ENGINEERING MATHEMATICS I

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ENGINEERING MATHEMATICS I

SPECIAL FUNCTIONS

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Class Content

- Gamma function - definition
- Properties of Gamma function
- Graph of Gamma function

Gamma function

- Gamma function is a definite integral whose integrand depends on one variable.

- It is also known as Euler's integral of second kind.

- It is the generalization of factorial notation from integer values to real numbers.

Definition

Gamma function is defined as

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx \; where \; n > 0 ------(1)$$

It defines a function of n for positive values of n

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Note:

Put n=1, in (1) then, $\Gamma(1) = \int_0^\infty e^{-x} x^{1-1} dx$

$$= \int_0^\infty e^{-x} dx$$

$$= \left[\frac{e^{-x}}{-1} \right]_0^\infty$$

$$e^{-\infty} = 0$$

$$e^{0} = 1$$

$$= 1$$

$$\boxed{\therefore \ \Gamma(1) = 1}$$

Alternate form

By definition, $\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx \; where \; n > 0$

Put $x = t^2, \; dx = 2tdt$

Then, $\Gamma(n) = \int_0^\infty e^{-t^2} \left(t^2\right)^{n-1} 2tdt$

$$= 2\int_0^\infty e^{-t^2} t^{2n-1} \, dt$$

Therefore another form of gamma function is,

$$\boxed{\Gamma(n) = 2\int_0^\infty e^{-t^2} \, t^{2n-1} dt}$$

Reduction formula

$\Gamma(n+1) = n\Gamma(n)$, where n is a real number.

Proof: By definition, $\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx$

$$\int uv dx = u \int v dx - \int \left(\frac{du}{dx} \int v dx \right) dx$$

$$\Rightarrow \Gamma(n+1) = \int_0^\infty e^{-x} x^{(n+1)-1} dx$$

$$= \int_0^\infty e^{-x} x^n dx$$

$$= \left(\frac{x^n e^{-x}}{-1} \right)_0^\infty + n \int_0^\infty e^{-x} x^{n-1} dx$$

$$= 0 + n\Gamma(n)$$

$$= n\Gamma(n)$$

$$\boxed{\therefore \Gamma(n+1) = n\Gamma(n)}$$

Reduction formula

When n is a negative non integer, the formula used to find $\Gamma(n)$ is,

$$\boxed{\Gamma(n) = \frac{\Gamma(n+1)}{n}}$$

This formula is used to compute $\Gamma(n)$, when n is a negative non integer.

value of $\Gamma(n)$ in terms of factorial

$\Gamma(n+1) = n!$ where n is a positive integer.

Proof: We know that, $\Gamma(n+1) = n\Gamma(n) ------(1)$

Therefore, $\Gamma(2) = 1\times\Gamma(1) = 1$

$\Gamma(3) = 2\times\Gamma(2) = 2 = 2!$

$\Gamma(4) = 3\times\Gamma(3) = 3\times 2! = 3!$

$\Gamma(5) = 4\times\Gamma(4) = 4\times 3! = 4!$

In general,

$$\boxed{\Gamma(n+1) = n!}$$

$\Gamma(n)$ is not defined when n is zero or a negative integer.

Value of $\Gamma\left(\frac{1}{2}\right)$

$$\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$$

Proof: We have, $\Gamma(n)=2\int_0^\infty e^{-x^2}\ x^{2n-1}dx$

$$n=\frac{1}{2}\Rightarrow\ \Gamma\left(\frac{1}{2}\right)=2\int_0^\infty e^{-x^2}dx$$

$$=2\int_0^\infty e^{-y^2}dy$$

$$\therefore\left(\Gamma\left(\frac{1}{2}\right)\right)^2=2\int_0^\infty e^{-x^2}dx\times 2\int_0^\infty e^{-y^2}dy$$

$$=4\int_0^\infty\int_0^\infty e^{-\left(x^2+y^2\right)}dxdy$$

Value of $\Gamma\left(\frac{1}{2}\right)$

put $x = r\cos\theta$ *and* $y = r\sin\theta$

then, $x^2 + y^2 = r^2$ and $dxdy = rdrd\theta$

$$i.e.\ \left(\Gamma\left(\frac{1}{2}\right)\right)^2 = 4\int_0^{\frac{\pi}{2}}\int_0^{\infty} e^{-r^2}\, rdrd\theta$$

$$= 4\int_0^{\frac{\pi}{2}} d\theta \int_0^{\infty} e^{-r^2}\, rdr$$

Value of $\Gamma\left(\frac{1}{2}\right)$

$$= 4.\frac{\pi}{2}\int_0^\infty e^{-r^2} r\,dr$$

$$r^2 = t$$

$$2r\,dr = dt$$

$$= 2\pi\left[\left(-\frac{1}{2}\right)e^{-r^2}\right]_0^\infty$$

$$= \pi$$

Thus $\boxed{\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}}$

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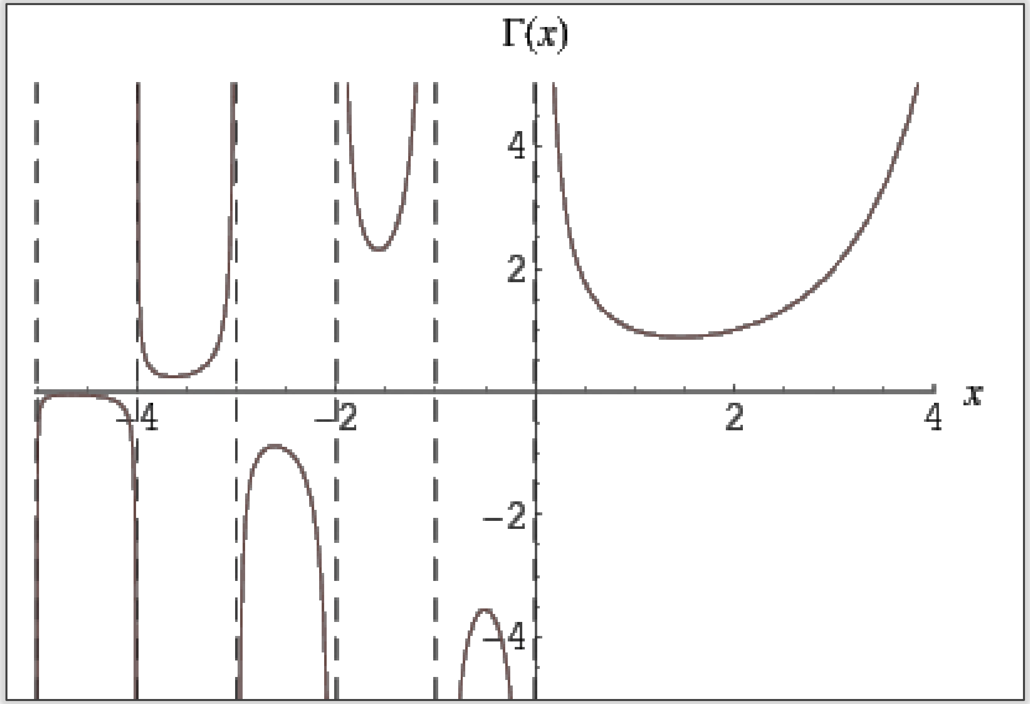
Table for Gamma Function

n	$\Gamma(n)$	n	$\Gamma(n)$	n	$\Gamma(n)$	n	$\Gamma(n)$
1.00	1.00000	1.25	0.90640	1.50	0.88623	1.75	0.91906
1.01	0.99433	1.26	0.90440	1.51	0.88659	1.76	0.92137
1.02	0.98884	1.27	0.90250	1.52	0.88704	1.77	0.92376
1.03	0.98355	1.28	0.90072	1.53	0.88757	1.78	0.92623
1.04	0.97844	1.29	0.89904	1.54	0.88818	1.79	0.92877
1.05	0.97350	1.30	0.89747	1.55	0.88887	1.80	0.93138
1.06	0.96874	1.31	0.89600	1.56	0.88964	1.81	0.93408
1.07	0.96415	1.32	0.89464	1.57	0.89049	1.82	0.93685
1.08	0.95973	1.33	0.89338	1.58	0.89142	1.83	0.93969
1.09	0.95546	1.34	0.89222	1.59	0.89243	1.84	0.94261
1.10	0.95135	1.35	0.89115	1.60	0.89352	1.85	0.94561
1.11	0.94739	1.36	0.89018	1.61	0.89468	1.86	0.94869
1.12	0.94359	1.37	0.88931	1.62	0.89592	1.87	0.95184
1.13	0.93993	1.38	0.88854	1.63	0.89724	1.88	0.95507
1.14	0.93642	1.39	0.88785	1.64	0.89864	1.89	0.95838
1.15	0.93304	1.40	0.88726	1.65	0.90012	1.90	0.96177
1.16	0.92980	1.41	0.88676	1.66	0.90167	1.91	0.96523
1.17	0.92670	1.42	0.88636	1.67	0.90330	1.92	0.96878
1.18	0.92373	1.43	0.88604	1.68	0.90500	1.93	0.97240
1.19	0.92088	1.44	0.88580	1.69	0.90678	1.94	0.97610
1.20	0.91817	1.45	0.88565	1.70	0.90864	1.95	0.97988
1.21	0.91558	1.46	0.88560	1.71	0.91057	1.96	0.98374
1.22	0.91311	1.47	0.88563	1.72	0.91258	1.97	0.98768
1.23	0.91075	1.48	0.88575	1.73	0.91466	1.98	0.99171
1.24	0.90852	1.49	0.88595	1.74	0.91683	1.99	0.99581
						2.00	1.00000

Graph Of Gamma Function



Graph of Gamma Function

$$\Gamma(n+1) = n\Gamma(n)$$

$$\Gamma(n) = \frac{\Gamma(n+1)}{n}$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

THANK YOU

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