PES University, Bangalore

(Established Under Karnataka Act 16 of 2013)

Department of Science and Humanities

Engineering Mathematics - I (UE25MA141A)

Question Bank

Unit - 4: Special Functions

1. Evaluate $\left(\int_0^{\pi/2}\sqrt{\tan\theta}\,d\theta\right)\times\left(\int_0^{\pi/2}\frac{1}{\sqrt{\tan\theta}}\,d\theta\right)=\frac{1}{4}\left[\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)\right]^2$.
2. Show that $\left(\int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta\right)\times\left(\int_0^{\pi/2}\frac{1}{\sqrt{\sin\theta}}\,d\theta\right)=\pi$.
3. Show that $\int_0^{\infty}x^n e^{-a^2x^2}\,dx=\frac{1}{2a^{n+1}}\Gamma\left(\frac{n+1}{2}\right)$. **Deduce that** $\int_0^{\infty}e^{-a^2x^2}\,dx=\frac{\sqrt{\pi}}{2a}$.
4. Prove that $\left(\int_0^1\frac{x^2\,dx}{\sqrt{1-x^4}}\right)\times\left(\int_0^1\frac{dx}{\sqrt{1+x^4}}\right)=\frac{\pi}{4\sqrt{2}}$.
5. Show that $\int_0^{\infty}\frac{x^c}{c^x}\,dx=\frac{\Gamma(c+1)}{(\log_e c)^{c+1}}$.
6. Prove that $\Gamma(m)\cdot\Gamma\left(m+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$.
7. Prove that $\beta(n,n)=\frac{\sqrt{\pi}\cdot\Gamma(n)}{2^{2n-1}\Gamma\left(n+\frac{1}{2}\right)}$.
8. Show that $\int_0^1\frac{(1-x^4)^{3/4}}{(1+x^4)^2}\,dx=\frac{1}{2^{9/2}}\,\beta\left(\frac{7}{4},\frac{1}{4}\right)$
9. Express $\int_0^1 x^m(1-x^n)^p\,dx$ in terms of gamma function and hence, evaluate $\int_0^1\frac{dx}{\sqrt{1-x^n}}$.
10. Express $\int_0^1 x^7(1-x^4)^9\,dx$ in terms of gamma function and evaluate.
11. Prove that $\Gamma\left(\frac{p+1}{q}\right)=qa^{\frac{p+1}{q}}\int_0^{\infty}x^pe^{-ax^q}\,dx$, p and q are positive constants..
12. Legendre duplication formula for gamma function: $\sqrt{\pi}\,\Gamma(2m)=2^{2m-1}\Gamma(m)\Gamma\left(m+\frac{1}{2}\right)$.
13. Prove that $\int_a^b(x-a)^m(b-x)^n\,dx=(b-a)^{m+n+1}\beta(m+1,n+1)$.
14. Show that $J_{-\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\cos x$.
15. Prove that $J_{5/2}(x)=\sqrt{\frac{2}{\pi x}}\left\{\frac{3-x^2}{x^2}\sin x-\frac{3}{x}\cos x\right\}$.
16. Prove that $\int xJ_0^2(x)\,dx=\frac{1}{2}x^2\left(J_0^2(x)+J_1^2(x)\right)$.
17. Find $J_0(2)$ and $J_1(1)$ correct to three decimal places.
18. Show that $\int_0^{\pi/2}\sqrt{\pi x}\,J_{1/2}(2x)\,dx=1$.

19. Show that $\int_0^\infty J_0(x)J_1(x)\,dx = -\frac{1}{2}\left[J_0(x)\right]^2$.

20. Show that $\int_0^\infty e^{-ax}J_0(bx)\,dx = \frac{1}{\sqrt{a^2+b^2}}$.