ENGINEERING PHYSICS

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Week #3 Class #9

- Well behaved wave function
- Normalization and Probability concepts
- Wave function as a state function
- Double slit experiment revisited
- Linear Superposition of wave functions

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➢ *Suggested Reading*

1. *Concepts of Modern Physics, Arthur Beiser, Chapter 5*
2. *Learning Material prepared by the Department of Physics*

➢ *Reference Videos*

1. *Video lectures : MIT 8.04 Quantum Physics I*

Wave functions

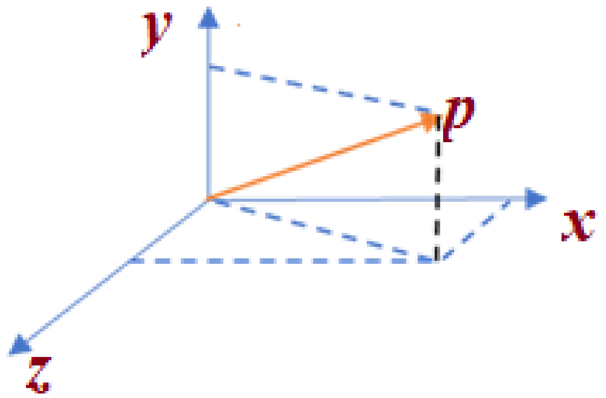
- *The state of a system in motion can be represented by a wave function -* Ψ
- Ψ *is a function of position and time -* $\Psi(x, y, z, t)$
- *In general -* Ψ *can be real or imaginary*
- *As inferred from the concept of a wave packet* Ψ *is a probability amplitude*
- *The wavefunction carries information about the system*
- *By itself has no other physical significance*

Wave functions

- *The three dimensional wave function in cartesian coordinates*

$$\Psi(x, y, z, t) = \psi(x).\phi(y).\chi(z).\varphi(t)$$

- *In general $\psi(x), \phi(y), \chi(z)$ are orthogonal functions*
- *Which are evaluated separately for the component of motion of the particle in the respective directions*

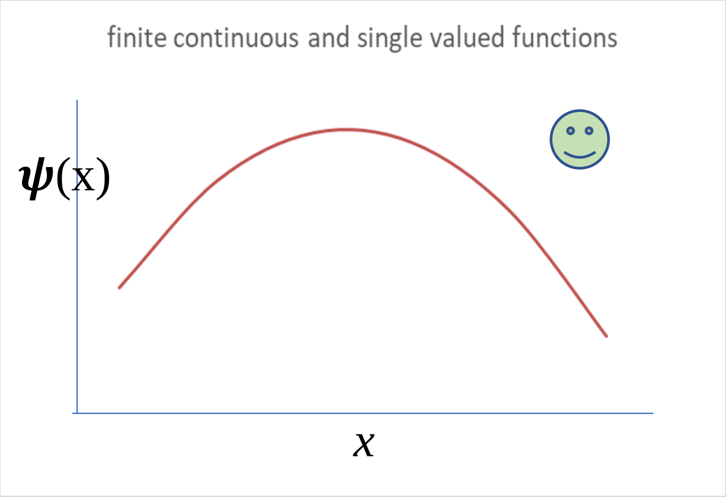


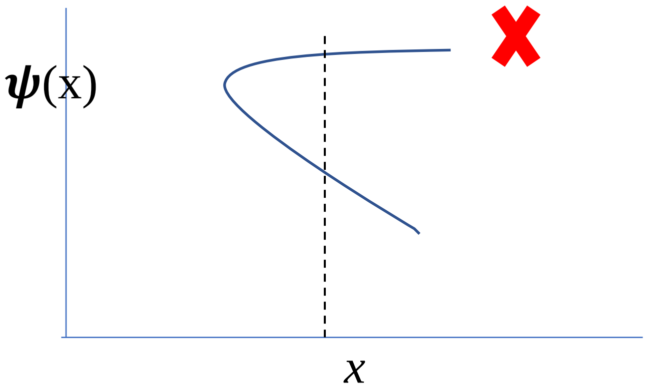
Characteristics of wave functions

- *A wave function in 1 dimension*

  $\psi = Ae^{i(kx-\omega t)}$ *where A can be real or imaginary*

- *The well-behaved wave function has to be*
  - *Finite, Continuous and Single valued*





Characteristics of wave functions

- ***The derivatives of Ψ with respect to the variables***

$$\frac{\partial\psi}{\partial x} = Ae^{i(kx-\omega t)}.\,ik = ik.\,\psi$$

$$\frac{\partial\psi}{\partial t} = Ae^{i(kx-\omega t)}.\,(-i\omega) = -i\omega.\,\psi$$

inherit the properties of Ψ and hence has to be

- ***Finite, Continuous and single valued***

Normalization of wave functions

- *Given the probability amplitude* $\psi = Ae^{i(kx-\omega t)}$ *which can be imaginary in the general case*
- ψ^* *is the complex conjugate of the wave function*

$$\psi^* = A^* e^{-i(kx-\omega t)}$$

- *Where* A^* *is the complex conjugate of* A
- *The product* $\psi^*\psi$ *is* $|\psi|^2$ *- a real number*
- *The square of the probability amplitude is the probability density*
- *Which gives the probability of finding the particle in unit length of space*

Normalization of wave functions

- *The integral* $\int \psi^* \psi \, dx = 1$

  should give the total probability in the range where the function is defined

- ψ *is a localized function ->* $\psi \rightarrow 0 \; as \; x \rightarrow \pm\infty$

- $\int_{-\infty}^{+\infty} \psi^* \psi \, dx = 1$

- *The given wave function must be normalizable*

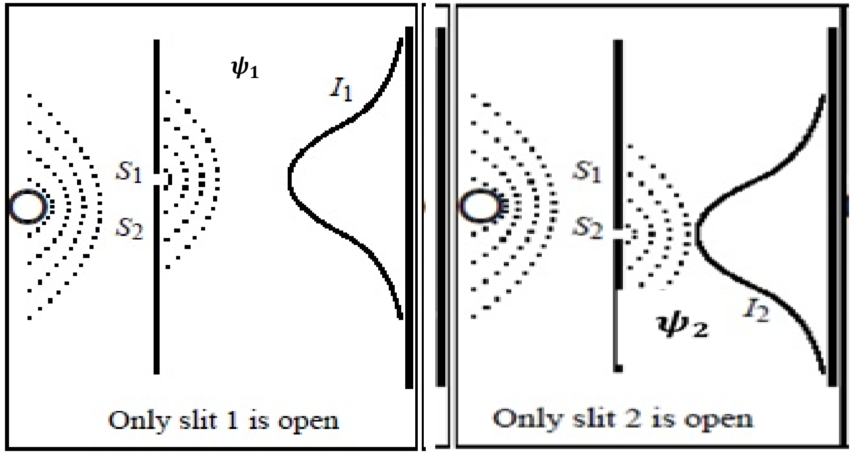
- *The process of normalization gets the right form of A*

Wave function as a state function

- ***The wave function satisfying all the conditions is a state function***
- $\psi = Ae^{i(kx-\omega t)}$
- $k = \frac{p}{\hbar}$ and $\omega = \frac{E}{\hbar}$
- ***The wave function*** $\psi = Ae^{\frac{i}{\hbar}(px-Et)}$
- ***The wave function can provide information about the state of the system***

Double slit experiment revisited

- ψ_1 *is the wave function for photons from slit 1*
- $I_1 = |\psi_1|^2$ *is probability the photon reaches the screen*
- ψ_2 *is the wave function for photons from slit 2*
- $I_2 = |\psi_2|^2$ *is probability the photon reaches the screen*

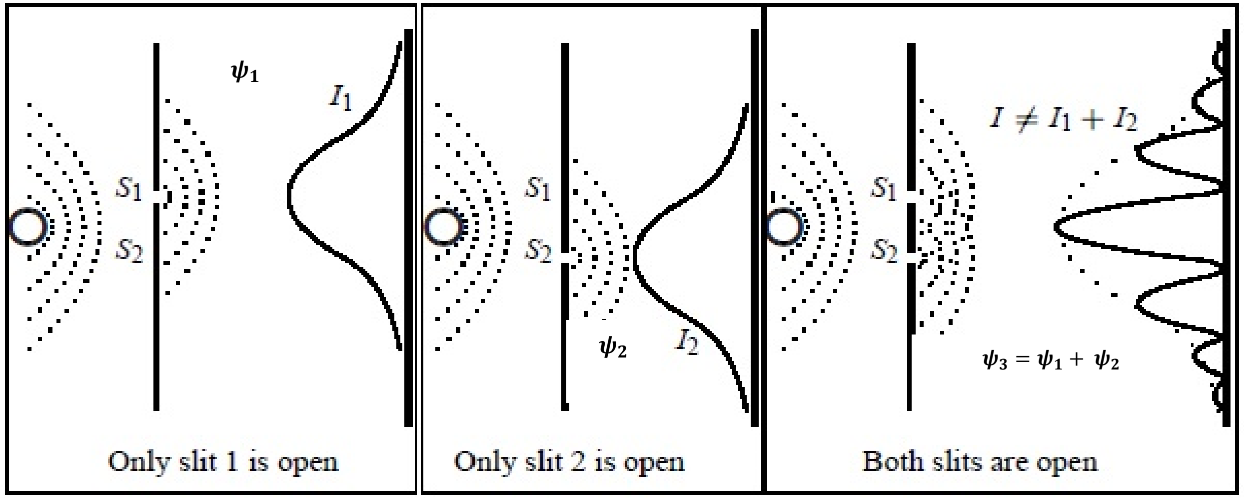


Double slit experiment revisited

- $\psi_3 = \psi_1 + \psi_2$ *is the superposed wave function for photons from both slits*

- $I_3 = |\psi_3|^2$ *is the combined probability of photons reaching the screen* $I_3 \neq I_1 + I_2$

- $|\psi_3|^2 = |\psi_1|^2 + |\psi_2|^2 + \psi_1^*\psi_2 + \psi_1\psi_2^* \neq |\psi_1|^2 + |\psi_2|^2$



Linear superposition of wave functions

- *The number of photons emerging from the slits can be different*
- *The superposed wave function* $\psi_3 = m.\psi_1 + n.\psi_2$
- *m and n are arbitrary constants*
- *This is the principle of linear superposition of wave functions*

Class #9 …….. Quiz….

The concepts of wave functions which are correct …..

1. Wave functions are always real
2. The amplitude of the wave function can be a complex number
3. Functions with a finite number of discontinuities are acceptable wave functions
4. The normalization process adjusts the amplitude of the wave function
5. In 3D, $\psi^*\psi$ gives the probability density of finding the particle in unit volume
6. The double slit experiment cannot explain linear superposition of wave functions.

THANK YOU

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