PES University, Bangalore

(Established Under Karnataka Act 16 of 2013)

Department of Science and Humanities

Engineering Mathematics - I
(UE25MA141A)

Assignment

Unit - 2: Higher-Order Differential Equations

Solve the following homogeneous linear differential equations

1. $\frac{d^4x}{dt^4} + 4x = 0.$
   Answer: $x = e^{-t}(c_1 \cos t + c_2 \sin t) + e^{t}(c_3 \cos t + c_4 \sin t).$

2. $\frac{d^2y}{dx^2} + (a+b)\frac{dy}{dx} + aby = 0.$
   Answer: $y = c_1 e^{-ax} + c_2 e^{-bx}$

3. $(D^2+1)^3(D^2+D+1)^2 y = 0.$
   Answer: $y = (c_1 + c_2 x + c_3 x^2)\cos x + (c_4 + c_5 x + c_6 x^2)\sin x +$
   $e^{-\frac{1}{2}x}\left[(c_7 + c_8 x)\cos\frac{\sqrt{3}}{2}x + (c_9 + c_{10}x)\sin\frac{\sqrt{3}}{2}x\right]$

4. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 29y = 0,$ given that when $x = 0$, $y = 0$ and $\frac{dy}{dx} = 15.$
   Answer: $y = 3e^{-2x}\sin 5x.$

Solve the following non-homogeneous linear differential equations

5. $(D^3 - 6D^2 + 11D - 6)y = e^{-2x} + e^{-3x}.$
   Answer: $y = \text{C.F.} + \text{P.I.} = c_1 e^{x} + c_2 e^{2x} + c_3 e^{3x} - \frac{1}{120}(2e^{-2x} + e^{-3x}).$

6. $(D+2)(D-1)^2 y = e^{-2x} + 2\sinh x.$
   Answer: $y = c_1 e^{-2x} + (c_2 + c_3 x)e^{x} + \frac{x}{9}e^{-2x} + \frac{x^2}{6}e^{x} + \frac{1}{4}e^{-x}.$

7. $(D-2)^2 y = 8(e^{2x} + \sin 2x).$
   Answer: $y = (c_1 + c_2 x)e^{2x} + 4x^2 e^{2x} + \cos 2x.$

8. $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} = e^{2x} + \sin 2x.$
   Answer: $y = c_1 + e^{x}(c_2 \cos\sqrt{3}x + c_3 \sin\sqrt{3}x) + \frac{1}{8}(e^{2x} + \sin 2x)$

9. $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 6\frac{dy}{dx} = 1 + x^2$
   Answer: $y = c_1 + c_2 e^{3x} + c_3 e^{-2x} - \frac{1}{18}\left(x^3 - \frac{x^2}{2} + \frac{25}{6}x\right)$

10. $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$
    Answer: $y = c_1 + c_2 e^{-x} + \frac{x^3}{3} + 4x.$

11. $\frac{d^2y}{dx^2} - 4y = x\sinh x.$
Answer: $y = c_1e^{2x} + c_2e^{-2x} - \frac{x}{3}\sinh x - \frac{2}{9}\cosh x.$

12. $\frac{d^4y}{dx^4} - y = \cos x\cosh x.$
Answer: $y = c_1e^{x} + c_2e^{-x} + c_3\cos x + c_4\sin x - \frac{1}{5}\cos x\cosh x.$

13. $\frac{d^2y}{dx^2} + y = x\sin x$
Answer: $y = c_1\cos x + c_2\sin x + \frac{x}{4}\sin x - \frac{x^2}{4}\cos x.$

14. $y'' - 2y' + 2y = e^x\tan x.$
Answer: $y = e^x(c_1\cos x + c_2\sin x) - e^x\cos x\log(\sec x + \tan x).$

15. $\frac{d^2y}{dx^2} + 4y = \tan 2x.$
Answer: $y = c_1\cos 2x + c_2\sin 2x - \frac{1}{4}\cos 2x\log(\sec 2x + \tan 2x).$

Solve the following Legendre's linear equation

16. $(3x+2)^2\frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1.$
Answer: $y = c_1(3x+2)^2 + c_2(3x+2)^{-2} + \frac{1}{108}\left[(3x+2)^2\log(3x+2) + 1\right].$

17. $(1+2x)^2\frac{d^2y}{dx^2} - 6(1+2x)\frac{dy}{dx} + 16y = 8(1+2x)^2$
Answer: $y = (1+2x)^2\left[c_1 + c_2\log(1+2x) + \{\log(1+2x)\}^2\right].$

Solve the following Cauchy's linear equation

18. $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0.$
Answer: $y = C_1x^2 + C_2x^2\ln x.$

19. $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + y = x^3.$
Answer: $y = C_1x^{(3+\sqrt{5})/2} + C_2x^{(3-\sqrt{5})/2} + x^3.$

Applications of linear differential equations

20. An electric circuit consists of an inductance of 0.1 henry, a resistance of 20 ohms and a condenser of capacitance 25 micro-farads. Find the charge q and the current i at any time t, given that at t = 0, q = 0.05 coulomb, $i = \frac{dq}{dt} = 0$ when t = 0.
Answer: $q = e^{-100t}\left[0.05\cos(624.5\,t) + 0.008\sin(624.5\,t)\right].$
$i = \frac{dq}{dt} = -100e^{-100t}\left[(c_1 - \sqrt{39}\,c_2)\cos 624.5\,t + (\sqrt{39}\,c_1 + c_2)\sin 624.5\,t\right].$
$\Rightarrow i = -0.32\,e^{-100t}\sin 624.5\,t. \qquad \left[\because \sqrt{39}\,c_2 = c_1\right].$

21. A body weighing 4.9 kg is hung from a spring. A pull of 10 kg will stretch the spring to 5 cm. The body is pulled down 6 cm below the static equilibrium position and then released. Find the displacement of the body from its equilibrium position at time t seconds, the maximum velocity and the period of oscillation.
Answer: $x = c_1\cos 20t + c_2\sin 20t$
Maximum velocity = ω (amplitude) = $20 \times 0.06 = 1.2$ m/sec.
Period of oscillation = $\frac{2\pi}{\omega} = \frac{2\pi}{20} = \frac{\pi}{10} = 0.314$ sec.