



ENGINEERING MATHEMATICS I

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ENGINEERING MATHEMATICS I

HIGHER ORDER DIFFERENTIAL EQUATIONS

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ENGINEERING MATHEMATICS I

NON HOMOGENEOUS LDE



CLASS CONTENT

- To solve a non - homogeneous linear differential equation of the type
 $f(D)y = x$ When $X = e^{ax}v$ where 'v' is a function of x.

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RULES TO FIND PARTICULAR INTEGRAL

- Type (iv); When $X = e^{ax} V$, where V is a function of x

$$\frac{1}{f(D)}(e^{ax}V) = e^{ax} \frac{1}{f(D+a)}V$$

- Type (v); When $X = xV$ where V is a function of x

$$\frac{1}{f(D)}(xV) = \left[\frac{x}{f(D)} - \frac{f'(D)}{(f(D))^2} \right] V$$

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1. Find the general solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x$

SOLUTION:

$$(D^2 - 2D + 4)y = e^x \cos x$$

$$\text{AE is } m^2 - 2m + 4 = 0$$

Roots are $1 \pm \sqrt{3}i$

$$y_c = e^x (c_1 \cos \sqrt{3}x + c_2 \sin \sqrt{3}x)$$



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$$y_p = \frac{e^x \cos x}{D^2 - 2D + 4}$$

$$= e^x \frac{\cos x}{(D+1)^2 - 2(D+1)+4}$$

$$= e^x \cdot \frac{\cos x}{D^2 + 3} = e^x \cdot \frac{\cos x}{-1^2 + 3}$$

$$\therefore y_p = \frac{e^x \cdot \cos x}{2}$$

Thus, $y = y_c + y_p$



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2. Find the general solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \cos x$

SOLUTION:

$$(D^2 - 2D + 1)y = xe^x \cos x$$

To find CF

$$AE \text{ is } m^2 - 2m + 1 = 0$$

Roots are $m = 1, 1$

$$\therefore y_c = (c_1 + c_2 x)e^x$$



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$$y_p = \frac{x e^x \cos x}{D^2 - 2D + 1}$$

$$= e^x \frac{x \cos x}{(D+1)^2 - 2(D+1) + 1}$$

$$= e^x \frac{x \cos x}{D^2 + 2D + 1 - 2D - 2 + 1}$$

$$= e^x \frac{x \cos x}{D^2}$$

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$$y_p = e^x \frac{1}{D} \int x \sin x dx$$

using integration by parts

$$y_p = e^x \frac{1}{D} \left[x \cdot (-\cos x) - \int (-\cos x) \cdot 1 dx \right]$$

$$y_p = e^x \int (-x \cos x + \sin x) dx$$

$$y_p = e^x \left[- (x \sin x + \cos x) - \cos x \right]$$

Thus, $y = y_c + y_p$





THANK YOU

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