ENGINEERING PHYSICS

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Class #49

- *Precession of J about B*
- *Quantum theory of paramagnetism*
- *Average magnetization at low B*
- *Average magnetization at high B*
- *Brillouin function*

Quantum theory of Paramagnetism

- ***Suggested Reading***

  1. *Quantum Physics of Atoms Nuclei and Molecules, Robert Eisberg, Robert Resnick, Wiley, 2nd edition, Ch 14, 2006.*
  2. *Learning material prepared by the Department of Physics*

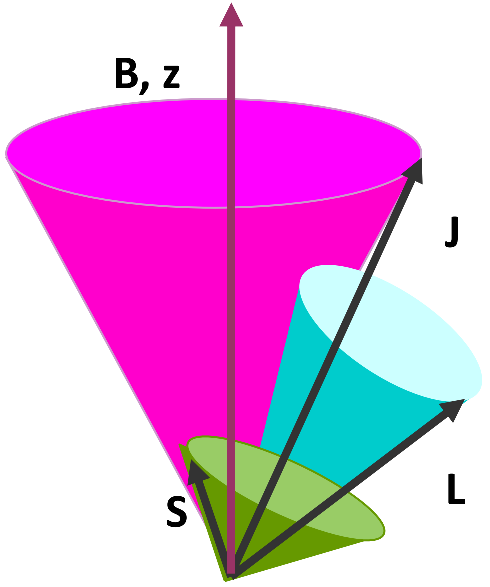
- ***Reference Videos***

  1. *https://nptel.ac.in/courses/115/106/115106061/*
  2. *https://nptel.ac.in/courses/115/104/115104109/*

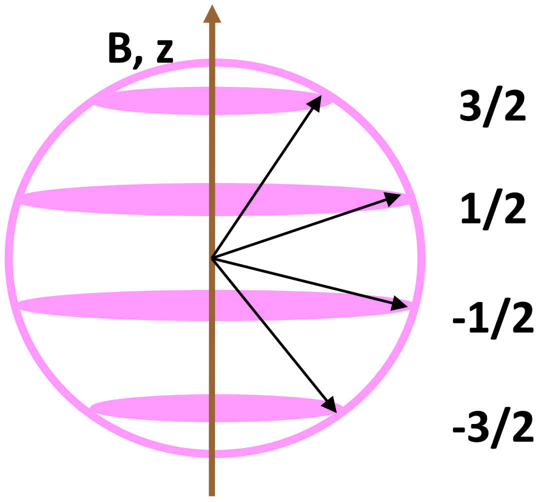
Precession of total angular momentum

- *Permanent magnetic moment of an atom cannot precess around the applied field freely*
- *Restricted to only a finite set of orientations*
- *For a given J, the allowed orientations are 2j+1*

  -j, -j+1…….. j-1, j



Precession of J about B



m_J values for L=1, S=1/2

Quantum theory of paramagnetism – Average magnetisation

- μ - component of total magnetic moment in the direction of the magnetic field
- Dipole moment

$$\mu = g\mu_B m_j$$

- Magnetic energy of interaction

$$E_j = \mu . B = g\mu_B m_j . B = g\mu_o \mu_B H m_j$$

Average magnetization

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- *Average value of a quantized physical parameter using statistical approach*

$$A = \left[\frac{\sum_{-i}^{+i} A_i * exp^{-\beta E_i}}{\sum_{-i}^{+i} exp^{-\beta E_i}}\right]$$

where $\beta = 1/k_BT$ and k_BT is the thermal energy

Average magnetization of the paramagnetic material consisting of N number of atoms per unit volume

$$\mathrm{M} = \mathrm{N}\frac{\sum_{m_j=-j}^{j} \mu \exp\left(\frac{-E}{k_BT}\right)}{\sum_{m_j=-j}^{j} exp\left(\frac{-E}{k_BT}\right)}$$

$$\Rightarrow \mathrm{M} = \mathrm{N}\frac{\sum_{m_j=-j}^{j} m_j\, g\mu_B \exp\left(\frac{m_j g\mu_o\mu_B H}{k_BT}\right)}{\sum_{m_j=-j}^{j} exp\left(\frac{m_j g\mu_o\mu_B H}{k_BT}\right)}$$

1. Average magnetization in paramagnetic materials at low B

- *If $E \ll k_BT$, i.e, the interaction energy is very small compared to thermal energy*
- $m_j g \mu_o \mu_B H \ll k_BT$

$$\Rightarrow exp\left(\frac{m_j g \mu_o \mu_B H}{k_BT}\right) \approx 1 + \frac{m_j g \mu_o \mu_B H}{k_BT}$$

$$\Rightarrow M \approx \frac{\sum_{m_j=-j}^{j} N m_j g \mu_B \left(1 + \frac{m_j g \mu_o \mu_B H}{k_BT}\right)}{\sum_{m_j=-j}^{j} 1 + \frac{m_j g_j \mu_o \mu_B H}{k_BT}}$$

- *Note*

$$\sum_{m_j=-j}^{j} 1 = 2j+1, \qquad \sum_{m_j=-j}^{j} m_j = 0, \qquad \sum_{m_j=-j}^{j} m_j^2 = \frac{j(j+1)(2j+1)}{3}$$

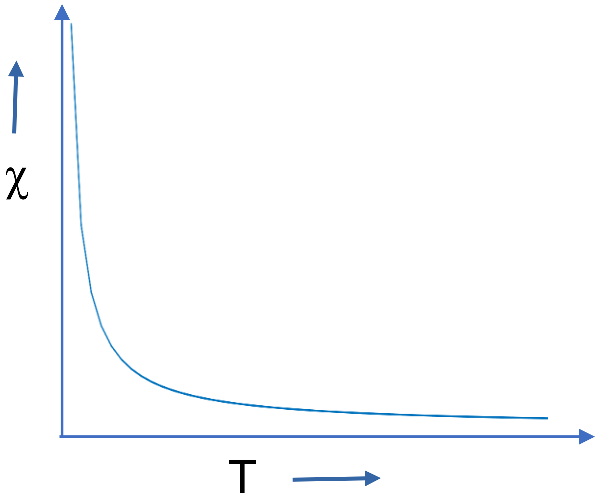
Susceptibility of paramagnetic materials: Low magnetization

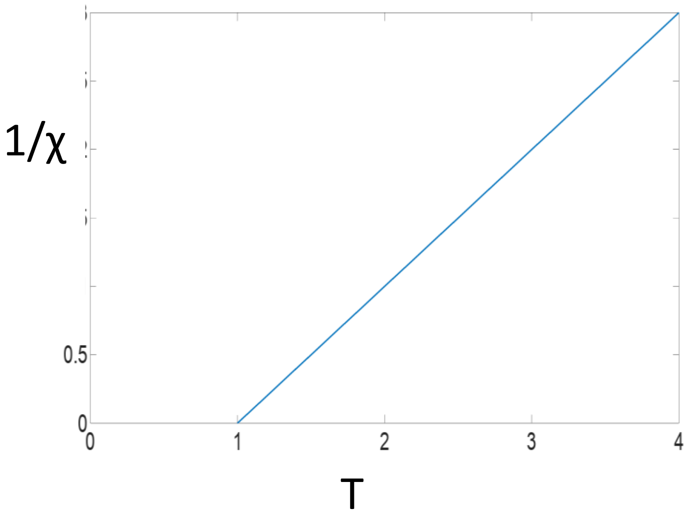
$$M = \frac{Ng^2\mu_o\mu_B^2}{k_BT}\,\frac{j(j+1)}{3}H = \chi H$$

$$\chi = \frac{Ng^2\mu_o\mu_B^2}{k_BT}\,\frac{j(j+1)}{3} = \frac{C}{T} \Rightarrow \chi\ \alpha\ \frac{1}{T}$$

Curie's law

where C is the Curie constant $C = \frac{Ng^2\mu_o\mu_B^2}{k_B}\,\frac{j(j+1)}{3}$.





Paramagnetic susceptibility in the case $E \ll K_BT$

2. Average magnetization in paramagnetic materials at high B

- *If $E \gg k_BT$, i.e, the interaction energy is high compared to thermal energy ie.., $m_j g\mu_o\mu_B H \gg k_BT$*

$$M = N\frac{\sum_{m_j=-j}^{j} m_j\, g\mu_B \exp\left(\frac{m_j g\mu_o\mu_B H}{k_BT}\right)}{\sum_{m_j=-j}^{j} exp\left(\frac{m_j g\mu_o\mu_B H}{k_BT}\right)}$$

- *Using* $x = \frac{g\mu_o\mu_B H}{k_BT}$ ⇨ $M = Ng\mu_B\frac{\sum_{m_j=-j}^{j} m_j \exp(m_j x)}{\sum_{m_j=-j}^{j} exp\,(m_j x)}$

$$\Rightarrow M = Ng\mu_B\frac{d}{dx}\left[\ln\sum_{m_j=-j}^{j} exp\,(m_j x)\right]$$

- *Terms inside the summation is a geometric series and the summation can be easily evaluated*

Brillouin function

$$M = Ng\mu_B j\left[\frac{2j+1}{2j}coth\left(\frac{(2j+1)a}{2j}\right) - \frac{1}{2j}coth\left(\frac{a}{2j}\right)\right]$$

- *Term in the square bracket is the Brillouin function*

  $B_j(a)$ **with** $a = jx$

- *At low temperatures, all dipoles align along* ***H***

  leading to saturation of magnetization M_s *with*

  $M_s = Ng\mu_B j$

$$\Rightarrow \frac{M}{M_s} = B_j(a) \qquad \textbf{or} \qquad M = M_s B_j(a)$$

Class #49 …….. Quiz….

The concepts related to quantum theory of paramagnetism which are true are……

1. Permanent magnetic moment of an atom can precess freely around the applied field
2. Magnetization of large number of atoms can be obtained by evaluating statistical average
3. Magnetic susceptibility of paramagnetic materials varies inversely with temperature
4. Brillouin function $B_j(x)$ is a function of x and exists for all values of j
5. At low temperature, all the dipoles align in the direction of applied field

THANK YOU

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