ENGINEERING MATHEMATICS - I

Unit - 3: Partial Differential Equations

Department of Science and Humanities

PES UNIVERSITY

Contents

1 Problems on Lagrange's Linear Equation

Solve $\dfrac{y^2 z}{x} p + xz\, q = y^2$

Given:

$$\frac{y^2 z}{x} p + xz\, q = y^2$$

This is Lagrange's equation $Pp + Qq = R$.
Here,

$$P = \frac{y^2 z}{x}, \quad Q = xz, \quad R = y^2$$

The subsidiary equations are: $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

$$\frac{dx}{y^2 z/x} = \frac{dy}{xz} = \frac{dz}{y^2} \implies \frac{x\, dx}{y^2 z} = \frac{dy}{xz} = \frac{dz}{y^2}$$

Considering the first two ratios:

$$\frac{x\, dx}{y^2 z} = \frac{dy}{xz} \implies x^2 dx = y^2 dy$$

Solve $\dfrac{y^2 z}{x}p + xz\,q = y^2$ (contd.)

Integrating,

$$\int x^2 dx = \int y^2 dy \implies \frac{x^3}{3} = \frac{y^3}{3} + c \implies x^3 - y^3 = 3c$$

$$\implies x^3 - y^3 = a \tag{1}$$

Considering the first and last ratios:

$$\frac{x\,dx}{y^2 z} = \frac{dz}{y^2} \implies x\,dx = z\,dz$$

Integrating,

$$\int x\,dx = \int z\,dz \implies \frac{x^2}{2} = \frac{z^2}{2} + c \implies x^2 - z^2 = 2c \implies x^2 - z^2 = b \tag{2}$$

General solution:

$$\therefore \quad \Phi(x^3 - y^3,\, x^2 - z^2) = 0$$

where Φ is arbitrary.

Solve $yzp - xzq = xy$

Solution:
Auxiliary equations are

$$\frac{dx}{yz} = \frac{dy}{-xz} = \frac{dz}{xy}$$

From first and second fractions, we get

$$\frac{dx}{y} = \frac{dy}{-x}$$

or

$$x\,dx + y\,dy = 0$$

Integrating,

$$x^2 + y^2 = c_1$$

From first and third fractions,

$$\frac{dx}{yz} = \frac{dz}{xy}$$

Solve $yzp - xzq = xy$ (contd.)

or

$$\frac{dx}{z} = \frac{dz}{x}$$

Integrating,

$$x^2 - z^2 = c_2$$

Thus the general solution is

$$\phi(x^2 + y^2, x^2 - z^2) = 0$$

Solve $z(z^2 + xy)(px - qy) = x^4$

Solution:
Auxiliary equations are

$$\frac{dx}{xz(z^2 + xy)} = \frac{dy}{-yz(z^2 + xy)} = \frac{dz}{x^4}$$

From first and second fractions, we get

$$\frac{dx}{x} = \frac{dy}{-y}$$

On integration, $xy = c_1$

Solve $z(z^2 + xy)(px - qy) = x^4$ (contd.)

From first and third fractions,

$$x^3 dx = (z^3 + xyz)dz$$

Using $xy = c_1$,

$$x^3 dx = (z^3 + c_1 z)dz$$

Integrating,

$$\int x^3 dx = \int (z^3 + c_1 z)dz$$

$$\frac{x^4}{4} = \frac{z^4}{4} + c_1 \frac{z^2}{2} + c_2$$

or

$$x^4 - z^4 - 2c_1 z^2 = c_2$$

Substituting for c_1,

$$x^4 - z^4 - 2(xy)z^2 = c_2$$

The general solution is

$$\phi(xy,\, x^4 - z^4 - 2xyz^2) = 0$$