## MA 201: MATHEMATICS III

End Semester Examination (July - November, 2014)

Duration: 3 hrs

Date: 25/11/2014

Total Marks: 50

Important Instructions: This paper contains SIX questions. Answer to each question should appear together. Please mention the answer page number for each question on the front page of the answer booklet. Give appropriate reasons wherever necessary.

## Wishing you all the best

- (a) By using residue theorem, evaluate the integral  $\int_{0}^{2\pi} \frac{1}{1+3\cos^2 t} dt$ . [4]
- (b) Find a Möbius transformation that maps  $z_1 = 1$ ,  $z_2 = 0$  and  $z_3 = -1$  onto the points  $w_1 = i, w_2 = \infty \text{ and } w_3 = 1.$ 
  - Find the image of  $\mathbb{H} = \{z = x + iy \in \mathbb{C} : y > 0\}$  under the Möbius transform  $S(z)=\frac{i-z}{i+z}.$

[3]

dz=it

2. (a) Use the method of characteristics to find the solution of the following initial value problem (IVP):

$$\frac{\partial u}{\partial y} + (1 - 2u)\frac{\partial u}{\partial x} = 0, \quad -\infty < x < \infty, \quad y > 0,$$

$$u(x, 0) = f(x) = \begin{cases} 0, & x \le 0, \\ x, & 0 < x \le 1, \\ 1, & 1 < x. \end{cases}$$

9+10-24P

- (b) Determine the form of the functions f(x) for which the first order partial differential equation  $\frac{\partial u}{\partial x}(x,y) - u(x,y) = 0$ , with side condition u(x,1) = f(x), has a solution.
- 3. (a) Use d'Alembert's formula to find the solution to the following initial boundary value problem (IBVP):

$$\begin{split} &\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \infty, \quad t > 0, \\ &u(0,t) = 2t, \quad t > 0, \\ &u(x,0) = f(x), \quad \frac{\partial u}{\partial t}(x,0) = g(x), \quad 0 \le x < \infty, \end{split}$$

where f(x) and g(x) are given functions.

(b) Show that the solution u(x,t) of the following IBVP

[3]

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0,$$

$$u(0,t) = 0, \quad u(\pi,t) = 0, \quad t > 0,$$

$$u(x,0) = \sin x + \frac{1}{2}\sin(2x), \quad 0 \le x \le \pi$$

satisfies  $0 \le u(x,t) \le \frac{3\sqrt{3}}{4}$  for all  $0 \le x \le \pi$ ,  $t \ge 0$ . d2 = d0 e10

7 = en coso tisino TI Easo - isino

$$x^{2} = \frac{\pi^{2}}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^{n}}{n^{2}} \cos(nx), \quad -\pi < x < \pi,$$

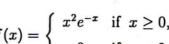
find the value of  $\sum_{n=0}^{\infty} \frac{1}{n^4}$ .

(b) Find the Fourier transform of



[3] [3]

[3]

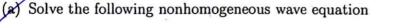


$$f(x) = \begin{cases} x^2 e^{-x} & \text{if } x \ge 0, \\ 0 & \text{if } x < 0. \end{cases}$$

(c) Find a function f(x) which solves the integral equation

$$f(x) + \int_0^\infty f(x-t)e^{-t}dt = \frac{1}{1+x^2}.$$

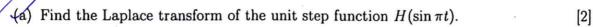
The following three problems are to be solved with the help of separation of variables method,



$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + Kx, \quad 0 < x < 1, \ t > 0, \quad \text{where} \quad K \ \text{ is a constant},$$

subject to 
$$u(0,t) = 0 = u(1,t), \ t > 0; \ u(x,0) = f(x), \ \frac{\partial u}{\partial t}(x,0) = 0, \ 0 \le x \le 1.$$

- (b) Consider the heat conduction problem in a thin metal bar of length L and thermal diffusivity  $\alpha$  with an initial temperature distribution f(x) and no heat loss at both ends. Formulate and solve the IBVP for all x and t > 0.
- Find the steady-state temperature  $u(r,\theta)$  in a disk of radius 1 if the upper half of the circumference is kept at 1000 and the lower half is kept at 00. What can you conclude about the temperature at the centre of the disk with respect to the temperature on the circumference? [4]



(b) By using Laplace transform, find the solution of the following IVP:

$$\frac{d^2y}{dt^2} - y = t * \cos t, \ y(0) = -1, \ \frac{dy}{dt}(0) = 0,$$

where \* denotes convolution.

[3]

[3]

Formulate and solve the following IBVP by using Laplace transform method:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \sin \pi x, \quad 0 < x < 1, \ t > 0,$$

with all zero conditions, where the boundary conditions are of Dirichlet type

\*\*\*\*\* Paper Ends \*\*\*\*\*