Introduction to cryptography and classical ciphers

Rocco DE NICOLA
IMT Lucca

https://cybersecnatlab.it

License & Disclaimer

License Information

Disclaimer

- We disclaim any warranties or representations as to the accuracy or completeness of this material.
- Materials are provided “as is” without warranty of any kind, either express or implied, including without limitation, warranties of merchantability, fitness for a particular purpose, and non-infringement.
- Under no circumstances shall we be liable for any loss, damage, liability or expense incurred or suffered which is claimed to have resulted from use of this material.

Outline

- Role of cryptography
- The shared key model
- Historical cyphers
- Substitution techniques
- The Vigenère cipher
- Permutation techniques

Introduction

- Cryptography is the practice and study of techniques for secure (secret) communication in the presence of third parties called opponents.
- More generally, cryptography concerns the construction and analysis of protocols that prevent third parties from reading or altering private messages.
- Various aspects of information security, such as confidentiality and data integrity, authentication and non-repudiation, are at the heart of modern cryptography.

Cryptography is ubiquitous

- Passwords hashing
- Secure credit-card transactions over the internet
- Encrypted WiFi
- Disk encryption
- Digitally signed software updates
- Bitcoin
- …

Some Key terms

- *Encryption algorithm*: transforms a plain text (comprehensible to a human or machine) into an encrypted text (ciphertext, incomprehensible).
- *Decryption algorithm*: takes the ciphertext and returns the plaintext.
- Each encryption algorithm must have a correspondent "inverse" decryption algorithm.
- The pair encryption/decryption algorithms is also called *encryption scheme*.

Cryptographic keys

- The encryption algorithm usually takes a *cryptographic key* as input.
- The cryptographic key:
  - is an information (a parameter) that determines the functional output of a cryptographic algorithm.
  - Is essential for determining the transformation of plain text into cipher text and cipher text into plain text.

Cryptography

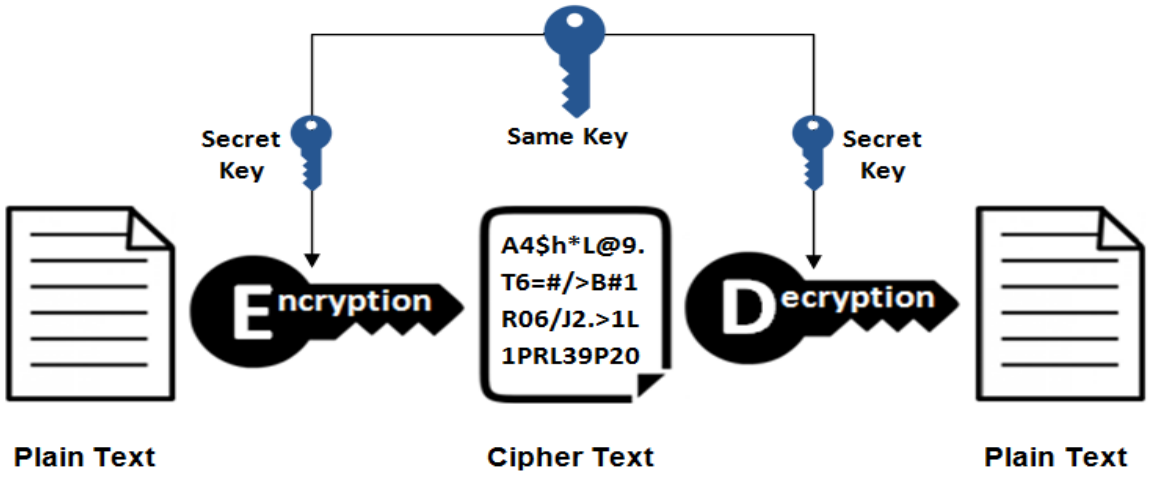
- Two main methods:
  - *Symmetric key* - Single key
  - *Public/Private key* - Double key

Symmetric key cryptography

- Also called *conventional* or *single key cryptography.*
- Universal technology to guarantee the confidentiality of transmitted or stored data.
- The only type of encryption in use before late 1970s when double key encryption was introduced.
- It remains the most common of the two types of encryption techniques.

Symmetric key cryptography

- Requires that both sender and recipient know the same key.
- An issue is how they do share it without meeting.



Symmetrical cryptographic systems

The cryptographic schemes are differentiated by:

- The *kind of operation* to transform the plaintext (P) into ciphertext (C):
  - *Replacements*: each element of P is replaced with another element
  - *Transpositions*: swappings are made between elements of P.
- The *way of "processing" P*:
  - *Block ciphers*: P is processed by the algorithm "one block at a time".
  - *Flow ciphers*: P is processed considering a single element at a time.

Private-key encryption

- A symmetric-*key encryption scheme* is defined by a message space $\mathcal{M}$ and three algorithms (Gen, Enc, Dec):
  - Gen (key-generation algorithm): outputs $k \in \mathcal{K}$
  - Enc (encryption algorithm): takes a key k and a message $m \in \mathcal{M}$ as input and outputs the ciphertext c: $c \leftarrow Enc_k(m)$
  - Dec (decryption algorithm): takes the key k and the ciphertext c as input and outputs m: $m := Dec_k(c)$

For all $m \in \mathcal{M}$ and k output by Gen, we have: $Dec_k(Enc_k(m)) = m$

Cryptanalysis

- *Cryptanalysis* is a set of techniques to test the robustness of the algorithm and of the key by trying to infer the key from the available ciphertext.

More in lecture CR_1.2

Encryption techniques

- Two basic mechanisms are applied to plaintext to obtain ciphertext:
  - *substitutions*
  - *permutations*
- The composition of these two techniques is at the basis of all modern symmetric encryption/decryption algorithms.

Substitution Techniques

- An encryption technique based on substitution relies on replacing each element of the plaintext with another element from the same alphabet.
- Replacements should be reversible, i.e., it must be possible to go back to the initial text.
- For each element in the plaintext, the key determines the element to be used to substitute it in the corresponding ciphertext.

Caesar's cipher

- The first known replacement cipher encryption is due to Julius Caesar.
- The algorithm requires replacing each letter of the alphabet with the one which is "three hops" away; when the alphabet ends it starts counting again.

PLAINTEXT	a	b	c	d	e	f	g	h	i	j	k	l	m
CIPHERTEXT	D	E	F	G	H	I	J	K	L	M	N	O	P
PLAINTEXT	n	o	p	q	r	s	t	u	v	w	x	y	z
CIPHERTEXT	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

attack

becomes

dwwdfn

Caesar's cipher

- Another example:

```
Plaintext:  meet me after the toga party
Ciphertext: PHHW PH DIWHU WKH WRJD SDUWB
```

A	B	C	D	E	F	G	H	I	J	K	L	M
0	1	2	3	4	5	6	7	8	9	10	11	12

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
13	14	15	16	17	18	19	20	21	22	23	24	25

With the association on the right, the number coding of ciphertext C in correspondence of the number p is given by:

$$C = E(3,P) = (P+3) \bmod 26.$$

Modular arithmetic

- x = y mod N if and only if N divides x-y
  - x mod N = the remainder when x is divided by N
  - the unique value $y \in \{0, \ldots, N-1\}$ such that x = y mod N
- Examples:
  - [25 mod 10] = [35 mod 10]
  - 25 ≠ [35 mod 10]
  - 5 = [35 mod 10]

Shift Ciphers

Generalizing Caesar's cipher

- $\mathcal{M}$ = {strings over lowercase English alphabet}
- Gen: choose uniform $k \in \{0, \ldots, 25\}$
- $Enc_k(m_1 \ldots m_t)$: output $c_1 \ldots c_t$, where
  $c_i := [m_i + k \bmod 26]$
- $Dec_k(c_1 \ldots c_t)$: output $m_1 \ldots m_t$, where
  $m_i := [c_i - k \bmod 26]$

Limits of Shift Ciphers

- From a cryptanalytical point of view, shift ciphers are rather weak, for two main reasons:
  - There are only 25 possible shifts to try
  - It is easy to see when the used key is the right one.
- Shift cyphers can be easily attacked by exhaustive searches.
- Their weakness is due to the fact that a specific substitution is used (e.g., a shift of 3)

Shift ciphers

- A cipher can be strengthened by allowing arbitrary substitutions, by introducing the possibility of not just shifting letters but arbitrarily changing them.
- For a given set of n elements, we have n! permutations; if S= {a,b,c} there are six permutations of S: abc, acb, bac, bca, cab, cba.
- With shift cipher we have instead only 3 possibility (the string “abc” could be mapped only to “abc”, “bca” and “cab”
- If message are encrypted by moving the plaintext according to one of the (26!) possible permutations it would be (much) more difficult to carry out brute force attacks.

Shift ciphers

- Given the following permutation of the 26 letters

  ABCDEFGHIJKLMNOPQRSTUVWXYZ

  DXUTNAVWKZFQGSIOYJBPLHCERM

- The plaintext «arrivano rinforzi» becomes :

  ARRIVANORINFORZI

  DJJKHDSIJKSAIJMK

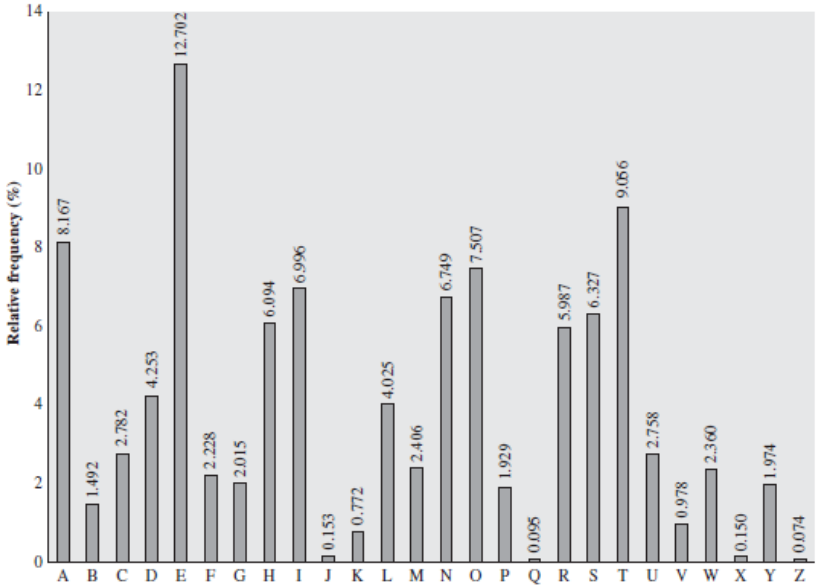
- The chosen permutation would be the key
- A brute force attack would require trying, in the worst case, 26! keys, instead of 25 keys as in the case of shift cipher.

Weakness of shift ciphers

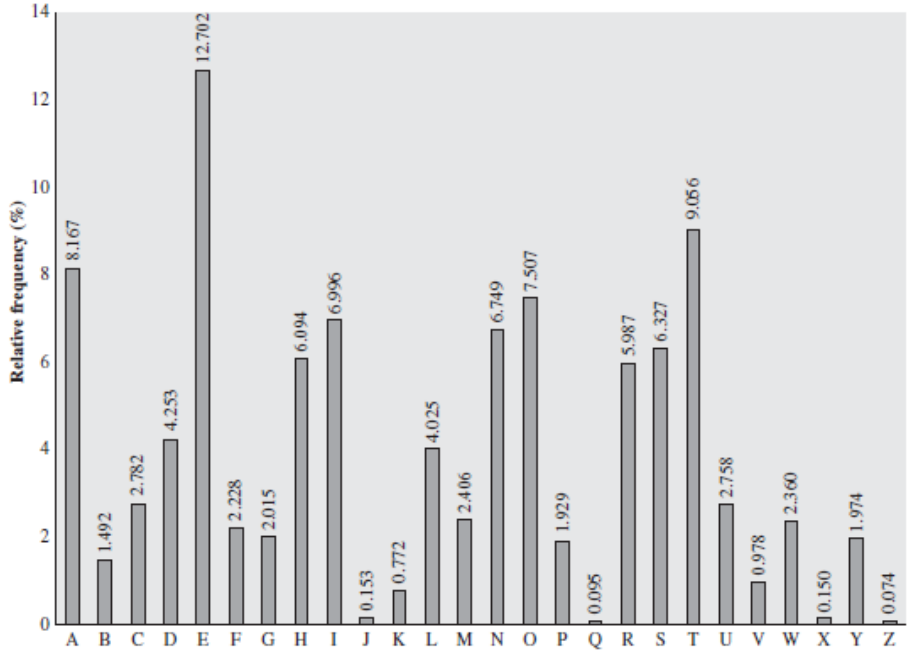
- Shift cipher are monoalphabetic: “same letter of plaintext always produces same letter of ciphertext “
- Regularities of languages such as:
  - frequency of individual cipher letters
  - pairs of letters close together for common words like "of"

  can be exploited

Letters frequency in English



Letters frequency in English



Most frequent letters in languages

ENGLISH		GERMAN		FINNISH		FRENCH		ITALIAN		SPANISH	
	%		%		%		%		%		%
E	12.31	E	18.46	A	12.06	E	15.87	E	11.79	E	13.15
T	9.59	N	11.42	I	10.59	A	9.42	A	11.74	A	12.69
A	8.05	I	8.02	T	9.76	I	8.41	I	11.28	O	9.49
O	7.94	R	7.14	N	8.64	S	7.90	O	9.83	S	7.60
N	7.19	S	7.04	E	8.11	T	7.26	N	6.88	N	6.95
I	7.18	A	5.38	S	7.83	N	7.15	L	6.51	R	6.25

Polyalphabetic cipher

- Polyalphabetic ciphers are an alternative to monoalphabetic ones, relying on multiple substitution alphabets.
- Proposed to disguise the plaintext letter frequency analysis
  - If P is the most frequent letter in a ciphertext of an English plaintext, one might suspect that P corresponds to E,
  - This is not possible if E is enciphered as different ciphertext letters at different points in the message.
- The Vigenère cipher is the best-known example of a polyalphabetic cipher.

The Vigenère cipher

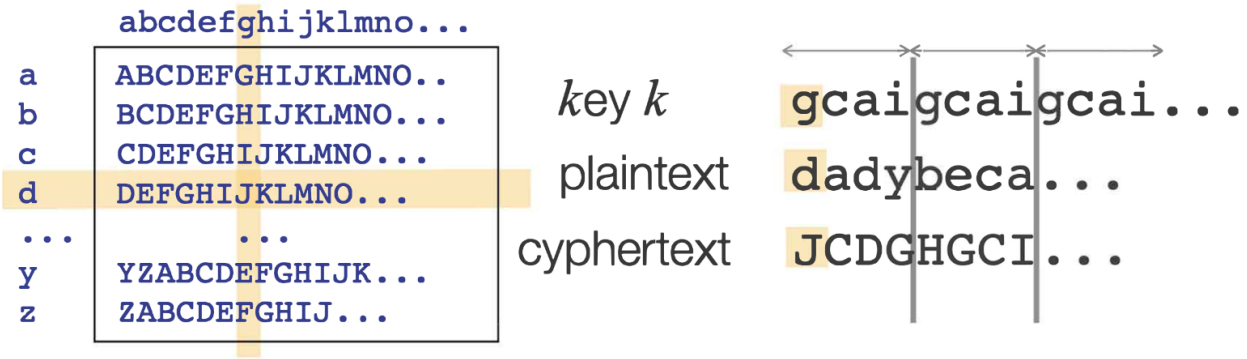
- The key is now a *string*, not just a character, and to each character is assigned a number according to the position in the alphabet (a:0, b:1, …, z:25)
- To encrypt, shift each character in the plaintext by the amount dictated by corresponding character of the key
  - Wrap around in the key as needed
- Decryption just reverses the process

```
tellhimaboutme
cafecafecafeca
veqpjiredozxoe
```

the key is cafe

The Vigenère cipher

- Substitutions are dictated by the symbol in the array cells determined by the pair < plaintext character, key character>
  - <d, g> yields J, <a, c> yields C, <d, a> yields D, …



Inspired "*rotor machines*" such as Enigma used during World War II

The Vigenère cipher and its weakness

- Robusteness
  - If keys are 14-character strings over the English alphabet, then the key space has size $26^{14} \approx 2^{66}$
  - If variable length keys are used, then the key space is even bigger
  - Brute-force search is infeasible
- Believed secure for many years …
- One weakness is the repeating nature of the key.
- If one guesses the key's length, the cipher text can be treated as interwoven Caesar ciphers, which can be broken individually.

Weaknesses of ENIGMA

- The operators, rather than picking a random key would sometimes pick three consecutive letters from the Enigma keyboard such as QWE or BNM.
- The predictable message keys became known as cillies, probably the initials of the operator's girlfriend, CIL, that was sometimes used.
- Before cracking Enigma the hard way, it became routine for the cryptanalysts to try out the cillies, and they would be sometimes successful.
- After building up a vast library of decrypted messages, Turing noticed that many of them conformed to a rigid structure.
- He could then predict part of the contents of an undeciphered message, based on when it was sent and its source:
  - Germans sent a regular enciphered weather report shortly after 6 A.M. each day.
  - An encrypted message intercepted at 6:05 A.M. could contain the word "wetter"

Variant of Vigenère cipher

Key: LOVEBIRD								
Plaintext:	BIRDS	LOVEW	HEATB	READB	ROWNR	ICEAN	DAWON	DERFU
Key :	LOVEB	IRDLO	VEBIR	DLOVE	BIRDL	OVEBI	RDLOV	EBIRD
Ciphertext:	MWMHT	TFYPK	CIBBS	UPOYF	SWNQC	WXIBV	UDHCI	HFZWX

Original: key (LOVEBIRD) is written repeatedly below the plaintext and this is changed as dictated by the key.

Key: LOVEBIRD								
Plaintext:	BIRDS	LOVEW	HEATB	READB	ROWNR	ICEAN	DAWON	DERFU
Key :	LOVEB	IRDMP	WFCJS	ENQXG	DKTFO	RYHEL	UGPSZ	IFMVH
Ciphertext:	MWMHT	TFYQL	DJCCT	VRQAH	UYPSF	ZALEY	XGLGM	LJDAB

Variant: the key is also shifted by 1 at each repetition:

LOVEBIRD becomes MPWFCJS (new key)

Permutation Techniques

With permutations techniques, ciphertext is obtained by *changing the order of the letters*.

If the plaintext is written in diagonal sequences and then read in line, the message "meet me after the toga party" becomes:

```
m e m a t r h t g p r y
 e t e f e t e o a a t
```

If read by line, the above message becomes

MEMATRHTGPRYETEFETEOAAT

Rail fence technique

Plain text is written downwards and diagonally on successive "rails" of an imaginary fence, moving down till the bottom rail is reached and then up till the top rail is reached, continuing until the whole plaintext is written out.

```
W . . . E . . . C . . . R . . . L . . . T . . . E
. E . R . D . S . O . E . E . F . E . A . O . C .
. . A . . . I . . . V . . . D . . . E . . . N . .
```

```
WECRLTEERDSOEEFEAOCAIVDEN
```

Other permutation techniques

An alternative is to write in a square the message line by line, and read it column by column by setting the order of reading between the columns

```
Key:          4 3 1 2 5 6 7
Plaintext:    a t t a c k p
              o s t p o n e
              d u n t i l t
              w o a m x y z
```

When columns are read in the order imposed by the key, we have

TTNAAPTMTSUOAODWCOIXKNLYPETZ

Introduction to cryptography and classical ciphers

Rocco DE NICOLA
IMT Lucca

https://cybersecnatlab.it