CL22_Q1. What is a linear harmonic oscillator? When can the oscillations become "anharmonic"?

Answer

Harmonic oscillator is one of the most fundamental systems in quantum mechanics which gives insight to a variety of problems such as the vibrational molecular spectroscopy as it describes the vibrations in molecules and their counterparts in solids, the phonons.
An oscillator that is not oscillating in harmonic motion, the oscillation becomes "anharmonic".

CL22_Q2. The lowest energy of the harmonic oscillator is non-zero. Explain why?

Answer

Eigen energy values of harmonic oscillator system is given by $\mathbf{E_n} = (\mathbf{n} + \frac{1}{2})\hbar\boldsymbol{\omega}$

This gives the allowed energy states as $\frac{1}{2}\hbar\boldsymbol{\omega}$, $\frac{3}{2}\hbar\boldsymbol{\omega}$, $\frac{5}{2}\hbar\boldsymbol{\omega}$, … …

Thus, the minimum energy state of the system is a non-zero $\frac{1}{2}\hbar\omega$ where ω is the fundamental frequency of vibration. The higher energy states are then equally spaced at $\hbar\omega$.

CL22_Q3. What are the classical "turning points" of an oscillator?

Answer

Oscillator moves between positive and negative turning points $\pm x_{max}$ where the total energy E equals the potential energy ½ kx^2_{max} while the kinetic energy is momentarily zero. When the oscillator moves past x=0, the kinetic energy reaches its maximum value while the potential energy equals zero.

CL22_Q4. The eigen functions of a particle performing linear harmonic oscillations is given by $\boldsymbol{\Psi}(x) = \left[2\mathbf{n}\left(\sqrt{\boldsymbol{\pi}}\right)\mathbf{n}!\right]^{-½} exp^{\left(-\frac{x^2}{2}\right)} Hn(x)$ where $H_n(x) = (-1)^n \exp(x^2)\, d^n/dx^n\, (\exp(-x^2))$. Write down the mathematical expressions for the Eigen functions for the first four quantum states.

Answer

Eigen energy values of harmonic oscillator system is $\mathbf{E_n} = (\mathbf{n} + \frac{1}{2})\hbar\boldsymbol{\omega}$

This gives the allowed energy states as $\frac{1}{2}\hbar\boldsymbol{\omega}$, $\frac{3}{2}\hbar\boldsymbol{\omega}$, $\frac{5}{2}\hbar\boldsymbol{\omega}$, … …

Thus, the minimum energy state of the system is a non-zero $\frac{1}{2}\hbar\omega$ where ω is the fundamental frequency of vibration. The higher energy states are then equally spaced at $\hbar\omega$.