Concept of degeneracy in the case of electrons in a 3D box.

QA

QA1:

The distribution of electrons are as follows:

ψ_{111}- 1 electron with energy $\frac{3h^2}{8mL^2}$

ψ_{211}- 1 electron with energy $\frac{6h^2}{8mL^2}$

ψ_{121}- 1 electron with energy $\frac{6h^2}{8mL^2}$

ψ_{112}- 1 electron with energy $\frac{6h^2}{8mL^2}$

ψ_{311}- 1 electron with energy $\frac{11h^2}{8mL^2}$

Hence the highest energy of the electron with energy $\frac{11h^2}{8mL^2}$ can occupy anyone of the following states: $\psi_{311}/\psi_{131}/\psi_{113}$

QA2:

For a 3 D infinite potential well of length L which is symmetric about the origin, the wavefunctions for all the combinations of the following three positive integer quantum numbers $(n_x, n_y, n_z) = (1,1,3)$ are:

$$\psi_{311} = \sqrt{\frac{8}{L^3}}\,\mathrm{Cos}\left(\frac{3\pi x}{L}\right)\mathrm{Cos}\left(\frac{1\pi y}{L}\right)\mathrm{Cos}\left(\frac{1\pi z}{L}\right)$$

$$\psi_{131} = \sqrt{\frac{8}{L^3}}\,\mathrm{Cos}\left(\frac{1\pi x}{L}\right)\mathrm{Cos}\left(\frac{3\pi y}{L}\right)\mathrm{Cos}\left(\frac{1\pi z}{L}\right)$$

$$\psi_{113} = \sqrt{\frac{8}{L^3}}\,\mathrm{Cos}\left(\frac{1\pi x}{L}\right)\mathrm{Cos}\left(\frac{1\pi y}{L}\right)\mathrm{Cos}\left(\frac{3\pi z}{L}\right)$$