ENGINEERING MATHEMATICS - I

Unit - 3: Partial Differential Equations

Department of Science and Humanities

PES UNIVERSITY

Contents

Solve $(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} = ly - mx$

The auxiliary equations are $\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx}$
Using multipliers x, y, z, we get
Each fraction

$$= \frac{x\,dx + y\,dy + z\,dz}{x(mz-ny) + y(nx-lz) + z(ly-mx)} = \frac{x\,dx + y\,dy + z\,dz}{0}$$

$$\implies x\,dx + y\,dy + z\,dz = 0$$

On integration, it gives

$$x^2 + y^2 + z^2 = c_1 \qquad (1)$$

Again using multipliers l, m, n, we get
Each fraction

$$= \frac{l\,dx + m\,dy + n\,dz}{l(mz-ny) + m(nx-lz) + n(ly-mx)} = \frac{l\,dx + m\,dy + n\,dz}{0}$$

$$\implies l\,dx + m\,dy + n\,dz = 0$$

On integration, it gives

$$lx + my + nz = c_2 \qquad (2)$$

Hence, from (1) and (2), the required general solution is

$$\phi(x^2 + y^2 + z^2, lx + my + nz) = 0$$

Solve $x(z^2 - y^2)\frac{\partial z}{\partial x} + y(x^2 - z^2)\frac{\partial z}{\partial y} = z(y^2 - x^2)$

Solution. The auxiliary equations are

$$\frac{dx}{x(z^2 - y^2)} = \frac{dy}{y(x^2 - z^2)} = \frac{dz}{z(y^2 - x^2)} \qquad (1)$$

Using multipliers x, y, z, each fraction of (1) is equal to

$$\frac{x\,dx + y\,dy + z\,dz}{x^2(z^2 - y^2) + y^2(x^2 - z^2) + z^2(y^2 - x^2)} = \frac{x\,dx + y\,dy + z\,dz}{0}$$

$$\implies x\,dx + y\,dy + z\,dz = 0$$

On integration

$$x^2 + y^2 + z^2 = C_1 \qquad (2)$$

Again, (1) can be written as

$$\frac{dx}{x(z^2-y^2)}=\frac{dy}{y(x^2-z^2)}=\frac{dz}{z(y^2-x^2)}$$

$$=\frac{\frac{dx}{x}+\frac{dy}{y}+\frac{dz}{z}}{(z^2-y^2)+(x^2-z^2)+(y^2-x^2)}=\frac{\frac{dx}{x}+\frac{dy}{y}+\frac{dz}{z}}{0}$$

$$\implies \frac{dx}{x}+\frac{dy}{y}+\frac{dz}{z}=0$$

$$\implies \log x+\log y+\log z=\log C_2$$

$$\implies \log xyz=\log C_2 \implies xyz=C_2 \qquad (3)$$

From (2) and (3), the general solution is

$$\phi(x^2+y^2+z^2, xyz)=0$$

Solve $(y^2 + z^2)p - xyq + zx = 0$

Solution:

Auxiliary equations are

$$\frac{dx}{y^2 + z^2} = \frac{dy}{-xy} = \frac{dz}{-zx}$$

From the 2nd and 3rd fractions,

$$\frac{dy}{y} = \frac{dz}{z} \quad \text{or} \quad \frac{y}{z} = c_1$$

Choosing multipliers as x, y, z:

$$x\,dx + y\,dy + z\,dz = x(y^2 + z^2) + y(-xy) + z(-zx) = 0$$

Solve $(y^2 + z^2)p - xyq + zx = 0$ (contd.)

Integrating,

$$x^2 + y^2 + z^2 = c_2$$

The general solution is

$$\phi\left(\frac{y}{z}, x^2 + y^2 + z^2\right) = 0$$

Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$

Here the auxiliary equations are

$$\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$$

Taking the last two members, we have

$$\frac{dy}{y} = \frac{dz}{z}$$

which on integration gives

$$\log y = \log z + \log a$$

or

$$\log \frac{y}{z} = \log a \quad \text{or} \quad \frac{y}{z} = a \qquad \cdots (1)$$

Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$ (contd.)

Using x, y, z as multipliers, we get
Each fraction

$$= \frac{x\,dx + y\,dy + z\,dz}{x(x^2 + y^2 + z^2)} = \frac{dz}{2xz}$$

or

$$\frac{2x\,dx + 2y\,dy + 2z\,dz}{x^2 + y^2 + z^2} = \frac{dz}{z}$$

which on integration gives

$$\log(x^2 + y^2 + z^2) = \log z + \log b$$

or

$$\log\left(\frac{x^2 + y^2 + z^2}{z}\right) = \log b \quad \text{or} \quad \frac{x^2 + y^2 + z^2}{z} = b \qquad \cdots (2)$$

From (1) and (2), the general solution is

$$\phi\left(\frac{x^2 + y^2 + z^2}{z}, \frac{y}{z}\right) = 0$$