CL18_Q1. Show that the energy of an electron confined in a 1-D symmetric potential well of length 'L' and infinite depth is quantized. Is the electron trapped in a potential well allowed to take zero energy? If not, why?

Ans:

The energy of the nth eigenstate is given by $E_n = \frac{h^2 n^2}{8mL^2}$ where n = 1,2 3…

The Eigen values are $E_1 = \frac{h^2}{8mL^2}$ $E_2 = \frac{h^2 2^2}{8mL^2}$ $E_3 = \frac{h^2 3^2}{8mL^2}$

Thus, the energies are quantized with n being the quantum number. The quantization is imposed by the boundary conditions and the requirement of normalizability. All bound quantum states are in fact quantized.

For an electron trapped within a one dimensional potential well, when n = 0, the wave function is zero for all values of x, i.e., it is zero even within the potential well. This would mean that the electron is not present within the well. Therefore the state with n = 0 is not allowed. As energy is proportional to n^2, the ground state energy cannot be zero since n = 0 is not allowed

CL18_Q2. What properties must a potential have in order that the wavefunctions have definite parity? If wavefunctions have definite parity, why is does the ground state always have *even* parity?

Ans:

A wavefunction has definite parity if ψ(−x) = ±ψ(x); this requires a symmetric potential, V(−x) = V(x). 2. Minimum energy implies maximum wavelength, so no nodes. No nodes condition implies parity is not odd (odd parity requires a central node), so if the wavefunction has definite parity it must be even.