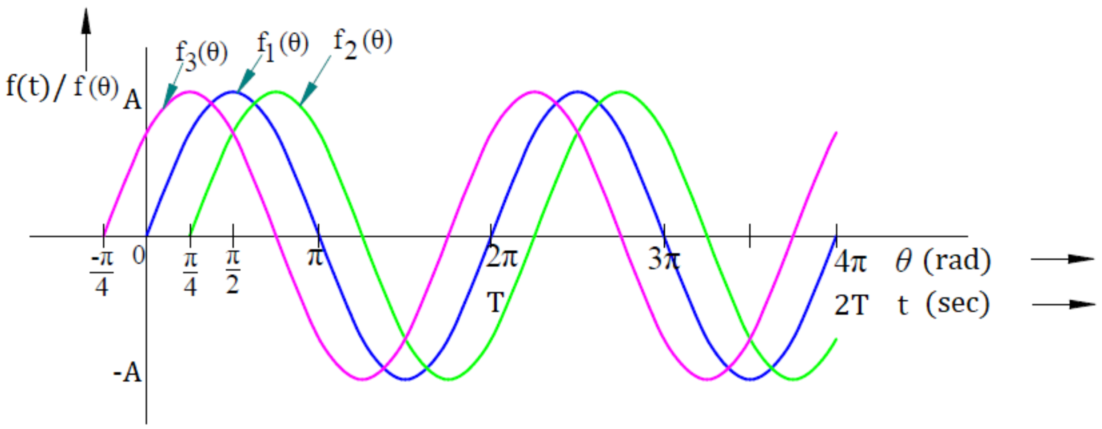
NOTES -Class 21

Concept of Phase Lag and Phase Lead:



$f_1(t) = A\sin(\omega t)$ represents a reference sine wave.

$f_2(t) = A\sin(\omega t - \frac{\pi}{4})$ lags reference sine wave by $\frac{\pi}{4}$ rad.

$f_3(t) = A\sin(\omega t + \frac{\pi}{4})$ leads reference sine wave by $\frac{\pi}{4}$ rad.

Also, $f_2(t)$ lags $f_3(t)$ by $\frac{\pi}{2}$ rad.

In general, sinusoidal function is represented as $A \sin(\omega t + \phi)$ where, ϕ represents the phase angle. If ϕ is positive, it leads the reference sine wave and lags if ϕ is negative.

Question 3:

Write an equation to represent the following sine waves of 50Hz frequency.

i) A sinusoidal current with RMS value 10A & starting at 5ms

ii) A sinusoidal current with peak value 20A & starting at -2.5ms

Also, comment on the phase relation between them.

Solution:

$\omega = 2\pi f = 100\pi rad/s$

Case (i) : Angle = $\omega*t = (100\pi*0.005) = \frac{\pi}{2}$ rad

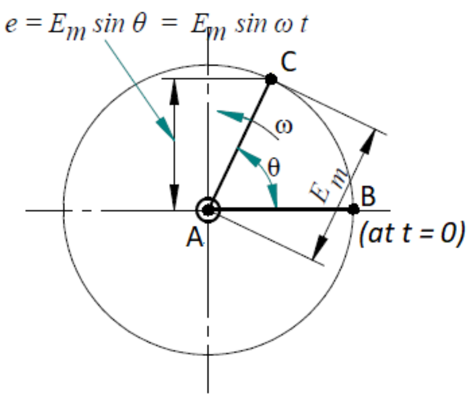
$i_1(t) = 10\sqrt{2}\sin(100\pi t - \frac{\pi}{2})A$

Case (ii) : Angle = $\omega*t = (100\pi*0.0025) = \frac{\pi}{4}$ rad

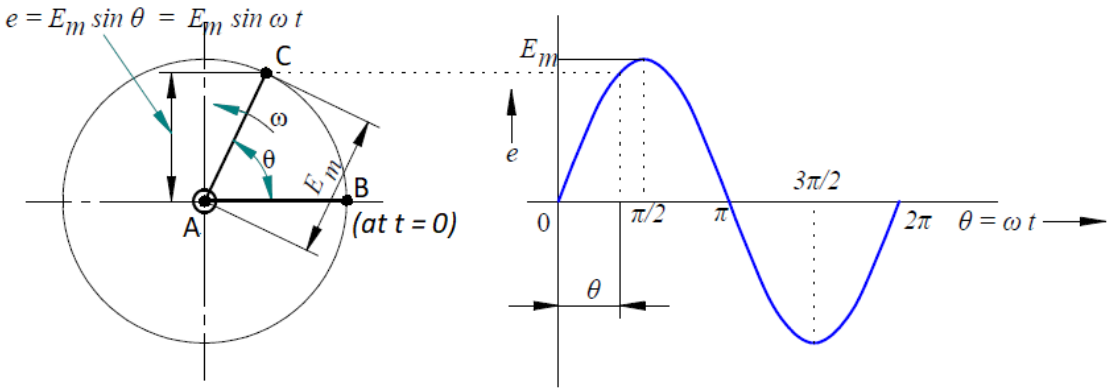
$i_2(t) = 20\sin(100\pi t + \frac{\pi}{4})A$

Concept of Phasor:

Let us consider a rotating crank of length E_m lying at 0° position at t = 0 and rotating anticlockwise at an angular speed of 'ω' rad/s.



At general time 't', it would be at an angle θ = ωt. Its vertical projection defines a sinusoidal function.



(a) Crank (b) Sinusoidal waveform

Thus the above rotating crank represents a sinusoidal function of the form $E_m\sin(\omega t)$.

Similarly, a sinusoidal function of the form $E_m\sin(\omega t + \phi)$ can be represented by another rotating crank of same length 'E_m' and rotating with same angular speed 'ω' rad/s anticlockwise but lying at an angle 'ϕ' at t = 0.

Thus, any sinusoidal function can be represented by a rotating crank and it is called '**Phasor representation**' of a sinusoidal function.

A **Phasor** is a rotating vector which effectively represents a sinusoidal function.

When a number of sinusoidal functions are to be represented as phasors, it is represented using a diagram called **phasor diagram**. While drawing a phasor diagram, all phasors must be represented corresponding to same point in time. It is usually preferred to represent them at a time t = 0. Then, angular position of each sinusoidal function corresponds to its phase angle.

Note: Only sinusoidal functions of same frequency can be represented together as a phasor diagram. Also, the length of the phasor is its RMS value.

Question 4:

Consider the following sinusoidal functions

a. $f_1(t) = 100\sin(100\pi t)$
b. $f_2(t) = 200\sin(100\pi t + 60°)$
c. $f_3(t) = 100\cos(100\pi t - 60°)$

Let us represent them using a phasor diagram.

Solution:

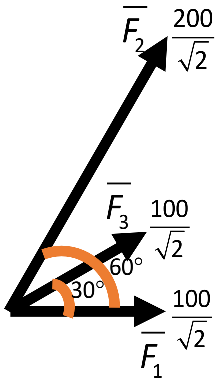
Note: Convert a cosine function to sine form before representing as a phasor.

For instance,

$f_3(t) = 100\cos(100\pi t - 60°)$

$= 100\sin(100\pi t - 60°+90°)$

$= 100\sin(100\pi t + 30°)$



Mathematical Representation of a Phasor:

A phasor is mathematically represented as

Phasor = Magnitude $\angle$Phase Angle

where, magnitude is the RMS value.

For instance, Consider these sinusoidal functions

i) $f_1(t) = 100\sin(100\pi t)$
ii) $f_2(t) = 200\sin(100\pi t + 60°)$
iii) $f_3(t) = 100\cos(100\pi t - 60°)$

Let us represent them using phasor representation.

$$f_1(t) = 100\sin(100\pi t) \quad \Rightarrow \overline{F_1} = \frac{100}{\sqrt{2}}\angle 0°$$

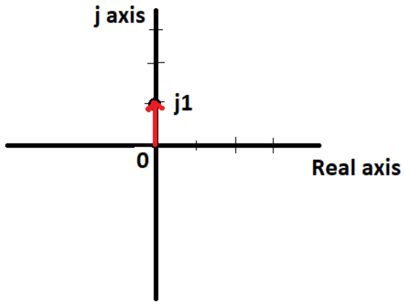
$$f_2(t) = 200\sin(100\pi t + 60°) \quad \Rightarrow \overline{F_2} = \frac{200}{\sqrt{2}}\angle 60°$$

$f_3(t) = 100\cos(100\pi t - 60°) = 100\sin(100\pi t + 30°)$

$$\Rightarrow \overline{F_3} = \frac{100}{\sqrt{2}}\angle 30°$$

j operator

'j' operator in phasor representation is analogous to 'i' operator in complex mathematics.



In rectangular form, j = (0 + j1)

In polar form, j = $1\angle 90°$

Conversion between the forms

Polar to Rectangular conversion :

Let us consider a polar number $r\angle\theta$

It can be converted to rectangular form (A + jB) using

$A = r\cos\theta$; $B = r\sin\theta$

Rectangular to Polar conversion :

Let us consider a rectangular number (A + jB)

It can be converted to polar form $r\angle\theta$ using

$$r = \sqrt{A^2 + B^2} \;;\; \theta = \mathrm{Tan}^{-1}\left(\frac{B}{A}\right)$$

θ will be positive if ‘B’ is positive and it is negative if ‘B’ is negative.

Addition, Subtraction, Multiplication & Division of Phasors:

Addition & Subtraction of Phasors:

Addition & subtraction of phasors would be easier in rectangular form.

For instance, let $\overline{F_1} = (A_1 + jB_1)$ & $\overline{F_2} = (A_2 + jB_2)$

$$\overline{F_1} + \overline{F_2} = (A_1 + A_2) + j(B_1 + B_2)$$

$$\overline{F_1} - \overline{F_2} = (A_1 - A_2) + j(B_1 - B_2)$$

Multiplication & Division of Phasors:

Multiplication & Division of phasors would be easier in Polar form.

For instance, let $\overline{F_1} = r_1\angle\theta_1$ & $\overline{F_2} = r_2\angle\theta_2$

$$\overline{F_1} * \overline{F_2} = r_1 * r_2\angle(\theta_1 + \theta_2)$$

$$\frac{\overline{F_1}}{\overline{F_2}} = \frac{r_1}{r_2}\angle(\theta_1 - \theta_2)$$

j operator properties:

'j' operator when multiplied to a phasor, does not change the magnitude of the phasor but rotates the phasor anticlockwise by 90°

For instance, if $\bar{F} = 3\angle 60°$, then $j\bar{F} = 1\angle 90° * 3\angle 60° = 3\angle 150°$

$$j^2 = 1\angle 90° * 1\angle 90° = 1\angle 180° = \cos(180°) + j\sin(180°) = -1$$

Similarly, $j^3 = j^2 * j = -j$

And $j^4 = j^2 * j^2 = 1$

Question 5:

There are 3 conducting wires connected to form a junction. The currents flowing into the junction in two wires are $i_1 = 10\sin 314t$ A and $i_2 = 15\cos(314t - 45°)$A. What is the current leaving the junction in the third wire? What is its value at t=0?

Solution: 1) Using Time-Domain Method

By KCL at the junction, $i_3(t) = i_1(t) + i_2(t)$

$$i_3(t) = 10\sin(314t) + 15\cos(314t - 45°)$$

$$i_3(t) = 10\sin(314t) + 15*(\cos 314t*\cos 45° + \sin 314t*\sin 45°)$$

$$i_3(t) = 20.61\sin(314t) + 10.61\cos(314t)$$

$$i_3(t) = 23.18*\left(\frac{20.61}{23.18}\sin(314t) + \frac{10.61}{23.18}\cos(314t)\right)$$

$$i_3(t) = 23.18*(\cos(27.24°)*\sin(314t) + \sin(27.24°)\cos(314t))$$

$$i_3(t) = 23.18*\sin(314t + 27.24°) \text{ A}$$

Its value at t = 0 is $i_3(0) = 23.18\sin(27.24°) = 10.61$A

Solution: 2) Using Phasor Domain Method

By KCL at the junction, $i_3(t) = i_1(t) + i_2(t)$

In Phasor form,

$$\bar{I}_3 = \bar{I}_1 + \bar{I}_2$$

$i_1(t) = 10\sin(314t) \Rightarrow \bar{I}_1 = \frac{10}{\sqrt{2}}\angle 0°A$

$i_2(t) = 15\cos(314t - 45°) = 15\sin(314t + 45°) \Rightarrow \bar{I}_2 = \frac{15}{\sqrt{2}}\angle 45°A$

$$\bar{I}_3 = \frac{10}{\sqrt{2}}\angle 0° + \frac{15}{\sqrt{2}}\angle 45° = 16.39\angle 27.24°A$$

$i_3(t) = 23.18*\sin(314t+27.24°)$ A

Its value at t = 0 is $i_3(0) = 23.18\sin(27.24°) = 10.61$A