CL28_Q1. Explain Fermi level and Fermi energy.

Answer

The energy corresponding to the highest occupied level in the valence band at 0 K is called Fermi energy. The corresponding energy level is called the Fermi level. At 0 K all the energy levels above the Fermi level are empty and all those below the Fermi level are completely filled.

CL28_Q2. Obtain an expression for Fermi energy using the concept of density of states.

Answer

The upper most occupied energy state at 0K is then termed as the Fermi energy of the metal. Thus, at 0K all the states below the Fermi energy are filled and all the states about the Fermi energy are empty.

We know that the density of states is given by

$$g(E)dE = \frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}}\, dE$$

The density of occupied states $N(E) = g(E) * F_d$ when evaluated for all levels from 0 to E_f, should result in the total count of free electrons in the metal, ie.,

The total free electronic concentration $n = \int_0^{E_f} N(E)\, dE = \int_0^{E_f} g(E) * F_d\, dE$.

We know that Fd has to be 1 for all energy levels below Ef at 0K and hence

$$n = \int_0^{E_f} N(E)\, dE = \int_0^{E_f} g(E)\, dE = \frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{\frac{3}{2}} \int_0^{E_f} E^{\frac{1}{2}}\, dE = \frac{\pi}{3}\left(\frac{8m}{h^2}\right)^{\frac{3}{2}} E_f^{3/2}$$

The Fermi energy can be estimated if the concentration of free electrons is known and can be evaluated as

$$\mathbf{E_f = \left(\frac{3}{\pi}\right)^{2/3}\left(\frac{h^2}{8m}\right) n^{2/3}}$$

CL 28 _Q3. Using the expression of density of states, show that average energy of electrons in a metal at 0K is $\frac{3}{5}\ E_F$

Answer

Since the distribution of electrons in the different energy states shows a non-linear variation, the average energy of the electron is not the simple average of the electron's max and min energy. From the graph of N(E) vs E we observe that N(E) states have energy E which implies that the total energy of all electrons in filled states upto E_f should the summation of all N(E)*E

The average energy of the electron = $\frac{\textbf{total energy of all electrons in different energy states}}{\textbf{total number of electrons}}$

$$\frac{\int_0^{E_f} g(E) * E * F_d\, dE}{\int_0^{E_f} g(E) * F_d\, dE} = \frac{\frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{\frac{3}{2}} \int_0^{E_f} E^{\frac{1}{2}}\, dE * E}{\frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{\frac{3}{2}} \int_0^{E_f} E^{\frac{1}{2}}\, dE}$$

This on integration gives the average energy $E = \frac{3}{5}\ E_f$