1. **Explain Compton shift and prove that for two photons of different initial energies scattered at the same angle, their Compton shifts are identical.**

The Compton shift is the change in the wavelength of a photon after scattering from a free electron due to energy and momentum transfer. It is given by $\boldsymbol{\Delta\lambda = (\lambda_f - \lambda_i)}$

Where λ_i is the unmodified component and λ_f is the modified component

From Compton's theory, $\boldsymbol{\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)}$

Where, h= Planck's constant; m_e= rest mass of electron; c= speed of light and $\boldsymbol{\theta}$ = scattering angle of the photon

From the above equation, $\boldsymbol{\Delta\lambda}$ is independent of the incident wavelength (λ) or photon energy $\left(E = \frac{hc}{\lambda}\right)$ and is dependent only on the angle of scattering

Therefore, For two photons with different initial wavelengths λ_1 and λ_2, if they scatter through the same angle $\boldsymbol{\theta}$, $\lambda_1 = \lambda_2 = \boldsymbol{\frac{h}{m_e c}(1 - \cos\theta)}$

So, no matter the initial photon energy (X-ray, gamma-ray, etc.), the Compton shift for a given scattering angle is the same. The scattered photon energy will differ, but the wavelength change remains constant.

2. **Discuss how the Compton Effect supports the concept of wave-particle duality**

The Compton Effect supports wave-particle duality by demonstrating the particle nature of light. In this effect, X-rays or gamma-ray photons collide with electrons and scatter, losing energy and increasing in wavelength. This can only be explained if photons have momentum and transfer it to electrons, like particles. The observed shift in wavelength agrees with the conservation of energy and momentum, which applies to particles—not waves. Thus, while light also shows wave behavior (like interference), the Compton effect confirms it also behaves like a particle, supporting the idea that light has both wave and particle properties.