NOTES -Class 24

Concept of Active, Reactive and Apparent Powers

Average Power in AC Circuits:

In AC systems, since both voltage v(t) and current i(t) are time varying, power is also time varying in nature.

Instantaneous power, p(t) = v(t)*i(t)

For the sake of energy calculations, it is useful to find average power.

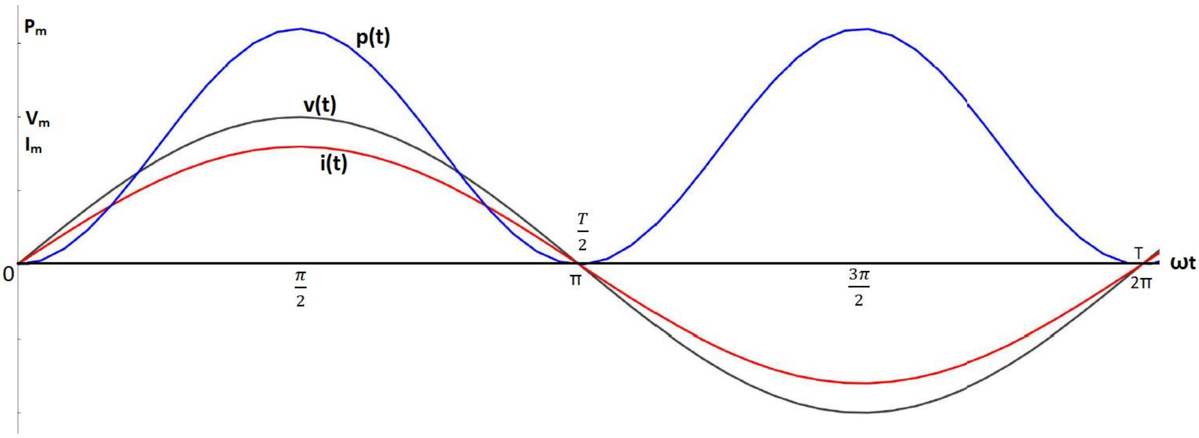
Average power, denoted by P is found out using the following equation:

$$P = \frac{1}{T}\int_0^T p(t)dt$$

It is measured in Watts (W).

Case 1: Resistive Load

For $v(t) = V_m \sin(\omega t)$, the current $i(t) = I_m \sin(\omega t)$; $p(t) = v(t)*i(t)$



$$P = \frac{1}{T}\int_0^T v(t)*i(t)dt = \frac{1}{T}\int_0^T V_m I_m \sin^2(\omega t)dt$$

$$= \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} * \frac{I_m}{\sqrt{2}} = V*I \quad \text{Watts}$$

where, V = RMS voltage and I = RMS current

In a resistive load, instantaneous power p(t) is always positive because a resistor consumes the power given to it by the source. A resistor dissipates the power absorbed as heat.

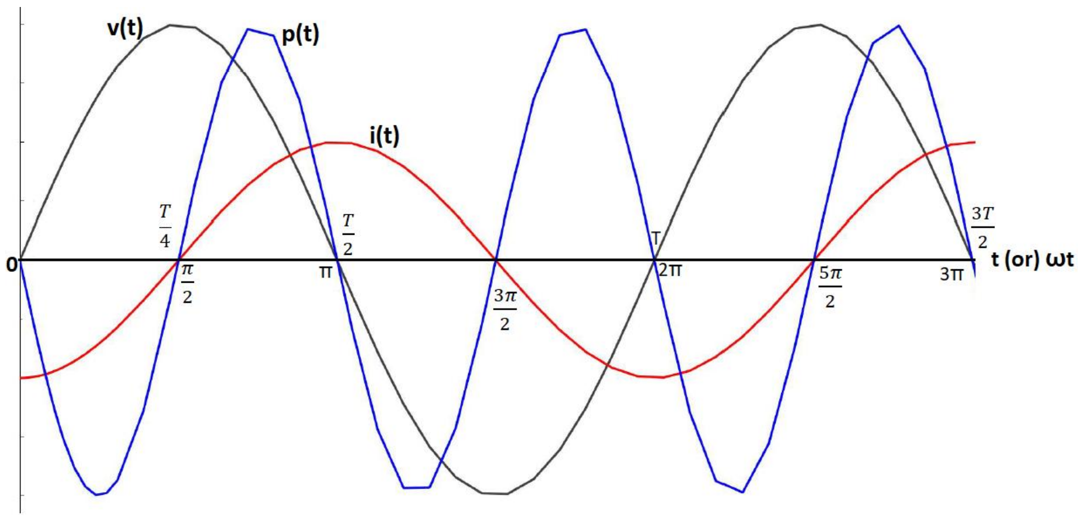
Average power is also called **active power** (or) **real power** and it is measured in Watts(W).

Case 2: Purely Inductive Load

For $v(t) = V_m \sin(\omega t)$, the current $i(t) = I_m \sin(\omega t - 90^\circ)$; $p(t) = v(t)*i(t)$

$$P = \frac{1}{T}\int_0^T v(t)*i(t)dt = \frac{1}{T}\int_0^T V_m I_m \sin(\omega t)\sin(\omega t - 90^\circ)dt$$

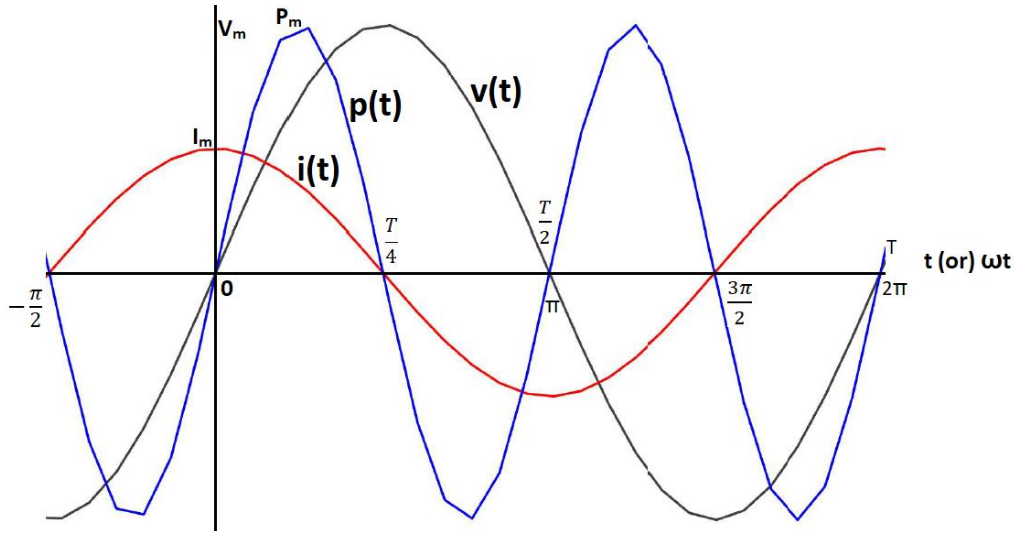
$$= 0$$



In a pure inductor, during one quarter cycle of voltage, power is positive i.e., it flows from source to inductor and gets stored in the magnetic field of the inductor & during the next quarter cycle of voltage, power is negative i.e. stored energy in the inductor flows back to the source.

Case 3: Purely Capacitive Load

For $v(t) = V_m\sin(\omega t)$, the current $i(t) = I_m\sin(\omega t+90^\circ)$; $p(t) = v(t)*i(t)$



$$P = \frac{1}{T}\int_0^T v(t)*i(t)dt = \frac{1}{T}\int_0^T V_m I_m \sin(\omega t)\sin(\omega t+90^\circ)dt$$

$$= 0$$

In a pure capacitor, during one quarter cycle of voltage, power is positive i.e., it flows from source to capacitor and gets stored in the electric field of the capacitor & during the next quarter cycle of voltage, power is negative i.e. stored energy in the capacitor flows back to the source.

Concept of Reactive Power:

In case of pure inductor and pure capacitor, average power is zero. In these elements, power circulates between the source and element but is not consumed. This type of AC power which is not consumed but circulates between the source and the element is called **Reactive Power.**

Accordingly, inductor and capacitor are called reactive elements.

Reactive Power is denoted by **Q** and is measured in **Volt-Amperes Reactive (VAR).**

Concept of Apparent Power:

The product of RMS value of voltage and RMS value of current of an element is called the **Apparent Power.**

Apparent Power is denoted by **S** and measured in **Volt-Amperes (VA).**

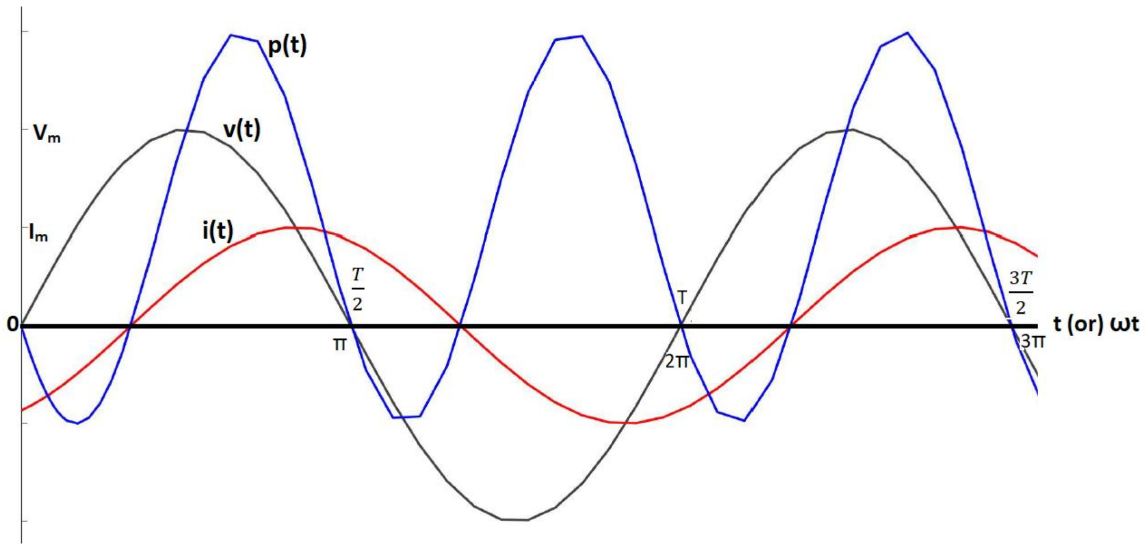
In case of a resistor, power absorbed is consumed. Hence, its reactive power is zero. Its power is completely in active form.

In case of a pure inductor or pure capacitor, power consumed is zero. Hence, its active power is zero. Its power is completely in reactive form i.e., it circulates between the source and the element.

Case iv) : General AC circuit:

Let us consider a general AC circuit, for instance series RL circuit.

For $v(t) = V_m \sin(\omega t)$, the current $i(t) = I_m \sin(\omega t+\phi)$ where ϕ is -ve.



In this case, average power is non-zero. This is because Resistor consumes power. Also, there is negative portion of power which is due to inductor. Thus, in this case there will be both active and reactive power.

$$P = \frac{1}{T}\int_0^T v(t)*i(t)dt = \frac{1}{T}\int_0^T V_m I_m \sin(\omega t)\sin(\omega t+\phi)dt$$

$$= \frac{V_m I_m}{2}\cos(\phi) = \frac{V_m}{\sqrt{2}} * \frac{I_m}{\sqrt{2}}\cos\phi = VI\cos\phi \quad W$$

Thus, in general in an AC circuit, active power is given by

$$P = VI\cos\phi \quad W$$

Where, V is the RMS voltage, I is the RMS current and ϕ is the angle between the voltage and current.

Similarly, reactive power in an AC circuit is

$$Q = VI\sin\phi \quad VAR$$

Thus, Apparent power is

$$S = \sqrt{P^2+Q^2} = VI \quad \text{Volt-Amperes}$$

Power Expressions for various cases:

Element/ Network	**Phase Angle (ϕ)**	**Active Power ($P = VI\cos\phi$)**	**Reactive Power ($Q = VI\sin\phi$)**	**Apparent Power ($S = VI$)**
R	0°	VI	0	VI
L	90°	0	VI	VI
C	-90°	0	-VI	VI
Series RL Circuit	$\tan^{-1}(\frac{X_L}{R})$	**$VI\cos\phi$**	**$VI\sin\phi$ (+ve)**	VI
Series RC Circuit	$-\tan^{-1}(\frac{X_C}{R})$	**$VI\cos\phi$**	**$VI\sin\phi$ (-ve)**	VI

Note: Conventionally, Inductive reactive power is positive and capacitive reactive power is negative.

Power factor in AC circuits

The ratio of Active Power to Apparent Power is termed as **Power factor** in AC systems.

$$\text{Power factor} = \frac{P}{S} = \cos\phi$$

Power factor is the cosine of phase angle of the network.

In case of series AC circuits, power factor can also be found as

$$\text{Power factor} = \frac{R_T}{|Z_T|}$$

Where, R_T is the total resistance in series and $|Z_T|$ is the magnitude of total impedance in series.

The power factor of a purely resistive circuit is **unity** since S = P for resistive circuits.

The power factor of a pure inductor is **Zero Lag** since P = 0 for a pure inductor. The word 'Lag' in the power factor indicates that current lags voltage.

The power factor of a pure Capacitor is **Zero Lead** since P = 0 for a pure capacitor.

The power factor of inductive circuits is between Zero and One and Lagging type & for capacitive circuits it is leading type and between Zero and One.

Ideally power factor must be as close to unity as possible.

Question 8:

A series RL circuit is connected to a sinusoidal voltage source v(t) = 100sin(ωt) V. It draws a current of 10sin(ωt - 60°) A.

Determine

i) Active, Reactive and Apparent Powers.

ii) Power factor of the circuit.

Solution:

$$V = \frac{V_m}{\sqrt{2}} = \frac{100}{\sqrt{2}}V$$

$$I = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}}A$$

Phase Angle, $\phi = \angle\overline{V} - \angle\bar{I} = 0° - (-60°) = 60°$

i)$P = VI\cos\phi = 250W$

$Q = VI\sin\phi = 433\ VAR$

$S = VI = 500\ VA$

ii) Power factor $= \cos\phi = \frac{P}{S} = 0.5Lag$