# ENGINEERING MATHEMATICS I

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# HIGHER ORDER DIFFERENTIAL EQUATIONS

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## CLASS CONTENT:

- Problems on **Legendre's Linear Differential Equation** and Cauchy's Linear differential Equation

1. Solve: $x^3\dfrac{d^3y}{dx^3}+2x^2\dfrac{d^2y}{dx^2}+2y=10\left(x+\dfrac{1}{x}\right)$

SOLUTION:

The given differential equation is Cauchy's LDE of order 3.

$$z=\log x\ ,\quad x\frac{dy}{dx}=D(y),\quad x^2\frac{d^2y}{dx^2}=D(D-1)y,\quad D=\frac{d}{dz}$$

$$x^3\frac{d^3y}{dx^3}=D(D-1)(D-2)y$$

$$[D(D-1)(D-2)+2D(D-1)+2]y=10\left(e^z+e^{-z}\right)$$

$$\left(D^3 - D^2 + 2\right)y = 10\left(e^z + e^{-z}\right)$$

This is a non homogeneous LDE with constant coefficients.

**To find CF**

AE is $m^3 - m^2 + 2 = 0$

Roots are $m = -1,\ 1 \pm i$

$$CF = c_1 e^{-z} + e^z\left(c_2 \cos z + c_3 \sin z\right)$$

$$\therefore CF = \frac{c_1}{x} + x\left(c_2 \cos(\log x) + c_3 \sin(\log x)\right)$$

TO FIND PI

$$PI = \frac{10\left(e^{z} + e^{-z}\right)}{D^3 - D^2 + 2} = \frac{10e^{z}}{D^3 - D^2 + 2} + \frac{10\,e^{-z}}{D^3 - D^2 + 2}$$

$$= \frac{10e^{z}}{1^3 - 1^2 + 2} + z.\frac{10\,e^{-z}}{3D^2 - 2D} = \frac{10e^{z}}{2} + z.\frac{10\,e^{-z}}{3(-1)^2 - 2(-1)}$$

$$\therefore\ PI = \frac{10e^{z}}{2} + z.\frac{10\,e^{-z}}{5}$$

**Thus, $y = CF + PI$**

2. Solve: $(2x+3)^2\dfrac{d^2y}{dx^2}-2(2x+3)\dfrac{dy}{dx}-12y=6x$

The given differential equation is Legendre's Linear Differential Equation.

$z=\log(2x+3),\qquad (2x+3)\dfrac{dy}{dx}=2.D(y)\qquad D=\dfrac{d}{dz}$

$2x+3=e^z\ ,\qquad (2x+3)^2\dfrac{d^2y}{dx^2}=4D(D-1)y$

$x=\dfrac{e^z-3}{2}$

$(4D(D-1)-2.2D-12)y=6.\dfrac{e^z-3}{2}$

$(D^2-2D-3)y=\dfrac{3}{4}(e^z-3)$

$$\left(D^2 - 2D - 3\right)y = \frac{3}{4}\left(e^z - 3\right) = \frac{3e^z}{4} - \frac{9}{4}$$

The above differential equation is LDE with constant coefficients.

**To find CF**

AE is $m^2 - 2m - 3 = 0$

Roots are $m = 3, -1$

$y_c = c_1 e^{3z} + c_2 e^{-z}$

Thus, $y_c = c_1 (2x+3)^3 + c_2 . \dfrac{1}{2x+3}$

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**To find PI**

$$y_p = \frac{3e^z}{4(D^2-2D-3)} - \frac{9}{4(D^2-2D-3)}$$

$$= \frac{3e^z}{4(1^2-2.1-3)} - \frac{9.e^0}{4(0^2-2.0-3)}$$

$$= \frac{3e^z}{4(-4)} - \frac{9.e^0}{4(-3)} = \frac{-3e^z}{16} + \frac{9.e^0}{12}$$

Thus, $y = y_c + y_p$

$$\text{Thus, } y_p = \frac{3}{4} - \frac{3(2x+3)}{16}$$

# THANK YOU

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