CL35_Q1. What are Einstein's coefficients? Show that the probabilities of induced absorption and stimulated emission are equal.

Ans:

In the analysis of interaction of radiation with matter, Einstein's coefficients describe the absorption and emission of photons via the electronic transitions in atoms.

In case of induced absorption, the rate equation is $\boldsymbol{R_{ind\,abs} = B_{12} * N_1 * \rho(\nu)}$, where B_{12} is the Einstein's coefficient for induced absorption.

The rate of spontaneous emission in terms of the population of excited states is given by, $\boldsymbol{R_{sp\,em} = A_{21} * N_2}$ **,** where A_{21} is the Einstein's coefficient for spontaneous emission.

The rate of stimulated emission is dependent on both the population of atoms in the excited state and the energy density of radiation (stimulating photon), represented as $\boldsymbol{R_{st\,em} = B_{21} * N_2 * \rho(\nu)}$ **,**where B_{21} is the Einstein's coefficient for stimulated emission.

The expression for the energy density of radiation in terms of Einstein's coefficients when the material is in thermal equilibrium with the radiation is given by,

$$\boldsymbol{\rho(\nu) = \frac{A_{21} * N_2}{(B_{12} * N_1 - B_{21} * N_2)} = \frac{A_{21}/B_{21}}{\left(\frac{B_{12}}{B_{21}} exp^{\frac{h\nu}{kT}} - 1\right)}} \qquad (1)$$

Comparing this with the Planck's expression for the energy density of radiation at any frequency and temperature

$$\boldsymbol{\rho(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{\left(exp^{\frac{h\nu}{kT}} - 1\right)}} \qquad (2)$$

Comparing term by term we observe that $\frac{\mathrm{B_{12}}}{\mathrm{B_{21}}} = \mathbf{1}$ and this implies that $\mathrm{B_{12}} = \mathrm{B_{21}} = \mathrm{B}$ i.e., the probabilities of induced absorption and stimulated emission are equal.

CL35_Q2. If R_1 is the rate of stimulated emission and R_2 is the rate of spontaneous emission between two energy levels, show that $\lambda = \frac{hc}{[kT\ ln\{(R_2/R_1)+1\}]}$

Ans:

$$Rate\ of\ stimulated\ emission\ \ R_1 is = B_{21}N_2\rho(\nu)$$

$$Rate\ of\ spontaneous\ emission\ \ R_2 is = A_{21}N_2$$

$\frac{R_1}{R_2} = \frac{B_{21}N_2\rho(\nu)}{A_{21}N_2} = \left(\frac{B_{21}}{A_{21}}\right)\frac{A_{21}/B_{21}}{\left(\frac{B_{12}}{B_{21}}exp^{\frac{h\nu}{kT}}-1\right)}$ where $B_{21} = B_{12}$

$$\frac{R_2}{R_1} = exp^{\frac{h\nu}{kT}} - 1$$

$$ln\left(\frac{R_2}{R_1}+1\right) = \frac{hc}{\lambda kT}$$

$$\lambda = \frac{hc}{kT\left\{ln\left(\frac{R_2}{R_1}+1\right)\right\}}$$

CL35_Q3. A source emits a radiation of wavelength of 400 nm, at what temperature the rates of spontaneous and stimulated emission will be equal.

$$Rate\ of\ stimulated\ emission\ = Rate\ of\ spontaneous\ emission$$

$B_{21}N_2\ \rho_\vartheta = A_{21}N_2$where $\rho(\nu) = \frac{A_{21}/B_{21}}{\left(\frac{B_{12}}{B_{21}}exp^{\frac{h\nu}{kT}}-1\right)}$ substituting this we get

$$\frac{\frac{A_{21}}{B_{21}}}{\left(exp^{\frac{h\nu}{kT}}-1\right)} = \frac{A_{21}}{B_{21}}$$

$$ln1 = \frac{hc}{\lambda kT}\ , T \to \infty$$