ENGINEERING MATHEMATICS I

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HIGHER ORDER DIFFERENTIAL EQUATIONS

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CLASS CONTENT

- **TO SOLVE A NON – HOMOGENEOUS LINEAR DIFFERENTIAL EQUATION OF THE TYPE $f(D)y = X$**
  WHEN $X =$.

RULES TO FIND PARTICULAR INTEGRAL

Type (ii) ; When $X = \sin(ax+b)$ or $\cos(ax+b)$

$$\frac{\sin(ax+b)}{f(D^2)} = \frac{\sin(ax+b)}{f(-a^2)} \quad \text{provided } f(-a^2) \neq 0$$

$$\frac{\cos(ax+b)}{f(D^2)} = \frac{\cos(ax+b)}{f(-a^2)} \quad \text{provided } f(-a^2) \neq 0$$

Type (ii) continued,

Case of failure; If $f(-a^2)=0$, then

$$\frac{\sin(ax+b)}{f(D^2)}=x.\frac{\sin(ax+b)}{f'(-a^2)}$$ provided $f'(-a^2)\neq 0$

$$\frac{\cos(ax+b)}{f(D^2)}=x.\frac{\cos(ax+b}{f'(-a^2)}$$ provided $f'(-a^2)\neq 0$

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Type (ii) continued,

If $f'(-a^2) = 0$, then

$$\frac{\sin(ax+b)}{f(D^2)} = x^2 . \frac{\sin(ax+b)}{f''(-a^2)}$$ provided $f''(-a^2) \neq 0$

$$\frac{\cos(ax+b)}{f(D^2)} = x^2 . \frac{\cos(ax+b)}{f''(-a^2)}$$ provided $f''(-a^2) \neq 0$

Type (ii) continued,

The same formula can be used to find the PI for the following functions also.

When $X = \sin ax . \cos bx$ or $\cos ax . \cos bx$

or $\sin ax . \sin bx$

When $X = \sin^2 ax$ or $\cos^2 ax$

When $X = \sin^3 ax$ or $\cos^3 ax$

Solve ; $(D^3+1)y = \cos(2x+3)$

To Find CF

AE is $m^3 + 1 = 0$

$(m+1)(m^2 - m + 1) = 0$

Roots are $m = -1$ and $\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

$$y_c = c_1 e^{-x} + e^{\frac{x}{2}}\left(c_2 \cos\left(\frac{\sqrt{3}}{2}x\right) + c_3 \sin\left(\frac{\sqrt{3}}{2}x\right)\right)$$

To Find PI

$$y_p = \frac{\cos(2x+3)}{D^3+1} = \frac{\cos(2x+3)}{D.D^2+1} = \frac{\cos(2x+3)}{D.(-2^2)+1}$$

$$= \frac{\cos(2x+3)}{(1-4D)} = \frac{(1+4D)\cos(2x+3)}{(1+4D)(1-4D)} = \frac{(1+4D)\cos(2x+3)}{(1-16D^2)}$$

$$= \frac{(1+4D)\cos(2x+3)}{65} = \frac{1}{65}(1+4D)\cos(2x+3)$$

$$= \frac{1}{65}(\cos(2x+3) - 8\sin(2x+3))$$

$$\therefore y = y_c + y_p$$

Solve; $(D-1)^2(D^2+1)y = e^x + \cos^2\left(\frac{x}{2}\right)$

To Find CF

AE is $(m-1)^2(m^2+1)=0$

Roots are $m = 1,\ 1,\ \pm i$

$CF = (c_1 + c_2 x)e^x + c_3 \cos x + c_4 \sin x$

To Find PI

$$PI = \frac{e^x}{(D-1)^2(D^2+1)} + \frac{\cos^2\frac{x}{2}}{(D-1)^2(D^2+1)}$$

$$= PI_1 + PI_2$$

where $PI_1 = \frac{e^x}{(D-1)^2(D^2+1)}$ and $PI_2 = \frac{\cos^2\frac{x}{2}}{(D-1)^2(D^2+1)}$

To Find PI_1

$$PI_1 = \frac{e^x}{(D-1)^2(D^2+1)} = \frac{1}{(D-1)^2}\left(\frac{e^x}{D^2+1}\right) = \frac{e^x}{2(D-1)^2}$$

$$= x^2\frac{e^x}{2.2} = \frac{x^2.e^x}{4}$$

$$\therefore PI_1 = \frac{x^2.e^x}{4}$$

To Find PI_2

$$PI_2 = \frac{\cos^2\frac{x}{2}}{(D-1)^2(D^2+1)} = \frac{(1+\cos x)/2}{(D-1)^2(D^2+1)}$$

$$= \frac{1}{2(D-1)^2(D^2+1)} + \frac{\cos x}{2(D-1)^2(D^2+1)}$$

$$= \frac{1}{4} + \frac{1}{2(D^2+1)}\left(\frac{\cos x}{D^2-2D+1}\right)$$

To Find PI_2

$$PI_2 = \frac{1}{4} + \frac{1}{2(D^2+1)}\left(\frac{\cos x}{-2D}\right) = \frac{1}{4} - \frac{\sin x}{4(D^2+1)}$$

$$= \frac{1}{4} - x\frac{\sin x}{4(2D)} = \frac{1}{4} + x\frac{\cos x}{8}$$

$$\therefore\ PI_2 = \frac{1}{4} + x\frac{\cos x}{8}$$

Thus, $PI = PI_1 + PI_2$ And the GS is $y = CF + PI$

THANK YOU

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