Topic 2: Firm Behaviour

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Learning Objectives

- The problem of the Firm
- Short and Long Run
- Technology of Firm: Production Function
- Profit maximization, input demand functions, supply function
- Cost minimization, conditional input demand function.
- Short and Long Run cost.
- Economies of scale and scope.
- Opportunity costs and sunk costs.

What is a Firm?

- To begin with, firms are black boxes which transforms various inputs into output. We will call this the engineering viewpoint of the firm.
- Inputs are transformed into output through production function, which is the description of technology.
- Example: a coffee shop owner uses pre-mix coffee, labor and a coffee machine to produce coffee. The technological rule that ties up the quantities of input with that of output is called production function.
- Objective of the firm is to choose inputs, given input and output prices, in such a way that maximises profit or minimises cost.

Production Function

- Assume that the output is q. Two inputs are L, K. Then the production function is a mathematical relation $q = f(L, K)$

Example

: $q = \ln L + \ln K$; $q = \sqrt{L} + \sqrt{K}$ etc.

- Production munction may look different in short and long run.

Short and Long Run

- One of the most important concepts of Economics
- In short run, you can change only a few factors of production: variable factors.
- Other factor is fixed factor.
- In short run, labour may be more variable than capital.
- Production function, $y = f(L, \bar{K})$
- In long run, all factors are variable.

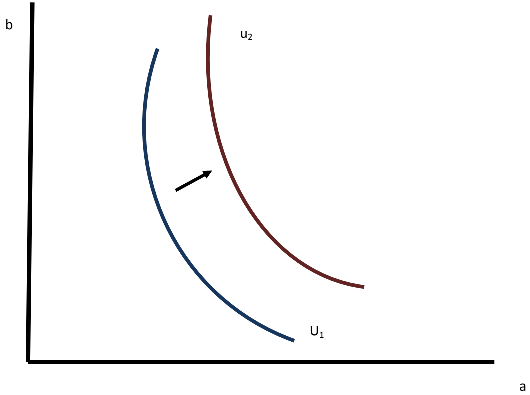
Marginal and Average Product

- Suppose one input is increased by a small amount. The effect onproduction is called marginal product For example, the marginal utility of L is $\frac{\Delta y}{\Delta L}$. At the limit, we can replace it by the partial derivative $MPL = \frac{\partial f}{\partial L} = f_L$
- Thus marginal productivity of a factor is positive, i.e. as input increases, output also increases
- But the rate at which output increases is first increasing (say, starting from zero input) and then diminishing: law of diminishing marginal product.
- Economically interesting input choice: where diminishing marginal productivity applies.
- Similarly, we can define average product $AP_L = \frac{y}{L}$.

Isoquants

- An isoquant is the locus of (L, K) such that, along the curve, y is a constant.
- Slope of isoquants: *negative* (since lower amount of L nececitates higher amount of K as compensation and vice versa)
- Slope of the isoquant: falls as we move to right (due to diminishing marginal productivity).
- $$\begin{aligned} \frac{dL}{dK} &= -\frac{f_K}{f_L} \\ \frac{d^2 L}{dK^2} &= -\frac{f_L f_{KK}^2 + f_K f_{LL}^2 - 2 f_{KL} f_L f_K}{f_L^3} \end{aligned}$$
- Higher isoquants mean higher output.

Isoquant Map



Other Production Functions

- In general, we assume that the function $f()$ is C^2, i.e. continuous second derivatives exist.
- In case of perfect substitutes: $f(L, K) = L + K$
- In case of perfect complements: $f(L, K) = \min(L, K)$
- Exercise: draw the isoquant map.

Long Run Production Function

- In 'long run' all factors are variable.
- Scale economies.
- If we increase all factors by same proportion, what is the effect on output?
- Suppose you double L and K. What is the effect on y?
- If y more than doubles, increasing returns to scale.
- If y doubles, constant returns to scale.
- If y increases, but less than double, decreasing or diminishing returns to scale.

Homogeneous Production Functions

- A production function $y = f(L, K)$ is homogeneous of degree k if

$$f(\lambda L, \lambda K) = \lambda^k f(L, K)$$

- If $k > 1$, then IRS
  $k = 1$,then CRS
  $k < 1$, then DRS
- Suppose you double L and K. What is the effect on y?
- If y more than doubles, increasing returns to scale.
- If y doubles, constant returns to scale.
- If y increases, but less than double, decreasing or diminishing returns to scale.

Example

Suppose $q = AK^{\alpha}L^{\beta}$. Show that the production function is homogeneous and comment on its scale properties.

- Profit (π) is the difference between revenue and cost.
- $\pi = pq - wL - rK = pf(L, K) - wL - rK$
- Choosing L and K, it can be shown that at optimum

$$\frac{f_L}{f_K} = \frac{w}{r}$$

- Why does this condition make sense?

Supply Function: Route 1

- Solving the first order conditions, we get $K = K(w, r, p)$ and $L = L(w, r, p)$.
- These are the input demand functions.
- Plugging back in the production function, $q = f\left(L(w, r.p).K(w, r, p)\right) = F(w, r, p)$
- The relashionship between q and p, keeping w, r constant, is called the supply function.

Problem with Profit Function

- Sometimes, perfectly well defined problems may not have a solution.

Example

$Max_{L,K}\ pK^{0.5}L^{0.5} - wL - rK$ does not have a solution.

- That is why economists usually focus on the Cost minimization, subject to an output constraint.

Cost Mimimization

- Given that you have a target output, how to choose your inputs such that the cost of production is minimised?
- Mathematical Programme

$$\begin{aligned} & \min_{L,K} \; wL + rK \\ \text{such that, } f(L,K) &= q \end{aligned}$$

- The first order condition from Lagrangian yields

$$\begin{aligned} w - \mu f_L &= 0 \\ r - \mu f_K &= 0 \\ f(L,K) - q &= 0 \end{aligned}$$

- Solving this, we express L, K as function of w,r and y.

Cost Function

- $L = L(w, r, y)$ and $K = K(w, r, y)$ are known as *conditional* input demand functions.
- Putting in the objective function $C = w * L(w, r, q) + r * K(w, r, q) = C(w, r, q)$
- The minimum cost of producing q units of output, given prices are w, r.
- *Ceteris paribus*, the relationship between cost and output $C = C(q)$ is known as the *cost function*.

Analysis of Cost

- Usually, the cost function will differ in short and long run.
- In short run, there are fixed factors of production, so the cost function will have 'fixed part'
- The part of the cost function which changes as output changes is known as variable cost.
- A typical short run cost function is $C = \Psi(y) + F$, where F is the fixed part and $\Psi(y)$ is the variable part.
- In long run, because all costs are variable, we must have $C = C(y)$.

Cost: Short Run

- The marginal cost is defined as $SMC(y) = \Psi'(y)$. In words, it is the extra cost incurred if one changes the production a little bit.
- The Average cost is the unit cost of production, defined as $SAC(y) = \frac{C}{y}$. In short run, it consists of two components.

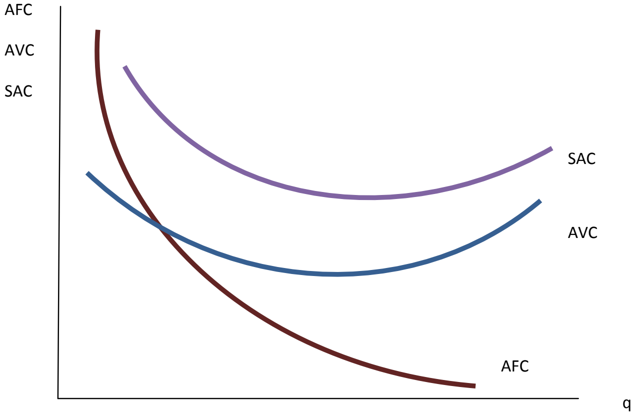
- Average Variable Cost (AVC) $= \frac{\Psi(y)}{y}$

-Average Fixed Cost (AFC) $= \frac{F}{y}$

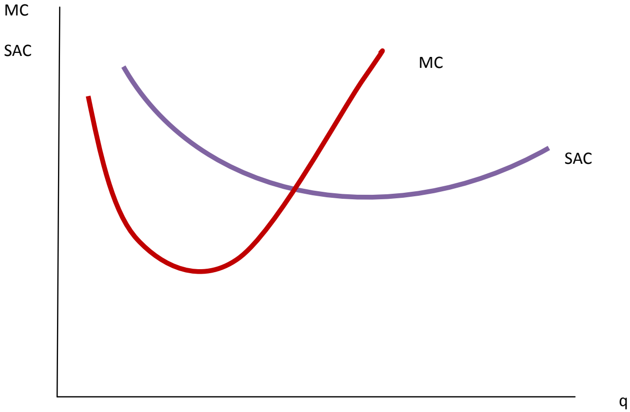
MC,AC and AVC

- If MC $>$ ATC, then AC is rising.
- If MC $>$ AVC, then AVC is rising.
- If MC $<$ ATC, then AC is falling.
- If MC $<$ AVC, then AVC is falling.
- If MC $=$ AVC and MC $=$ AC, then AVC and AC are at their minimum points.

Geometry Of Cost Curves (General Case-1)



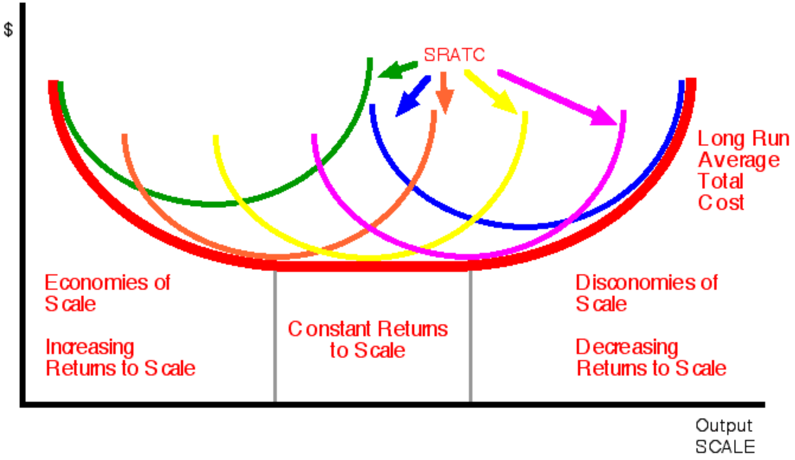
Geometry Of Cost Curves (General Case-2)



Geometry Of Cost Curves (General Case-3)

- AVC is U shaped because of increasing and then decreasing marginal product.
- AFC is a rectangular hyperbola.
- SAC is U shaped due to the interplay between AFC and AVC.
- MC is U shaped due to increasing and decreasing marginal product.

Long Run AC: Lower Envelope of Short Run AC



Economics of Scale and Scope

- Economies of scale is associated with long run production.
- No variable costs.
- Economies of scale if AC is falling, diseconomies of scale if AC is rising.

Definition

A cost function exhibits economies of scale if
$C(q_1 + q_2) \leq C(q_1) + C(q_2)$

Fact

If the production function shows (decreasing)increasing returns to scale, then the cost function exhibits (dis)economies of scale.

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- Economies of scope is related to joint production of goods. For example, it is less costly to produce wax and honey together (share the inputs) than to produce them separately.

Application of Cost Concepts

- Applications of cost concepts have to wait upto the next topic.
- Even now, we can think about the importance of cost concepts.
- Assume that two firms are merging.
- Merger is beneficial if, and only if, there is economies of scale due to
  -indivisible inputs
  -firm specific specialization.
  -avoidance of duplicate set up (fixed costs).
- *Natural Monopoly* occurs when the AC is downward sloping throughout, so it is better to have a single firm serving the market rather than many firms.

Opportunity Cost

- Opportunity cost of a business is the option you forgo to pursue a decision..
- Suppose you invest \$10000 to buy a machine. The accounting cost is \$10000.
- The opportunitu cost is the interest income that you forego, e.g at 10% interest rate, \$1000.
- Economic cost is a sum of opportunity cost and accounting cost.
- In future, we will talk about 'zero profit condition. That does not mean that accounting profit (Revenue net of accounting cost) is zero, but economic profit (Revenue -accounting cost- opportunity cost) is zero.

Sunk Cost

- A sunk cost is a fixed cost that can not be recovered, but not all fixed costs are sunk cost.
- Sunk costs can not be avoided, regardless of the choices.
- They must be ignored.
- An example: suppose you know, just prior to a murder-mystery movie, the identity of the killer.
- Should you watch the movie just because you have purchased the ticket?
- If an industry has high sunk cost, then it acts as a significant barrier to entry.

References

- Cabral, online edition, chapter 3, page 1-8; Mankiw page 247-48