Take Home Exam 1

1a. $((p \vee \neg q) \wedge (q \vee \neg r)) \leftrightarrow (p \wedge \neg r)$

p	$\vee$	$\neg q$	$\wedge$	q	$\vee$	$\neg r$	$\leftrightarrow$	p	$\wedge$	$\neg r$
1	1	0	1	1	1	0	0	1	0	0
1	1	0	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	1	1	0	0
1	1	1	1	0	1	1	1	1	1	1
0	0	0	0	1	1	0	1	0	0	0
0	0	0	0	1	1	1	1	0	0	1
0	1	1	0	0	0	0	1	0	0	0
0	1	1	1	0	1	1	0	0	0	1

Not a tautology

b. $((p \rightarrow q) \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r))$

p	$\rightarrow$	q	$\rightarrow$	r	$\rightarrow$	p	$\rightarrow$	q	$\rightarrow$	r
1	1	1	1	1	1	1	1	1	1	1
1	1	1	0	0	1	1	1	1	0	0
1	0	0	1	1	1	1	0	0	1	1
1	0	0	1	0	1	1	0	0	1	0
0	1	1	1	1	1	0	1	1	1	1
0	1	1	0	0	1	0	1	1	0	0
0	1	0	1	1	1	0	1	0	1	1
0	1	0	0	0	1	0	1	0	0	0

Tautology ✓✓

Scanned with CamScanner

c. $(((p \wedge q) \rightarrow r) \rightarrow r) \leftrightarrow (\neg r \rightarrow p)$

p	∧	q	→	r	→	r	↔	¬r	→	p
1	1	1	1	1	1	1	1	0	1	1
1	1	1	0	0	1	0	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	0	1	0	0	0	0	1	1	1
0	0	1	1	1	1	1	1	0	1	0
0	0	1	1	0	0	0	1	1	0	0
0	0	0	1	1	1	1	1	0	1	0
0	0	0	1	0	0	0	1	1	0	0

Not a tautology

2a. $\cancel{pqr}$, $\cancel{pq\bar{r}}$, $\cancel{p\bar{q}r}$, $\cancel{p\bar{q}\bar{r}}$, $\bar{p}qr$, $\bar{p}q\bar{r}$, $\overline{pq}r$, $\overline{pqr}$

$\neg p$

$\bar{p}qr$, $\cancel{\bar{p}q\bar{r}}$, $\cancel{\overline{pq}r}$, $\overline{pqr}$

$q \leftrightarrow r$

$\bar{p}qr$, $\overline{pqr}$

$(p \rightarrow (q \wedge \neg r)) \rightarrow (\neg q \vee r)$

Tautology

$\bar{p}qr$, $\overline{pqr}$

$r \rightarrow p$

$\boxed{\text{Does not entail}}$

3a.

i. No, since they can be false in every given row.

ii. φ could always be false.

iii. $\varphi \wedge \psi$ is a tautology, so they are true in every row. So this is consistency.

iv. $(\varphi \vDash \neg\psi)$ is not the case. So there is one situation.

v. Does not guarantee both are true.

$\boxed{\text{iii and iv.}}$

b.

i. True. They have the same truth value.

ii. False. They could always be false.

iii. False. It is one way, so isn't equivalent.

iv. False. They are both true, but they don't necessarily have to be.

v. False. Not necessary that all cases where ψ is true, φ is true.

4. S_1 = The first sentence is true.

S_2 = The second sentence is true.

S_3 = The third sentence is true.

a. $S_2 \to S_3$

$S_1 \to \neg S_3$

$S_2 \to S_3$

b.

S_1	S_2	S_3	$S_2 \to S_3$	$S_1 \to \neg S_3$	$S_2 \to 3$
1	1	1	1	0	1
1	1	0	0	1	0
1	0	1	1	0	1
1	0	0	1	1	1
0	1	1	1	1	1
0	1	0	0	1	0
0	0	1	1	1	1
0	0	0	1	1	1

c. The second. First and third can't be since they are the same.

5. $p \wedge q \wedge r$ row 1
$p \wedge \neg p \wedge \neg r$ row 4
$\neg p \wedge q \wedge \neg r$ row 6
$\neg p \wedge \neg q \wedge \neg r$ row 8

$(p \wedge q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r$

6a.

p	q	$\neg$	$(p$	$\wedge$	$q)$
1	1	0	1	1	1
1	0	1	1	0	0
0	1	1	0	0	1
0	0	1	0	0	0

p	$\rightarrow$	$(p$	$\oplus$	$q)$
1	0	1	0	1
1	1	1	1	0
0	1	0	1	1
0	1	0	0	0

$$\boxed{p \rightarrow (p \oplus q)}$$

b.

$\neg$	p
0	1
1	0

p	$\rightarrow$	$(p$	$\oplus$	$p)$
1	0	1	0	1
0	1	0	0	0

$$\boxed{p \rightarrow (p \oplus p)}$$

c. Yes. We can get negation, and we already have the conditional, which those two are expressively complete, so the whole thing is.

7.

a. $(p \to (q \to p))$ Axiom 1

$(p \to (q \to p)) \to ((p \to q) \to (p \to p))$ Axiom 2

$\neg p$ Given

$p \to q$ Modus Ponens

b. $\neg p$ Part A

$p \to q$ Modus Ponens

$\neg p \to (p \to q)$ Modus Ponens