ENGINEERING MECHANICS

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ENGINEERING MECHANICS

Centroid

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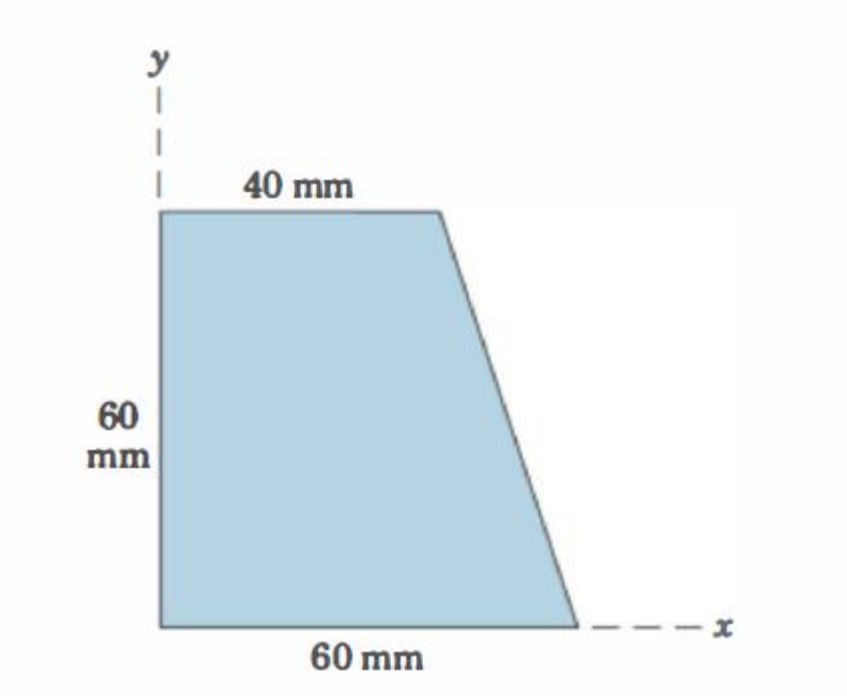
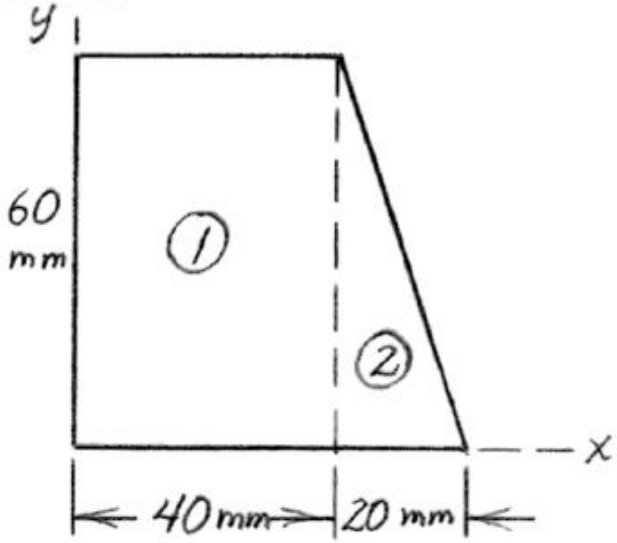
5/47 Determine the coordinates of the centroid of the trapezoidal area shown.



5/47



① $A_1 = 40(60) = 2400\ mm^2$

$\bar{x}_1 = 20\ mm,\ \bar{y}_1 = 30\ mm$

② $A_2 = \frac{1}{2}(20)(60) = 600\ mm^2$

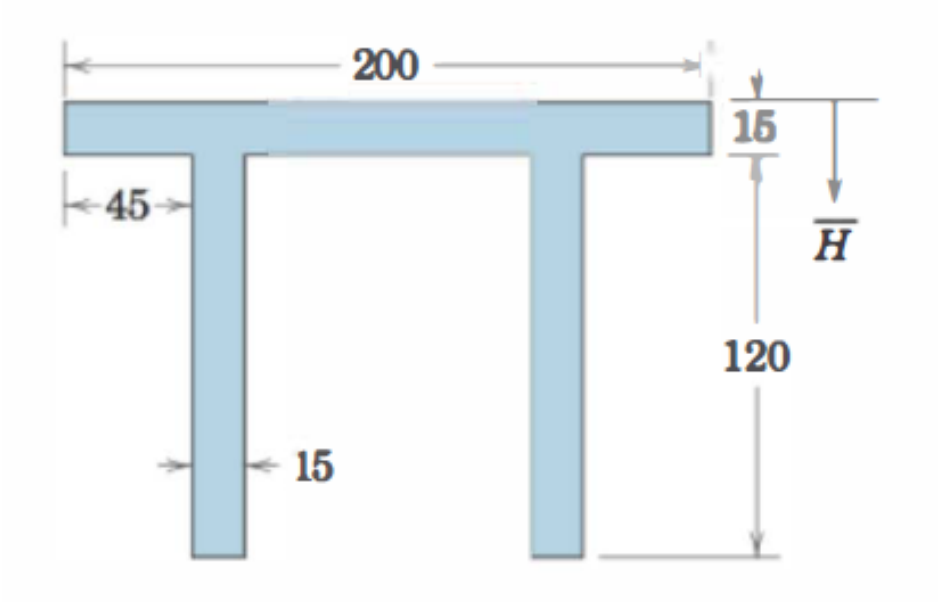
$\bar{x}_2 = 40 + \frac{20}{3} = 46.7\ mm$

$\bar{y}_2 = \frac{60}{3} = 20\ mm$

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{2400(20) + 600(46.7)}{2400 + 600} = \underline{25.3\ mm}$$

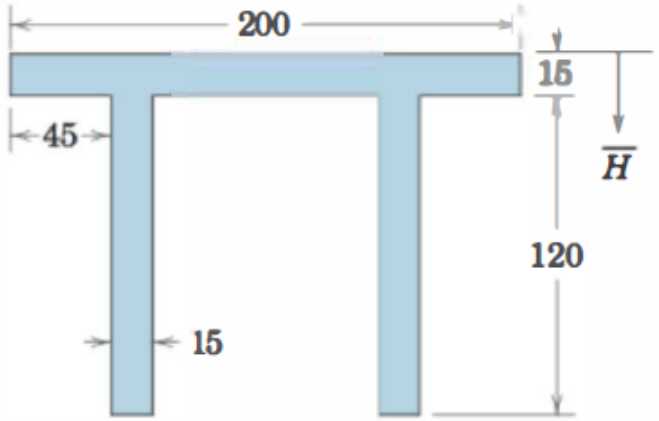
$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{2400(30) + 600(20)}{2400 + 600} = \underline{28.0\ mm}$$

5/48 Determine the distance H from the upper surface of the symmetric double-T beam cross section to the location of the centroid.



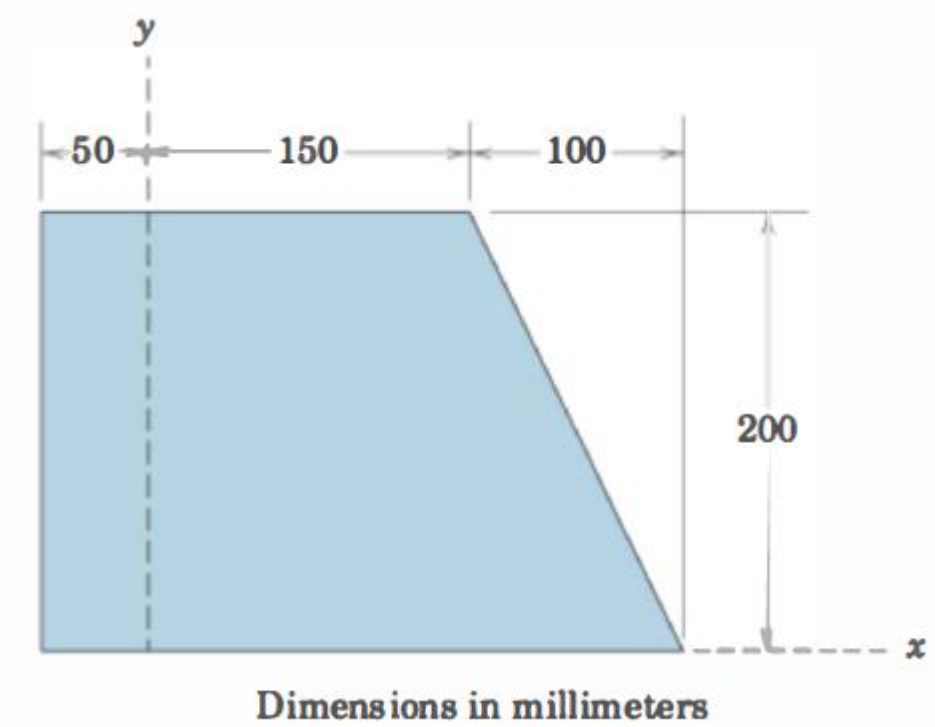
ENGINEERING MECHANICS

Centroid



$$\bar{H} = \frac{\Sigma A \bar{h}}{\Sigma A} = \frac{200(15)\left(\frac{15}{2}\right) + 2(15)(120)\left(15 + \frac{120}{2}\right)}{200(15) + 2(15)(120)} \longrightarrow \bar{H} = 44.3 \text{ mm}$$

5/49 Determine the x- and y-coordinates of the centroid of the shaded area.



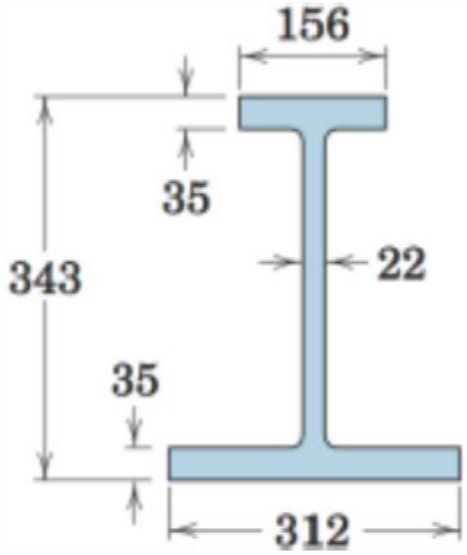
$$\overline{X} = \frac{\Sigma A \bar{x}}{\Sigma A} = \frac{50(200)(-25) + 150(200)(75) + \frac{1}{2}(100)(200)\left(150 + \frac{100}{3}\right)}{50(200) + 150(200) + \frac{1}{2}(100)(200)}$$

$$\overline{X} = 76.7 \text{ mm}$$

$$\overline{Y} = \frac{\Sigma A \bar{y}}{\Sigma A} = \frac{50(200)(100) + 150(200)(100) + \frac{1}{2}(100)(200)\left(\frac{200}{3}\right)}{50(200) + 150(200) + \frac{1}{2}(100)(200)}$$

$$\overline{Y} = 93.3 \text{ mm}$$

5/50 Determine the height above the base of the centroid of the cross-sectional area of the beam. Neglect the fillets.



Dimensions in millimeters

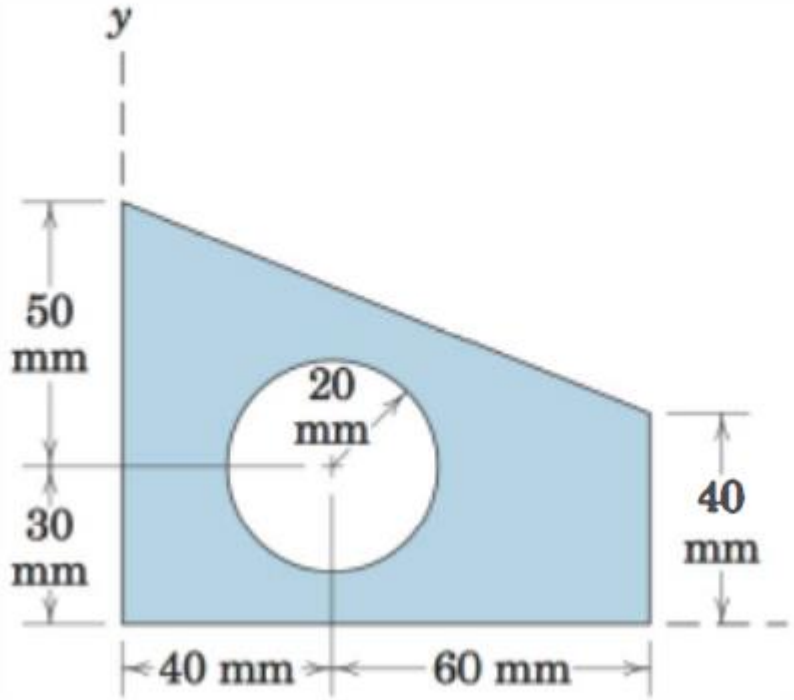
Centroid

Comp.	$A\,(mm^2)$	$\bar{y}\,(mm)$	$A\bar{y}\,(mm^3)$
①	$312(35)$	$\frac{35}{2}$	191 100
②	$273\,(22)$	$35+\frac{273}{2}$	1 030 000
③	$156\,(35)$	$35+273+\frac{35}{2}$	1 777 000

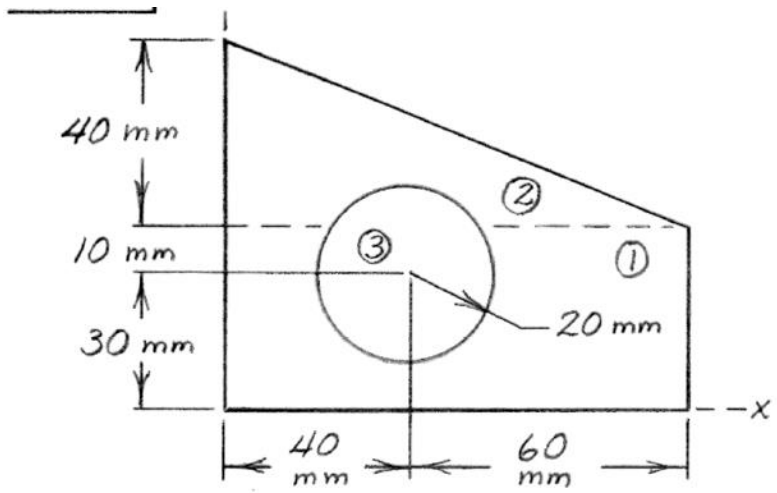
$$\Sigma A = 22\,400 \qquad \Sigma A\bar{y} = 3\,000\,000$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{3\,000\,000}{22400} = 133.9\ mm$$

5/51 Determine the x- and y-coordinates of the centroid of the shaded area.



ENGINEERING MECHANICS
Centroid

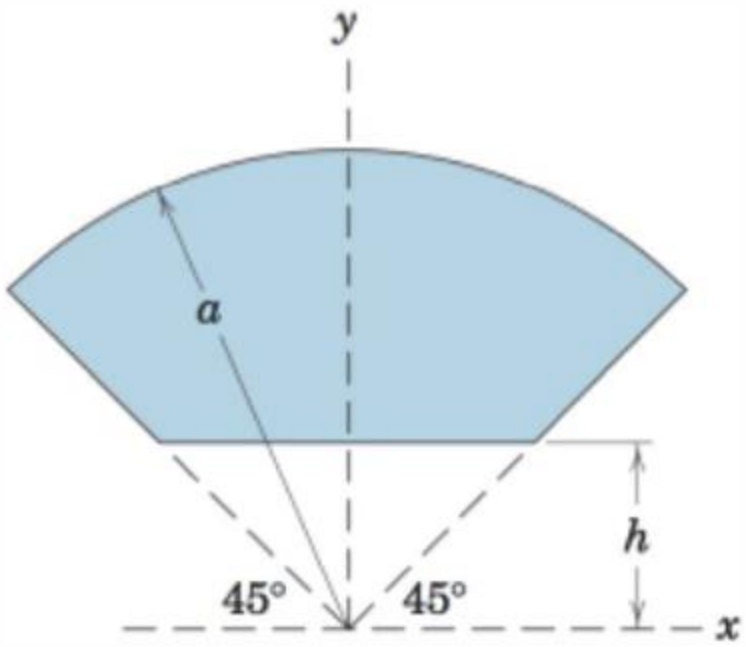


Part	A (mm^2)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$A\bar{x}$ (mm^3)	$A\bar{y}$ (mm^3)
1	4000	50	20	$200(10^3)$	$80(10^3)$
2	2000	100/3	$40+\frac{40}{3}$	$66.7(10^3)$	$106.7(10^3)$
3	$-\pi(20^2)$	40	30	$-50.3(10^3)$	$-37.7(10^3)$
Totals	4740			$216(10^3)$	$149.0(10^3)$

$$\bar{X}=\frac{\Sigma A\bar{x}}{\Sigma A}=\frac{216(10^3)}{4740}=45.6\ mm$$

$$\bar{Y}=\frac{\Sigma A\bar{y}}{\Sigma A}=\frac{149.0(10^3)}{4740}=31.4\ mm$$

5/55 Determine the y-coordinate of the centroid of the shaded area.



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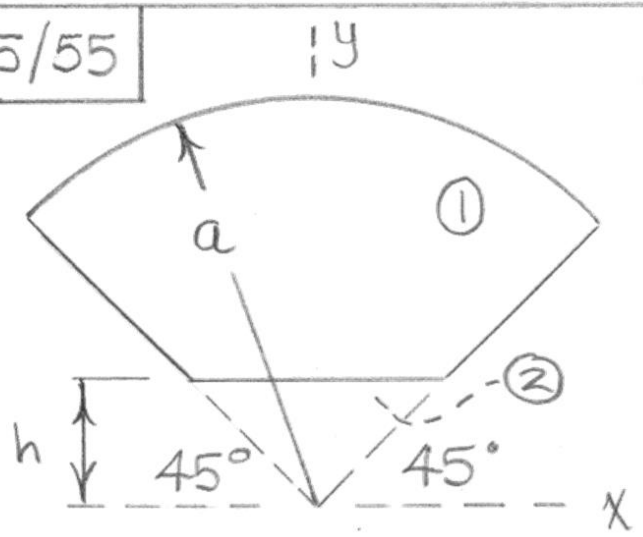


Circular sector (full) ①:

$$A_1 = \frac{\pi}{4}a^2$$

$$\bar{y}_1 = \frac{2}{3}a\frac{\sin 45^\circ}{\pi/4}$$

$$= \frac{4\sqrt{2}}{3\pi}a$$

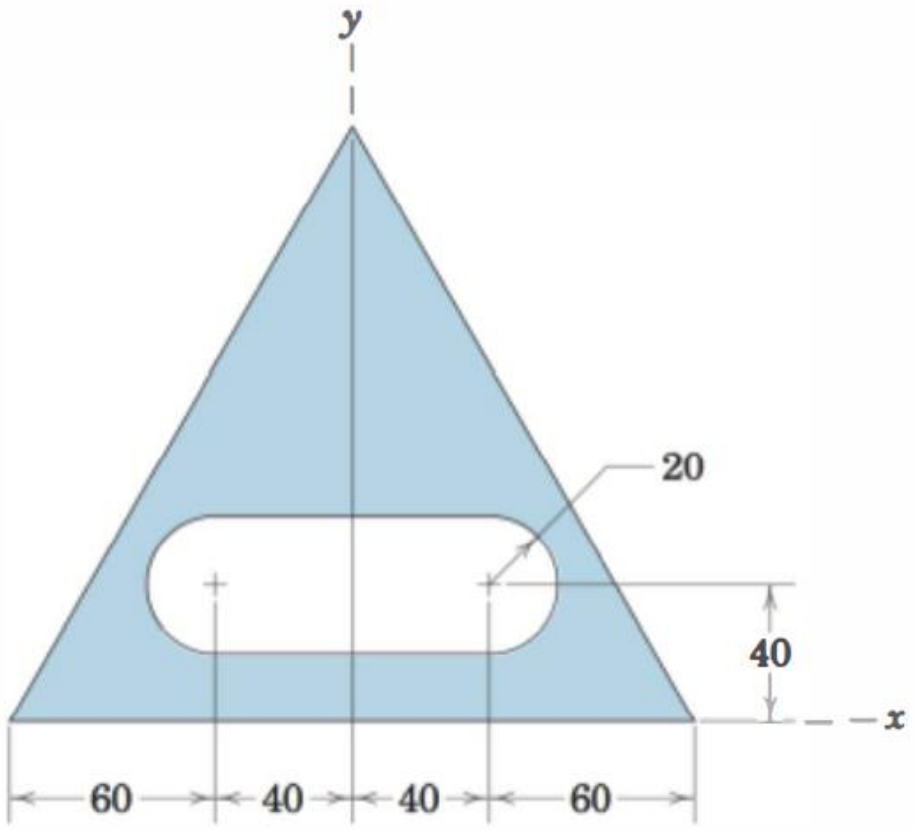
Triangular "hole" ②:

$$A_2 = \frac{1}{2}h(2h) = h^2 \qquad \bar{y}_2 = \frac{2}{3}h$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{\frac{\pi}{4}a^2\left(\frac{4\sqrt{2}}{3\pi}a\right) - h^2\left(\frac{2}{3}h\right)}{\frac{\pi}{4}a^2 - h^2}$$

$$= \frac{4\left(\sqrt{2}a^3 - 2h^3\right)}{3\left(\pi a^2 - 4h^2\right)}$$

5/56 Determine the y-coordinate of the centroid of the shaded area. The triangle is equilateral.



Dimensions in millimeters

ENGINEERING MECHANICS
Centroid

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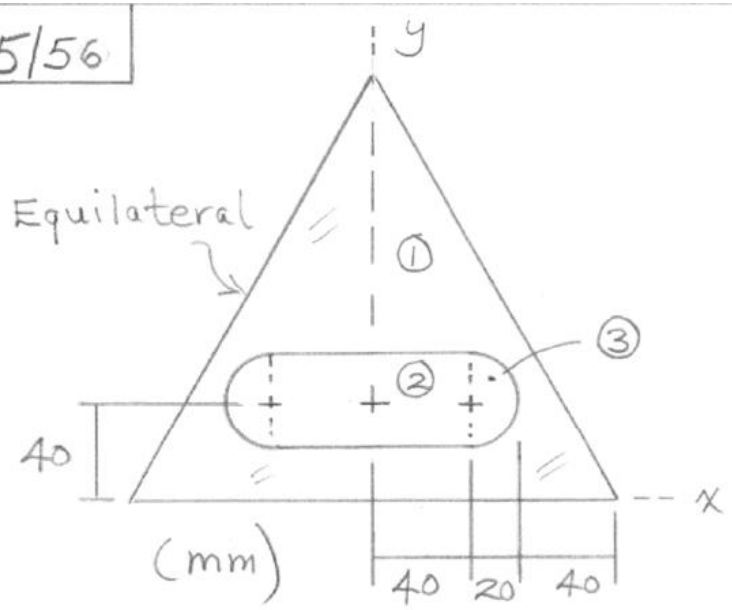
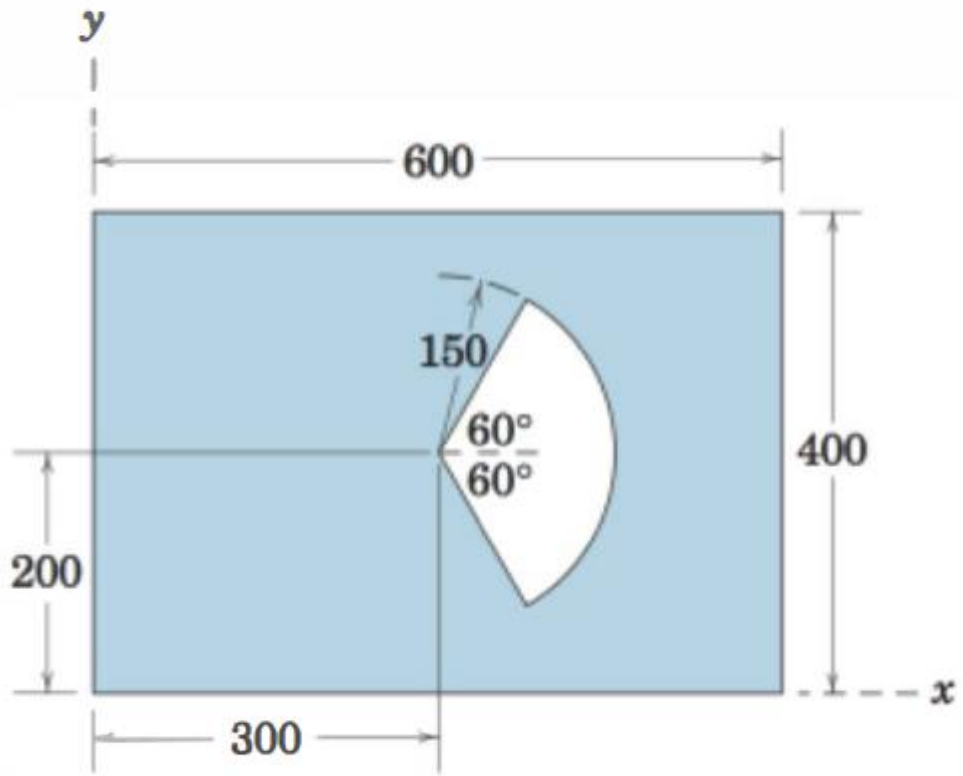


Component	A (mm^2)	$\bar{y}$ (mm)	$\bar{y}A$ (mm^3)
Triangle 1	17 320	57.7	10^6
Rectangle 2	− 3200	40	− 128 000
2 semicircles 3	− 1257	40	− 50,300
	$\Sigma A = 12\,860$		$\Sigma \bar{y}A = 822\,000$

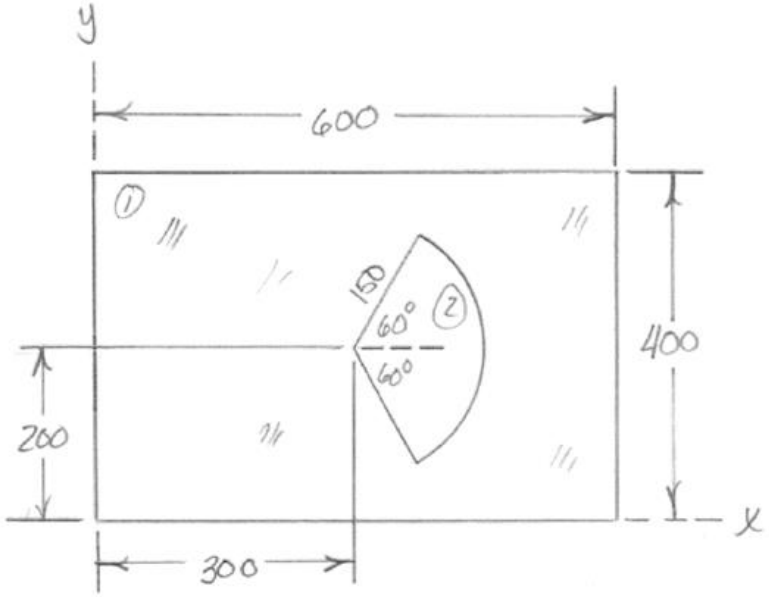
$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} \quad \frac{822\,000}{12\,860} = 63.9 \text{ mm}$$

5/57 Determine the x- and y-coordinates of the centroid of the shaded area.



Dimensions in millimeters

ENGINEERING MECHANICS
Centroid

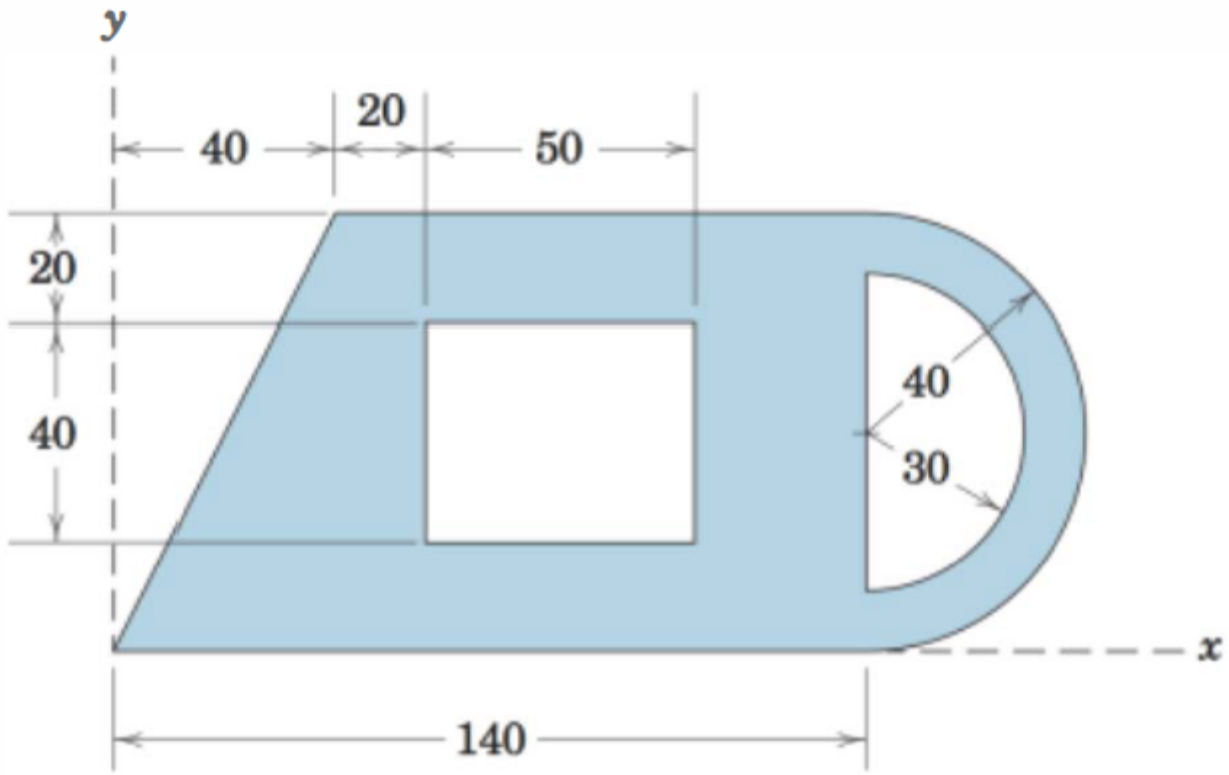


$$\overline{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{400(600)(300) - \frac{1}{3}\pi(150)^2\left(300 + \frac{2}{3}(150)\frac{\sin 60}{\pi/3}\right)}{400(600) - \frac{1}{3}\pi(150)^2}$$

$$\overline{X} = 291 \text{ mm}$$

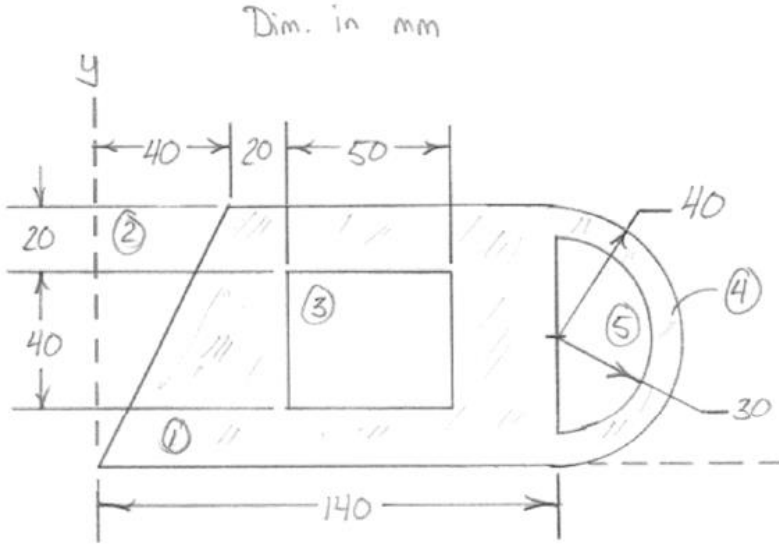
$$\overline{Y} = 200 \text{ mm} \quad \text{(INSPECTION)}$$

5/58 Determine the coordinates of the centroid of the shaded area.



Dimensions in millimeters

ENGINEERING MECHANICS
Centroid



	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$A\bar{x}$, mm^3	$A\bar{y}$, mm^3
①	$140(80) = 11200$	70	40	784×10^3	448×10^3
②	$-\frac{1}{2}(40)(80) = -1600$	$\frac{40}{3} = 13.33$	$\frac{2}{3}(80) = 53.3$	-21.3×10^3	-85.3×10^3
③	$-40(50) = -2000$	85	40	-170×10^3	-80×10^3
④	$\frac{\pi(40)^2}{2} = 800\pi$	$140+\frac{4(40)}{3\pi} = 157.0$	40	395×10^3	32000π
⑤	$-\frac{\pi(30)^2}{2} = -450\pi$	$140+\frac{4(30)}{3\pi} = 152.7$	40	-216×10^3	-18000π
Σ	8700			771×10^3	327×10^3

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{771\times10^3}{8700} \rightarrow \bar{X} = 88.7\ mm \qquad \bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{327\times10^3}{8700} \rightarrow \bar{Y} = 37.5\ mm$$

THANK YOU

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