

CL10_Q1. Explain the concept of “expectation value of a physical measurable quantity” in quantum mechanics?

Answer

Expectation value is the average value that you would expect to get for some observable quantity like x or p if you measured it many times.

Quantum mechanics deals with probabilities and hence predicts only the most probable values of the observables of a physical system which are called the expectation values. These expectation values could be the average of repeated measurements on the system.

In the general case for an operator \hat{A} when and hence the expectation value of the parameter A can be written as $\langle A \rangle = \frac{\int \psi^* \hat{A} \psi dV}{\int \psi^* \psi dV}$.

CL10_Q2. Explain operators and observables.

Answer

Observables: The physical parameters associated with the particle such as energy, momentum, kinetic energy, spin, etc. are observables of the state of a system.

Operators: The values of the observables can be obtained using a mathematical operator operating on the wave function.

CL10_Q3. Find the expectation value $\langle X \rangle$ of the position of a particle trapped in a box ‘L’ wide.

Answer

$$\begin{aligned}\langle X \rangle &= \int_{-\infty}^{\infty} x[\psi]^2 dx \\ &= \frac{2}{L} \int_0^L x \sin^2\left(\frac{n\pi}{L}x\right) dx = \frac{2}{L} \left[\frac{L^2}{4} \right] = \frac{L}{2}\end{aligned}$$

CL10_Q4. Describe an Eigen value equation explaining each term.

Answer

Eigen Value Equation

$$\hat{G} \psi_n = \lambda_n \psi_n$$

Where \hat{G} is an operator, λ_n are called Eigen values (Real) and ψ_n are called Eigen wave functions. The above equation holds only for a discrete set of λ_n and ψ_n . The Eigen values are the possible outcomes of the measurement on the observable represented by the operator \hat{G} . Any measurement cannot result in a value which is not an Eigen value of the equation. Eigen functions give us the probability of finding the particle for that given Eigen value.

CL10_Q5. Write any five operators associated with dynamical variables.

Answer

Observables and Operators

A dynamical physical quantity which can be measured in an experiment is called an observable.

For example, position, momentum, kinetic energy etc.

All observables are represented by operators. Generally, such operators are either matrices or they involve some type of differential operators.

Observable	Operator
Position. x	$\hat{x} = x$
Momentum p	$\hat{p} = -i\hbar \frac{\partial}{\partial x}$
Kinetic energy K	$\hat{K} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$
Potential energy V	$\hat{V} = V(x)$
Total energy E	$\hat{E} = i\hbar \frac{\partial}{\partial t}$

Total energy
Hamiltonian H

$$\hat{H} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x)$$

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