ENGINEERING PHYSICS

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Barrier Potential – quantum tunneling

Class # 26

- In class 25 we discussed the solutions of Schrodinger's equation in the

➢ Region before the barrier – we got $\psi_I(x) = Ae^{ik_Ix} + Be^{-ik_Ix}$

➢ Region within the barrier – we got $\psi_{II}(x) = Ce^{\alpha x} + De^{-\alpha x}$

➢ Region after the barrier – we got $\psi_{III}(x) = Ge^{ik_{III}x}$

Barrier Potential – quantum tunneling

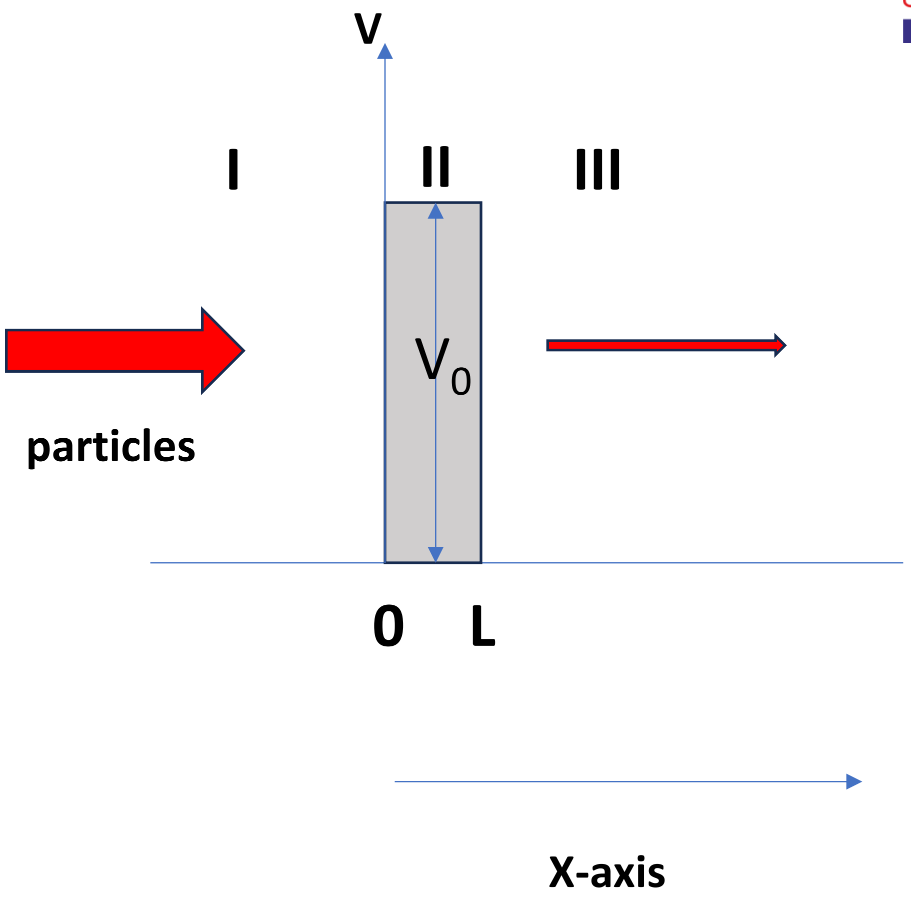
$\psi_I(x) = Ae^{ik_I x} + Be^{-ik_I x}$ says that particles are incident on the barrier as the first term in the incident wave and the particles get reflected as given by the reflected wave which is the second term

$\psi_{II}(x) = Ce^{\alpha x} + De^{-\alpha x}$ indicate an exponential decay of wavefunction but no definite conclusion as to what might be happening

$\psi_{III}(x) = Ge^{ik_{III} x}$ indicates a wave moving to the right which appears to be the transmitted wave. The conclusion is that real particles seem to appear in the region after the barrier – very COUNTERINTUITIVE

Barrier potential – quantum tunneling

- ***What is really going on?***
- ***The particles definitely did not have the energy to go over the barrier and somehow land themselves on the other side***
- ***QM says this is TUNNELING. The particles tunnel through the barrier and appear on the other side. The question is can we compute the transmission coefficient and the answer is yes***



Barrier potential- transmission coefficient

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When we solve the equations

1. $A + B = C + D$
2. $ik(A - B) = \alpha(C - D)$
3. $Ce^{\alpha L} + De^{-\alpha L} = Ge^{ikL}$
4. $\alpha\left(Ce^{\alpha L} - De^{-\alpha L}\right) = ikGe^{ikL}$

We can get the ratio G/A. The transmission coefficient is the defined as $\frac{\psi_t^*\psi_t}{\psi_i^*\psi_i}$ *where* ψ_i *is the incident wave function and*

ψ_t *is the transmitted wave function*

Barrier potential- transmission coefficient

Transmission coefficient $\mathrm{T} = \dfrac{\psi_t^* \psi_t}{\psi_i^* \psi_i}$

$\psi_i = Ae^{ikx}$ and $\psi_i^* = A^* e^{-ikx}$

$\psi_t = Ge^{ikx}$ and $\psi_t^* = G^* e^{-ikx}$

Hence $\mathrm{T} = \dfrac{\psi_t^* \psi_t}{\psi_i^* \psi_i} = \dfrac{G^* G}{A^* A} = \left(\dfrac{G}{A}\right)^* \dfrac{G}{A}$

Computing the ratio G/A we get the following

Barrier potential – transmission coefficient

- *It is easier to get the T^{-1}*
- $$T^{-1} = \frac{A^*A}{G^*G} = 1 + \frac{\sinh^2(\alpha L)}{4\left(\frac{E}{V_0}\right)\left(1-\frac{E}{V_0}\right)}$$
- **This can be simplified as follows**
- $$T^{-1} = 1 + \frac{\left(e^{\alpha L}-e^{-\alpha L}\right)^2}{16\left(\frac{E}{V_0}\right)\left(1-\frac{E}{V_0}\right)} = 1 + \frac{e^{2\alpha L}-e^{-2\alpha L}-2}{16\left(\frac{E}{V_0}\right)\left(1-\frac{E}{V_0}\right)}$$
- **If E << V_0 then we can make the following simplification**
- $$T^{-1} = 1 + \frac{\left(e^{\alpha L}-e^{-\alpha L}\right)^2}{16\left(\frac{E}{V_0}\right)\left(1-\frac{E}{V_0}\right)} = 1 + \frac{e^{2\alpha L}}{1} \approx e^{2\alpha L} \text{ or } T \approx e^{-2\alpha L}$$

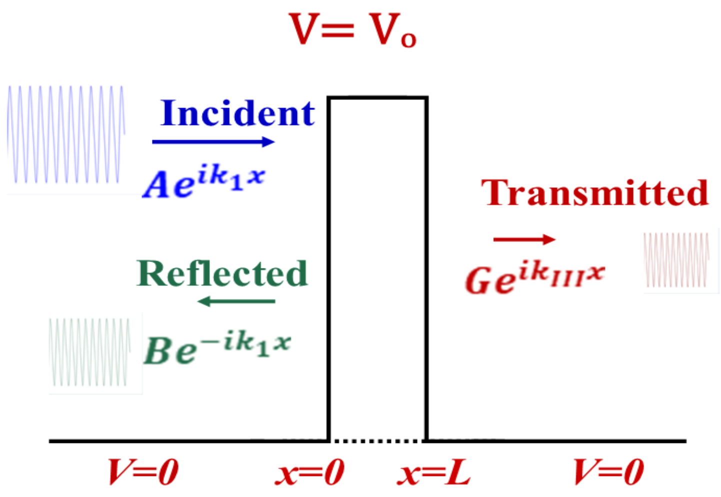
Barrier potential

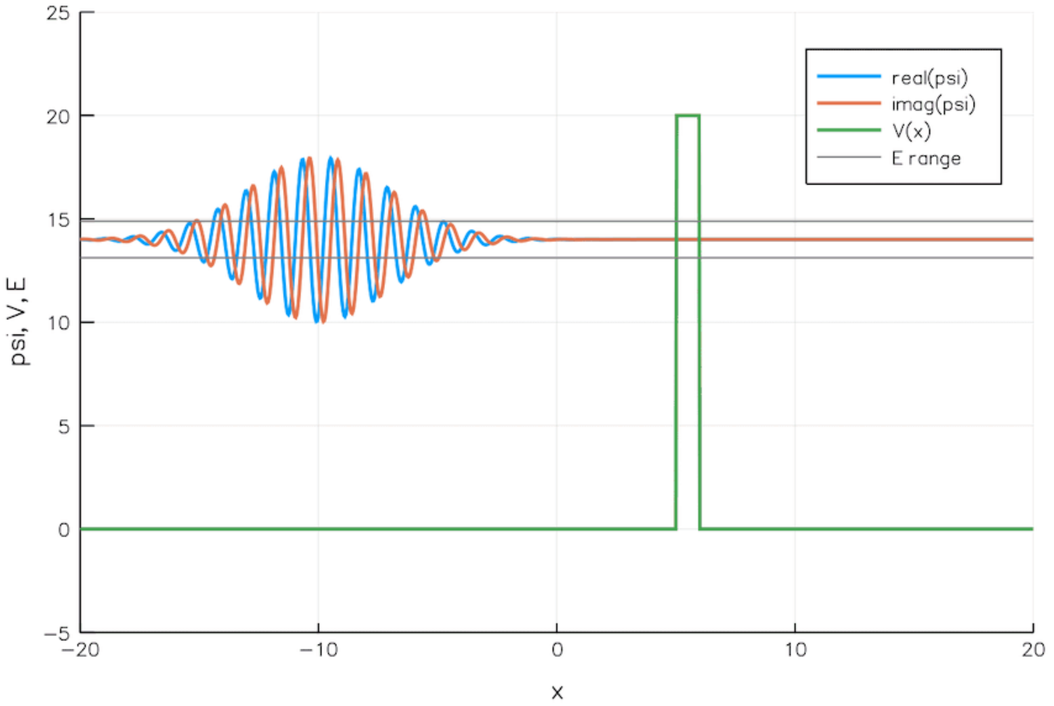
- ***T* depends on the following**
- ***The difference in the energy E and the barrier height V_0***
- ***The width of the barrier***
- ***Mass of the particle***

It is clear that for a given particle energy E if the height of the barrier increases T decreases. Further If the width of the barrier increases T decreases. For a given E, V_0 and L, heavier particles are less likely to tunnel than their lighter counterparts

Barrier Tunneling (E < V_o)- animation

The transmission probability is higher if the penetration depth is greater than the width of the barrier $\boldsymbol{\Delta x > L}$





https://commons.wikimedia.org/wiki/File:E14-V20-B1.gif

Barrier Tunneling - animation

Quantum Tunneling

When a wave packet strikes a barrier, part of it reflects and part tunnels through.

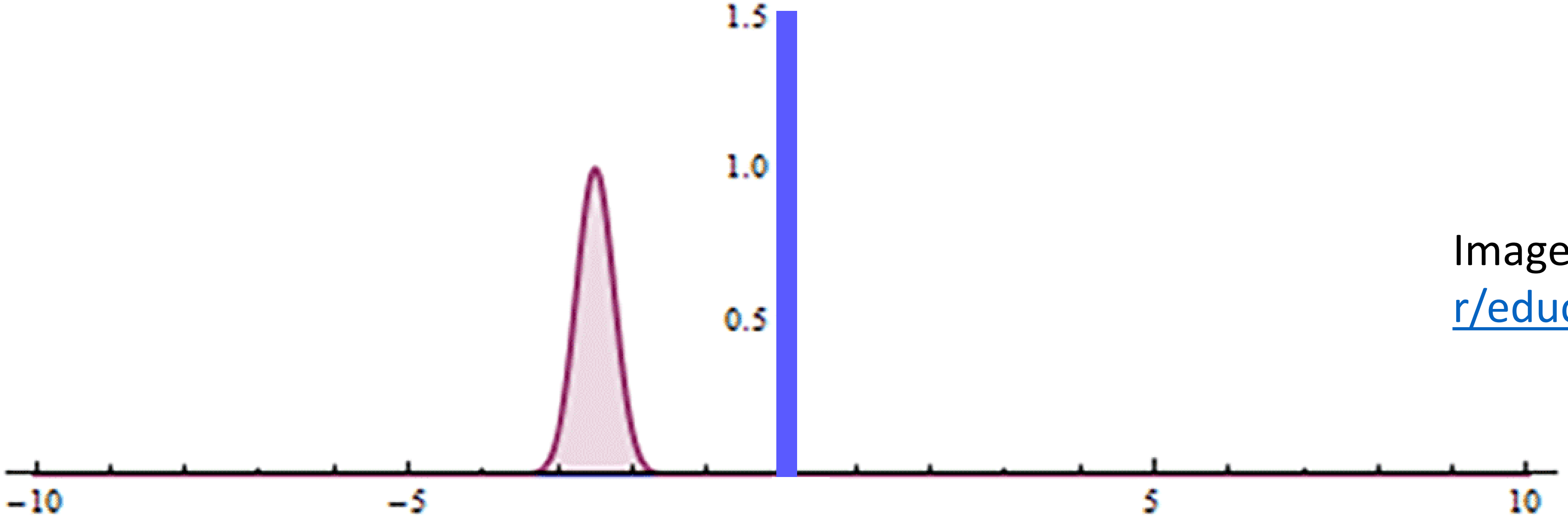


Image courtesy:
r/educationalgifs

Barrier Tunneling - animation

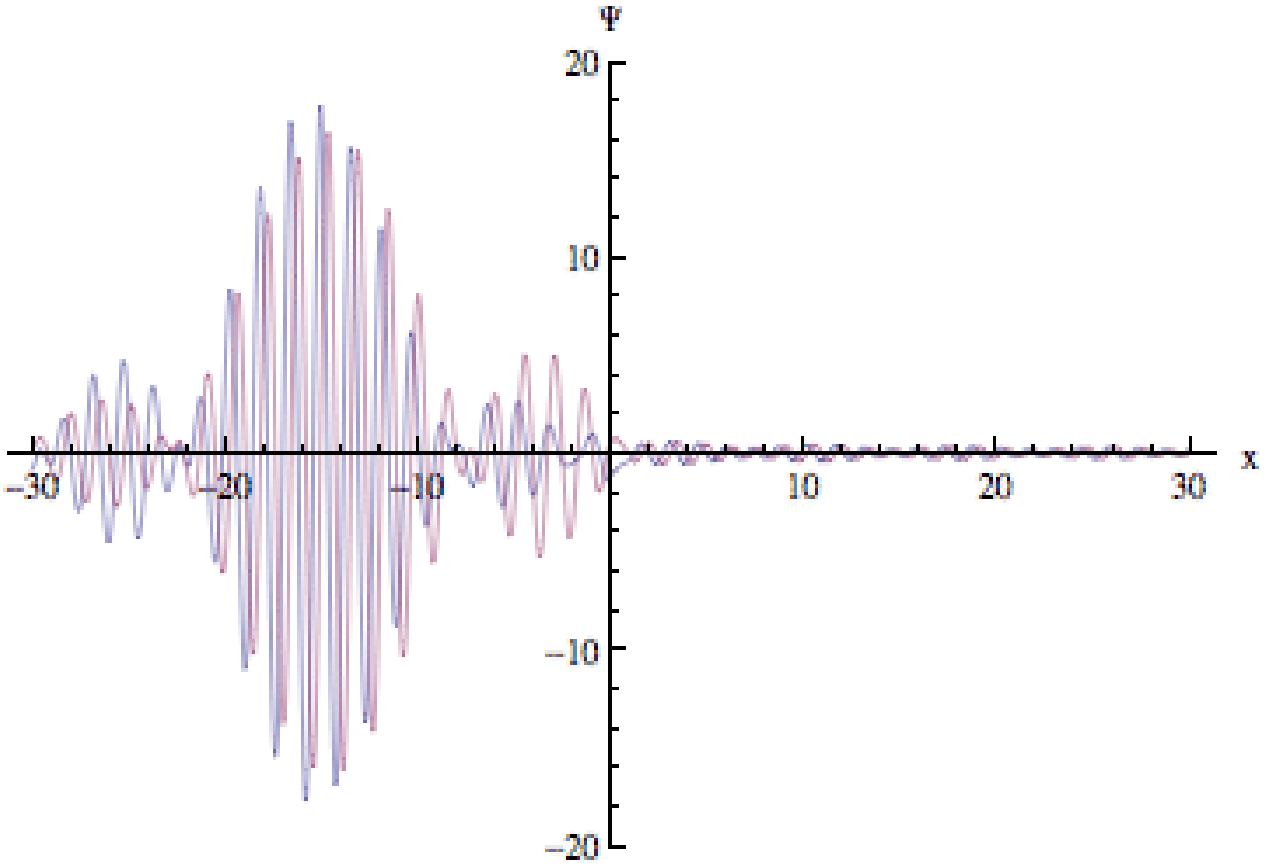


Image courtesy: facultystaff.Richmond.edu

Barrier Tunneling - some insights

The particle cannot be physically present in region II.

The particle tunneling through the barrier is almost instantaneous

The particle borrows ΔE *energy from the field and moves through the barrier in a time* Δt*, such that* $\Delta E \times \Delta t \geq \frac{h}{4\pi}$

Δt *typically < pico seconds*

The process is instantaneous - very short time spans for barrier tunneling!

Summarizing : Barrier tunneling E < V_o

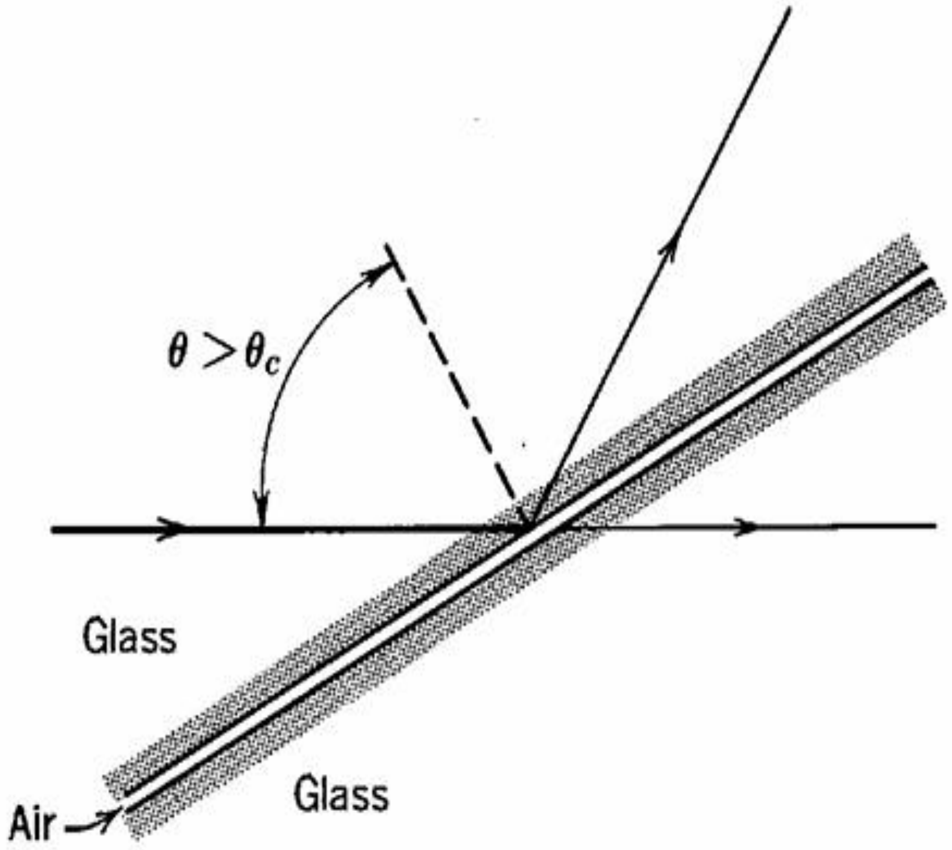
- *Region I*
- $\psi_I(x) = Ae^{ik_I x} + Be^{-ik_I x}$
- $k_I = \sqrt{\frac{2mE}{\hbar^2}}$
- $E = \frac{\hbar^2 k_I^2}{2m} = KE$
- $P_I = \hbar k_I$
- $\lambda_I = \frac{h}{\sqrt{2mE}}$

- *Region II*
- $\psi_{II}(x) = De^{-\alpha x}$
- $\alpha = \sqrt{\frac{2m(V_o - E)}{\hbar^2}}$
- $\Delta x = \sqrt{\frac{\hbar^2}{2m(V_o - E)}}$
- $KE = E - V_o$ -ve

- *Region III*
- $\psi_{III}(x) = Ge^{ik_{III} x}$
- $k_{III} = \sqrt{\frac{2mE}{\hbar^2}}$
- $E = \frac{\hbar^2 k_{III}^2}{2m} = KE$
- $P_{III} = \hbar k_{III}$
- $\lambda_{III} = \frac{h}{\sqrt{2mE}}$

- *The transmission probability* $T \cong e^{-2\alpha L}$

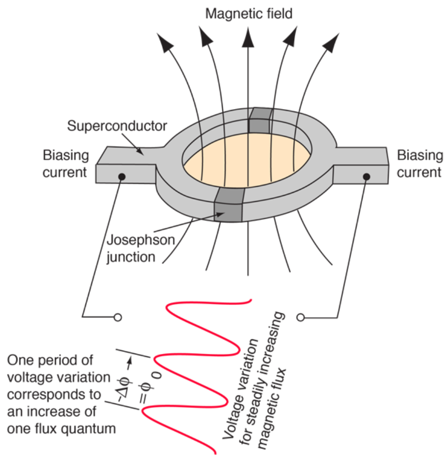
Barrier tunneling – optical equivalent

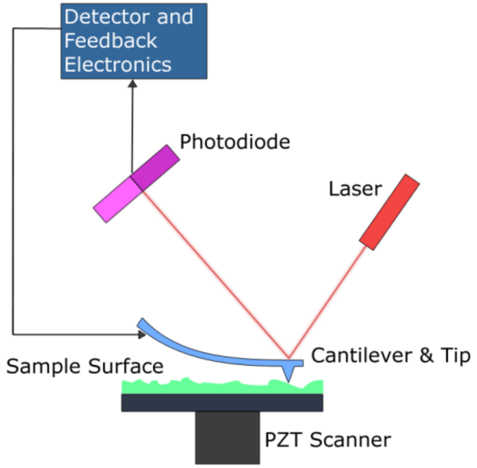


Illustrating frustrated total internal reflection. Some of the light ray is transmitted through the air gap if the gap is sufficiently narrow.

Barrier tunneling applications

SQUIDs in MRI scan sensor



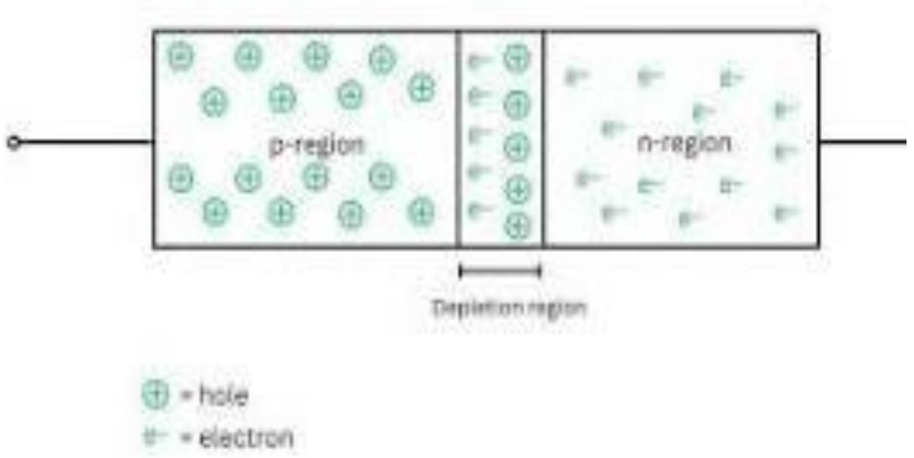


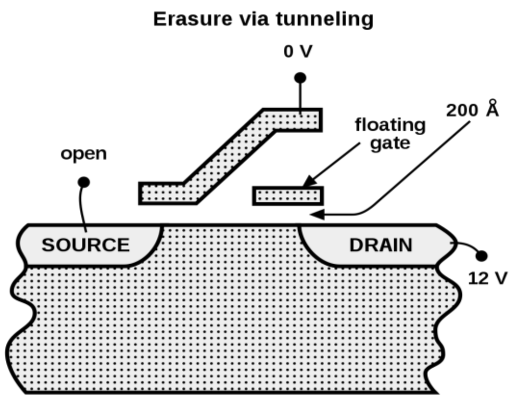
Electron tunneling current as an image in Atomic force microscopes

http://hyperphysics.phy-astr.gsu.edu/

Barrier tunneling applications

Tunnel diode for high frequency oscillators





Flash memory devices

https://commons.wikimedia.org/

Quiz

A bunch of particles are incident at a potential barrier of width L. If the transmission coefficient is given by $T = e^{-2\alpha L}$, which of the following expressions represents the reflection coefficient?

- $\frac{Sin(\alpha L)}{e^{\alpha L}}$
- $\frac{Sinh(\alpha L)}{e^{\alpha L}}$
- $\frac{2Sin(\alpha L)}{e^{\alpha L}}$
- $\frac{2Sinh(\alpha L)}{e^{\alpha L}}$

THANK YOU

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