# ENGINEERING MATHEMATICS I

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# HIGHER ORDER DIFFERENTIAL EQUATIONS

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### CLASS CONTENT

- **To Solve a Higher Order Homogeneous Linear Differential Equation with constant coefficients**

- Case 1. When the Auxiliary Equation has Real and Distinct Roots

- Case 2. When the Auxiliary Equation has Real and Equal Roots

- Case 3. When the Auxiliary Equation has Imaginary Roots

## GENERAL SOLUTION OF SECOND ORDER LINEAR HOMOGENEOUS DIFFERENTIAL EQUATION

**AUXILIARY EQUATION**:

- Consider, + + y = 0
- Let us assume y =
- = and ’ =

## GENERAL SOLUTION OF SECOND ORDER LINEAR HOMOGENEOUS DIFFERENTIAL EQUATION

**AUXILIARY EQUATION:**

- Substituting into the above equation, we see that y = is a solution if
- (+ m+ ) = 0
- **+ m+ = 0 is the AUXILIARY EQUATION of (** + )y = 0

## GENERAL SOLUTION OF SECOND ORDER LINEAR HOMOGENEOUS DIFFERENTIAL EQUATION

Consider a second order homogeneous differential equation

$+ \quad + y = 0$

The auxiliary polynomial of this differential equation is f(m)=0

i.e, $m \quad = 0$

The solution of the differential equation depends on the nature of roots of the auxiliary equation.

## GENERAL SOLUTION OF SECOND ORDER HOMOGENEOUS LINEAR DIFFERENTIAL EQUATION

### CASE I

- Let the roots of the auxiliary equation be $m_1$ and $m_2$
- If the roots $m_1$ and $m_2$ are real and distinct,

then the general solution of the differential equation

$\frac{d^2y}{dx^2}+k_1\frac{dy}{dx}+k_2y=0$ is $y=c_1e^{m_1x}+c_2e^{m_2x}$

ILLLUSTRATION: Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}-3y=0$

Auxiliary equation $m^2+2m-3=0$

Roots $m=-3,1$

GS is $y=c_1e^{-3x}+c_2e^x$

# ENGINEERING MATHEMATICS I

## GENERAL SOLUTION OF SECOND ORDER LINEAR DIFFERENTIAL EQUATION

### CASE II

- If the roots $m_1$ and $m_2$ are real and equal,

then the general solution of the differential equation

$$\frac{d^2y}{dx^2} + k_1\frac{dy}{dx} + k_2y = 0 \text{ is} \qquad y = (c_1 + c_2x)e^{mx}$$

ILLLUSTRATION: Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$

Auxiliary equation $m^2 + 4m + 4 = 0$

Roots $m = -2, -2$

GS is $y = (c_1 + c_2x)e^{-2x}$

## GENERAL SOLUTION OF SECOND ORDER LINEAR HOMOGENEOUS DIFFERENTIAL EQUATION

### CASE III

If the roots $m_1$ and $m_2$ are complex

Let the roots be $m_1 = \alpha + i\beta$ and $m_2 = \alpha - i\beta$

then the general solution of the differential equation

$$\frac{d^2y}{dx^2} + k_1\frac{dy}{dx} + k_2 y = 0 \text{ is } y = e^{\alpha x}\left(c_1 \cos\beta x + c_2 \sin\beta x\right)$$

ILLLUSTRATION: Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0$

AE is $m^2 - 4m + 13 = 0$

Roots are $2 \pm 3i$

The GS is $y = e^{2x}\left(c_1 \cos 3x + c_2 \sin 3x\right)$

## GENERAL SOLUTION OF HOMOGENEOUS LINEAR DIFFERENTIAL EQUATION OF ORDER "n"

Consider the differential equation

$$\left(D^n + k_1 D^{n-1} + k_2 D^{n-2} + \ldots\ldots\ldots\ldots\ldots + k_n\right)y = 0$$

**Let the roots of the AE**

$$m^n + k_1 m^{n-1} + k_2 m^{n-2} + \ldots\ldots\ldots\ldots + k_n = 0 \text{ be}$$

$$m_1, m_2, - - - - - m_n$$

## GENERAL SOLUTION OF HOMOGENEOUS LINEAR DIFFERENTIAL EQUATION OF ORDER "n"

### Case(i)

If the roots $m_1, m_2, - - - - - m_n$ are real and distinct,

then the general solution is

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + - - - + c_n e^{m_n x}$$

## GENERAL SOLUTION OF HOMOGENEOUS LINEAR DIFFERENTIAL EQUATION OF ORDER "n"

### Case(ii)

If the roots $m_1, m_2, \ldots\ldots\ldots, m_n$ are real and equal (say equal to $m$),

then general solution is

$$y = \left(c_1 + c_2 x + \ldots\ldots\ldots + c_n x^{n-1}\right) e^{mx}$$

### Case (iii)

If the roots $m_1, m_2, m_3, m_4$ are real and equal (say equal to $m$) and the remaining $(n-4)$ roots are real and distinct, then the general solution is

$$y = \left((c_1 + c_2 x + c_3 x^2 + c_4 x^3) e^{mx} + c_5 e^{m_5 x} + c_6 e^{m_6 x} - - - - - - + c_n e^{m_n x}\right)$$

## GENERAL SOLUTION OF HOMOGENEOUS LINEAR DIFFERENTIAL EQUATION OF ORDER "n"

### Case (iv)

If there are two pairs of imaginary roots $\alpha \pm i\beta$, $\gamma \pm i\delta$, then the general solution is

$$y = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x) + e^{\gamma x}(c_3 \cos\delta x + c_4 \sin\delta x) + c_5 e^{m_5 x} + - - + c_n e^{m_n x}$$

### Case (v)

If a pair of complex roots say $\alpha \pm i\beta$ occuring twice, then the GS is

$$y = e^{\alpha x}\left((c_1 + c_2 x)\cos\beta x + (c_3 + c_4 x)\sin\beta x\right) + c_5 e^{m_5 x} + - - + c_n e^{m_n x}$$

## HOMOGENEOUS LINEAR DIFFERENTIAL EQUATION OF ORDER "n"

Solve: $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = 0$

$$(D^3 + 4D)y = 0$$

AE is $m^3 + 4m = 0$

Roots are $m = 0, \pm 2i$

$$y = c_1 e^{0x} + e^{0x}(c_2 \cos 2x + c_3 \sin 2x)$$

Thus, $y = c_1 + c_2 \cos 2x + c_3 \sin 2x$

# THANK YOU

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