CL18_Q1. Show that the energy of an electron confined in a 1-D symmetric potential well of length 'L' and infinite depth is quantized. Is the electron trapped in a potential well allowed to take zero energy? If not, why?

Answer

The energy of the nth eigenstate is given by $E_n = \frac{h^2 n^2}{8mL^2}$ where n = 1,2 3...

The Eigen values are $E_1 = \frac{h^2}{8mL^2}$ $E_2 = \frac{h^2 2^2}{8mL^2}$ $E_3 = \frac{h^2 3^2}{8mL^2}$

Thus, the energies are quantized with n being the quantum number. The quantization is imposed by the boundary conditions and the requirement of normalizability. All bound quantum states are in fact quantized.
For an electron trapped within a one dimensional potential lwell, when n = 0, the wave function is zero for all values of x, i.e., it is zero even within the potential well. This would mean that the electron is not present within the well. Therefore the state with n = 0 is not allowed. As energy is proportional to n^2, the ground state energy cannot be zero since n = 0 is not allowed

CL18_Q2. Elaborate the concept of parity as applied to Eigen functions. When is it possible to describe the parity aspect of Eigen functions?

Answer

The parity of a function is determined by changing the sign of the variable. If the function remains unchanged, then it is defined as an even parity function and if the function changes sign then it is an odd parity function. If $\psi(-x) = \psi(x)$, then the function has an even parity and if $\psi(-x) = -\psi(x)$, then the function has an odd parity.

CL18_Q3. Write the wave functions for the states n = 1, n = 2 and n = 3, for a particle in an infinite square well potential.

Answer

The eigen wave functions of the particle in box

$$\boldsymbol{\psi_n(x) = \sqrt{\frac{2}{a}} cos\left(\frac{n\pi}{a}x\right) \quad for\ n\ odd} \qquad \text{(even parity)}$$

$$\boldsymbol{\psi_n(x) = \sqrt{\frac{2}{a}} sin\left(\frac{n\pi}{a}x\right) \quad for\ n\ even} \qquad \text{(odd parity)}$$

The eigen wave functions of the particle in box in the first four states can be written as

$$\psi_1(x) = \sqrt{\frac{2}{a}}\cos\left(\frac{1\pi}{a}x\right) \quad n = 1 \qquad \text{(even parity)}$$

$$\psi_2(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{2\pi}{a}x\right) \quad n = 2 \qquad \text{(odd parity)}$$

$$\psi_3(x) = \sqrt{\frac{2}{a}}\cos\left(\frac{3\pi}{a}x\right) \quad n = 3 \qquad \text{(even parity)}$$