ENGINEERING PHYSICS

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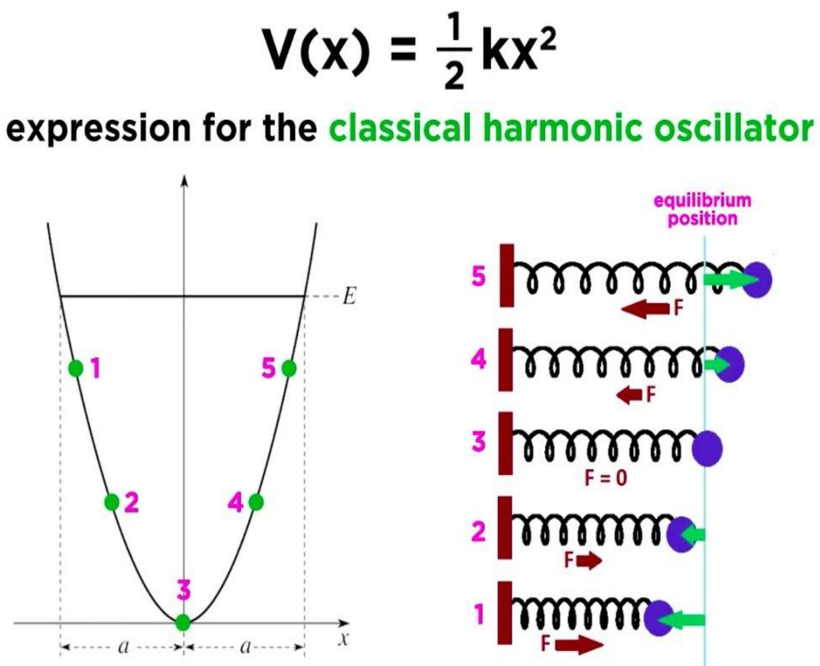
QHO anharmonicity and artificial levels as QUBITs

Class # 35

- **Review of Harmonic oscillator**
- **Potential energy is given by (same as classical oscillator)**

$$V = \frac{1}{2}Cx^2$$



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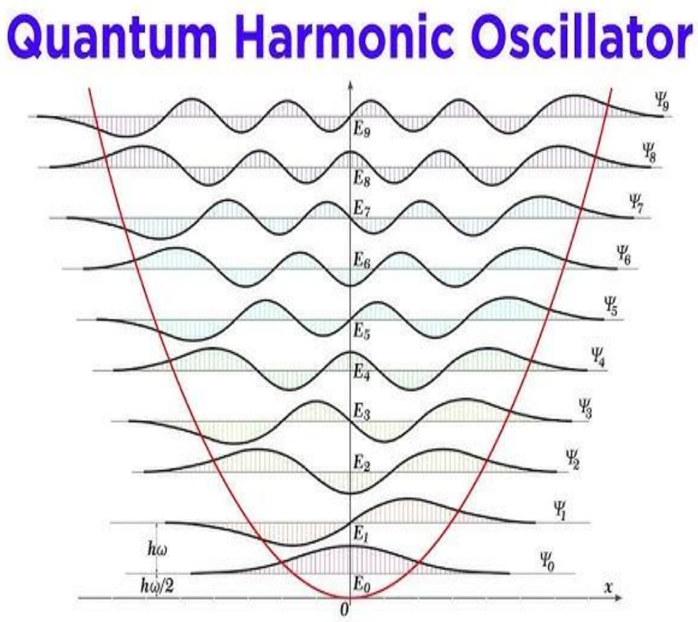
- ***Eigen values:*** $E_n = \left(n + \frac{1}{2}\right) h\nu$, ***n = 0,1,2,3,….***
- ***Eigen functions:*** $\psi_n = Ae^{-\frac{x^2}{2}} H_n(x)$ ***, where H is a polynomial***



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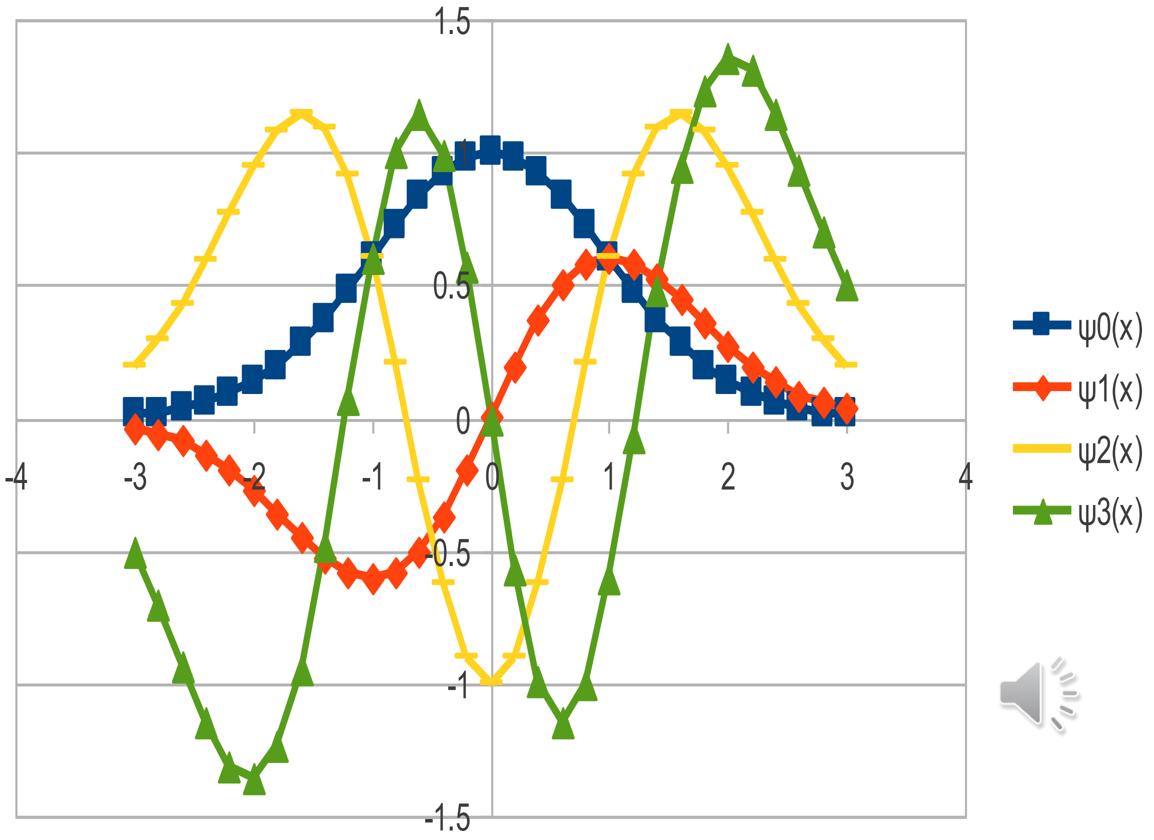
- ***The first four eigen functions (shown only between the turning points)***



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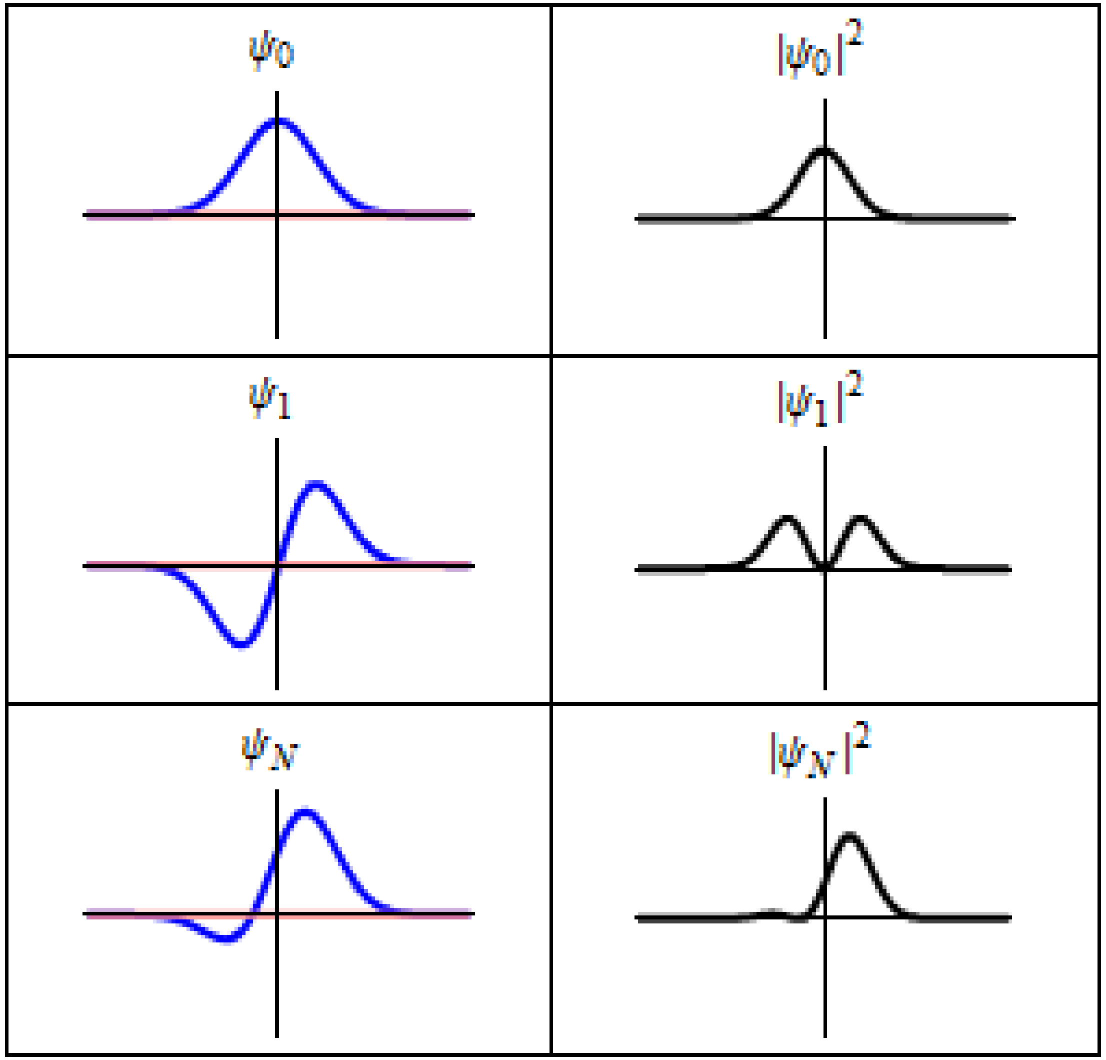
- ***Animation of Eigen functions and the probability densities***



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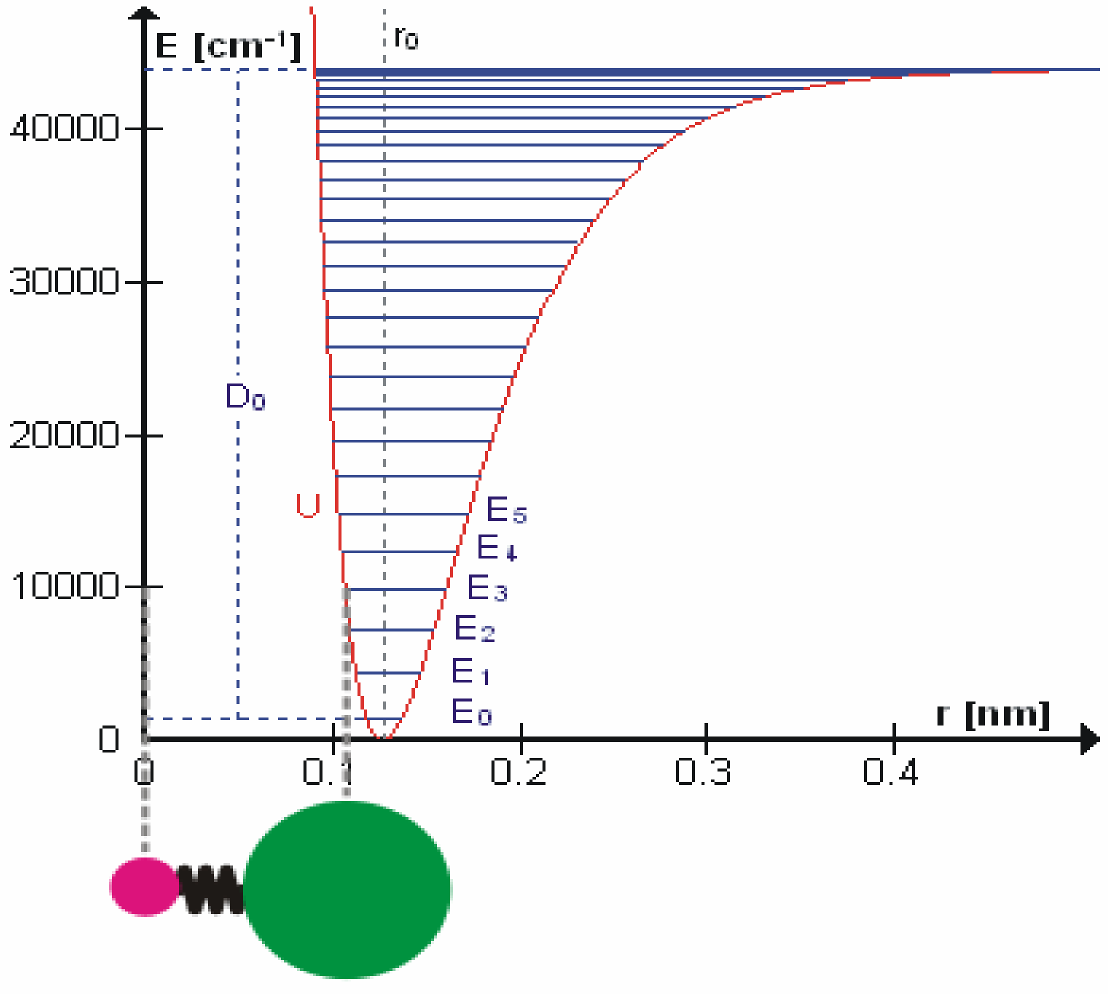
- ***An example of a quantum mechanical oscillator is a diatomic molecules where the atoms can vibrate.***
- ***How does the actual potential curve look like?***
- ***Shown here is the behaviour only near the equilibrium point*** $V = \frac{1}{2}Cx^2$ **(parabolic)**
- ***If we move away from the equilibrium point, we see that V is non-parabolic***



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- ***Two questions:***
- ***How do we mathematically represent V?***
- ***What is the consequence of this potential energy behavior?***

- ***Answer to first question: There could be several forms***

➢ ***Morse function:*** $V = D_{eq}\left(1 - e^{a\left(r_{eq}-r\right)^2}\right)$

➢ ***Polynomial form:*** $V = \alpha x^2 + \beta x^3 + \gamma x^4 + \cdots..$

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- **Answer to second question: The form of eigen energy equation $E_n = \left(n+\frac{1}{2}\right)h\nu - \chi_1\left(n+\frac{1}{2}\right)^2 h\nu + \chi_2\left(n+\frac{1}{2}\right)^3 h\nu + \cdots$.**
- **A simplified version of the eigen energies is obtained by dropping terms beyond the second term. Thus**
- $E_n = \left(n+\frac{1}{2}\right)h\nu - \chi_1\left(n+\frac{1}{2}\right)^2 h\nu$

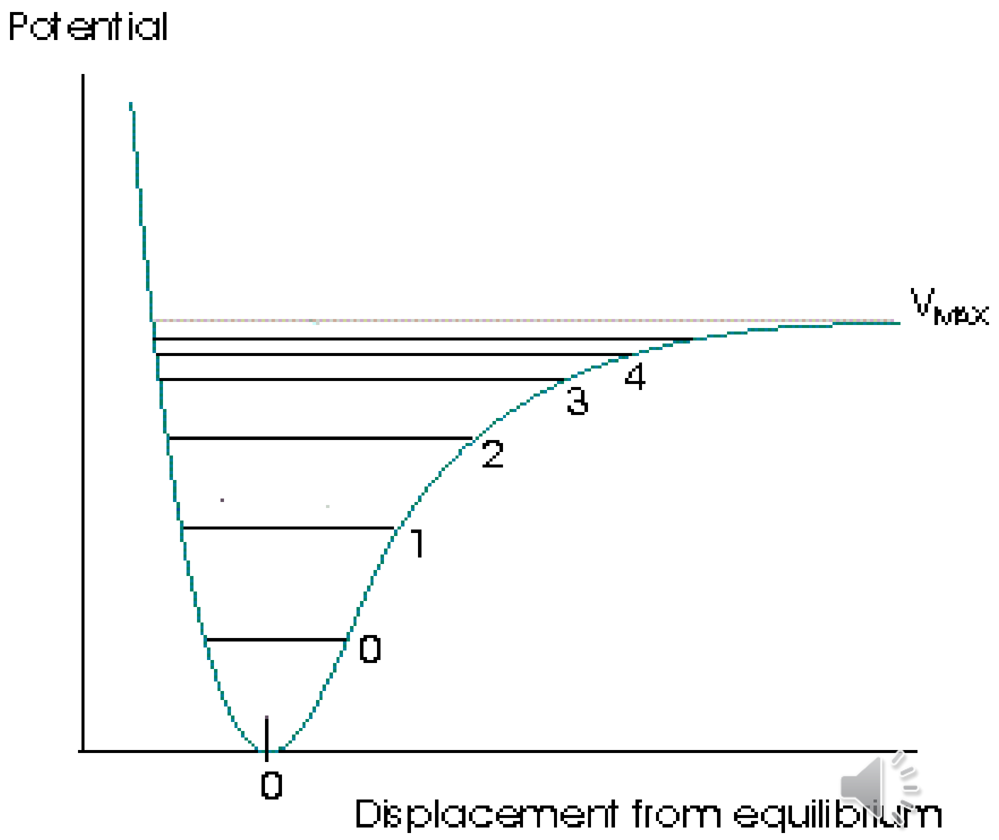

Image courtesy: Chemistry revision site

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- **All real oscillators are in general anharmonic. Only when displacements are very small about the equivalent point do we get harmonic oscillators**
- **Why are anharmonic oscillators useful? Well, they are potential candidates for realizing qubits**

- **Recall that a qubit is given by $|\psi\rangle = \mathrm{a}|0\rangle + b|1\rangle$.**
- **Now $|0\rangle$ could refer to a particular energy level and $|1\rangle$ is generally the next energy level**
- **The problem in harmonic oscillators is that the spacing between the energy levels is same and hence choosing two unique levels is impossible**

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- **In anharmonic oscillators the spacing between energy levels is different**
- **Therefore, it possible to choose two unique levels and E_1 and E_2 and label them as $|0\rangle\ and |1\rangle$.**
- **Hence anharmonic oscillators become useful for designing a qubit.**
- **The making of an oscillator and realizing qubits happens to be a matter of engineering**

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- **Has a qubit based on anharmonic oscillators realized in practice?**
- **Yes, using LC based oscillators. However, the inductor has to be non-linear**

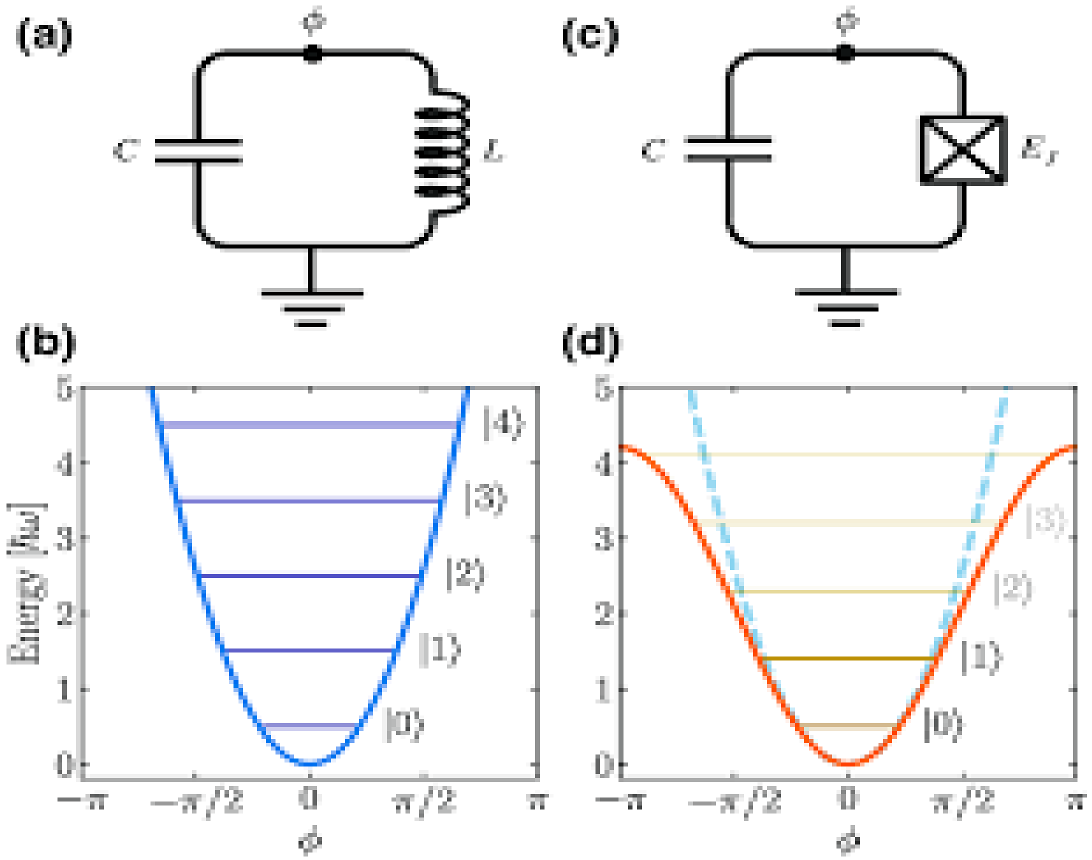

Image courtesy: Researchgate

THANK YOU

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