CL2_Q1: Which are the Maxwell's equations that contain 'sources'?

Answer

- $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_o}$
- $\vec{\nabla}x\vec{B} = \mu_o\vec{J} + \mu_o\varepsilon_o \frac{\partial\vec{E}}{\partial t}$

CL2_Q2: How do Maxwell's equations describe electromagnetic waves?

Answer

Using Maxwell's equations we can construct wave equations for both electric and magnetic fields as

$$\nabla^2\vec{\mathrm{E}} = \left(\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}\right) \text{ and } \nabla^2\vec{\mathrm{B}} = \left(\frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2}\right)$$

Maxwell concluded that they should be electric and magnetic vector in free space travelling at the speed of light $c = \sqrt{\frac{1}{\mu_o\varepsilon_o}}$

The electric and magnetic waves must therefore be representing light and hence Maxwell proposed that light could be treated as electromagnetic waves, where the electric and magnetic vectors are mutually perpendicular and perpendicular to the direction of propagation of the radiation.

CL2_Q3: Discuss the phase correlation and direction of the E and B fields of an EM Wave.

Answer

The electric field and the magnetic field are described by

$E = E_0 \sin(\omega t - kx)$ *and* $B = B_0 \sin(\omega t - kx)$.

There is no phase difference between them. However, they are perpendicular to each other.

CL2_Q4. Starting from Maxwell's equations, obtain the wave equation of a transverse electric wave in free space and compare this with the corresponding plane magnetic wave.

Answer

Maxwell's equations for free space,

$\nabla.E = 0$ (1); $\nabla.B = 0$ (2); $\nabla \times E = -(\partial B/\partial t)$ (3); $\nabla \times B = (\mu_o \varepsilon_o \partial E/\partial t)$ (4)

Taking the curl of curl of the electric field of the equation $\nabla \times E$,

$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \times \left(-\frac{\partial B}{\partial t}\right)$ this reduces to

$$\vec{\nabla}(\vec{\nabla}.\overrightarrow{\mathrm{E}}) - \nabla^2 E = \left(-\frac{\partial(\vec{\nabla} \times \vec{B})}{\partial t}\right)$$

Since $\vec{\nabla}.\vec{E} = 0$, this can be written as, $-\nabla^2\vec{\mathrm{E}} = \left(-\frac{\partial \vec{\nabla} \times \vec{B}}{\partial t}\right)$

Substituting for curl of B from equation (4), the above equation simplifies to

$\nabla^2\vec{\mathrm{E}} = \left(\mu_o \varepsilon_o \frac{\partial^2 E}{\partial t^2}\right)$.

Where, $\mu_o \varepsilon_o = \frac{1}{c^2}$, c *is the speed of light*.

$\nabla^2\vec{\mathrm{B}} = \left(\frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2}\right)$ which is the general form of a wave equation for electric field vector.

The corresponding transverse magnetic wave can be expressed as,

$\nabla^2\vec{\mathrm{B}} = \left(\frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2}\right)$ which is the general form of a wave equation in B.