# ENGINEERING MECHANICS - STATICS

**Rashmi B A**

Department of Civil Engineering

# ENGINEERING MECHANICS - STATICS
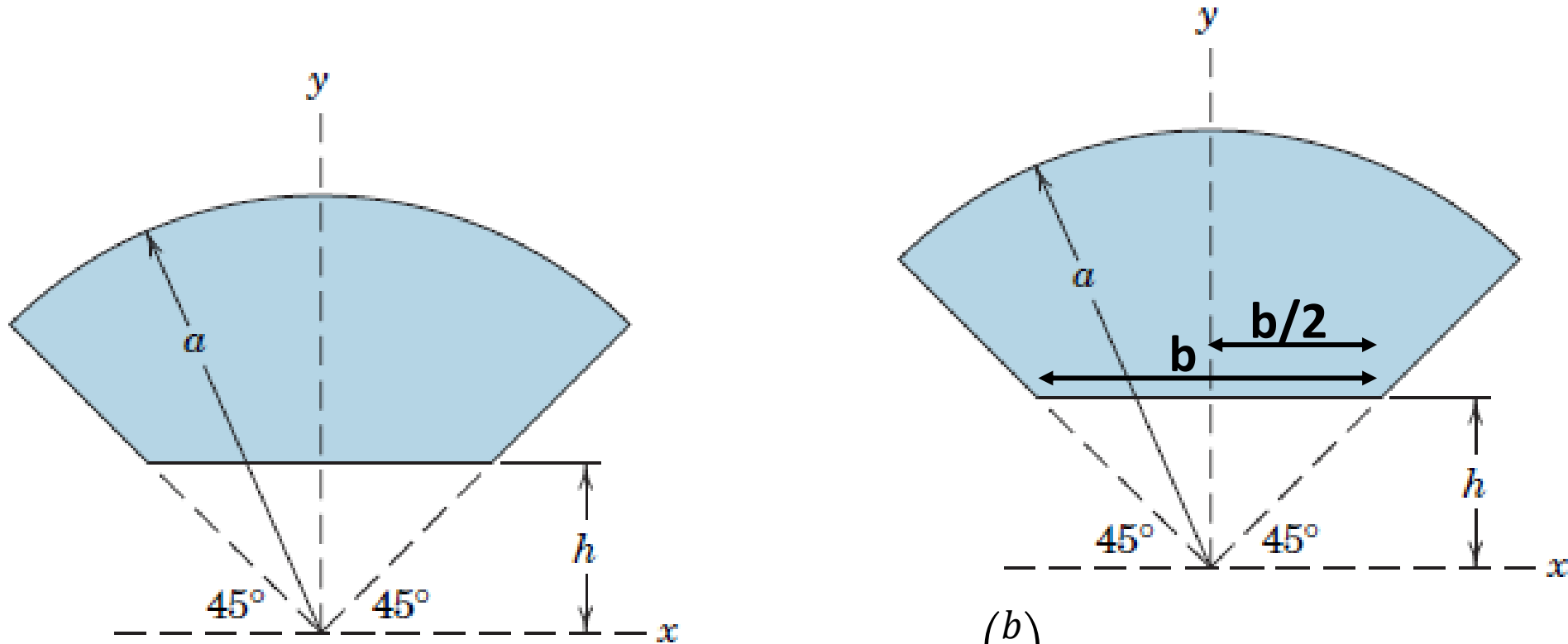
## DISTRIBUTED FORCES

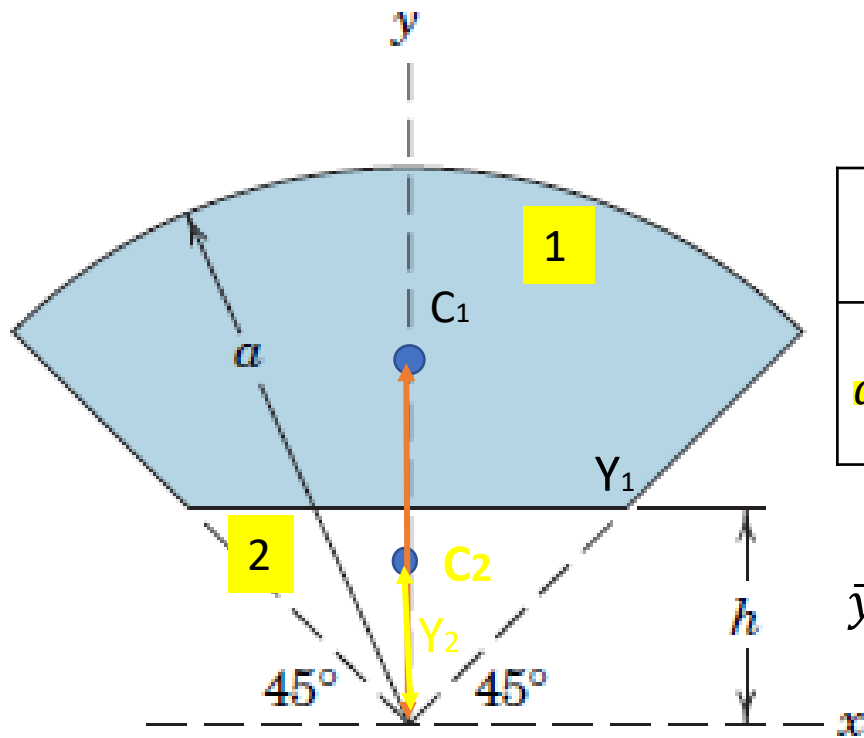### Session- 4

**Rashmi B A**

Department of Civil Engineering

## Centroid- Numericals

**Problem: 5/55.** Determine the y-coordinate of the centroid of the shaded area.



$$\tan(45) = \frac{opp}{adj} = \frac{\left(\frac{b}{2}\right)}{h}$$

$$b = 2h$$

## Centroid- Numericals

1. Sector $(R = a, \alpha = \pi/4)$
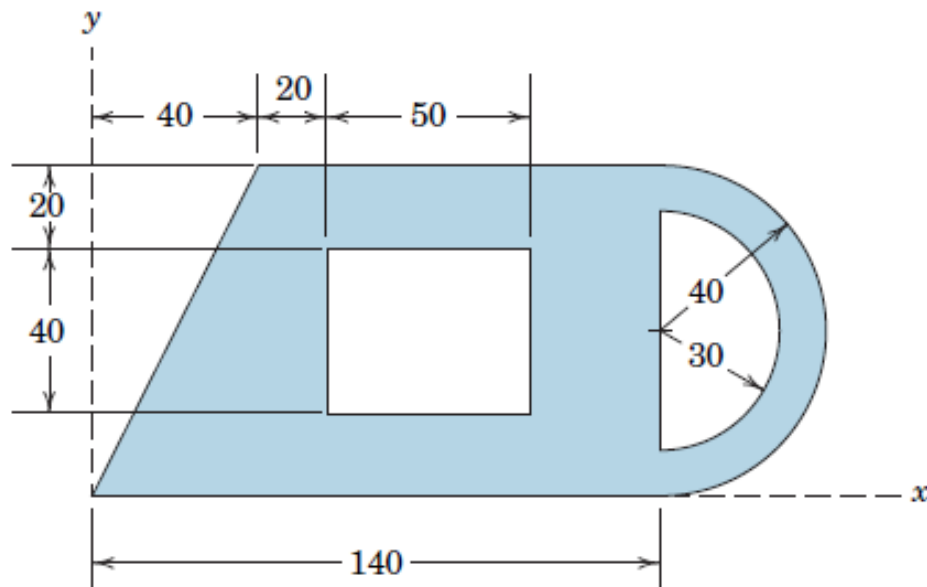2. Triangle $(b = 2h, h = h)$

$$\bar{y} = \frac{\sum a_i y_i}{A}$$

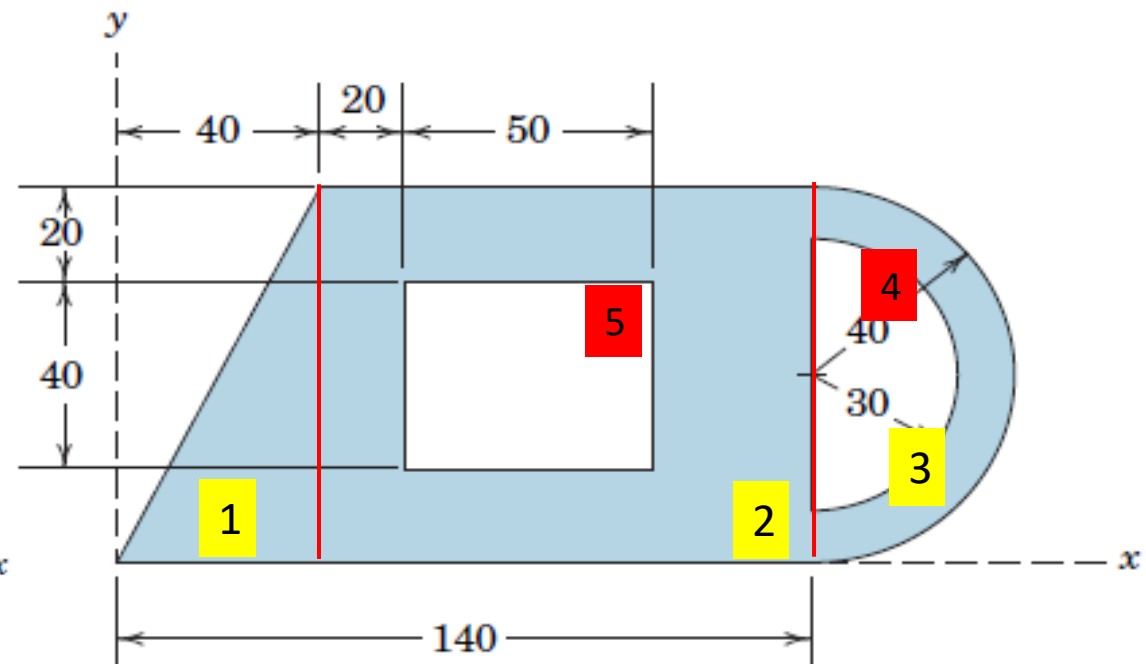| | |
|---|---|
| $a_1 = R^2 \alpha = a^2 \frac{\pi}{4}$ | $y_1 = \frac{2r \sin \alpha}{3\alpha} = \frac{4\sqrt{2}\, a}{3\pi}$ |
| $a_2 = -\left(\frac{(bh)}{2}\right) = \frac{-(2h \times h)}{2} = -h^2$ | $y_2 = \frac{2h}{3}$ |

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{A} = \frac{\left(a^2 \frac{\pi}{4}\right)\left(\frac{4\sqrt{2}\,a}{3\pi}\right) + \left(-h^2 \times \frac{2h}{3}\right)}{a^2 \frac{\pi}{4} - h^2} = \frac{\left(\frac{\sqrt{2}a^3}{3}\right) - \left(\frac{2h^3}{3}\right)}{\frac{\pi a^2 - 4h^2}{4}}$$

$$\boxed{\bar{y} = \frac{4\left(\sqrt{2}a^3 - 2h^3\right)}{3(\pi a^2 - 4h^2)}}$$

## Centroid - Numericals

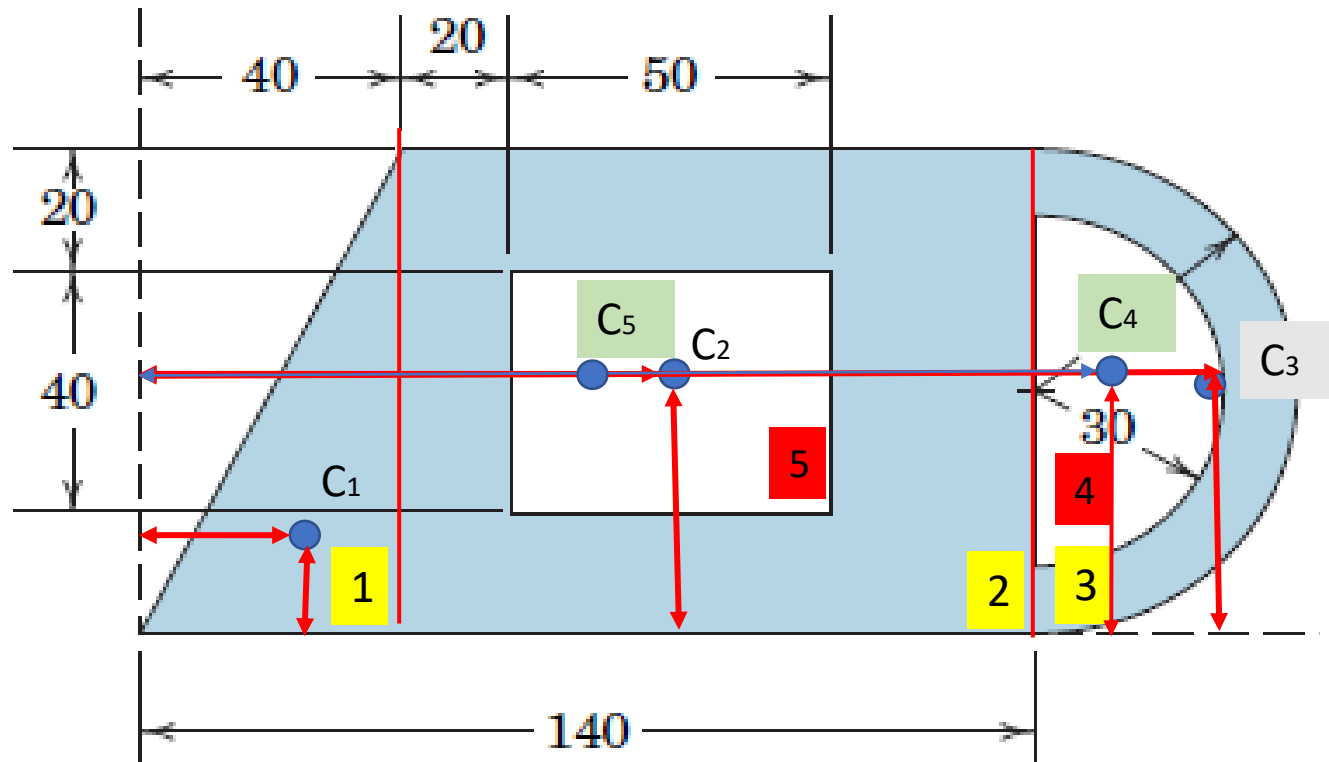**Problem: 5/58.** Determine the coordinates of the centroid of the shaded area.



Dimensions in millimeters



Dimensions in millimeters

| | | |
|---|---|---|
| $a_1 = \dfrac{bh}{2} = \dfrac{40(80)}{2} = 1600$ | $x_1 = \dfrac{2b}{3} = \dfrac{2(40)}{3} = 26.67$ | $y_1 = \dfrac{h}{3} = \dfrac{80}{3} = 26.67$ |
| $a_2 = bh = 100 \, X \, 80 = 8000$ | $x_2 = 40 + \dfrac{b}{2} = 40 + \dfrac{(140-40)}{2} = 90$ | $y_2 = \dfrac{h}{2} = \dfrac{80}{2} = 40$ |
| $a_3 = \dfrac{\pi R^2}{2} = \dfrac{\pi 40^2}{2} = 800\pi$ | $x_3 = 140 + \dfrac{4R}{3\pi} = 140 + \dfrac{4(40)}{3\pi} = 157$ | $y_3 = R = 40$ |
| $a_4 = -\left(\dfrac{\pi R^2}{2}\right) = \dfrac{\pi 30^2}{2} = -450\,\pi$ | $x_4 = 140 + \dfrac{4R}{3\pi} = 140 + \dfrac{4(30)}{3\pi} = 152.7$ | $y_4 = R = 40$ |
| $a_5 = -(bh) = -(50 \, X \, 40)$ $= -2000$ | $x_5 = 40 + 20 + \dfrac{b}{2} = 60 + \dfrac{50}{2} = 85$ | $y_5 = \dfrac{h}{2} = \dfrac{80}{2} = 40$ |

| | | |
|---|---|---|
| $x_1 = \dfrac{2b}{3} = \dfrac{2(40)}{3} = 26.67$ | $y_1 = \dfrac{h}{3} = \dfrac{80}{3} = 26.67$ | $a_1 = \dfrac{bh}{2} = \dfrac{40(80)}{2} = 1600$ |
| $x_2 = 40 + \dfrac{b}{2} = 40 + \dfrac{(140-40)}{2} = 90$ | $y_2 = \dfrac{h}{2} = \dfrac{80}{2} = 40$ | $a_2 = bh = 100\,X80 = 8000$ |
| $x_3 = 140 + \dfrac{4R}{3\pi} = 140 + \dfrac{4(40)}{3\pi} = 157$ | $y_3 = R = 40$ | $a_3 = \dfrac{\pi R^2}{2} = \dfrac{\pi 40^2}{2} = 800\pi$ |
| $x_4 = 140 + \dfrac{4R}{3\pi} = 140 + \dfrac{4(30)}{3\pi} = 152.7$ | $y_4 = R = 40$ | $a_4 = -\left(\dfrac{\pi R^2}{2}\right) = \dfrac{\pi 30^2}{2} = -450\,\pi$ |
| $x_5 = 40 + 20 + \dfrac{b}{2} = 60 + \dfrac{50}{2} = 85$ | $y_5 = \dfrac{h}{2} = \dfrac{80}{2} = 40$ | $a_5 = -(bh) = -(50\,X\,40)$ $= -2000$ |

$$\bar{x} = \frac{\sum a_i x_i}{\sum a_i} = \frac{(1600\,X\,26.67) + (8000\,X\,90) + (800\pi\,X\,157) + (-450\pi\,X\,152.7) + (-2000\,X\,85)}{1600 + 8000 + 800\pi - 450\pi - 2000} = \frac{771381.5}{8699.557} = 88.67\ mm$$

$$\bar{y} = \frac{\sum a_i y_i}{\sum a_i} = \frac{(1600\,X\,26.67) + (8000\,X\,90) + (800\pi\,X\,157) + (-450\pi\,X\,152.7) + (-2000\,X\,85)}{1600 + 8000 + 800\pi - 450\pi - 2000} = \frac{326654.3}{8699.557} = 37.55\ mm$$

$$\boxed{\bar{x} = 88.67mm, \bar{y} = 37.55mm}$$

# THANK YOU

**Rashmi B A**

Department of Civil Engineering

**rashmiba@pes.edu**