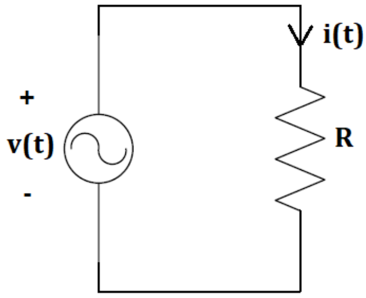
NOTES -Class 23

Response of Resistive Load to Sinusoidal Supply:



Let the supply voltage be $v(t) = V_m \sin(\omega t)$

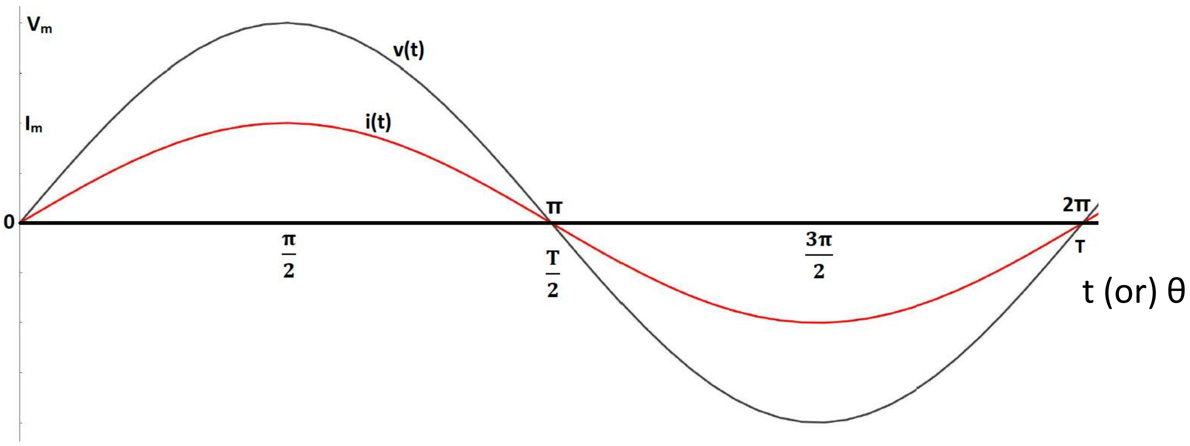
where, V_m is the peak value of voltage

By Ohm's Law, $i(t) = \dfrac{v(t)}{R}$

Hence current will be of the form, $i(t) = I_m \sin(\omega t)$

where, $I_m = \dfrac{V_m}{R}$ is the peak value of current

Time domain representation:



Phasor Representation and Phasor Diagram:

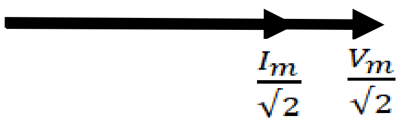
$$v(t)=V_m \sin(\omega t) \Rightarrow \bar{V}=\frac{V_m}{\sqrt{2}}\angle 0^\circ$$

$$i(t)=I_m \sin(\omega t) \Rightarrow \bar{I}=\frac{I_m}{\sqrt{2}}\angle 0^\circ$$

$$\text{Impedance,} \quad Z=\bar{V}/\bar{I}=R\angle 0^\circ = R\ \Omega$$

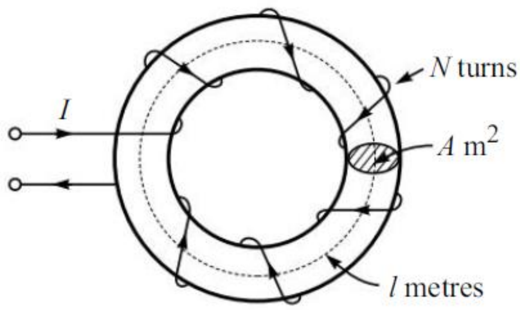
Impedance is the ratio of voltage phasor to current phasor. It is analogous to resistance in DC networks. It is the opposition offered by an AC element or network to the flow of AC currents.

Phasor Diagram:



Inductor & the concept of inductance:

An inductor is obtained by winding the conductor into a coil.



A current carrying coil sets up a magnetic field around it.

Magnetic field is expressed as magnetic flux ϕ around the coil.

$$\text{Magnetic flux } \phi = \frac{\text{Magnetomotive Force}}{\text{Reluctance}} = \frac{N*I}{S}$$

Where, S = Reluctance = $\dfrac{\text{length}}{(\text{Permeability*Area})}$

Magnetic flux, ϕ is directly proportional to the current in the inductor coil.

i.e., $\phi \propto i$

i.e., $N\phi \propto i$

Where, $N\phi$ is called **flux linkages** denoted by ψ

Therefore, ψ is proportional to i

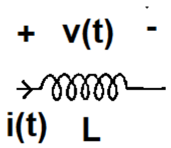
$$\Rightarrow \psi = Li$$

Where, L is the proportionality constant called 'Inductance' of the inductor.

$$L = \frac{\psi}{i} = \frac{N\phi}{i}$$

Inductance is measured in Henrys (H).

Voltage – Current relationship in an inductor:



The voltage across the terminals of an inductor is directly proportional to rate of change of flux linkages.

i.e., $v(t) \propto \dfrac{d}{dt}(\psi)$

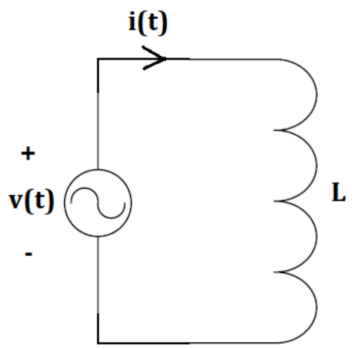
$$v(t) = \frac{d}{dt}(\psi) = \frac{d}{dt}(N\phi) = N\frac{d\phi}{dt} = \frac{d}{dt}(Li) = L\frac{di}{dt}$$

i.e., voltage v(t) is related to current i(t) as $v(t) = L\frac{di(t)}{dt}$

Therefore, i(t) can be expressed as

$$i(t) = \frac{1}{L}\int v(t)dt$$

Response of Pure Inductive Load to Sinusoidal Supply:



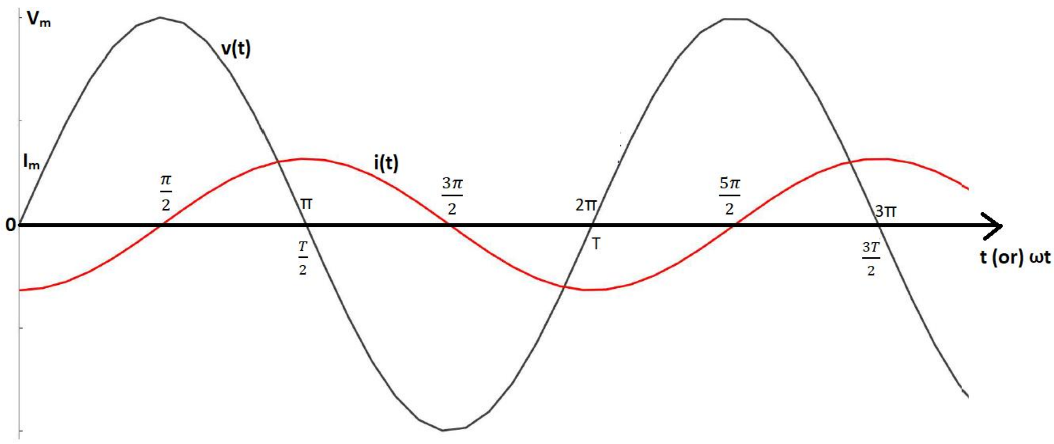
Let the supply voltage be $v(t) = V_m \sin(\omega t)$

In a pure inductor, $i(t) = \frac{1}{L}\int v(t)dt$

$$= \frac{-V_m}{\omega L}\cos(\omega t)$$

$$= I_m \sin(\omega t - 90°)$$

where, $I_m = \frac{V_m}{\omega L}$ is the peak value of current

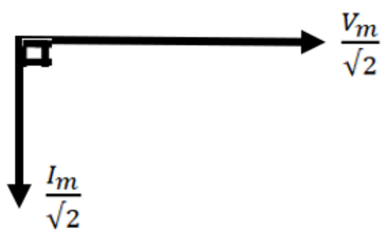


In a pure inductor, current **lags** voltage by 90°

Phasor Diagram:

$$v(t)=V_m\sin(\omega t) \quad \Rightarrow \quad \bar{V} = \frac{V_m}{\sqrt{2}}\angle 0^\circ$$

$$i(t)=I_m\sin(\omega t-90^\circ) \quad \Rightarrow \quad \bar{I} = \frac{I_m}{\sqrt{2}}\angle -90^\circ$$

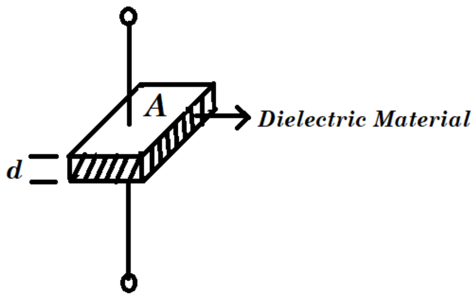


$$Z = \frac{\bar{V}}{\bar{I}} = \frac{\frac{V_m}{\sqrt{2}}\angle 0^\circ}{\frac{I_m}{\sqrt{2}}\angle -90^\circ} = \omega L\angle 90^\circ = jX_L \quad \Omega$$

Where, X_L is called '**Inductive Reactance**'.

Capacitor & the concept of Capacitance:

A Capacitor is obtained by placing a dielectric medium between the conducting plates.



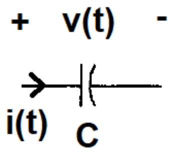
Capacitance, C = $\frac{\epsilon A}{d}$ Farad

Where, A is the area of each of the plates in m^2

d is the distance between the plates in m

ε is the permittivity of the dielectric medium in F/m

Voltage – Current relationship in a Capacitor:



The charge on the plates of a capacitor is directly proportional to the voltage across its terminals.

$$\text{i.e., } q(t) \propto v(t) \Rightarrow q(t) = Cv(t)$$

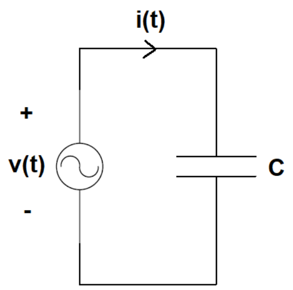
The constant of proportionality 'C' is called Capacitance of the Capacitor.

Hence, current, $i(t) = \frac{dq(t)}{dt} = C\frac{dv(t)}{dt}$

Therefore, v(t) can be expressed as

$$v(t) = \frac{1}{C}\int i(t)dt$$

Response of Pure Capacitor to Sinusoidal Supply:



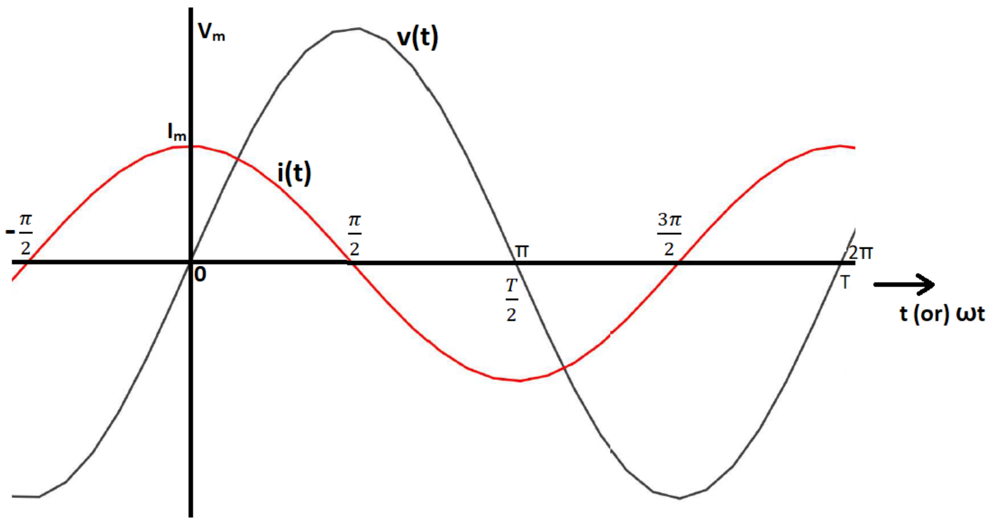
Let the supply voltage be $v(t) = V_m \sin(\omega t)$

In a pure capacitor, $i(t) = C\dfrac{dv(t)}{dt}$

$$= CV_m \omega \cos(\omega t)$$

$$= I_m \sin(\omega t + 90°)$$

Where, $I_m = V_m \omega C$ is the peak value of current

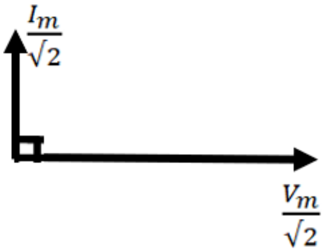


In a pure capacitor, current **leads** voltage by 90°

Phasor Diagram:

$$v(t)=V_m \sin(\omega t) \Rightarrow \bar{V} = \frac{V_m}{\sqrt{2}} \angle 0^\circ$$

$$i(t)=I_m \sin(\omega t+90^\circ) \Rightarrow \bar{I} = \frac{I_m}{\sqrt{2}} \angle 90^\circ$$



$$Z = \frac{\bar{V}}{\bar{I}} = \frac{\frac{V_m}{\sqrt{2}}\angle 0^\circ}{\frac{I_m}{\sqrt{2}}\angle 90^\circ} = \frac{1}{\omega C}\angle -90^\circ = -jX_C \ \ \Omega$$

Where, $X_C = \frac{1}{\omega C}$ is called '**Capacitive Reactance**'.

Question 6:

A Capacitor of Capacitance 100µF is connected across an AC voltage source 100sin(100πt) V. Determine

i) Capacitive Reactance
ii) Impedance
iii) Instantaneous expression for the current

Also, draw the phasor diagram.

Solution:

Given, V(t) = 100sin(100πt) V

Hence, $\omega = 100\pi$ rad/s

i) Capacitive Reactance, $X_C = \frac{1}{\omega C} = 31.83\Omega$

ii) Impedance, $Z = -jX_C = -j31.83\Omega$

iii) Instantaneous current, $i(t) = V_m \omega C \sin(\omega t + 90°)$ A

$= 3.14\sin(\omega t + 90°)$ A

Phasor Diagram:

$\bar{V} = \frac{100}{\sqrt{2}}\angle 0°$ V

$\dot{I} = \frac{3.14}{\sqrt{2}}\angle 90°$ A