CL30_Q1. Mention the expressions for electrical and thermal conductivities of a metal and hence obtain the Wiedemann-Franz law.

Answer

The thermal conductivity of the metal $K = \frac{\pi^2}{3}.n.\frac{k_B{}^2T}{m}.\tau$

The electrical conductivity of the metal is then given by $\sigma = \frac{ne^2\tau}{m}$

The ratio of the thermal conductivity to electrical conductivity can be calculated as

$$\frac{\mathbf{K}}{\boldsymbol{\sigma}} = \frac{\boldsymbol{\pi}^2}{\mathbf{3e}^2}\mathbf{k_B}^2\mathbf{T}$$

This is the Wiedemann-Franz law.

CL30_Q2. Mention the demerits of Quantum free electron theory of metals?

Answer

Quantum free electron theory fails to explain the following,

1. The origin of the band gap in semiconductors and insulators and differences in conduction mechanism in metal, semiconductor and insulator
2. Experimentally observed positive Hall co-efficient observed in some metals like Zinc.

CL30_Q3. Obtain the relation between thermal and electrical conductivities of a metal.

Answer: Electrons close to the Fermi energy are responsible for the electrical or thermal conduction and hence these two quantities must be related to each other.

It is known that the thermal conductivity of the metal $K = \frac{1}{3}.\frac{C_{el}}{V}.v.L$

Where C_{el} is the electronic specific heat given by $C_{el} = \frac{\pi^2}{2}N.\frac{{k_B}^2T}{E_f}$,

V the volume, v is the velocity of electrons and L the mean free path.

The mean free path of electrons is given by $L = v.\tau$.

Taking velocity to be the Fermi velocity v_F (since most of the conduction electrons are located about the Fermi energy) the expression for the thermal conductivity can be written as

$K = \frac{1}{3}.\frac{1}{V}.\frac{\pi^2}{2}N.\frac{{k_B}^2T}{E_f}.v_F.v_F\tau = \frac{\pi^2}{6}.n.\frac{{k_B}^2T}{E_f}.v_F^2.\tau.$

where $n = \frac{N}{V}$ is the concentration of free electrons.

$$K = \frac{\pi^2}{3}.n.\frac{{k_B}^2T}{E_f}.\frac{mv_F^2}{2m}.\tau = \frac{\pi^2}{3}.n.\frac{{k_B}^2T}{m}.\tau$$

The electrical conductivity of the metal is then given by $\sigma = \frac{ne^2\tau}{m}$

The ratio of the thermal conductivity to electrical conductivity can be calculated as

$\frac{K}{\sigma} = \frac{\pi^2}{3e^2}{k_B}^2T = \mathrm{LT}$ where L is the Lorenz number, $L = \frac{\pi^2}{3e^2}{k_B}^2$

This is the Wiedemann-Franz law, which relates thermal conductivity of a metal to its electrical conductivity.