ENGINEERING MATHEMATICS I

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HIGHER ORDER DIFFERENTIAL EQUATIONS

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CLASS CONTENT:

- Higher Order Linear Differential Equations With Variable Coefficients
- Legendre’s Linear Differential Equation
- Cauchy’s Linear differential Equation

LINEAR DIFFERENTIAL EQUATIONS WITH VARIABLE COEFFICIENTS

- We consider two forms of differential equations with variable coefficients which can be reduced to linear differential equations with constant coefficients by suitable substitutions.

- Legendre's Linear Differential equation; An equation of the form

$$k_0(ax+b)^n \frac{d^n y}{dx^n} + k_1(ax+b)^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + k_2(ax+b)^{n-2} \frac{d^{n-2} y}{dx^{n-2}} + \cdots + k_n y = X$$

where k_i's are constants and X is a function of x.

- Cauchy's Linear differential equation; An equation of the form

$$k_0 x^n \frac{d^n y}{dx^n} + k_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + k_2 x^{n-2} \frac{d^{n-1} y}{dx^{n-1}} + - - - - + k_n y = X,$$

where X is a function of x.

Legendre's Linear Differential Equation

- An nth order Legendre's Linear Differential Equation is of the form

$$k_0(ax+b)^n \frac{d^n y}{dx^n} + k_1(ax+b)^{n-1}\frac{d^{n-1}y}{dx^{n-1}} + k_2(ax+b)^{n-2}\frac{d^{n-1}y}{dx^{n-1}} + ----+k_n\, y = X$$

where k's are constants and X is a function of x.

- Such equations can be reduced to linear differential equations with constant coefficients by the substitution

$$ax+b=e^z \quad \text{or} \quad z=\log(ax+b)$$

$$\frac{dz}{dx}=\frac{a}{ax+b}$$

$$\frac{dy}{dx} = \frac{dy}{dz}\cdot\frac{dz}{dx} = \frac{dy}{dz}\cdot\frac{a}{ax+b}$$

$$(ax+b).\frac{dy}{dx} = a.\frac{dy}{dz} = a.D(y) \quad \text{where } D = \frac{d}{dz}$$

$$(ax+b)^2\frac{d^2y}{dx^2} = a^2 D(D-1)y$$

$$(ax+b)^3\frac{d^3y}{dx^3} = a^3 D(D-1)(D-2)y$$

using these results in Legendre's Differential Equation,

we get Linear Differential Equation with constant coefficients

which can be solved by the methods discussed before.

Second order Legendre’s Linear Differential Equation

- Consider Second order Legendre's Linear Differential Equation

$$(ax+b)^2\frac{d^2y}{dx^2}+k_1(ax+b)\frac{dy}{dx}+k_2\,y=X$$

- By the substitution $z=\log(ax+b)$, the above equation reduces to

$$\left(a^2D(D-1)+k_1aD+k_2\right)y=f(Z)$$

- It is a second order linear non homogeneous differential equation with constant coefficients which can be solved.

Cauchy's Linear Differential Equation

- An nth order Cauchy's Linear Differential Equation is of the form

$$x^n \frac{d^n y}{dx^n} + k_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + k_2 x^{n-2} \frac{d^{n-1} y}{dx^{n-1}} + - - - - + k_n\, y = X$$

where k's are constants and X is a function of x.

- Cauchy's Linear Differential Equation is a special case of Legendre's Linear Differential Equation in which

$a = 1$ and $b = 0$

Such equations can be reduced to linear differential equations with constant coefficients by the substitution

$$x = e^z \quad \text{or} \quad z = \log x$$

$$x.\frac{dy}{dx} = \frac{dy}{dz} = D(y) \quad \text{where } D = \frac{d}{dz}$$

$$x^2.\frac{d^2y}{dx^2} = D(D-1)y$$

$$x^3.\frac{d^3y}{dx^3} = D(D-1)(D-2)y$$

Second order Cauchy's Linear Differential Equation

- Consider Second order Cauchy's Linear Differential Equation

$$x^2 \frac{d^2y}{dx^2} + k_1 x \frac{dy}{dx} + k_2\, y = X$$

- By the substitution $z = \log x$, the above equation reduces to

$$\left(D.(D-1) + k_1 D + k_2\right) y = f(Z)$$

- It is a second order linear differential equation with constant coefficients which can be solved.

THANK YOU

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