PES University, Bangalore

(Established Under Karnataka Act 16 of 2013)

Department of Science and Humanities

Engineering Mathematics - I
(UE25MA141A)

Assignment

Unit - 3: Partial Differential Equations

Form PDE by elimination of arbitrary functions

1. $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$.
   Answer: $x^2p + yq = 2y^2$.

2. $z = (x + y)f(x^2 - y^2)$.
   Answer: $py + qz = z$.

3. $z = f(x + at) + g(x - at)$.
   Answer: $z_{tt} = a^2 z_{xx}$.

4. $f(x^2 + y^2, z - xy) = 0$.
   Answer: $xq - yp = x^2 - y^2$.

Solve the following Lagrange's linear PDE of first order: $Pp + Qq = R$

5. $pz - qz = z^2 + (x + y)^2$.
   Asnwer: $\phi\left[x + y,\ \log\left(x^2 + y^2 + z^2 + 2xy\right) - 2x\right] = 0$.

6. $2yzp + zxq = 3xy$.
   Answer: $\phi(x^2 - 2y^2, 3y^2 - z^2) = 0$.

7. $y^2p - xyq = x(z - 2y)$.
   Answer: $\phi(x^2 + y^2, y(z - y)) = 0$.

8. $2p + q = \sin(x - 2y)$.
   Answer: $\phi\left(x - 2y, y - \frac{z}{\sin(x-2y)}\right) = 0$.

9. $xzp + yzq = xy$.
   Answer: $\phi\left(\frac{x}{y}, xy - z^2\right) = 0$.

Find the solution of the following PDEs by the method of separation of variables

10. Solve $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} = 0$.
    Answer: $u = ce^{\left(\frac{k}{x} + \frac{c}{y}\right)}$.

11. $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given that $u(0, y) = 2e^{5y}$.
Answer: $u = 2e^{-\frac{x}{2}+5y}$.

Solve the following higher order linear equations with constant coefficients

The following notations are used:
$p = \frac{\partial x}{\partial x} = Dz; q = \frac{\partial z}{\partial y} = D'z; D^2z = \frac{\partial^2 z}{\partial x^2}; D'^2z = \frac{\partial^2 z}{\partial y^2}; DD'z = \frac{\partial^2 z}{\partial x \partial y}$.

12. $\left(D^4 - 2D^2D^2 + D^4\right) z = 0$.
Answer: $z = \phi_1(x - y) + x\,\psi_1(x + 2y) + \psi_2(x + 2y)$.

13. $\left[D^2 + D\,D' - 2\left(D'\right)^2\right] z = 5e^{x+2y}$.
Answer: $z = \phi_1(x + y) + \phi_2(2x - y) - e^{x+2y}$.

14. $\left[3D^2 + 10\,D\,D' + 3\,(D')^2\right] z = e^{x-y}$.
Answer: $\phi_1(x - 3y) + \phi_2(3x - y) - \frac{1}{4}e^{x-y}$.

15. $(2D^2 + 5\,D\,D' - 3\,(D')^2)\,z = \sin\left(2x - y\right)$.
Answer: $z = \phi_1(3x - y) + \phi_2(x + 2y) + \frac{1}{5}\sin\left(2x - y\right)$.

16. $\left[D^2 + 3\,D\,D' + 2\,(D')^2\right] z = 84\cos\left(x + 3y\right)$.
Answer: $z = \phi_1(x - y) + \phi_2(2x - y) - 3\cos\left(x + 3y\right)$.

17. $\left[2D^2 - 7\,D\,D' + 6\,(D')^2\right] z = x\,y$
Answer: $z = \phi_1\left(2x + y\right) + \phi_2\left(3x + 2y\right) + \frac{1}{96}\,x^3\left(8y + 7x\right)$.

18. $\left[D^2 - D\,D' - 2\,(D')^2\right] z = 16\,x\,e^{2y}$.
Answer: $z = \phi_1(x - y) + \phi_2(2x + y) + \frac{1}{2}\,(1 - 4x)\,e^{2y}$.