ENGINEERING MATHEMATICS I

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HIGHER ORDER DIFFERENTIAL EQUATIONS

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UNIT CONTENT:

- Linear Differential Equation of higher order
- Homogeneous and Non homogeneous Linear Differential Equation with constant coefficients
- Linear Differential Equation of higher order with variable coefficients
- Method of variation of parameters to solve second order Linear Differential Equation
- Applications to solve problems on Electric Circuits and SHM.

ENGINEERING MATHEMATICS I

HIGHER ORDER DIFFERENTIAL EQUATIONS

CLASS CONTENT:

- Introduction to Linear Differential Equation
- Higher Order Linear Differential Equation
- Different Forms of Linear Differential Equation
- Linear Differential Equation with constant coefficients and variable coefficients
- Complementary Function and Particular Integral

Ordinary Differential Equation

Linear Differential Equation

$$\text{Eg. } \frac{dy}{dx} + 2xy = x^2$$

Non - Linear Differential Equation

$$\text{Eg. } y^2 . \frac{dy}{dx} + \sin y = x^2 e^{2y}$$

A differential equation in which the dependent variable and its derivative occur in first degree and are not multiplied together is a Linear Differential Equation.

$$\frac{dy}{dx} + P.\,y = Q \quad \text{where } P \text{ and } Q \text{ are functions of } x \text{ alone}$$

LINEAR DIFFERENTIAL EQUATIONS OF DIFFERENT ORDER

A linear differential equation of first order is of the form

$$\frac{dy}{dx} + P.\,y = Q \quad \text{where } P \text{ and } Q \text{ are functions of } x \text{ alone.}$$

A linear differential equation of order two is of the form

$$\frac{d^2y}{dx^2} + P_1\frac{dy}{dx} + P_2 y = X \quad \text{where } P_1 \text{ and } P_2 \text{ and } X \text{ are functions of } x \text{ alone.}$$

A linear differential equation of order'n' is of the form

$$\frac{d^ny}{dx^n} + P_1\frac{d^{n-1}y}{dx^{n-1}} + P_2\frac{d^{n-2}y}{dx^{n-2}} + \text{-------} + P_n y = X$$

where $P_1, P_2, \text{---} P_n$ and X are functions of x alone.

HIGHER ORDER LINEAR DIFFERENTIAL EQUATION

A general linear differential equation of order n is of the form

$$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + \text{-------} + a_n y = X$$

where $a_1, a_2, \text{---} a_n$ are functions of x or constants

and X is a function of x alone.

Two forms of Higher Order linear differential equation of order n

1. Linear differential equation with constant coefficients

In this type, the coefficient of derivatives are constants.

$$Eg.\ 2\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + y = x^2$$

Two forms of Higher Order linear differential equation of order n

2. Linear differential equation with variable coefficients

$$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + \dots\dots\dots\dots + a_n y = X$$

In this type, the coefficient of derivatives a_1, a_2 -- a_n are functions of x.

Eg. $$2x^3 \frac{d^3 y}{dx^3} - (5x^2 + 2)\frac{d^2 y}{dx^2} + x\frac{dy}{dx} + y = x^2 \sin x$$

Linear differential equation with constant coefficients

$$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + \ldots\ldots\ldots\ldots\ldots + a_n y = X$$

HOMOGENEOUS LDE

RHS X is zero

Eg. $2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 9y = 0$

NON HOMOGENEOUS LDE

RHS X is a nonzero function of x

Eg. $2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 9y = x.\sinh x$

HIGHER ORDER LINEAR DIFFERENTIAL EQUATION

DIFFERENTIAL OPERATOR

$$D = \frac{d}{dx}, \qquad \frac{d^2}{dx^2} = D^2, \quad \frac{d^3}{dx^3} = D^3, \; - - \; \frac{d^n}{dx^n} = D^n$$

Thus, in terms of operator notation

$$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1}y}{dx^{n-1}} + a_2 \frac{d^{n-2}y}{dx^{n-2}} + \dots\dots\dots + a_n y = X \text{ can be written as}$$

$$D^n y + a_1 D^{n-1} y + a_2 D^{n-2} y + \dots\dots\dots + a_n y = X$$

$$f(D)y = X$$

$$\text{where} \quad f(D) = D^n + a_1 D^{n-1} + a_2 D^{n-2} + \dots\dots\dots + a_n$$

called linear differential operator.

Consider a non homogeneous equation of the type

$$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + - - - + a_n y = X$$

$$\mathbf{f(D)y = X}$$

The general solution of the non homogeneous linear differential equation has two components:
Complementary Function and particular Integral.

The complementary function is the complete solution of homogeneous linear differential equation.

Linear Dependence and Linear Independent Functions

A set of functions $f_1(x), f_2(x), - - f_n(x)$ are said to be linearly dependent on an interval 'I' if there exists constants $c_1, c_2 - - c_n$ not all zero, such that $c_1f_1 + c_2f_2 + - - - + c_nf_n = 0.$

If not linearly dependent, the functions are called linearly independent functions.

In other words, if the set is linearly independent, then

$$c_1f_1 + c_2f_2 + - - - - + c_nf_n = 0 \quad \Rightarrow \quad c_1 = c_2 = c_3 = - - -- = c_n = 0$$

Linear Dependent and Linear Independent Functions

Two functions and are linearly independent when +
0 for non zero constants and .

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SECOND ORDER HOMOGENEOUS LINEAR DIFFERENTIAL EQUATION WITH CONSTANT COEFFICIENTS:

Consider a second order LDE with constant coefficients of the type

$$\frac{d^2y}{dx^2} + k_1 \frac{dy}{dx} + k_2 y = 0$$

If and are two linearly independent solutions of a second order homogeneous linear differential equation, then + is also a solution of the differential equation.

It is the complete solution of the differential equation.

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EXTENSION OF THE PROPERTY TO A LINEAR DIFFERENTIAL EQUATION OF ORDER 'n' :

If the nth order LDE with constant coefficients

$$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + - - - + a_n y = 0 \text{ has 'n' linearly independent solutions}$$

$y_1, y_2, y_3 - - - y_n$, then their linear combination

$c_1 y_1 + c_2 y_2 + - - - + c_n y_n$ is also a solution of the differential equation.

$y = c_1 y_1 + c_2 y_2 + - - - + c_n y_n$, the complete solution is called

COMPLEMENTARY FUNCTION of the differential equation.

NON HOMOGENEOUS LINEAR DIFFERENTIAL EQUATION WITH CONSTANT COEFFICIENTS:

Consider a non homogeneous linear differential equation $f(D)y = X$

If $y = y_c$ is the complete solution of $f(D)y = 0$ and $y = y_p$ is a particular solution of the differential equation $f(D)y = X$, then the complete solution is $y = y_c + y_p$

The particular solution $'y_p'$ is called PARTICULAR INTEGRAL.

THANK YOU

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