

CL9_Q1. Give physical interpretation of the wave function.**Answer**

The wave function $\psi(x, y, z, t)$ is a probability amplitude and the intensity of the wave (the point at which the energy of the wave is likely to be concentrated) is the square of the probability amplitude. Since the wave function can be a real or an imaginary function, it is evident that the square of the wave function $|\psi|^2 = \psi^* \cdot \psi$. ψ^* is the complex conjugate of the wave function. Thus the product is representative of the intensity of the wave or the probability of finding the particle at any point in the wave packet and is called the probability density.

CL9_Q2. Prove that $\psi^*(x, t) \psi(x, t)$ is necessarily real and either positive or zero.**Answer**

As we know that wave functions are usually complex in nature, it consists of real and imaginary parts. But the probability must be a real and positive quantity.

Let $\psi(x, t) = A + iB$, then its complex conjugate is $\psi^*(x, t) = A - iB$

$$\psi^*(x, t) \psi(x, t) = \psi^2$$

$$\psi^2 = \psi^* \psi = (A - iB)(A + iB) = A^2 - i^2 B^2$$

$$\psi^2 = \psi^* \psi = A^2 + B^2$$

Hence the proof.

CL9_Q3. Mention important properties of wave function.**Answer**

1. ψ must be finite, continuous and single valued in the regions of interest
2. The derivatives of the wave function must be finite, continuous and single valued in the regions of interest.

3. The wave function ψ must be normalisable. i.e. $\int_{-\infty}^{+\infty} \psi^* \psi dV = 1$

CL9_Q4. What is the difference between probability density and probability?

Answer

Particles exhibit wave like properties, such as interference and diffraction. Though we do not see a physical wave, it is evident that a certain wavelength has to be introduced with each particle to explain experiments like interference and diffraction with regard to particles. The wave associated with a particle is not observable, however, mathematically quantified in terms of de Broglie wavelength. As with every wave a certain quantity is expressed as a function of position and time, with constant parameters like frequency ω and propagation constant k .

$$\psi = A \sin (\omega t - k x) \quad \text{or generally}$$

$$\psi = A e^{i(\omega t - k x)}$$

There is no physical quantity which ψ can be associated with. However, it is seen that $|\psi|^2$ will be related to probability of locating particles. This can be thought of as the square of the resultant amplitude in Young's double slit experiment giving us intensity; intensity is directly related to probability.

$$|\psi|^2 = \psi^* \psi \quad \text{where } \psi^* \text{ is the complex conjugate of } \psi.$$

To obtain a complex conjugate replace all (i) in ψ with $(-i)$

$$|\psi|^2 \quad \text{is called Probability Density}$$

$$|\psi|^2 \Delta x, \text{ gives the probability of finding a particle in a region } \Delta x$$

$$|\psi|^2 \Delta A, \text{ gives the probability of finding a particle in a region } \Delta A$$

$$|\psi|^2 \Delta V \text{ gives the probability of finding a particle in a region } \Delta V$$

PES University