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$$= e^{-\lambda}\left[\frac{\lambda^2}{0!} + \frac{\lambda^3}{1!} + \frac{\lambda^4}{2!} + \cdots\right] + e^{-\lambda}\left[\frac{\lambda}{0!} + \frac{\lambda^2}{1!} + \cdots\right]$$

$$= e^{-\lambda}\lambda^2\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots\right] + e^{-\lambda}\lambda\left[1 + \frac{\lambda}{1!} + \cdots\right]$$

Nithin Davis

22.2.16 Monday

Fitting of poisson distribution

Poisson distribution has only 1 parameter λ.

Estimation of λ

Let $\mu = \lambda$.

Mean $\bar{x} = \dfrac{\sum x_i O_i}{\sum O_i}$

i.e, $\mu = \bar{x} = \lambda$

Now calculate $f(x) = \dfrac{e^{-\lambda} \cdot \lambda^x}{x!}$

calculate $E_i = N f(x_i)$ $N = \sum O_i$

Finally check $\sum O_i = \sum E_i$

1) Fit a poisson distribution to the following data:

x:	0	1	2	3	4
f:	123	59	14	3	1

Ans:

$$\mu = \lambda = \frac{(0\times123) + (1\times59) + (2\times14) + (3\times3) + (4\times1)}{123+59+14+3+1}$$

$$= \frac{1}{2}$$

$$= 0.5$$

x	$f(x) = \dfrac{e^{-1/2} \cdot (\frac{1}{2})^x}{x!}$	$E_i = \Sigma O_i \, f(x_i)$
0	$f(0) = 0.6065$	121.3
1	$f(1) = 0.3032$	60.64
2	$f(2) = 0.0758$	15.16
3	$f(3) = 0.0126$	2.52
4	$f(4) = 1.5763 \times 10^{-3}$ $= 0.0016$	0.32 ~~315.26~~
		$\Sigma E_i = 199.94$

$\Sigma E_i = 199.94 \simeq 200$

$\therefore \Sigma E_i \simeq N$

2. $n = 5$

Let x be no. of successes.

$X \sim b(x : n, p)$

$f(x) = {}^nC_x \, p^x q^{n-x} \rightarrow ①$

$f(1) = 0.4096 \rightarrow ②$

$f(2) = 0.2048 \rightarrow ③$

Put ② in ①,

$0.4096 = 5C_1 \cdot p^1 q^4 \longrightarrow ④$

put ③ in ①,

$0.2048 = 5C_2 \cdot p^2 \cdot q^3 \longrightarrow ⑤$

$$\frac{5p^1q^4}{10p^2q^3} = \frac{0.4096}{0.2048}$$

$$\frac{q}{2p} = 2$$

$q = 4p$

$1-p = 4p$

$5p = 1$

$p = 1/5$, $q = 4/5$

Mean $= np = 5 \times \frac{1}{5} = 1$

Variance $= npq = 1 \times \frac{4}{5} = 4/5$

$P[x \text{ exceeds } 3] = P[x = 4, 5]$

$= P[x=4] + P[x=5]$

$= f(4) + f(5)$

$= 5C_4 \cdot \left(\frac{1}{5}\right)^4 \left(\frac{4}{5}\right)^1 + 5C_5 \cdot \left(\frac{1}{5}\right)^5 \left(\frac{4}{5}\right)^0$

$= 5 \cdot \left(\frac{1}{5}\right)^4 \left(\frac{4}{5}\right) + \left(\frac{1}{5}\right)^5$

$= 6.72 \times 10^{-3}$

2. $X \sim b(x : n, p)$

$f(x) = {}^nC_x \, p^x \, q^{n-x}$

Mean $= np = 2.4$

$S.D = \sqrt{\text{variance}} = \sqrt{npq} = 1.2$

$npq = 1.44$

$\frac{np}{npq} = \frac{2.4}{1.44}$

$\frac{1}{q} = \frac{2.4}{1.44}$

$\underline{\underline{q = 0.6}}$

$\underline{\underline{p = 0.4}}$

$n = \frac{2.4}{0.4}$

$= \underline{\underline{6}}$

$\frac{6!}{5!\,1!}$

$P[X \geq 5] = P[X=5] + P[X=6]$

$= {}^6C_5 (0.4)^5 (0.6)^1 + {}^6C_6 (0.4)^6 (0.6)^0$

$= \underline{\underline{0.04096}}$

3. $n = 6$.

x be the no: of bombs that will strike the target.

$X \sim b(x : n, p) \quad f(x) = {}^nC_x \, p^x \, q^{n-x}$

$p = \frac{1}{5}$.

$\therefore \; q = \frac{4}{5}$.

$\frac{6!}{2!\,4!}$

$f(2)$ P(exactly two bombs strikes) $= f(2)$

$= {}^6C_2 \left(\frac{1}{5}\right)^2 \cdot \left(\frac{4}{5}\right)^4$

$= \underline{\underline{0.24576}}$

5. $n = 7$

x be the no: of workmen who are suffering from occupational disease.

$X \sim b(x : n, p) \quad f(x) = {}^nC_x \, p^x \, q^{n-x}$.

$P(5 \text{ or more sufferers}) = P[X \geq 5]$

$= P[X=5] + P[X=6] + P[X=7]$

$p = \frac{10}{100} = \underline{\underline{0.1}}$

$q = \underline{\underline{0.9}}$

$P[x \geq 5] = {}^7C_5 \cdot (0.1)^5 (0.9)^2 + {}^7C_6 \cdot (0.1)^6 (0.9)^1 +$

${}^7C_7 \cdot (0.1)^7 (0.9)^0$

$= \underline{\underline{1.765 \times 10^{-4}}}$

6. $n = 500 \Rightarrow > 30 \Rightarrow$ Poisson distribution.

$x \to$ no: of defective screws.

$X \sim P(x : \lambda)$, $f(x) = \dfrac{e^{-\lambda} \cdot \lambda^x}{x!}$

$p = \dfrac{2}{100}$

$\lambda = np$.

$= 500 \times \dfrac{2}{100}$

$= \underline{\underline{10}}$

P(a box contains 15 defecti. screws)

$= P[x = 15]$

$= f(15)$

$= \dfrac{e^{-10} \cdot 10^{15}}{15!}$

$= \underline{\underline{0.0847}}$

26/2/16 Friday

8

$n = 10$

x be the event of getting a literate person from a group.

$x \sim b(x : n, p)$ $\quad f(x) = nC_x \, p^x q^{n-x}$

$p = 80\% = 0.8$

$q = 0.2$

P[1 group contains 8 or less literate persons]

$= P[x = 0, 1, 2, 3]$

$f(0) + f(1) + f(2) + f(3)$.

$= 10C_0 \cdot (0.8)^0 (0.2)^{10} + 10C_1 (0.8)^1 (0.2)^9 +$

$10C_2 (0.8)^2 (0.2)^8 + 10C_3 (0.8)^3 (0.2)^7$

$= \underline{8.56 \times 10^{-4}}$

No: of investigators with this probability

$= 8.56 \times 10^{-4} \times 200$

$= 0.1712$

$\cong \underline{\underline{0}}$

1

Note:

If A & B are independent, then,

$P(A \text{ or } B) = P(A) + P(B)$

$P(A \text{ and } B) = P(A) \cdot P(B)$

9. $n = 6400$

Let X be the event of getting 6 heads with 6 coins.

$X \sim P(x:\lambda)$, $f(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$

$P[X=3] = ?$

$= \frac{e^{-\lambda}\lambda^3}{3!}$

$p = $ [Prob of getting 6 heads]

$= P(H \& H \& H \& H \& H \& H)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \left(\frac{1}{2}\right)^6$ (2)

$\lambda = np$

$= 6400 \times \frac{1}{2^6}$

$= 100.$

$P[X=3] = \frac{e^{-100} \cdot 100^3}{3!}$

$= 6.2 \times 10^{-39}$

10. $n = 400$

X be the event that a man will die b/w the age of 35 & 40.

$X \sim P(x:\lambda)$ $f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$p = 0.018$

$P[2 \text{ men die}] = P[X=2]$

$= f(2) = \frac{e^{-\lambda}\lambda^2}{2!}$

$\lambda = np$

$= 400 \times 0.018$

$= 7.2$ (3)

$f(2) = \frac{e^{-7.2}\, 7.2^2}{2!}$

$= 0.01935$

11. Bag contains 20 balls.

Out of 20, 4 balls are drawn $\bar{c}$ replacement.

so, $n = 4$.

x be the event of getting white ball.

$X \sim b(x : n, p)$

p = getting a white ball

$= \frac{\text{fav. no}}{\text{tot. no}} = \frac{5}{20} = \frac{1}{4}$.

$\therefore q = \frac{3}{4}$.

(i) P[none is white] $= P[X=0]$

$= 4C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^4$

$= \underline{\underline{0.316}}$

(ii) P[Atleast one white]

$= 1 - P[X<1]$

$= 1 - P[X=0]$

$= 1 - 0.316$

$= \underline{\underline{0.683}}$

4

(iii) P[all are white] $= P[X=4]$

$= 4C_4 \left(\frac{1}{4}\right)^4 \left(\frac{8}{4}\right)^0$

$= \underline{\underline{0.0039}}$

(iv) P[only two are white] $= P[X=2]$

$= 4C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^2$

$= \underline{\underline{0.21}}$

Assignment Wednesday

18) $P[X=2]$ is two thirds of $P[X=1]$

$X \sim P(x : \lambda)$ $\quad f(x) = \frac{e^{-\lambda} \lambda^x}{x!}$

$P(x=2) = \frac{2}{3} P(x=1)$

$f(2) = \frac{2}{3} f(1)$

5

$\frac{\cancel{e^{-\lambda}} \lambda^2}{2!} = \frac{2}{3} \frac{\cancel{e^{-\lambda}} \cancel{\lambda}}{1!}$

$\Rightarrow \lambda = 4/3$.

(i) $P[X=0] = \frac{e^{-4/3} \lambda^0}{0!} = e^{-4/3} = 0.2637$

(ii) $P[X=3] = \frac{e^{-4/3} \left(\frac{4}{3}\right)^3}{3!} = 0.1042$

(ii) $P[x \text{ exceeds } 3] = P[X>3]$

$= 1-P[X \leq 3]$

$= 1 - [f(0)+f(1)+f(2)+f(3)]$

$= \underline{\underline{0.046}}$

20. $P(x=2) = 9P(x=4)+90P(x=6)$

$f(2) = 9f(4)+90f(6)$

$$\frac{e^{-\lambda}\lambda^2}{2!} = 9 \cdot \frac{e^{-\lambda}\lambda^4}{4!} + 90 \cdot \frac{e^{-\lambda} \cdot \lambda^6}{6!}$$

$$\frac{1}{2} = \frac{9\lambda^2}{4!} + \frac{90\lambda^4}{6!}$$

4 solns -

put $\lambda^2 = m$.

$$\Rightarrow \frac{1}{2} = \frac{9m}{4!} + \frac{90m^2}{6!}$$

29.2.16 Monday

21. x be the event of getting a head

$P = \frac{1}{2}$

10 coins are tossed simultaneously is same as 1 coin is tossed 10 times.

$N = 10$

$X \sim b(x:n,p)$ $f(x) = {}^nC_x \cdot p^x q^{n-x}$

$P[\text{getting atleast 7 heads}] = P(x \geq 7) = {}^{10}C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3 +$

${}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 + {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^1 +$

${}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0$

$= \left(\frac{1}{2}\right)^{10} \left[{}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}\right]$

$= \underline{\underline{0.171875}}$

24. let x be the event ~~that~~ ~~a~~ ~~selected~~ page of getting an error.

10 pgs are selected means $n = 10$.

$X \sim b(x:n,p)$ $f(x) = {}^nC_x p^x q^{n-x}$

$p = \frac{43}{585} = 0.073$.

$q = 1 - \frac{43}{585} = 0.926$

$P(x=0) = {}^{10}C_0 (0.073)^0 (0.926)^{10}$

$= \underline{\underline{0.469}}$

25. x be the event of getting a girl.

$N = 4$.

$X \sim (b: x; n,p)$. $b(x) = {}^nC_x p^x q^{n-x}$

$p = 1/2 \quad q = 1/2$

(i) $P[2 \text{ boys } \& \text{ } 2 \text{ girls}] = P[x=2]$

$= 4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2$

$= 4C_2 \cdot \left(\frac{1}{2}\right)^4$

$= \underline{\underline{0.375}}$

$\therefore$ tot no: of families with this prob $= 0.375 \times 800$

$= \underline{\underline{300}}$

(ii) $P[\text{atleast two boys}] = P[\cancel{x \geq 2}]$

Boys	2	3	4
Girls	2	1	0

$= P[\text{almost two girls}]$

$= P[x = 0, 1, 2]$

$= 4C_0 \cdot \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4 + 4C_1 \cdot \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^3 +$

$4C_2 \cdot \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2$

$= \left(\frac{1}{2}\right)^4 \left[4C_0 + 4C_1 + 4C_2\right]$

$= 0.6875$

$\therefore \quad 0.6875 \times 800 = \underline{\underline{550}}$

(iii) $P[\text{no girl}] = P[x=0]$

$= 4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4$

$= \frac{1}{16}$

$\frac{1}{16} \times 800 = \underline{\underline{50}}$

(iv) $P[\text{almost two girls}] = P[\text{atleast 2 boys}]$ (already did)

$= 0.6875$

$0.6875 \times 800 = \underline{\underline{550}}$

26. X be the no: of demands for a car.

$X \sim P(x; \lambda)$

$\lambda = 1.5$

(i) $P[X=0] = \frac{e^{-1.5} \times 1.5^0}{0!}$

$= \underline{\underline{0.223}}$

(ii) $P[\text{demand is refused}] = P[x > 2]$

$= 1 - P[x \leq 2] = 1 - (p(0) + p(1) + p(2))$

$= 1 - \left(\frac{e^{-1.5} \times 1.5^2}{2!} + \frac{e^{-1.5} \times 1.5^1}{1!} + \frac{e^{-1.5} \times 1.5^0}{0!}\right)$

$= \underline{\underline{0.1911}}$

Continuous Distributions

1) Uniform distribution

A random variable X is said to have a continuous unfm distrbn over an (a,b) if its prob density fn is given by:

$$f(x) = \begin{cases} \dfrac{1}{b-a}, & a < x < b \\ 0, & \text{elsewhere} \end{cases}$$

Note:

Here a & b are called parameters of the unfm distbn & denoted by $X \sim U(a,b)$, ~~a & b~~

Mean & variance of uniform distribution

If $X \sim U(a,b)$, then $f(x) = \begin{cases} \dfrac{1}{b-a} & a < x < b \\ 0 & \text{elsewhere} \end{cases}$

$$\text{Mean} = E(X)$$

$$= \int x f(x)\, dx$$

$$= \int_a^b x \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a} \int_a^b x\, dx$$

$$= \frac{1}{b-a}\left(\frac{x^2}{2}\right)_a^b$$

$$= \frac{1}{2(b-a)}(b^2 - a^2)$$

$$= \frac{1}{2}(b+a)$$

$$\boxed{\text{Mean} = \frac{a+b}{2}}$$

$$\text{variance} = E(x^2) - [E(X)]^2.$$

$$E(x^2) = \int x^2 f(x)\, dx$$

$$= \int_a^b x^2 \times \frac{1}{b-a}\, dx$$

$$= \frac{1}{3(b-a)}\left(x^3\right)_a^b$$

$$= \frac{1}{3(b-a)}(b^3 - a^3)$$

$$= \frac{1}{3(b-a)}\left[(b-a)(b^2 + ab + a^2)\right]$$

$$= \frac{b^2 + ab + a^2}{3}$$

$$V(x) = E(x^2) - [E(x)]^2$$

$$= \frac{b^2 + ab + a^2}{3} - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{b^2 + ab + a^2}{3} - \frac{a^2 + 2ab + b^2}{4}$$

$$= \frac{4b^2 + 4ab + 4a^2 - 3a^2 - 6ab - 3b^2}{12}$$

$$= \frac{b^2 - 2ab + a^2}{12}$$

$$\boxed{V(x) = \frac{(b-a)^2}{12}}$$

[$b-a$ is always +ve in (a,b).]

~~27~~ In a sampling of large no. of parts manufactured by a machine, the mean no. of defectives in ,

27. X be the event of getting a defective part.

$N = 1000$ $\quad X \sim P(a : x)$ $\quad f(x) = \frac{e^{-\lambda} \lambda^x}{x!}$

$p = \frac{2}{20} = \frac{1}{10}$

$q = \frac{9}{10}$

$\lambda = np$

$= 1000 \times \frac{1}{10} = 100$

(i) $P[x=2] = \frac{e^{-100} \cdot 100^2}{2!}$

$=$

(ii) $P[x \geq 2] = 1 - P[x < 2]$

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - \left[\frac{e^{-100} \cdot 100^0}{0!} + \frac{e^{-100} \cdot 100^1}{1!}\right]$

$=$

(iii) $P[x \leq 2] = P[x=0] + P[x=1] + P[x=2]$

$= \frac{e^{-100} \cdot 100^0}{0!} + \frac{e^{-100} \cdot 100^1}{1!} + \frac{e^{-100} \cdot 100^2}{2!}$

$=$

(iv) $P[x=3] + P[x=4] + P[x=5]$

$= \frac{e^{-100} \cdot 100^3}{3!} + \frac{e^{-100} \cdot 100^4}{4!} + \frac{e^{-100} \cdot 100^5}{5!}$

$=$

(v) $P[x > 2] = 1 - P[x \leq 1]$

18/1/16 Friday

Problems
Uniform Distribution

1. Mean $= \frac{a+b}{2}$

$\frac{a+b}{2} = 2$

$a+b = 4 \longrightarrow (1)$

$v = \frac{(b-a)^2}{12}$

$\Rightarrow \frac{(b-a)^2}{12} = 3$.

$(b-a)^2 = 36$

$b-a = \pm 6$

Take $b-a = +6 \; \hookrightarrow (2)$ [coz in an interval $b>a$ $\therefore$ +ve]

Adding (1) & (2),

$2b = 10$

$\underline{\underline{b = 5}}$

$\underline{\underline{a = -1}}$

$X \sim U(a,b)$

$\Rightarrow X \sim U(-1,5)$

$P(X<1) = ?$

$$f(x) = \begin{cases} \frac{1}{b-a}, & a<x<b \\ 0, & \text{elsewhere} \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} \frac{1}{6}, & -1<x<5 \\ 0, & \text{otherwise} \end{cases}$$

$P(X<1) = \int_{-1}^{1} f(x)\,dx\,dx$

$= \int_{-1}^{1} \frac{1}{6}\,dx$

$= \frac{1}{6}(x)_{-1}^{1}$

$= \frac{2}{6}$

$= \underline{\underline{1/3}}$

2. $X \sim U[-3,3]$

(i) $P(X<0)$

$$f(x) = \begin{cases} \frac{1}{3-(-3)} = \frac{1}{6}, & -3<x<3 \\ 0, & \text{otherwise} \end{cases}$$

$P(X<0) = \int_{-\infty}^{0} f(x)\,dx = \int_{-3}^{0} \frac{1}{6}\,dx$

$= \frac{1}{6}(x)_{-3}^{0} = \frac{1}{6}(3)$

$= \frac{3}{6} = \underline{\underline{1/2}}$

(ii) $P\left[|x-1| \geq \frac{1}{2}\right]$

Note: $|x| \leq a \Rightarrow -a \leq x \leq a$

Make $\geq$ into a $<$ function.

$P\left[|x-1| \geq \frac{1}{2}\right] = 1 - \left[P\left[|x-1| < 1/2\right]\right]$

$= 1 - P\left[-1/2 < (x-1) < 1/2\right]$

$= 1 - P\left[-1/2 + 1 < x < 1/2 + 1\right]$

$= 1 - P\left[1/2 < x < \frac{3}{2}\right]$

$= 1 - \int_{1/2}^{3/2} f(x)\,dx$

$= 1 - \int_{+1/2}^{3/2} \frac{1}{6}\,dx$

$= 1 - \frac{1}{6}(3/2 - 1/2)$

$= 1 - \frac{1}{6} \times 1$

$= 1 - 1/6$

$= \underline{\underline{5/6}}$

3. $(-a, a)$ $\quad a > 0$

$X \sim U(-a, a)$

$$f(x) = \begin{cases} \frac{1}{2a}, & -a < x < a \\ 0, & \text{otherwise.} \end{cases}$$

$P(X > 1) = 1/4.$

$\int_1^a f(x)\,dx = \frac{1}{4}$

$\int_1^a \frac{1}{2a}\,dx = \frac{1}{4}$

$\frac{1}{2a}(a-1) = \frac{1}{4}$

$\frac{1}{a}(a-1) = \frac{1}{2}$

$a = 2a - 2$

$\underline{\underline{a = 2}}$

$\therefore \underline{\underline{X \sim U(-2, 2)}}$

4. $$f(x) = \begin{cases} 1/4 &: |x| < 2 \\ 0 &: \text{elsewhere.} \end{cases}$$

$|x| < 2 \Rightarrow -2 < x < 2$

(i) $P(X<1)$

$$= \int_{-2}^{1} \frac{1}{4}\, dx$$

$$= \frac{1}{4}(1+2)$$

$$= \frac{3}{4}$$

(ii) $P[|x|>1]$

$$= 1 - P[|x| \le 1]$$

$$= 1 - P[-1 \le x \le 1]$$

$$= 1 - \int_{-1}^{1} f(x)\, dx$$

$$= 1 - \int_{-1}^{1} \frac{1}{4}\, dx$$

$$= 1 - \frac{1}{4} \times 2$$

$$= 1 - 1/2$$

$$= 1/2$$

(iii) $P[(2x+3)>5]$

$2x+3>5$

$2x>2$

$x>1$

$$P[x>1] = \int_{1}^{2} \frac{1}{4}\, dx$$

$$= \frac{1}{4} \cancel{\tfrac{2}{2}} \times 1$$

$$= \frac{1}{4}$$

5. Let X be the waiting time.

Max waiting time $= 40$ min

$\therefore$ time interval for waiting $(0, 40)$

$\therefore X \sim U[0, 40]$

$$f(x) = \begin{cases} \frac{1}{40} & 0 \le x \le 40 \\ 0 & \text{otherwise.} \end{cases}$$

(i) $P[\text{atleast } 10 \text{ min}] = P[X \ge 10]$

$$= \int_{10}^{40} f(x)\, dx$$

$$= \int_{10}^{40} \frac{1}{40}\, dx = \frac{1}{40} \times 30 = \frac{3}{4}$$

(ii) P [b/w 10 and 30 minutes]

$$= \int_{10}^{30} f(x)\,dx$$

$$= \int_{10}^{30} \frac{1}{40}\,dx$$

$$= \frac{1}{40} \times 20$$

$$= \underline{\underline{1/2}}$$

6. x be the driving time from house to bus station.

$X \sim U(10, 50)$

$$f(x) = \begin{cases} \frac{1}{40} & 10 \leq x \leq 50 \\ 0 & \text{otherwise.} \end{cases}$$

~~Max~~ time gap b/w 6 pm & 5:43 pm is 17 min.

Max driving time = 17 - 2 = 15 min

~~for~~ P [driving time is almost ~~50~~ 15 mints]

$$= \int_{10}^{15} f(x)\,dx$$

$$= \int_{10}^{15} \frac{1}{40}\,dx = \frac{1}{40}(15-10)$$

$$= \frac{1}{40} \times 5 = \underline{\underline{1/8}}$$

7. $X \sim U$

x be the time to failure after the warranty period.

Time to failure after warranty period is $(0, 2)$ [OR $(t, t+2)$]

$X \sim U(0, 2)$ → years

i.e. $X \sim U(0, 24)$ → months.

$$f(x) = \begin{cases} \frac{1}{24} & 0 < x < 24 \\ 0 & \text{Otherwise} \end{cases}$$

P [failure happens within 6 months] = $P[X \leq 6]$

$$= \int_{0}^{6} \frac{1}{24}\,dx$$

$$= \frac{1}{24} (x)_0^6$$

$$= \frac{1}{24} \times 6$$

$$= \underline{\underline{1/4}}$$

Statistical Table

Exponential Distribution

It is the distribution of amount of time until some specific event occurs.

A continuous random variable x whose pdf is given by:

$$f(x) = \begin{cases} \theta e^{-\theta x} & x \geq 0 \\ 0 & x < 0 \end{cases}, \quad \text{where } \theta > 0.$$

Symbolically we can write

$X \sim E(\theta)$, where θ is the parameter of the exponential distbn.

Mean and variance of exponential distbn

$$\text{Mean} = E(X) = \int x f(x)\, dx$$

$$= \int_0^\infty x\, \theta e^{-\theta x}\, dx$$

$$= \theta \int_0^\infty \overset{\mathrm{I}}{x} (\overset{\mathrm{II}}{e^{-\theta x}})\, dx$$

$$= \theta \left[x \times \left(\frac{e^{-\theta x}}{-\theta} \right) - (1) \left(\frac{e^{-\theta x}}{\theta^2} \right) \right]_0^\infty$$

$$= \theta \left[0 - 0 - \left(0 - \frac{1}{\theta^2} e^0 \right) \right]$$

$$= \theta \left(1/\theta^2 \right)$$

$$= \frac{1}{\theta}$$

$$\boxed{\text{Mean} = \frac{1}{\theta}}$$

$$\text{Variance} = E(x^2) - [E(x)]^2.$$

$$E(x^2) = \int_0^\infty x^2 f(x)\, dx$$

$$= \int_0^\infty \underset{\mathrm{I}}{x^2} \cdot \underset{\mathrm{II}}{\theta e^{-\theta x}}\, dx$$

$$= \theta \left[x^2 \cdot \frac{e^{-\theta x}}{-\theta} - 2x \times \frac{e^{-\theta x}}{\theta^2} + 2 \times \frac{e^{-\theta x}}{-\theta^3} \right]_0^\infty$$

$$= \theta \left[0 - 0 + 0 - \left(0 - 0 + \frac{2}{-\theta^3} e^0 \right) \right]$$

$$= \theta \left[\frac{2}{\theta^3} \right]$$

$$= \frac{2}{\theta^2}$$

$$\therefore \text{Variance} = E(x^2) - [E(x)]^2$$

$$= \frac{2}{\theta^2} - \frac{1}{\theta^2}$$

$$= \frac{1}{\theta^2}$$

$$\boxed{\text{Variance} = V(x) = \frac{1}{\theta^2}}$$

11.3.16
FRIDAY

Exponential Distribution [problems]

1) $X \sim E(\theta)$

then, $f(x) = \begin{cases} \theta e^{-\theta x} & , x \geq 0 \\ 0 & x < 0 \end{cases}$

$\theta = 1/5$

Let x be the waiting time.

(i) P[atleast 5 minutes]

$= P[x \geq 5]$

$= \int_5^\infty f(x)\,dx$

$= \int_5^\infty \frac{1}{5} e^{-\frac{x}{5}}\,dx$

$= \frac{1}{5} \int_5^\infty e^{-\frac{x}{5}}\,dx$

$= \frac{1}{5} \left(\frac{e^{-\frac{x}{5}}}{-1/5} \right)_5^\infty$

$= -1 \left(e^{-\infty/5} - e^{-1} \right)$

$= -1 \left(0 + e^{-1} \right.$

$= \underline{\underline{0.367}}$

(ii) P[b/w 5 & 15 mints]

$= \int_5^{15} \frac{1}{5} e^{-\frac{x}{5}}\,dx$

$= \frac{1}{5} \left(\frac{e^{-x/5}}{-1/5} \right)_5^{15}$

$= -1 \left(e^{-15/5} - e^{-5/5} \right)$

$= \underline{\underline{0.318}}$

(iii) P[atmost 10 mints]

$= P[x \leq 10]$

$= \int_0^{10} f(x)\,dx$

$= \int_0^{10} \frac{1}{5} e^{-x/5}\,dx$

$= 1/5 \left(\frac{e^{-x/5}}{-1/5} \right)_0^{10}$

$= -1 \left(e^{-10/5} - e^{-0/5} \right)$

$= \underline{\underline{0.864}}$

2. $X \sim E(\theta)$

$\theta = 1/3$

Let x be the repair time.

$$P[x \text{ exceeds } 4] = P[x > 4]$$

$$= \int_4^\infty f(x)\,dx$$

$$= \int_4^\infty \frac{1}{3} e^{-x/3}\,dx$$

$$= \frac{1}{3}\left(\frac{e^{-x/3}}{-1/3}\right)_4^\infty$$

$$= -1\left(e^{-\infty/3} - e^{-4/3}\right)$$

$$= \underline{\underline{0.263}}$$

Normal Distribution

A continous random variable X which takes values in the interval $(-\infty, \infty)$ is said to follow a normal distribution with parameters μ & σ if its PDF is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\left(\frac{x-\mu}{\sigma}\right)^2}{2}}$$

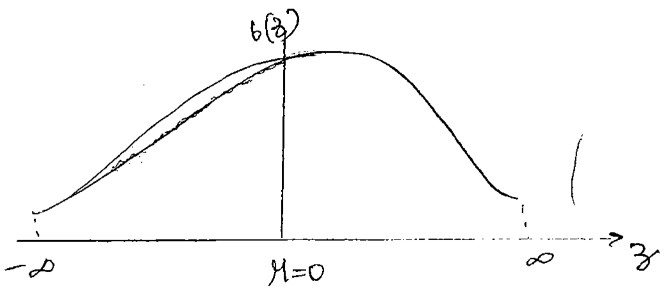
Symbolically, $X \sim N(\mu, \sigma)$

Standard Normal Distribution

Normal distribution with mean 0 and SD 1 is known as std. normal distbn.

$Z = \frac{X-\mu}{\sigma}$ is called standardized normal random variable.

Standard Normal curve



Properties

1. It is a bell shaped graph.
2. Continous graph.
3. Total area under the curve above the x-axis is 1.
4. It is symmetric abt $\mu = 0$.
5. On either side of the line thr O, the area under the curve is 0.5.

Steps to find out prob from standard normal table

1) Convert the normal random variable X into mean μ & SD σ to std normal random variable Z using formula

$$Z = \frac{X-\mu}{\sigma}$$

2) Convert the reqd prob starting from 0. It's general form is:

$$P[0<Z<a]$$

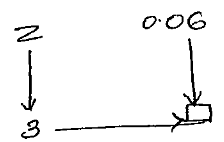
eg: $P[2<Z<3]$

$$\Rightarrow P[0<Z<3] - P[0<Z<2]$$

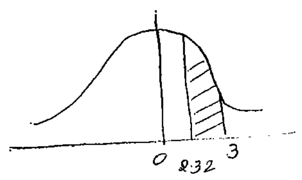
3) $P[0<Z<a]$ = the value corresponding to pt a in the std normal table.

1. Determine the following probs:

$P[0<Z<3.06] = 0.4989$



2. Find $P[2.32<Z<3]$



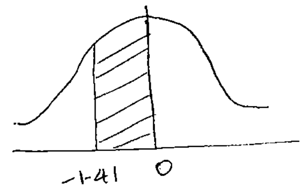
$$= P[0<Z<3] - P[0<Z<2.32]$$

$$\begin{array}{r} 2.3+ \\ .02 \\ \hline 2.32 \end{array}$$

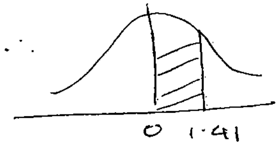
$$= 0.49865 - 0.4898$$

$$= 8.85 \times 10^{-3}$$

3. Find $P[-1.41<Z<0]$



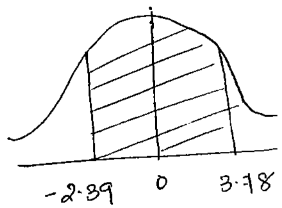
Assume same area in right.



$\therefore \Rightarrow P[0<Z<1.41]$

$= 0.4207$

4. $P[-2.39<Z<3.78]$

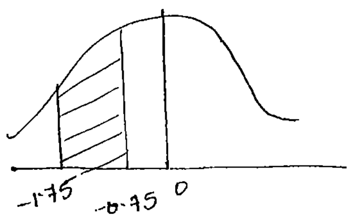


$P[0<Z<2.39] + P[0<Z<3.78]$

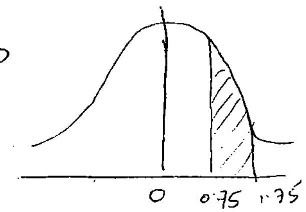
$= 0.4916 + 0.4999$

≈ 0.9915

5) $P[-1.75 < z < -0.75]$



$\Rightarrow$

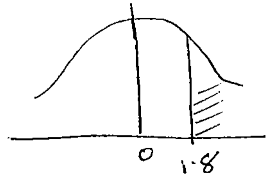


$P[0 < z < 1.75] - P[0 < z < 0.75]$

$= 0.4599 - 0.2734$

$= \underline{\underline{0.1865}}$

6) $P[z > 1.8]$

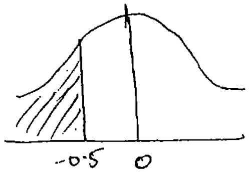


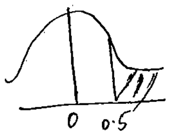
$= 0.5 - P[0 < z < 1.8]$

$= 0.5 - \cancel{0.3599}\ 0.4641$

$= \cancel{0.1401}\ \underline{\underline{0.0359}}$

7) $P[z < -0.5]$



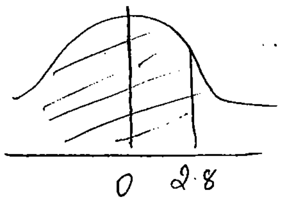


8) $P[z < 2.8]$

$= 0.5 + P[0 < z < 2.8]$

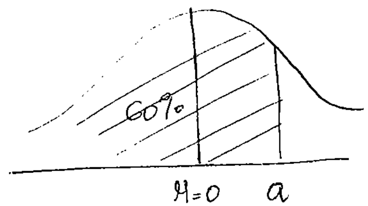
$= 0.5 + 0.4974$

$= \underline{\underline{0.9974}}$



14/3/16 Monday

1) Find the value of a such that 60% of the area is below a.



$P[z < a] = 60\%$

$0.5 + P[0 < z < a] = .6$

$P[0 < z < a] = .6 - .5$

$= \underline{\underline{0.1}}$

0.0987	.1026
.2 + .05	.2 + .06
.25	.26

$\frac{.25 + .26}{2} = \underline{\underline{.255}}$

$\therefore a = \underline{\underline{.255}}$

Problems - Normal Distribution

2. $X \sim N(\mu, \sigma)$

$\sigma = 0.002$ cm

$\mu = 0.7515$

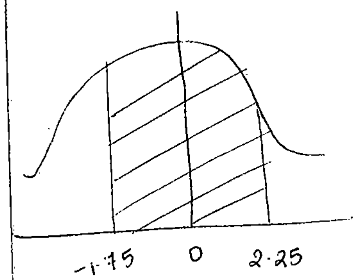
Approved diameter is 0.752 ± 0.004

$P[\text{acceptance of a plug}] = P[0.748 < X < 0.756]$

$= P\left[\frac{0.748-\mu}{\sigma} < Z < \frac{0.756-\mu}{\sigma}\right]$ $\left(Z = \frac{X-\mu}{\sigma}\right)$

$= P\left[\frac{0.748-0.7515}{0.002} < Z < \frac{0.756-0.7515}{0.002}\right]$

$= P[-1.75 < Z < 2.25]$



$= P[0 < Z < 1.75] + P[0 < Z < 2.25]$

$= 0.4599 + 0.4878$

$= \underline{\underline{0.9477}}$

$P[\text{rejection of 1 plug}] = 1 - 0.9477$

$= \underline{\underline{0.0523}}$

No: of plugs rejected $= 0.0523 \times 1000$

$= 52.3$

$\approx \underline{\underline{52}}$

3) Mean, μ $X \sim N(\mu, \sigma)$

$\mu = 50$

$\sigma = 15$.

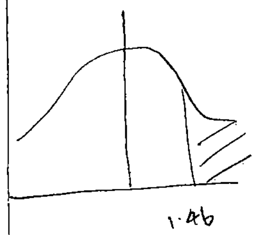
Let x be the lamp which is replaced.

P[a lamp is to be replaced after completing 72 hrs] $= P[X > 72]$

$= P\left[Z > \frac{72-\mu}{\sigma}\right]$

$= P\left[Z > \frac{72-50}{15}\right]$

$= P[Z > 1.46]$



$= 0.5 - P[0 < Z < 1.46]$

$= 0.5 - 0.4279$

$= \underline{\underline{0.0721}}$

No: of lamps $= 5000$

$\therefore$ tot no: of lamps replaced $= 0.0721 \times 5000$

$= 360.5$

$\approx \underline{\underline{361}}$

4) $X \sim N(\mu, \sigma)$

[Note: Both average & S.D are given together $\therefore$ normal distbn.]

Let tot. mark for one paper $= 100$

$\mu = 40\% = 40$

$\sigma = 10\% = 10$

[μ & σ are not written as % usually]

(i) $P[X \geq 50]$

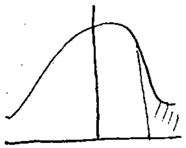
If 50% is fixed as minimum pass mark, Prob p for a student to pass is $P[X \geq 50]$

$P[X \geq 50]$

$= P\left[Z \geq \frac{50-\mu}{\sigma}\right]$

$= P\left[Z \geq \frac{50-40}{10}\right]$

$= P[Z \geq 1]$



$= 0.5 - P[0 < Z < 1]$

$= 0.5 - 0.3413$

$= \underline{\underline{0.1587}}$

No: of students $= 500$

Tot no: of students who will pass if 50% is minimum $= 0.1587 \times 500$

$= 79.35$

$\approx \underline{\underline{79}}$

(ii) Let 'a' be the minimum mark

Prob for 350 candidates are to pass if 'a' is the min mark $= \dfrac{\text{fav no: of cases}}{\text{tot no: of cases}}$

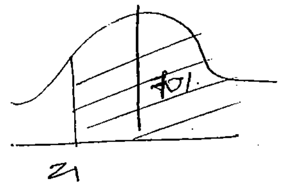
$= \dfrac{350}{500}$

$\Rightarrow P[X \geq a] = 0.7$

$\Rightarrow P\left[Z \geq \frac{a-\mu}{\sigma}\right] = 0.7.$

$P\left[Z \geq \underbrace{\frac{a-40}{10}}_{z_1}\right] = 0.7.$

$P[Z \geq z_1] = 0.7$ (z_1 is on left. Since 70% is needed)



$\Rightarrow 0.5 + P[0 < Z < z_1] = 0.7.$

$\therefore P[0 < Z < z_1] = 0.2$

0.1985	0.2019
0.52	0.53

$z_1 = \dfrac{0.52 + 0.53}{2}$

$= 0.525$

Since z_1 is taken to left, it should be (-ve).

$\Rightarrow z_1 = -0.525$

$\frac{a-40}{10} = -0.525$

$a = (-0.525 \times 10) + 40$

$= 34.75$

$\approx \underline{\underline{35}}$

∴ Minimum pass mark if 350 students are to pass $= \underline{\underline{35}}$

(iii) $P[x > 60]$

15.3.16 Tuesday

5) $X \sim N(\mu, \sigma)$

$\mu = 5$

$\sigma = 2$.

Prob that a thermostat is replaced at the end of 2^{nd} year →

$P[x \le 2] = P\left[z \le \frac{2-\mu}{\sigma}\right]$

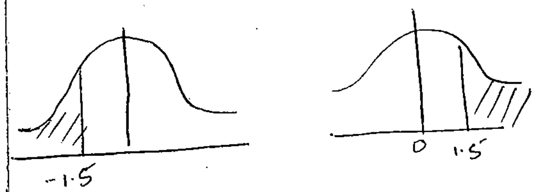
$= P\left[z \le \frac{2-5}{2}\right]$

$= P[z \le -1.5]$

$= 0.5 - P[0 < z < 1.5]$

$= 0.5 - 0.4332$

$\approx \underline{\underline{0.0668}}$



1000 units were sold out every month. So at the end of 2^{nd} year, total units sold out $= 1000 \times 24$

$= 24000$

∴ No: of thermostats replaced at the end of 2^{nd} yr $= 0.0668 \times 24000$

$= 1603.2$

$\approx \underline{\underline{1603}}$

6) $X \sim N(\mu, \sigma)$

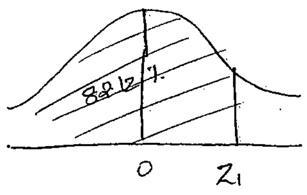
$\sigma = 10$

$P[X < 82.5] = 0.8212$

$\Rightarrow P\left[\cancel{Z} < \underbrace{\frac{82.5 - \mu}{10}}\right] = 0.8212$

$P[Z < z_1] = 0.8212$

↳ 82.12% lies behind pt z_1.



$50\% + P[0 < z < z_1] = 82.12\%$

$P[0 < z < z_1] = 32.12\%$

$\Rightarrow 0.3212$

$\Rightarrow z_1 = \underline{\underline{0.92}}$

$\frac{82.5 - \mu}{10} = 0.92$

$\Rightarrow \mu = 82.5 - (0.92 \times 10)$

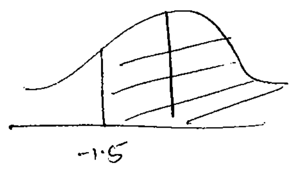
$= \underline{\underline{72.3}}$

$P[x > 58.3] = ?$

$= \cancel{P}\left[z > \cancel{\frac{58.3 - 73.3}{10}}\right]$ $\quad P\left[z > \frac{58.3 - 73.3}{10}\right]$

$= P[\cancel{z > -1.5}]$ $\quad = P[z > -1.5]$

~~50 - p[~~



$= 0.5 + P[0 < z < 1.5]$

$= 0.5 + 0.4332$

$= \underline{\underline{0.9332}}$

8) $X \sim N(\mu, \sigma)$

$\mu = 500$

$\sigma = 50$

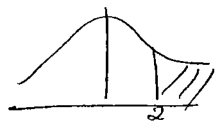
Prob that a person getting above 600 $= \frac{228}{n}$

$P[X > 600] = \frac{228}{n}$

(let 'n' be the total no: of persons)

$P\left[z > \frac{600 - 500}{50}\right] = \frac{228}{n}$

$P[z > 2] = \frac{228}{n}$



$\Rightarrow 0.5 - P[0 < z < 2] = \frac{228}{n}$

$0.5 - 0.4772 = \frac{228}{n}$

$\frac{228}{n} = 0.0228$

$n = \frac{228}{0.0228} = \underline{\underline{10000}}$

9) $X \sim N(\mu, \sigma)$

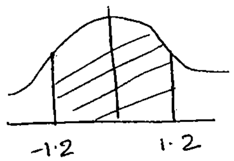
$\mu = 5.02$

$\sigma = 0.05$

$P[4.95 < X < 5.08]$ (Prob that a washer is good)

$= P\left[\frac{4.96 - 5.02}{0.05} < z < \frac{5.08 - 5.02}{0.05}\right]$

$= P[-1.2 < z < 1.2]$



$= P[0 < z < 1.2] + P[0 < z < 1.2]$

$= 0.3849$

$= 0.7698$

$\therefore$ Prob that a washer is defective

$= 1 - 0.7698$

$= 0.2302$

$\therefore$ tot no. of defective washers $= 0.2302 \times 200$

$= 46.04$

$\approx 46\%$

10) $X \sim N(\mu, \sigma)$

$\mu = 70$

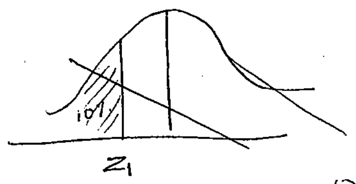
$\sigma = 5$

Prob that a person gets highest payment $= \frac{100}{1000} = 0.1$

Let 'a' be lowest weekly wages of highest paid workers.

$P[x \geq a] = 0.1$ [Prob that a person gets salary $\geq$ 'a']

$P\left[z \geq \frac{a - 70}{5}\right] = 0.1$



$P[z \geq z_1] = 0.1$

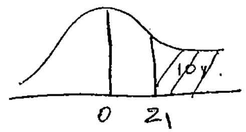
(10% lies ~~below~~ above z_1)

~~(sign of z_1 is '−')~~

~~P[0~~ 50% $- P[0 < z < z_1] = 10\%$

$P[0 < z < z_1] = 40\%$

$\Rightarrow 0.4$



0.3997	0.4015
1.28	1.29

$z_1 = 1.285$

$\frac{a - 70}{5} = 1.285$

$\Rightarrow a = (1.285 \times 5) + 70$

$= 76.425$

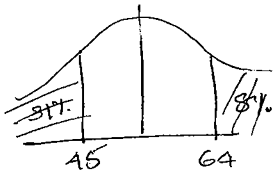
≈ 76

Q) In a normal distbn, 31% of items are under 45 and 8% are over 64. Find the mean & SD of the distbn.

Ans: $x \sim N(\mu, \sigma)$

$P[x < 45] = 31\%$ $\quad P[x > 64] = 8\%$

$P\left[z < \frac{45-\mu}{\sigma}\right] = 0.31$ $\quad P\left[z > \frac{64-\mu}{\sigma}\right] = \cancel{0.8} \; 0.08$



$P[z < z_1] = 0.31$ $\quad P[z > z_2] = 0.08$

$\downarrow$ (-ve) $\quad \downarrow$ (+ve)

$0.5 - P[0 < z < z_1] = 0.31$

$\Rightarrow P[0 < z < z_1] = 0.19$

$A = \frac{3}{4} + (0.3433 \cos 2\theta) \ldots$

$= 0.75$ proved.

Amplitude $= \sqrt{a_1^2 + b_1^2} =$

8\. Obtain the first 3 harmonics in any 1 period & finite ... from the following of max & minima.

x : 0 1 2

y : 4 8 15

$N = 6$ $\quad l = 6$ $\quad \cos n\ldots$

x	y	$\cos \pi x/6$
0	4	1
1	8	0.860
2	15	0.5
3	7	0
4	6	-0.5
5	2	[illegible]