- Transducers are used in electronic drum pads to detect the impact of the drummer's sticks.



### Pyroelectric materials

Pyroelectricity is the ability of certain materials to generate an electrical potential when they are heated or cooled. Non centrosymmetric crystals with a net spontaneous polarization can be sensitive to external temperatures.

The change in temperature modifies the positions of the atoms slightly within the crystal structure, such that the polarization of the material changes. This polarization change gives rise to a voltage across the crystal. If the temperature stays constant at its new value, the pyroelectric voltage gradually disappears due to leakage current (the leakage can be due to electrons moving through the crystal, ions moving through the air, current leaking through a voltmeter attached across the crystal, etc.).

The pyroelectric coefficient may be described as the change in the spontaneous polarization vector  $P_s$  with temperature  $T$

$$p_i = \frac{\partial P_s}{\partial T} \quad \text{where } p_i (\text{Cm}^{-2}\text{K}^{-1}) \text{ is the vector for the pyroelectric coefficient.}$$

It can be used to create a flow of current in an external circuit connected to a device using pyroelectric materials. The effect has been known for many years, but it is only since about 1960 that its technological applications have been seriously considered.

These have been almost entirely in the field of the detection of electromagnetic radiation, especially in the two “atmospheric window” infrared (IR) bands of 3-5  $\mu\text{m}$  and 8-14  $\mu\text{m}$ . The ambient temperature operation of pyroelectric detectors, leading to low power consumption, low cost, compactness.

Pyroelectric detectors have found a huge range of applications in products ranging from fire alarms to intruder detectors, in instrumentation such as gas analysis and laser beam characterization and in military/paramilitary applications such as thermal imaging.

### **Ferroelectric materials**

Charge displacements in dielectric and para electric ceramics, such as  $\text{Al}_2\text{O}_3$ , mica,  $\text{TiO}_2$ , are totally reversible, as ions / electron cloud return to their original position once an applied field is removed, and the polarization effect is linear.

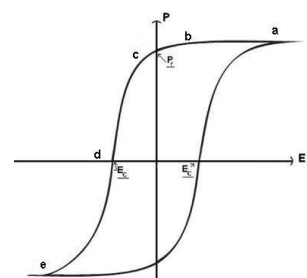
Ferroelectrics are a class of non-centro symmetric crystals which are also a subclass of the pyro electric / piezoelectric materials. These materials show a spontaneous polarization even in the absence of an electric field.

The spontaneous polarization observed in ferroelectric materials depends on the crystalline phase of the material (generally referred to as perovskites).  $\text{BaTiO}_3$  is a classic example of a perovskite material.

The dielectric susceptibility of the material is highly temperature dependent and is given by  $\chi = \frac{C}{T-T_c}$

for all  $T > T_c$  where  $C$  is a constant dependent on the material and  $T_c$  is the curie temperature. At temperature below  $T_c$ , the material shows spontaneous polarization and is classified as a ferroelectric material. At temperatures greater than  $T_c$ , the material is para electric with the susceptibility inversely proportional to temperature.

Ferroelectric dielectrics differ from dielectric materials in their response to external applied electric field. Ferroelectrics display a nonlinear response of polarization to changing electric fields and display a hysteresis in the  $P$  versus  $E$  variations. The hysteresis loop is caused by the existence of permanent electric dipoles in classes of materials, which develop spontaneously below the Curie temperature. The polarisation state of the ferroelectric material has a memory effect and hence is used extensively in DRAMs and SRAMS.



Ferro electric materials find numerous applications as sensors and actuators taking advantage of the temperature and field dependencies of the susceptibility (dielectric constant).

**Solved examples**

1. **The surface density of charge of a parallel plate capacitor is  $7.2 \times 10^{-10} \text{ C m}^{-2}$ . A dielectric medium with  $\epsilon_r = 12$  is introduced between the plates of the capacitor. Estimate the induced surface density of charge on the dielectric surface and the electric fields between the plates with and without the dielectric material.**

The electric field across the plates of the capacitor without dielectric  $E_o = \frac{\sigma_o}{\epsilon_o} = 81.35 \text{ V/m}$

The electric field across the dielectric due to surface charges  $= E_p = \frac{\sigma_p}{\epsilon_o}$

$$\sigma_p = (\epsilon_r - 1) \frac{\sigma_o}{\epsilon_r} = 6.6 \times 10^{-10} \text{ C m}^{-2}.$$

Electric field across dielectric  $= \frac{\sigma_p}{\epsilon_o} = 74.57 \text{ V/m}$

Electric field across the plates with the dielectric  $= 6.78 \text{ V/m}$

2. **A dielectric medium with  $\epsilon_r = 25$  is introduced between the plates of the capacitor with surface density of charge on the plates equal to  $8.00 \times 10^{-10} \text{ C m}^{-2}$ . Estimate, the electric fields between the plates with and without the dielectric material and the surface density of charge on the dielectric.**

The electric between the plates of the capacitor without the dielectric

$$E_o = \frac{\sigma_o}{\epsilon_o} = 90.40 \text{ V/m}$$

The electric field between the plates with the dielectric  $= \frac{E_o}{\epsilon_r} = 3.62 \text{ V/m}$

The electric field across the dielectric  $E' = 86.78 \text{ V/m}$

The surface density of charge on the dielectric  $\sigma = E' * \epsilon_o = 7.68 \times 10^{-10} \text{ C m}^{-2}$

3. **An elemental dielectric has  $\epsilon_r = 10$  and contains  $7.5 \times 10^{29}$  atoms /  $\text{m}^3$ . Calculate the electronic polarisability of the material..**

$$P = \epsilon_o(\epsilon_r - 1)E = N\alpha_e E \quad \text{Given } \epsilon_r = 10$$

$$\alpha_e = \frac{\epsilon_o(\epsilon_r - 1)}{N} = \frac{8.85 \times 10^{-12} \times 9}{7.5 \times 10^{29}} = 10.62 \times 10^{-41} \text{ F m}^2$$

**Problem Set:**

1. If the molecular dipoles in a  $10^{-3} \text{ m}$  radius water drop are pointed in the same direction, calculate the polarisation of the water drop. Dipole moment of water molecule is  $6 \times 10^{-30} \text{ C m}$ .
2. Assuming that the polarisability of Kr atom is  $2.18 \times 10^{-40} \text{ F m}^2$ , calculate its dielectric constant. Kr has  $2.7 \times 10^{25}$  atoms per unit volume at NTP.
3. An elemental dielectric has  $\epsilon_r = 12$  and contains  $5 \times 10^{29}$  atoms /  $\text{m}^3$ . Calculate the electronic polarisability of the material.
4. Find the total polarisability of  $\text{CO}_2$  if its susceptibility is  $0.985 \times 10^{-3}$ . Density of  $\text{CO}_2$  is  $1.977 \text{ Kg m}^{-3}$ .
5. On being polarised an oxygen atom shows a dipole moment of  $0.5 \times 10^{-22} \text{ C-m}$ . If the distance of the center of the -ve charge cloud from the nucleus is  $4 \times 10^{-17} \text{ m}$ , calculate the polarisability of oxygen.

6. A parallel plate capacitor without a dielectric is charged such that the surface charge density on the plates is  $10^6 \text{ C-m}^{-2}$ . If a slab of a material with dielectric constant 10 is inserted between the plates, calculate the polarisation in the material and the electric field due to induced surface charge on the dielectric.
7. A dielectric material has one species having an atomic polarizability value of  $10^{-30} \text{ Cm}^2\text{V}^{-1}$ . It is found that when the dielectric sample is kept in a uniform electric field, the field reduces to one tenth of its original value. Estimate the number of atoms per unit volume of the material.
8. Electronic polarisability of Sulphur is  $3.28 \times 10^{-40} \text{ F-m}^2$ . Calculate the dielectric constant of Sulphur if the density and atomic weight of Sulphur are  $2.08 \times 10^3 \text{ kg-m}^{-3}$  and 32 respectively.

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