ENGINEERING MATHEMATICS - I

Unit - 3: Partial Differential Equations

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Contents

Find the solutions of the following partial differential equations

(i) $(D^4 - 2D^2D'^2 + D'^4)z = 0$,

(ii) $(4D^3 - 3DD'^2 + D'^3)z = 0$.

Solution

(i) We have

$$(D^4 - 2D^2D'^2 + D'^4)z = (D^2 - D'^2)^2 z = (D - D')^2(D + D')^2 z = 0.$$

For the factor $(D - D')^2$, we have $a_1 = 1$, $b_1 = -1$, $c_1 = 0$.
For the factor $(D + D')^2$, we have $a_2 = 1$, $b_2 = 1$, $c_2 = 0$.
If we set $m = D/D'$, we get $m^4 - 2m^2 + 1 = 0$. The roots are $m^2 = 1$, or $m = \pm 1$ which are double roots. Hence, the factors are $(D - D')^2$ and $(D + D')^2$.
Therefore, the general solution as

$$z = [x\phi_1^*(-x - y) + \phi_2^*(-x - y)] + [x\psi_1(x - y) + \psi_2(x - y)]$$

or

$$z = x\phi_1(x + y) + \phi_2(x + y) + x\psi_1(x - y) + \psi_2(x - y).$$

Problems (contd.)

(ii) We have

$$(4D^3 - 3DD'^2 + D'^3)z = (D + D')(4D^2 - 4DD' + D'^2)z$$

$$= (D + D')(2D - D')^2 z = 0.$$

For the factor $D + D'$, we have $a_1 = 1$, $b_1 = 1$, $c_1 = 0$.
For the factor $(2D - D')^2$, we have $a_2 = 2$, $b_2 = -1$, $c_2 = 0$.
The general solution as

$$z = \phi_1(x - y) + x\psi_1^*(-x - 2y) + \psi_2^*(-x - 2y)$$

or

$$z = \phi_1(x - y) + x\psi_1(x + 2y) + \psi_2(x + 2y).$$

3. Find the solutions of the following partial differential equation $(D^2 - DD' - 6D'^2)z = 0$

Solution: Let $m = \frac{D}{D'}$. Then the auxiliary equation becomes:

$$m^2 - m - 6 = 0.$$

Solving:

$$m = \frac{1 \pm \sqrt{1+24}}{2} = \frac{1 \pm 5}{2} \Rightarrow m = 3, -2.$$

- For $m = 3$: $a_1 = 1,\ b_1 = -3 \Rightarrow$ characteristic variable $-3x - y$,
- For $m = -2$: $a_2 = 1,\ b_2 = 2 \Rightarrow$ characteristic variable $2x - y$.

Hence, the general solution is:

$$z = \phi_1^*(-3x - y) + \phi_2(2x - y) = \phi_1(3x + y) + \phi_2(2x - y)$$

4. Find the solutions of the following partial differential equations

(i) $(D^2 - 2DD' + D'^2)^2 z = 0$,

(ii) $(D^3 - 3D^2D' + 3DD'^2 - D'^3)z = 0$.

Solution: (i) We observe that

$$(D^2 - 2DD' + D'^2)^2 z = (D - D')^4 z = 0.$$

So the repeated factor is $(D - D')$, four times.
This corresponds to:

$$a = 1, \quad b = -1, \quad \text{(repeated root)}.$$

Hence, the general solution is:

$$z = x^3\phi_1(x+y) + x^2\phi_2(x+y) + x\phi_3(x+y) + \phi_4(x+y)$$

Problems (contd.)

(ii) We notice that

$$(D^3 - 3D^2D' + 3DD'^2 - D'^3)z = (D - D')^3 z = 0.$$

So the repeated factor is $(D - D')$, three times.
Hence, the general solution is:

$$z = x^2\phi_1(x+y) + x\phi_2(x+y) + \phi_3(x+y)$$