ENGINEERING MATHEMATICS - I

Unit - 3: Partial Differential Equations

Department of Science and Humanities

PES UNIVERSITY

Contents

Problem 1

$z = (x+y)\,\phi(x^2 - y^2)$

Solution: Differentiating

$$z_x = 1 \cdot \phi + (x+y)2x \cdot \phi' \tag{1}$$

$$z_y = 1 \cdot \phi + (x+y)(-2y)\phi' \tag{2}$$

From (2),

$$\phi' = \frac{\phi - z_y}{2y(x+y)} \tag{3}$$

Substituting (3) in (1):

$$z_x = \phi + 2x(x+y)\left[\frac{\phi - z_y}{2y(x+y)}\right]$$

$$p = \phi + \frac{x}{y}(\phi - q)$$

$$p = \left(\frac{x+y}{y}\right)\phi - \frac{x}{y}q$$

Problem 1 (contd.)

From the given equation, $\phi = \frac{z}{x+y}$. Substituting ϕ,

$$p = \frac{x+y}{y} \cdot \frac{z}{x+y} - \frac{x}{y}q = \frac{z}{y} - \frac{x}{y}q$$

$$\text{or} \quad yp + xq = z$$

Problem 2

$z = x^n f\left(\frac{y}{x}\right)$

Solution: By differentiation,

$$z_x = nx^{n-1} f + x^n \left(-\frac{y}{x^2}\right) f'$$
$$z_y = x^n \cdot \frac{1}{x} f' \qquad \text{or} \quad f' = \frac{z_y}{x^{n-1}}$$

Eliminating f',

$$z_x = nx^{n-1} f - x^{n-2} y \cdot \frac{z_y}{x^{n-1}}$$
$$xp = nx^n f - yq$$
$$\text{or} \quad xp = nz - yq$$

Problem 3

$xyz = f(x+y+z)$

Solution: Differentiating w.r.t. x and y

$$yz + xyz_x = 1 \cdot f' + f' \cdot z_x \tag{1}$$
$$xz + xyz_y = 1 \cdot f' + f' \cdot z_y \tag{2}$$

From (2),

$$f' = \frac{xz + xyz_y}{1 + z_y} = \frac{xz + xyq}{1 + q} \tag{3}$$

Put (3) in (1)

$$yz + xyp = (1+p)f' = (1+p)\left(\frac{xz + xyq}{1+q}\right)$$

$$(1+q)(yz + xyp) = (1+p)(xz + xyq)$$

$$\text{or} \quad x(y-z)p + y(z-x)q = z(x-y)$$

Problem 4

$z = f(x)\, g(y)$

Solution: Differentiating w.r.t. x and y, we get

$$z_x = f'(x)\, g(y)$$
$$z_y = f(x)\, g'(y)$$

$$z_x \cdot z_y = f'(x)\, g(y) \cdot f(x)\, g'(y) = f(x)g(y)f'(x)g'(y) = z\, f'(x)g'(y)$$

But

$$z_{xy} = f'(x)g'(y)$$

so

$$z_x \cdot z_y = z\, z_{xy}$$

or

$$pq = zs$$