1. When the wave function is an eigenfunction of the operator $\hat{A}$ then show that the expectation value of $\hat{A}$ is the eigenvalue of the wave function.

Answer: Eigenvalue equation for operator $\hat{A}$ is as follows

$$\hat{A}\,\psi(x) = a\psi(x)$$

Then expectation value of operator $\hat{A}$ is given by

$$\langle A\rangle = \int \psi^{*}\,\hat{A}\,\psi\,dx$$

$$= \int \psi^{*}\,a\,\psi\,dx$$

$$= a\int \psi^{*}\,\psi\,dx$$

$$\Rightarrow \qquad \langle A\rangle = a \qquad \left[\because \int \psi^{*}\,\psi\,dx = 1\right]$$

2. Find the eigenvalue of momentum corresponding to the wavefunction $\psi(x) = Ae^{-ikx}$

Answer: Eigenvalue equation for momentum operator $\hat{P}$ is as follows

$$\hat{P}\,\psi(x) = p\psi(x)$$

Momentum operator is $-i\hbar\frac{\partial}{\partial x}$

Therefore $-i\hbar\frac{\partial}{\partial x}\,(Ae^{-ikx}) = -i\hbar\,A(-ik)\,e^{-ikx} = -\hbar\,k\,(A\,e^{-ikx})$

$\Rightarrow$ The eigenvalue of momentum is $-\hbar\,k$