Unit 4 Lasers

LASER: **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation

The first laser (ruby laser) was invented by Maiman in 1960. Although Einstein, in 1920s had built some theoretical framework of lasers, experimental development took so long to build the first laser. Lasers have resurrected the field of optics and find multitude of applications in science and technology.

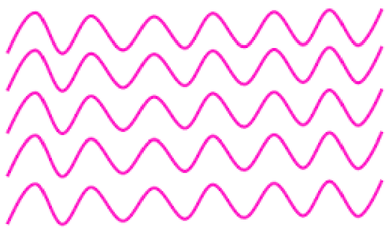
Properties of a laser beam

1. Coherence

Coherence of a laser beam refers to the phase inter-relationship between the waves of electromagnetic fields of light.

Spatial coherence

If the electric fields at different spatial locations across the laser beam has a strong phase correlation, then the laser is said to be spatially coherent, which means, generally, that the waves oscillate in a correlated way.



Coherent light waves

Temporal Coherence

If the electric fields of a laser are sampled at different times and if the samples exhibit a well defined phase correlation, then the laser is said to be temporally coherent. A perfectly monochromatic emission for example has perfect temporal coherence.
An experiment such as optical interference of beams derived from the same laser, but travelling different distances before they interfere can provide information about the temporal coherence of the beam.

Coherence length ($\boldsymbol{l_c}$) The distance upto which a beam exhibits temporal coherence. For example a pure sine wave has a coherence length of infinity.
Coherence time (τ_c): The time duration upto which a laser emission maintains its temporal coherence. The coherence time is given by

$\boldsymbol{\tau_c = \frac{1}{\Delta\upsilon}}$ **where $\Delta\upsilon$ is the line width of the laser**

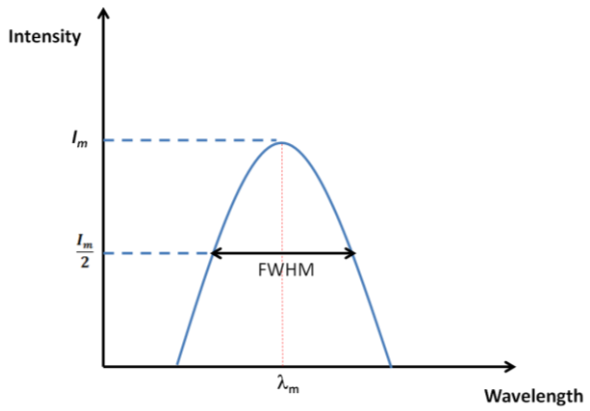
We also have $l_c = c\ \tau_c$

2. Monochromaticity

Laser is a highly monochromatic source of light. However it still has a certain amount of wavelength spread called line width defined generally as
$\Delta\lambda$ = FWHM, called the Full Width ant Half Maximum in the intensity versus wavelength plot as shown the fig. It can also be written in terms of

$$\Delta\upsilon = \frac{c\,\Delta\lambda}{\lambda_m^2}$$

where c is the speed of light and λ_m is the central wavelength; with maximum intensity.

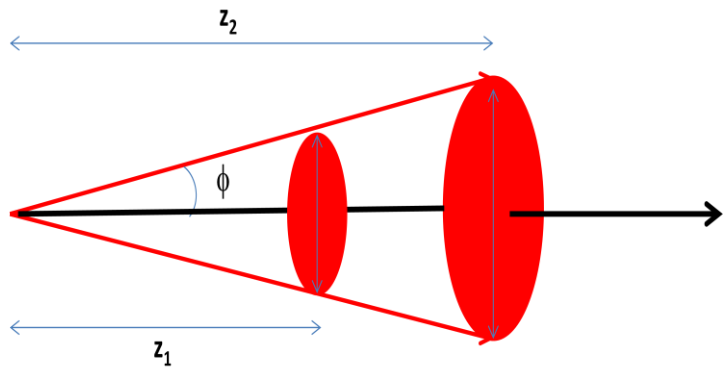


3. Directionality and (low) divergence

Lasers generally do not diverge much and therefore travel long distances without much loss in intensity. The divergence is given by

$$\phi_d = \frac{d_2 - d_1}{z_2 - z_1}$$

Here d_1 and d_2 the diameters of the beam at distances z_1 and z_2 respectively.



- $\textit{Spot size}\ \ s = \frac{\lambda}{\pi d_0}$

where d_0 is the diameter of the beam at the narrowest position
called the beam waist.

4. High intensity

Laser beams possess very high intensities because of the concentration of light energy in small beam cross sections and because of very small divergence.

Interaction of radiation with matter and Einstein coefficients

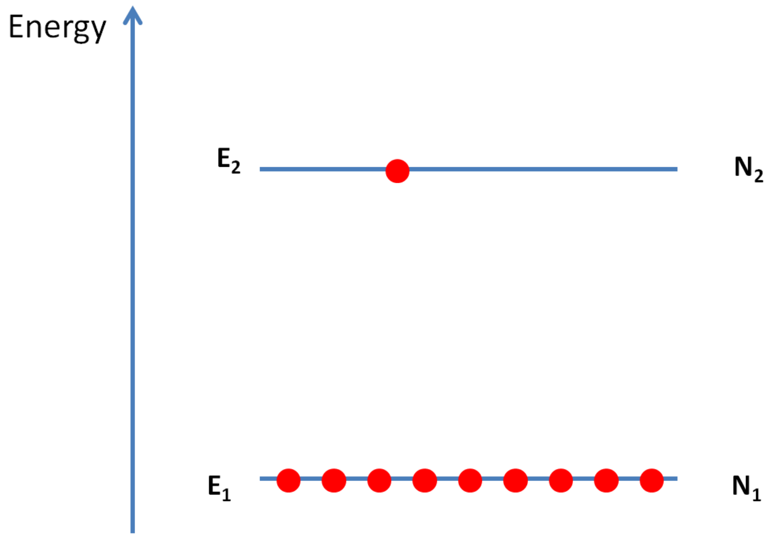
Consider a two level system with two states E_1 and E_2 with populations N_1 and N_2. The populations, under thermal equilibrium, are related by the Boltzman equation given by

$$\frac{N_2}{N_1} = e^{\frac{-(E_2-E_1)}{kT}}$$

Under thermal equilibrium, the ground state populations N_1 is much greater than the excited state population N_2.

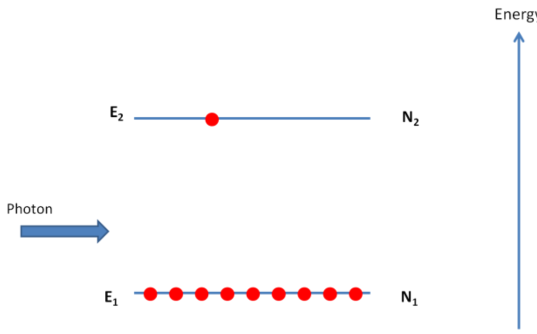
$N_2 >> N_1$

1. Light Absorption

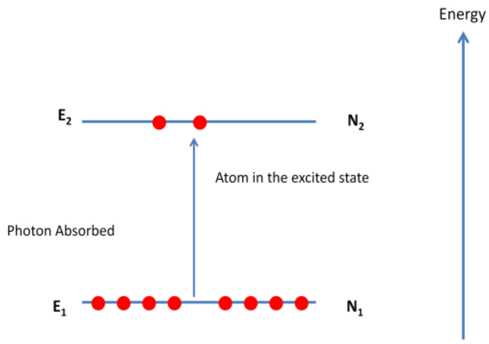


Consider a two state system under thermal equilibrium as shown in the fig.

When radiation with photons of energy equal to $(E_2 – E_1)$ is incident on such a system, there is a probability that an atom in the ground state can absorb the photon and jump to the excited state.



Before photon absorption



After photon absorption

The rate of absorption $$\boldsymbol{R_{Ab} = B_{12} N_1 U\ (\upsilon)}$$

where ***U (υ)*** is the energy density and $\mathbf{B_{12}}$ is the Einstein's absorption coefficient.

2. Spontaneous emission

An atom in excited state generally has life time of the order of 10^{-8} s, after which it de excites to the ground state spontaneously. This will release a photon of energy equal to ($E_2 - E_1$). The photons emitted during such de excitations are random in direction and there is no defined phase relationships between them. The rate of spontaneous emission is given by

$$\boldsymbol{R_{Sp.Em} = A_{21} N_2}$$

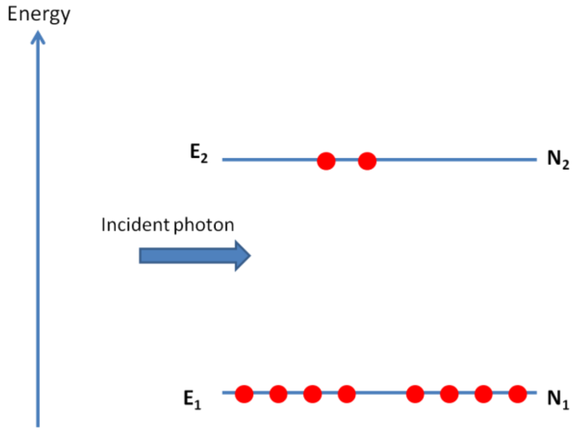
where $\mathbf{A_{21}}$ is called the Einstein's Spontaneous emission coefficient.

3. Stimulated emission

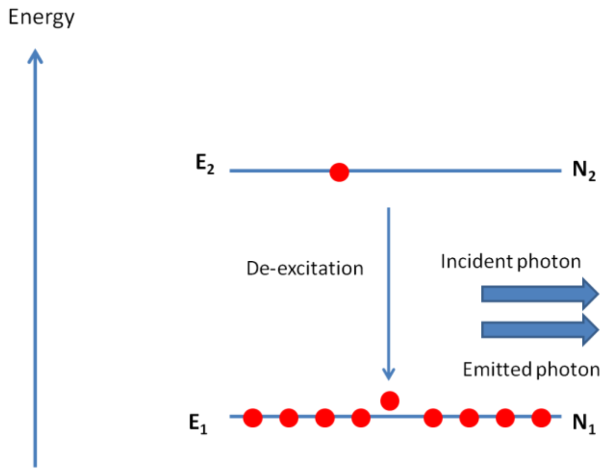
A system with atoms in the excited state may undergo de excitations in the presence of photons (of an electromagnetic field). If these stimulating photons have an energy equal to ($E_2 - E_1$), then the probability of de-excitation is very high. The rate of stimulated emission is given by

$$\boldsymbol{R_{St.Em} = B_{21}\ N_2\ U\ (\upsilon)}$$

Where B_{21} is Einstein's coefficient of stimulated emission.



Before emission



After stimulated emission

The emitted photons during stimulated emission are always in phase with the photons that caused the emission in the first place and also travel in the same direction. These photons cause a chain reaction of stimulated emission in the system and the intensity of the emitted beam increases exponentially. This beam is coherent and unidirectional.

Under a given situation which of the three processes happens is left to the probabilities based on the rates.

However because the number of atoms is fixed, and for continued process of excitations and de excitations under thermal equilibrium we must have

$$R_{Sp\ Em} + R_{St\ Em} = R_{Abs}$$

$$\mathrm{B_{12}N_1\ U(\upsilon)} = A_{21}N_1 + B_{21}\,N_2U\,(\upsilon)$$

$$U(\upsilon) = \frac{\mathrm{A_{21}N_1}}{\mathrm{(B_{12}N_1 - B_{21}\,N_2)}} \qquad (1)$$

This equation can be compared to Planck's radiation expression from radiation energy density in a cavity where energy transfer happens between radiation and matter.

$$U(\upsilon) = \frac{8\pi h\upsilon^3/c^3}{\left(e^{\left(\frac{h\upsilon}{kT}\right)} - 1\right)} \qquad (2)$$

To make equation (1) look like (2) make the following changes and also substitute from the Boltzmann equation

$$U(\upsilon) = \frac{\frac{A_{21}}{B_{12}}}{\left(\frac{N_1}{N_2} - \frac{B_{21}}{B_{12}}\right)}$$

$$U(\upsilon) = \frac{\frac{A_{21}}{B_{12}}}{\left(e^{\left(\frac{h\upsilon}{kT}\right)} - \frac{B_{21}}{B_{12}}\right)} \qquad (3)$$

Comparing (2) and (3)

$$\frac{A_{21}}{B_{12}} = \frac{8\pi h\upsilon^3}{c^3} \quad \text{and} \quad \frac{B_{21}}{B_{12}} = 1$$

Which implies that $B_{21} = B_{12} = B$

$A_{21} = A$

A and B are Einstein's coefficients

This makes at thermal equilibrium

$R_{Abs} > R_{St\,Em}$ for a given $U(\upsilon)$

and since $N_1 > N_2$ and $B_{21} = B_{12} = B$

which implies that at thermal equilibrium absorption dominates over Stimulated emission, crucial for laser emission. To obtain a situation where $R_{St\,Em} > R_{Abs}$, we need a condition where $N_2 > N_1$ which is impossible under thermal equilibrium.

Population Inversion [Achieving $N_2 > N1$]

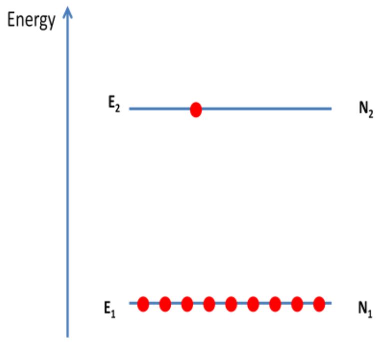
Population inversion, $N_2 > N1$, is only possible if extremely high amount of energy is pumped into the system and the system is not under thermal equilibrium.

The system or mechanism which helps in achieving population inversion is called an energy pump. There are different types of mechanisms, such as, optical, electrical, chemical and other which are chosen based on the effectiveness in achieving population inversion.

For example, in a ruby laser, a xenon flash lamp; in He-Ne and CO2 laser, electric discharge using a high voltage supply, serve as the energy pump mechanism. In semiconductor laser a high forward bias current in a heavily doped diode does the job.

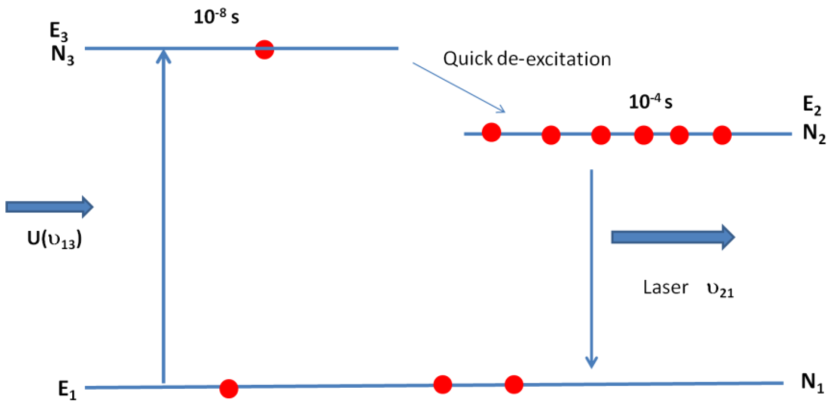
Two level, three level and four level lasers

1. Two level laser



In a two level laser population inversion is very hard to achieve as increasing the pump energy increases both absorption and stimulated emission simultaneously, unless an unusually extreme energy is pumped in an ultra short time. However such systems are inefficient and not practical.

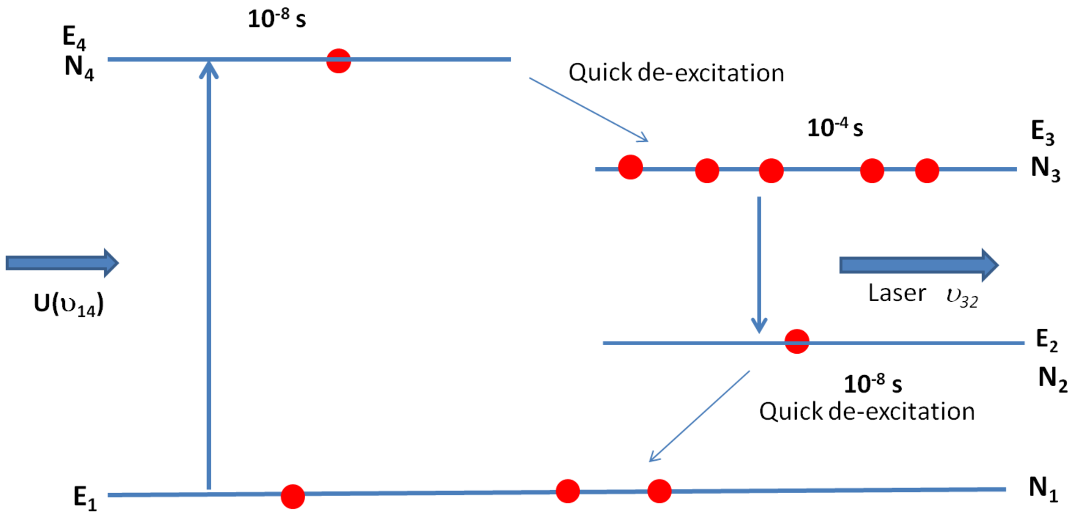
2. Three level laser



In a three level laser, the frequency **(υ_{13}) of E_3-E_1** and **(υ_{21}) of E_2-E_1** transitions are decoupled. This allows in achieving an excitation into E_3 by using a pump **$U(\upsilon_{13})$.** If **E_3** is normal state (life time ~ 10^{-8} s) that quickly undergoes a transition to **E_2 and if E_2** is a metastable state (life time ~ 10^{-4} s) then a population inversion can be achieved between **E_2 and E_1.**

Stimulated inversion can therefore work producing a laser of frequency $\upsilon_{21.}$ However, since we are depleting the ground state population to achieve this, continuous laser operation is not possible and only a pulsed laser can be obtained.

3. Four level laser



- In a four level laser, the pump frequency is υ_{14} and the laser frequency is υ_{32}.

- **E_4** is a short life time state, **E_3** is a metastable state. **E_3** can be populated easily if an energy pump can excite atoms into the **E_4** state, from where they would quickly de-excite to **E_3.**

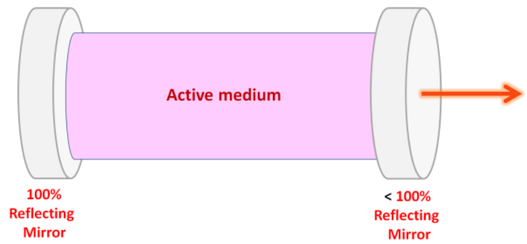
- Population inversion is achieved between **E_3 and E_2** and laser emission is possible.

- Since **E_2** is not the ground state, and since we do not have to draw a large number of atoms from the ground state **E_1** to achieve population inversion between **E_3 and E_2,** we do not need a very powerful energy pump.

- There is always a reservoir of atoms in the ground state **E_1** at any given point of time and therefore the laser can work continuously.

Basic requirements of a laser system

Active medium

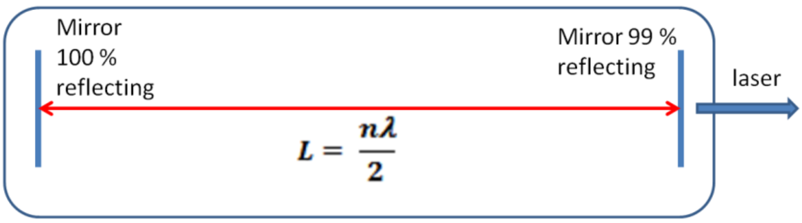


The active medium is the material medium (solid or liquid or gas) which acts as the host with suitable energy levels amongst which transitions can take place and population inversion can be achieved. The presence of the meta stable states increases the probability of population inversion which is a prime condition for laser action.

Energy pump

To achieve population inversion in an active medium, an external energy source is needed. The external energy sources could be optical, electrical or chemical depending on the type of lasers. In the case of gas lasers, generally an electrical discharge is a sufficient source for exciting the medium.

Resonating Cavity

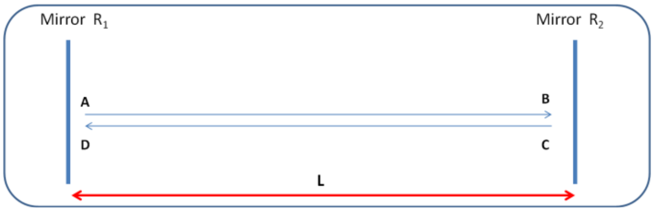


Once the lasing action is initiated it is essential that the stimulated emission in the desired wavelength is amplified to get a sustainable laser action of sufficient intensity. The design of the optical cavity is an important aspect of the laser system. In general the optical cavity has to be a narrow region whose length in the direction of propagation is a multiple of the desired wavelength.

$$L = \frac{n\lambda}{2}$$

This also helps in eliminating undesired wavelengths which may be present in the lasing process and increase the monochromaticity of the system.

Round Trip Gain and Threshold gain



Let the reflection coefficients of the mirrors be $\mathbf{R_1}$ **and** $\mathbf{R_2}$**.**
Let the beam start with an intensity

$$I_{initial} = I_0$$

at point A situated at mirror R_1. As the beam travels, because of stimulated emission, the intensity grows exponentially as function of length. But, because of the losses the intensity decreases exponentially as well. Let the gain factor be ***g*** and the loss factor be $\boldsymbol{\alpha}$**.**

By the time the beam reaches point located at mirror R_2, the beam would have travelled a distance L and the intensity at B can be written as

$$I_B = I_0\, e^{(g-\alpha)L}$$

After reflection from R2, the intensity reduces by a factor R2 and the intensity at point C is given by

$$I_C = R_2 I_0\, e^{(g-\alpha)L}$$

The beam travels a distance L again to reach point D, so the intensity is now

$$I_D = R_2 I_0\, e^{(g-\alpha)2L}$$

It is now reflected at R_1 and its intensity just before joining point A is

$$I_{final} = R_1 R_2 I_0\, e^{(g-\alpha)2L}$$

So in a round trip in the cavity, the round trip gain in intensity is

$$\text{Gain } \beta = \frac{I_{final}}{I_{initial}}$$

$$\beta = \frac{R_1 R_2 I_0\, e^{(g-\alpha)2L}}{I_0} = R_1 R_2\, e^{(g-\alpha)2L}$$

Because are very lossy systems, a round trip gain of slightly above 1 is very practical and should be enough as over millions of such trips per second (remember the speed of light).

Assuming the value of 1 to be the threshold gain beyond which a laser is possible, we can write

$$R_1 R_2\, e^{(g_{th}-\alpha)2L} = 1$$

Where we have called the gain coefficient as g_{th}, the threshold gain factor.

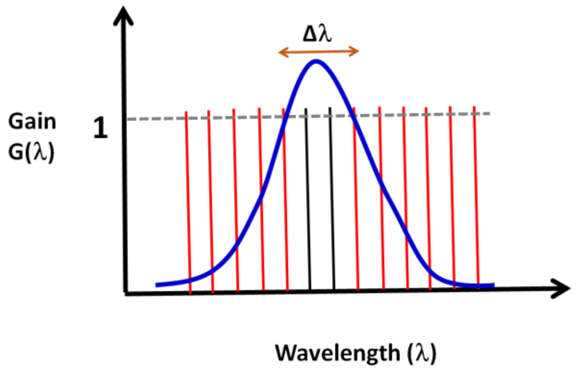
$$e^{(g_{th}-\alpha)2L} = \frac{1}{R_1 R_2}$$

$$g_{th} - \alpha = \frac{1}{2L}\, ln\left(\frac{1}{R_1 R_2}\right)$$

The threshold gain factor is given by

$$g_{th} = \alpha + \frac{1}{2L}\, ln\left(\frac{1}{R_1 R_2}\right)$$

Frequency Comb and Gain Curve

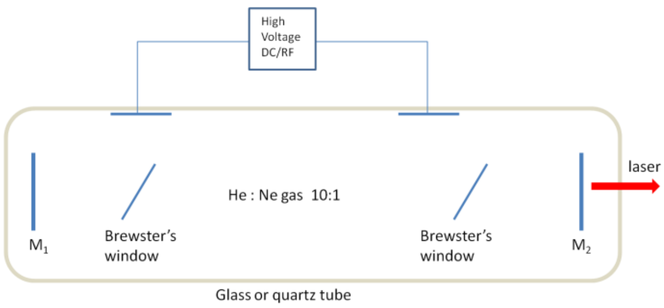


He Ne Laser

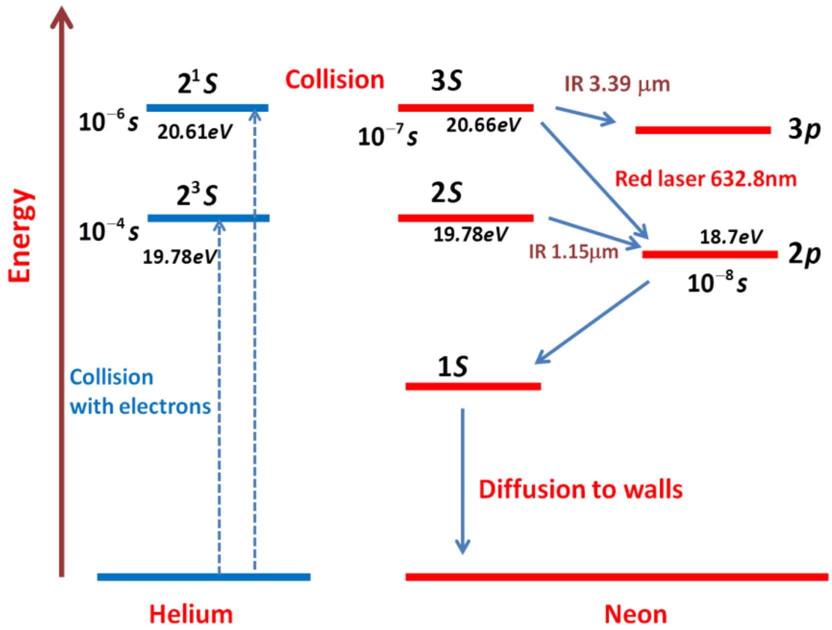
- First gas laser
- Atomic excitations of electrons in the atoms of Ne
- 632.8 nm laser (visible)
- Continuous Four level laser
- High quality beam
- Power ~ a few mW

Construction

A DC or RF discharge is established through a gas mixture containing typically 1 mmHg of He and 0.1 mmHg of Ne. Energetic electrons in the discharge excite electrons in the helium atoms to 2_3S and 2_1S states which are metastable. When these helium atoms collide with the neon atoms they excite electrons of neon atoms into 2S and 3S states (which nearly coincide with the 23S and 21S of He).



Energy level diagram
(Yariv A, Optical electronics)



Collision of the first kind $e^* + He \rightarrow e + He^*$

Collision of the second kind $He^* + Ne \rightarrow He + Ne^*$

Where * indicates energetic/excited.

632. 8 nm laser: The transition is from ***Ne 3S level to the 2p***. The *2p* decays to the long lived *1S* state. This 1S state if not emptied quickly can allow excitation back into *2p* involving collisions with fast moving electrons, hurting population inversion of the lasing states *3S and 2p*.
To depopulate the 1S state, a long narrow glass tube is used to house the gases whose larger internal surface area causes more Neon atoms to collide (with the glass tube) and de-excite to the ground state. For this reason the gain of 632.8 nm radiation increases with decreasing diameter of the tube.

This He Ne red laser with 632.8 nm is a very popular one and the beam quality is also excellent in general. The power is in a few mW.

1.15 μm radiation. A transition develops across ***2S to 2p*** and emits an infra red radiation of wavelength 1.15 μm. To suppress this radiation (dominant) a small amount of methane gas is also introduced. Brewster windows can also absorb some amount of IR.

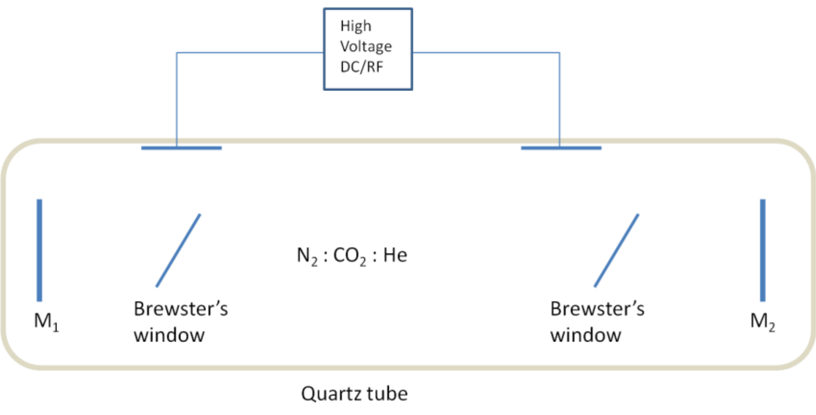
3.35 μm radiation. *This involves* $\mathbf{3S-3p}$ transition and thus uses the same upper level 3S. As this also is in IR range, it needs to be absorbed so that 632.8 nm laser has the dominating gain.

The Brewster windows apart from absorbing IR also act as laser beam polarizers.

The CO_2 laser

- **First molecular laser**
- **Infra Red laser**
- **Transitions across molecular energy levels (Not electronic excitations)**
- **Extremely powerful (Can go up to kW)**
- **Finds great deal of applications in industry**

Construction

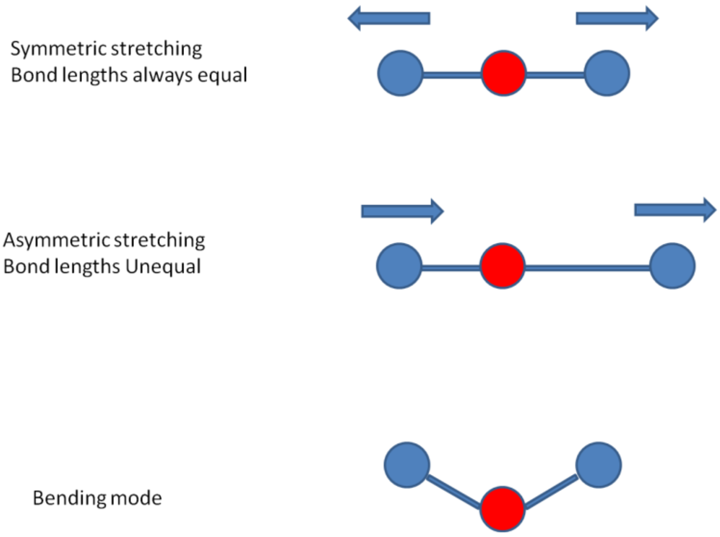


A quartz tube with N_2, CO_2 and He gas mixture is used. The mirrors and Brewster windows are made of semiconducting materials such as Ge to avoid IR absorption.

CO2 molecules has three modes of vibration: Symmetric stretching, Asymmetric stretching and Bending mode.

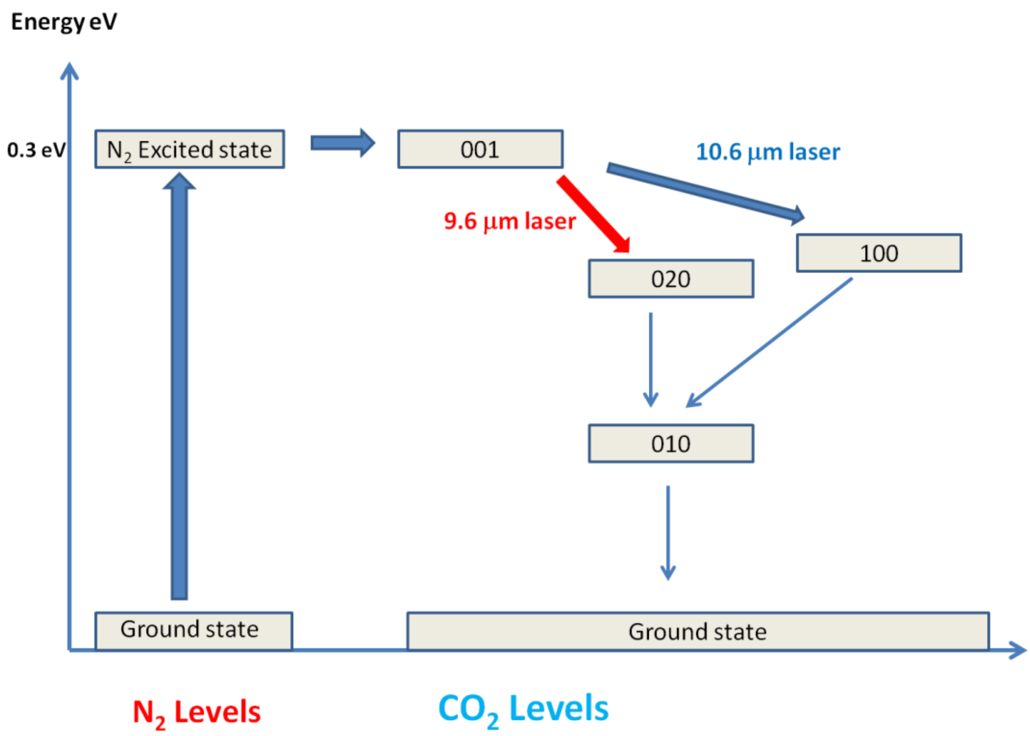
- The quantized energies of the symmetric stretching are denoted as (n00)
- The quantized energies of the asymmetric stretching are denoted as (00n)
- The quantized energies of the bending mode are denoted as (0n0).

where n is a positive integer.



Energy level diagram

Fast moving electrons from the discharge collide with the more numerous N_2 molecules and excite them to their first excited state at 0.3 eV. These excited N_2 molecules then collide with CO_2 molecules and selectively excite them to the asymmetric 001 state.



Collision of the first kind $e^* + N_2 \rightarrow e + N_2{}^*$

Collision of the second kind $N_2{}^* + CO_2 \rightarrow N_2 + CO_2{}^*$

Where star indicates energetic/excited.

10.6 μm laser. The laser transition of 10.6 μm takes place between the 001 state and the 100 state.

9.6 μm radiation. This radiation is emitted when transitions take place between 001 level and 020 level.

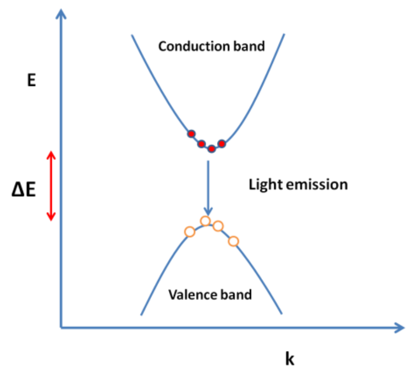
From 100 and 020 levels the CO2 molecule would quickly de-excite to 010 level which is metastable state. This 010 state is depopulated by collisions of CO2 with a buffer gas such as He.

Out of 10.6 μm and 9.6 μm radiation one of them can be chosen by creating a resonating cavity of suitable length L.

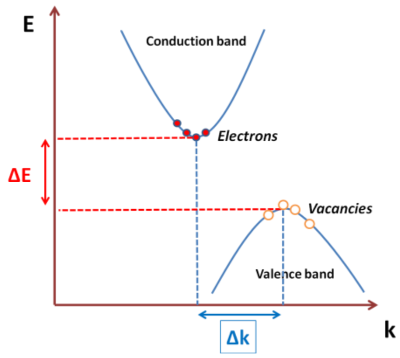
Because these lasers emit very high power (kW) external cooling is necessary, through means such as by circulating water.

Semiconductor lasers

Semiconductor lasers diode are created using direct band gap semiconductors such GaAs. AlGaAs, GaP, InGaP etc.



Direct band Gap



Indirect Band Gap

Though a single junction diode might emit light due to electron recombination when an electron moves from the **n** region to the **p** region, the probability of recombination is quite small and stimulation emission rate is negligible. Light emission is only possible at high currents such as hundreds of amperes.

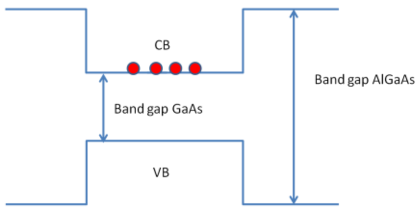
Heterojunction laser

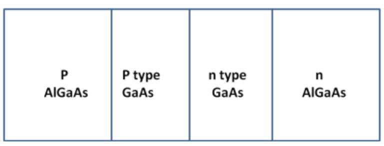
A heterojunction laser has two advantages over a homo-junction laser.

1. Charge confinement

1. A double hetero structure diode has two junctions which are between two different band gap semiconductors (GaAs and AlGaAs). The GaAs active layer has a lower band gap than the AlGaAs layers on either side. This results in a population of electrons in the conduction band of the GaAs layer from the n type AlGaAs layer and a population of holes in the GaAs layer from the p type AlGaAs. The population of the electrons and hole in the GaAs layer can recombine in the forward bias condition resulting in stimulated emission of photons. This requires less current to establish the required concentration of electrons for population inversion.

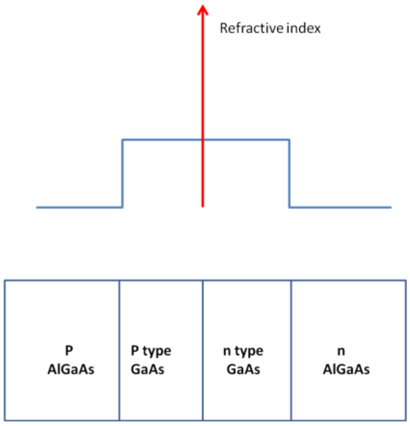
$The\ band\ gap\ E_g = \frac{hc}{\lambda}$. From this expression we can determine wavelength of the laser.





2. Photon confinement

The other aspect of confining all the emitted photons to a narrow region can be achieved by constructing a dielectric waveguide around the optical gain region and increase the probability of stimulated emission. The n and p type AlGaAs on either side have lower refractive index than the GaAs region which result in an increase in the number of photons travelling along the cavity axis by total internal reflection.



Semiconductor lasers generally operate at low currents and can produce up to a few watts of power. Though the beam quality is not as good as that of gas lasers, the technology is catching up.

With band gap engineering of semiconductor materials, we have been able to obtain up to blue emission, which in turn facilitates white light emission.

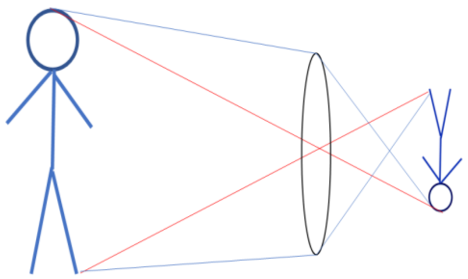
Holography

Photography

Normal photography only produces a 2D picture which lacks depth information. It only captures the intensity of light reflected from an object on a photographic film(or CCD). There is no information of the phase of the electromagnetic field falling on the film.

The intensity $I(x,y) = |E(x,y)|^2$

where E(x, y) is the electric field



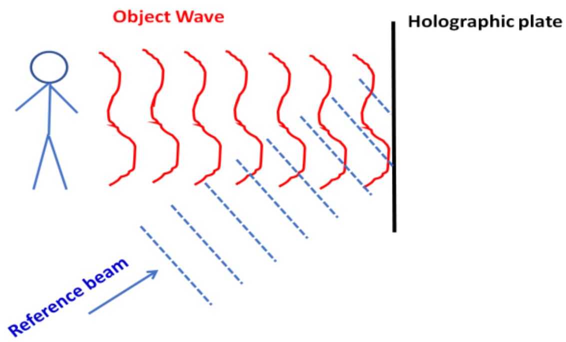
2D Photography

- **2D projection of 3D object**
- **Only intensity *I (x,y)* is captured**
- **Phase information is lost**
- **No depth information**

Holography

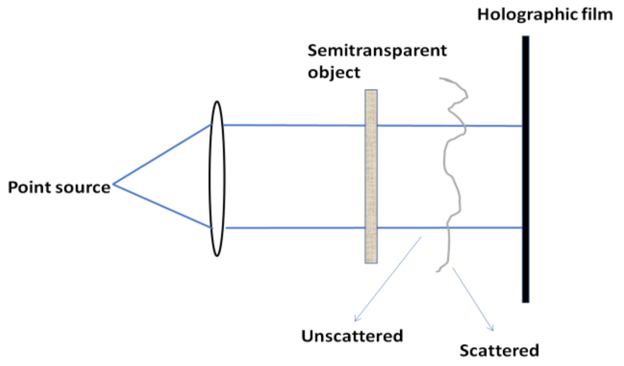
Holography creates 3D images of objects. By capturing the phase and intensity, it can produce true 3D image, which looks like the real object. To capture the phase information, a reference beam is used. The interference of the reference beam with the light reflected from the object produces a pattern on a special film called a hologram.

This hologram with the recorded interference pattern can produce a 3D image when the reference beam is passed through.



To retain phase information we create an Interference pattern on a photographic plate (hologram) *of light scattered from the object* and a *reference beam*

Inline holography



Consider an object wave (scattered from the object) represented by $E_o(x, y)$ and a reference beam (unscattered) represented by E_R which remains constant at all (x, y).

Intensity at the holographic plate

$$I = \left|E_O(x,y) + E_R\right|^2$$

$$I = \left(E_O^*(x,y) + E_R^*\right)\left(E_O(x,y) + E_R\right)$$

$$I = \left|E_O(x,y)\right|^2 + \left|E_R\right|^2 + E_O^*(x,y)E_R + E_O(x,y)E_R^* \text{------}(1)$$

Imagine that the holographic plate's transparency is proprtional to the light intensity

Transparency $T(x,y) = a + b\,I\,(x,y)$

where a and b are constants

The light passing through the hologram when illuminated only by the reference beam

$$E(x,y) = E_R\ T(x,y)$$

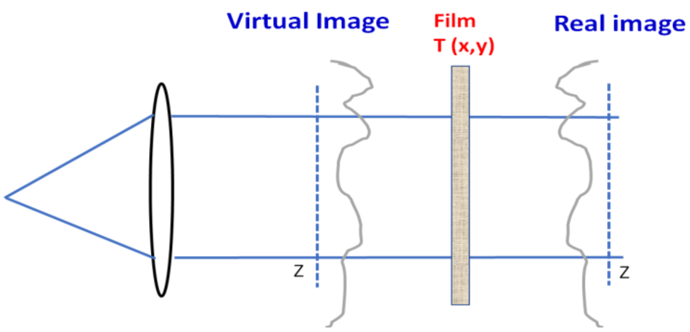
$$E = E_R(a + b\,I)$$

$$E = aE_R + b\,E_R\,I \qquad \textit{Substitute for I from eq(1)}$$

$$E = aE_R + bE_R(\left|E_O(x,y)\right|^2 + \left|E_R\right|^2 + E_O^*(x,y)E_R + E_O(x,y)E_R^*)$$

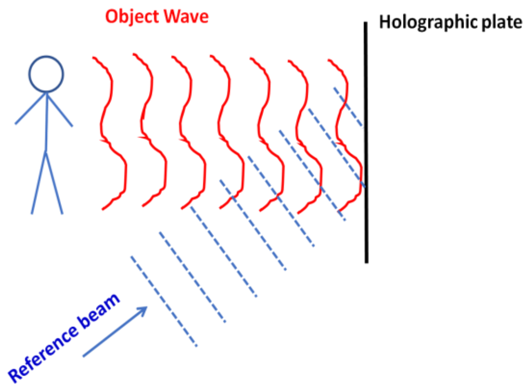
$$E(x,y) = aE_R + bE_R\left|E_R\right|^2 + bE_R\left|E_O(x,y)\right|^2 + bE_O^*(x,y)\left|E_R\right|^2 + E_O(x,y)E_R^2$$

$$
\begin{aligned}
E = \; & aE_R + bE_R\left|E_R\right|^2 && \text{Constant Term as } E_R \text{ is constant} \\
& + bE_R\left|E_o(x,y)\right|^2 && \text{Scattered: Negligible} \\
& + bE_o^*(x,y)\left|E_R\right|^2 && \text{Image of the object} \\
& + bE_o(x,y)E_R^2 && \text{Image of the object}
\end{aligned}
$$



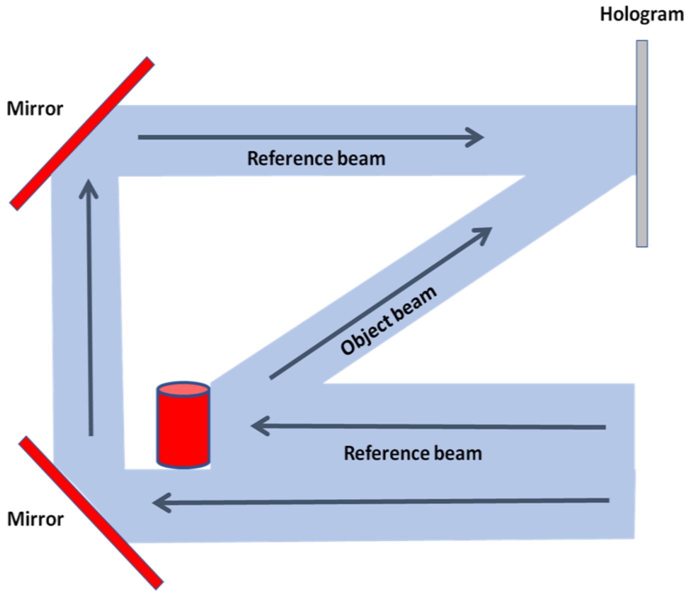
The twin images, the problem with inline holography

Off axis holography



Off- axis Holography solves the twin image problem

Construction of the hologram



Creation of the image