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# **ENGINEERING MECHANICS**

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# ENGINEERING MECHANICS

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## Moment of Inertia

**P. Ramchandra**

Department of Civil Engineering

- **Area moment of inertia**
- **Rectangular moment of inertia/ second moment of inertia**
- **Polar moment of inertia**
- **Radius of Gyration(Rectangular radius of gyration and polar radius of gyration)**
- **Parallel axis Theorem**
- **Derivation of moment of inertia for different geometric figures**

- First area moment of inertia is either positive or negative or zero
- Second area moment of inertia is the Moment of first area moment of inertia
- but Second area moment of inertia is always positive about the axis
- Second moment of inertia is denoted by capital 'I'
- Unit of second area moment of inertia is mm<sup>4</sup>

### Rectangular and Polar Moments of Inertia

Consider the area  $A$  in the  $x$ - $y$  plane, Fig. A/2. The moments of inertia of the element  $dA$  about the  $x$ - and  $y$ -axes are, by definition,  $dI_x = y^2 dA$  and  $dI_y = x^2 dA$ , respectively. The moments of inertia of  $A$  about the same axes are therefore

$$I_x = \int y^2 dA$$

$$I_y = \int x^2 dA$$

(A/1)

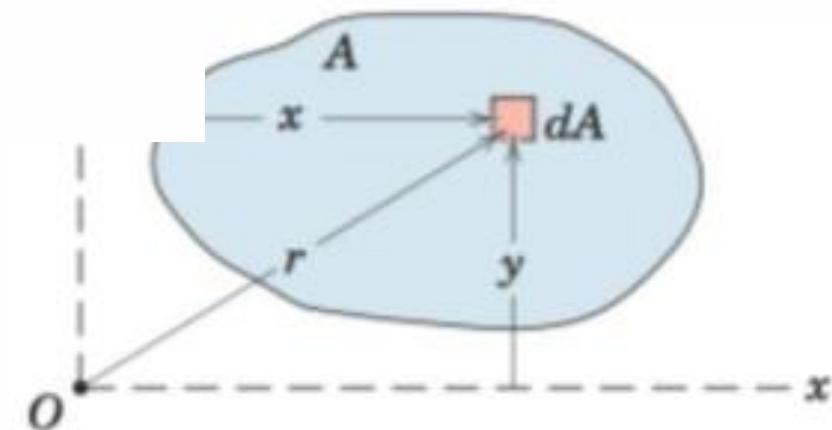


Figure A/2

The moment of inertia of  $dA$  about the pole  $O$  ( $z$ -axis) is, by similar definition,  $dI_z = r^2 dA$ . The moment of inertia of the entire area about  $O$  is

$$I_z = \int r^2 dA \quad (\text{A/2})$$

The expressions defined by Eqs. A/1 are called *rectangular* moments of inertia, whereas the expression of Eq. A/2 is called the *polar* moment of inertia.\* Because  $x^2 + y^2 = r^2$ , it is clear that

$$I_z = I_x + I_y \quad (\text{A/3})$$

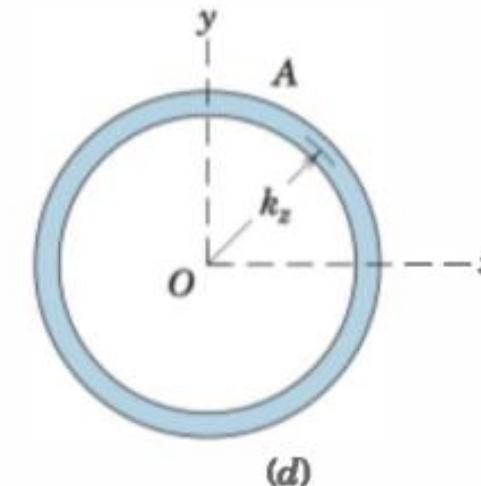
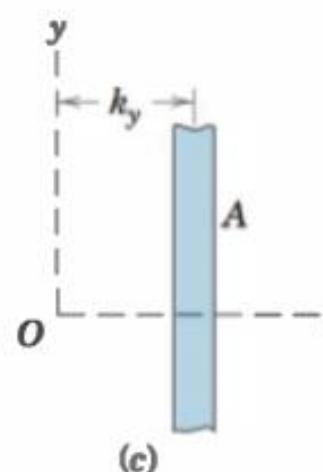
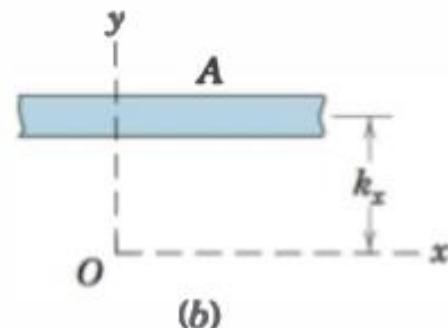
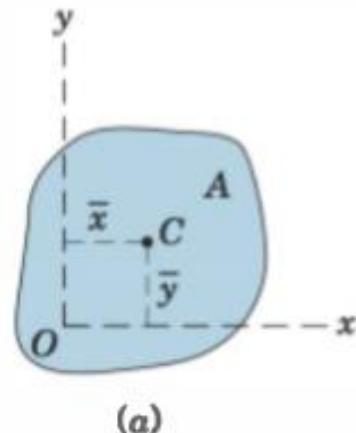


Figure A/3

$$I_x = k_x^2 A$$

$$I_y = k_y^2 A$$

$$I_z = k_z^2 A$$

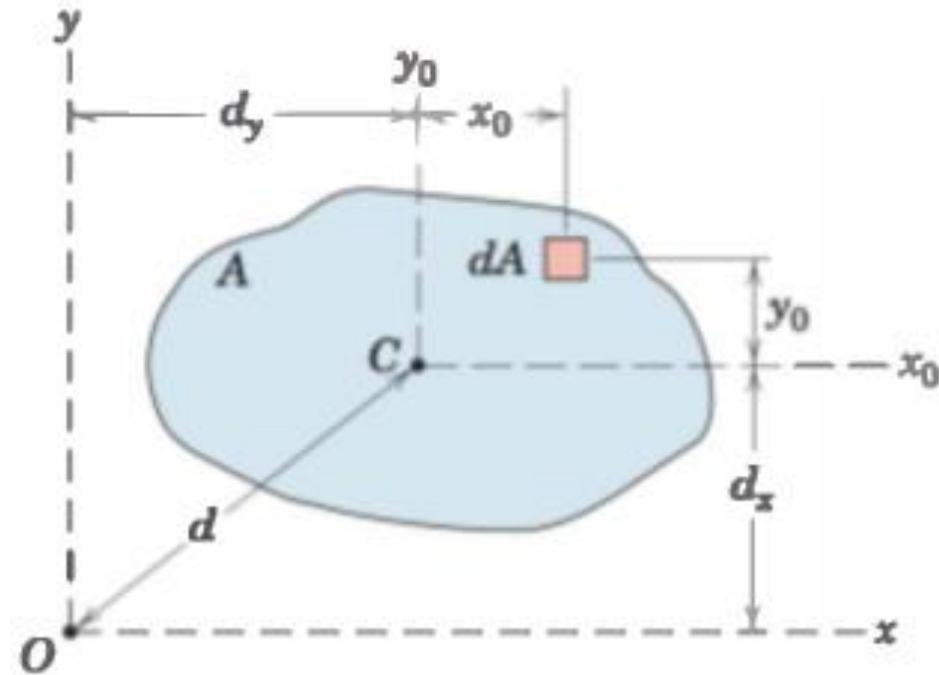
or

$$k_x = \sqrt{I_x/A}$$

$$k_y = \sqrt{I_y/A}$$

$$k_z = \sqrt{I_z/A}$$

$$k_x^2 = k_x^2 + k_y^2$$



- The moment of inertia of an area about a non centroidal axis may be easily expressed in terms of the moment of inertia about a parallel centroidal axis.

- By definition, the moment of inertia of the element  $dA$  about the x-axis is given by

$$dI_x = (y_0 + d_x)^2 dA$$

Expanding and integrating give us

$$I_x = \int y_0^2 dA + 2d_x \int y_0 dA + d_x^2 \int dA$$

- We see that the first integral is by definition the moment of inertia  $\bar{I}_x$  about the centroidal  $x_0$ -axis. The second integral is zero, since  $\int y_0 dA = A\bar{y}_0$  and  $\bar{y}_0$  is automatically zero with the centroid on the  $x_0$ -axis. The third term is simply  $Ad_x^2$ . Thus, the expression for  $I_x$  and the similar expression for  $I_y$  become

$$\begin{aligned} I_x &= \bar{I}_x + Ad_x^2 \\ I_y &= \bar{I}_y + Ad_y^2 \end{aligned} \tag{A/6}$$

By Eq. A/3 the sum of these two equations gives

$$I_z = \bar{I}_z + Ad^2 \tag{A/6a}$$

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- **Conditions for parallel axis theorem**
  1. **two axis should be there and two axis must be parallel to each other**
  2. **Between two axis, one axis has to pass through the centroidal axis**

$$[I_x = \int y^2 dA]$$

$$\bar{I}_x = \int_{-h/2}^{h/2} y^2 b dy = \frac{1}{12}bh^3$$

*Ans.*

By interchange of symbols, the moment of inertia about the centroidal  $y_0$ -axis is

$$\bar{I}_{y_0} = \frac{1}{12}hb^3$$

*Ans.*

The centroidal polar moment of inertia is

$$\bar{I}_z = \bar{I}_x + \bar{I}_{y_0}$$

$$\bar{I}_z = \frac{1}{12}(bh^3 + hb^3) = \frac{1}{12}A(b^2 + h^2)$$

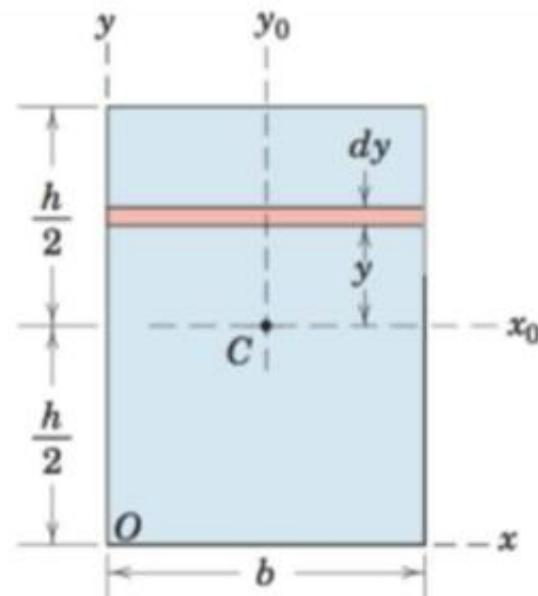
*Ans.*

By the parallel-axis theorem, the moment of inertia about the  $x$ -axis is

$$[I_x = \bar{I}_x + Ad_x^2]$$

$$I_x = \frac{1}{12}bh^3 + bh\left(\frac{h}{2}\right)^2 = \frac{1}{3}bh^3 = \frac{1}{3}Ah^2$$

*Ans.*



**Solution.** A strip of area parallel to the base is selected as shown in the figure, and it has the area  $dA = x \, dy = [(h - y)b/h] \, dy$ . By definition

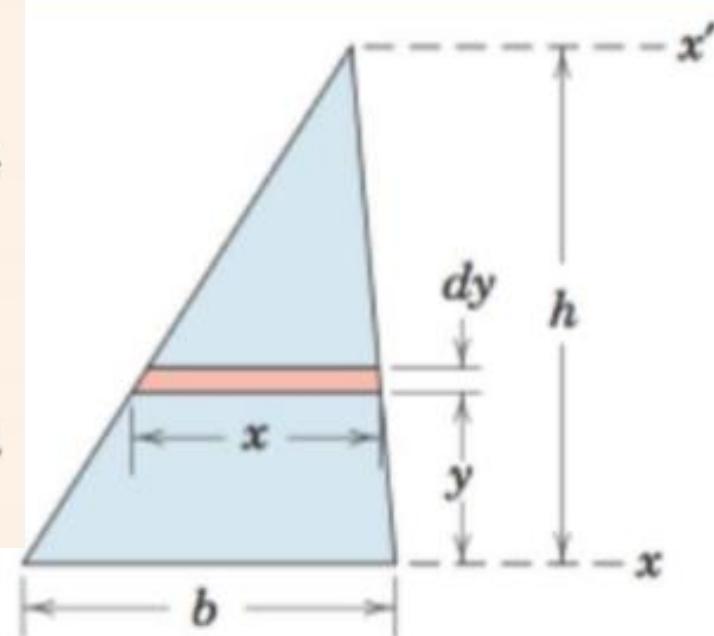
$$[I_x = \int y^2 \, dA] \quad I_x = \int_0^h y^2 \frac{h-y}{h} b \, dy = b \left[ \frac{y^3}{3} - \frac{y^4}{4h} \right]_0^h = \frac{bh^3}{12} \quad \text{Ans.}$$

By the parallel-axis theorem, the moment of inertia  $\bar{I}$  about an axis through the centroid, a distance  $h/3$  above the  $x$ -axis, is

$$[\bar{I} = I - Ad^2] \quad \bar{I} = \frac{bh^3}{12} - \left( \frac{bh}{2} \right) \left( \frac{h}{3} \right)^2 = \frac{bh^3}{36} \quad \text{Ans.}$$

A transfer from the centroidal axis to the  $x'$ -axis through the vertex gives

$$[I = \bar{I} + Ad^2] \quad I_{x'} = \frac{bh^3}{36} + \left( \frac{bh}{2} \right) \left( \frac{2h}{3} \right)^2 = \frac{bh^3}{4} \quad \text{Ans.}$$



**Solution.** A differential element of area in the form of a circular ring may be used for the calculation of the moment of inertia about the polar  $z$ -axis through  $O$  since all elements of the ring are equidistant from  $O$ . The elemental area is  $dA = 2\pi r_0 dr_0$ , and thus,

$$[I_z = \int r^2 dA] \quad I_z = \int_0^r r_0^2 (2\pi r_0 dr_0) = \frac{\pi r^4}{2} = \frac{1}{2}Ar^2$$

Ans.

The polar radius of gyration is

$$\left[ k = \sqrt{\frac{I}{A}} \right]$$

$$k_z = \frac{r}{\sqrt{2}}$$

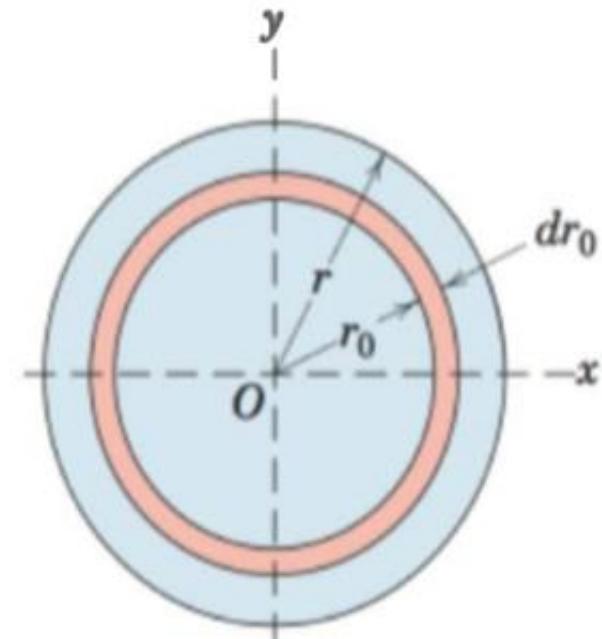
Ans.

By symmetry  $I_x = I_y$ , so that from Eq. A/3

$$[I_z = I_x + I_y]$$

$$I_x = \frac{1}{2}I_z = \frac{\pi r^4}{4} = \frac{1}{4}Ar^2$$

Ans.



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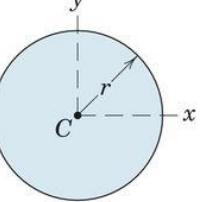
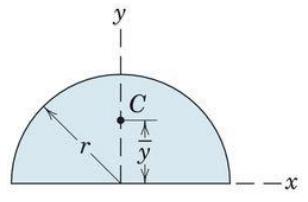
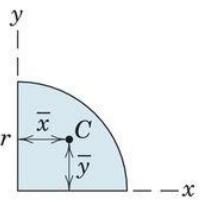
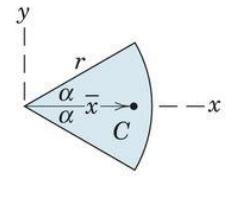
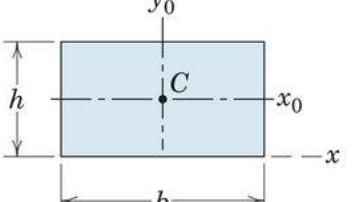
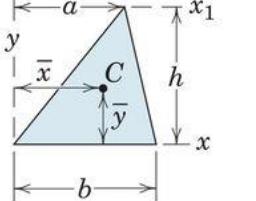
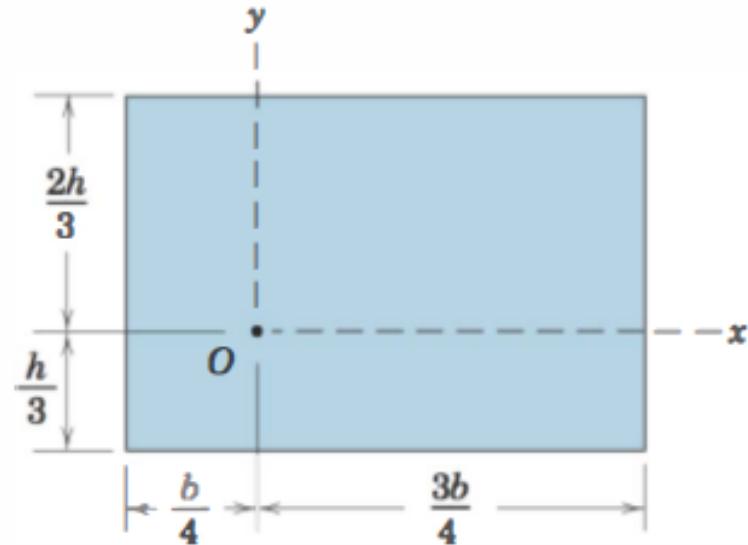
FIGURE	AREA MOMENTS OF INERTIA
Circular Area 	$I_x = I_y = \frac{\pi r^4}{4}$ $I_z = \frac{\pi r^4}{2}$
Semicircular Area 	$I_x = I_y = \frac{\pi r^4}{8}$ $\bar{I}_x = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4$ $I_z = \frac{\pi r^4}{4}$
Quarter-Circular Area 	$I_x = I_y = \frac{\pi r^4}{16}$ $\bar{I}_x = \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right) r^4$ $I_z = \frac{\pi r^4}{8}$
Area of Circular Sector 	$I_x = \frac{r^4}{4} (\alpha - \frac{1}{2} \sin 2\alpha)$ $I_y = \frac{r^4}{4} (\alpha + \frac{1}{2} \sin 2\alpha)$ $I_z = \frac{1}{2} r^4 \alpha$

FIGURE	AREA MOMENTS OF INERTIA
Rectangular Area 	$I_x = \frac{bh^3}{3}$ $\bar{I}_x = \frac{bh^3}{12}$ $\bar{I}_z = \frac{bh}{12} (b^2 + h^2)$
Triangular Area 	$I_x = \frac{bh^3}{12}$ $\bar{I}_x = \frac{bh^3}{36}$ $I_{x_1} = \frac{bh^3}{4}$

A/1 Determine the moments of inertia of the rectangular area about the x- and y-axes and find the polar moment of inertia about point O.

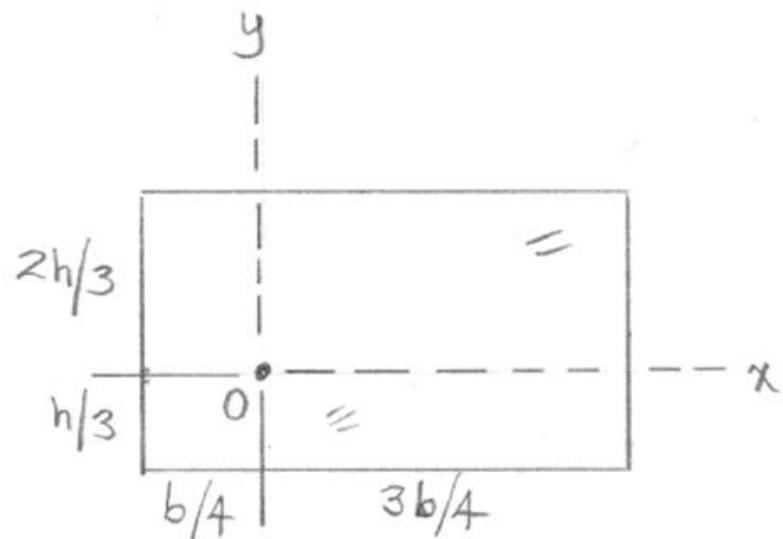


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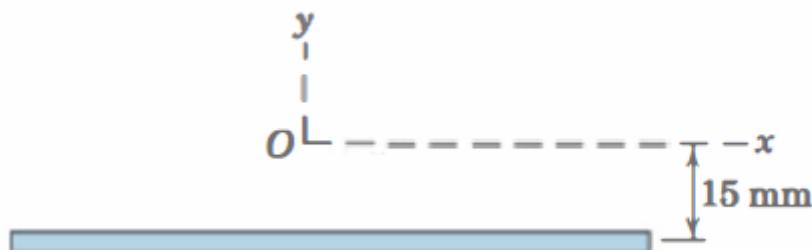


$$I_x = \bar{I}_x + Adx^2 = \frac{1}{12}bh^3 + bh\left(\frac{h}{6}\right)^2$$
$$= \frac{1}{9}bh^3$$

$$I_y = \bar{I}_y + Ady^2 = \frac{1}{12}hb^3 + bh\left(\frac{b}{4}\right)^2$$
$$= \frac{7}{48}hb^3$$

$$I_z = I_x + I_y = bh\left(\frac{h^2}{9} + \frac{7b^2}{48}\right)$$

A/3 The narrow rectangular strip has an area of  $300 \text{ mm}^2$ , and its moment of inertia about the y-axis is  $35(10^3) \text{ mm}^4$ . Obtain a close approximation to the polar radius of gyration about point O.



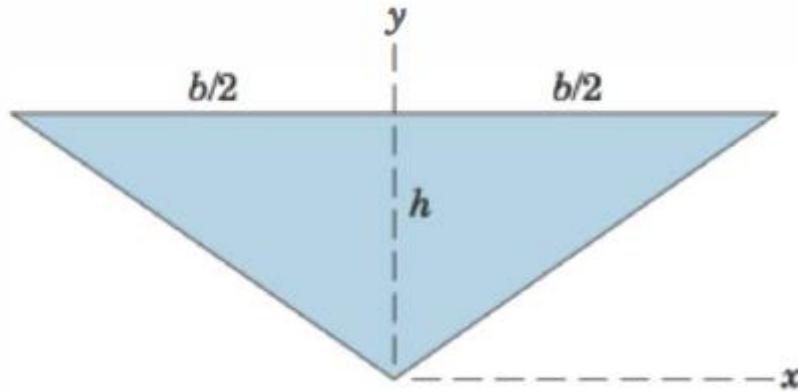
$$\mathcal{I}_x \approx Ad^2 = 300(15)^2 = 67.5(10^3) \text{ mm}^4$$

$$J_o = \mathcal{I}_x + \mathcal{I}_y = 67.5(10^3) + 35(10^3) = 102.5(10^3) \text{ mm}^4$$

$$k_o = \sqrt{J_o/A} = \sqrt{\frac{102.5(10^3)}{300}} = 18.48 \text{ mm}$$



A/4 Determine the ratio  $b/h$  such that  $I_x = I_y$  for the area of the isosceles triangle.

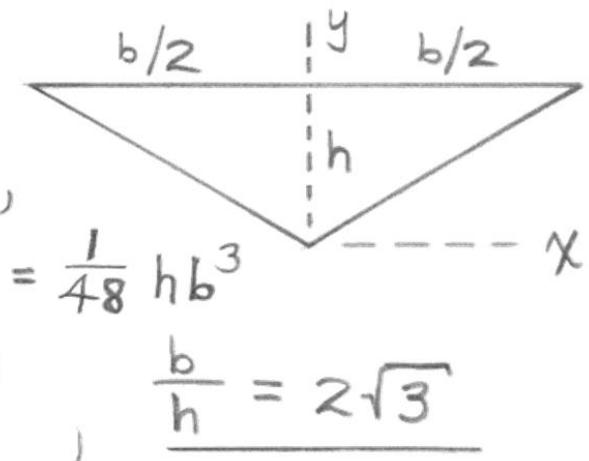


$$\frac{A}{4}$$

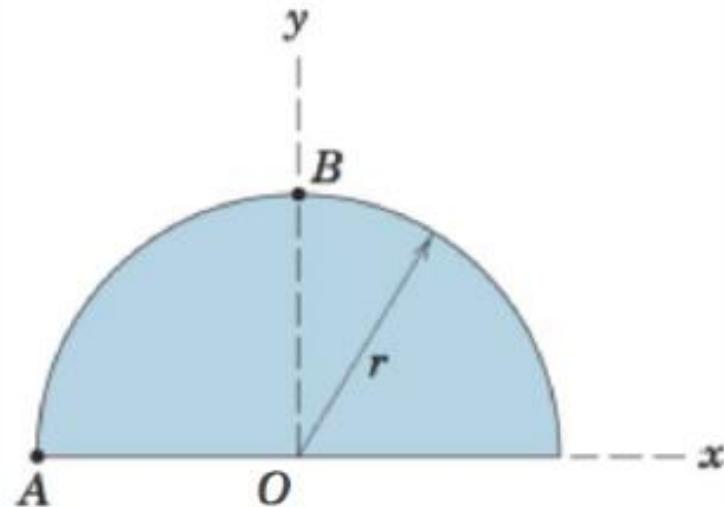
From Sample Problem A/2,

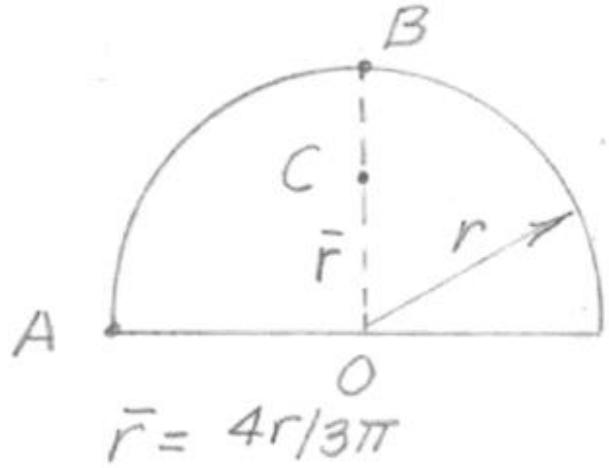
$$I_x = \frac{1}{4}bh^3, I_y = 2\left\{\frac{1}{12}h\left(\frac{b}{2}\right)^3\right\} = \frac{1}{48}hb^3$$

$$I_x = I_y \text{ if } \frac{1}{4}bh^3 = \frac{1}{48}hb^3, \frac{b}{h} = 2\sqrt{3}$$



A/6 Determine the polar moments of inertia of the semicircular area about points A and B.





For complete circle

$$I_A = I_o + Ar^2 = \frac{1}{2}Ar^2 + Ar^2 \\ = \frac{3}{2}Ar^2$$

For half circle

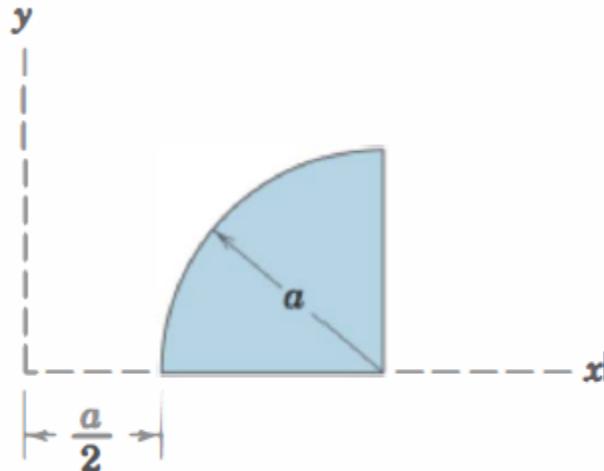
$$I_A = \frac{1}{2} \left( \frac{3}{2} \pi r^4 \right) = \underline{\frac{3}{4} \pi r^4}$$

For half circle,  $I_o = \frac{1}{4} \pi r^4$

$$I_B = I_c + A(r - \bar{r})^2 = I_o - A\bar{r}^2 + A(r - \bar{r})^2 \\ = I_o + A(r^2 - 2r\bar{r}) \\ = \frac{1}{4}\pi r^4 + \frac{\pi r^4}{2} \left( 1 - \frac{8}{3\pi} \right) = r^4 \left( \frac{3\pi}{4} - \frac{4}{3} \right)$$

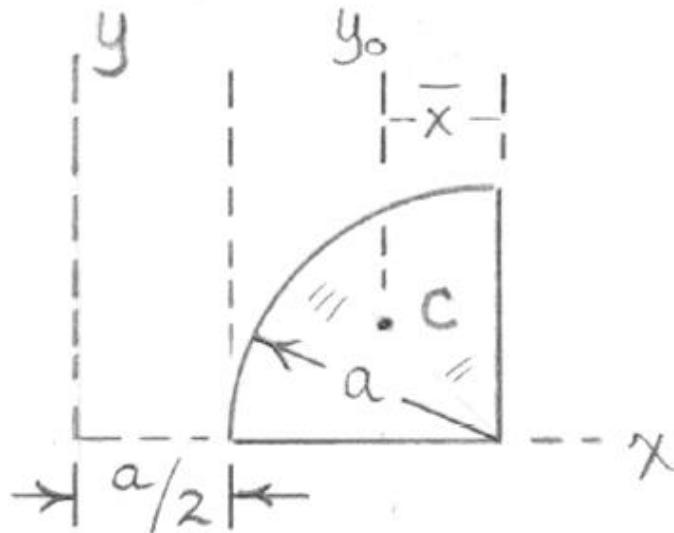


A/7 Determine the moment of inertia of the quarter circular area about the y-axis.





$$\frac{A/7}{|}$$



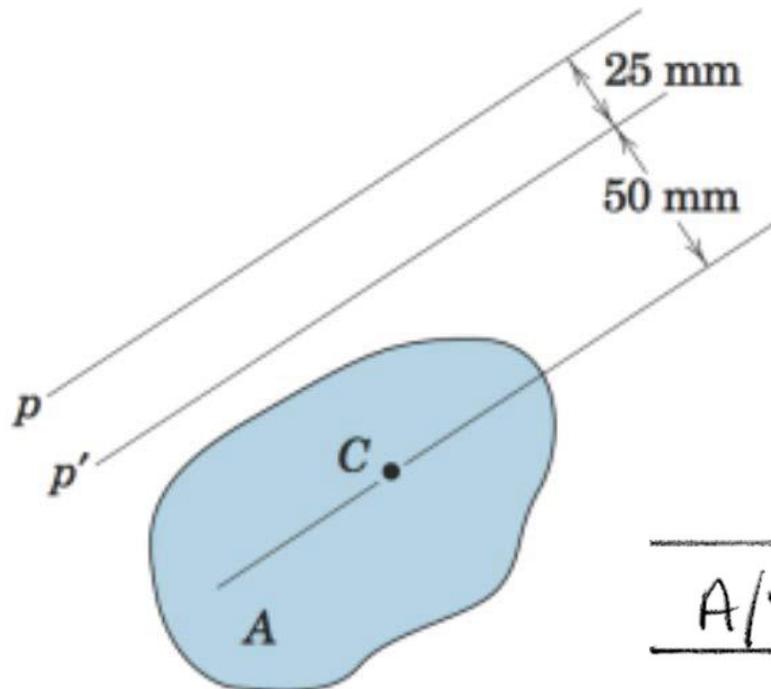
From Table D/3:

$$\bar{I}_y = \left( \frac{\pi}{16} - \frac{4}{9\pi} \right) a^4$$

$$\bar{x} = \frac{4a}{3\pi}$$

$$\begin{aligned}
 I_y &= \bar{I}_y + A dy^2 \\
 &= \left( \frac{\pi}{16} - \frac{4}{9\pi} \right) a^4 + \frac{\pi a^2}{4} \left[ \frac{a}{2} + \left( a - \frac{4a}{3\pi} \right) \right]^2 \\
 &= \left[ \frac{5\pi}{8} - 1 \right] a^4
 \end{aligned}$$

A/9 The moments of inertia of the area A about the parallel p- and p'-axes differ by  $15(10^6)$  mm<sup>4</sup>. Compute the area A, which has its centroid at C.

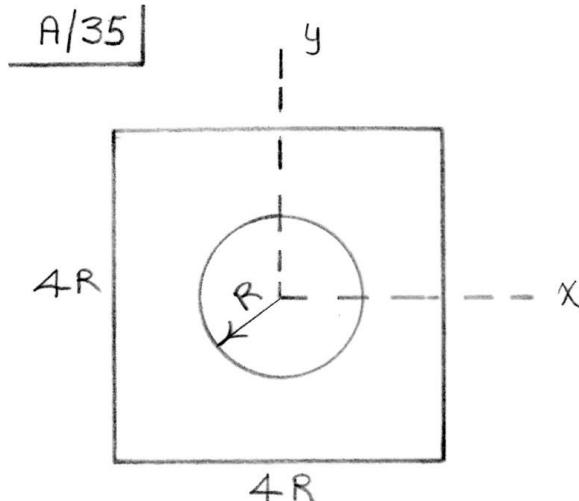
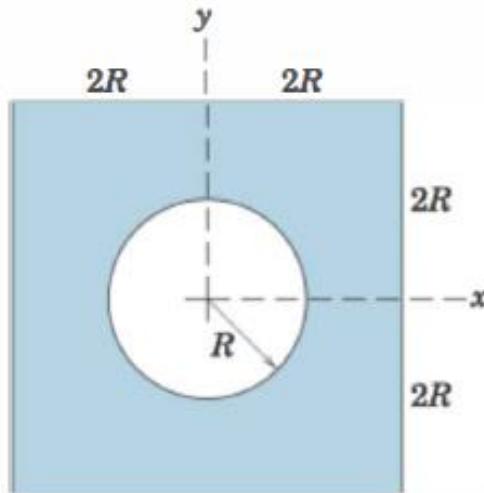


$$\boxed{A/9} \quad I_p = I_c + A(75)^2, \quad I_{p'} = I_c + A(50)^2$$

$$I_p - I_{p'} = 15(10^6) = A[(75)^2 - (50)^2]$$
$$A = 4800 \text{ mm}^2$$



A/35 Determine the moment of inertia about the x-axis of the square area without and with the central circular hole.



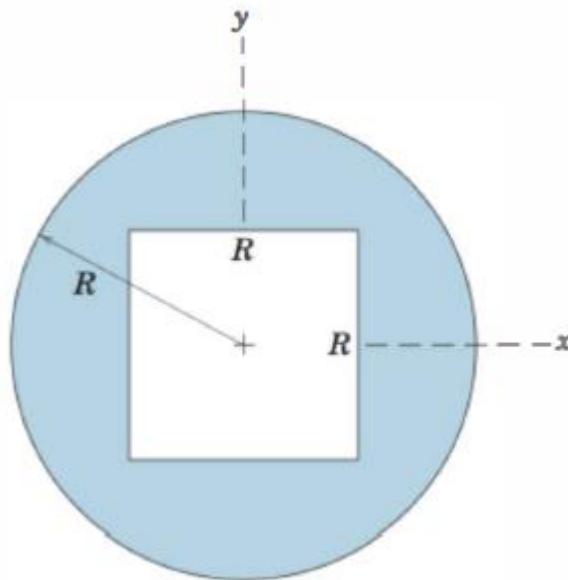
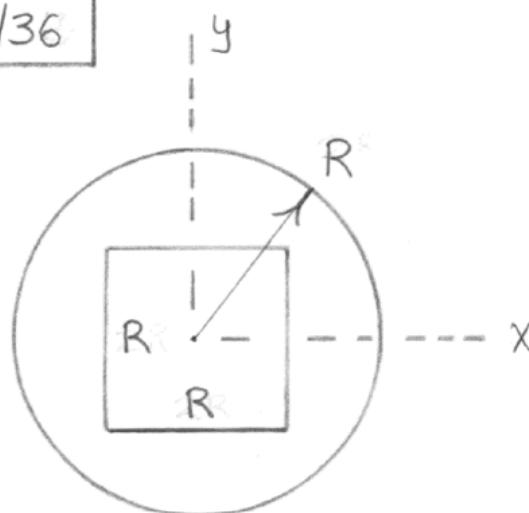
$$\text{Without hole, } I_x = \frac{1}{12} (4R)(4R)^3 = \frac{64}{3} (21.3 R^4)$$

$$\begin{aligned}\text{With hole, } I_x &= \frac{64}{3} R^4 - \frac{1}{4} (\pi R^2) R^2 \\ &= \underline{\underline{20.5 R^4}}\end{aligned}$$

(a 3.68% reduction)

A/36 Determine the polar moment of inertia of the circular area without and with the central square hole.

A/36



Without square hole:

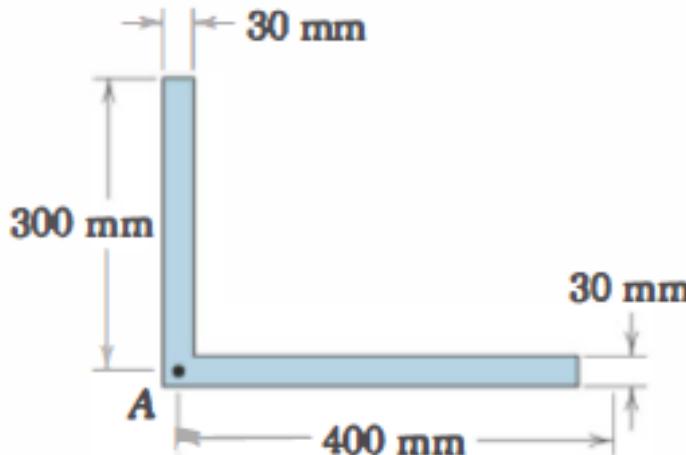
$$I_z = 2I_x = 2 \left( \frac{1}{4} \pi R^2 \cdot R^2 \right) = \underline{1.571 R^4}$$

With hole:

$$I_z = 1.571 R^4 - 2 \left( \frac{1}{12} R \cdot R^3 \right) = \underline{1.404 R^4}$$

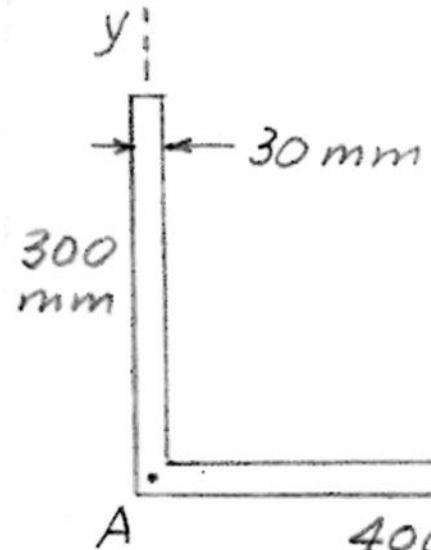
(a reduction of 10.61%)

A/37 Calculate the polar radius of gyration of the area of the angle section about point A Note that the width of the legs is small compared with the length of each leg.



A/37

$$I_x \approx \frac{1}{3}(30)(300)^3 + 0 = 270(10)^6 \text{ mm}^4$$



$$I_y \approx \frac{1}{3}(30)(400)^3 + 0 = 640(10)^6 \text{ mm}^4$$

$$J_A = I_x + I_y = 910(10)^6 \text{ mm}^4$$

$$k_A = \sqrt{J_A/A} = \sqrt{\frac{910(10)^6}{30(300+400)}}$$

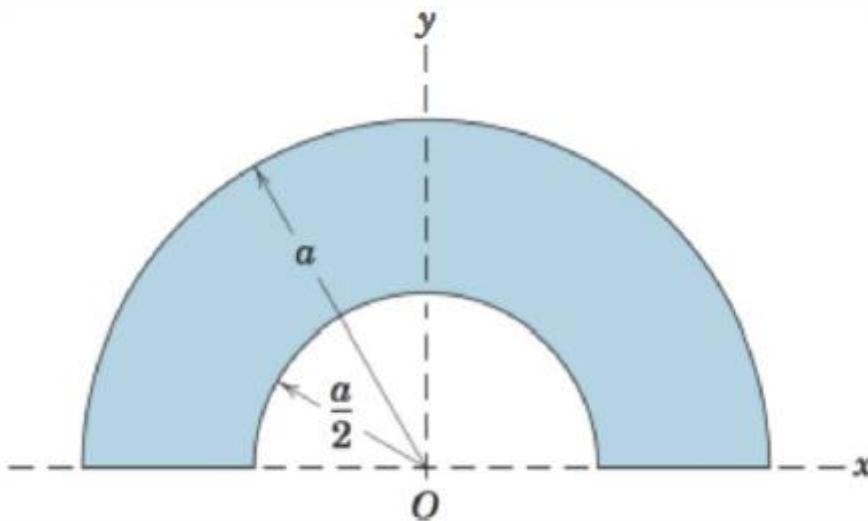
400 mm

$$k_A = \underline{208 \text{ mm}}$$



A/38 Calculate the polar radius of gyration of the area of the angle section about point O.

A Note that the width of the legs is small compared with the length of each leg.



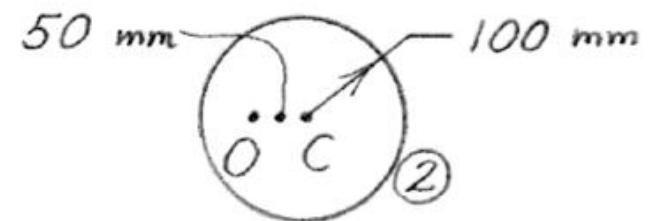
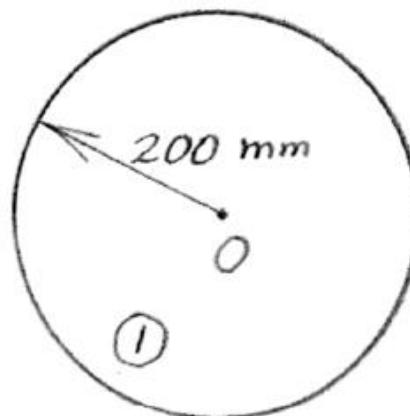
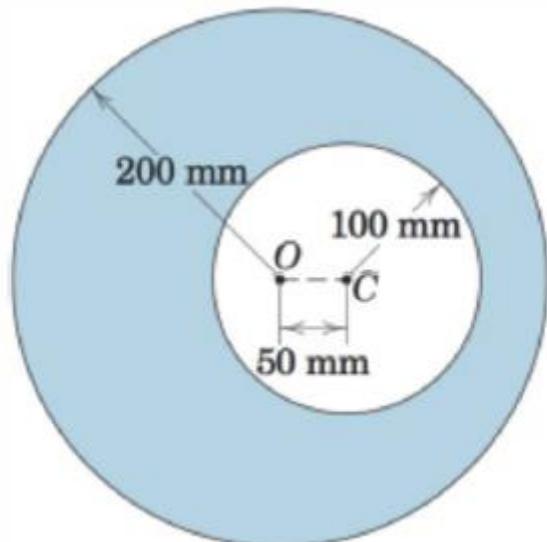
$$\boxed{\text{A/38}} \quad I_z = \frac{1}{2} \left[ \frac{\pi a^4}{2} - \frac{\pi (\frac{a}{2})^4}{2} \right] = \frac{15}{64} \pi a^4$$

$$k_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{\frac{15}{64} \pi a^4}{\frac{3}{8} \pi a^2}} = \frac{\sqrt{10}}{4} a$$

From  $k_x^2 + k_y^2 = k_z^2$  and the fact that  $k_x = k_y$  for the present case,

$$2k_x^2 = \left(\frac{\sqrt{10}}{4} a\right)^2, \quad k_x = k_y = \frac{\sqrt{5}}{4} a$$

A/39 Calculate the polar radius of gyration of the shaded area about the center O of the larger circle.



$$\text{Area } A = A_1 - A_2 = \pi(200^2 - 100^2) = 3(10^4)\pi \text{ mm}^2$$

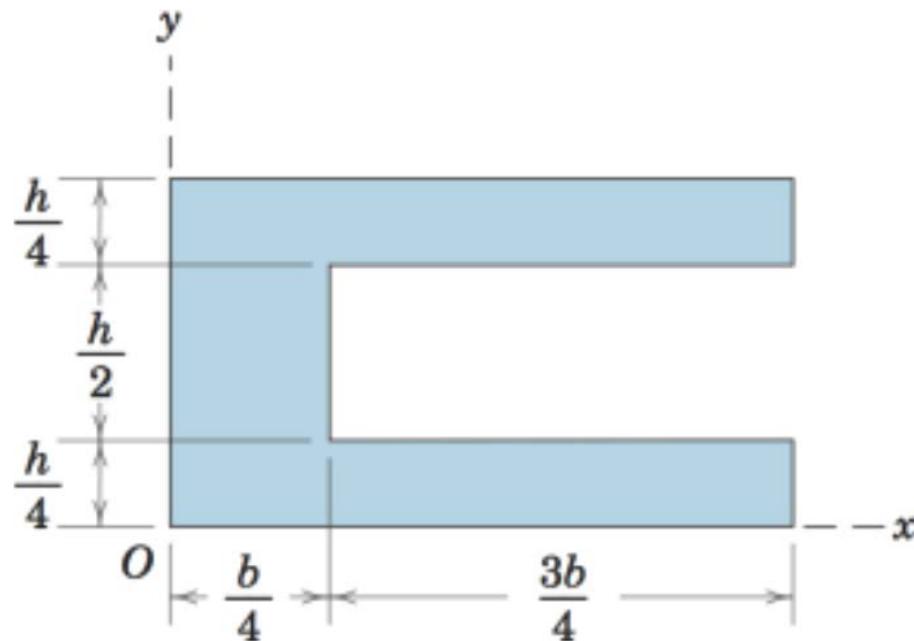
$$\textcircled{1} I_{O_1} = \frac{1}{2}(\pi \cdot 200^2)(200^2) = 8(10^8)\pi \text{ mm}^4$$

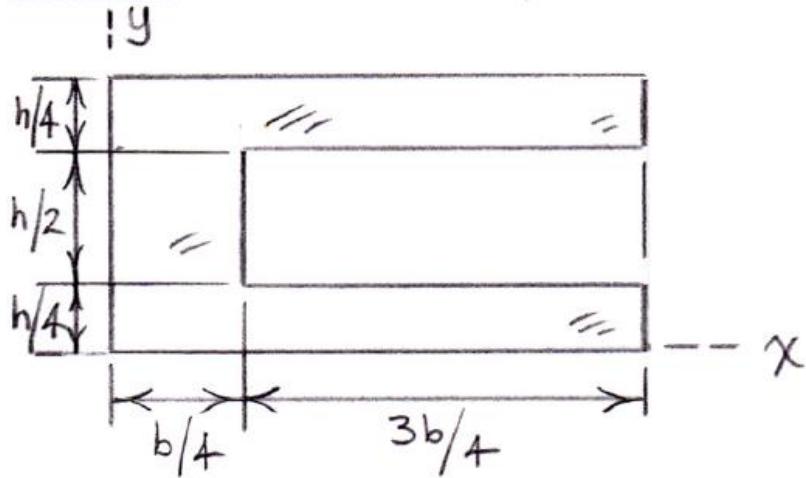
$$\textcircled{2} I_{O_2} = \frac{1}{2}(\pi \cdot 100^2)(100^2) + \pi(100^2)(50^2) = 0.75(10^8)\pi \text{ mm}^4$$

$$\text{So } I_O = I_{O_1} - I_{O_2} = 7.25(10^8)\pi \text{ mm}^4$$

$$k_O = \sqrt{\frac{I_O}{A}} = \sqrt{\frac{7.25(10^8)\pi}{3(10^4)\pi}} = \underline{155.5 \text{ mm}}$$

A/40 Determine the percent reductions in both area and area moment of inertia about the y-axis caused by removal of the rectangular cutout from the rectangular plate of baseband height  $h$ .





Full rectangle :  $A = bh$ ,  $I_y = \frac{1}{3}hb^3$

With cutout :  $A = bh - \frac{3b}{4}\left(\frac{h}{2}\right) = \frac{5}{8}bh$

$$I_y = \frac{1}{3}hb^3 - \left[ \frac{1}{12} \frac{h}{2} \left( \frac{3b}{4} \right)^3 + \frac{3}{8}bh \left( \frac{b}{4} + \frac{3b}{8} \right)^2 \right]$$

$$= \frac{65}{384}hb^3$$

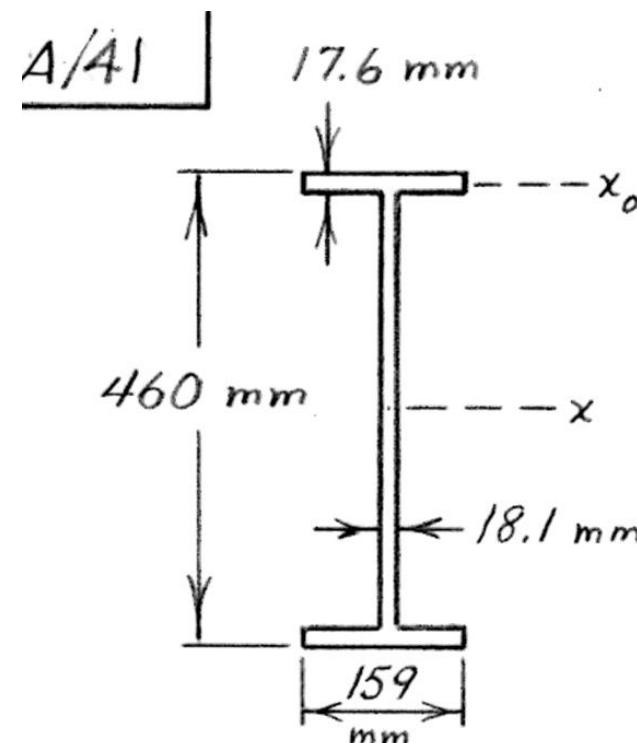
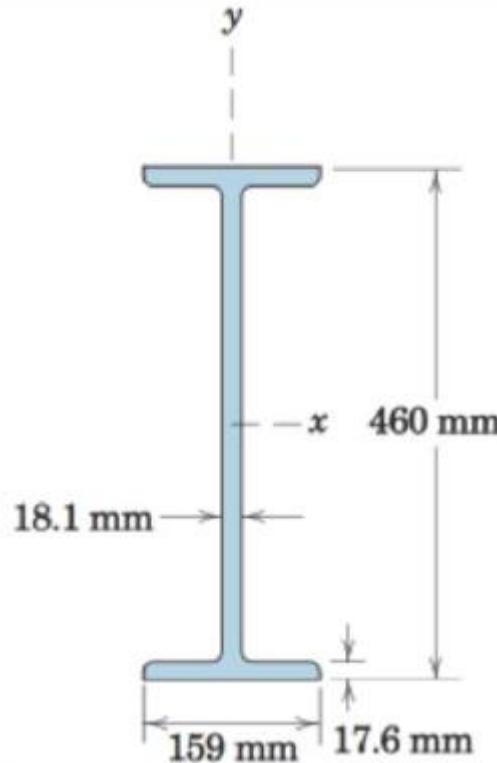
Percent reductions :

$$n_A = \frac{bh - \frac{5}{8}bh}{bh} (100\%) = 37.5\%$$

$$n_{I_y} = \frac{\frac{1}{3}hb^3 - \frac{65}{384}hb^3}{\frac{1}{3}hb^3} = 49.2\%$$



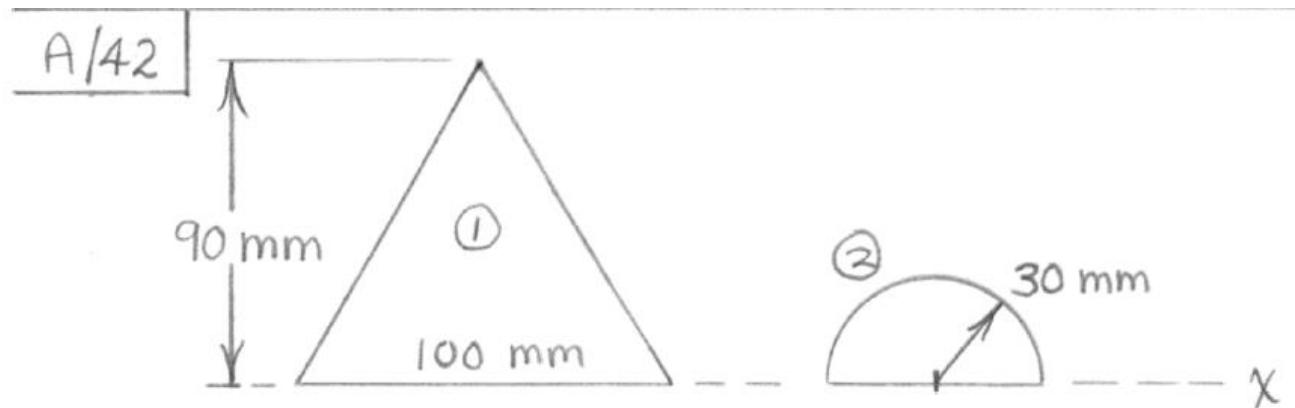
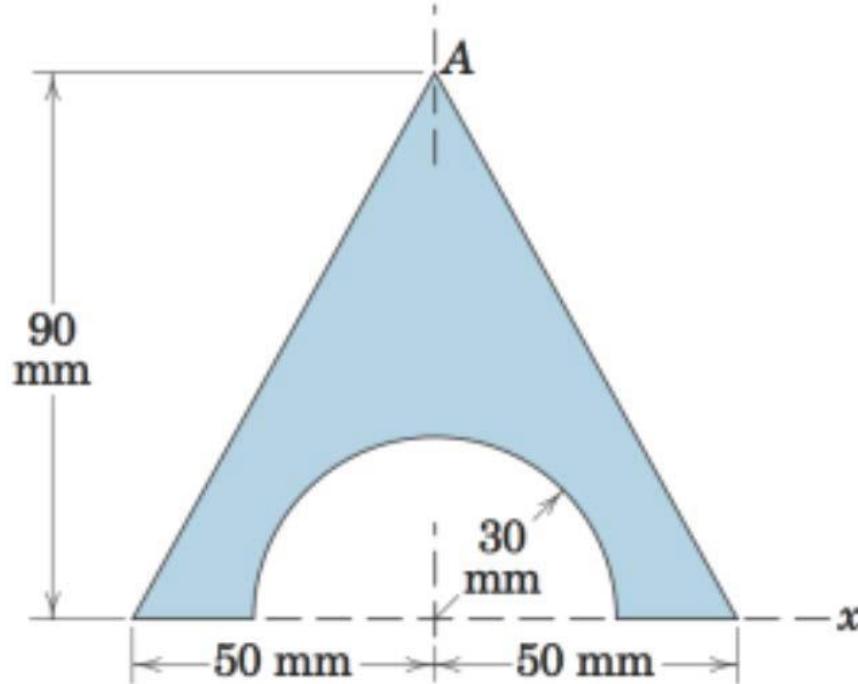
A/41 The cross-sectional area of an I-beam has the dimensions shown. Obtain a close approximation to the handbook value of  $I_x = 385(10^6) \text{ mm}^4$  by treating the section as being composed of three rectangles.



$$\begin{aligned} \text{Flanges: } \bar{I}_x &= I_{x_0} + Ad^2 \\ &= 2 \left\{ \frac{1}{12} (159)(17.6^3) + 159(17.6)(230 - \frac{17.6}{2})^2 \right\} \\ &= 2 \left\{ 7.22(10^4) + 1.369(10^8) \right\} \text{ mm}^4 \\ &= 2.74(10^8) \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} \text{Web: } \bar{I}_x &= \frac{1}{12} (18.1)(460 - 2[17.6])^3 \\ &= 1.156(10^8) \text{ mm}^4 \\ \text{Total } \bar{I}_x &= 3.90(10^8) \text{ mm}^4 \end{aligned}$$

A/42 Calculate the moment of inertia of the shaded area about the x-axis.



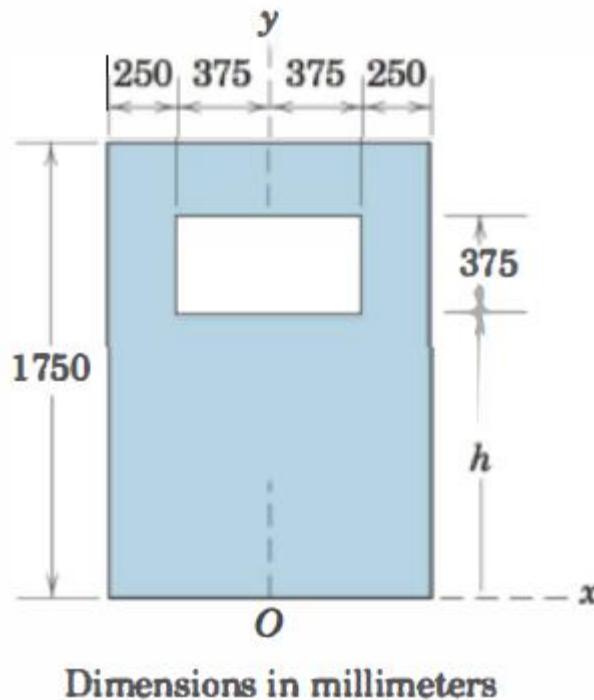
$$I_{x_1} = \frac{1}{12}(100)(90^3) = 6.08(10^6) \text{ mm}^4$$

$$I_{x_2} = -\frac{\pi(30^4)}{8} = -0.318(10^6) \text{ mm}^4$$

$$\text{So } I_x = (6.08 - 0.318)10^6 = \underline{\underline{5.76(10^6) \text{ mm}^4}}$$



A/43 The variable  $h$  designates the arbitrary vertical location of the bottom of the rectangular cutout within the rectangular area. Determine the area moment of inertia about the  $x$ -axis for (a)  $h = 1000$  mm and (b)  $h = 1500$  mm.



$$(a) h = 1000 \text{ mm } (\text{hole complete})$$

$$I_x = \frac{1}{3} (1250)(1750^3) - \left[ \frac{1}{12} (750)(375)^3 + 750(375)(1000 + \frac{375}{2})^2 \right]$$

$$= \underline{1.833(10^{12}) \text{ mm}^4 \text{ or } 1.833 \text{ m}^4}$$

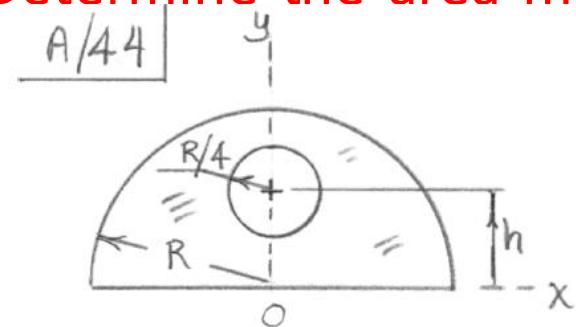
$$(b) h = 1500 \text{ mm } (250 \text{ mm of hole in play})$$

$$I_x = \frac{1}{3} (1250)(1750^3) - \left[ \frac{1}{12} (750)(250)^3 + 750(250)(1500 + \frac{250}{2})^2 \right]$$

$$= \underline{1.737(10^{12}) \text{ mm}^4 \text{ or } 1.737 \text{ m}^4}$$



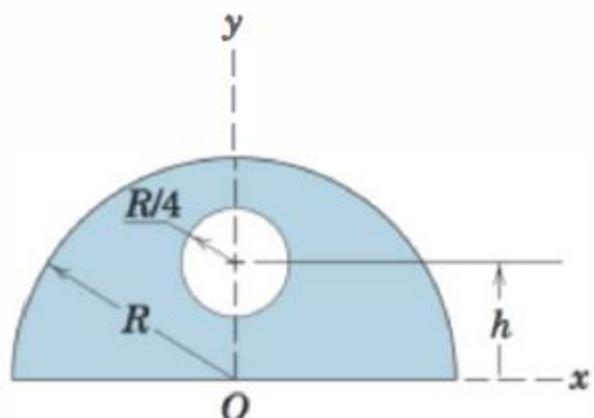
A/44 The variable  $h$  designates the arbitrary vertical location of the center of the circular cutout within the semicircular area. Determine the area moment of inertia about the  $x$ -axis for (a)  $h = 0$  and (b)  $h = R/2$ .



(a)  $h = 0$  (One-half of hole considered)

$$I_x = \frac{\pi R^4}{8} - \frac{\pi (R/4)^4}{8} = \frac{255}{2048} \pi R^4$$

$$(0.391 R^4)$$

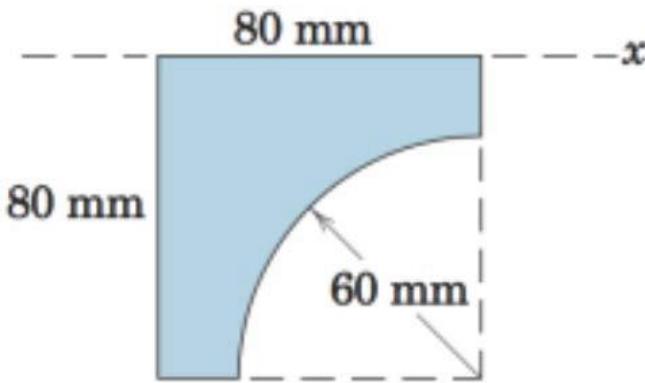


(b)  $h = \frac{R}{2}$  (Entire hole now in play)

$$I_x = \frac{\pi R^4}{8} - \left[ \frac{\pi (R/4)^4}{4} + \pi \left(\frac{R}{4}\right)^2 \left(\frac{R}{2}\right)^2 \right]$$

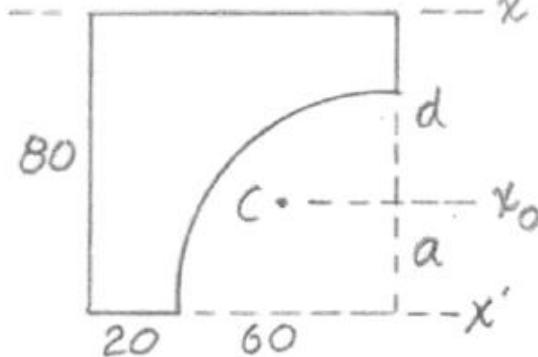
$$= \frac{111}{1024} \pi R^4 (0.341 R^4)$$

A/45 Calculate the moment of inertia of the shaded area about the x-axis.



A/45

Dimen. in mm



$$\text{Square: } I_x = \frac{1}{3} b^4 = \frac{1}{3} (80)^4 = 13.65(10^6) \text{ mm}^4$$

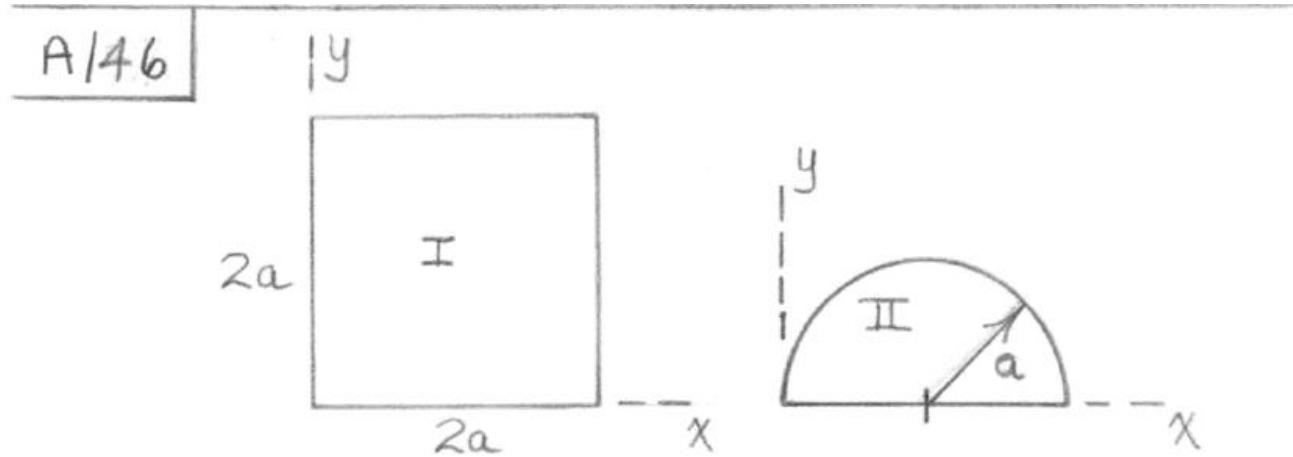
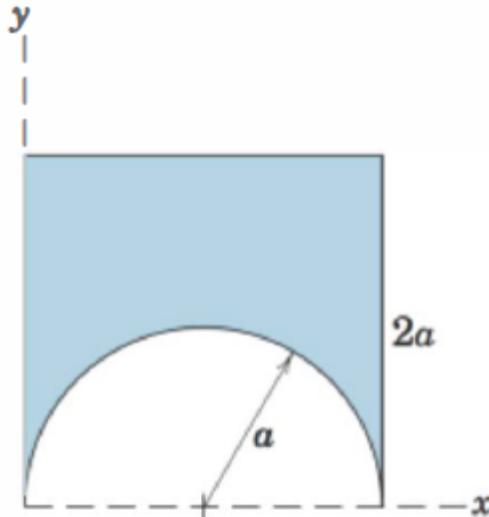
$$\text{Quarter-circle: } a = \frac{4r}{3\pi} = \frac{4(60)}{3\pi} = 25.46 \text{ mm}$$

$$d = 80 - 25.46 = 54.54 \text{ mm}$$

$$\begin{aligned} I_x &= I_{x_0} + Ad^2 = I_{x_0} - Aa^2 + Ad^2 \\ &= \frac{-1}{4} \frac{\pi r^4}{4} - \frac{\pi r^2}{4} (d^2 - a^2) = -\frac{\pi r^2}{4} \left( \frac{r^2}{4} + d^2 - a^2 \right) \\ &= -\frac{\pi (60)^2}{4} \left[ \frac{60^2}{4} + (54.54)^2 - (25.46)^2 \right] \\ &= -9.120 (10^6) \text{ mm}^4 \end{aligned}$$

$$\text{Total } I_x = (13.65 - 9.120)(10^6) = \underline{4.53 (10^6) \text{ mm}^4}$$

A/46 Determine the moments of inertia of the shaded area about the x- and y-axes.



I. Square     $I_x = I_y = \frac{1}{3} (4a^2)(2a)^2 = \frac{16}{3}a^4$

II. Semicircle     $I_x = \frac{1}{8}\pi a^4$

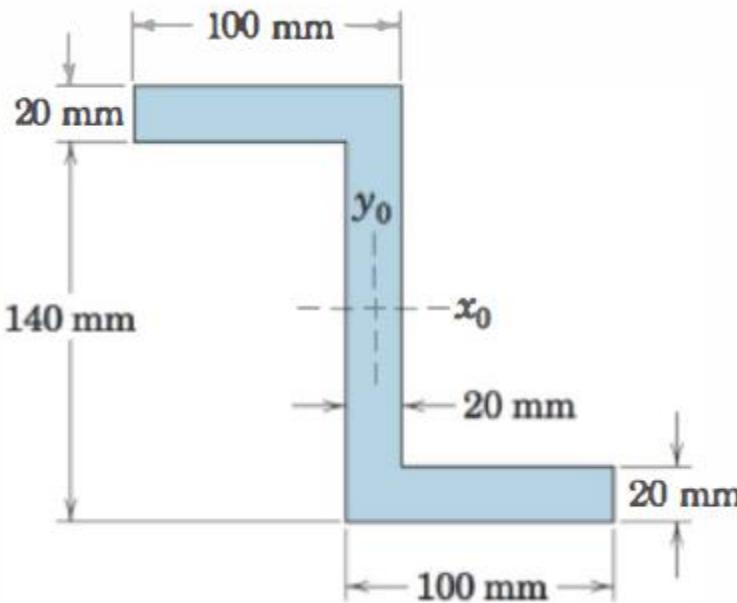
$$I_y = \frac{1}{8}\pi a^4 + \frac{1}{2}\pi a^2(a^2) = \frac{5}{8}\pi a^4$$

Combined:  $I_x = \frac{16}{3}a^4 - \frac{\pi}{8}a^4 = \underline{4.94a^4}$

$$I_y = \frac{16}{3}a^4 - \frac{5}{8}\pi a^4 = \underline{3.37a^4}$$



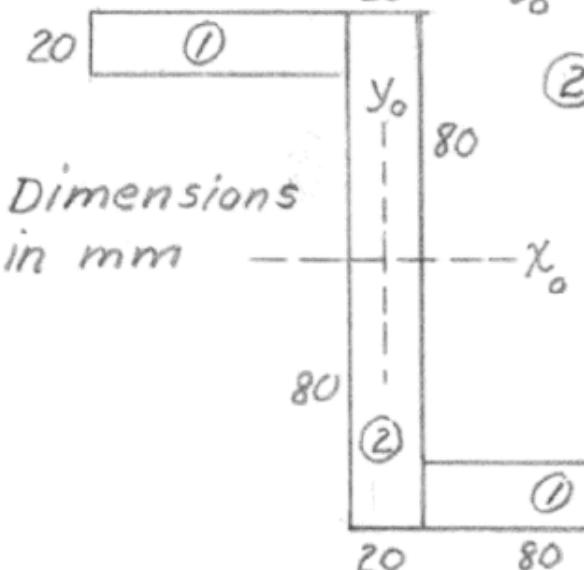
A/48 Determine the moments of inertia of the Z-section about its centroidal  $x_0$ - and  $y_0$ -axes.



A/48

$$\textcircled{1} I_{x_0} = \frac{1}{12} (80)(20)^3 + (80)(20)(70)^2 = 7.89(10^6) \text{ mm}^4$$

$$I_{y_0} = \frac{1}{12} (20)(80)^3 + (20)(80)(50)^2 = 4.85(10^6) \text{ mm}^4$$



Dimensions  
in mm

$$\textcircled{2} I_{x_0} = \frac{1}{12} (20)(160)^3 = 6.83(10^6) \text{ mm}^4$$

$$I_{y_0} = \frac{1}{12} (160)(20)^3 = 0.1067(10^6) \text{ mm}^4$$

Total  $\bar{I}_x = [2(7.89) + 6.83](10^6)$

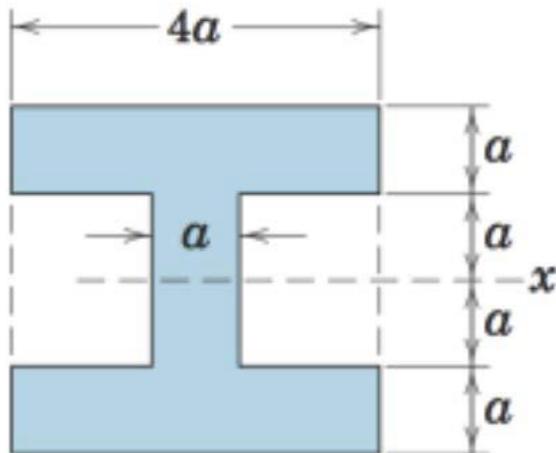
$$= 22.6(10^6) \text{ mm}^4$$

$$\bar{I}_y = [2(4.85) + 0.1067](10^6)$$

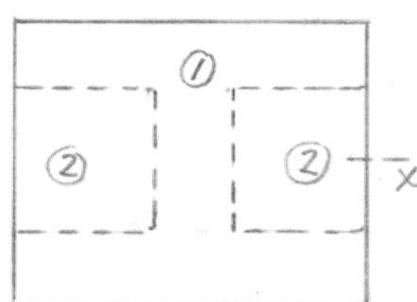
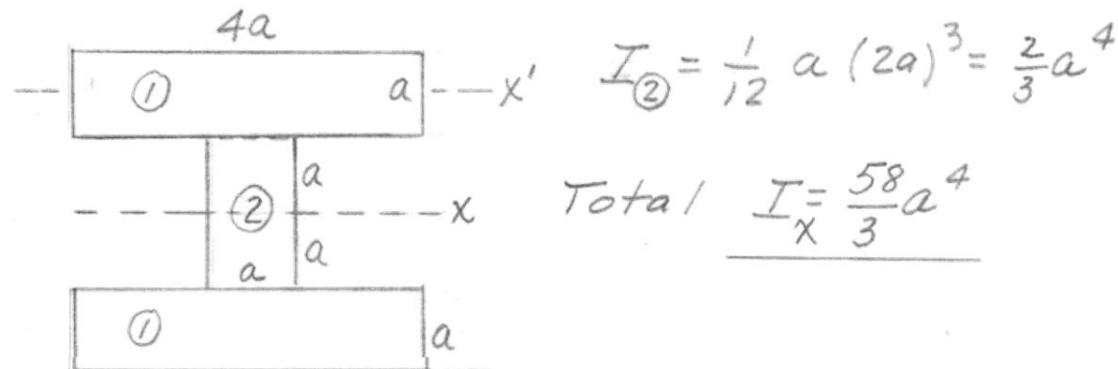
$$= 9.81(10^6) \text{ mm}^4$$



A/49 Determine the moment of inertia of the shaded area about the x-axis in two different ways.



A/49 Sol. I  $I_{\text{Total}} = 2 \left[ \frac{1}{12} 4a(a^3) + 4a^2 \left(\frac{3a}{2}\right)^2 \right] = \frac{56}{3}a^4$



Sol. II

$$I_{\text{Total}} = I_{\text{Part 1}} + I_{\text{Part 2}}$$

$$I_{\text{Part 1}} = \frac{1}{12}(4a)(4a)^3 = \frac{64}{3}a^4$$

$$I_{\text{Part 2}} = -\frac{1}{12}(3a)(2a)^3 = -2a^4$$

$$\text{Total} = \left(\frac{64}{3} - \frac{6}{3}\right)a^4 = \frac{58}{3}a^4$$



**THANK YOU**

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