ENGINEERING PHYSICS

Radhakrishnan S, Ph.D.

Department of Science and Humanities

ENGINEERING PHYSICS

Unit I : Review of concepts leading to Quantum Mechanics

Week #1

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Electromagnetic wave equations

Class #2

- *Maxwell's equation in differential form*

- *Maxwell's Equations in free space*

- *Ideas of Electric and Magnetic waves*

- *Wave equation*

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Electromagnetic wave equations

➢ *Suggested Reading*

1. *Fundamentals of Physics, Halliday, Resnik, Chapter 34*

2. *NCERT Physics Book I grade 12 – Chapter 8*

➢ *Reference Videos*

1. *https://nptel.ac.in/courses/108/106/108106073/*

2. *Engineering Physics week1 class1*

Gauss's law for Electric and Magnetic fields

Electric fields

the divergence of the electric field is proportional to the charge density

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

Magnetic Fields

the divergence of the magnetic flux is uniformly zero

$$\nabla \cdot \vec{B} = 0$$

Faraday's law of electromagnetic induction

The curl of the induced electric field in a closed loop is proportional to the rate of change of magnetic flux linked with the loop

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

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Ampere - Maxwell's law

Ampere - Maxwell circuital law

The curl of the magnetic field in a closed loop is equal to the sum of the current density and the displacement current due to the time varying electric field

$$\nabla \times \vec{B} = \mu_o \vec{J} + \mu_o \varepsilon_o \frac{\partial \vec{E}}{\partial t}$$

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Maxwell's equations in free space

Free space implies – charges and currents do not exist

$$\vec{\nabla}.\vec{E} = 0 \qquad\qquad 1.$$

$$\vec{\nabla}.\vec{B} = 0 \qquad\qquad 2.$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad\qquad 3$$

$$\vec{\nabla} \times \vec{B} = +\mu_o \varepsilon_o \frac{\partial \vec{E}}{\partial t} \qquad\qquad 4.$$

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Electric waves in free space

The curl of equation 3. $\quad \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \times \left(-\frac{\partial \vec{B}}{\partial t} \right)$

$$\vec{\nabla}(\vec{\nabla}.\vec{E}) - \nabla^2\vec{E} = \left(-\frac{\partial \vec{\nabla} \times \vec{B}}{\partial t} \right)$$

For free space $\quad \vec{\nabla}.\vec{E} = 0$

$$-\nabla^2\vec{E} = \left(-\frac{\partial \vec{\nabla} \times \vec{B}}{\partial t} \right)$$

Substituting for curl of B from equation 4.

$$\nabla^2\vec{E} = \left(\mu_o \varepsilon_o \frac{\partial^2 E}{\partial t^2} \right)$$

With $\mu_o \varepsilon_o = \dfrac{1}{c^2}$
$$\nabla^2\vec{E} = \left(\frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} \right) \quad \ldots\ldots$$

A wave equation for electric wave propagating in free space !

Magnetic waves in free space

- *The curl of Maxwell's equation 4.*

$$\vec{\nabla} \times \vec{\nabla} \times \vec{B} = \vec{\nabla} \times (+\mu_o \varepsilon_o \frac{\partial \vec{E}}{\partial t}) \quad \rightarrow$$

$$\nabla^2 \vec{B} = \left(\frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} \right) \quad \rightarrow$$

   a magnetic wave propagating in free space with speed of light

- *Both electric and magnetic waves propagate with the speed of light*

- *Maxwell's predication of light (radiation) as electromagnetic waves*

Electromagnetic waves in free space

*Light waves are transverse waves ---*

Electromagnetic waves are transverse waves !

The electric and magnetic fields are perpendicular to the direction of propagation of the electromagnetic wave

Electromagnetic waves in free space

- *Let a plane EM wave be propagating along the z direction* $(E_z = 0)$

- *Instantaneous electric field* $E_x = E_{xo} \sin(\omega t + kz)$

Electromagnetic waves in free space
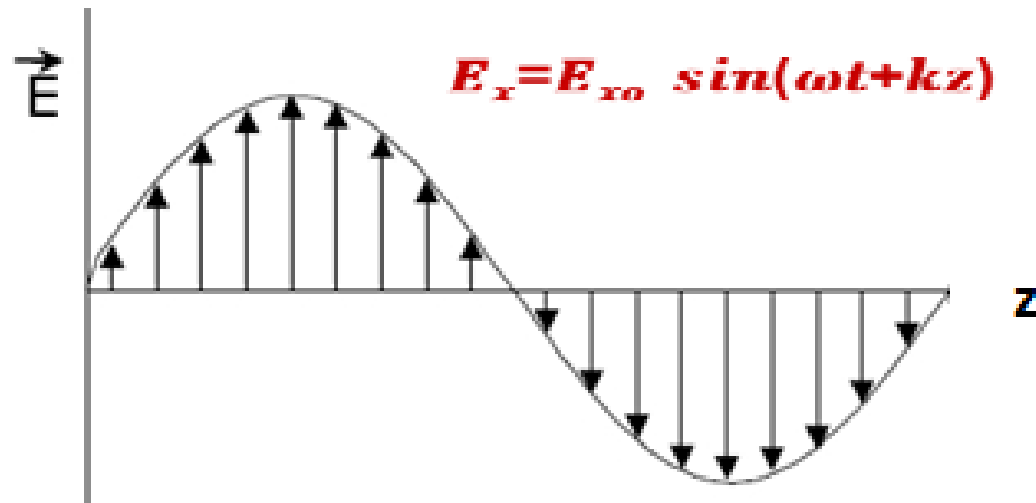
- *Plane wave implies $E_y = 0$*

- *From the third Maxwell's equation*

- $$\vec{\nabla} \times \overrightarrow{E_x} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -\hat{\jmath}\,\dfrac{\partial E_x}{\partial z} = -\dfrac{\partial \vec{B}}{\partial t}$$

- $\dfrac{\partial \vec{B}}{\partial t}$ *can be evaluated since E_x is known*

Electromagnetic waves in free space

- *Integrate $\dfrac{\partial \vec{B}}{\partial t}$ with respect to time to obtain*

- $B_y = \hat{j}\,\dfrac{E_{xo}}{\omega/k}\,sin(\omega t + kz)$

- $\dfrac{\omega}{k} = the\ velocity\ of\ the\ wave = c$

- *magnetic field is oriented in the* **y direction**

- *perpendicular to the Electric field (x direction)*

- *E and B are mutually perpendicular to each other*

Electromagnetic waves in free space

- $E_x = \hat{i} E_{xo} \sin(\omega t + kz)$

- $B_y = \hat{j} \dfrac{E_{xo} \sin(\omega t + kz)}{\omega/k}$

- *in phase with the electric field variations*

- $\omega/k = c$ *is the phase velocity of the wave*

- *Thus* B_y *is perpendicular to E and* $|B_y| = \dfrac{|E_x|}{c}$

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Electromagnetic waves in free space

- *Electromagnetic wave*

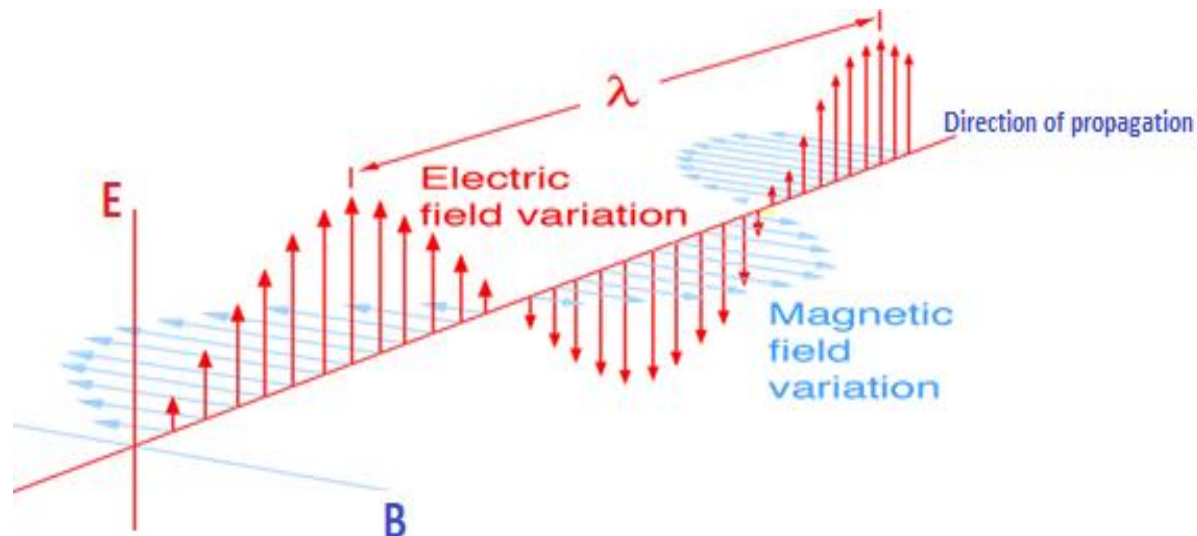- *E and B are mutually perpendicular to each other and the direction of propogation*

Image courtesy **Wikipedia**

Class 2 .   Quiz …

The concepts which are not correct are….

1. Electric waves in free space are longitudinal

2. Magnetic waves in free space are transverse

3. The curl of a magnetic field is uniformly zero

4. The divergence of a magnetic field can be non zero

5. The curl of an electric field is always linked to an time varying magnetic field

6. The divergence of a vector field is a also a scalar

THANK YOU

Radhakrishnan S, Ph.D.

Professor, Department of Science and Humanities

sradhakrishnan@pes.edu

+91 80 21722683 Extn 759