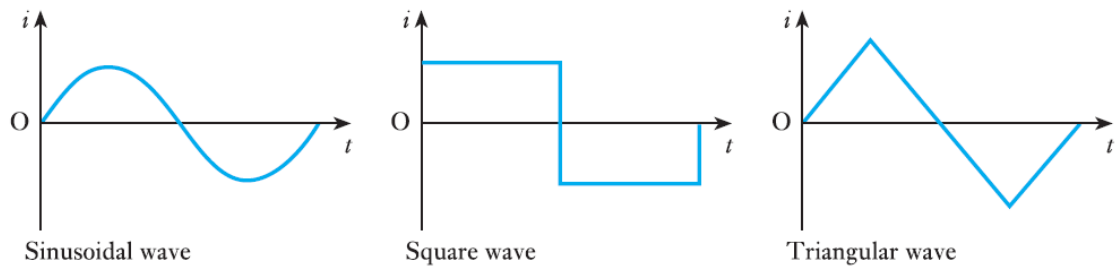
NOTES -Class 20

Introduction:

Due to the invention of transformer, AC systems have gained popularity over DC Systems for Power Generation, Transmission and Distribution.

AC Stands for 'Alternating Current'. An AC waveform is a periodic waveform which alternates i.e., which has alternately positive and negative portions in the waveform.

For instance, the following waveforms are examples of AC waveforms.



Some Basic Definitions:

1) Periodic waveform:

A periodic waveform is one which repeats itself after certain time interval.

2) Time Period(T):

The time taken to complete one cycle of a periodic waveform is termed as its time period. It is measured in Seconds.

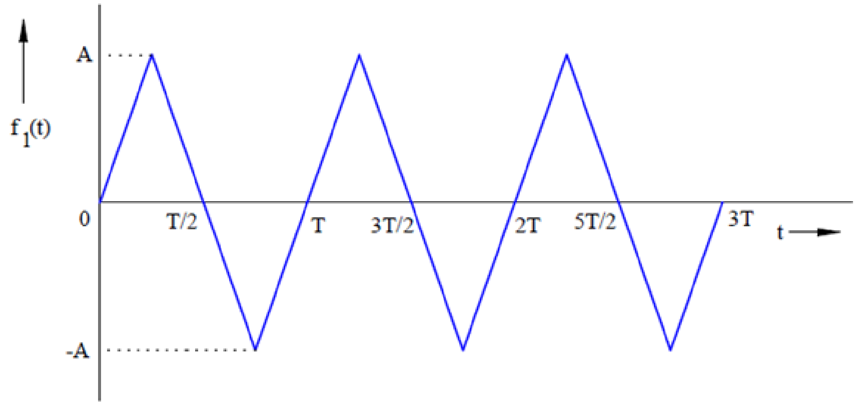
3) **Frequency(f):**

The number of cycles completed in one second of a periodic waveform is termed as its frequency. It is measured in Hz (or) cycles/sec.

Concept of Pure AC waveform:

A pure AC waveform is one whose average value is zero i.e., each positive area is equally matched by a corresponding negative area over one Time period.

For instance, consider the following waveform $f_1(t)$:



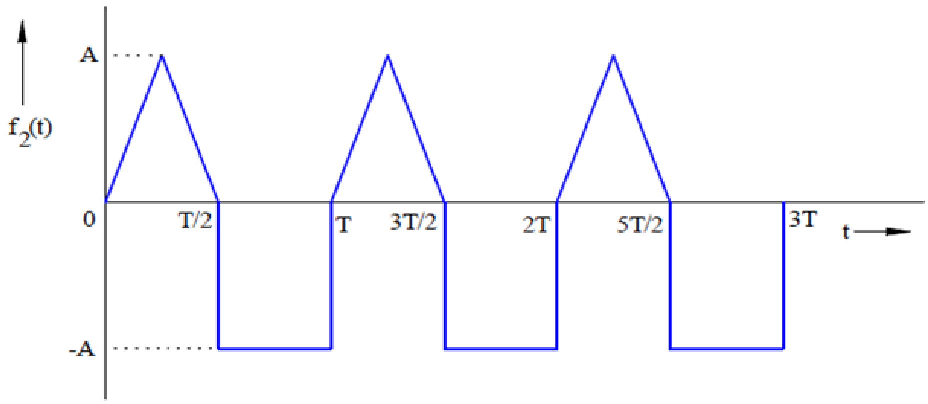
Positive Area = (1/2)*(T/2)*A = AT/4

Negative Area = (1/2)*(T/2)*(-A) = -AT/4

Net Area over one Time Period = 0

Hence, Average value is zero. Therefore, it is a Pure AC waveform.

Consider another waveform $f_2(t)$ as shown below:



Positive Area = (1/2)*(T/2)*A = AT/4

Negative Area = (T/2)*(-A) = -AT/2

Net Area over one Time Period = -AT/4

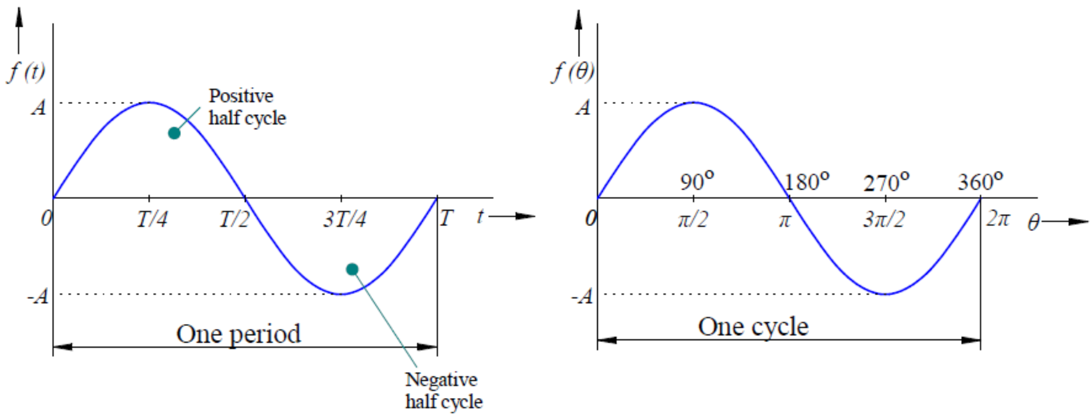
Hence, Average value is non-zero. Therefore, it is not a Pure AC waveform.

Sinusoidal waveform:

Since power generation, transmission and distribution happens as sinusoidal AC power, our discussion in this chapter confines to Sinusoidal waveform.

A sinusoidal waveform for one complete cycle is shown below:

It can be expressed as a function of time in seconds (or) angle in radians.



Accordingly, one cycle completes in T seconds (or) 2π radians.

The following table gives relation between time in seconds and angle in radians.

Time (sec)	Angle θ (Rad)
T	2π
T/2	Π
1	(2π/T)
t	(2π/T)*t

From the above table it can be concluded that at a general angle 't' seconds, the corresponding angle θ is (2π/T)*t radians.

A sinusoidal function is usually expressed as a function of angle as

$$e(\theta) = E_m\sin(\theta)$$

$$= E_m\sin((2\pi/T)*t) = E_m\sin(\omega t) = e(t)$$

where, $\omega = 2\pi/T = 2\pi f$ is called the angular frequency of the sine wave in rad/s.

In general, the standard representation of a sinusoidal function is $E_m\sin(\omega t+\phi)$ where ϕ is called the phase angle which can be either positive or negative.

Numerical Example 1

Question:

For a Sinusoidal function of frequency 50 Hz, find

i) Half time period

ii) Angular frequency

Solution:

Time period, T = 1/f = 1/50 = 0.02s = 20 ms

i) Half time period T/2 = 20/2=10 ms

ii) Angular frequency (ω)

$\omega = 2\pi f = 2\pi(50) = 100\,\pi = 314.159$ rad/sec

Numerical Example 2

Question:

The maximum value of a sinusoidal alternating current of frequency 50Hz is 25 A. Write the equation for the instantaneous expression of current,. Determine its value at 3ms and 14 ms.

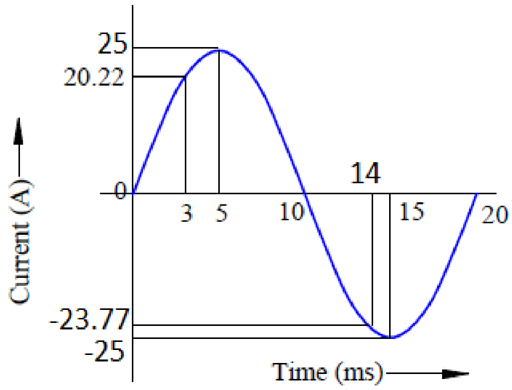
Solution:

Angular frequency, $\omega = 2\pi f = 100\pi$rad/s

$i(t) = 25\sin(100\pi t)$ A

$i(3ms) = 25\sin(100*\pi*0.003) = 20.22A$

Similarly, $i(14ms) = -23.77A$



Note: If radian scale is selected then substitute 'π' symbol in above equation. If degree scale is selected then don't use 'π' symbol, but substitute 180 in place of 'π'.

Average value of a Sinusoidal Function:

In general, the average value of an AC waveform f(t) is given by

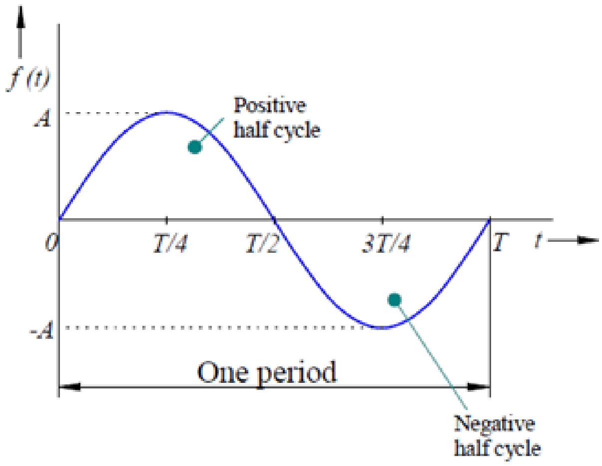
$$F_{avg} = \frac{1}{T}\int_0^T f(t)dt$$

The average value of a sinusoidal function f(t) = Asin(ωt) is

$$F_{avg} = \frac{1}{T}\int_0^T A\sin(\omega t)dt$$

$$= \frac{A}{T}\left(\frac{-\cos(\omega t)}{\omega}\right)_0^T = 0$$

Also, we can conclude the same result from the waveform as well.



Over one Time period, Net Area = 0. Hence, $F_{avg} = 0$

Effective (or) Root Mean Square (RMS) Value of an AC function:

Consider an AC Voltage v(t) connected across a resistor R for 'T' seconds.

Energy consumed by the resistor during this period is

$$E_{AC} = \int_0^T p(t)dt$$

$$= \int_0^T \frac{[v(t)]^2}{R}dt$$

Now, excite this resistor using a DC Voltage source of voltage 'V' for same time 'T' seconds.

Energy consumed by the resistor in this case is

$$E_{DC} = \frac{V^2}{R}.T$$

That value of DC voltage 'V' for which $E_{AC} = E_{DC}$ is said to be the Effective value of the AC voltage v(t).

Hence,

$$\int_0^T \frac{[v(t)]^2}{R} dt = \frac{V^2}{R}.T$$

Therefore, Effective value

$$V = \sqrt{\frac{1}{T}\int_0^T [v(t)]^2 dt}$$

Mathematically the operations involved are

i) Square of the function

ii) Mean (Average) of the function

iii) Square root of the function

Hence, it is also called Root Mean Square (RMS) value.

Effective (or) Root Mean Square (RMS) Value of Sine Wave:

Consider a sinusoidal voltage

$$v(t)=V_m \sin(\omega t)$$

$$\sqrt{\frac{1}{T}\int_0^T [V_m \sin\omega t]^2 dt}$$

Its RMS value, V =

$$= \sqrt{\frac{V_m^{\ 2}}{T}\int_0^T [\sin^2 \omega t]dt}$$

$$= \sqrt{\frac{V_m^{\ 2}}{T} * \frac{T}{2}}$$

$$= \frac{V_m}{\sqrt{2}}$$

Major advantage of finding effective (or) RMS value of an AC function is that it makes power calculations easy.

Power consumed in AC circuits, p(t) = v(t)*i(t)

Average power consumed, P = $\dfrac{\int_0^T p(t)dt}{T} = \dfrac{\int_0^T \frac{[v(t)]^2}{R}}{T} = \dfrac{\int_0^T \frac{[v(t)]^2}{T}}{R} = \dfrac{V^2}{R}$

Where V = RMS value of voltage.

Similarly, average power consumed is also equal to I^2*R where, I = RMS current.

Also, Energy consumed in 't' seconds = P*t

i.e.,

$(I^2R)t$ (or) $\dfrac{V^2}{R}t$