DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

MA201 Mathematics III

First Semester of Academic Year 2015 - 2016

Tutorial Sheet - 6

Date of Tutorial Class: September 29, 2015

Conformal Mappings, Mobius Transformations

1. State where the following mappings are conformal.
   (i) $w = \sin z$ (ii) $w = z^2 + 2z$.

2. Show that the mapping $w = \cos z$ is not conformal at $z_0 = 0$.

3. Let $T(z) = \frac{(1-i)z+2}{(1+i)z+2}$. Find $T^{-1}(z)$.

4. Find a bilinear transformation which maps 2, i, -2 onto 1, i, -1.

5. Find a Mobius transformation which maps 0, 1, ∞ onto i, -1, $-i$.

6. Find a Mobius transformation which maps i, -1, 1 onto 0, 1, ∞.

7. Find a bilinear transformation which maps ∞, i, 0 onto 0, i, ∞.

8. Find the image of the right half plane $\mathrm{Re}(z) > 0$ under $w = i(1-z)/(1+z)$.

9. Show that a bilinear transformation maps circles (include straight lines) onto circles (include straight lines).

10. Show that the transformation $w = \dfrac{z-i}{1-iz}$ maps the interior of the circle $|z| = 1$ onto the lower halfplane $\mathrm{Im}\,(w) < 0$.

11. Find the image of the straight line $\mathrm{Re}\,(z) = a$ (constant) in the z-plane under the mapping $w = \dfrac{z-1}{z+1}$. (This map is used to display the rang of impedance of an electrical circuit. See: Impedance Smith Chart. This problem indicates constant resistance contours.)

Supplementary Problems

12. Read Section 88 from the Brown and Churchill Book, 7th edition, to understand the derivation of *the most general bilinear transformation that maps the upper halfplane Im* $(z) > 0$ *in the z-plane onto the unit open disk* $|w| < 1$ *in the w-plane.*
    By imitating the arguments, derive *the most general bilinear transformation that maps the right halfplane Re* $(z) > 0$ *in the z-plane onto the unit open disk* $|w| < 1$ *in the w-plane.*

13. Find a bilinear transformation that maps the crescent-shaped region that lies inside the disk $|z-2| < 2$ and outside the circle $|z-1| = 1$ onto a horizontal strip $0 < \mathrm{Im}(z) < 1$.