ENGINEERING MATHEMATICS - I

Unit - 3: Higher Order Differential Equation

Department of Science and Humanities

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Contents

Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 sin(logx)$

Solution:

$[D(D-1)y - 3Dy + 5y] = e^{2t} \cdot sint$

Substitute

put $x = e^t$ OR $t = \log x$.

Then

$$\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) - 3\frac{dy}{dt} + 5y = e^{2t}sint$$

$$\Downarrow$$

$[D^2 - 4D + 5]y = e^{2t} \cdot sint$

$$\implies \frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 5y = e^{2t}sint$$

C.F. : Auxillary Equation is given by

$$m^2 - 4m + 5 = 0 \Rightarrow m = 2 \pm i \begin{bmatrix} \alpha = 2 \\ \beta = 1 \end{bmatrix}$$

Complimentary Function is given by

$$y_c(t) = e^{2t}(c_1cost + c_2sint) \rightarrow \log x$$

Particular Integral is given by

$P.I = \frac{1}{f(D)}X$

$$y_p = \frac{1}{D^2 - 4D + 5}e^{2t}.sint$$

4.

Replacing D by $D + 2$

$e^{2t} \cdot \frac{1}{(D+2)^2 - 4(D+2)+5} \cdot sint$

$$y_p = \frac{e^{2t}}{D^2 + 1}sint$$

2.

$D^2 = -a^2 = -1$

$$\implies y_p = e^{2t}\left(-\frac{tcost}{2}\right)$$

$P.I = t.\frac{e^{2t}}{2D}.sint$

$= \frac{t.e^{2t}}{2}\int sint.dt$

$-cost$

The general solution is

$$y = e^{2t}\underbrace{(c_1cost + c_2sint)}_{C.F} + e^{2t}\underbrace{\left(-\frac{tcost}{2}\right)}_{P.I}$$

Replacing t by lnx

$$y(x) = x^2(c_1cos(lnx) + c_2sin(lnx)) - logx\frac{x^2}{2}cos(lnx)$$ #

Solve $x\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = \frac{1}{x}$

Given D.E. is, $x\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = \frac{1}{x}$
Multiplying by x^2 we get,

$$\boxed{x^3\frac{d^3y}{dx^3} + x^2\frac{d^2y}{dx^2} = x} \tag{1}$$

Put $t = lnx$ or $e^t = x$. Then

$$x\frac{dy}{dx} = Dy, \quad x^2\frac{d^2y}{dx^2} = D(D-1)y, \quad x^3\frac{d^3y}{dx^3} = D(D-1)(D-2)y$$

where $D = \frac{d}{dt}$.
Equation (1) becomes

$$\Big[D(D-1)(D-2) + \cancel{D}D(D-1)\Big]y = e^t$$

Contd.

The A.E. is given by

$$m(m-1)(m-2) + m(m-1) = 0$$
$$m(m-1)^2 = 0$$
$$\implies \boxed{m = 0, 1, 1}$$

Therefore C.F. is given by

$$y_c = c_1 + (c_2 + c_3 t)e^t$$

Particular Integral is given by

$$y_p = \frac{e^t}{D^3 - 2D^2 + D} \quad \rightarrow 1 \qquad D = 1.$$
$$= t\frac{e^t}{3D^2 - 4D + 1} \qquad (denominator = 0)$$
$$= t^2 \frac{e^t}{6D - 4} = \frac{t^2 e^t}{2} \quad \#$$

The general solution is

$$y = c_1 + (c_2 + c_3 t)e^t + \frac{t^2 e^t}{2}$$

Replacing $t = lnx$, we get

$$y = c_1 + (c_2 + c_3 \ln x)x + \frac{x(\ln x)^2}{2}$$

Solution: This is a Legendre's Linear equation.

Put $1+x = e^t$, i.e., $t = \log(1+x)$ so that

$$(1+x)\frac{dy}{dx} = Dy, \quad (1+x)^2\frac{d^2y}{dx^2} = D(D-1)y$$

where $D = \frac{d}{dt}$.

Then the Legendre equation becomes

$$(D^2+1)y = 2\sin t$$

Its auxiliary equation is

$c.F = c_1 \cos t + c_2 \sin t.$

$$m^2 + 1 = 0 \quad \Rightarrow \quad m = \pm i$$

$D^2 = -1.$

$$y_p = 2 \cdot \frac{1}{D^2+1}\sin t = \cancel{2}t \cdot \frac{1}{\cancel{2}D}\sin t \quad = -t \cdot \cos t$$

The solution is

$$y = c_1 \cos t + c_2 \sin t - t \cos t$$

On replacing t by $\log(1+x)$, we get

$$y(x) = c_1 \cos[\log(1+x)] + c_2 \sin[\log(1+x)] - \log(1+x)\cos[\log(1+x)]$$

Solve $(2x-1)^2\frac{d^2y}{dx^2}+(2x-1)\frac{dy}{dx}-2y=8x^2-2x+3$

Solution: This is a Legendre's Linear equation.

Put $2x-1=e^t$, i.e., $t=\log(2x-1)$ so that

$x = \frac{e^t+1}{2}$

$$(2x-1)\frac{dy}{dx}=2Dy, \quad (2x-1)^2\frac{d^2y}{dx^2}=4D(D-1)y$$

where $D=\frac{d}{dt}$.

Then the Legendre equation becomes

$$\boxed{2D^2y-Dy-y=e^{2t}+\frac{3}{2}e^t+2} \quad 1$$

Its A.E. is

$$2m^2-m-1=0 \implies m=1,-\frac{1}{2}$$

$$C.F.=c_1e^t+c_2e^{-\frac{t}{2}}$$

Particular Integral is given by

$$P.I. = \frac{1}{2D^2 - D - 1}\left(e^{2t} + \frac{3}{2}e^t + 2\right)$$

$$= \frac{1}{5}e^{2t} + \frac{3t}{2}\frac{1}{4-1}e^t - 2 = \frac{1}{5}e^{2t} + \frac{t}{2}e^t - 2$$

Hrnce the solution is

$$y = c_1e^t + c_2e^{-\frac{t}{2}} + \frac{1}{5}e^{2t} + \frac{t}{2}e^t - 2$$

On replacing t by $(2x-1)$

$$y = c_1(2x-1) + c_2(2x-1)^{-\frac{1}{2}} + \frac{1}{5}(2x-1)^2 + \frac{1}{2}(2x-1)\log(2x-1) - 2$$