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MATH 221-003

# MATH221 (Homework-2)

* 2.4

4. gcd (8370, 465)

⇒ gcd (465, 8370%465)
⇒ gcd (465, 0)

gcd (8370, 465) is 465.

∴ we can say: $8370 = 18 \times 465$
$465 = 1 \times 465$

∴ Linear Combination is $= \left( \frac{\frac{8370}{18} + \frac{465}{1}}{2} \right)$

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26. Let 'n' be a positive integer

Let gcd (n, n+1) = g
then $\frac{g}{n}$ and $\frac{g}{n+1}$

⇒ $\frac{g}{n+1-n} = \frac{g}{1} = g$.

The only possible value for g that divides 1 is 1.

$\therefore g = 1$

$\therefore \gcd(n, n+1) = 1$

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48.

(a) $n = 8 = 3 + 5$

(b) $n = 12 = 5 + 7$

(c) $n = 28 = 5 + 23$

In all 3 solutions, both the adding values are prime, that verifies Goldbach's conjecture.

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* 6.1

2.

(a) Yes, the graph is simple because it does not have any loop or multiple edges.

(b) No, the graph is incomplete because every pair of distinct vertices is not connected by a

pair of unique edges.

(c) Yes, the graph is connected because there is a path between every pair of vertex.

(d) Yes, (3-5-6 , 3-4-5-6)

(e) Yes, (3-4-5-3)

(f) Yes

(g) Yes

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4.

(a) 4 and 5

(b) shortest length = 2 , path = 3-5-6

(c) 1-2-1-2-2-3-4-5-6

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8.

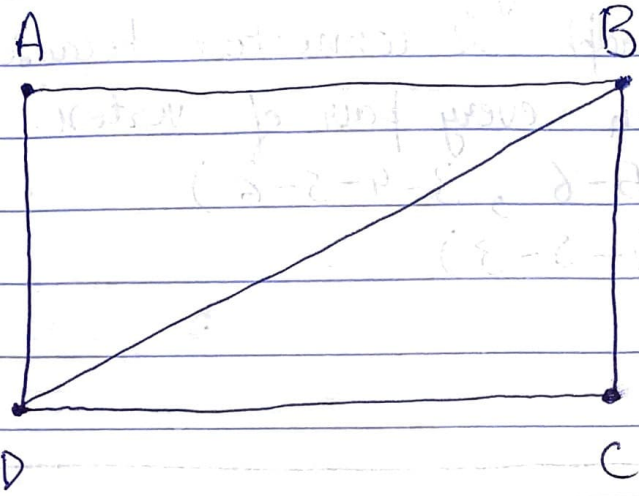
(a) If the graph exists, the sum of degree of vertices is $7 \times 3 = 21$.

For any graph to exist, the sum of degrees of each vertex should be even. And in this case, it is odd. ∴ The graph cannot exist.

(b) The sum of these nodes = $2 \times 2 + 2 \times 3$

$= 10$

As the sum of degrees of each vertex is even, The graph is possible.

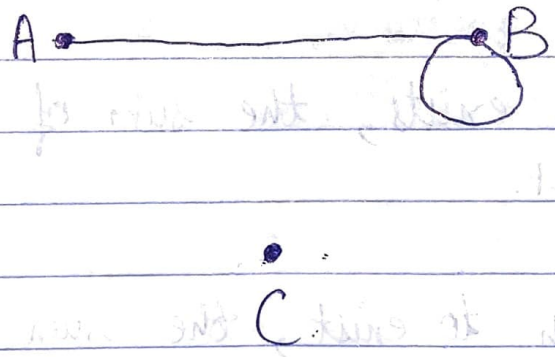



$\deg(A) = \deg(C) = 2$

$\deg(B) = \deg(D) = 3$

(c) The sum of these nodes = $0 + 1 + 3 = 4$

As the sum of degrees of each ~~node~~ vertex is even, This graph is possible.




[$\deg(A) = 1$, $\deg(B) = 3$, $\deg(C) = 0$]

(d) For a complete graph of 4 nodes, it should have a degree of 3 at each vertices so that the other vertices are connected.

∴ The graph is not possible.