ENGINEERING MATHEMATICS - I

Unit - 3: Partial Differential Equations

Department of Science and Humanities

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Contents

Definition: Differential Equation

A **Differential Equation** is an equation that involves:

- an **independent variable**,
- a **dependent variable**, and
- **derivatives of the dependent variable with respect to the independent variable**.

Example (ODE):

$$\frac{dy}{dx} + y = e^x$$

- x: independent variable
- y: dependent variable
- $\frac{dy}{dx}$: derivative of y with respect to x

Classification of Differential Equations

- Differential Equations
  - Ordinary Differential Equations (ODEs)
  - Partial Differential Equations (PDEs)

ODEs vs PDEs

Ordinary Differential Equations (ODEs)

- Having Derivatives with respect to only **one independent variable**
- The unknown function depends on a **single variable**

Example:

$$\frac{dy}{dx} + y = e^x$$

Partial Differential Equations (PDEs)

- Involves **partial derivatives** with respect to **two or more independent variables**
- The unknown function depends on **multiple variables**

Example:

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$

Partial Differential Equations

- A partial differential equation (PDE) is an equation that involves partial derivatives of a function with several independent variables
- In contrast to ordinary differential equations (ODEs), which have one independent variable, PDEs involve two or more
Examples:

$$\text{One-dimensional heat conduction equation:} \quad u_t = u_{xx}$$

$$\text{Laplace equation:} \quad u_{xx} + u_{yy} = 0$$

$$\text{One-dimensional wave equation:} \quad u_{tt} = u_{xx}$$

Why Use PDEs?

Many physical and social problems have more than two independent variables. Because of this, partial differential equations (PDEs) are the best tool to study these problems. PDEs help us understand many real-world phenomena, including:

- Waves, like those in vibrating strings
- Heat flow
- Fluid movement
- Electromagnetic fields

Order and Degree of Partial Differential Equations

- **Order:** The order of a partial differential equation is the order of the highest order partial derivative occurring in the equation
- **Degree:** The degree of a partial differential equation is the degree of the highest order partial derivative occurring in the equation after the equation has been made free of radicals and fractions with respect to the partial derivatives

  Example: $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z \quad (\text{Order} = 1, \text{Degree} = 1)$

  $a^2\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right) = \frac{\partial^2 z}{\partial t^2} \quad (\text{Order} = 2, \text{Degree} = 1)$

  $\left(\frac{\partial z}{\partial x}\right)^3 + \frac{\partial^2 z}{\partial y^2} = \cos(x+y) \quad (\text{Order} = 2, \text{Degree} = 1)$

  $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0 \quad (\text{Order} = 2, \text{Degree} = 1)$

Standard Notations

Let z be a dependent variable and x, y be independent variables, that is, $z = z(x, y)$. Then,

- $p = \partial z/\partial x = z_x$
- $q = \partial z/\partial y = z_y$
- $r = \partial^2 z/\partial x^2 = z_{xx}$
- $s = \partial^2 z/\partial x \partial y = z_{xy}$
- $t = \partial^2 z/\partial y^2 = z_{yy}$

- An equation containing x, y, z, p, q defines a first order partial differential equation, that is

$$f(x, y, z, p, q) = 0 \tag{1}$$

This equation is linear, if it is linear in p, q

- An equation containing x, y, z, p, q, r, s, t defines a second order partial differential equation, that is

$$g(x, y, z, p, q, r, s, t) = 0 \tag{2}$$

This equation is linear, if it is linear in p, q, r, s, t