ENGINEERING MATHEMATICS I

Department of Science and Humanities

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HIGHER ORDER DIFFERENTIAL EQUATIONS

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CLASS CONTENT:

- TO SOLVE A NON – HOMOGENEOUS LINEAR DIFFERENTIAL EQUATION OF THE TYPE f(D)y = X WHEN X =

INVERSE DIFFERENTIAL OPERATOR

The inverse differential operator is denoted as $\frac{1}{f(D)}$.

$\frac{1}{f(D)}(X)$ is that function of x which is free from arbitarary constants which when operated upon by $f(D)$ gives X.

Thus, $f(D)$ and $\frac{1}{f(D)}$ are inverse operators.

$$f(D)\left\{\frac{1}{f(D)}X\right\} = X$$

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INVERSE DIFFERENTIAL OPERATOR

$$\text{Theorem: } \frac{1}{f(D)}(X) \text{ is the particular solution of } f(D)y = X$$

$$\text{Proof : The given equation is } f(D)y = X$$

$$\text{Let } y = \frac{1}{f(D)}(X)$$

$$f(D)\left[\frac{1}{f(D)}X\right] = X \Rightarrow X = X$$

$$\text{Thus, } \frac{1}{f(D)}(X) \text{ is the particular solution of } f(D)y = X$$

Consider the differential equation $f(D)y = X$

The general solution of the differential equation is

y = Complimentary function + Particular integral

The Particular Integral is given by $\frac{1}{f(D)}(X)$

SOME IMPORTANT RESULTS

- $\frac{1}{D}(X) = \int X.dx$ where X is a function of x.
- $\frac{1}{D-a}(X) = e^{ax} \int e^{-ax}.X.dx$ where X is a function of x.
- If $f(D) = ((D-a)(D-b))$

then $\frac{1}{f(D)}(X) = \frac{1}{(D-a)(D-b)}(X) = \frac{1}{D-a}\left(\frac{1}{D-b}(x)\right)$

RULES TO FIND PARTICULAR INTEGRAL

Type (i); $X = e^{ax}$

$$\frac{1}{f(D)}\left(e^{ax}\right) = \frac{e^{ax}}{f(a)} \quad \text{provided} \quad f(a) \neq 0$$

Cases of failure;

When $f(a) = 0$, then $\frac{1}{f(D)}\left(e^{ax}\right) = x.\frac{e^{ax}}{f'(a)}$ provided $f'(a) \neq 0$

When $f'(a) = 0$, then $\frac{1}{f(D)}\left(e^{ax}\right) = x^2.\frac{e^{ax}}{f''(a)}$ provided $f''(a) \neq 0$

Extension of Type (i)

When $X = K$, $\boxed{\dfrac{K}{f(D)} = K\dfrac{e^{0x}}{f(D)} = \dfrac{K}{f(0)}}$

When $X = \sinh ax$ $\quad \sinh ax = \dfrac{e^{ax} - e^{-ax}}{2}$

$$\boxed{\frac{\sinh ax}{f(D)} = \frac{1}{2}\left[\frac{e^{ax} - e^{-ax}}{f(D)}\right] = \frac{1}{2}\left[\frac{e^{ax}}{f(a)} - \frac{e^{-ax}}{f(-a)}\right]}$$

Extension of Type (i)

When $X = \cosh ax$ $\quad \cosh ax = \dfrac{e^{ax} + e^{-ax}}{2}$

$$\frac{\cosh ax}{f(D)} = \frac{1}{2}\left[\frac{e^{ax} + e^{-ax}}{f(D)}\right] = \frac{1}{2}\left[\frac{e^{ax}}{f(a)} + \frac{e^{-ax}}{f(-a)}\right]$$

When $X = a^x$ $\quad \dfrac{a^x}{f(D)} = \dfrac{e^{\log a.x}}{f(D)} = \dfrac{e^{\log a.x}}{f(\log a)}$

$$\text{Thus, } \frac{a^x}{f(D)} = \frac{a^x}{f(\log a)}$$

Solve ; $(D^2 - 2D + 2)y = a^{-x} + e^x$

TO FIND COMPLEMENTARY FUNCTION

AE is $m^2 - 2m + 2 = 0$

Roots are $m = 1 \pm i$

$y_c = e^x (c_1 \cos x + c_2 \sin x)$

TO FIND PARTICULAR INTEGRAL

$$y_p = \frac{a^{-x}}{\left(D^2 - 2D + 2\right)} + \frac{e^x}{\left(D^2 - 2D + 2\right)}$$

$$y_p = \frac{a^{-x}}{\left((-\log a)^2 - 2(-\log a) + 2\right)} + \frac{e^x}{\left(1^2 - 2.1 + 2\right)}$$

Thus, $y = y_c + y_p$

THANK YOU

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