



ENGINEERING MECHANICS

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Moment of Inertia

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- **Area moment of inertia**
- Rectangular moment of inertia/ second moment of inertia
- Polar moment of inertia
- Radius of Gyration (Rectangular radius of gyration and polar radius of gyration)
- **Parallel axis Theorem**
- **Derivation of moment of inertia for different geometric figures**

- First area moment of inertia is either positive or negative or zero
- Second area moment of inertia is the Moment of first area moment of inertia
- but Second area moment of inertia is always positive about the axis
- Second moment of inertia is denoted by capital 'I'
- Unit of second area moment of inertia is mm⁴

Rectangular and Polar Moments of Inertia

Consider the area A in the x - y plane, Fig. A/2. The moments of inertia of the element dA about the x - and y -axes are, by definition, $dI_x = y^2 dA$ and $dI_y = x^2 dA$, respectively. The moments of inertia of A about the same axes are therefore

$$I_x = \int y^2 dA$$

$$I_y = \int x^2 dA$$

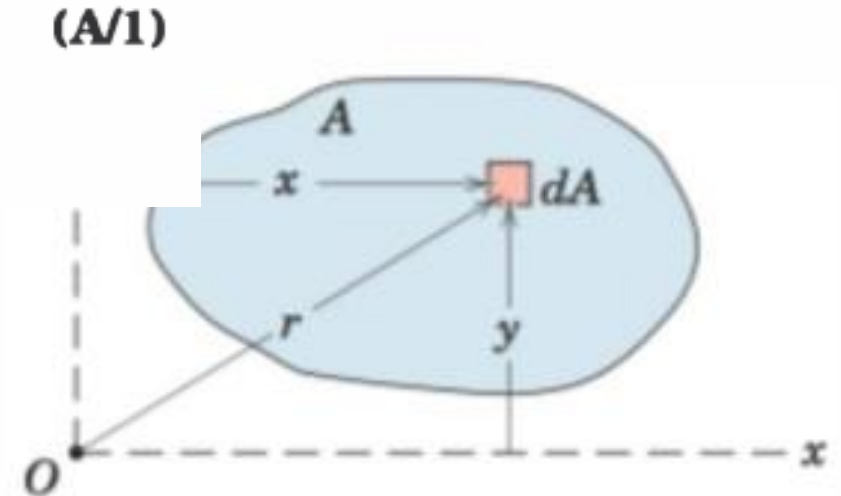


Figure A/2

The moment of inertia of dA about the pole O (z -axis) is, by similar definition, $dI_z = r^2 dA$. The moment of inertia of the entire area about O is

$$I_z = \int r^2 dA \quad (A/2)$$

The expressions defined by Eqs. A/1 are called *rectangular* moments of inertia, whereas the expression of Eq. A/2 is called the *polar* moment of inertia.* Because $x^2 + y^2 = r^2$, it is clear that

$$I_z = I_x + I_y \quad (A/3)$$

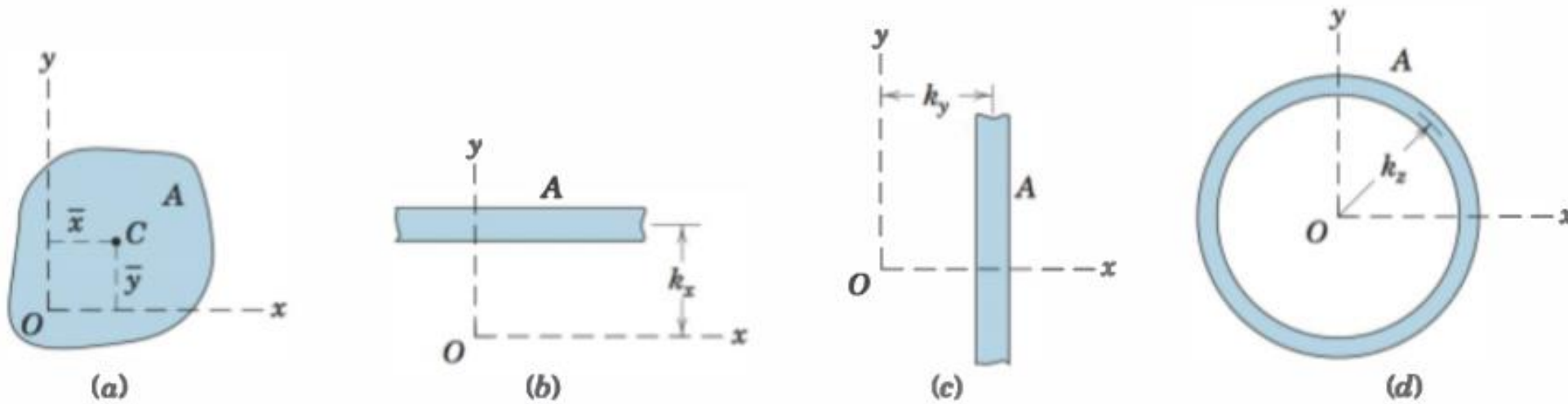


Figure A/3

$$I_x = k_x^2 A$$

$$I_y = k_y^2 A$$

$$I_z = k_z^2 A$$

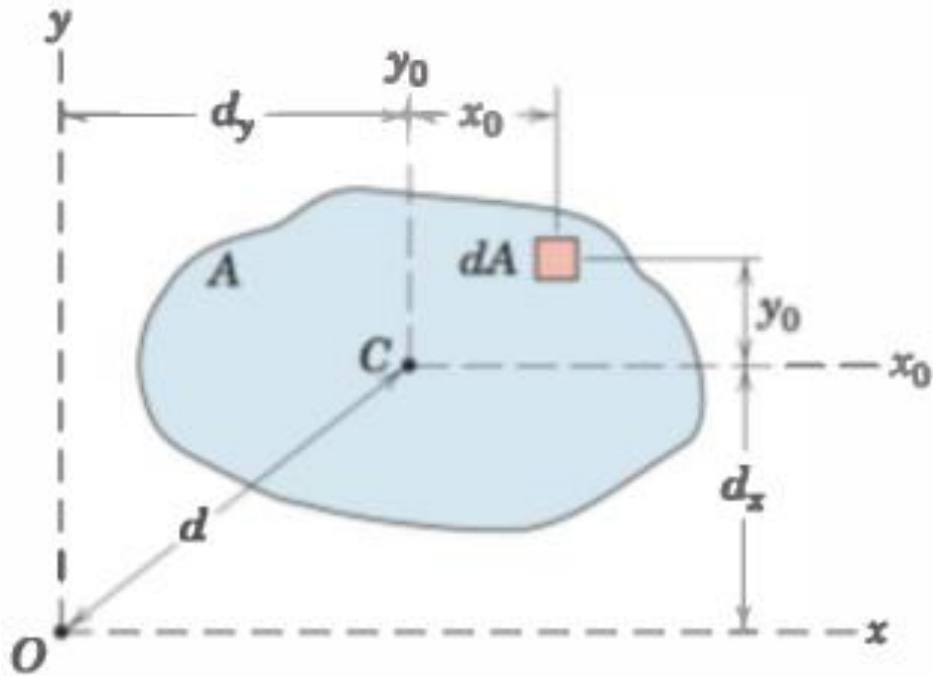
or

$$k_x = \sqrt{I_x/A}$$

$$k_y = \sqrt{I_y/A}$$

$$k_z = \sqrt{I_z/A}$$

$$k_z^2 = k_x^2 + k_y^2$$



- The moment of inertia of an area about a non centroidal axis may be easily expressed in terms of the moment of inertia about a parallel centroidal axis.

- By definition, the moment of inertia of the element dA about the x -axis is given by

$$dI_x = (y_0 + d_x)^2 dA$$

Expanding and integrating give us

$$I_x = \int y_0^2 dA + 2d_x \int y_0 dA + d_x^2 \int dA$$

- We see that the first integral is by definition the moment of inertia \bar{I}_x about the centroidal x_0 -axis. The second integral is zero, since $\int y_0 dA = A\bar{y}_0$ and \bar{y}_0 is automatically zero with the centroid on the x_0 -axis. The third term is simply Ad_x^2 . Thus, the expression for I_x and the similar expression for I_y become

$$\begin{aligned} I_x &= \bar{I}_x + Ad_x^2 \\ I_y &= \bar{I}_y + Ad_y^2 \end{aligned} \quad (A/6)$$

By Eq. A/3 the sum of these two equations gives

$$I_z = \bar{I}_z + Ad^2 \quad (A/6a)$$

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- **Conditions for parallel axis theorem**
 1. **two axis should be there and two axis must be parallel to each other**
 2. **Between two axis, one axis has to pass through the centroidal axis**

$$[I_x = \int y^2 dA] \quad \bar{I}_x = \int_{-h/2}^{h/2} y^2 b dy = \frac{1}{12} b h^3 \quad \text{Ans.}$$

By interchange of symbols, the moment of inertia about the centroidal y_0 -axis is

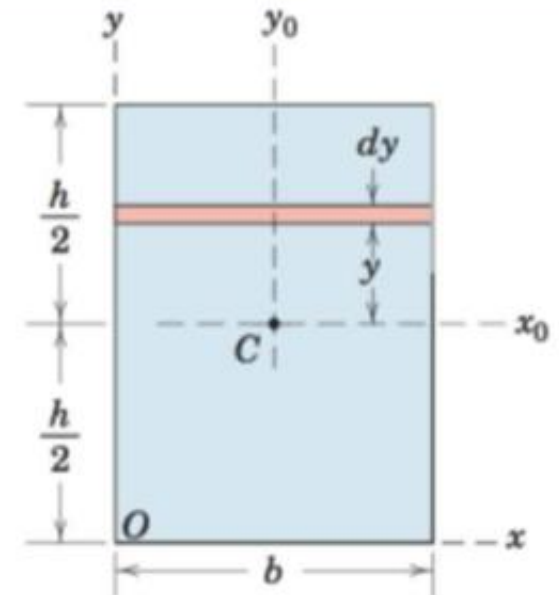
$$\bar{I}_y = \frac{1}{12} h b^3 \quad \text{Ans.}$$

The centroidal polar moment of inertia is

$$[\bar{I}_z = \bar{I}_x + \bar{I}_y] \quad \bar{I}_z = \frac{1}{12} (b h^3 + h b^3) = \frac{1}{12} A (b^2 + h^2) \quad \text{Ans.}$$

By the parallel-axis theorem, the moment of inertia about the x -axis is

$$[I_x = \bar{I}_x + A d_x^2] \quad I_x = \frac{1}{12} b h^3 + b h \left(\frac{h}{2} \right)^2 = \frac{1}{3} b h^3 = \frac{1}{3} A h^2 \quad \text{Ans.}$$



Solution. A strip of area parallel to the base is selected as shown in the figure, and it has the area $dA = x dy = [(h - y)b/h] dy$. By definition

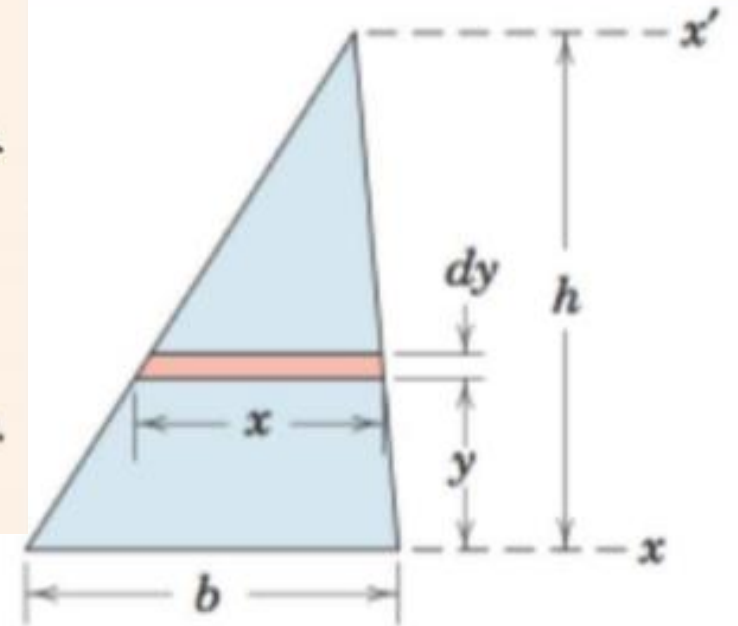
$$[I_x = \int y^2 dA] \quad I_x = \int_0^h y^2 \frac{h-y}{h} b dy = b \left[\frac{y^3}{3} - \frac{y^4}{4h} \right]_0^h = \frac{bh^3}{12} \quad \text{Ans.}$$

By the parallel-axis theorem, the moment of inertia \bar{I} about an axis through the centroid, a distance $h/3$ above the x -axis, is

$$[\bar{I} = I - Ad^2] \quad \bar{I} = \frac{bh^3}{12} - \left(\frac{bh}{2} \right) \left(\frac{h}{3} \right)^2 = \frac{bh^3}{36} \quad \text{Ans.}$$

A transfer from the centroidal axis to the x' -axis through the vertex gives

$$[I = \bar{I} + Ad^2] \quad I_{x'} = \frac{bh^3}{36} + \left(\frac{bh}{2} \right) \left(\frac{2h}{3} \right)^2 = \frac{bh^3}{4} \quad \text{Ans.}$$



Solution. A differential element of area in the form of a circular ring may be used for the calculation of the moment of inertia about the polar z -axis through O since all elements of the ring are equidistant from O . The elemental area is $dA = 2\pi r_0 dr_0$, and thus,

$$[I_z = \int r^2 dA] \quad I_z = \int_0^r r_0^2 (2\pi r_0 dr_0) = \frac{\pi r^4}{2} = \frac{1}{2} A r^2$$

Ans.

The polar radius of gyration is

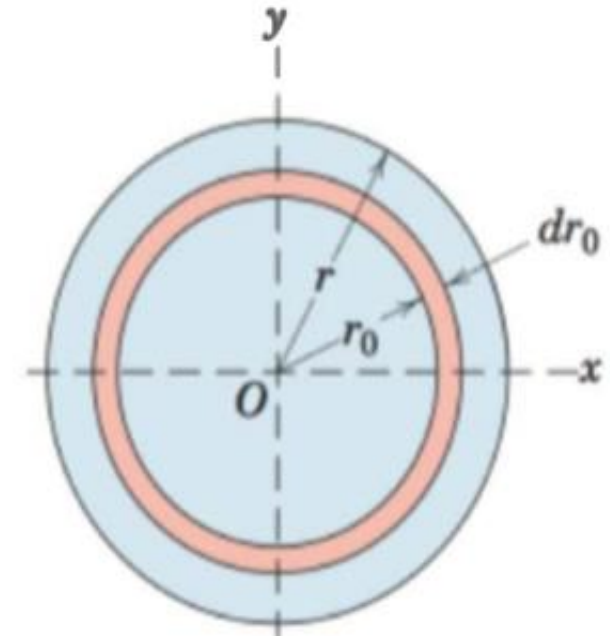
$$\left[k = \sqrt{\frac{I}{A}} \right] \quad k_z = \frac{r}{\sqrt{2}}$$

Ans.

By symmetry $I_x = I_y$, so that from Eq. A/3

$$[I_z = I_x + I_y] \quad I_x = \frac{1}{2} I_z = \frac{\pi r^4}{4} = \frac{1}{4} A r^2$$

Ans.



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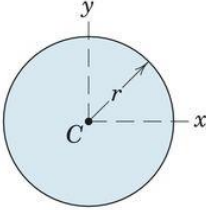
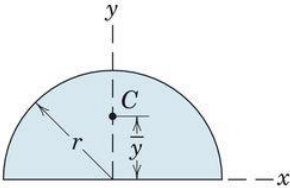
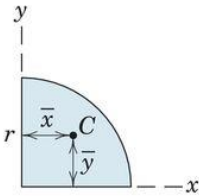
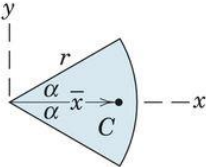
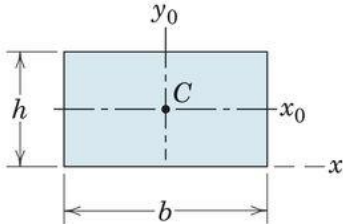
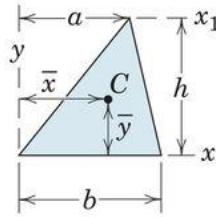
FIGURE	AREA MOMENTS OF INERTIA
<p>Circular Area</p> 	$I_x = I_y = \frac{\pi r^4}{4}$ $I_z = \frac{\pi r^4}{2}$
<p>Semicircular Area</p> 	$I_x = I_y = \frac{\pi r^4}{8}$ $\bar{I}_x = \left(\frac{\pi}{8} - \frac{8}{9\pi} \right) r^4$ $I_z = \frac{\pi r^4}{4}$
<p>Quarter-Circular Area</p> 	$I_x = I_y = \frac{\pi r^4}{16}$ $\bar{I}_x = \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi} \right) r^4$ $I_z = \frac{\pi r^4}{8}$
<p>Area of Circular Sector</p> 	$I_x = \frac{r^4}{4} \left(\alpha - \frac{1}{2} \sin 2\alpha \right)$ $I_y = \frac{r^4}{4} \left(\alpha + \frac{1}{2} \sin 2\alpha \right)$ $I_z = \frac{1}{2} r^4 \alpha$

FIGURE	AREA MOMENTS OF INERTIA
<p>Rectangular Area</p> 	$I_x = \frac{bh^3}{3}$ $\bar{I}_x = \frac{bh^3}{12}$ $\bar{I}_z = \frac{bh}{12} (b^2 + h^2)$
<p>Triangular Area</p> 	$I_x = \frac{bh^3}{12}$ $\bar{I}_x = \frac{bh^3}{36}$ $I_{x_1} = \frac{bh^3}{4}$

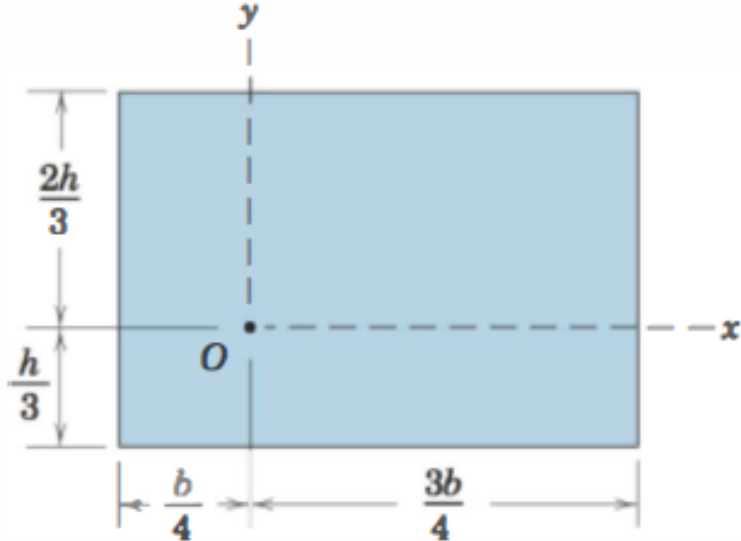
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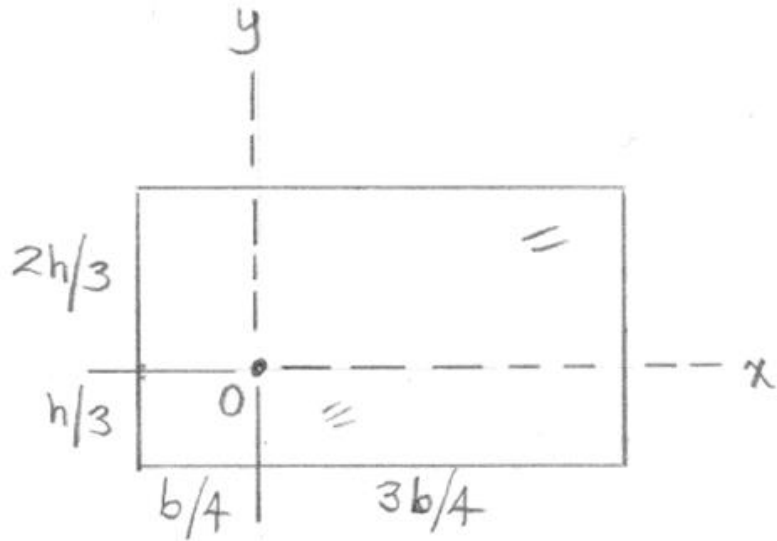
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A/1 Determine the moments of inertia of the rectangular area about the x- and y-axes and find the polar moment of inertia about point O.



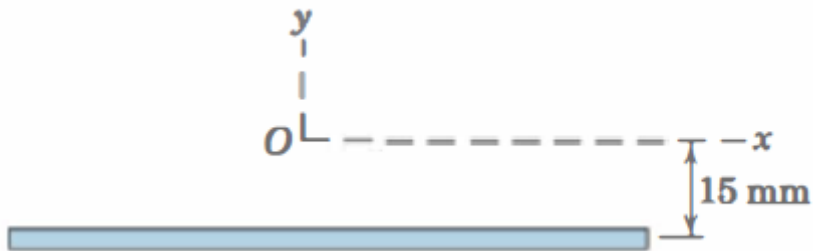


$$I_x = \bar{I}_x + Ad_x^2 = \frac{1}{12}bh^3 + bh\left(\frac{h}{6}\right)^2$$
$$= \frac{1}{9}bh^3$$

$$I_y = \bar{I}_y + Ad_y^2 = \frac{1}{12}hb^3 + bh\left(\frac{b}{4}\right)^2$$
$$= \frac{7}{48}hb^3$$

$$I_z = I_x + I_y = bh\left(\frac{h^2}{9} + \frac{7b^2}{48}\right)$$

A/3 The narrow rectangular strip has an area of 300 mm^2 , and its moment of inertia about the y -axis is $35(10^3) \text{ mm}^4$. Obtain a close approximation to the polar radius of gyration about point O .



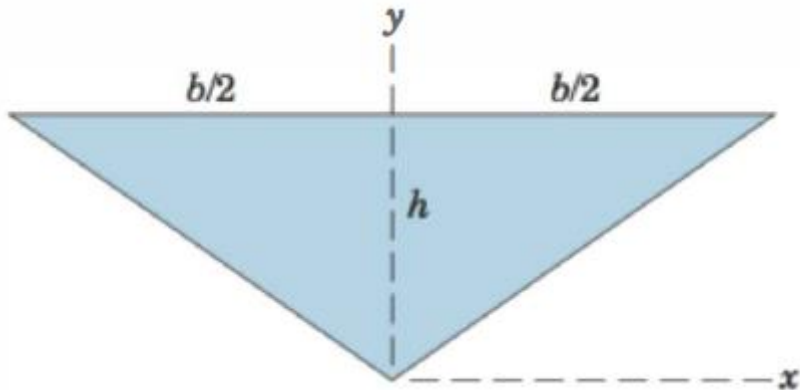
$$I_x \approx Ad^2 = 300(15)^2 = 67.5(10^3) \text{ mm}^4$$

$$J_o = I_x + I_y = 67.5(10^3) + 35(10^3) = 102.5(10^3) \text{ mm}^4$$

$$k_o = \sqrt{J_o/A} = \sqrt{\frac{102.5(10^3)}{300}} = \underline{18.48 \text{ mm}}$$



A/4 Determine the ratio b/h such that $I_x = I_y$ for the area of the isosceles triangle.

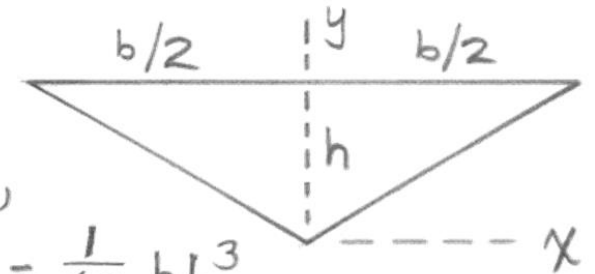


A/4

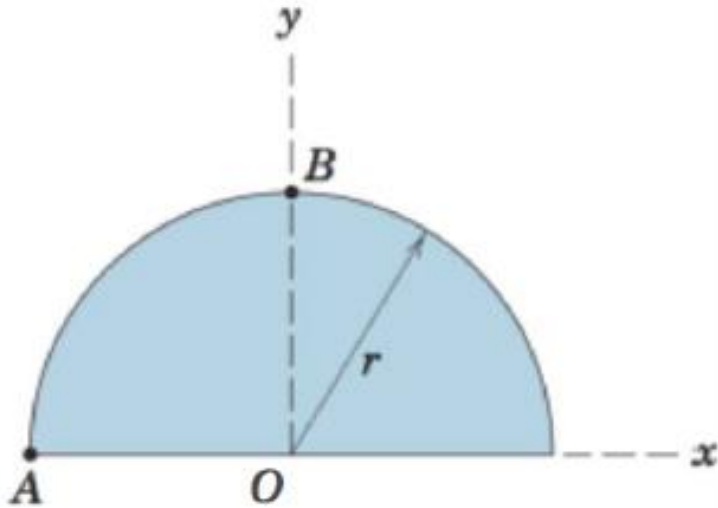
From Sample Problem A/2,

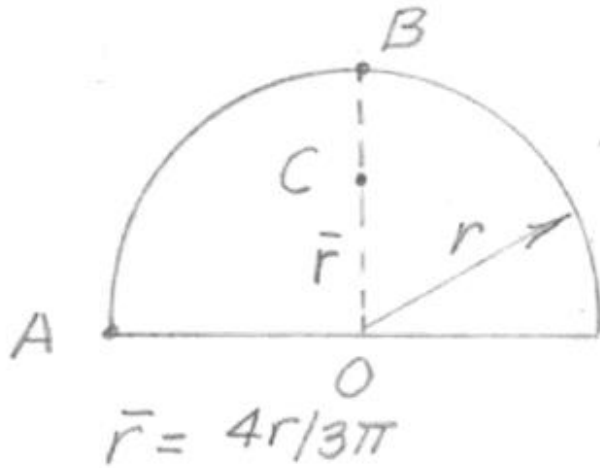
$$I_x = \frac{1}{4}bh^3, \quad I_y = 2\left\{\frac{1}{12}h\left(\frac{b}{2}\right)^3\right\} = \frac{1}{48}hb^3$$

$$I_x = I_y \quad \text{if} \quad \frac{1}{4}bh^3 = \frac{1}{48}hb^3, \quad \underline{\underline{\frac{b}{h} = 2\sqrt{3}}}$$



A/6 Determine the polar moments of inertia of the semicircular area about points A and B.





For complete circle

$$I_A = I_O + Ar^2 = \frac{1}{2}Ar^2 + Ar^2 \\ = \frac{3}{2}Ar^2$$

For half circle

$$I_A = \frac{1}{2} \left(\frac{3}{2} \pi r^4 \right) = \underline{\underline{\frac{3}{4} \pi r^4}}$$

For half circle, $I_O = \frac{1}{4} \pi r^4$

$$I_B = I_C + A(r - \bar{r})^2 = I_O - A\bar{r}^2 + A(r - \bar{r})^2$$

$$= I_O + A(r^2 - 2r\bar{r})$$

$$= \frac{1}{4} \pi r^4 + \frac{\pi r^4}{2} \left(1 - \frac{8}{3\pi} \right) = r^4 \left(\frac{3\pi}{4} - \frac{4}{3} \right)$$

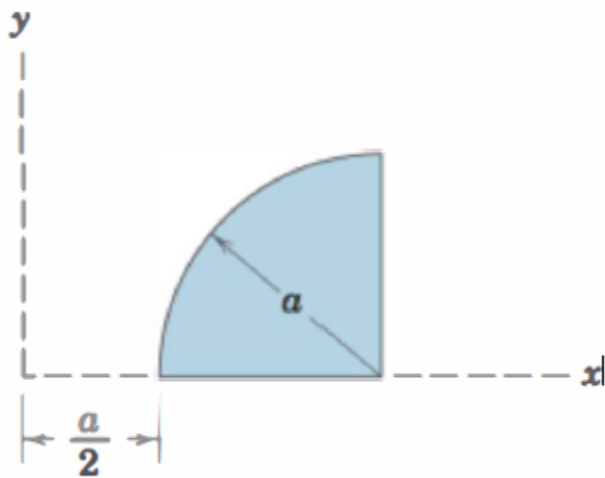
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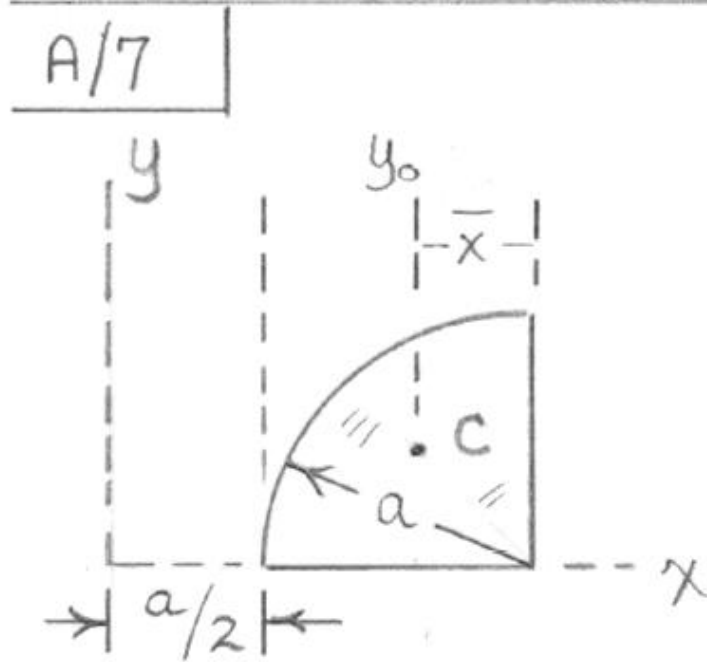
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A/7 Determine the moment of inertia of the quarter circular area about the y-axis.





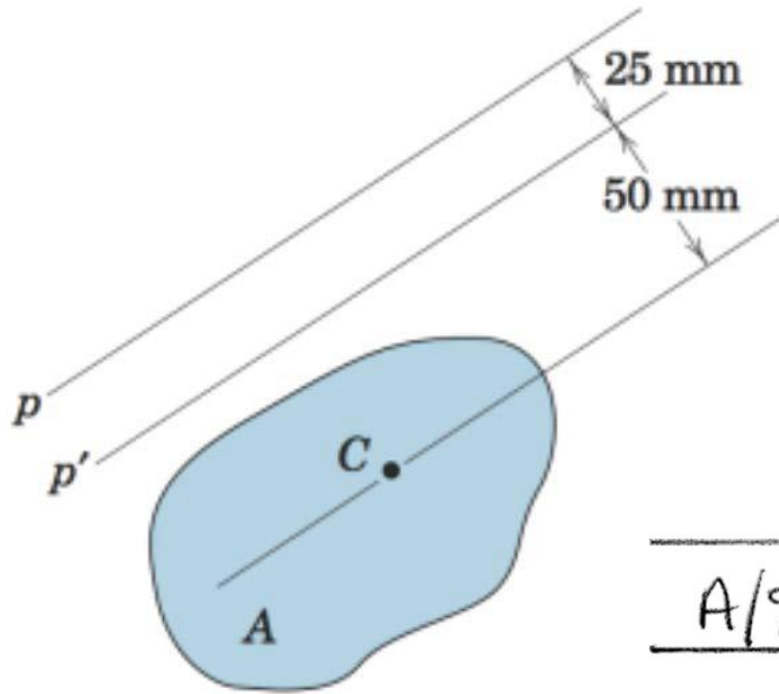
From Table D/3:

$$\bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi} \right) a^4$$

$$\bar{x} = \frac{4a}{3\pi}$$

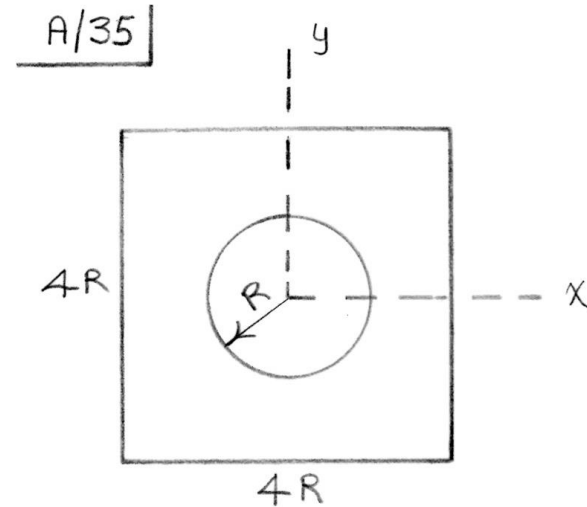
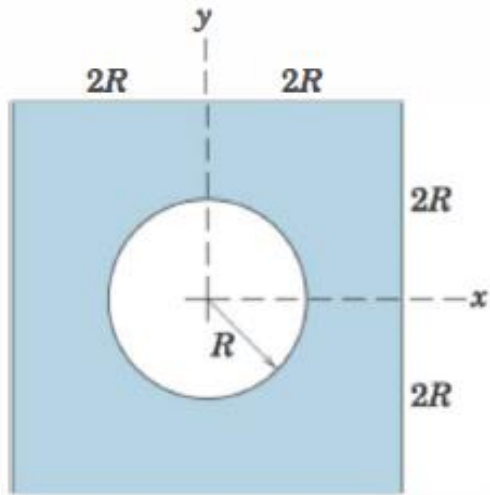
$$\begin{aligned} I_y &= \bar{I}_y + A d_y^2 \\ &= \left(\frac{\pi}{16} - \frac{4}{9\pi} \right) a^4 + \frac{\pi a^2}{4} \left[\frac{a}{2} + \left(a - \frac{4a}{3\pi} \right) \right]^2 \\ &= \left[\frac{5\pi}{8} - 1 \right] a^4 \end{aligned}$$

A/9 The moments of inertia of the area A about the parallel p - and p' -axes differ by $15(10^6) \text{ mm}^4$. Compute the area A , which has its centroid at C .



$$\begin{aligned} \text{A/9} \quad I_p &= I_c + A(75)^2, \quad I_{p'} = I_c + A(50)^2 \\ I_p - I_{p'} &= 15(10^6) = A[(75)^2 - (50)^2] \\ A &= 4800 \text{ mm}^2 \end{aligned}$$

A/35 Determine the moment of inertia about the x-axis of the square area without and with the central circular hole.

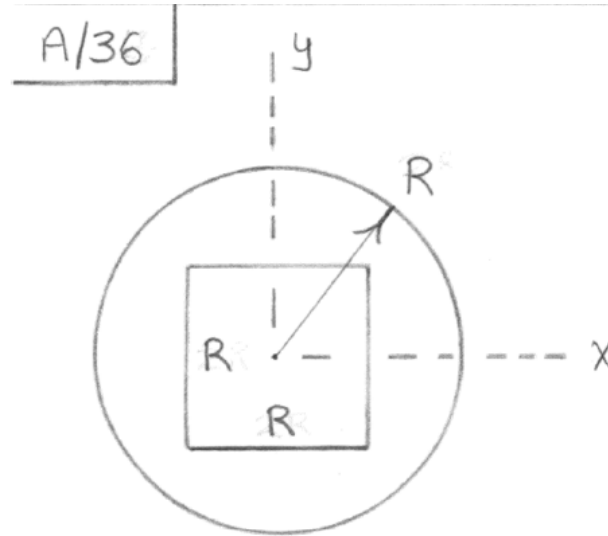
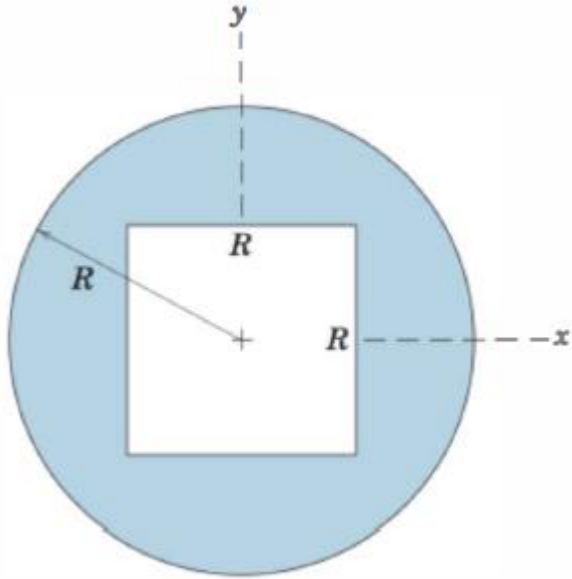


Without hole, $I_x = \frac{1}{12} (4R)(4R)^3 = \frac{64}{3}$
 $(21.3 R^4)$

With hole, $I_x = \frac{64}{3} R^4 - \frac{1}{4} (\pi R^2) R^2$
 $= \underline{20.5 R^4}$

(a 3.68% reduction)

A/36 Determine the polar moment of inertia of the circular area without and with the central square hole.



Without square hole:

$$I_z = 2I_x = 2 \left(\frac{1}{4} \pi R^2 \cdot R^2 \right) = \underline{1.571 R^4}$$

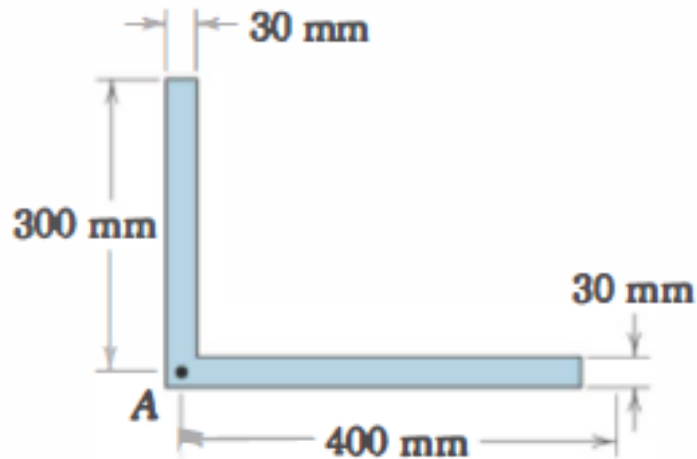
With hole:

$$I_z = 1.571 R^4 - 2 \left(\frac{1}{12} R \cdot R^3 \right) = \underline{1.404 R^4}$$

(a reduction of 10.61%)



A/37 Calculate the polar radius of gyration of the area of the angle section about point A. Note that the width of the legs is small compared with the length of each leg.



A/37

$$I_x \approx \frac{1}{3}(30)(300)^3 + 0 = 270(10)^6 \text{ mm}^4$$

$$I_y \approx \frac{1}{3}(30)(400)^3 + 0 = 640(10)^6 \text{ mm}^4$$

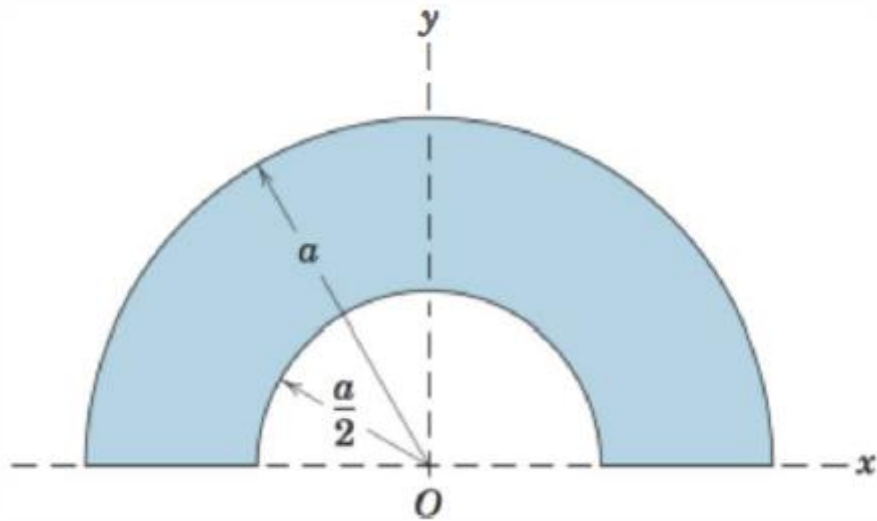
$$J_A = I_x + I_y = 910(10)^6 \text{ mm}^4$$

$$k_A = \sqrt{J_A/A} = \sqrt{\frac{910(10)^6}{30(300+400)}}$$

$$k_A = \underline{208 \text{ mm}}$$



A/38 Calculate the polar radius of gyration of the area of the angle section about point O. Note that the width of the legs is small compared with the length of each leg.

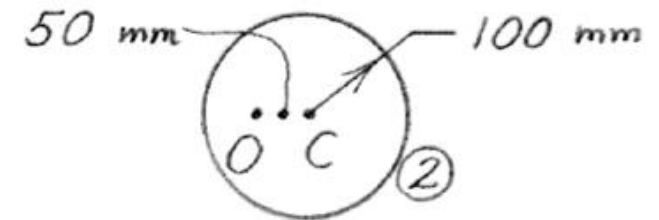
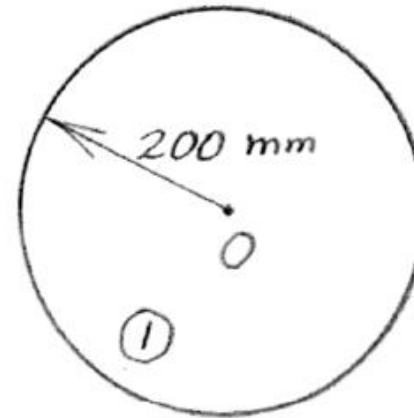
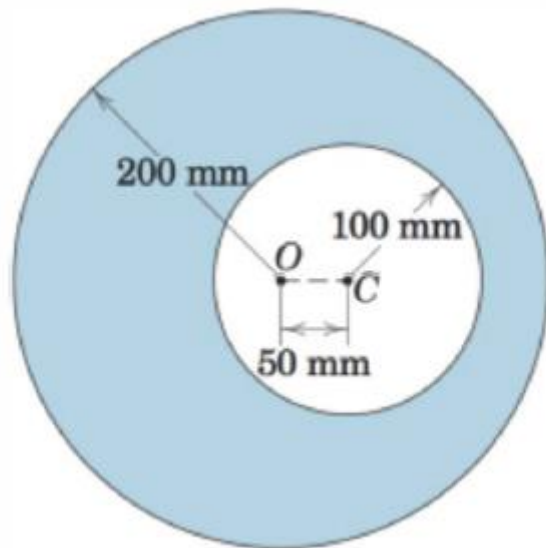


$$\begin{aligned} \text{A/38} \quad I_z &= \frac{1}{2} \left[\frac{\pi a^4}{2} - \frac{\pi \left(\frac{a}{2}\right)^4}{2} \right] = \frac{15}{64} \pi a^4 \\ k_z &= \sqrt{\frac{I_z}{A}} = \sqrt{\frac{\frac{15}{64} \pi a^4}{\frac{3}{8} \pi a^2}} = \frac{\sqrt{10}}{4} a \end{aligned}$$

From $k_x^2 + k_y^2 = k_z^2$ and the fact that $k_x = k_y$ for the present case,

$$2k_x^2 = \left(\frac{\sqrt{10}}{4} a\right)^2, \quad k_x = k_y = \frac{\sqrt{5}}{4} a$$

A/39 Calculate the polar radius of gyration of the shaded area about the center O of the larger circle.



$$\text{Area } A = A_1 - A_2 = \pi(200^2 - 100^2) = 3(10^4)\pi \text{ mm}^2$$

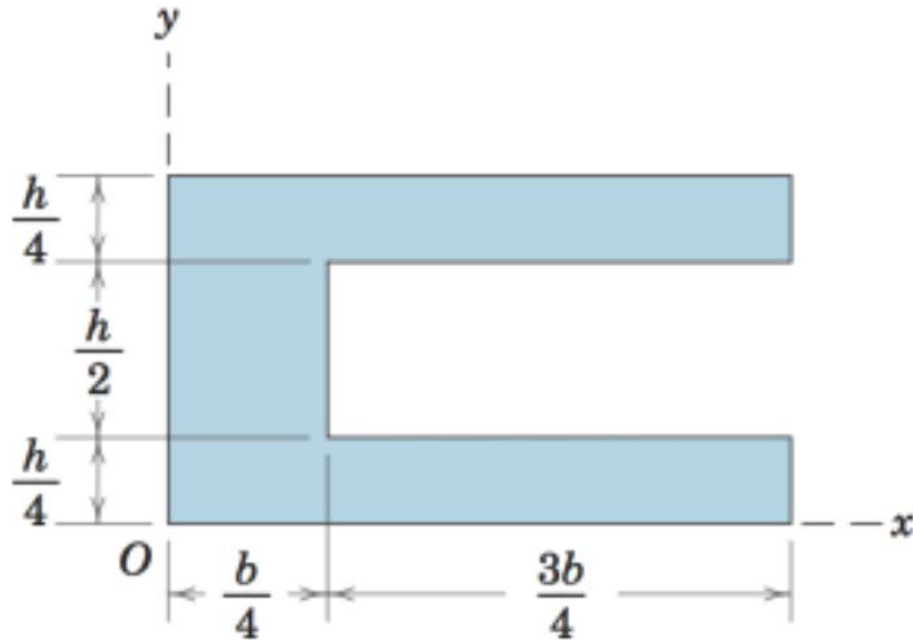
$$\textcircled{1} I_{O_1} = \frac{1}{2}(\pi \cdot 200^2)(200^2) = 8(10^8)\pi \text{ mm}^4$$

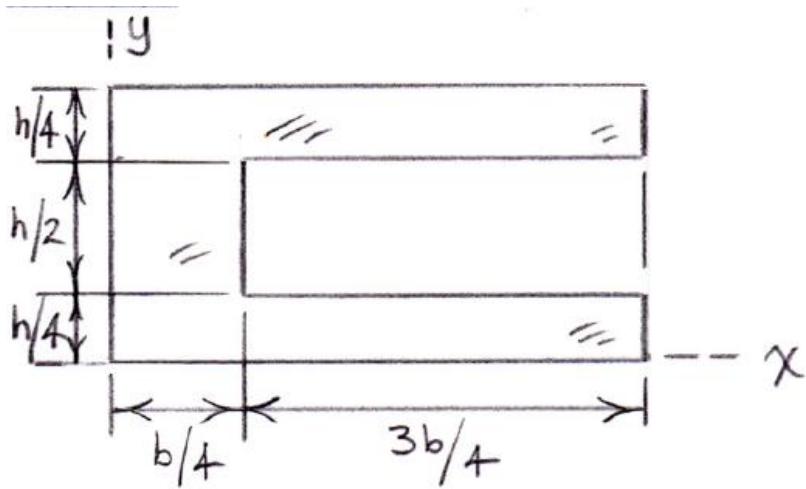
$$\textcircled{2} I_{O_2} = \frac{1}{2}(\pi \cdot 100^2)(100^2) + \pi(100^2)(50^2) = 0.75(10^8)\pi \text{ mm}^4$$

$$\text{So } I_O = I_{O_1} - I_{O_2} = 7.25(10^8)\pi \text{ mm}^4$$

$$k_o = \sqrt{I_O / A} = \sqrt{\frac{7.25(10^8)\pi}{3(10^4)\pi}} = \underline{155.5 \text{ mm}}$$

A/40 Determine the percent reductions in both area and area moment of inertia about the y-axis caused by removal of the rectangular cutout from the rectangular plate of base and height h .





Percent reductions :

$$n_A = \frac{bh - \frac{5}{8}bh}{bh} (100\%) = \underline{37.5\%}$$

$$n_{I_y} = \frac{\frac{1}{3}hb^3 - \frac{65}{384}hb^3}{\frac{1}{3}hb^3} = \underline{49.2\%}$$

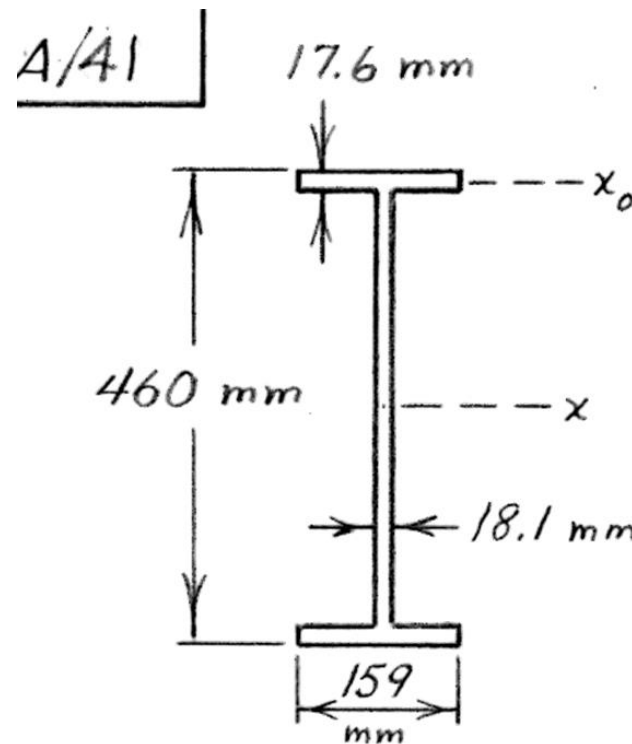
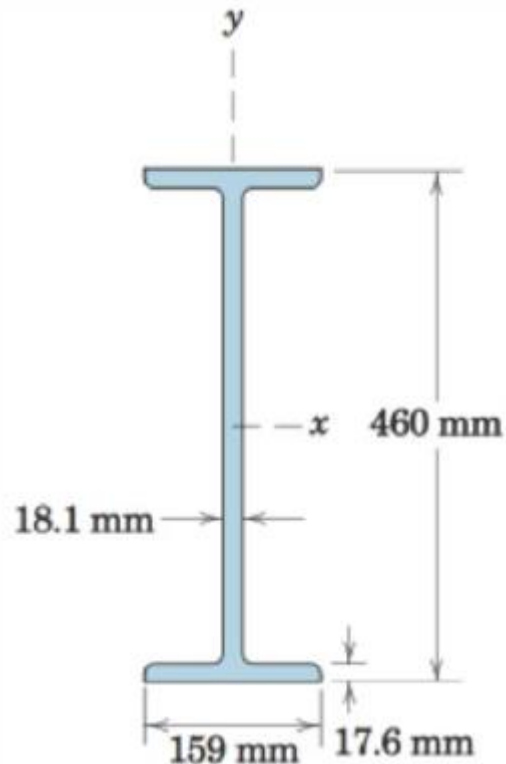
Full rectangle: $A = bh$, $I_y = \frac{1}{3}hb^3$

With cutout: $A = bh - \frac{3b}{4}\left(\frac{h}{2}\right) = \frac{5}{8}bh$

$$I_y = \frac{1}{3}hb^3 - \left[\frac{1}{12} \frac{h}{2} \left(\frac{3b}{4}\right)^3 + \frac{3}{8}bh \left(\frac{b}{4} + \frac{3b}{8}\right)^2 \right]$$
$$= \frac{65}{384}hb^3$$



A/41 The cross-sectional area of an I-beam has the dimensions shown. Obtain a close approximation to the handbook value of $I_x = 385(10^6) \text{ mm}^4$ by treating the section as being composed of three rectangles.



Flanges: $\bar{I}_x = I_{x_0} + Ad^2$

$$= 2 \left\{ \frac{1}{12} (159)(17.6^3) + 159(17.6) \left(230 - \frac{17.6}{2} \right)^2 \right\}$$

$$= 2 \left\{ 7.22(10^4) + 1.369(10^8) \right\} \text{ mm}^4$$

$$= 2.74(10^8) \text{ mm}^4$$

Web: $\bar{I}_x = \frac{1}{12} (18.1)(460 - 2[17.6])^3$

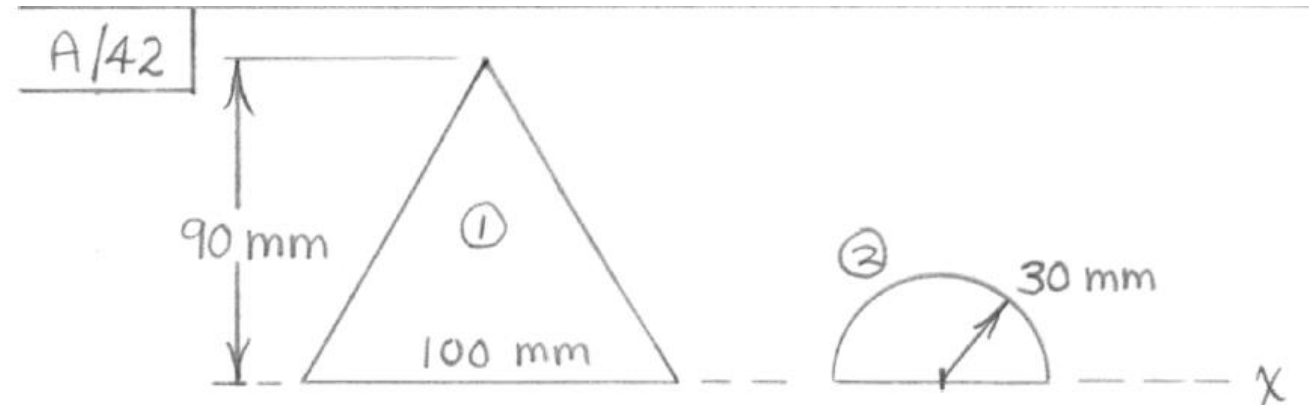
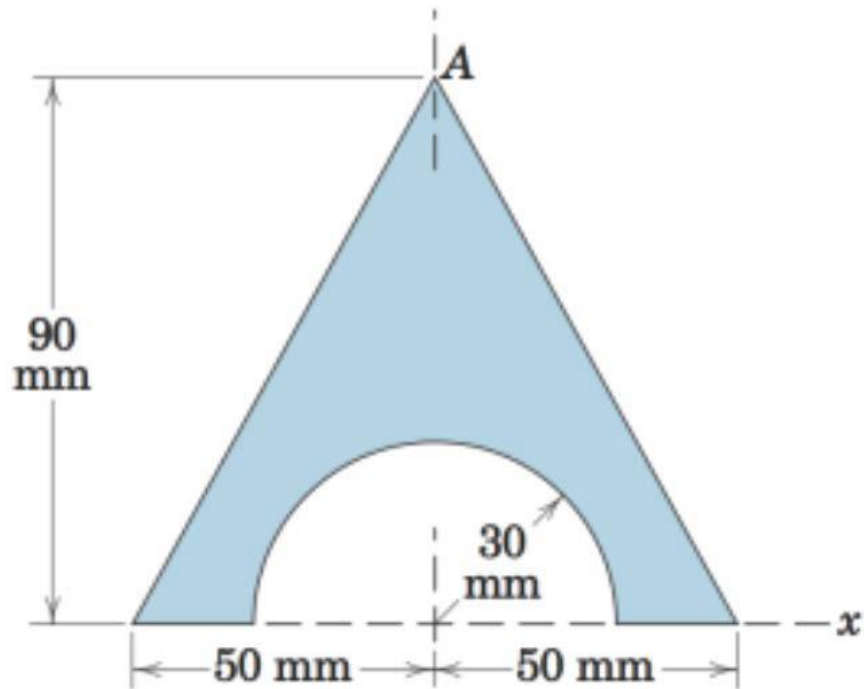
$$= 1.156(10^8) \text{ mm}^4$$

Total $\bar{I}_x = 3.90(10^8) \text{ mm}^4$

ENGINEERING MECHANICS

Moment of Inertia

A/42 Calculate the moment of inertia of the shaded area about the x-axis.



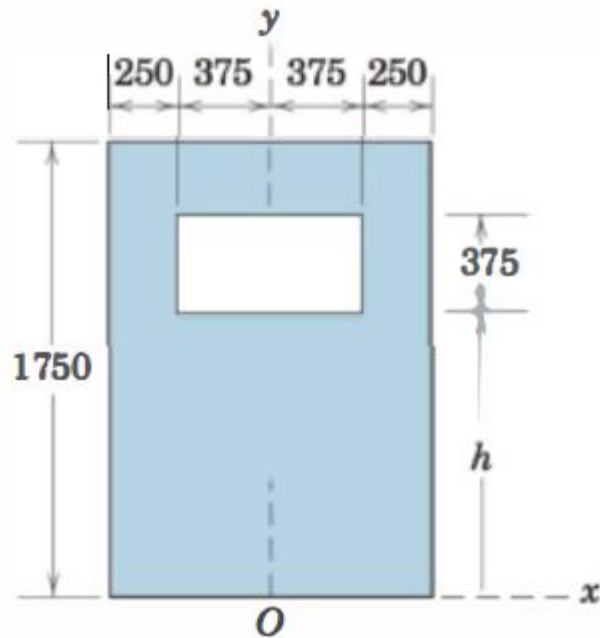
$$I_{x_1} = \frac{1}{12} (100) (90^3) = 6.08 (10^6) \text{ mm}^4$$

$$I_{x_2} = - \frac{\pi (30^4)}{8} = -0.318 (10^6) \text{ mm}^4$$

$$\text{So } I_x = (6.08 - 0.318) 10^6 = \underline{5.76 (10^6) \text{ mm}^4}$$



A/43 The variable h designates the arbitrary vertical location of the bottom of the rectangular cutout within the rectangular area. Determine the area moment of inertia about the x -axis for (a) $h = 1000$ mm and (b) $h = 1500$ mm.



Dimensions in millimeters

(a) $h = 1000$ mm (hole complete)

$$I_x = \frac{1}{3} (1250)(1750^3) - \left[\frac{1}{12} (750)(375)^3 + 750(375) \left(1000 + \frac{375}{2} \right)^2 \right]$$

$$= 1.833 (10^{12}) \text{ mm}^4 \text{ or } 1.833 \text{ m}^4$$

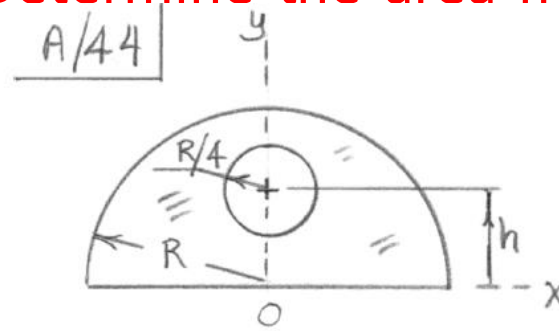
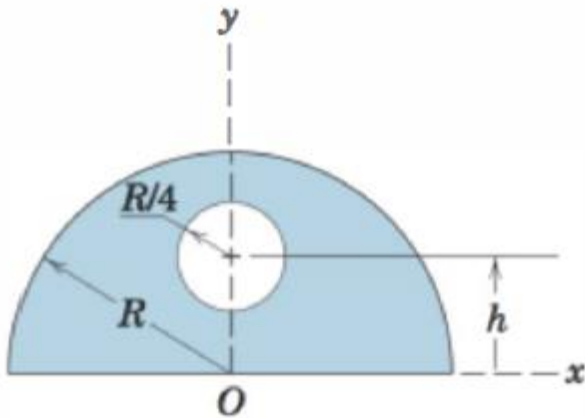
(b) $h = 1500$ mm (250 mm of hole in play)

$$I_x = \frac{1}{3} (1250)(1750^3) - \left[\frac{1}{12} (750)(250)^3 + 750(250) \left(1500 + \frac{250}{2} \right)^2 \right]$$

$$= 1.737 (10^{12}) \text{ mm}^4 \text{ or } 1.737 \text{ m}^4$$



A/44 The variable h designates the arbitrary vertical location of the center of the circular cutout within the semicircular area. Determine the area moment of inertia about the x -axis for (a) $h = 0$ and (b) $h = R/2$.



(a) $h = 0$ (One-half of hole considered)

$$I_x = \frac{\pi R^4}{8} - \frac{\pi (R/4)^4}{8} = \frac{255}{2048} \pi R^4$$

$(0.391 R^4)$

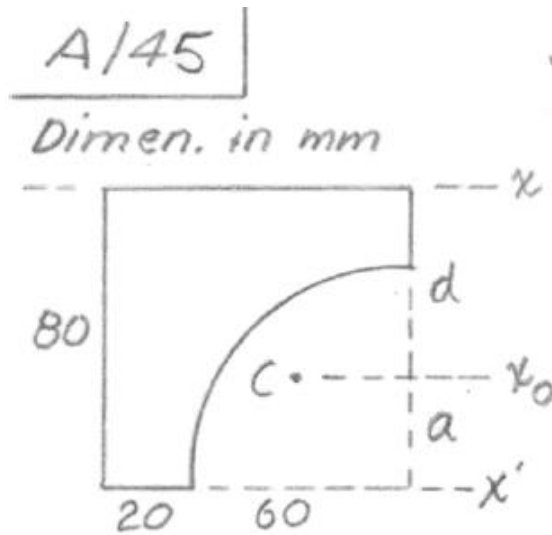
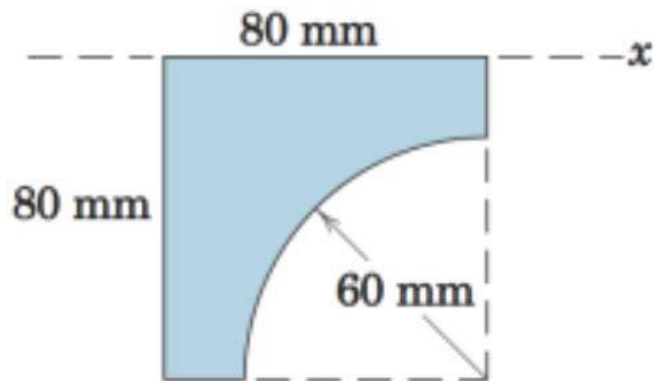
(b) $h = \frac{R}{2}$ (Entire hole now in play)

$$I_x = \frac{\pi R^4}{8} - \left[\frac{\pi (R/4)^4}{4} + \pi \left(\frac{R}{4}\right)^2 \left(\frac{R}{2}\right)^2 \right]$$

$$= \frac{111}{1024} \pi R^4 \quad (0.341 R^4)$$



A/45 Calculate the moment of inertia of the shaded area about the x-axis.



Square: $I_x = \frac{1}{3} b^4 = \frac{1}{3} (80)^4 = 13.65 (10^6) \text{ mm}^4$

Quarter-circle: $a = \frac{4r}{3\pi} = \frac{4(60)}{3\pi}$
 $= 25.46 \text{ mm}$

$d = 80 - 25.46 = 54.54 \text{ mm}$

$$I_x = I_{x_0} + Ad^2 = I_{x'} - Aa^2 + Ad^2$$

$$= -\frac{1}{4} \frac{\pi r^4}{4} - \frac{\pi r^2}{4} (d^2 - a^2) = -\frac{\pi r^2}{4} \left(\frac{r^2}{4} + d^2 - a^2 \right)$$

$$= -\frac{\pi (60)^2}{4} \left[\frac{60^2}{4} + (54.54)^2 - (25.46)^2 \right]$$

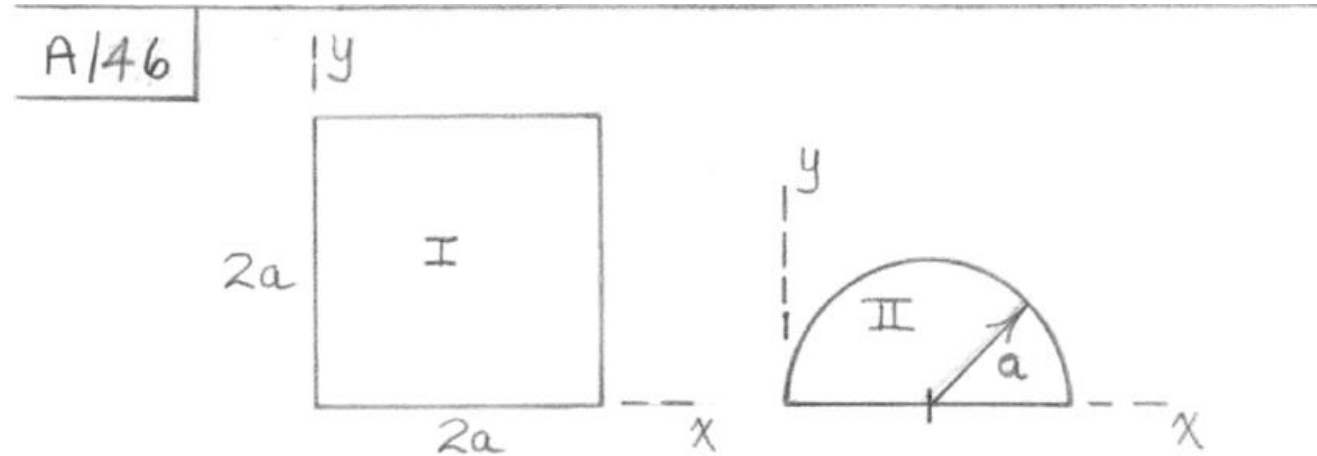
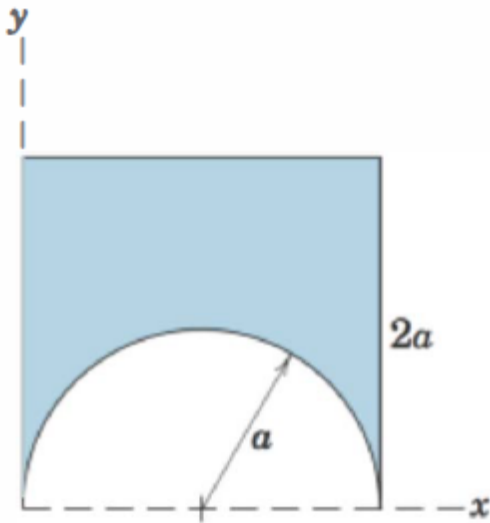
$$= -9.120 (10^6) \text{ mm}^4$$

Total $I_x = (13.65 - 9.120) (10^6) = \underline{4.53 (10^6) \text{ mm}^4}$

ENGINEERING MECHANICS

Moment of Inertia

A/46 Determine the moments of inertia of the shaded area about the x- and y-axes.



I. Square $I_x = I_y = \frac{1}{3} (4a^2) (2a)^2 = \frac{16}{3} a^4$

II. Semicircle $I_x = \frac{1}{8} \pi a^4$

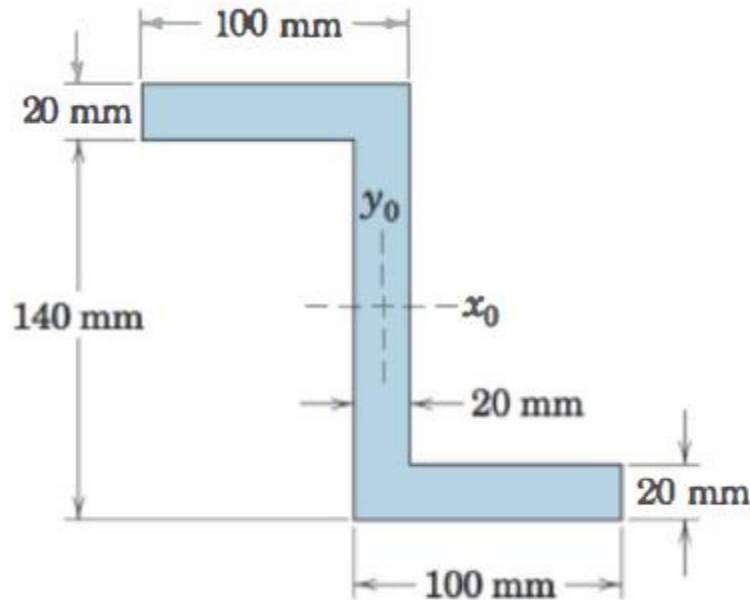
$$I_y = \frac{1}{8} \pi a^4 + \frac{1}{2} \pi a^2 (a^2) = \frac{5}{8} \pi a^4$$

Combined: $I_x = \frac{16}{3} a^4 - \frac{\pi}{8} a^4 = \underline{4.94 a^4}$

$$I_y = \frac{16}{3} a^4 - \frac{5}{8} \pi a^4 = \underline{3.37 a^4}$$



A/48 Determine the moments of inertia of the Z-section about its centroidal x_0 - and y_0 -axes.



A/48

$$\textcircled{1} I_{x_0} = \frac{1}{12} (80)(20)^3 + (80)(20)(70)^2 = 7.89(10^6) \text{ mm}^4$$

$$I_{y_0} = \frac{1}{12} (20)(80)^3 + (20)(80)(50)^2 = 4.85(10^6) \text{ mm}^4$$

Dimensions in mm

$$\textcircled{2} I_{x_0} = \frac{1}{12} (20)(160)^3 = 6.83(10^6) \text{ mm}^4$$

$$I_{y_0} = \frac{1}{12} (160)(20)^3 = 0.1067(10^6) \text{ mm}^4$$

$$\text{Total } \bar{I}_x = [2(7.89) + 6.83](10^6)$$

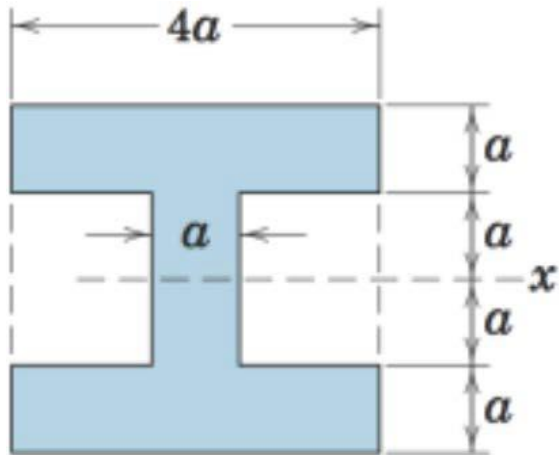
$$= \underline{22.6(10^6) \text{ mm}^4}$$

$$\bar{I}_y = [2(4.85) + 0.1067](10^6)$$

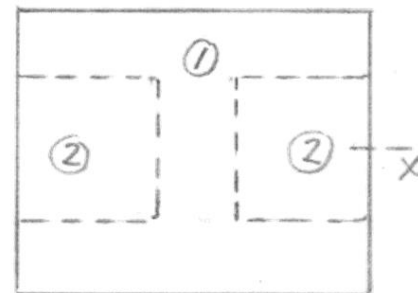
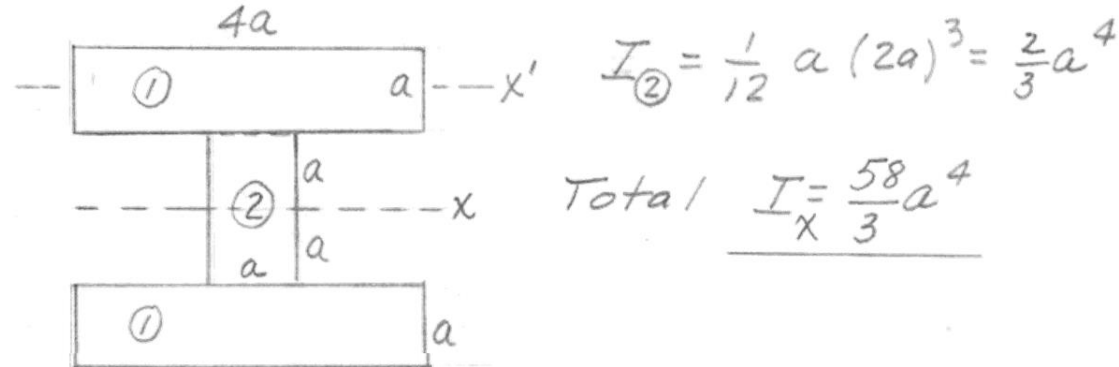
$$= \underline{9.81(10^6) \text{ mm}^4}$$



A/49 Determine the moment of inertia of the shaded area about the x-axis in two different ways.



A/49 Sol. I $I_{\textcircled{1}} = 2 \left[\frac{1}{12} 4a(a^3) + 4a^2 \left(\frac{3a}{2} \right)^2 \right] = \frac{56}{3} a^4$



Sol. II
 $I_{\textcircled{1}} = \frac{1}{12} (4a)(4a)^3 = \frac{64}{3} a^4$
 $I_{\textcircled{2}} = -\frac{1}{12} (3a)(2a)^3 = -2a^4$
 Total $I_x = \left(\frac{64}{3} - \frac{6}{3} \right) a^4 = \frac{58}{3} a^4$



THANK YOU

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