UE25MA141A: ENGINEERING MATHEMATHICS - I

Unit - 2: HIgher Order Ordinary Differential Equations

Department of Science and Humanities

PES UNIVERSITY

Finding Particular Integral of ODE
Type 1: when $X = e^{ax}$

For the differential equation $f(D)Y = X$, the particular integral is defined as

$$P.I = \frac{1}{f(D)}X$$

Type 1: When $X = e^{ax}$

$$P.I = \frac{e^{ax}}{f(D)} = \frac{e^{ax}}{f(a)}, \quad \text{provided} \quad f(a) \neq 0$$

If $f(a) = 0$, then P.I$=x\frac{e^{ax}}{f'(a)}$, provided $f'(a) \neq 0$
If $f'(a) = 0$, then P.I$=x^2\frac{e^{ax}}{f''(a)}$, provided $f''(a) \neq 0$, similarly, it proceeds on when $f''(a) = 0$......

Problem 1

Find the Particular Integral (PI) of the differential equation:

$$D^2y + 2Dy + 5y = 1$$

where $D = \frac{d}{dx}$ is the differential operator.

Solution:
We can write $X = 1 = e^{0x}$

$$\text{Then, P.I} = \frac{X}{f(D)} = \frac{e^{0x}}{D^2 + 2D + 5} = \frac{1}{5}, \quad \text{where} \quad D = a = 0$$

$$\boxed{\text{PI} = \frac{1}{5}}$$

Problem 2

Find the Particular Integral (PI) of the differential equation:

$$y''(x) + 4y'(x) + 4y = e^{-2x}$$

Solution:

Here $X = e^{-2x}$, so $a = -2$ and $f(D) = D^2 + 4D + 4$
When $a = -2$, $f(a) = 0$ then find $f'(D) = 2D + 4 \Rightarrow f'(a) = 0$
Since $f'(a) = 0$ then $f''(D) = 2 \Rightarrow f''(a) = 2 \neq 0$

$$\boxed{\text{PI} = \frac{x^2 e^{-2x}}{f''(a)} = \frac{x^2 e^{-2x}}{2}}$$

Problem 3

Find the Particular Integral (PI) of the differential equation:

$$D^3y - 3Dy + 2y = 2\sinh x$$

where $D = \frac{d}{dx}$ is the differential operator.

Solution:

We express the right-hand side using the identity:

$$\sinh x = \frac{e^x - e^{-x}}{2} \Rightarrow 2\sinh x = e^x - e^{-x}$$

So the differential equation becomes:

$$f(D) = (D^3 - 3D + 2)y = e^x - e^{-x}$$

$$\text{Then, PI} = \frac{1}{D^3 - 3D + 2}(e^x - e^{-x}) = \text{PI}_1 + \text{PI}_2$$

where:

$$\text{PI}_1 = \frac{1}{D^3 - 3D + 2}e^x, \quad \text{PI}_2 = \frac{-1}{D^3 - 3D + 2}e^{-x}$$

$$\text{In } \mathrm{PI}_1, a = 1 \quad \text{and } f(a) = 0, f'(a) = 0 \quad \text{so, find} f''(a)$$

$$f''(D) = 6D = 6, \Rightarrow f''(a) = 6$$

$$\mathrm{PI}_1 = \frac{x^2 e^x}{6} \quad \text{then}$$

$$\text{In } \mathrm{PI}_2, \quad a = -1 \quad \text{and} \quad f(a) = 4 \quad \text{then, } \mathrm{PI}_2 = \frac{e^{-x}}{4}$$

Final Particular Integral is

$$\boxed{\mathrm{PI} = \frac{x^2 e^x}{6} - \frac{e^{-x}}{4}}$$

Type 2: When $X = sin(ax+b)/cos(ax+b)$

When $X = sin(ax+b)$

$$P.I = \frac{sin(ax+b)}{f(D^2)} = \frac{sin(ax+b)}{f(-a^2)}, \quad \text{provided} \quad f(-a^2) \neq 0$$

where $D^2 = -a^2$

If $f(-a^2) = 0$, then P.I$=x\frac{sin(ax+b)}{f'(a)}$, provided $f'(-a^2) \neq 0$

If $f'(-a^2) = 0$, then P.I$=x^2\frac{sin(ax+b)}{f''(a)}$, provided $f''(-a^2) \neq 0$,
similarly, it proceeds on when $f''(a) = 0$......

Problem ~~2~~ 1

Find the Particular Integral (PI) of the differential equation:

$$(D^2+3)y = cos\sqrt{3}x$$

Solution:

$$P.I = \frac{cos\sqrt{3}x}{D^2+3} = x\frac{cos\sqrt{3}x}{2D}, \quad \text{since} \quad f(D^2) = f(-a^2) = 0$$

$$P.I = \frac{x}{2}\int cos\sqrt{3}x dx$$

$$\boxed{P.I = \frac{xsin\sqrt{3}x}{2\sqrt{3}}}$$

Problem 2

Find the Particular Integral (PI) of the differential equation:

$$(D^3 - 2D^2 + D)y = sin^2x$$

Solution:

$$\text{We can write} X = sin^2x = \frac{1 - cos2x}{2}$$

$$\text{P.I} = \frac{e^{0x}}{2(D^3 - 2D^2 + D)} - \frac{cos2x}{2(D^3 - 2D^2 + D)}$$

$$\text{P.I} = \frac{x}{2} - \frac{cos2x}{2(-4D + 8 + D)} = \frac{x}{2} - \frac{(8 + 3D)cos2x}{2(64 - 9D^2)}$$

$$\boxed{\text{P.I} = \frac{x}{2} - \frac{8cos2x}{100} + \frac{6sin2x}{100}}$$

THANK YOU