CL26_Q1. Explain the concept of Fermi factor and discuss its temperature dependence.

Answer

Under thermal equilibrium the free electrons occupy various energy levels in accordance with a statistical rule known as Fermi – Dirac statistics. Fermi – Dirac statistics enables the evaluation of probability of finding electron in energy levels over a certain range of energy values. The evaluation is done with the help of a quantity called Fermi factor F_d given by:

$$F_d = \frac{1}{\left(e^{\left(\frac{E-E_f}{k_B T}\right)} + 1\right)}$$

Effect of temperature on Fermi factor

- If $E<E_f$ then for $E-E_f$ is negative, then the Fermi factor $F_d = \frac{1}{\left(e^{-\left(\frac{\Delta E}{k_B T}\right)}+1\right)}$.

  At 0K this becomes $F_d = \frac{1}{(e^{-(\infty)}+1)} = 1$.

  This implies that at 0K all electron states below the Fermi level are filled states.

- If $E>E_f$ then for $E-E_f$ is positive, then the Fermi factor $F_d = \frac{1}{\left(e^{\left(\frac{\Delta E}{k_B T}\right)}+1\right)}$.

  At 0K this becomes $F_d = \frac{1}{(e^{(\infty)}+1)} = 0$.

  This implies that at 0K all electron states above the Fermi level are empty states.

- For T>0 and E=Ef the Fermi factor $F_d = \frac{1}{\left(e^{\left(\frac{E-E_f}{k_B T}\right)}+1\right)} = \frac{1}{e^0+1} = \frac{1}{2} = 0.5$.

  This gives a probability of occupation of 50% for the Fermi energy.

CL26_Q2. Show that the probability of occupancy of an energy level ΔE below the Fermi leve is the same as that of the probability of non-occupancy of an energy level ΔE above the Fermi level.

Answer

The Fermi distribution function is

$$F(E) = \frac{1}{1 + e^{\left(\frac{E-E_f}{k_B T}\right)}}$$

For $E-E_F = -\Delta E$, $F(E) = x$

$$\therefore \qquad x = \frac{1}{1 + e^{\left(\frac{-\Delta E}{k_B T}\right)}}$$

For $E-E_F = \Delta E$,

$$F(E + \Delta E) = \frac{1}{1 + e^{\left(\frac{\Delta E}{k_B T}\right)}}$$

F(E+ΔE) is the probability of occupancy of energy level ΔE above the Fermi level. Therefore the probability of non-occupancy of energy level ΔE above the Fermi level is

$$1 - F(E + \Delta E) = 1 - F(E + \Delta E) = \frac{1}{1 + e^{\left(\frac{\Delta E}{k_B T}\right)}}$$

$$= \frac{1 + e^{\left(\frac{\Delta E}{k_B T}\right)} - 1}{1 + e^{\left(\frac{\Delta E}{k_B T}\right)}}$$

$$= \frac{e^{\left(\frac{\Delta E}{k_B T}\right)}}{1 + e^{\left(\frac{\Delta E}{k_B T}\right)}}$$

$$= \frac{1}{1 + e^{\left(\frac{-\Delta E}{k_B T}\right)}}$$

(dividing numerator and denominator by $e^{\left(\frac{\Delta E}{k_B T}\right)}$)

$$1 - F(E + \Delta E) = x$$