# UE25MA141A: ENGINEERING MATHEMATHICS - I

## Unit - 2: Higher Order Differential Equations

Department of Science and Humanities

PES UNIVERSITY

# Problem 1

Find the particular integral for the differential equation

$$(D+1)(D+2)y = sin(e^x).$$

Solution:

$$y_p = \frac{1}{(D+1)(D+2)}[sin(e^x)]$$

Consider $\dfrac{1}{(D+1)(D+2)} = \dfrac{A}{(D+1)} + \dfrac{B}{(D+2)} = \dfrac{1}{(D+1)} - \dfrac{1}{(D+2)}$

$$\therefore y_p = \left[\frac{1}{(D+1)} - \frac{1}{(D+2)}\right] sin(e^x) = \frac{1}{(D+1)} sin(e^x) - \frac{1}{(D+2)} sin(e^x)$$

Consider $\dfrac{1}{(D+1)} sin(e^x) = e^{-x} \int e^x sin(e^x) dx$

using the substitution $t = e^x \Rightarrow dt = e^x dx$

$$e^{-x} \int e^x sin(e^x) dx = e^{-x} \int sint \ dt = e^{-x}(-cost) = -e^{-x} cos(e^x)$$

# Problem 1 (Contd...)

Consider $\dfrac{1}{D+2}\sin(e^x) = e^{-2x}\int e^{2x}\sin(e^x)\,dx$

using the substitution $t = e^x \Rightarrow dt = e^x dx$

$$e^{-2x}\int e^{2x}\sin(e^x)\,dx = e^{-2x}\int t\ sint\,dt = e^{-2x}\left[-t\cos(t) + \int \cos(t)\,dt\right]$$

$$= e^{-2x}\left[-t\cos(t) + sint\right] = e^{-2x}\left[-e^x\cos(e^x) + \sin(e^x)\right]$$

$$= -e^{-x}\cos(e^x) + e^{-2x}\sin(e^x)$$

$$\therefore y_p = \left[\frac{1}{(D+1)} - \frac{1}{(D+2)}\right] sin(e^x)$$

$$= \left[-e^{-x}cos(e^x)\right] - \left[-e^{-x}\cos(e^x) + e^{-2x}\sin(e^x)\right]$$

Thus, $y_p = -e^{-2x}\sin(e^x)$

# Problem 2

Find the particular integral for the differential equation $(D^2 + 5D + 6)y = e^{2x}\sin(e^x)$.

Solution:
Given: $(D^2 + 5D + 6)y = e^{2x}\sin(e^x)$

$$\Rightarrow (D+3)(D+2)y = e^{2x}\sin(e^x)$$

$$\Rightarrow y_p = \frac{1}{(D+2)(D+3)}e^{2x}\sin(e^x)$$

Using partial fractions:

$$\frac{1}{(D+2)(D+3)} = \frac{A}{D+2} + \frac{B}{D+3} = \frac{1}{D+2} - \frac{1}{D+3}$$

# Problem 2 (Contd...)

$$y_p = \left[\frac{1}{D+2} - \frac{1}{D+3}\right] e^{2x}\sin(e^x)$$

$$= e^{-2x}\int e^{4x}\sin(e^x)dx - e^{-3x}\int e^{5x}\sin(e^x)dx$$

Substitute $t = e^x \Rightarrow dt = e^x dx \Rightarrow dx = \frac{dt}{t}$

$$= e^{-2x}\int t^3 \sin t\, dt - e^{-3x}\int t^4 \sin t\, dt$$

Consider $e^{-2x}\int t^3 \sin t\, dt$
Using integration by parts:

$$= e^{-2x}\left[t^3(-\cos t) - 3t^2(-\sin t) + 6t\cos t - 6\sin t\right]$$

$$= e^{-2x}\left[-t^3\cos t + 3t^2\sin t + 6t\cos t - 6\sin t\right]$$

$$= -e^{-x}\cos(e^x) + 3\sin(e^x) + 6e^{-x}\cos(e^x) - 6e^{-2x}\sin(e^x) - -- \qquad (1)$$

Consider $e^{-3x}\int t^4 \sin t\, dt$

$$= e^{-3x}\left[-t^4\cos t + 4t^3\sin t + 12t^2\cos t - 24t\sin t - 24\cos t\right]$$

$$= -e^{x}\cos(e^x) + 4\sin(e^x) + 12e^{-x}\cos(e^x) - 24e^{-2x}\sin(e^x) - 24e^{-3x}\cos(e^x) \qquad (2)$$

# Problem 2 (Contd...)

$$y_p = (1) - (2)$$
$$y_p = [-e^{-x}\cos(e^x) + 3\sin(e^x) + 6e^{-x}\cos(e^x) - 6e^{-2x}\sin(e^x)] -$$

$$[-e^{x}\cos(e^x) + 4\sin(e^x) + 12e^{-x}\cos(e^x) - 24e^{-2x}\sin(e^x) - 24e^{-3x}\cos(e^x)]$$

$$\therefore \boxed{y_p = -\sin(e^x) - 6e^{-x}\cos(e^x) + 18e^{-2x}\sin(e^x) + 24e^{-3x}\cos(e^x)}$$