ENGINEERING MECHANICS - STATICS

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Department of Civil Engineering

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DISTRIBUTED FORCES

Session- 2

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Centroid of the area of a circular sector:

Due to symmetry $\bar{y} = 0$ & $\bar{x} = \dfrac{\int x_c dA}{A}$ ----(1)

Consider an element at a distance r_o from the centre O of a sector, radial width being dr_o & bound by radii at θ & $(\theta + d\theta)$

$dA = r_o d\theta\, dr_o \qquad x_c = r_o \cos\theta \qquad A = R^2 \alpha$
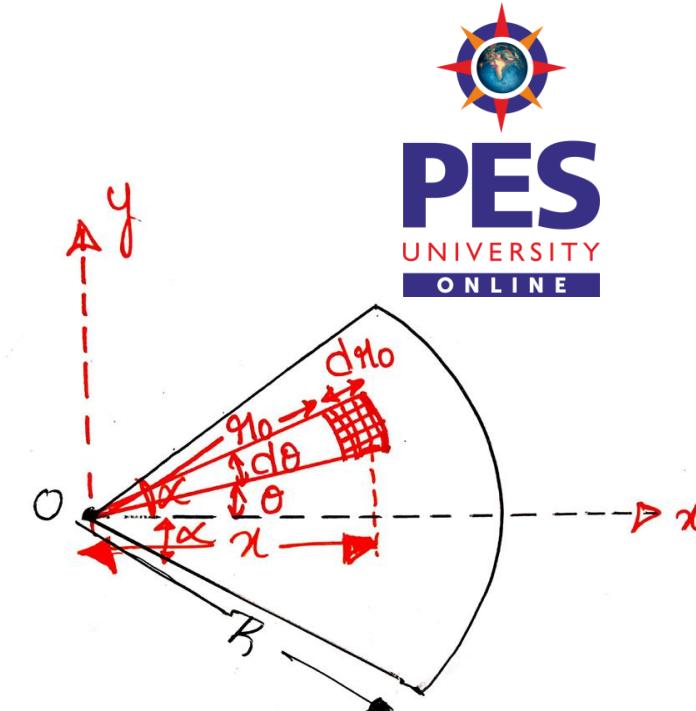
$\qquad\qquad Substituting\ in\ equation(1)$

$$\bar{x} = \frac{\int r_o\ cos(\theta)\ r_o d\theta\ dr_o}{A} = \frac{\int_{-\alpha}^{+\alpha}\int_0^R r_o^2\ dr_o\ cos\theta\ d\theta}{A} = \frac{\frac{R^3}{3} 2\ sin\alpha}{A}$$

$$= \frac{\frac{R^3}{3} 2\ sin\alpha}{R^2\,\alpha} = \frac{2R\ sin\alpha}{3\alpha}$$

Here α is in radians.

$$\boxed{\bar{x} = \frac{2R\ sin\alpha}{3\alpha}\ \&\ \ \bar{y} = 0}$$

Locate the centroid of the area of a semicircle
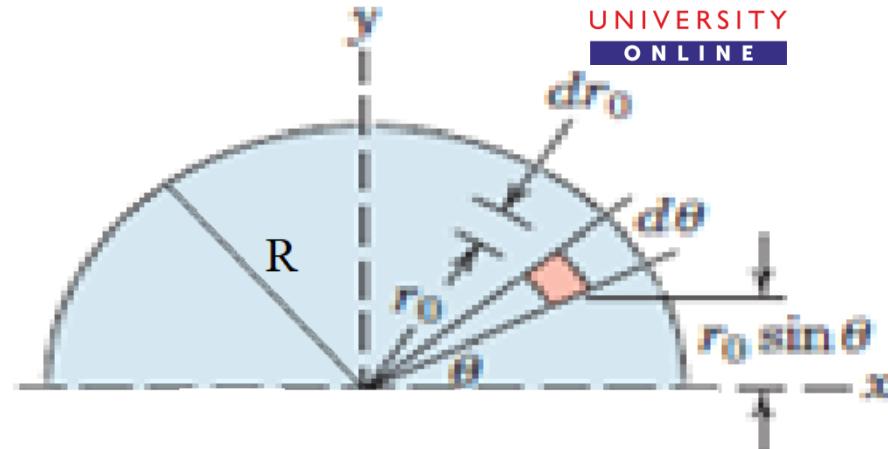
Due to symmetry $\overline{x} = 0$ & $\overline{y} = \dfrac{\int y_c dA}{A}$----(1)

Consider an element at a distance r_o from the centre O of a semicircle, radial width being dr_o & bound by radii at θ & $\theta + d\theta$

$dA = r_o d\theta \, dr_o \qquad y_c = r_o \sin\theta \qquad A = \dfrac{\pi R^2}{2}$ $\;Substituting\;in\;equation\;(1)$
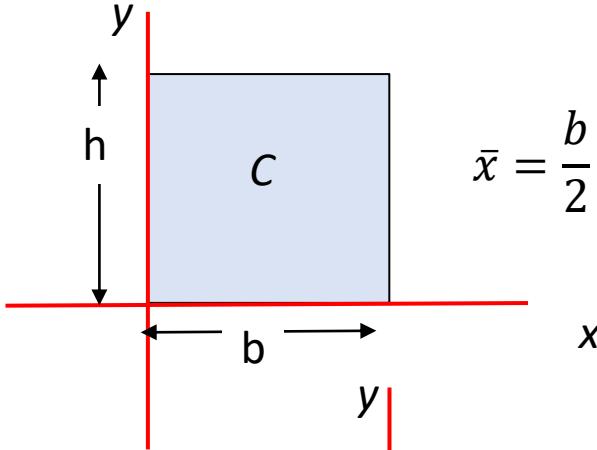
$$\overline{y} = \frac{\int r_o \sin\theta \, r_o d\theta \, dr_o}{A} = \frac{\int_0^\pi \int_0^R r_o{}^2 \sin\theta \, d\theta \, dr_o}{A} = \frac{\frac{2R^3}{3}}{A} = \frac{\frac{2R^3}{3}}{\frac{\pi R^2}{2}} = \frac{4R}{3\pi}$$

$$\boxed{\overline{x} = 0 \;\&\; \overline{y} = \frac{4R}{3\pi}}$$
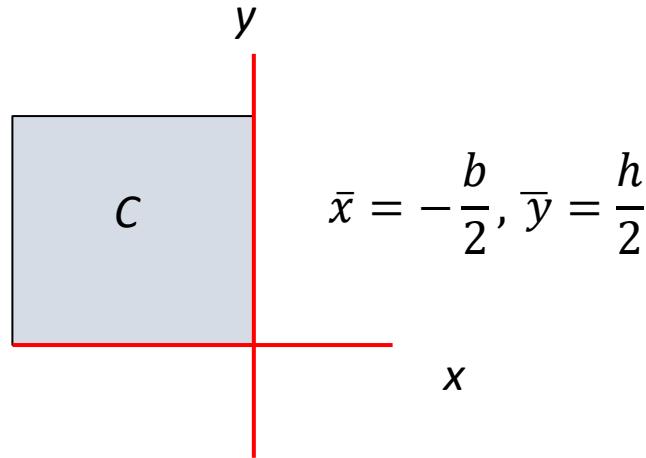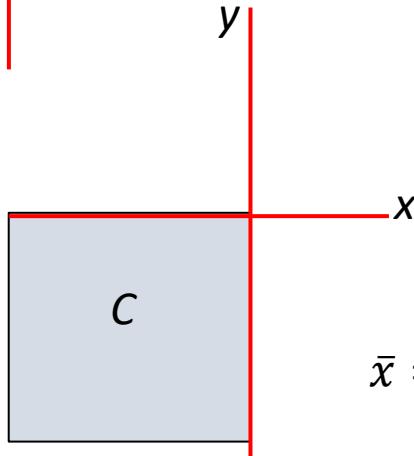
Centroid

$$\bar{x} = \frac{b}{2}, \ \overline{y} = \frac{h}{2}$$

$$\bar{x} = -\frac{b}{2}, \ \overline{y} = \frac{h}{2}$$

$$\bar{x} = -\frac{b}{2}, \ \overline{y} = -\frac{h}{2}$$

Centroid

$$\overline{y} = \frac{h}{3}$$

$$\overline{x} = \frac{b}{3} \; , \; \overline{y} = \frac{h}{3}$$

$$\overline{x} = -\frac{2b}{3}, \overline{y} = -\frac{2h}{3}$$

$$\overline{x} = -\frac{2b}{3}, \overline{y} = \frac{h}{3}$$

Centroid

$\bar{x} = R, \bar{y} = R$

$\bar{x} = 0, \bar{y} = 0$

$\bar{x} = -R , \bar{y} = 0$

$\bar{x} = 0, \bar{y} = R$
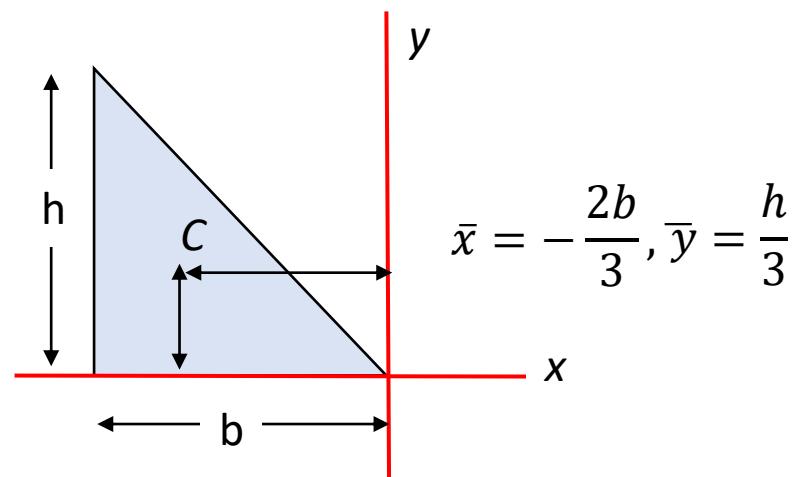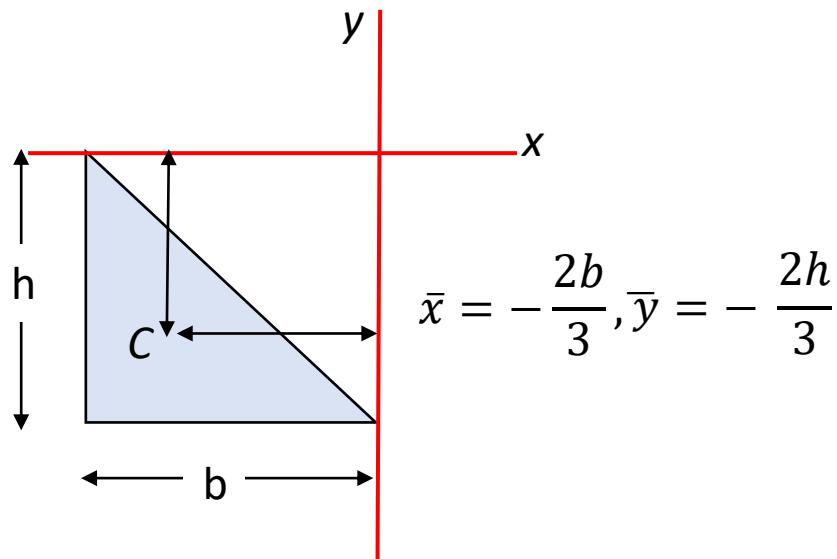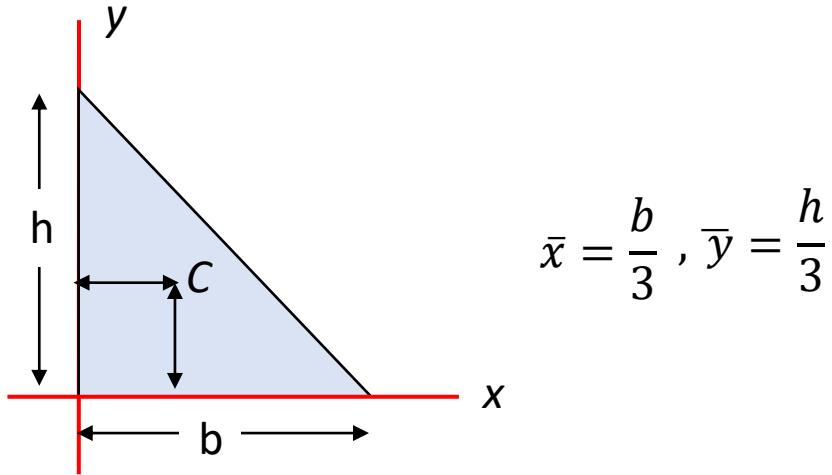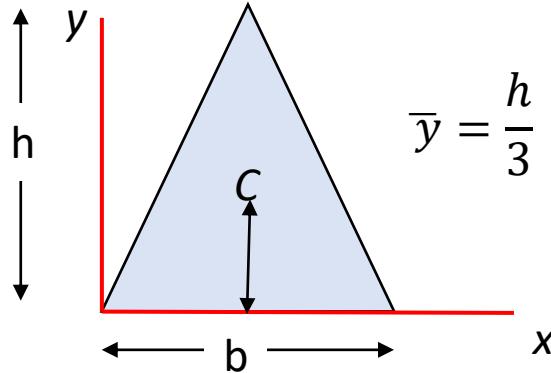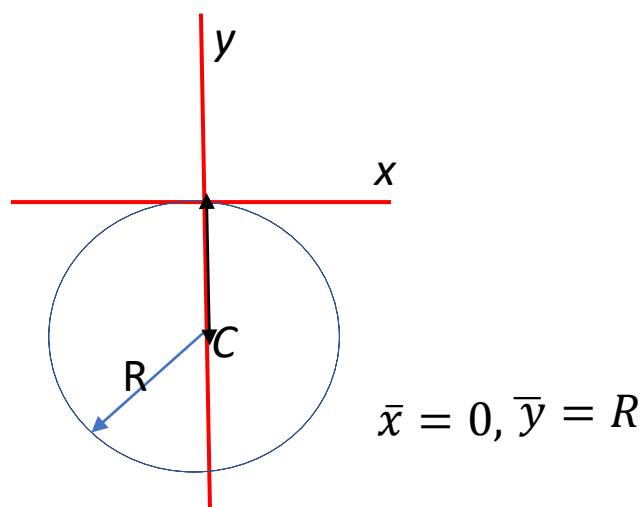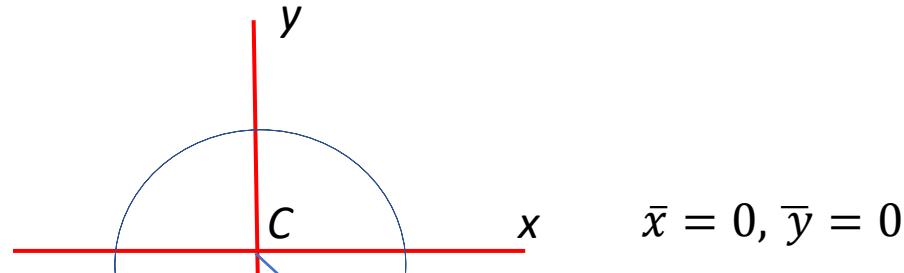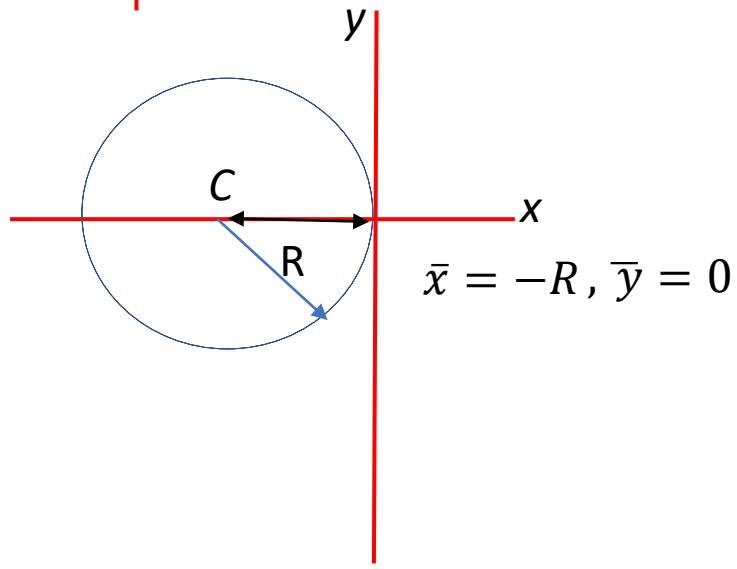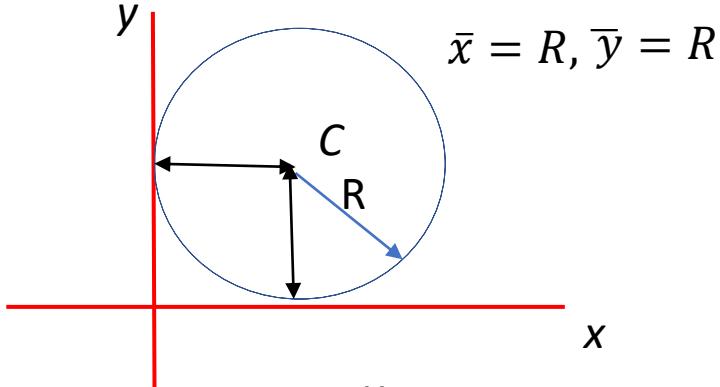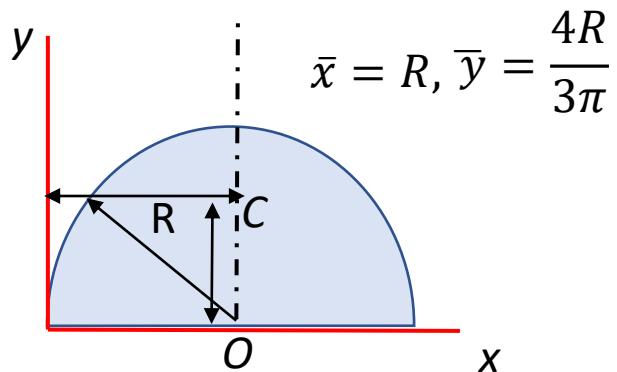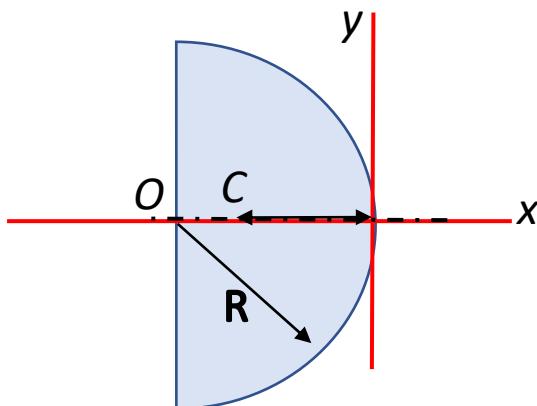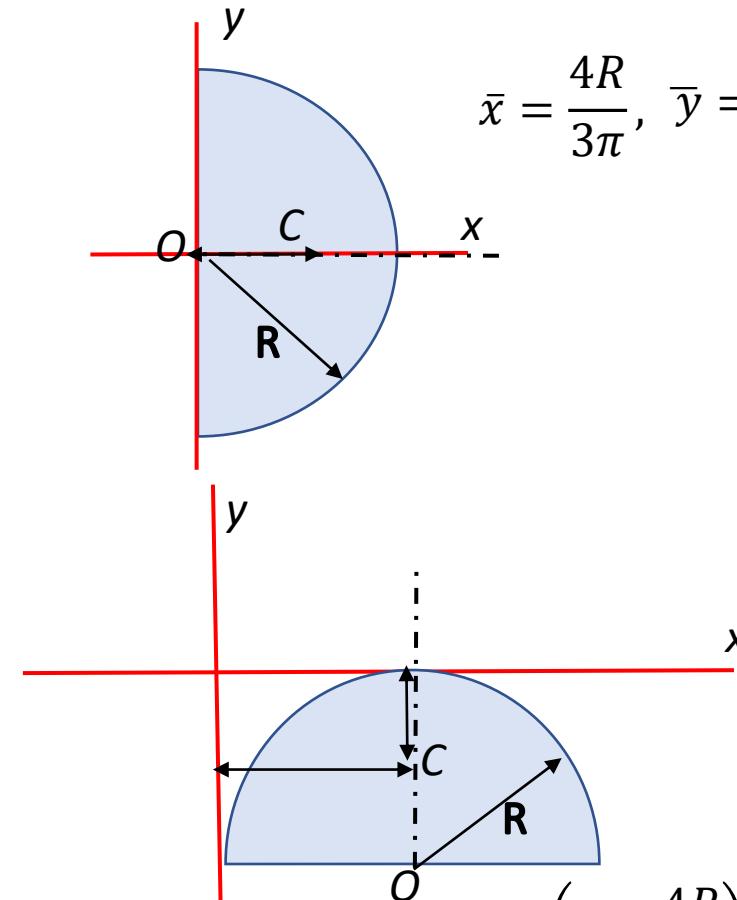
Centroid

$$\bar{x} = R, \quad \overline{y} = \frac{4R}{3\pi}$$
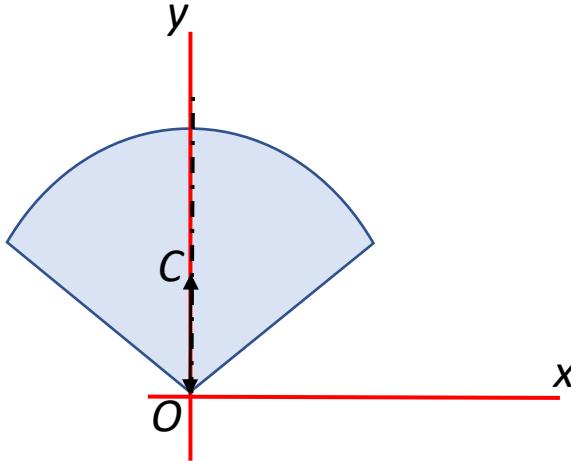
$$\bar{x} = \frac{4R}{3\pi}, \quad \overline{y} = 0$$

$$\bar{x} = -\left(R - \frac{4R}{3\pi}\right), \quad \overline{y} = 0$$
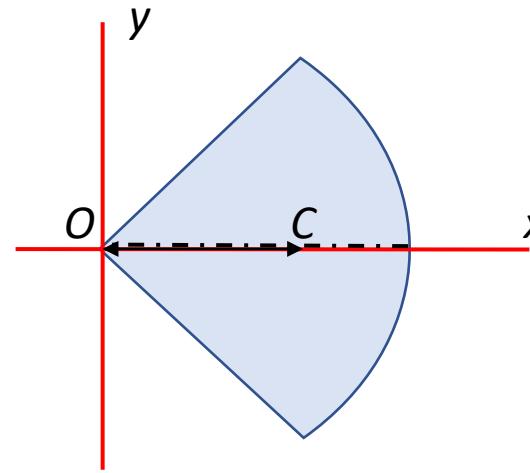
$$\bar{x} = R, \quad \overline{y} = -\left(R - \frac{4R}{3\pi}\right)$$

Centroid

$$\bar{x} = 0 \ , \ \overline{y} = \frac{2R \ sin\alpha}{3\alpha}$$
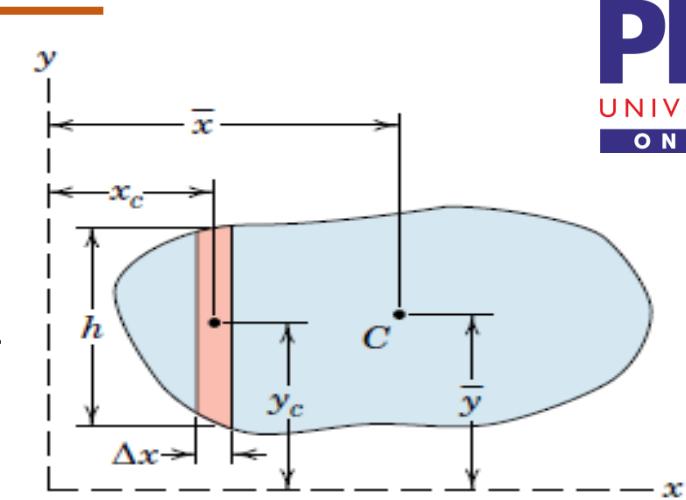
$$\bar{x} = \frac{2R \ sin\alpha}{3\alpha} \ , \ \overline{y} = 0$$

Composite Bodies And Figures; Approximations

The area is divided into strips of width *dx* and variable height h. The area A of each strip, such as the one shown in red, is h *dx* and is multiplied by the coordinates x_c and y_c of its centroid to obtain the moments of the element of area. The sum of the moments for all strips divided by the total area of the strips will give the corresponding centroidal coordinate. A systematic tabulation of the results will permit an orderly evaluation of the total area $\sum A$, the sums $\sum A x_c$ and $\sum A y_c$, and the centroidal coordinates

$$\bar{x} = \frac{\sum_{i=1}^{i=n} a_i x_i}{\sum_{i=1}^{i=n} a_i}, \bar{y} = \frac{\sum_{i=1}^{i=n} a_i y_i}{\sum_{i=1}^{i=n} a_i} \quad \sum_{i=1}^{i=n} a_i = A$$

THANK YOU

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