Vistrit Pandey
vp433

MATH 22-003

MATH 221 HOMEWORK-1

14.

(a) True
(b) True
(C) ~~True~~ False (because $\{1,3,\pi\} \not\subseteq \{\{1,3,\pi\}, 1\}$)
(d) True
(e) True
(f) True
(g) False (because $\{1\} \in S$, $\{1\} \not\subseteq \{1,3,9,10\}$)

22.

(a) False (it should be $\phi \subset A$)
(b) False ($\phi \neq \{\phi\}$, both are different)
(C) True
(d) False (A and C can be equal but not equal to B)
(e) False

26. We can prove it using Mathematical Induction.

Base: $n=3$

A set with 3 elements has exactly 1 subset with 3 elements, i.e., the set itself.

$\Rightarrow$ ~~n(n-1)(n-2)/6~~ If we put 3 in $\frac{n(n-1)(n-2)}{6}$,

we get:

$$\frac{3(3-1)(3-2)}{6} = \frac{3\times 2\times 1}{6} = 1$$

$\therefore$ The base is proved

Assuming for k elements.

$\therefore n = k.$

$$\Rightarrow \frac{k(k-1)(k-2)}{6}.$$

$\therefore$ for $(k+1)$ elements

$$\Rightarrow \frac{(k+1)\cdot k\cdot (k-1)}{6}$$

Let x be a member of set $(k+1)$ elements

If we take the value from ex. 25; we get $\left(\frac{k(k-1)}{2}\right)$ subsets, for pairing x with 2 element subsets of k-element set.

$\therefore$ The number of 3 element subset is:

$$\frac{k(k-1)(k-2)}{6} + \frac{k(k-1)}{2}$$

$$= \frac{k(k-1)(k-2) + 3(k-1)}{6}$$

$$= \frac{k(k-1)(k-2+3)}{6}$$

$$= \frac{k(k-1)(k+1)}{6}$$ (which is equal to subsets in $k+1$)

$\therefore$ L.H.S = R.H.S

Hence Proved

---

36. Let $x \in P(A)$

$\therefore$ x is a subset of A.

As $x \subseteq A$ and $A \subseteq B$, $\therefore x \subseteq B$

If x is a subset of B, It is a member of $P(B)$

$x \in P(B)$

$\therefore P(A) \subseteq P(B)$

~~$x \in P(A) \Leftrightarrow x \subseteq A$~~

~~$A \subseteq B \Rightarrow x \subseteq B$~~

Hence Proved

40.

(a) Unary operation

(b) No because the closure fails ($3 \cdot 3 \notin S$)

(c) No because 2 different fractions could have same denominator and $\therefore$ Nothing unique.

(d) Binary operation

42.

(b) No, the closure fails because the sum of 2 non-consecutive fibonacci series is not a Fibonacci number.

(c) Unary operation

(d) No, the closure fails because the sum of 2 irrational number can be rational.