CL29_Q1. Define effective number of electrons.

Answer

Only small fractions of electrons close to the Fermi energy level participate in the conduction, these electrons are called effective number of electrons.
This number can be estimated as (in one mole of the metal for a mono valent metal)

$$\mathbf{n}_{\mathbf{eff}} = \frac{\mathbf{N_a}}{\mathbf{E_f}}.\mathbf{k_B T}$$

CL29_Q2. The electrons near the Fermi level contribute to the specific heat of metals. Explain.

Answer

According to classical free electron theory, all the conduction electrons are capable of absorbing the heat energy and participated in the conduction process. Hence the theory predicts a large value for specific heat. But according quantum free electron theory, it is only small fractions of electrons that are occupying energy levels very close to the Fermi energy level which can absorb the heat energy and participated in the conduction process. This number can be estimated as the effective number of electrons given by

$$n_{\,eff} = \frac{N_a}{E_f}.k_B T.$$

Hence, if the average thermal energy of the electrons is taken to be $\frac{3}{2}\mathrm{k_B T}$ then the total energy of electrons in one mole of the material $\mathrm{U = n_{eff}.\ \frac{3}{2}k_B T = \frac{3}{2}k_B T\ \frac{N_a}{E_f}.k_B T = \frac{3}{2}.\frac{N_a}{E_f}.k_B{}^2 T^2}$

Hence the specific heat $\mathrm{C_{el} = \frac{dU}{dT} = 3.\frac{N_a}{E_f}.k_B{}^2 T = 3R.\frac{kT}{E_f}}$.

Thus the electronic specific heat is a fraction of the value predicted by the CFET (since $\mathrm{\frac{kT}{E_f}}$ is a fraction less than 1% for most metals) and is temperature dependent . This analysis gives the correct correlation with the experimental results.

CL 29_Q3. Explain the contribution of free electrons to the specific heat of metals on the basis of quantum free electron theory.

Answer

Only electrons close to the Fermi level (participate in the conduction process) contribute to the electronic specific heat. Hence the heat absorption happens due to that fraction of electrons. This number can be estimated as the effective number of electrons (in one mole of the metal for a mono valent metal) in the conduction process as $\mathrm{n_{\,eff} = \frac{N_a}{E_f}.k_B T}$.

Hence, if the average thermal energy of the electrons is taken to be $\frac{3}{2}\mathrm{k_B T}$ then the total energy of electrons in one mole of the material

$$\mathrm{U = n_{eff}.\ \frac{3}{2}k_B T = \frac{3}{2}k_B T\ \frac{N_a}{E_f}.k_B T = \frac{3}{2}.\frac{N_a}{E_f}.k_B{}^2 T^2}$$

Hence the specific heat $\mathrm{C_{el} = \frac{dU}{dT} = 3.\frac{N_a}{E_f}.k_B{}^2 T = 3R.\frac{kT}{E_f}}$.

Thus, the electronic specific heat is a fraction of the value predicted by the CFET (since $\frac{kT}{E_f}$ is a fraction less than 1% for most metals) and is temperature dependent. This analysis gives the correct correlation with the experimental results.