

Unit IV LASERS**Contents :**

1. Introduction – interaction of radiation with matter – Einstein's Coefficient
2. Conditions for laser action
 - a. Two level system
 - b. Three level system
 - c. Four level system
3. Basic requirement of a laser system
4. Round trip gain in a laser medium
5. Properties of LASERS.
6. Atomic LASER - Ruby Laser and He Ne LASER system
7. Molecular LASER – CO₂ LASER system
8. Semiconductor LASER – CO₂ LASER system
9. Dielectric Polarisation
10. Non-Linear Dielectrics

Introduction

LASER is the acronym for Light Amplification by Stimulated Emission of Radiation. Einstein analyzed the interaction of radiation with matter and formalized the rate equations for induced absorption and spontaneous emission. He also conceptualized stimulated emission as a probable emission mechanism emission from excited atoms. From the discussions that follow it will be evident that the process of stimulated emission is the key to a LASER system.

1. Interaction of radiation with matter – Einstein's coefficients

The interaction of radiation with matter can be explained by the three processes namely

- **Induced absorption (stimulated absorption)**

In the induced absorption process an atom in the ground state / lower energy state (E_1) absorbs radiation and is excited to the higher state (E_2). The rate of absorption is dependent on the population of the ground state N_1 / lower energy state and the energy density of radiation ($\rho(hv)$) of the appropriate frequency such that $hv = E_2 - E_1$.

$$\text{The rate of induced absorption } R_{ind\ abs} = B_{12} * N_1 * \rho(v)$$

where B_{12} is the Einstein's coefficient for induced absorption.

- **spontaneous emission**

An atom in the higher energy / excited state cannot normally remain in the excited state for a long time and generally de-excites to the lower energy state spontaneously. The lifetimes of the excited states are generally of the order of nanoseconds. The rate of spontaneous emission is dependent on the population of atoms in the excited state N_2 only and

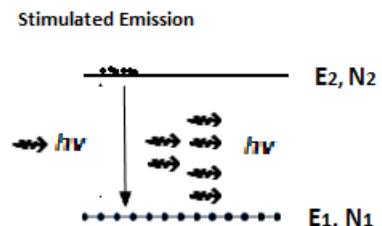
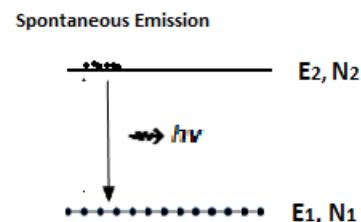
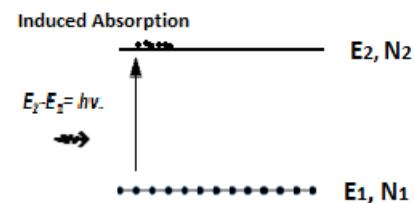
$$R_{sp\ em} = A_{21} * N_2 \quad \text{where } A_{21} \text{ is the Einstein's coefficient for spontaneous emission.}$$

$$\text{If the process of spontaneous emission is predominant we can infer that } R_{sp\ em} = -\frac{dN_2}{dt} = A_{21} * N_2.$$

From this we can infer that $N_2 = N_2(0)e^{-A_{21}t}$ and the Einstein's co-efficient for spontaneous emission can be understood to be $A_{21} = \frac{1}{\tau}$ where τ is the average life time of electrons in the upper energy state for spontaneous emission.

- **Stimulated emission.**

An atom in the excited state can have a life time in the excited state for longer periods of time of the order of milliseconds to few seconds. These states are referred to as Meta stable states. Such excited atoms have to be stimulated to return to the lower energy state with an external intervention in the form of a photon whose energy is equal to $E_2 - E_1$. In this process the energy of the excited



atom is released as a photon whose characteristics remain the same as that of the stimulating photon. This process sets in a chain of photon emission where all the photons are in the same state. The rate of stimulated emission is then dependent on the population of atoms in the excited state and the energy density of radiation is given by

$$R_{stim} = B_{21} * N_2 * \rho(\nu) \quad B_{21} \text{ is the Einstein's coefficient for stimulated emission.}$$

When the material is in thermal equilibrium with the radiation, the rate of absorption should be equal to the rates of emission due to different processes ie., $B_{12} * N_1 * \rho(\nu) = A_{21} * N_2 + B_{21} * N_2 * \rho(\nu)$

This gives

$$\rho(\nu)(B_{12} * N_1 - B_{21} * N_2) = A_{21} * N_2$$

$$\rho(\nu) = \frac{A_{21} * N_2}{(B_{12} * N_1 - B_{21} * N_2)} = \frac{A_{21}/B_{21}}{\left(\frac{B_{12} * N_1}{B_{21} * N_2} - 1\right)}$$

The distribution of electrons in the energy states are described by the Maxwell Boltzmann distribution laws and are given by $\frac{N_1}{N_2} = \exp\frac{(E_2 - E_1)}{kT} = \exp\frac{hv}{kT}$. Substitution of this in the equation for energy density gives the expression for the energy density of radiation as

$$\rho(\nu) = \frac{A_{21} * N_2}{(B_{12} * N_1 - B_{21} * N_2)} = \frac{A_{21}/B_{21}}{\left(\frac{B_{12}}{B_{21}} \exp\frac{hv}{kT} - 1\right)} \quad (1)$$

Comparing this with the Planck's expression for energy density of radiation at any frequency and temperature

$$\rho(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{\left(\exp\frac{hv}{kT} - 1\right)} \quad (2)$$

Comparing term by term we observe that $A_{21}/B_{21} = \frac{8\pi h\nu^3}{c^3}$ and $\frac{B_{12}}{B_{21}} = 1$. This implies that $B_{12} = B_{21} = B$ i.e., the induced absorption coefficient is equal to the stimulated emission coefficient and the ratio of the coefficient of spontaneous emission to the coefficient of stimulated emission is proportional to ν^3 .

For a system to have a predominant stimulated photon emission, then the ratio of the rate of stimulated emission to the rate of spontaneous emission should be greater than 1.

Thus $\frac{B * N_2 * \rho(\nu)}{A * N_2} = \frac{\rho(\nu)}{\frac{A}{B}} = \frac{1}{\left(\exp\frac{hv}{kT} - 1\right)} \approx \exp\frac{hv}{kT} = \frac{N_2}{N_1}$. (Since $hv \gg kT$, $\exp\frac{hv}{kT} \gg 1$ for visible radiations at normal temperatures).

This implies that the rate of stimulated emission will be predominant over rate of spontaneous emission if and only if $N_2 > N_1$ or the population of the higher energy state is higher than the lower energy state. This condition is referred to as the population inversion.

2. Conditions for the lasing action

Two level systems

The basic requirement for light amplification to occur is that the stimulated emission is the predominant emission mechanism over the spontaneous emission mechanism (which is the natural response of a system).

From the discussion it is evident that stimulated emission is possible when the upper energy state has a higher population of occupation than the lower energy state. For a two level laser system this requires $N_2 > N_1$ or population inversion has to be established between the higher and lower energy states. But from the MB distribution function we find that

$$\frac{N_1}{N_2} = \exp^{\frac{hv}{kT}} \gg 1.$$

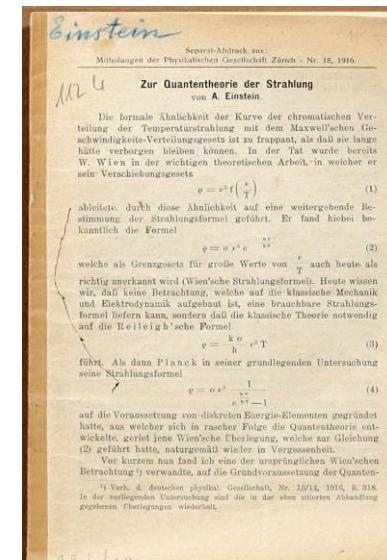
For a two level system in equilibrium this implies that T has to be negative if N_2 has to be greater than N_1 . Hence it is not possible to obtain population inversion between E_2 and E_1 in a two level system and it may not be possible to get a LASER beam from absorption and emission between two energy levels. The understanding is that it is not possible to get a LASER if the same levels are involved in both the emission and absorption process.

b. Three level systems:

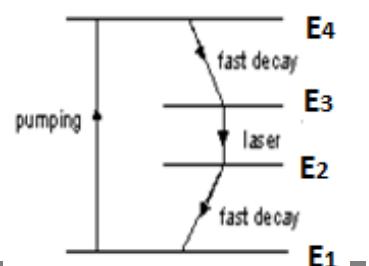
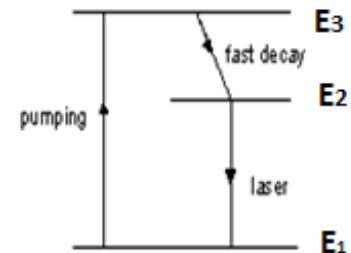
The introduction of an intermediate level between the ground state and the upper excited state can result in decoupling the emission process and absorption process levels. The absorption process is between the ground state E_1 and the upper excited state E_3 . The electrons from the upper energy state decays non-radiatively to the intermediate meta stable state E_2 . If this state is a meta stable state (lifetime of the electrons $\approx 10^{-3}$ seconds), electrons can accumulate in this state and the population of electrons in the meta stable state could be higher than the population of the ground state in a very short time resulting in a favorable condition for stimulated emission from E_2 to E_1 . However, the drawback is that the ground state is quickly depleted resulting in a discontinuous phenomenon of stimulated emission. Generally, three level systems give a pulsed LASER. This is because the ground state is still a common factor in the absorption and emission process.

c. Four level systems:

A four level system can effectively decouple the absorption levels and the emission levels. In a four level system the absorption is between the lower (ground) state E_1 and the higher excited state E_4 . The electrons in the excited state decays non radiatively to the intermediate meta stable state E_3 . The electrons are stimulated to transit to a lower energy state E_2 (above E_1).



Einstein's original paper on radiations 1917.



Finally the electrons from the level E_2 fall back to the ground state maintaining the population of the lower E_1 so that the process of excitation can continue. The absorption is between E_1 and E_4 whereas the stimulated emission is between E_3 and E_2 . Thus the energy states in the two processes are completely decoupled. In this way the system can behave in a continuous mode and can produce a continuous LASER.

3. Basic requirements of a laser system

- I. **Active medium** – The active medium consists of the medium which possess the appropriate energy levels which are meta stable states. The presence of the meta stable states increases the probability of population inversion which is a prime condition for laser action. The active medium could be solids, liquids or gases depending on the type of lasers.
- II. **Energy pump** – The constituents of the active medium have to suitably excited to the lasing high energy state from an external energy source. The external energy sources could be optical, thermal, electrical or chemical depending on the type of lasers. In the case of gas lasers, generally an electrical discharge is a sufficient source for exciting the medium.
- III. **Resonating Cavity** – Once the lasing action is initiated it is essential that the stimulated emission in the desired wavelength is amplified to get a sustainable laser action of sufficient intensity. The design of the optical cavity is an important aspect of the laser system.

The optical cavity may be the active medium of a suitable dimension (in the case of a solid state laser system) or a suitable enclosure for the gas or liquid as the case may be.

The emitted photons are contained in the cavity through multiple reflections by highly aligned mirrors at the end of the cavity. The reflectivity of one of the mirrors could be close to 100% while the second mirror could be 90-98% reflective.

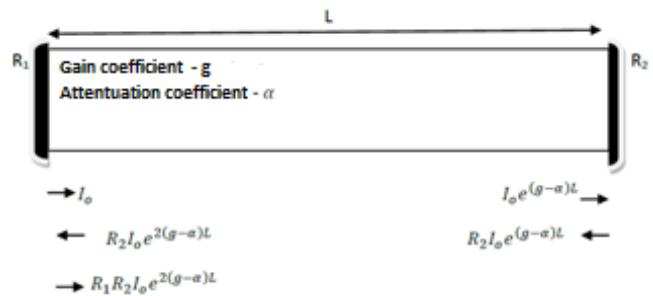
In general the optical cavity has to be a narrow region whose length in the direction of propagation is a half integer multiple of the desired wavelength.

This helps in the reflected beam to be in phase with the incident beam thus preventing photon loss due to destructive **interference**.

4. Round trip gain in a laser medium

The stimulated emission in the medium provides for gain with a optical feedback mechanism of reflecting mirrors on both ends of the cavity. This arrangement results in multiple travel of the trapped optical beam in the medium and ideally the beam should have a high intensity after few reflections. The gain of photons as the beam progresses is given by the intensity increasing as $I = I_o e^{gx}$ where g is the gain coefficient.

However, there could be also losses in the medium due to absorption, scattering and the partial transmission from one of the mirrors. The reduction in the intensity due to scattering and absorption is described by $I = I_o e^{-\alpha x}$ where α is the loss coefficient.



In order to reach a steady-state with non zero intensity (oscillation) the gain due to stimulated emission must be sufficient to overcome these losses.

In general the intensity of the photon beam as it travels a distance x in the medium can be written as

$$I = I_0 e^{(g-\alpha)x}$$

Consider a beam of photons of intensity I_0 originating from one of the cavity of length L at $x=0$.

One round trip of the photons is then the process where

- the photons starting from the face of the first mirror travel the length L to the second mirror
- gets reflected at the mirror with reflectivity R_2
- travels back to the first mirror covering a distance of L
- and reflected back from the mirror with reflectivity R_1

The intensity of the beam after one round trip gain is $I = I_0 R_1 R_2 e^{2(g_0 - \alpha)L}$

The amplification factor is then the ratio of the output intensity to the input intensity and should be equal to $R_1 R_2 e^{2(g_0 - \alpha)L}$.

If $R_1 R_2 e^{2(g_0 - \alpha)L} > 1$, oscillations can build up and the laser is said to be above the threshold. The threshold of laser oscillations is then defined by $R_1 R_2 e^{2(g_{th} - \alpha)L} = 1$

$$g_{th} = \frac{1}{2L} (2\alpha L - \ln(R_1 R_2))$$

This implies that the gain of the system can be tuned with the length of the cavity and the reflection coefficients of the two mirrors.

5. Properties of LASERs.

The most important properties of a LASER are attributed to the stimulated emission of photons (BOSONS which display identical properties)

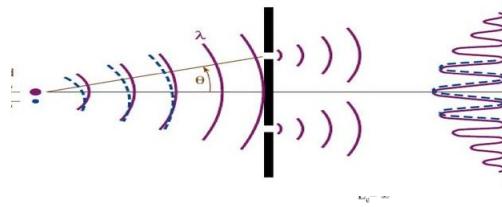
1. Monochromaticity (spectral line broadening): Light from a laser typically comes from an atomic transition with a single precise wavelength. So, the laser light has a single spectral color and is almost the purest monochromatic light available. However, the laser light is not truly monochromatic.

The emission line widths are also limited by the uncertainty principle which limits the accuracy of the energy (ΔE) of the photons emitted by electrons which spend times with a spread in time (Δt). Generally, LASER line widths $\Delta\lambda$ are very small of the order of 10^{-6} Å as compared to 1 Å for ordinary monochromatic sources.

The spectral emission line has a finite width due to Doppler Effect of the moving atoms or molecules from which the transitions occur. Since the wavelength of the light is extremely small compared to the size of the laser cavities used, many resonant modes of the tiny spectral bandwidth can exist in the laser cavity, however these are still very small compared to the line width of ordinary light sources.

Light source	Center Wavelength λ_0 (Å)	FWHM Line width $\Delta\lambda_0$ (Å)	FWHM line width $\Delta\nu$ (Hz)
Ordinary discharge lamp	5896	≈1	9X10 ¹⁰
Cadmium low-pressure lamp	6438	≈0.013	9.4X10 ⁸
Helium-neon laser	6328	≈10 ⁻⁷	7.5X10 ³

2. **Coherence** - Coherence is a unique property of laser light. In the stimulated emission process triggered by a common, the emitted photons are "in phase" or have a definite phase relation to each other. This coherence is essential to produce high quality interference, which is used to produce holograms.

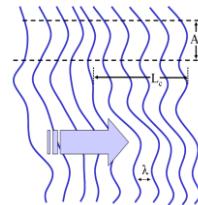


Ordinary light is incoherent because it comes from independent atoms, which emit on time scales of about 10^{-8} seconds. There is a degree of coherence in sources like the mercury green line and some other useful spectral sources, but their coherence does not approach that of a laser.

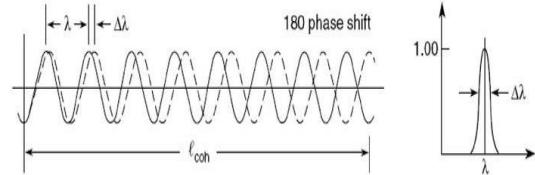
Coherence can be of two types of temporal coherence and spatial coherence.

- **Temporal coherence** refers to the correlation between the field at a point and the field at the same point after an elapse of time. If the phase difference between the two fields is constant during the period (of the order of microseconds), the wave is said to have temporal coherence. If the phase difference changes many times and in an irregular way during the period of observation, the wave is said to be non-coherent.

Temporal coherence is characteristic of a single beam of light. The temporal coherence is evaluated as $\tau_c = \frac{1}{\Delta\nu}$ where $\Delta\nu$ is the spread in the frequency of the Laser. The coherence length defines the largest distance for which interference can be well defined and is given by $l_c = \tau_c \cdot c$ where c is the velocity of light.



The length in which the coherence exists may be of the order of kilometers for LASERs compared to few centimeters for ordinary light.



- **Spatial coherence** - Two fields at two different points of a wave front is said to be spatially coherent if they preserve a constant phase difference over any time t . Two beams of light originating from different parts of a source will have been emitted by different groups of atoms. Each beam will be time incoherent and will have random phase changes. Two such beams are said to be spatially incoherent, and the interference pattern produced by these will have a poor visibility. When visibility of the interference pattern as a function of the size of the source then we have spatial coherence and is described by the coherence width $l_w \approx \frac{\lambda D}{d}$.

3. **Divergence** (directionality) – LASER is characterized by a very low divergence which ensures that the beam profile is small over long distances. The divergence of a LASER beam is given by $\theta = \frac{\lambda_o}{\pi\omega_o}$ where λ_o is the wavelength, and ω_o is the spot size. Typically, the divergence is of the order of mill radians (0.001°). A common lab laser beam of a wavelength of 532nm and a radius of 1mm on the surface of

the earth would have a diameter of 6.50 km on the surface of the moon. ($\theta = (2/\pi) * (532 \text{ e-9} / 2 * 10^{-3}) = 1.7 * 10^{-4}$ this is then multiplied by the distance to the moon ($3.844 * 10^8 \text{ m}$), which gives the spot size to be 65192 m.)

4. **Intensity** – The high intensity of a Laser arises out of the properties of monochromaticity, coherence and low divergence. Typically, very low power LASERS of about 1 to 2mW output with a beam diameter of 1 mm can result in an intensity of about 10 kW/m^2 as against an intensity of 10W/m^2 produced by a 20W bulb.

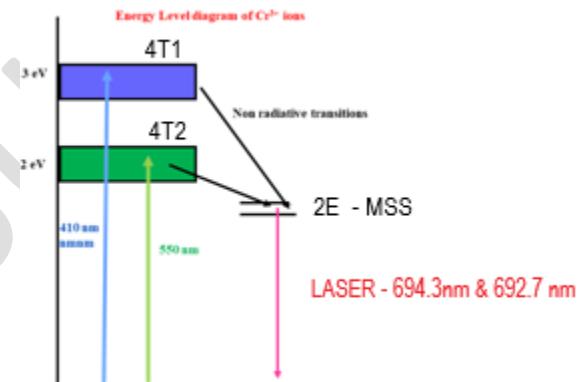
5. Atomic LASER

i) Ruby Laser

The first working laser was a Ruby laser made by Theodore H. Maiman at Hughes Research Laboratories in 1960. This is an example of an atomic three-level laser which is a pulsed laser and deliver very high energy.

Active Medium

The active medium of the system is a Ruby crystal – Al_2O_3 doped with Cr^{3+} ions (typical concentration is about 0.05%). The Al_2O_3 is a passive matrix which supports the Cr^{3+} ions.



The lasing action basically originates from the absorption and emission energy levels of the Cr^{3+} ions. The Cr^{3+} ions have absorption energy levels around 2eV and 3 eV where are normally de-exciting states (lifetime of electrons in the time frame of 10^{-9}s). These absorptions energies correspond to photons with 550 nm (green) and 410nm (blue) wavelengths as the absorption wavelengths.

These excited electrons transit from the upper absorption energy levels to the intermediate meta stable state around 1.74eV through non-radiative transitions. The Meta stable state typically has a lifetime of the order of 5ms. This leads to an increase in the population of the intermediate state while the population of the ground state is being depleted rapidly.

For a short duration of time the population of the intermediate state > the population of the intermediate state leading to POPULATION INVERSION. Transitions from the intermediate state to the ground state can lead to photons of the stimulated type leading to a LASER with a wavelength of 694.3 nm.

Typical ruby laser pulse widths are of the order of a millisecond.

Energy Pump

The energy pump for the Ruby Laser is a high-power white light flash lamp capable of delivering energy densities up to 21 kJ m^{-2} . A capacitor is charged to high voltages and discharged through the lamp (like the photo flash). Typically, the discharge results in 675-770 J of energy.

Two types of flash lamp arrangements are commonly used. The simplest arrangement is that of a helical flash lamp concentric with the ruby rod (at the center)

The other arrangement has an elliptical mirror where the flash lamp and the ruby rods are at the two foci of mirror arrangement. This arrangement is found to be more efficient than the first arrangement.

Resonant Cavity

The resonant cavity of the system is the ruby rod of appropriate length with its ends highly polished and within a quarter of a wavelength of the output light. The two polished faces are parallel to each other within a few seconds of arc.

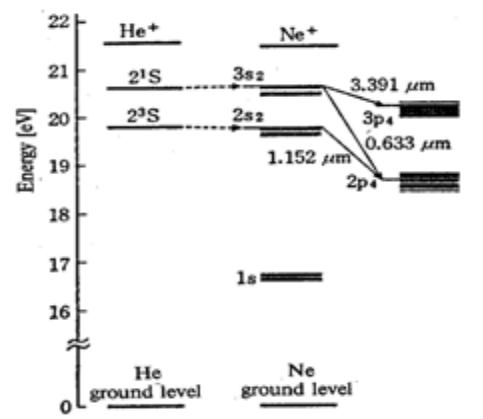
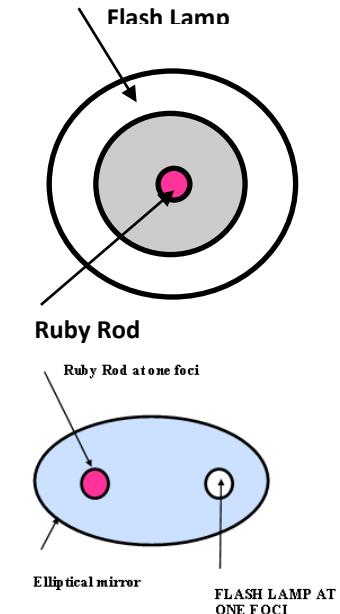
Silver coatings of the polished ends of the rod form the mirror arrangement. Antireflection coatings and external dielectric mirrors are also some of the design variations found. The ends cut and polished at Brewster's angle also eliminates the need for mirrors.

They are still used in a number of applications where short pulses of red light are required. Holographers around the world produce holographic portraits with ruby lasers, in sizes up to a meter square. Because of its high pulsed power and good coherence length, the red 694 nm laser light is preferred over other solid state lasers.

ii) He-Ne LASER system

Active medium: The He-Ne LASER is a atomic species laser where the active medium is the He-Ne gas mixture contained in a quartz tube of narrow diameter and maintained at a low pressure which forms the active medium.

He and Ne mixed in the ratio of 10:1 is the active medium where the absorption levels are in the He atoms and the lasing levels are in the Ne atomic transitions. The He atoms are excited with an electrical discharge and the two excited states of helium atom, the 2^3S and 2^1S which are Meta stable. These excited He atoms transfer their energy to Ne atoms by collisions and the excites the Neon atoms to the $2s_2$ and $3s_2$ levels as



the energy levels of these states are close to the He excited states. (This process is referred to resonant energy transfer.)

A large number of Ne atoms due to collision with He atoms get to the excited state create a population inversion with the ground state. The excited states of Ne are not meta stable and hence de-excites to the ground states through the intermediate states of 3p and 2p. The transition between the 3s to the 2p intermediate states gives the characteristic red laser of Ne with a wavelength of 632.8 nm. The transitions from the 3s to 3p and 2s to 2p lines give rise to radiations with wavelengths in the Infrared of 3.39 micrometers and 1.152 micrometers.

The transitions from the 3p and 2p levels to the 1s intermediate level (close to the ground state) is non radiative. However, the 1s state is a Meta stable state and has to be quickly depopulated. This is achieved by making the tube narrow, increasing the collision probability of the atoms with the sides of the walls of the tube.

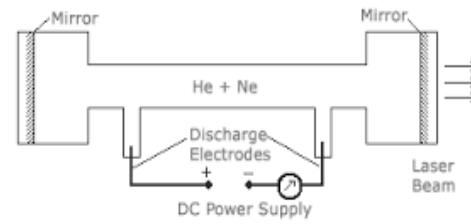
Once in the ground state the Ne atoms are pumped back, and the system gives a continuous output.

Energy pump: The energy pump is enabled by maintaining an electrical discharge across the length of the Quartz tube by either a high voltage DC source or a RF source.

Resonant cavity: The cavity is the Quartz tube of appropriate lengths with reflecting mirrors on both ends of the axis of the tube. The cavity consists of reflecting mirrors and the path length adjusted for the visible radiation at 632.8 nm, which also suppress the IR radiations.

Additional polarizers (Brewster's windows) may be placed in the path of the beam to ensure a polarized beam of LASER. However, the addition of the Brewster's window would eventually lead to a reduction in the output by a factor of 40% to 50%.

Light from the system can be partially polarized (the polarization state of the stimulating photon). The addition of Brewster's windows at the ends of the discharge tube before the reflecting mirrors would ensure that the emitted beam would be fully polarized in the plane of incidence. Additionally, some gases which have absorption in the Infra-red are added in small quantities to suppress the IR radiations.



7. Molecular laser

Molecular vibrations in the infra-red are very interesting in that the molecular excited states generally have lifetimes of the order of 1s to a few milli seconds. It would be therefore possible to excite the molecules to the higher energy state and make them relax to the ground state to get stimulated emission quite naturally if the population of the higher energy state can be managed to be greater than the lower energy states. The

CO₂ laser is an example of a molecular laser where the Laser emission is in the infrared. Interestingly these are high power continuous wave lasers with the efficiencies reaching up to 40%.

Carbon dioxide laser

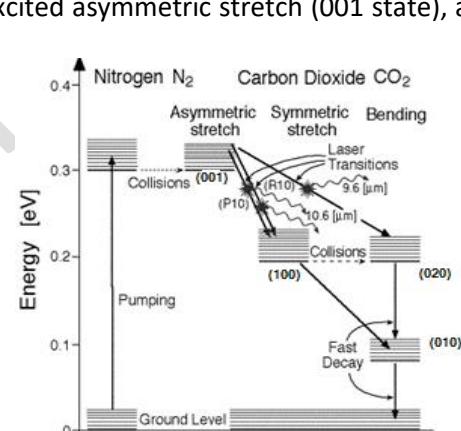
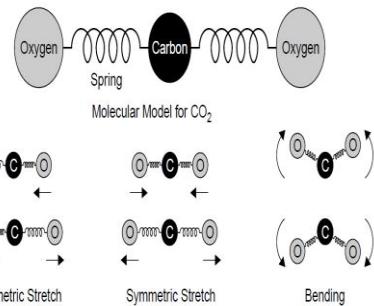
In the CO₂ molecule, the Oxygen atoms are bound to the Carbon atom by the bonding force which acts like a harmonic oscillator. Molecules can be excited to vibrate about their mean positions. Additionally, the molecules may rotate and spin because they are in a gaseous state. The rotational and vibrational states are quantized.

Transitions between vibrational energy states/levels results in photon emission in the infrared, while transitions between rotational states emit photons in the microwave region.

If the CO₂ molecules are excited and made to relax they emit in the infra red producing heat. This mode of emission could be mimicked to a **stimulated emission** if the population of molecules in the excited states is greater than the population in the ground state, thus creating a LASER with infra red wavelengths.

Carbon dioxide molecule has three possible vibrational states – an excited asymmetric stretch (001 state), a lower symmetric stretch (100 state) and bending states (020 and 010 states). The asymmetric stretch states have a higher life time (molecular excited states have higher life times of the order of 1ms to a fraction of a second) and higher energy than the symmetric and bending modes. An excited carbon dioxide molecule in the higher anti symmetric stretch state can relax into the symmetric stretch state giving a radiation at 10.6 μm (0.117eV) and into the bending mode with emission of IR at 9.6 μm (0.129 eV).

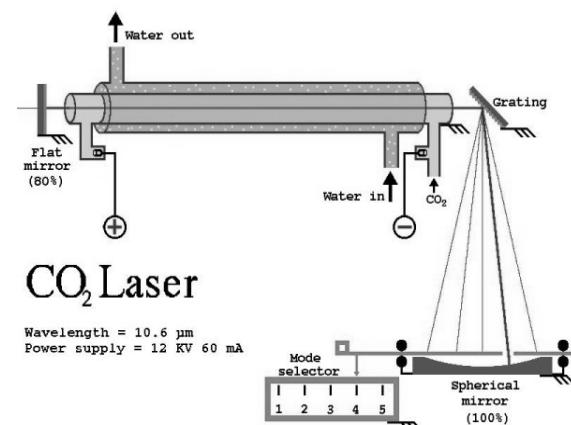
The carbon dioxide laser is a high power gas laser with immense industrial applications.



Construction and Principle of Operation

All lasers consist of three components: a gain (or laser) medium, an energy source (also known as a pump) and an optical resonating cavity. The three components of a Carbon dioxide laser system comprise of :

- **THE ACTIVE MEDIUM** - A mixture of carbon dioxide, nitrogen, and helium gases serve as the gain medium. Typical gas mixtures have an CO₂: N₂: He ratio of 1:2:8. The N₂ molecules are excited with energy close to the excited states of CO₂ which results in the excitation of CO₂ to the asymmetric stretch mode.
- **THE ENERGY PUMP** - Electrical discharge current — serving as the laser pump — which excites the gas medium to higher energy states through the electrical discharge of the He gas, which collides with



the N₂ gas to excite them into the higher energy states.

- **OPTICAL CAVITY** - A specialized optical resonator. Because CO₂ lasers operate solely within the infrared spectrum and can attain high power outputs, their optical components are typically made of specialized (and often expensive) materials such as Germanium, Zinc Selenide, Silver, Gold, and Diamond. Since the CO₂ lasers work in the Infra-red region all parts connected with the laser cavity must have suitable infra-red absorption coatings and an effective cooling system is required for the system as a whole. The hot gas mix must then be cooled to maintain a population inversion (a sufficient difference between excited and lower energy atoms to produce optical gain) with the excited carbon dioxide molecule.

8. Semiconductor lasers - Homo junction

Light emitting diodes work on the principle of recombination of electron and holes in the depletion region of a pn junction diode which result in the emission of a photon. The photon emission is possible only if the semiconductor is of the direct band gap type such as GaAs, InP etc (Si and Ge have an indirect band gap and are not suitable for such applications). The photon emission is a case of spontaneous emission which is highly non coherent and has no directionality.

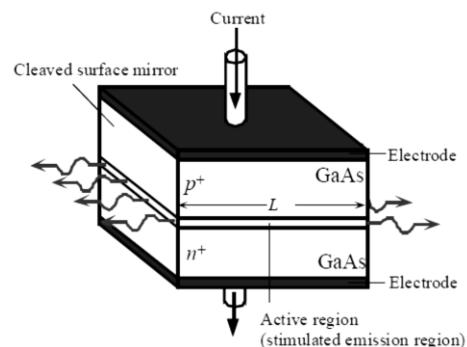
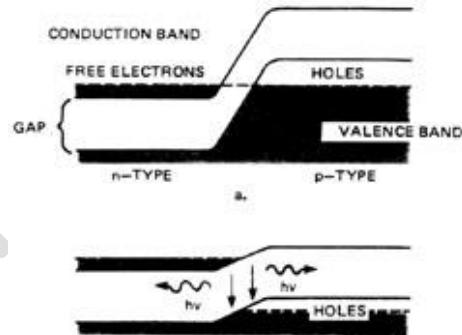
To convert a LED into a laser it is essential conditions of population inversion in the depletion region, stimulated emission and a resonating cavity are satisfied.

Semiconductor lasers use heavily doped direct band gap semiconductors like GaAs which is the active medium. The heavy doping results in an extremely thin depletion region and, moves the Fermi level of the n type into the conduction band and the Fermi level of the p type into the valence band.

Figure shows the depletion region of a heavily doped PN junction in the unbiased condition where the in electrons and holes are present in the “depletion region”, however they are not in a favorable state for recombination.

The energy pump in these devices is the large forward bias current when the PN junction is forward bias. The larger number of electrons in the n side and holes in the p side are in a favorable state for recombination in a narrow region referred to as the active region. This recombination result in the generation of photons in the action region. This emission is of the stimulated type. A suitably designed laser cavity with appropriate dimensions (proportional to $\lambda/2$) and cleaving the surface carefully for maximum reflection at opposite ends, results in the emission of laser from the active region.

The homo junction lasers are not very efficient and require a very high forward current density of the order of 10000 A cm⁻² at room temperature and hence are operated at very low temperatures or in the pulsed condition.



This is attributed to the low charge concentrations in the active region and not all the photons produced by the electron hole recombination are part of the laser output. This results in a device with extremely low efficiencies.

Heterojunction lasers.

The problems in a homojunction laser can be overcome with the design of a hetero junction laser. Hetero-structures makes it possible to solve the problems in a homo junction laser since in hetero junctions the fundamental parameter of semiconductor crystals and devices like band gaps, effective masses of the charge carriers and their mobilities, refractive indices, etc could be effectively tailored.

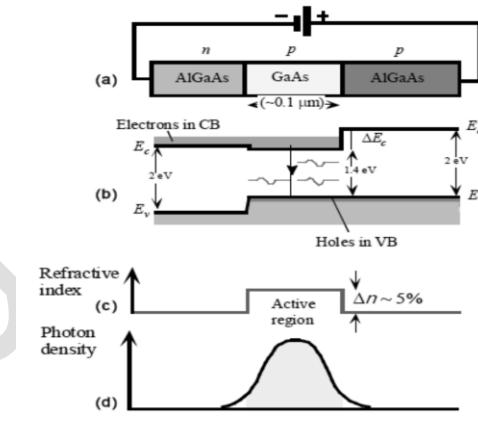
The double heterojunction device ensures higher efficiency by

1. **Carrier confinement:** A double hetero structure diode has two junctions which are between two different band gap semiconductors (GaAs and AlGaAs). The GaAs active layer has a lower band gap than the AlGaAs layers on either side. This results in a population of electrons in the conduction band of the GaAs layer from the n type AlGaAs layer and a population of holes in the GaAs layer from the p type AlGaAs. The population of the electrons and hole in the GaAs layer can recombine in the forward bias condition resulting in stimulated emission of photons. This requires less current to establish the required concentration of electrons for population inversion.
2. **Photon confinement:** The other aspect of confining all the emitted photons to a narrow region can be achieved by constructing a dielectric waveguide around the optical gain region and increase the probability of stimulated emission. The n and p type AlGaAs on either side have lower refractive index than the GaAs region which result in an increase in the number of photons traveling along the cavity axis by total internal reflection.

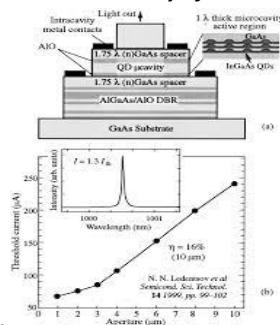
Single hetero junction lasers require a current density of about 1500 A cm^{-2} whereas double hetero junction lasers operate at lower currents of 600 A cm^{-2} .

Practical hetero junction lasers however consist of many layers to improve the efficiency of the carrier and photon confinements and operate at much lower operating currents at room temperatures.

Applications of Lasers



Nobel Lecture: The double heterostructure concept and its applications in physics, electronics, and technology Zhores I. Alferov A. F. Ioffe Physico-Technical Institute, Russian Academy of Sciences, St. Petersburg 194021,*



Russian Federation (Published 22 October 2001)

1. Frequency comb.

A resonant cavity oscillator can support multiple wavelengths (frequencies). Given a cavity of length L (which is much greater than the wavelength of visible radiations) the resonant frequencies, which are referred to as the cavity modes, can be represented as shown.

The Laser line width indicates that there are multiple frequencies in output as well. The gain curve of the laser gives the laser output in all the frequencies with a g_{th} greater than 1.

The output then is a frequency comb, which has multiple applications as demonstrated by TW Hansch and J Hall, the Nobel Prize winners in 2005.

Optical Frequency combs (OFC) with mode locked femtosecond pulsed lasers are perfect time or frequency references. OFC also are found to be the most accurate time references and are the most stable clock references for integrated opto electronic systems.

Laser spectroscopy using frequency combs have become a standard analytical tool in analysis of materials. These systems are used in a variety of astronomical measurements and is considered to be a potential standard clock reference.

2. Principles of Holography

Information storage with light has been known from the days of photography where in the wavelength and intensity of light are recorded on a photographic plate or digitally on pixels. The photographic information storage has a limitation of being a 2D creation of the information of the object being photographed.

Fundamentally light is fully characterized by wavelength λ , phase ϕ and intensity I at any point. Hence, capturing all the three parameters would result in storing all the information about an object on a medium or a digital medium. This is the fundamental idea of Holography.

Capturing phase information is best done with an interference. Interference requires two wavefronts - one is a reference beam (whose phase information is fixed) and the second is the wavefront reflected by the object, which carries the phase information from different points on an object. Thus, the object beam contains light from every point on the object carries all required information of wavelength, phase and intensity. When the two beams interfere, the resultant interference pattern has all information about all the points on the object.

The recording and reconstruction process can be explained in the language of Mathematics as follows. The complex amplitude of the reference wave can be represented by

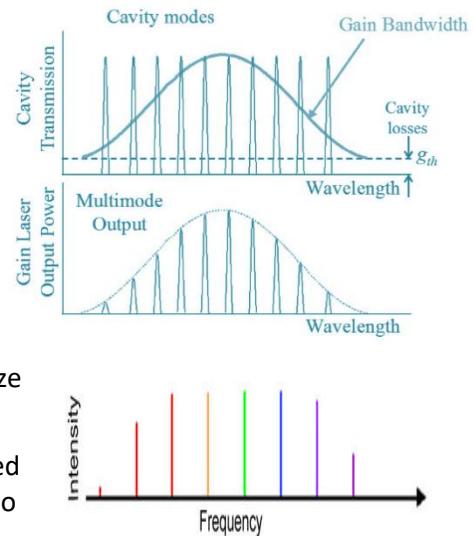
$$R(x, y) = R_0(x, y) * e^{i\psi(x, y)}$$

where with real amplitude $R_0(x, y)$ and phase $\psi(x, y)$.

The wave which originates from the reflection from a surface similarly is given by

$$O(x, y) = O_0(x, y) * e^{i\phi(x, y)}$$

with real amplitude $O_0(x, y)$ and phase $\phi(x, y)$.



The interference of the two waves on the holographic plate resulting in a complex interference pattern and the intensity distribution can be evaluated as

$$I(x, y) = [O(x, y) + R_r(x, y)]^2 = [O_o(x, y)]^2 + [O_r(x, y)]^2 + O(x, y) \cdot R_r(x, y)^* + O(x, y)^* \cdot R_r(x, y)$$

Where the last term is $O(x, y) \cdot R_r(x, y)^* + O(x, y)^* \cdot R_r(x, y) = 2O_o(x, y) \cdot R_r(x, y) \cdot \cos(\psi(x, y) - \phi(x, y))$

includes both the amplitude and phase of the object wave fronts, i.e., $O_o(x, y)$ and $\phi(x, y)$.

The reconstruction of the image will require the recorded holographic film and the reference source. The beam that is transmitted is sensitive to the intensity of the source.

The transmission function of optical recording devices including photographic film is sensitive to intensity. We will assume that the sensitivity is linear in intensity. The reference $R_r(x, y)$ is assumed to be constant, and equal to A_r , which is a plane wave incident perpendicular to the hologram. The transmission function (also known as the hologram function) of such a device contains the stored information and can be written as

$$t(x, y) = t_0 + \beta\tau\{[O_o(x, y)]^2 + [A_r(x, y)]^2 + OR^*_r + O^*R_r\}$$

where β and t_0 are constants. The constant β is the slope of the amplitude transmittance versus exposure characteristic of the light sensitive material. For photographic emulsions β is negative. τ is the exposure time and t_0 is the amplitude transmission of the unexposed plate. $t(x, y)$ is known as the hologram function.

If the hologram created as above, is illuminated by another reference wave $R_p(x, y)$ as shown in the figure

The wave emanating from the hologram can be written as

$$\begin{aligned} U(x, y) &= [t(x, y) * R_p(x, y)] \\ &= [t_0 + \beta\tau\{O_o(x, y)^2 + [A_r(x, y)]^2 + OR^*_r + O^*R_r\}] * R_p(x, y) \end{aligned}$$

$$R_t = U_1 + U_2 + U_3 + U_4$$

$$\text{where, } U_1 = (t_0 + \beta\tau[O_o(x, y)^2]) \cdot R_p(x, y)$$

$$U_2 = \beta\tau[[A_r(x, y)]^2] \cdot R_p(x, y)$$

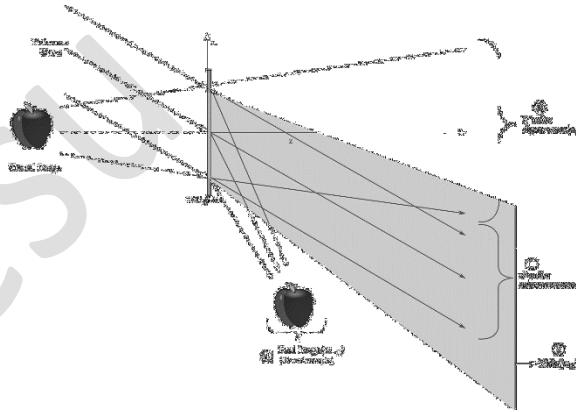
$$U_3 = \beta\tau[OR^*_r] \cdot R_p(x, y)$$

$$U_4 = \beta\tau[O^*R_r] \cdot R_p(x, y)$$

Suppose that R_r and R_p are the same and are constant, as in the case of a plane wave perpendicular to the direction of propagation. Then, U_3 is proportional to O , and U_4 is proportional to O^* .

The first term U_1 refers to the intensity reduction of the reconstruction wave by the factor $t_0 + \beta\tau|A_r(x, y)|^2$ during reconstruction.

The second term is small assuming that we choose $A_o(x, y) < A_r(x, y)$ during recording. This term is distinguished from the first term by its spatial variation $|A_o(x, y)|^2$. The $|A_o(x, y)|^2$ term contains low spatial frequencies which have small diffraction angles and create a so-called halo around the reconstruction wave.



The size of the halo is given by the angular dimension of the object. These first two terms form the zeroth diffraction order in equation.

The third term U_3 denotes the object wave $U(x, y)$ multiplied with the constant factor $\beta\tau R_r^2$. An observer who registers this wave in his eye therefore sees the virtual image of the object (not present). This is the most important term and represents the first diffraction order. The wave travels divergent from the hologram thus creating a virtual image at the position of the original object. It is a virtual image because the wave is not converging to form a real image. This image cannot be captured on a screen. The intensity (square of amplitude) of the image does not depend on the sign of β . Therefore, it is unimportant whether the hologram is processed “positive” or “negative”.

The fourth term U_4 is essentially the complex conjugate of the object wave U^* except for a multiplicative term. This represents the -1 st diffraction order. Since it is complex conjugated wave, the phase changes its sign with respect to $U(x, y)$. As a consequence, the wave $U^*(x, y)$ travels convergent and forms a real image. The conjugated real image U_4 is usually located at the opposite side of the hologram with respect to U_3 . U_3 and U_4 are called twin images or also represented as virtual and real images, respectively. All these reconstructed wavefronts are represented in Figure.

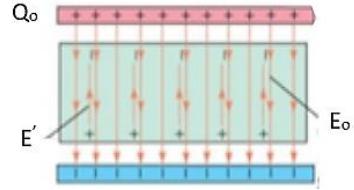
Polarisation in dielectrics

Dielectrics are a class of material which respond to external electric fields, and find extensive use in the electronics and electrical industry.

From our knowledge of capacitors, the dielectric constant of a material is

- $\epsilon_r = \frac{\text{capacitance of a parallel plate capacitor with the material}}{\text{capacitance of the capacitor without the material}} = \frac{C}{C_0}$
- $\epsilon_r = \frac{\text{field across a parallel plate capacitor without the material}}{\text{field across the capacitor with the material}} = \frac{E_0}{E}$

where E_0 is the applied electric field and E is the net electric field across the dielectric.



An external electric field can cause the center of positive and negative charges in the crystal to shift from their mean position. This separation of charges results in creation of dipoles in the system. The net dipole moment created per unit volume is called the polarization \mathbf{P} . The polarization in the material is dependent on the net electric field experienced by the dielectric.

If E_0 is the applied electric field applied to a parallel plate capacitor the surface charge density on the plates of the capacitor $\sigma = \epsilon_0 E_0$

If a dielectric material is placed between the plates of the capacitor, the surface density of charge due to polarization $\sigma_p = \epsilon_0 E'$ where E' is called the depolarization field.

The net electric field E between the plates is reduced by a factor which is the dielectric constant ϵ_r of the material ie., $E = \frac{E_0}{\epsilon_r}$ or $E_0 = \epsilon_r E$

$$\text{The net electric field } E = E_0 - E' = \epsilon_r E - \frac{\sigma_p}{\epsilon_0}$$

$$\text{Simplifying } \sigma_p = \epsilon_0 \epsilon_r E - \epsilon_0 E = \epsilon_0 (\epsilon_r - 1) E$$

Hence the polarization in the material due to a net electric field is given by

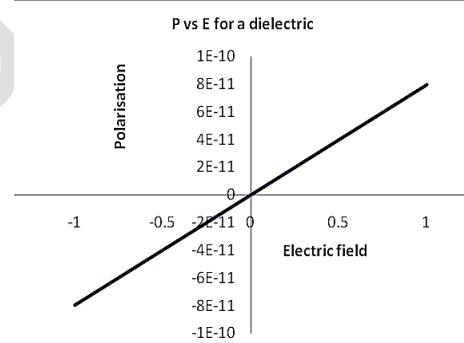
$$P = \sigma_p = \epsilon_0 (\epsilon_r - 1) E$$

The polarisation of the material can also be written as

$P = \epsilon_0 \chi E$ where χ is the dielectric susceptibility of the material and is a measure of the ability to create dipoles in the material. The polarization is linearly proportional to the applied electric field for pure dielectrics.

This gives us the relation between susceptibility and dielectric constant

$$\chi = (\epsilon_r - 1) \text{ or } \epsilon_r = 1 + \chi.$$



If N is the number of atoms per unit volume and α_e is the polarisability of the individual dipoles then the polarization can be written as $\mathbf{P} = \epsilon_0 \chi \mathbf{E} = N \alpha_e \mathbf{E}$.

$$\text{Or } \chi = \frac{N \alpha_e}{\epsilon_0}$$

Polarization in dielectrics is thus proportional to the net electric field.

Electric fields in a dielectric material

To understand the polarization mechanism in totality, it is essential to understand the components of the electric fields which exist in a real dielectric, apart from the applied electric field \mathbf{E} and the depolarization field \mathbf{E}' . In general the local electric field inside a dielectric material comprises of four components of electric field.

$$\mathbf{E}_{loc} = \mathbf{E}_o + \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3$$

\mathbf{E}_o is the external applied electric field,

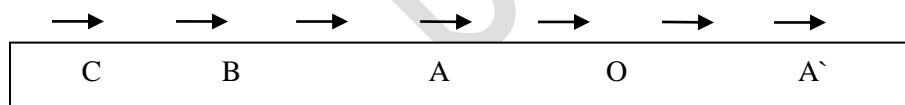
\mathbf{E}_1 is the depolarization field,

\mathbf{E}_2 is the Lorentz field on the surface of the spherical cavity (scooped out of the dielectric and

\mathbf{E}_3 is the field due to other dipoles lying within the sphere.

{ In the case of cubic structures the net Lorentz field can be shown to be zero. The electric field due to the dipoles inside the spherical cavity can be estimated as follows.

Consider a linear array of dipoles at A, B, C, A', B' and C' with dipole moment μ at regular intervals of d .



The electric field at A and A' are equal in strength and direction and equal to

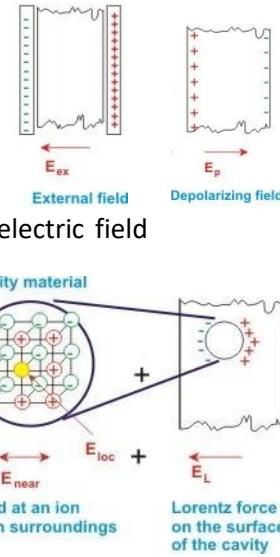
$$E_A = E_{A'} = \frac{2\mu}{4\pi\epsilon_0 d^3} \quad \text{and the net electric field } E_{AA'} = \frac{\mu}{\pi\epsilon_0 d^3}.$$

The electric field at O due to the pair of dipoles equidistant from O can be written as

$E = E_{AA'} + E_{BB'} + E_{CC'} \dots \dots = \frac{\mu}{\pi\epsilon_0} \left(\frac{1}{d^3} + \frac{1}{(2d)^3} + \frac{1}{(3d)^3} \dots \right) = \frac{\mu}{\pi\epsilon_0 d^3} \sum \frac{1}{n^3} = \frac{1.2 \mu}{\pi\epsilon_0 d^3} = \frac{\gamma P}{\epsilon_0}$ where γ is a constant that depends on the geometry of the material. From the series we can evaluate the contribution of the nearest neighbors and find that only first few pair of neighbors contributes significantly to the electric field at any site. ***

In the case of a dielectric with a cubic structure the effective field can be written $E_{in} = \frac{P}{3\epsilon_0}$

Thus the net electric field across the dielectrics will be greater by this factor and given by



$$E_{loc} = E + E_{in} = E + \frac{P}{3\epsilon_0}$$

From the definition of polarization we have $P = N\alpha_e E_{loc} = N\alpha_e \left(E + \frac{P}{3\epsilon_0} \right) = \epsilon_0(\epsilon_r - 1)E$

Solving this we get $\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha_e}{3\epsilon_0}$. This is the Clausius Mosotti relation which relates the macroscopic dielectric constant to the microscopic polarisability of the material.

It is obvious that the effective microscopic polarisability is greater when the internal fields are considered and should evolve in a more realistic dielectric constant of the material.

[derivation of the Clausius Mosotti relation:

$$P = N\alpha_e \left(E + \frac{P}{3\epsilon_0} \right)$$

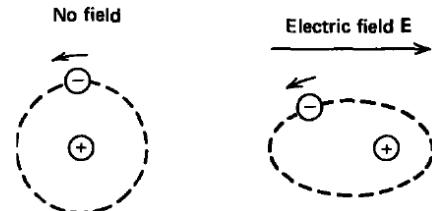
Rearranging terms we get $P \left(1 - \frac{N\alpha_e}{3\epsilon_0} \right) = N\alpha_e E$

$$P = \frac{N\alpha_e E}{\left(1 - \frac{N\alpha_e}{3\epsilon_0} \right)} = \epsilon_0(\epsilon_r - 1)E \text{ simplifying} \quad \frac{N\alpha_e/\epsilon_0}{\left(1 - \frac{N\alpha_e}{3\epsilon_0} \right)} = (\epsilon_r - 1) \text{ Taking the inverse}$$

$$\frac{\left(1 - \frac{N\alpha_e}{3\epsilon_0} \right)}{N\alpha_e/\epsilon_0} = \frac{1}{\frac{N\alpha_e}{\epsilon_0}} - \frac{1}{3} = \frac{1}{(\epsilon_r - 1)} \text{ leading to} \quad \frac{1}{\frac{N\alpha_e}{\epsilon_0}} = \frac{1}{3} + \frac{1}{(\epsilon_r - 1)} = \frac{(\epsilon_r + 2)}{3(\epsilon_r - 1)} \quad \text{Hence } \frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha_e}{3\epsilon_0} \quad]$$

Polarisation mechanisms

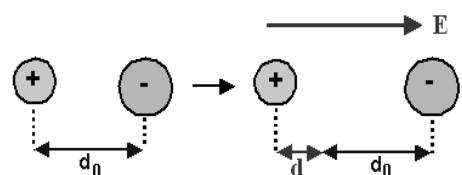
Electronic polarization : When an atom is subjected to an electric field E , the nucleus and the electron cloud will try to move in opposite directions. The nucleus moves in the direction of the applied field and the electron cloud in the direction opposite to that of the applied field. The centre of the negatively charged electron cloud no longer coincides with the positive nucleus and hence results in an induced dipole. The polarization produced due to this induced dipole is called "Electronic polarization".



The electronic polarization is $\alpha_e = 4\pi\epsilon_0 R^3$ where R is the radius of the atom. Electronic polarization is temperature independent since the size of the atom does not depend on temperature.

Ionic Polarisation

Dielectric materials with ionic bonding there exists regular arrangement of positive and negative ions. Ions between any two bonds is then a dipole. Because of the symmetric arrangement of the dipoles in the material, the net dipole moment in the material



may be zero.

In the presence of an external electric field the elongation of the bond will result in an increased the dipole moment along the direction of the field. However, when the ions are displaced from their equilibrium position, a restoring force in the form of the columbic force of attraction tends to constrain the bond elongation.

The displacement of the ions results in a net induced dipole moment per unit volume, or the electric field induces polarization in the material. The ionic polarisability α_i is independent of temperature and depends on the Young's modulus of the material.

Orientational polarization:

Polar molecules have permanent dipole moments since the net charge centers are separated by a small distance. These dipoles can move and rotate freely (polar liquids - such as the water molecule). However due to the random motion of the molecules at normal temperatures, there is no net polarization in the material.

In the presence of an external field the dipoles tend to align in the direction of the field. This results in a net dipole moment for the material and is called orientational polarization.

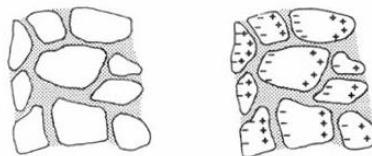
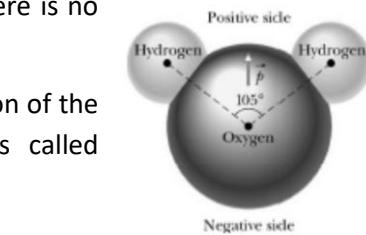
$$\text{The polarisation is given by } P = \frac{N\mu^2 E}{3kT}$$

Where N is the number of dipoles per unit volume, μ is the dipole moment of the individual molecules and T the temperature.

Increase in temperature increases the random motion of the dipoles and hence the orientational polarization is highly temperature dependent.

Space charge limited polarisation

In general, when two materials of different conducting phases meet at an interface, a potential barrier layer is formed resulting in an accumulation of charges of the opposite polarities on either side of the interface. This can be modeled to be a polarization of charges in the bulk of the material. The width of the barrier is generally very small and can result in reasonably high capacitance. These are, however, very sensitive to voltages and frequencies and may not be effective at normal working conditions.



Frequency dependence of dielectric constant.

The response of the dielectric constant to an applied ac electric field can be understood by considering the ability of the individual dipoles in the material to follow the external AC field. The dielectric constant of an ideal dielectric which has all mechanism contributing to the polarization can be expressed as

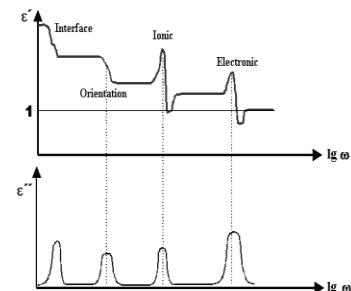
$$\epsilon_r = \epsilon_{sp} + \epsilon_{or} + \epsilon_{io} + \epsilon_e$$

The AC electric field induces the dipoles due to the individual polarization mechanisms have to align in the direction of the field. At low frequencies, the dipoles due to all mechanisms tend to align with the electric field easily. As the frequency of the applied electric field increases, the dipoles alignment tends to lag behind the field (depending on the inertia of the dipole). This results in a effective decrease in the polarization of the material as the frequency increases.

The AC response of the dielectric constant can be represented as $\epsilon = \epsilon' + j\epsilon''$ where ϵ' is the real part of the dielectric constant and ϵ'' is the imaginary part of the dielectric constant which describes the losses in the dielectric.

At low frequencies of about 100Hz, the contribution to the dielectric constant from the space charge polarization disappears and hence the dielectric constant decreases. Beyond this and up to about 10^9 Hz remains constant implying all the other three polarization mechanisms are active. At round 10^9 Hz (the microwave region) the orientation polarization stops responding to the electric field and the contribution from this mechanism to the dielectric constant become zero and the effective dielectric constant of the material decreases to a lower value.

At this point only two polarization mechanisms are contributing to the dielectric constant – the ionic and electronic polarization mechanisms.



Around 10^{13} Hz (far infra red) the frequency of the AC field is in close to the vibrational frequencies of the bond and resonance phenomena sets in. The dielectric constant shows an initial increase and exhibits a resonance type of behavior around the resonant frequency.

Beyond this range the dielectric constant is only due to the electronic polarization. As the frequency of the AC approach 10^{15} Hz - the visible range which corresponds to the visible absorption frequency and a resonant condition sets in with a behavior similar to the ionic range.

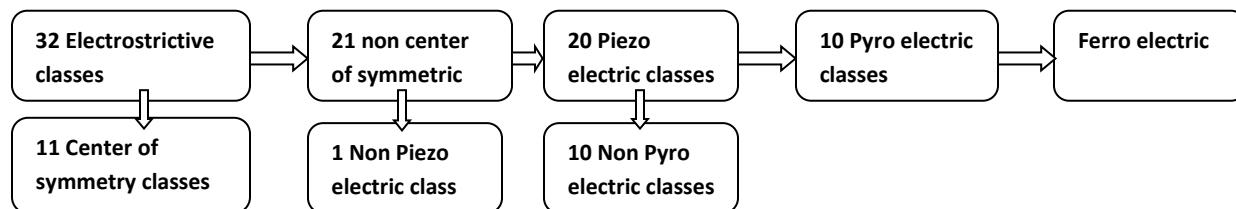
The material may be treated as transparent to frequencies $> 10^{16}$ Hz and the dielectric constant should be that of vacuum or free space.

The dielectric loss ϵ'' represents the energy loss in the dipoles following the electric fields. As the frequency increases generally the inertia of the dipoles becomes significant and at particular frequency ranges the dipoles become inert to the increasing ac frequency. } } # # #

Classification of non linear dielectric materials

Nonlinear dielectrics

Dielectrics can be exposed to different types of stresses such pressure variations (S), temperature (T) variations and electric field variations (E). Invariably all these stresses tend to change the state of polarization of the materials in a nonlinear way and are in some way interrelated and are termed as the electrostrictive class of materials. The classification of these materials reveals that these observed polarization response to a large extend is related to the solid state phase transitions of the second order, where the polarization changes rapidly at phase transition temperatures.



Non Centro-symmetric system

Out of a total of 32 crystal point groups, 21 are non-centrosymmetric *i.e.* crystals not having a center of symmetry.

20 classes of material show piezo electric behaviour. Out of these 20 classes 10 are pyro electric and all the pyro electric classes are also ferro electric classes.

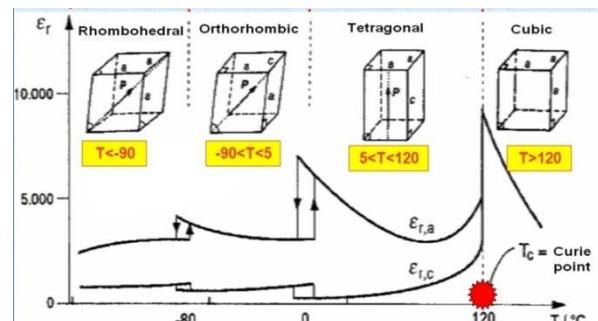
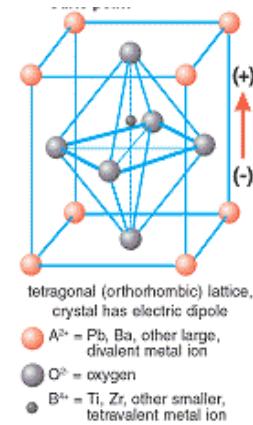
Center of symmetry for a crystal can be evaluated by finding the center of positive and negative charges associated with the atoms at the lattice points.

BaTiO_3 is a good example of a non-centro symmetric crystal. The unit cell is cubic above the curie temperature of about 120 C, when the material is centro symmetric and behaves as a paraelectric. Between 5C and 120C the material is in the tetragonal phase exhibiting ferroelectric behavior. Below 5C and -90C the material has a orthorhombic phase and below -90C the material is in the rhombohedral phase. In all these phases also, the material exhibits ferroelectric behavior in varying proportions.

These dipoles arise due to the fact that in the tetragonal unit cell of BaTiO_3 , the Ti^{4+} cation is surrounded by six O^{2-} anions in a slightly deformed octahedral configuration and can occupy one of two asymmetrical sites. In either position, the Ti^{4+} cation is not coincident with the negative charge center of the oxygen anions by a small fraction of nm, creating an electric dipole for the unit cell. The polarization which is the dipole moment per unit volume turns out to be quite considerable.

The energy barrier between the two possible Ti atom positions is sufficiently low to permit motion of the atom between sites by the coercion of an electric field, and the material can thus be directionally polarized with ease.

Non centro symmetric crystals can respond to an external stimulus producing polarization or surface charges which show up as a potential across the element. However, these properties are highly anisotropic and could be described as a tensor (which describes the direction dependent properties of materials). Piezoelectric behavior is the response of these crystals to external mechanical pressures; Pyroelectric behavior is the response to thermal changes and Ferroelectric behavior is a response of the material to external electric fields.

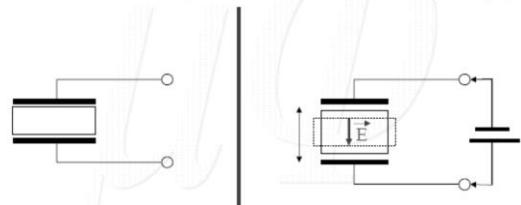


Piezoelectric materials

Piezoelectric Effect is appearance of an electrical potential across some faces of some crystals when a pressure is applied to the crystal. Pierre Curie and his brother Jacques discovered the effect in 1880. Subsequently the inverse piezoelectric effect was observed as a distortion in the crystal when an electrical field is applied.

The term piezoelectricity refers to the fact that, when a crystal is strained, an electric field is produced within the substance. As a result of this field, a potential difference develops across the sample, and by measuring this potential one may determine the field. The inverse effect - that an applied field produces strain - has also been observed. It is explained by the displacement of ions, causing the electric polarization of the crystal's structural units. When an electrical field is applied, the ions are displaced by electrostatic forces, resulting in the mechanical deformation of the whole crystal.

– Mechanical strain (stress) ⇔ Electric field (charge)



The piezoelectric effect is very small. A field of 10^3 V/cm in quartz (SiO_2) produces a strain of only 10^{-7} . That is, a rod 1 cm long changes its length by 10\AA . Conversely, even small strains can produce enormous electric fields.

The piezoelectric effect is often used to convert electrical energy into mechanical energy, and vice versa; i.e., these materials are used to make transducers. For instance, an electric signal applied to the end of a quartz rod generates a mechanical strain, which consequently leads to the propagation of a mechanical wave - a sound wave - down the rod.

Piezo electric materials like PZT ($\text{Pb}(\text{Zr},\text{Ti})\text{O}_3$), and Quartz find extensive application in electronics as sensors and actuators.

- Piezoelectric microbalances are used as very sensitive chemical and biological sensors.
- Atomic force and scanning tunneling microscopes for precision manipulation of sample at nano meter levels
- Transient pressure measurement to study explosives, internal combustion engines (knock sensors), and any other vibrations, accelerations, or impacts.
- Energy Harvesting from impact on the ground
- Electric igniters
- Transducers are used in electronic drum pads to detect the impact of the drummer's sticks.

Pyroelectric materials

Pyroelectricity is the ability of certain materials to generate an electrical potential when they are heated or cooled. Non centrosymmetric crystals with a net spontaneous polarization can be sensitive to external temperatures.

The change in temperature modifies the positions of the atoms slightly within the crystal structure, such that the polarization of the material changes. This polarization change gives rise to a voltage across the

crystal. If the temperature stays constant at its new value, the pyroelectric voltage gradually disappears due to leakage current (the leakage can be due to electrons moving through the crystal, ions moving through the air, current leaking through a voltmeter attached across the crystal, etc.).

The pyroelectric coefficient may be described as the change in the spontaneous polarization vector P_s with temperature T

$$p_i = \frac{\partial P_s}{\partial T} \quad \text{where } p_i (\text{Cm}^{-2}\text{K}^{-1}) \text{ is the vector for the pyroelectric coefficient.}$$

It can be used to create a flow of current in an external circuit connected to a device using pyroelectric materials. The effect has been known for many years, but it is only since about 1960 that its technological applications have been seriously considered.

These have been almost entirely in the field of the detection of electromagnetic radiation, especially in the two "atmospheric window" infrared (IR) bands of 3-5 pm and 8-14 pm. The ambient temperature operation of pyroelectric detectors, leading to low power consumption, low cost, compactness.

Pyroelectric detectors have found a huge range of applications in products ranging from fire alarms to intruder detectors, in instrumentation such as gas analysis and laser beam characterization and in military/paramilitary applications such as thermal imaging.

Ferroelectric materials

Charge displacements in dielectric and para electric ceramics, such as Al_2O_3 , mica, TiO_2 , are totally reversible, as ions / electron cloud return to their original position once an applied field is removed, and the polarization effect is linear.

Ferroelectrics are a class of non-centro symmetric crystals which are also a subclass of the pyro electric / piezoelectric materials. These materials show a spontaneous polarization even in the absence of an electric field.

The spontaneous polarization observed in ferroelectric materials depends on the crystalline phase of the material (generally referred to as perovskites). BaTiO_3 is a classic example of a perovskite material.

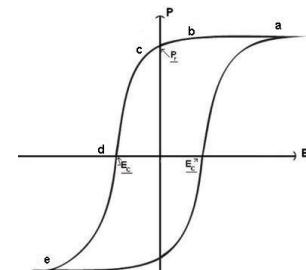
The dielectric susceptibility of the material is highly temperature dependent

$$\text{and is given by } \chi = \frac{C}{T - T_c}$$

for all $T > T_c$ where C is a constant dependent on the material and T_c is the curie temperature. At temperature below T_c , the material shows spontaneous polarization and is classified as a ferroelectric material. At temperatures greater than T_c , the material is para electric with the susceptibility inversely proportional to temperature.

Ferroelectric dielectrics differ from dielectric materials in their response to external applied electric field.

Ferroelectrics display a nonlinear response of polarization to changing electric fields $p_e = \frac{\partial P_s}{\partial E}$ and display a hysteresis in the P versus E variations. The hysteresis loop is caused by the existence of permanent



electric dipoles in classes of materials, which develop spontaneously below the Curie temperature. The polarisation state of the ferroelectric material has a memory effect and hence is used extensively in DRAMs and SRAMs.

Ferro electric materials find numerous applications as sensors and actuators taking advantage of the temperature and field dependencies of the susceptibility (dielectric constant).

NON-LINEAR OPTICAL MATERIALS AND SECOND HARMONIC GENERATION

Polarization in the dielectric materials is linearly proportional to the applied electric field when the field is relatively small.

$$P = \epsilon_0 \chi E$$

This applies as well to the interaction of electromagnetic radiation with matter, the electric field of the radiation can cause polarisation in the medium.

However, with the advent of high energy laser sources the underlying polarisation is now understood to be a nonlinear function of the applied electric field of the electromagnetic radiation.

This in simple mathematical terms can written as

$$P = \epsilon_0 (\chi^1 E + \chi^2 E^2 + \chi^3 E^3 \dots)$$

$\epsilon_0 \chi^1 E$ refers to the conventional linear term, $\epsilon_0 \chi^2 E^2$ refers to the second order term, $\epsilon_0 \chi^3 E^3$ refers to the third order term etc.

The susceptibility terms relates to the dielectric constant and hence the refractive index of the material.

Thus χ^2 and χ^3 are referred to as the second and third order nonlinear susceptibilities respectively.

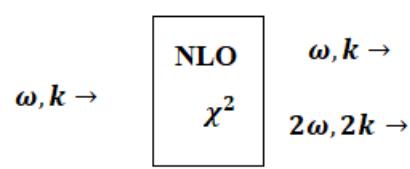
χ^2 and χ^3 are extremely small compared χ ($\chi \gg \chi^2 \gg \chi^3$) and do not contribute significantly when the electric fields of the electromagnetic waves (optical signals) are small.

However, the interaction of matter with high intensity electromagnetic waves (as in the case of high power lasers) can result in a high electric fields. This results in increased nonlinear the polarization of the medium and the contribution of the higher order terms becomes significant.

Consider a high frequency electric field given by $E = E_0 e^{-i\omega t} + cc$ incident on a nonlinear optical material. The contribution of the second order polarisation can be expressed as

$$P^{(2)} = \epsilon_0 \chi^2 E^2 = \epsilon_0 \chi^2 (E_0 e^{-i\omega t} + cc)^2 = \epsilon_0 \chi^2 (E_0^2 e^{-i2\omega t} + 2E_0 e^{-i\omega t} \times cc + cc^2)$$

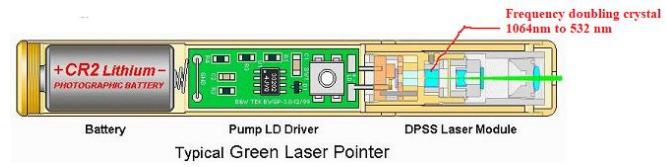
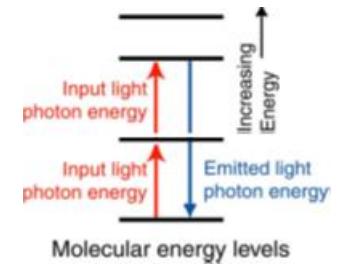
Solving the electromagnetic wave equation for the second order polarisation, we realize that the strong interaction of the electric field with the crystal can result in the generation of an additional wave with twice the frequency of the input wave.



The second term relates to a second harmonic generation – or photons with twice the frequency of the input frequency is present in the output.

Second-order non-linearity is exhibited by crystals which do not possess a center of inversion symmetry (the non-centro symmetric ferroelectric / perovskite family of crystals) like lithium niobate (LiNbO_3), lithium tantalite (LiTaO_3), potassium titanyl phosphate (KTiPO_4), and potassium dihydrogen phosphate (KH_2PO_4). The parametric oscillator uses the second-order non-linearity for optical amplification and oscillation.

A common application of the SHG in NLO is the making of the green pointer laser. In this system the basic lasing system is in the IR (1064nm) and using a frequency doubling crystal a green wavelength of 532nm is the output of the crystal.



Solved Numericals:

1. An emission system has two levels which gives rise to an emission wavelength of 546.1 nm. If the population of the lower state is 4×10^{22} at 600 K, estimate the population of the higher energy state.

Let N_1 and N_2 be the populations of energy states E_1 and E_2 .

According to the Maxwell Boltzmann distribution $\frac{N_1}{N_2} = \exp^{\frac{(E_2 - E_1)}{kT}}$.

$$\text{Hence } N_2 = N_1 \exp^{-\frac{(E_2 - E_1)}{kT}} = N_1 \exp^{-\frac{hc}{\lambda kT}} = 8.088 \times 10^{-20}$$

2. The ratio of population between the high energy states to the lower energy state is 5×10^{-19} at 400K. Find the emission wavelength between two states and the ratio A/B.

$$\text{Given } N_2/N_1 = 5 \times 10^{-19} \quad N_1/N_2 = 2 \times 10^{18} = e^{hv/kT}$$

$$\lambda = \frac{hc}{kT \ln \left(\frac{N_1}{N_2} \right)} = 854.56 \text{ nm}$$

$$\frac{A}{B} = \frac{8\pi h}{\lambda^3} = 2.667 \times 10^{-14}$$

3. The ratio of population of the upper excited state to the lower energy state in a system at 300K is found to be 1.2×10^{-19} . Find the wavelength of the radiation emitted and the energy density of radiation.

$$\frac{N_1}{N_2} = \exp^{\frac{(E_2 - E_1)}{kT}} = \exp^{\frac{hc}{\lambda kT}}$$

$$\text{Wavelength } \lambda = 1.06 \times 10^{-6} \text{ m}$$

$$\rho(v) = \frac{8\pi hv^3}{c^3} \frac{1}{\left(\exp^{\frac{hv}{kT}} - 1 \right)} = 1.987 \times 10^{-33} \text{ Js/m}^3.$$

4. A laser emission from a certain laser has an output power of 10 milli watts. If the wavelength of the emission is 632.8nm, find the rate of emission of the stimulated photons.

$$\text{Power of Laser } P = n \times hv = n \times h \times \frac{c}{\lambda} \text{ where } n \text{ is the rate of stimulated emission.}$$

$$n = \frac{P \times \lambda}{h \times c} = 3.18 \times 10^{16} \text{ per second}$$

Problem set

- 1 The ratio of the population of two energy levels is 1.5×10^{30} . The upper level corresponds to a meta stable state. Find the wavelength of light emitted at 330K (**Ans 628 nm**)
- 2 An hypothetical atom has energy levels uniformly separated by 1.2 ev. Find the ratio of the no of atoms in the 7th excited state to that in the 5th excited state. (**Ans: 5.22×10^{-41}**)
- 3 A pulsed laser has a power of 1mW and lasts for 10 ns. If the no. of photons emitted per second is 3.491×10^7 , calculate the wavelength of the photons. (**Ans: 693 nm**)
- 4 If R_1 is the rate of stimulated emission and R_2 is the rate of spontaneous emission between two energy levels, show that $\lambda = hc / [kT \ln\{(R_2/R_1)+1\}]$.
- 5 Find the ratio of the rate of stimulated emission to the rate of spontaneous emission for a system emitting a wavelength of 632.8 nm at 300K. (**Ans: 1.11×10^{-33}**)
- 6 If $B_{10} = 2.7 \times 10^{19} \text{ m}^3/\text{W}\cdot\text{s}^3$ for a particular atom, find the life time of the 1 to 0 transition at (a) 550nm and (b) 55nm (answer: (a)370ns (b) 0.37ns)
- 7 The energy levels in a two-level atom are separated by 2eV. There are 3×10^{18} atoms in the upper level and 1.7×10^{18} atoms in the ground level. The coefficient of stimulated emission is $3.2 \times 10^5 \text{ m}^3/\text{W}\cdot\text{s}^3$ and the spectral radiance is $4 \text{ W/m}^2\cdot\text{Hz}$. Calculate the stimulated emission rate?
- 8 For an ordinary source, the coherence time $\tau_c = 10^{-10}$ second. Obtain the degree of non-monochromaticity for wavelength $\lambda_0 = 5400 \text{ \AA}$.
- 9 Calculate the coherence length of a laser beam for which the band width $\Delta v = 3000 \text{ Hz}$.
- 10 The lifetime of transitions in a Na atoms emitting wavelength of 589.6nm is estimated to be 16.4ns. Calculate the Einstein's coefficients A and B. Calculate spectral broadening and the coherence length of radiations.
- 11 The spectral line width of a HeNe laser emitting 632.8 nm is 10^{-16} m . Calculate Einstein's coefficients A and B and the coherence length of radiations.
- 12 Calculate the threshold gain factor of a helium–neon laser, which has a loss factor of 0.05 m^{-1} if the configuration of the system is as follows:
 - (a) A 50-cm tube with one mirror 99% reflecting and the output coupler 90% reflecting
 - (b) A 20-cm tube with one mirror 99% reflecting and the output coupler 95% reflecting
 - (c) A 20-cm tube with one mirror 99% reflecting and the output coupler 97% reflecting

Comment on the results obtained.

Solved examples

1. **The surface density of charge of a parallel plate capacitor is $7.2 \times 10^{-10} \text{ C m}^{-2}$. A dielectric medium with $\epsilon_r = 12$ is introduced between the plates of the capacitor. Estimate the induced surface density of charge on the dielectric surface and the electric fields between the plates with and without the dielectric material.**

The electric field across the plates of the capacitor without dielectric $E_0 = \frac{\sigma_0}{\epsilon_0} = 81.35 \text{ V/m}$

The electric field across the dielectric due to surface charges = $E_p = \frac{\sigma_p}{\epsilon_0}$

$$\sigma_p = (\epsilon_r - 1) \frac{\sigma_0}{\epsilon_r} = 6.6 \times 10^{-10} \text{ C m}^{-2}.$$

Electric field across dielectric = $\frac{\sigma_p}{\epsilon_0} = 74.57 \text{ V/m}$

Electric field across the plates with the dielectric = 6.78 V/m

2. **A dielectric medium with $\epsilon_r = 25$ is introduced between the plates of the capacitor with surface density of charge on the plates equal to $8.00 \times 10^{-10} \text{ C m}^{-2}$. Estimate, the electric fields between the plates with and without the dielectric material and the surface density of charge on the dielectric.**

The electric between the plates of the capacitor without the dielectric

$$E_0 = \frac{\sigma_0}{\epsilon_0} = 90.40 \text{ V/m}$$

The electric field between the plates with the dielectric = $\frac{E_0}{\epsilon_r} = 3.62 \text{ V/m}$

The electric field across the dielectric $E' = 86.78 \text{ V/m}$

The surface density of charge on the dielectric $\sigma = E' * \epsilon_0 = 7.68 \times 10^{-10} \text{ C m}^{-2}$

3. **An elemental dielectric has $\epsilon_r = 10$ and contains $7.5 \times 10^{29} \text{ atoms /m}^3$. Calculate the electronic polarisability of the material..**

$$P = \epsilon_0(\epsilon_r - 1)E = N\alpha_e E \quad \text{Given } \epsilon_r = 10$$

$$\alpha_e = \frac{\epsilon_0(\epsilon_r - 1)}{N} = \frac{8.85 \times 10^{-12} \times 9}{7.5 \times 10^{29}} = 10.62 \times 10^{-41} \text{ F m}^2$$

Problem Set:

- If the molecular dipoles in a 10^{-3} m radius water drop are pointed in the same direction, calculate the polarisation of the water drop. Dipole moment of water molecule is $6 \times 10^{-30} \text{ C m}$.
- Assuming that the polarisability of Kr atom is $2.18 \times 10^{-40} \text{ F m}^2$, calculate its dielectric constant. Kr has 2.7×10^{25} atoms per unit volume at NTP.

3. An elemental dielectric has $\epsilon_r = 12$ and contains 5×10^{29} atoms /m³. Calculate the electronic polarisability of the material.
4. Find the total polarisability of CO₂ if its susceptibility is 0.985×10^{-3} . Density of CO₂ is 1.977 Kgm^{-3} .
5. On being polarised an oxygen atom shows a dipole moment of 0.5×10^{-22} C-m. If the distance of the center of the -ve charge cloud from the nucleus is 4×10^{-17} m, calculate the polarisability of oxygen.
6. A parallel plate capacitor without a dielectric is charged such that the surface charge density on the plates is 10^6 C-m^{-2} . If a slab of a material with dielectric constant 10 is inserted between the plates, calculate the polarisation in the material and the electric field due to induced surface charge on the dielectric.
7. A dielectric material has one species having an atomic polarizability value of $10^{-30} \text{ Cm}^2\text{V}^{-1}$. It is found that when the dielectric sample is kept in a uniform electric field, the field reduces to one tenth of its original value. Estimate the number of atoms per unit volume of the material.
8. Electronic polarisability of Sulphur is $3.28 \times 10^{-40} \text{ F-m}^2$. Calculate the dielectric constant of Sulphur if the density and atomic weight of Sulphur are $2.08 \times 10^3 \text{ kg-m}^{-3}$ and 32 respectively.

pesu