

Key Exchange & Diffie-Hellman

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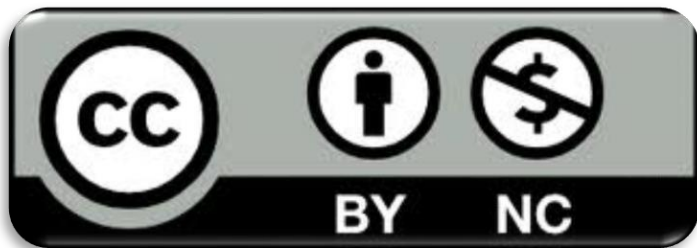
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Prerequisites

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➤ Lectures:

- *CR_0.1 - Number Theory and modular arithmetic*
- *CR_1.1 – Introduction to cryptography and classical ciphers*

Recap

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- At this point we know:
 - How symmetric ciphers work
 - How to protect communications using a shared secret key
- Goal of this lecture:
 - Find a way to exchange keys
 - Expose the possible issues in key-exchange

Outline

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- The Key Exchange problem
- The Diffie-Hellman protocol
- Issues

Outline

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The Key Exchange Problem

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- Problem settings:
 - N users that want to communicate with each others
 - Communication must be independent from the others but in a shared channel:
 - User C can see the messages exchanged by user A and user B
 - User C cannot decrypt them
- Naïve solution: every user stores $N - 1$ different keys shared with every single other user

A Better Solution

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- The naïve solution doesn't seem efficient as the number of keys grows quadratically with the number of users
- A better way to solve this problem, is to have a **trusted 3rd party** (TTP):
 - Every user has to remember a single key to speak with the TTP
 - The TTP manages the creation of shared keys between users

TTP – Toy Example

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- Alice wants to speak with Bob using a TTP, who knows both Alice's key k_a and Bob's key k_b :
 - Alice says to TTP: "I want to talk with Bob"
 - TTP randomly creates a key k_{AB}
 - TTP sends to Alice $E(k_A, k_{AB})$ and $E(k_B, k_{AB})$
 - Alice sends to Bob the second one and they start communicating

TTP - Issues

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- This kind of scheme has two main problems:
 - It relies on the TTP being always online
 - The TTP knows all the keys: it is a single point of failure for the whole system
- There are contexts in which TTP makes sense:
 - Inside a company/university or a whatever closed environment
 - A similar reasoning is in fact at the basis of Kerberos (See lecture CP_1.3 - Kerberos)

The Key Exchange Problem

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- Key question: can we generate online keys without a TTP?
- Some protocols have been proposed during the years:
 - Merkle puzzles (1974)
 - Diffie-Hellman (1976)
 - RSA (1977)
 - Identity-based encryption (2001)
 - Functional encryption (2011)

Merkle Puzzles

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- Merkle puzzles are a way to exchange keys using block ciphers
 - The idea is that Alice sends N puzzles to Bob, containing the messages *“This is message X . This is the symmetrical key Y ”*
 - Bob randomly choose one of them, solve it, and gives back the number X to Alice
 - Alice and Bob both know which puzzle X Bob solved and its symmetrical key Y , but whoever listen to the conversation do not
- Issue¹: to make this work, the number of puzzles N must be very big
- Issue²: An eavesdropper need only a linear time factor of $O(N)$, compared to Alice and Bob, to find the key exchanged
- Our goal is to have a key exchange algorithm which needs exponential time to be broken

Outline

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- The Key Exchange problem
- **The Diffie-Hellman protocol**
- Issues

The Diffie-Hellman Protocol

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*“We stand today on the brink
of a revolution in cryptography.”*

*[Whitfield Diffie and Martin Hellman,
“New directions in Cryptography”, November 1976]*

The Diffie-Hellman Protocol

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- Diffie and Hellman in 1976 give a better solution using number theory
 - Alice and Bob agree (publicly!) on a prime number p and a generator g in $\{2, 3, \dots, p - 1\}$
 - Alice generates a number a in $\{2, 3, \dots, p - 1\}$ and Bob does the same with b
 - Alice sends the value $A = g^a \bmod p$ and, in the same way, Bob sends $B = g^b \bmod p$

The Diffie-Hellman Protocol

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- At this point:
 - Alice knows a and B
 - Bob knows b and A
 - They both can calculate $g^{ab} = A^b = B^a \bmod p$, that will be their shared key
 - An eavesdropper that only knows $g^a \bmod p$ and $g^b \bmod p$ can't compute $g^{ab} \bmod p$ efficiently!

The Diffie-Hellman Protocol

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➤ Example:

- Take $p = 37$ and $g = 2$
- Alice generates the number 7 and sends $2^7 \bmod 37 = 17$
- Bob generates the number 21 and sends $2^{21} \bmod 37 = 29$
- Both can compute $2^{7*21} = 29^7 = 17^{21} = 8 \bmod 37$
- 8 will be Alice and Bob's shared key

How hard is to break DH?

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- It is believed that the only way to break DH is to recover one of a and b , computing so a discrete logarithm
- The best-known algorithm for discrete logarithms is the General Number Field Sieve (GNFS) that runs in $O(e^{(\sqrt[3]{n})})$

How hard is to break DH?

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- To give some practical numbers:
 - Breaking DH with p of 1024 bits is roughly equivalent to break a block cipher with 80-bit security
 - If p is of 3072 bits it is equivalent to a 128-bit block cipher (like AES-128!)
 - ...and so on
- These numbers are even better in the case of variations of the classic DH: for example, a 256-bit instance of DH over Elliptic Curves is enough to get the security of AES-128

Outline

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- The Key Exchange problem
- The Diffie-Hellman protocol
- **Issues**

Issues

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- There are two main issues in using Diffie-Hellman:
 - Reaching high level of security requires very big keys
 - Diffie-Hellman Protocol is vulnerable to active attacks like the *Man-in-the-Middle*

Insecurity against Man-in-the-Middle

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- An eavesdropper, who can listen and modify the communication, can:
 - Intercept $g^a \bmod p$ from Alice and substitute it with $g^{a'} \bmod p$
 - Do the same with Bob, using $g^{b'} \bmod p$
 - Craft the keys $(g^{a'b} \bmod p, g^{ab'} \bmod p)$ (now Alice and Bob have different “shared” keys!)
 - Decrypt every communication, read it, and re-encrypt it with the correct key (without letting know Alice and Bob anything!)

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