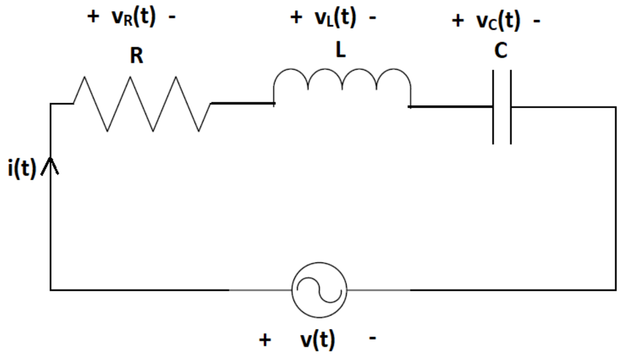
NOTES -Class 27

Analysis of Series RLC circuit & Impedance and Power Triangles

Series RLC Circuit:



By KVL, $v(t) = v_R(t) + v_L(t) + v_C(t)$

In Phasor form, $\overline{V} = \overline{V_R} + \overline{V_L} + \overline{V_C}$

$\overline{V_R} = \bar{I} * R \qquad \overline{V_L} = \bar{I} * (jX_L) \qquad \overline{V_C} = \bar{I} * (-jX_C)$

$\overline{V} = \bar{I} * (R + jX_L - jX_C)$

$Z_T = \frac{\overline{V}}{\bar{I}} = (R + jX_L - jX_C) = \sqrt{R^2 + (X_L - X_C)^2} \angle \mathrm{Tan}^{-1}\left(\frac{X_L - X_C}{R}\right)$

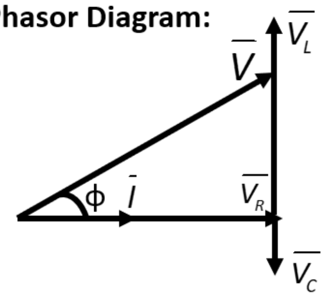
Case 1: $X_L > X_C$

If $X_L > X_C$ then $IX_L > IX_C$

i.e., $|\overline{V_L}| > |\overline{V_C}|$

The circuit behaves effectively as inductive circuit i.e., series RL type.

Phasor Diagram:



$\phi = \mathrm{Tan}^{-1}\left(\frac{|\overline{V_L}| - |\overline{V_C}|}{|\overline{V_R}|}\right) = \mathrm{Tan}^{-1}\left(\frac{V_L - V_C}{V_R}\right)$

$= \mathrm{Tan}^{-1}\left(\frac{X_L - X_C}{R}\right)$

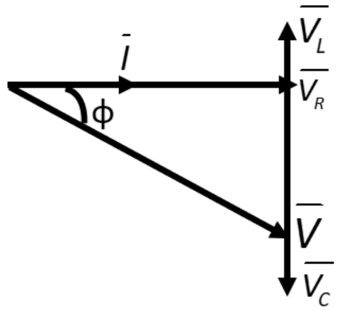
Case 2: $X_C > X_L$

If $X_C > X_L$ then $IX_C > IX_L$

i.e., $|\overline{V_C}| > |\overline{V_L}|$

The circuit behaves effectively as a capacitive circuit i.e., series RC type.

Phasor Diagram:



$$\phi = \text{Tan}^{-1}\left(\frac{|\overline{V_L}|-|\overline{V_C}|}{|\overline{V_R}|}\right) = \text{Tan}^{-1}\left(\frac{V_L - V_C}{V_R}\right)$$

$$= \text{Tan}^{-1}\left(\frac{X_L - X_C}{R}\right)$$

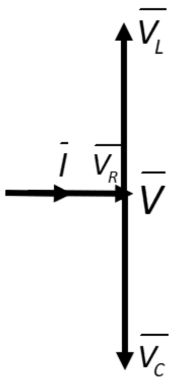
Note: ϕ will be negative in this case since $X_L < X_C$

Case 3: $X_L = X_C$

If $X_L = X_C$ then $IX_L = IX_C$ i.e., $|\overline{V_L}| = |\overline{V_C}|$

The circuit behaves effectively as a purely resistive circuit. This case is called **'Series Resonance'** case.

Phasor Diagram:



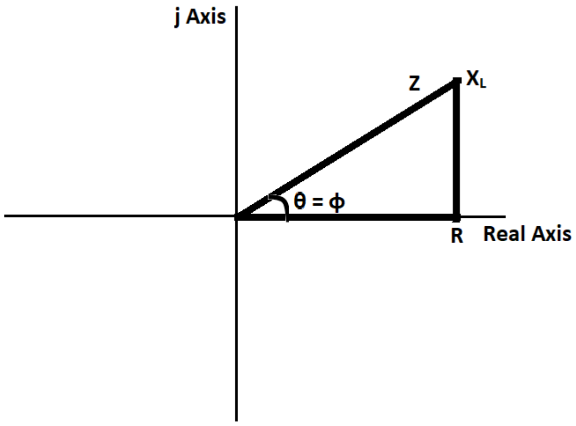
$\overline{V} = \overline{V_R}$

$Z = R$

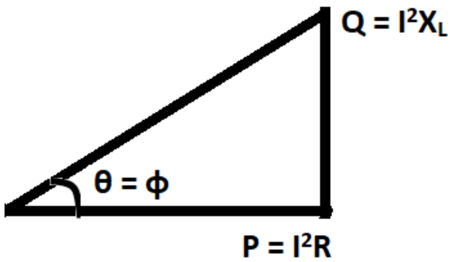
$\phi = 0°$

Impedance & Power Triangles – Series RL Circuit

For a series RL circuit, $Z = R + jX_L = \sqrt{R^2+X_L^2}\angle Tan^{-1}(\frac{X_L}{R})$



Impedance Triangle of a series RL circuit lies Quadrant I of complex plane.



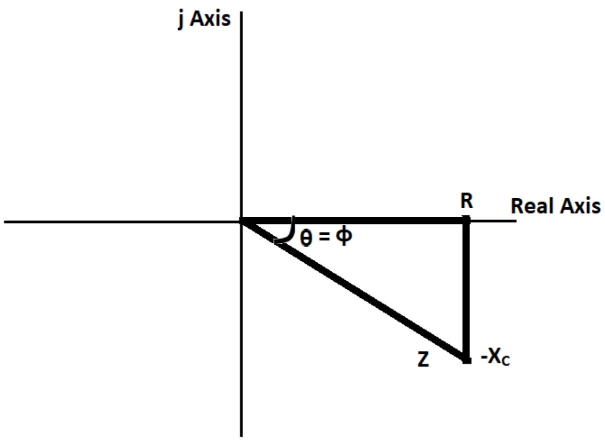
$$P = VI\cos\phi = (I.|Z|).I.\frac{R}{|Z|} = I^2R$$

$$Q = VI\sin\phi = (I.|Z|).I.\frac{X_L}{|Z|} = I^2X_L$$

$$S = VI = (I.|Z|).I = I^2\,|Z|$$

Impedance & Power Triangles – Series RC Circuit

For a series RC circuit, $Z = R - jX_C = \sqrt{R^2+X_C^2}\angle -Tan^{-1}(\frac{X_C}{R})$



Impedance Triangle of a series RC circuit lies Quadrant IV of complex plane.

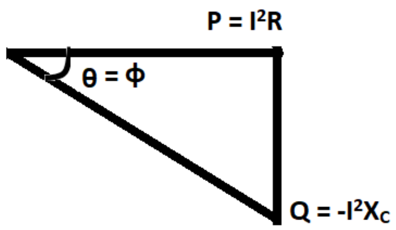


Fig: Power Triangle

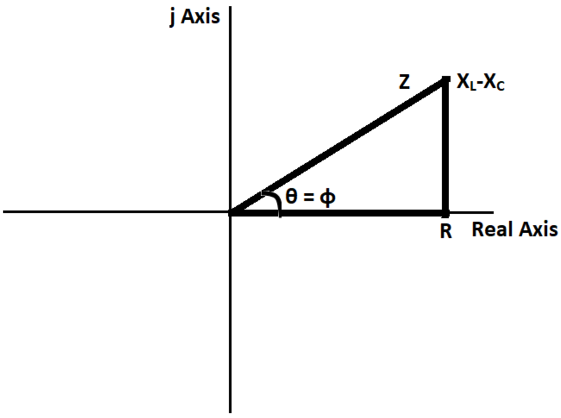
$$P = VI\cos\phi = (I.|Z|).I.\frac{R}{|Z|} = I^2R$$

$$Q = VI\sin\phi = (I.|Z|).I.\frac{-X_C}{|Z|} = -I^2X_C$$

$$S = VI = (I.|Z|).I = I^2\,|Z|$$

Impedance & Power Triangles – Series RLC Circuit

For a series RLC circuit, $Z = R + j(X_L - X_C) = \sqrt{R^2 + (X_L - X_C)^2}\angle \mathrm{Tan}^{-1}\left(\frac{X_L - X_C}{R}\right)$



Impedance Triangle of a series RLC circuit for $X_L > X_C$ lies in Quadrant I of complex plane.

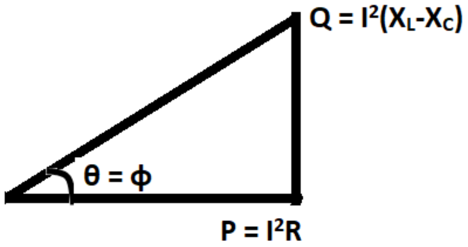


Fig: Power Triangle

$$P = VI\cos\phi = (I.|Z|).I.\frac{R}{|Z|} = I^2R$$

$$Q = VI\sin\phi = I^2(X_L - X_C)$$

$$S = VI = (I.|Z|).I = I^2|Z|$$

Question 9:

A series RLC circuit draws a current of 20A when connected to 200V, 50Hz supply. If the total active power drawn from the source is 500W and the circuit behaves effectively like an inductive circuit (series RL type), determine

i) Power factor of the circuit

ii) Inductance in the circuit if Capacitance is 100µF

Solution:

Given, V = 200V , I = 20A & P = 500W

i) Since $P = I^2R$,

$R = 1.25\Omega$

$$|Z| = \frac{V}{I} = 10\Omega$$

$$\text{Therefore, Power factor} = \frac{R}{|Z|} = 0.125 \text{ Lag}$$

ii) Net Reactance, $X = (X_L - X_C) = \sqrt{Z^2 - R^2} = 9.92\Omega$

$X_C = 31.83\Omega$

Hence, $X_L = 41.75\Omega$

Therefore, L = 132.89mH