# ENGINEERING MATHEMATICS I

**T. R.Geetha**
Department of Science and Humanities

ENGINEERING MATHEMATICS I

# HIGHER ORDER DIFFERENTIAL EQUATIONS

**T R Geetha**

Department of Science and Humanities

## NON – HOMOGENEOUS LDE

### CLASS CONTENT:

- **To solve a non – homogeneous linear differential equation of the type f(D)y = x when x = , 'n' being a positive integer**

PES UNIVERSITY ONLINE

## RULES TO FIND PARTICULAR INTEGRAL

- Type (iii); When $X = x^n$ , 'n' being a positive integer.
- Here $PI = \frac{1}{f(D)}\left(x^n\right)$
- Take the lowest degree term common from f(D) to get an expression of the form $(1 \pm \varphi(D))$ in the denominator . Take this factor to the numerator to get $(1 \pm \varphi(D))^{-1}$.
- Expand $(1 \pm \varphi(D))^{-1}$ using Binomial theorem upto nth power as $(n+1)^{th}$ derivative of $x^n$ is zero.
- Operate on the numerator term by term by taking $D = \frac{d}{dx}$.

## NON – HOMOGENEOUS LDE

The following expressions will be useful to expand $(1 \pm \varphi(D))^{-1}$ in ascending powers of D.

- $(1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots\ldots\ldots\ldots$
- $(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots\ldots\ldots\ldots\ldots$
- $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots\ldots\ldots\ldots\ldots\ldots$
- $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots\ldots\ldots\ldots\ldots\ldots$

Find the general solution of $\frac{d^2y}{dx^2} - y = 5x - 2$

$$(D^2 - 1)y = 5x - 2$$

## To Find CF

**AE is** $m^2 - 1 = 0$

Roots are $m = \pm 1$

$$y_c = c_1 e^x + c_2 e^{-x}$$

## To Find PI

$$y_p = \frac{5x - 2}{D^2 - 1} = -\left(1 - D^2\right)^{-1}(5x - 2)$$

$$= -\left(1 - D^2 + D^4 - - - -\right)(5x - 2) = -(5x - 2)$$

$$\therefore y_p = 2 - 5x$$

And the GS is $y = y_c + y_p$

## NON – HOMOGENEOUS LDE

Find the particular integral of $(D^2 + 5D + 4)y = x^2 + 7x + 9$

$$PI = \frac{x^2 + 7x + 9}{D^2 + 5D + 4}$$

$$= \frac{x^2 + 7x + 9}{4\left(1 + \frac{1}{4}\left(D^2 + 5D\right)\right)}$$

$$= \frac{1}{4}\left(1 + \frac{1}{4}\left(D^2 + 5D\right)\right)^{-1}\left(x^2 + 7x + 9\right)$$

$$= \frac{1}{4}\left(1 - \frac{1}{4}\left(D^2 + 5D\right) + \left(\frac{1}{4}\left(D^2 + 5D\right)\right)^2\right)\left(x^2 + 7x + 9\right)$$

$$= \frac{1}{4}\left(1 - \frac{1}{4}\left(D^2 + 5D\right) + \left(\frac{1}{4}\left(D^2 + 5D\right)\right)^2\right)\left(x^2 + 7x + 9\right)$$

$$= \frac{1}{4}\left(1 - \frac{D^2}{4} - \frac{5D}{4} + \frac{25D^2}{16}\right)\left(x^2 + 7x + 9\right)$$

$$= \frac{1}{4}\left(x^2 + 7x + 9 - \frac{2}{4} - \frac{5(2x + 7)}{4} + \frac{25.2}{16}\right)$$

$$\therefore \text{PI} = \frac{1}{4}\left(x^2 + \frac{9}{2}x + \frac{23}{8}\right)$$

# THANK YOU

**T R Geetha**

Department of Science and Humanities