PES University, Bangalore

(Established Under Karnataka Act 16 of 2013)

Department of Science and Humanities

Engineering Mathematics - I (UE25MA141A)

Question Bank

Unit - 2: Higher-Order Differential Equations

1. Solve the differential equation:

$$y''' - 6y'' + 11y' - 6y = 0$$

with initial conditions:

$$y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 0.$$

Answer: $y(x) = 3e^x - 3e^{2x} + e^{3x}$.

2. Solve the differential equation:

$$y^{(4)} + 4y''' + 6y'' + 4y' + y = 0$$

with initial conditions:

$$y(0) = 1, \quad y'(0) = -1, \quad y''(0) = 0, \quad y'''(0) = 1.$$

Answer:

$$y(x) = \left(1 - \frac{1}{2}x^2 + \frac{1}{6}x^3\right) e^{-x}$$

3. Solve the differential equation:

$$y^{(4)} - 5y'' + 4y = 0$$

with initial conditions:

$$y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = -1.$$

Answer:

$$y(x) = \frac{2}{3}e^x - \frac{2}{3}e^{-x} - \frac{1}{6}e^{2x} + \frac{1}{6}e^{-2x}$$

4. Solve the differential equation:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = -2\cosh x$$

Answer:

$$y = e^{-2x}\left[C_1 \cos x + C_2 \sin x\right] - \frac{e^x}{10} - \frac{e^{-x}}{2}$$

5. Solve the differential equation:

$$(D^3 - 12D + 16)y = (e^x + e^{-2x})^2$$

Answer:

$$y = (C_1 + C_2 x)e^{2x} + C_3 e^{-4x} + \frac{x^2 e^{2x}}{12} + \frac{2}{27}e^{-x} + \frac{xe^{-4x}}{36}$$

6. Solve the differential equation:

$$(D^2 - 4D + 3)y = \sin 3x \cos 2x$$

Answer:

$$y = C_1 e^x + C_2 e^{3x} + \frac{10\cos 5x - 11\sin 5x}{884} + \frac{\sin x + 2\cos x}{20}$$

7. Solve the differential equation:

$$(D^2 - 3D + 2)y = 2\cos(2x + 3) + 2e^x$$

Answer:

$$y = C_1 e^x + C_2 e^{2x} + \frac{3\sin(2x+3) + \cos(2x+3)}{10} - 2xe^x$$

8. Solve the differential equation:

$$(D^3 - D^2 - D + 1)y = 1 + x^2$$

Answer:

$$y = (C_1 + C_2 x)e^x + C_3 e^{-x} + x^2 + 2x + 5$$

9. Solve the differential equation:

$$(D^2 + 4)y = x^4 + \cos^2 x$$

Answer:

$$y = C_1 \cos 2x + C_2 \sin 2x + \frac{x^4}{4} - \frac{3x^2}{4} + \frac{1}{2} + \frac{x\sin 2x}{8}$$

10. Solve the differential equation:

$$(D^3 + 3D^2 + 2D)y = x^2 + 1$$

Answer:

$$y = C_1 + C_2 e^{-x} + C_3 e^{-2x} + \frac{x^3}{6} - \frac{3x^2}{4} + \frac{9x}{4}$$

11. Solve the differential equation:

$$(D^2 - 2D + 2)y = e^x x^2 + 5 + e^{-2x}$$

Answer:

$$y = e^x(C_1 \cos x + C_2 \sin x) + e^x(x^2 - 2) + \frac{5}{2} + \frac{e^{-2x}}{10}$$

12. Solve the differential equation:

$$(D^2 + 2D + 1)y = \frac{e^{-x}}{x^2}$$

Answer:

$$y = e^{-x}\left(C_1 + C_2 x - \ln|x|\right)$$

13. Solve the differential equation:

$$(D^2 - 4)y = \cosh(2x - 1) + 3^x$$

Answer:

$$y = C_1 e^{2x} + C_2 e^{-2x} + \frac{x \sinh(2x-1)}{4} + \frac{3^x}{(\ln 3)^2 - 4}$$

14. Solve the differential equation:

$$\frac{d^2y}{dx^2} + y = x \sin x$$

Answer:

$$y = C_1 \cos x + C_2 \sin x - \frac{x^2}{4} \cos x + \frac{x \sin 2x}{4} \cos x + \frac{\cos 2x}{8} \cos x - \frac{x \cos 2x}{4} \sin x + \frac{\sin 2x}{8} \sin x$$

15. Solve the differential equation:

$$\frac{d^2y}{dx^2} + y = cosecx \cot x$$

Answer:

$$y = C_1 \cos x + C_2 \sin x - \cos x \log|\sin x| - \sin x \cot x - x \sin x$$

16. Solve the differential equation:

$$\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + \frac{1}{x^2}y = \log x$$

Answer:

$$y(x) \;=\; C_1 x \;+\; C_2 x \ln x \;+\; \ln x + 2.$$

17. Solve

$$(x+2)^2\frac{d^2y}{dx^2} - (x+2)\frac{dy}{dx} + y = 3x+4.$$

Answer:

$$y = (C_1 + C_2 z)e^z + \frac{3z^2}{2}e^z - 2.$$

Substituting back $z = \log(x+2)$:

$$y = C_1(x+2) + C_2(x+2)\log(x+2) + \frac{3}{2}(x+2)\log^2(x+2) - 2.$$

18. Solve

$$(3x+2)^2\frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1.$$

Answer:

$$y = C_1e^{2z} + C_2e^{-2z} + \frac{ze^{2z}}{108} + \frac{1}{108}$$

Substitute $z = \log(3x+2)$:

$$e^{2z} = (3x+2)^2, \quad e^{-2z} = \frac{1}{(3x+2)^2}, \quad ze^{2z} = (3x+2)^2\log(3x+2)$$

$$y = C_1(3x+2)^2 + \frac{C_2}{(3x+2)^2} + \frac{(3x+2)^2\log(3x+2)}{108} + \frac{1}{108}$$

19. An uncharged condenser of capacity C is charged by applying an e.m.f $E\sin\left(\frac{t}{\sqrt{LC}}\right)$ through leads of self inductance L and negligible resistance. Prove that, at time t, the charge q on one of the plates is:

$$q = \frac{EC}{2}\left[\sin\left(\frac{t}{\sqrt{LC}}\right) - \left(\frac{t}{\sqrt{LC}}\right)\cos\left(\frac{t}{\sqrt{LC}}\right)\right]$$

Note: The differential equation governing the circuit is:

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = \frac{E}{L}\sin\left(\frac{t}{\sqrt{LC}}\right).$$

Take $\omega^2 = \frac{1}{LC}$.

20. A particle executes a simple harmonic motion such that in two of its positions on the same side of the mean position, its velocities are u, v and the corresponding accelerations are α, β. Show that the distance between these positions is: $\frac{v^2-u^2}{\alpha+\beta}$.