1. How is group velocity linked to phase velocity in a dispersive medium?

Answer: The expressions for group velocity and phase velocity are given as

$v_g = \frac{d\omega}{dk}$ and $v_{ph} = \omega/k$ respectively.

➔ $\frac{d}{dk}(v_{ph}.k) = v_{ph} + k\frac{dv_{ph}}{dk}$

However, $\boldsymbol{\frac{dv_{ph}}{dk} = \frac{dv_{ph}}{d\lambda}.\frac{d\lambda}{dk}}$ and since $\frac{d\lambda}{dk} = -\frac{2\pi}{k^2}$

$$\boldsymbol{v_g = v_{ph} - \frac{2\pi}{k}\frac{dv_{ph}}{d\lambda} = v_{ph} - \lambda\frac{dv_{ph}}{d\lambda}}$$

2. In a medium, $v_p = a\lambda^{1/4}$, then express group velocity in terms of phase velocity?

Answer: W.K.T group velocity is given as $v_g = v_{ph} - \lambda\frac{dv_{ph}}{d\lambda}$

Since $\frac{dv_{ph}}{d\lambda} = \frac{a\lambda^{1/4}}{4} = \frac{v_{ph}}{4}$

$v_g = v_{ph} - \frac{v_{ph}}{4} = \frac{3}{4}v_{ph}$