



# ENGINEERING PHYSICS

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# ENGINEERING PHYSICS

## Unit I : Review of concepts leading to Quantum Mechanics

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### Week #3 Class #10

- Concept of observables of the state of a system
- Operators in quantum mechanics
- Expectation values of observables

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### ➤ *Suggested Reading*

1. *Concepts of Modern Physics, Arthur Beiser, Chapter 5*
2. *Learning Material prepared by the Department of Physics*

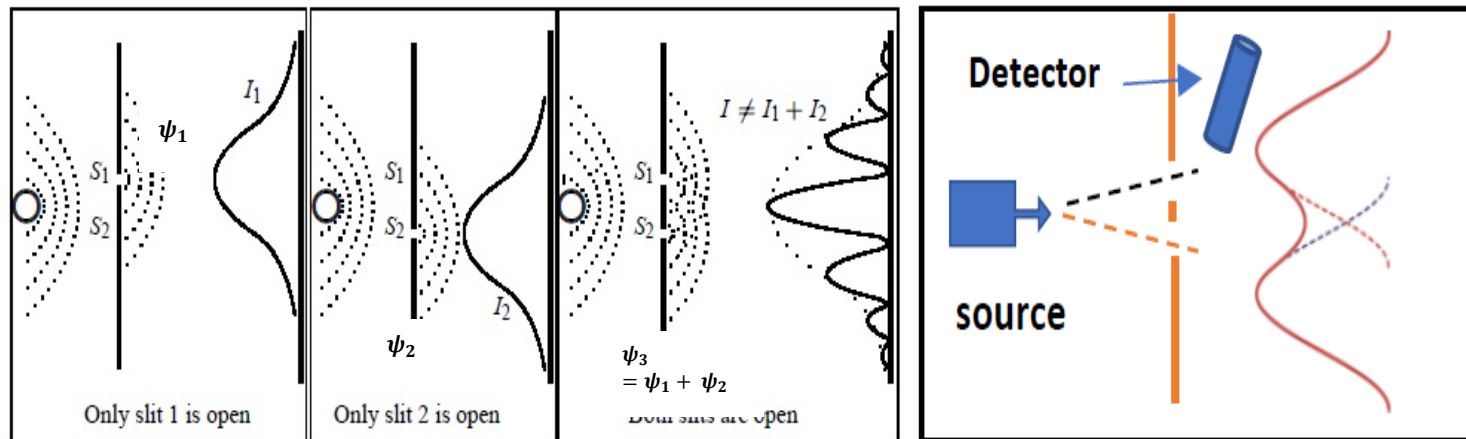
### ➤ *Reference Videos*

1. *Video lectures : MIT 8.04 Quantum Physics I*

- *All experimentally measurable parameters of a physical system are observables*
  - *Position*
  - *momentum*
  - *Energy of a state*
  - *life time of electrons*
  - *Spin of a system*
- *Multiple measurements yield average values of the parameters*

## Double slit experiment revisited

- *Quantum mechanically observations on physical systems do not yield information about the system*
- *Observations lead to a collapse of the wave function - double slit experiment with an “**observer**”*
- *Observables in quantum mechanics have to be estimated probabilistically*



## Operators

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- *Wave functions contain information about the quantum system*
- *Mathematical operators can be used to extract information about the physical state in terms of the observables*

## Operators

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- A normalized wave function contain information about the quantum system  $\psi = Ae^{\frac{i}{\hbar}(px-Et)}$  - eigen function
- A mathematical operator  $\hat{G}$  operating on the wavefunction can result in the eigen value  $G$  of the observable
- The eigen value equation  $\hat{G}\psi = G\psi$

### *Momentum operator -*

- *The partial derivative of  $\psi$  with respect to position yields*

$$\frac{\partial \psi}{\partial x} = \left(\frac{ip}{\hbar}\right)\psi$$

$$\left\{-i\hbar \frac{\partial}{\partial x}\right\} \psi = p \psi$$

- *The momentum operator  $\hat{p} = \left\{-i\hbar \frac{\partial}{\partial x}\right\}$*

*Operating on the eigen function yields the momentum eigen value*



*Kinetic energy operator -*

- The second derivative of  $\psi$  with respect to position yields*

$$\frac{\partial^2 \psi}{\partial x^2} = \left( \frac{ip}{\hbar} \right)^2 \psi$$

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = \frac{p^2}{2m} \psi = KE\psi$$

*The kinetic energy operator  $\widehat{KE} = \left\{ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \right\}$*

*operating on the eigen function yields the eigen value of the kinetic energy of quantum system*

*Total energy operator -*

- The derivative of  $\psi$  with respect to time yields*

$$\frac{\partial \psi}{\partial t} = \left(-\frac{iE}{\hbar}\right)\psi$$
$$\left\{i\hbar \frac{\partial}{\partial t}\right\} \psi = E\psi$$

*The total energy operator  $\hat{E} = \left\{i\hbar \frac{\partial}{\partial t}\right\}$*

*Operating on the eigen function yields the eigen value of the total energy of quantum system*

*This is also called as the Hamiltonian operator  $\hat{H}$*

### *Position operator -*

- *The position operator has to be discussed in the momentum space*
- *The position operator  $\hat{x}$  operating on  $\psi$*   
$$\hat{x}\psi = x\psi$$
- *yields the eigen value of position of the quantum system*

### *Potential energy operator -*

- *Potential energy operator is not explicitly described*
- *The eigen value of the potential energy can be inferred as the difference of the total energy and the kinetic energy*
- *The eigen value equation for the potential energy is*

$$\hat{V}\psi = V\psi$$

Quantum mechanics predicts only the most probable values of the observables of a physical system

- the expectation values  $\equiv$  the average of repeated measurements on the system.

The eigen value equation for momentum

$$\hat{p}\psi = p\psi$$

The operation  $\psi^*\hat{p}\psi = \psi^*p\psi = p\psi^*\psi$

$\psi^*\psi$  is the probability density

$p\psi^*\psi$  should be the probability of the eigen value

The spread in the wave packet can yield a range of  $p\psi^*\psi$

Integrated over the range of  $x$  for the extend of the wave packet

$$\int \psi^* \hat{p} \psi dx = \int \psi^* p \psi dx = \langle p \rangle \int \psi^* \psi dx$$

$\langle p \rangle$  is the most probable value of the momentum.

Thus the expectation value of the momentum is written as

$$\langle p \rangle = \frac{\int \psi^* \hat{p} \psi dx}{\int \psi^* \psi dx}$$

In general an operator  $\hat{G}$  of the observable  $g$

Gives the expectation value of the observable

$$\langle g \rangle = \frac{\int \psi^* \hat{G} \psi dx}{\int \psi^* \psi dx}$$

And in three dimensional space

$$\langle g \rangle = \frac{\int \Psi^* \hat{G} \Psi dV}{\int \Psi^* \Psi dV}$$

### Understanding concepts of observables, operators and expectation values .....

1. Observables in quantum mechanics are obtained as results of experiments
2. The total energy operator is  $\left\{i\hbar \frac{\partial}{\partial t}\right\}$
3. The position operator as obtained from the momentum space would be  $\left\{-i\hbar \frac{\partial}{\partial p}\right\}$
4. The total energy operator is Hamiltonian operator
5. Expectation value of position in 1d is  $\langle x \rangle = \frac{\int \psi^* \hat{x} \psi dx}{\int \psi^* \psi dx}$





# THANK YOU

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