Equilibrium, in Physics the condition of a system when neither its state of motion nor its internal energy state tends to change with time. A simple mechanical body is said to be in equilibrium if it experiences neither linear acceleration nor angular acceleration; unless it is disturbed by an outside force, it will continue in that condition indefinitely.

A rigid body is said to be when the resultant of all forces acting on it is zero.

Equilibrium equations are as follows.

$$\boxed{\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_O = 0}$$

Examples include a weight suspended by a spring or a brick lying on a level surface.

System Isolation and Free Body Diagram

For analyzing an actual physical system, first we need to create an idealized model. The object should be separated from its surroundings. Then we need to draw a free-body diagram showing all the external (active and reactive) forces. Finally, we need to apply the equations of equilibrium to solve for any unknowns.

The rigid body to be analyzed should be separated from rest of the system which is called as System Isolation.

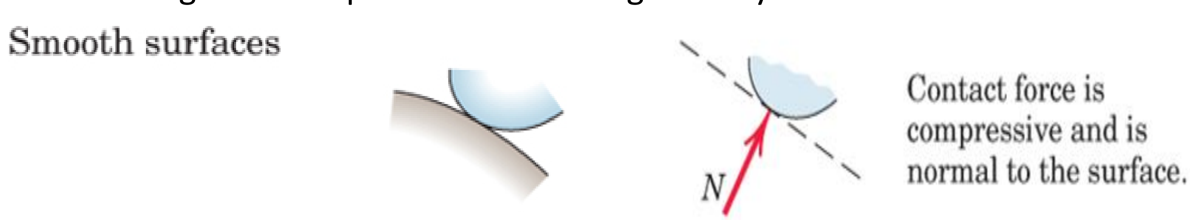
Further to sketch FBD , all the forces applied on it externally , the contact forces and the support reactions are included. Here always one should consider the characteristics of the forces given.

A convenient reference coordinate system should be choosen.

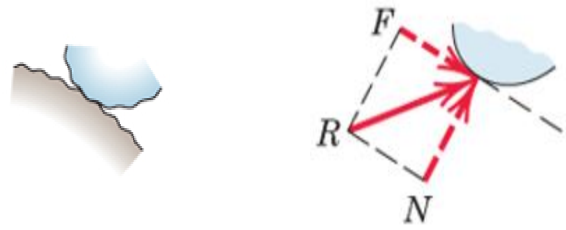
Based on the given force system the equilibrium equations are employed to solve the problem.

Modeling

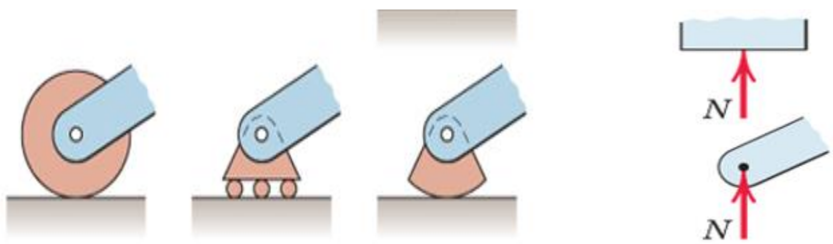
The following sketch explains the modeling in analysis.



Rough surfaces



Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.

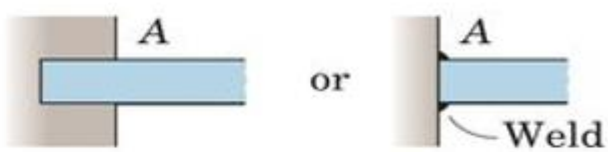


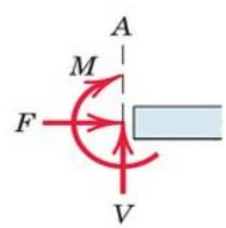
Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.

Gravitational attraction

The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center of gravity G.

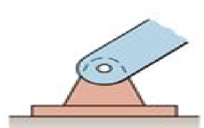
Built-in or fixed support

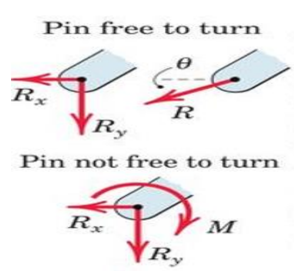




A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.

Pin connection



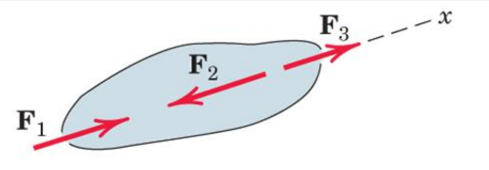
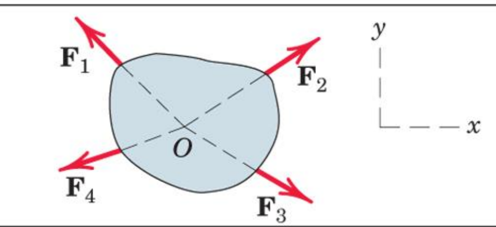
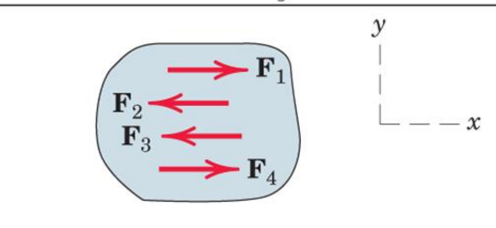
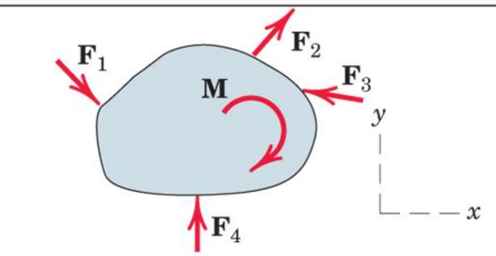


A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.

For more details refer Text book , page nos 111- 120.

Equilibrium of different force system.

CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS

Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_z = 0$ $\Sigma F_y = 0$

From above table it is self explanatory that to bring equilibrium in a system of Collinear forces , summation of forces in the direction of given forces must be zero. For a Concurrent force system we know that net moment of the system zero, therefore sum of forces in the reference axis direction must be zero.
Like this one should identify the necessary equilibrium equations / conditions to establish equilibrium, and the unknown forces , moments can be found.

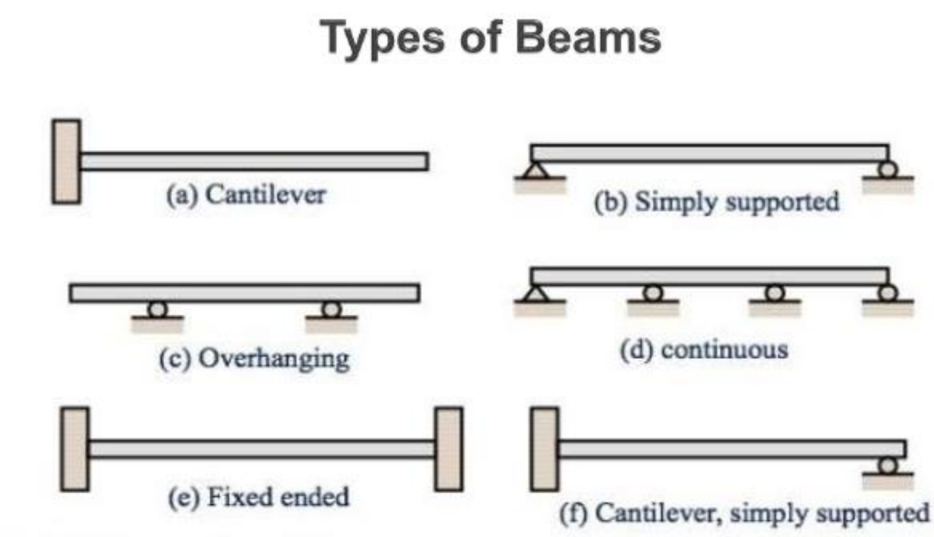
Statically Determinate and Indeterminate Structures:

- A rigid body with more external supports than required for maintaining equilibrium condition is said to be Statically indeterminate.
- A rigid body with minimum number of supports required for maintaining equilibrium condition is said to be Statically determinate.

Also, if a structure which can be analysed with the necessary equilibrium equations will be called as Statically determinate structures, otherwise Statically indeterminate structures.

The excess supports given to any structure are called as Redundant. The structure maintains equilibrium if the redundant is removed. But it may be required to impart stiffness, geometry to it.

More details refer Text book page no 122 – 126.



In the above figure different types of beam a structure is given,

These structures are also classified depending on the analysis into Determinate beam and Indeterminate beam.

Cantilever beam is one which has one end fixed (restrained in all the directions).

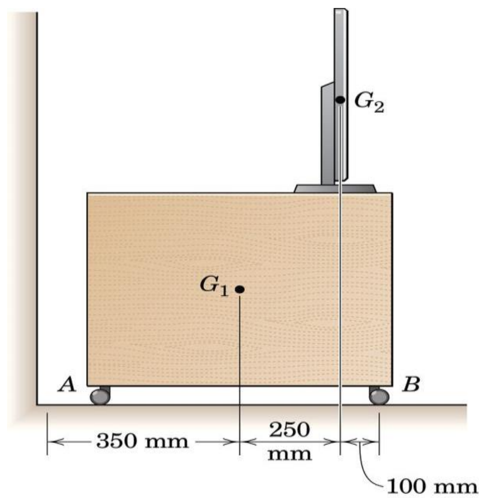
So, the necessary equilibrium equations which are applied to solve unknowns are 3 i.e, $\boxed{\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_O = 0}$ are sufficient to find the 3 unknown reactions.

In case of Simply supported beam, which is one end hinged other end roller supported 3 unknown reactions 2 at hinged support and 1 at roller support can be found with the available equilibrium equations. These are examples of Determinate structures.

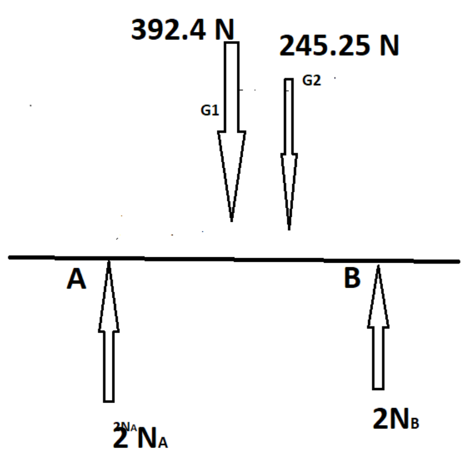
Where as Fixed beam (both the ends are fixed or built-in) which has three unknown support reactions at both ends totally 6, can not be analysed with 3 equilibrium equations. Therefore, it is an example of Indeterminate structure.

Examples:

In the side view of a 25 kg flat – screen television resting on a 40-kg cabinet, the respective centers of mass are labeled G2 and G1. Assume symmetry into the paper and calculate the normal reaction force at each of the four casters.



Solution:



FBD:

$$\boxed{\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_O = 0}$$

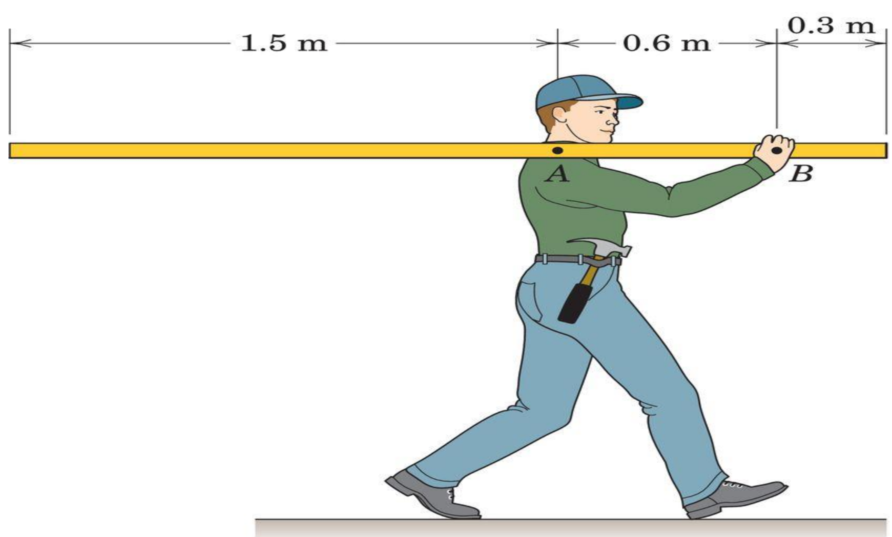
$$M_A = -392.4X0.35 - 245.25X0.6 + 2N_B\ X0.7 = 0$$

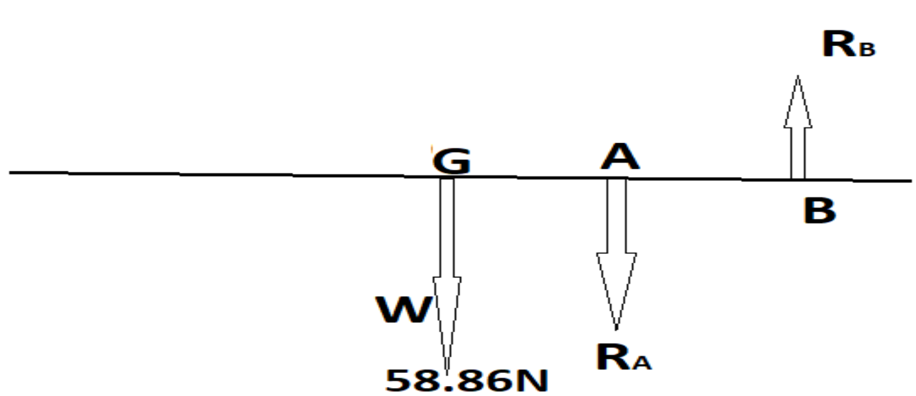
$$\Sigma F_y = 0$$

$$-392.4 - 245.25 + 2\,N_A + 2N_B = 0$$

$$N_A = 115.62\ N$$

2. A carpenter carries a 6-kg uniform board as shown. What downward force does he feel on his shoulder at A ?





$$\Sigma M_B = 0$$

$$58.86\ X\ 0.9 + N_A\ X\ 0.6 = 0$$

$$N_A = -88.29\ N$$
$$N_A = 88.29\ N\ upward$$

*Upward reaction found is w.r.t the board and the same is downward on his shoulder.