Mariya Vaughew

9

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx \, dx$$

$$= \frac{1}{\pi} \int_0^{2\pi} \sqrt{2} \sin x/2 \cdot \cos nx \, dx.$$

$$= \frac{\sqrt{2}}{\pi} \int_0^{2\pi} \sin x/2 \cdot \cos nx \, dx$$

$$= \frac{\sqrt{2}}{2\pi} \int_0^{2\pi} 2 \cos nx \cdot \sin x/2 \, dx$$

$$= \frac{1}{\sqrt{2}\pi} \int_0^{2\pi} \sin (n+1/2) x - \sin (n-1/2) x \, dx$$

$$= \frac{1}{\sqrt{2}\pi} \left[\frac{-2}{2n+1} \cos \frac{(2n+1)x}{2} + \frac{2}{2n-1} \frac{\cos (2n-1)x}{2} \right]_0^{2\pi}$$

$(-1)^{(2n+1)2}$ $(-1)^{2n}$

$$= \frac{2}{\sqrt{2}\pi} \left[-\frac{1}{2n+1} \left[\cos \frac{(2n+1)2\pi - 1}{2} \right] + \frac{1}{2n-1} \left[\cos \frac{(2n-1)2\pi - 1}{2} \right] \right.$$

$$= \frac{\sqrt{2}}{\pi} \left(\frac{2}{2n+1} - \frac{2}{2n-1} \right) = \frac{-4\sqrt{2}}{\pi (4n^2-1)}$$

$\cos (2n+1)\pi = \cos (2n-1)\pi = -1$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} \sqrt{2} \sin \frac{x}{2} \cdot \sin nx \, dx$$

$4n-2-4n$
-2

$$= \frac{\sqrt{2}}{2\pi} \int_0^{2\pi} 2 \sin nx \cdot \sin \frac{x}{2} \, dx$$

Mariya Vaughew (2)
CSE - A
10.

$$= \frac{1}{\sqrt{2}\pi} \int_0^{2\pi} [\cos (n-1/2) x - \cos (n+1/2) x] \, dx$$

$$= \frac{1}{\sqrt{2}\pi} \left[\frac{2}{2n-1} \cdot \sin \frac{(2n-1)x}{2} - \frac{2}{2n+1} \cdot \sin \frac{(2n+1)x}{2} \right]_0^{2\pi}$$

$$= \frac{\sqrt{2}}{\pi} \left[\frac{1}{2n-1} \left(\sin (2n-1)\pi - 0 \right) - \frac{1}{2n+1} \left(\sin (2n+1)\pi - 0 \right) \right]$$

$$= 0$$

$$\therefore \sqrt{1-\cos x} = \frac{2\sqrt{2}}{\pi} + \sum_{n=1}^{\infty} \frac{-4\sqrt{2}}{\pi (4n^2-1)} \cdot \cos nx$$

$$\sqrt{1-\cos x} = \frac{2\sqrt{2}}{\pi} - \frac{4\sqrt{2}}{\pi} \left[\frac{\cos x}{4-1} + \frac{\cos 2x}{16-1} + \frac{\cos 3x}{36-1} + \cdots \right]$$

$$\sqrt{1-\cos x} = \frac{2\sqrt{2}}{\pi} - \frac{4\sqrt{2}}{\pi} \left[\frac{\cos x}{3} + \frac{\cos 2x}{15} + \frac{\cos 3x}{35} + \cdots \right]$$

When $x=0$,

$$\sqrt{1-\cos 0} = \frac{2\sqrt{2}}{\pi} - \frac{4\sqrt{2}}{\pi}\left[\frac{\cos 0}{3} + \frac{\cos 0}{15} + \frac{\cos 0}{35} + \cdots\right]$$

$$0 = \frac{2\sqrt{2}}{\pi} - \frac{4\sqrt{2}}{\pi}\left[\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \cdots\right]$$

$$\frac{4\sqrt{2}}{\pi}\left[\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \cdots\right] = \frac{2\sqrt{2}}{\pi}$$

$$\therefore \frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \cdots = \frac{2\sqrt{2}}{\pi} \times \frac{\pi}{4\sqrt{2}}$$

$$= \frac{1}{2}$$

ie, $$\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \cdots = \frac{1}{2}$$

Functions with finite discontinuities.

8. Expand $f(x) = -\pi$, $-\pi < x < 0$.

$= x$, $0 < x < \pi$ and deduce

that $\frac{1}{1^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{8}$.

Here, the function is defined in the interval $-\pi$ to π. and there is a discontinuity at $x=0$.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx).$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi\,dx + \int_{0}^{\pi} x\,dx\right]$$

$$= \frac{1}{\pi}\left[-\pi\,[x]_{-\pi}^{0} + \left[\frac{x^2}{2}\right]_0^{\pi}\right]$$

$$= \frac{1}{\pi}\left[-\pi(0--\pi) + \frac{\pi^2}{2}\right]$$

$$= \frac{1}{\pi}\left[-\pi^2 + \frac{\pi^2}{2}\right]$$

$$= \frac{1}{\cancel{\pi}} \times -\frac{\pi^{\cancel{2}}}{2}$$

$$= -\frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\cdot\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi\cdot\cos nx\,dx + \int_{0}^{\pi} x\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[-\pi\cdot\left[\frac{\sin nx}{n}\right]_{-\pi}^{0} + \left[x\cdot\frac{\sin nx}{n} - 1\cdot -\frac{\cos nx}{n^2}\right]_{0}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{n}\times 0 + \frac{1}{n^2}\left(\cos n\pi - \cos 0\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n^2}\left((-1)^n - 1\right)\right]$$

$$a_n = \frac{(-1)^n - 1}{n^2\pi} \left.\begin{matrix} = \frac{-2}{n^2\pi}, \text{ if } n = \text{odd} \\ = 0 \quad \text{if } n = \text{even}\end{matrix}\right.$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi\sin nx\,dx + \int_{0}^{\pi} x\sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[-\pi\left[-\frac{\cos nx}{n}\right]_{-\pi}^{0} + \left[x\cdot\frac{-\cos nx}{n} - 1\cdot -\frac{\sin nx}{n^2}\right]_{0}^{\pi}\right]$$

$$\cancel{= \frac{1}{\pi}\left[\frac{\pi}{n}\left[\cos 0 - \cos(n\pi)\right] + -\frac{\pi}{n}\cdot\left(\cos n\pi - \cos 0\right)\right]}$$

$$\cancel{= \frac{1}{\pi}\left[\frac{\pi}{n}(1 - -1) - \frac{\pi}{n}\left((-1)^n - 1\right)\right]}$$

$$\cancel{= \frac{1}{n}\left[2 - (-1)^n + 1\right]}$$

$$\cancel{= \frac{3 - (-1)^n}{n}}$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n}\left(\cos 0 - \cos(-n\pi)\right) - \frac{1}{n}\left\{\pi \cos n\pi - 0\right\}\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n}\left(1 - (-1)^n\right) - \frac{\pi}{n}(-1)^n\right]$$

$$= \frac{1}{n}\left[1 - (-1)^n \cancel{- (-1)^n}\right]$$

$$b_n = \frac{1 - 2\cdot(-1)^n}{n} \begin{cases} = \frac{3}{n}, & \text{if } n = \text{odd} \\ = -\frac{1}{n}, & \text{if } n = \text{even.} \end{cases}$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx.$$

$$f(x) = -\frac{\pi}{4} + -\frac{2}{\pi}\cos x - \frac{2}{9\pi}\cos 3x - \frac{2}{25\pi}\cos 5x + \dots + 3\sin x - \frac{1}{2}\sin 2x + \frac{3}{3}\sin 3x - \frac{1}{4}\sin 4x + \dots$$

At a point of discontinuity, $x = c$

$$f(x) = f(c) = \frac{1}{2}\left(f(x-c) + f(x+c)\right)$$

$$f(x) = -\frac{\pi}{4} - \frac{2}{\pi}\left[\cos x + \frac{\cos 3x}{9} + \frac{\cos 5x}{25} + \dots\right] + 3\left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots\right) - \left(\frac{\sin 2x}{2} + \frac{\sin 4x}{4} + \dots\right)$$

Put $x = 0$.

since $x=0$ is a point of discontinuity,

$$\therefore f(x) = \frac{1}{2}\left[f(x+0) + f(x-0)\right]$$

(do we need discontinuity if it is not to deduce)?

R.H. limit $f(x+0) = f(0+0)$

$$= 0$$

L.H limit $f(x-0) = f(0-0)$

$$= -\pi$$

$$\therefore f(0) = \frac{1}{2}\left[0 - \pi\right]$$

$$f(0) = \frac{-\pi}{2}$$

when $x=0$, F.S becomes,

$$f(0) = -\frac{\pi}{4} - \frac{2}{\pi}\left[\cos 0 + \frac{\cos 0}{9} + \frac{\cos 0}{25} + \cdots\right]$$

$$-\frac{\pi}{2} = -\frac{\pi}{4} - \frac{2}{\pi}\left[1 + \frac{1}{9} + \frac{1}{25} + \cdots\right]$$

$$\frac{2}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right] = -\frac{\pi}{4} + \frac{\pi}{2}$$

$$" = \pi\left(-\frac{1}{4} + \frac{1}{2}\right)$$

$$" = \pi\left(\frac{-2+4}{8}\right)$$

$$" = \pi \times \frac{\cancel{2}^{1}}{\cancel{8}_{4}}$$

$$\frac{2}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right] = \frac{\pi}{4}$$

$$\therefore \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi}{4} \times \frac{\pi}{2}$$

$$= \frac{\pi^2}{8}$$

9. Find the fourier series to represent the function $f(x) = |\sin x|$, $-\pi < x < \pi$.

$|\sin x|$ is an even function.

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx.$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx.$$

$$= \frac{2}{\pi}\int_0^{\pi} |\sin x|\,dx.$$

$\sin x > 0$ in $0 < x < \pi$

$$= \frac{2}{\pi}\int_0^{\pi} \sin x\,dx$$

$$= \frac{2}{\pi}\left[-\cos x\right]_0^{\pi}$$

$$= -\frac{2}{\pi}\left[\cos \pi - \cos 0\right]$$

$$= -\frac{2}{\pi}\left[-1-1\right]$$

$$= -\frac{2}{\pi} \times -2$$

$$= \frac{4}{\pi}$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx \, dx$$

$$= \frac{2}{\pi} \int_0^{\pi} |\sin x| \cos nx \, dx$$

$$= \frac{2}{\pi} \int_0^{\pi} \sin x \cos nx \, dx$$

$$= \frac{1}{\pi} \int_0^{\pi} 2 \cos nx \cdot \sin x \, dx$$

$$= \frac{1}{\pi} \int_0^{\pi} \left[\sin (n+1) x - \sin (n-1) x \right] dx$$

$$= \frac{1}{\pi} \left[- \frac{\cos (n+1) x}{n+1} - - \frac{\cos (n-1) x}{n-1} \right]_0^{\pi}$$

$$= \frac{1}{\pi} \left[- \frac{\cos (n+1) x}{n+1} + \frac{\cos (n-1) x}{n-1} \right]_0^{\pi}$$

$$= \frac{1}{\pi} \left[\frac{-1}{n+1} \left\{ \cos (n+1)\pi - \cos 0 \right\} + \frac{1}{n-1} \left\{ \cos (n-1)\pi - \cos 0 \right\} \right]$$

$$\boxed{\text{see}} = \frac{1}{\pi} \left[-\frac{1}{n+1} \left((-1)^{n+1} - 1 \right) + \frac{1}{n-1} \left((-1)^{n-1} - 1 \right) \right]$$

$$= \frac{1}{\pi} \left[- \frac{(-1)^{n+1}}{n+1} + \frac{1}{n+1} + \frac{(-1)^{n-1}}{n-1} - \frac{1}{n-1} \right]$$

$$= \frac{1}{\pi} \left[- \frac{(-1)^{n+1}}{n+1} + \frac{(-1)^{n-1}}{n-1} + \frac{1}{n+1} - \frac{1}{n-1} \right]$$

$$= \frac{1}{\pi} \left[(-1)^n \left\{ \frac{1}{n+1} - \frac{1}{n-1} \right\} + \frac{1}{n+1} - \frac{1}{n-1} \right]$$

$$= \frac{1}{\pi} \times \left((-1)^n + 1 \right) \left\{ \frac{1}{n+1} - \frac{1}{n-1} \right\}$$

$$= \frac{(-1)^n + 1}{\pi} \left[\frac{\cancel{n} - 1 - \cancel{n} - 1}{n^2 - 1} \right]$$

$$= \frac{(-1)^n + 1}{\pi} \times \frac{-2}{n^2 - 1}$$

$$= \frac{-2 \left((-1)^n + 1 \right)}{\pi \left(n^2 - 1 \right)}$$

$$\therefore \left|\sin x\right| = \frac{2}{\pi} + \sum_{n=1}^{\infty} \frac{-2\left((-1)^n+1\right)}{\pi\,(n^2-1)}\cos nx$$

$$= \frac{2}{\pi} + 0 + \frac{-2\times 2}{\pi}\cdot\frac{\cos 2x}{2^2-1}$$

$$+ 0 - \frac{2\times 2}{\pi}\cdot\frac{\cos 4x}{4^2-1} + \cdots$$

$$\left|\sin x\right| = \frac{2}{\pi} - \frac{4}{\pi}\left(\frac{\cos 2x}{3} + \frac{\cos 4x}{15} + \cdots + \frac{\cos 2nx}{4n^2-1} + \cdots\right)$$

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16. Obtain the Fourier series for,

U.Q

$$f(x) = 0, \quad -\pi \le x \le 0$$
$$= \sin x, \quad 0 \le x \le \pi$$

and deduce $\frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} \cdots = \frac{\pi-2}{4}$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos nx + b_n\sin nx.$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi}\sin x\,dx\right]$$

$$= \frac{1}{\pi}\left[-\cos x\right]_0^{\pi}$$

$$= -\frac{1}{\pi}\left[\cos\pi - \cos 0\right]$$

$$= -\frac{1}{\pi}\left[-1-1\right]$$

$$= \frac{2}{\pi}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\cdot\cos nx\,dx + \int_{0}^{\pi}\sin x\cdot\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\int_{0}^{\pi}\sin x\cdot\cos nx\,dx$$

$$= \frac{1}{2\pi}\int_{0}^{\pi} 2\cos nx\cdot\sin x\,dx.$$

$$= \frac{1}{2\pi} \int_0^{\pi} \left(\sin (n+1)x - \sin (n-1)x \right) dx$$

$$= \frac{1}{2\pi} \left[\frac{-\cos (n+1)x}{n+1} - \frac{-\cos(n-1)x}{n-1} \right]_0^{\pi}$$

$$= \frac{1}{2\pi} \left[\frac{-\cos (n+1)x}{n+1} + \frac{\cos (n-1)x}{n-1} \right]_0^{\pi}$$

$$= \frac{1}{2\pi} \left[\frac{-1}{n+1} \left(\cos (n+1)\pi - \cos 0 \right) + \frac{1}{n-1} \left(\cos (n-1)\pi - \cos 0 \right) \right]$$

$$= \frac{1}{2\pi} \left[-\frac{1}{n+1} \left((-1)^{n+1} - 1 \right) + \frac{1}{n-1} \left((-1)^{n-1} - 1 \right) \right]$$

$$= \frac{1}{2\pi} \left[\frac{-(-1)^{n+1}}{n+1} + \frac{1}{n+1} + \frac{(-1)^{n-1}}{n-1} - \frac{1}{n-1} \right]$$

$$= \frac{1}{2\pi} \left[\frac{-(-1)^{n+1}}{n+1} + \frac{(-1)^{n-1}}{n-1} + \frac{1}{n+1} - \frac{1}{n-1} \right]$$

$$= \frac{1}{2\pi} \left[(-1)^n \left\{ \frac{1}{n+1} - \frac{1}{n-1} \right\} + \frac{1}{n+1} - \frac{1}{n-1} \right]$$

$$= \frac{(-1)^n + 1}{2\pi} \left[\frac{1}{n+1} - \frac{1}{n-1} \right]$$

$$= \frac{(-1)^n+1}{2\pi} \times \frac{-2}{n^2-1}$$

$n-1-n-1$
-2
$\frac{-2}{\pi(n^2-1)}$

$$= -\frac{1}{\pi(n^2-1)} \cdot \left((-1)^n + 1 \right) \quad \text{if } n \neq 1$$

$$a_n = \begin{cases} 0 & \text{when } n \text{ is odd} \\ \dfrac{-2}{\pi(n^2-1)} & \text{when } n \text{ is even} \end{cases}$$

$$a_1 = \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} f(x) \cdot \cos x \, dx$$

$$= \frac{1}{\pi} \cdot \int_0^{\pi} \sin x \cdot \cos x \, dx.$$

$$= \frac{1}{2\pi} \cdot \int_0^{\pi} 2 \sin x \cos x \, dx.$$

$$= \frac{1}{2\pi} \int_0^{\pi} \sin 2x \cdot dx.$$

$$= \frac{1}{2\pi} \left(-\frac{\cos 2x}{2} \right)_0^{\pi}$$

$$= -\frac{1}{4\pi} \left[\cos 2\pi - \cos 0 \right]$$

$$= \underline{\underline{0}}$$

$$b_n = \frac{1}{\pi} \cdot \int_{-\pi}^{\pi} f(x) \sin nx\,dx$$

$$= \frac{1}{\pi} \cdot \left[\int_{-\pi}^{0} 0 \sin nx\,dx + \int_0^{\pi} \sin x \sin nx\,dx \right]$$

$$= \frac{1}{\pi} \cdot \int_0^{\pi} \sin x \cdot \sin nx\,dx.$$

$$= \frac{1}{2\pi} \int_0^{\pi} 2 \sin nx \cdot \sin x\,dx.$$

$$= \frac{1}{2\pi} \int_0^{\pi} \left(\cos (n-1)x - \cos (n+1)x \right) dx.$$

$$= \frac{1}{2\pi} \left[\frac{\sin (n-1)x}{n-1} - \frac{\sin (n+1)x}{n+1} \right]_0^{\pi}$$

$$= \underline{\underline{0}} \quad n \neq 1 \quad = ?$$

$\therefore$ ~~$f(x) = \frac{1}{\pi} + \sin x \sum_{n=1}^{\infty}$~~

$$f(x) = \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos 2nx}{(2n)^2 - 1}$$

$$f(x) = \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi} \left[\frac{\cos 2x}{3} + \frac{\cos 4x}{15} + \cdots \right]$$

$$b_1 = \frac{1}{\pi} \cdot \int_0^{\pi} \sin x \cdot \sin x\,dx.$$

$$= \frac{1}{\pi} \cdot \int_0^{\pi} \left(\frac{1 - \cos 2x}{2} \right) dx.$$

$$= \frac{1}{2\pi} \left[x - \frac{\sin 2x}{2} \right]_0^{\pi}$$

$$= \underline{\underline{\frac{1}{2}}}$$

$x = 0 \Rightarrow$ discontinuity.

Put $x = 0$,

$$f(0) = \frac{1}{2} \left(f(x-0) + f(x+0) \right)$$
$$f(0-0) + f(0+0)$$

$$= \frac{1}{2} \left[0 + \sin 0 \right]$$

$$= \frac{\sin 0}{2} = 0$$

we can write $f(x)$ as,

$$f(x) = \frac{1}{\pi} + \frac{1}{2}\sin x - \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{\cos 2nx}{(2n)^2 - 1}$$

$$f(x) = \frac{1}{\pi} + \frac{1}{2}\sin x - \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{\cos 2nx}{4n^2 - 1}$$

when $x = 0$,

$$0 = \frac{1}{\pi} - \frac{2}{\pi}\left(\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \cdots\right)$$

$$\therefore \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \cdots = \frac{1}{\cancel{\pi}} \times \frac{\cancel{\pi}}{2} = \frac{1}{2}$$

when $x = \pi/2$

$$f(\pi/2) = \cancel{\ldots} \sin \pi/2 = 1$$

$\therefore$ F.S becomes,

$$1 = \frac{1}{\pi} + \frac{1}{2} - \frac{2}{\pi}\left[-\frac{1}{1.3} + \frac{1}{3.5} - \frac{1}{5.7} + \cdots\right]$$

$$\frac{1}{2} = \frac{1}{\pi} + \frac{2}{\pi}\left(\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} + \cdots\right)$$

$$\frac{1}{2} - \frac{1}{\pi} = \frac{2}{\pi}\left(\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots\right)$$

$$\therefore \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots = \frac{\pi - 2}{2\cancel{\pi}} \times \frac{\cancel{\pi}}{2} = \frac{\pi - 2}{4}$$

11. Find fourier series of the function;

$$f(x) = +\pi x, \quad 0 \le x \le 1$$
$$= \pi(2x), \quad 1 \le x \le 2. \quad \text{and deduce}$$

$$\frac{1}{1^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{8}.$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}.$$

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx.$$

$2l = 2$
$\therefore l = 1$

$$= \frac{1}{1}\left[\int_0^2 f(x)\,dx\right]$$

$$= \int_0^1 \pi x\,dx + \int_1^2 \pi(2-x)\,dx.$$

$$= \pi\left[\frac{x^2}{2}\right]_0^1 + \pi\left[2x - \frac{x^2}{2}\right]_1^2$$

$$= \frac{\pi}{2} + \pi\left[\cancel{\ldots}\; 2 - \frac{1}{2}\times 3\right]$$

$$= \frac{\pi}{2} + \frac{\pi}{2}$$

$$= \pi$$

$2 - \frac{3}{2}$
$= \frac{4-3}{2}$
$= \frac{1}{2}$

$$a_n = \frac{1}{l} \int_0^{2l} f(x) \cos \frac{n\pi x}{l} \, dx.$$

$$= \frac{1}{1} \int_0^{2} f(x) \cdot \frac{\cos n\pi x}{l} \, dx.$$

$$= \int_0^1 \pi x \cdot \cos \frac{n\pi x}{1} \, dx + \int_1^2 \pi(2-x) \cdot \frac{\cos n\pi x}{1} dx.$$

$$= \int_0^1 \pi x \cdot \cos n\pi x \, dx + \int_1^2 \pi(2-x) \cdot \cos n\pi x \, dx$$

$$= \pi \cdot \left[x \cdot \frac{\sin n\pi x}{n\pi} - 1 \cdot \frac{-\cos n\pi x}{n^2\pi^2} \right]_0^1$$

$$+ \pi \left[2 \cdot \frac{\sin n\pi x}{n\pi} - x \cdot \frac{\sin n\pi x}{n\pi} - 1 \cdot \frac{\cos n\pi x}{n^2\pi^2} \right]_1^2$$

$$= \pi \left[\frac{(\cancel{\sin n\pi}^{0} - 0)}{n\pi} + \frac{\cos n\pi - \cos 0}{n^2\pi^2} \right]$$

$$+ \pi \left[\frac{2}{n\pi}(\cancel{\sin 2n\pi}^{0} - \sin n\pi) - \frac{2\sin 2n\pi - 1\sin n\pi}{n\pi} - \left(\frac{\cos 2n\pi - \cos n\pi}{n^2\pi^2} \right) \right]$$

$$= \pi \left[\frac{(-1)^n - 1}{n^2\pi^2} \right] + \pi \left[-\frac{1}{n^2\pi^2} \left((-1)^{2n} - (-1)^n \right) \right]$$

$$= \frac{\pi}{n^2\pi^2} \left[(-1)^n - 1 \right] - \frac{\pi}{n^2\pi^2} \left[(-1)^{2n} - (-1)^n \right]$$

$$= \frac{\cancel{\pi}}{n^2\pi^{\cancel{2}}} \left[(-1)^n - 1 - (-1)^{2n} + (-1)^n \right]$$

$$= \frac{1}{n^2\pi} \left[(-1)^n - 1 - 1 + (-1)^n \right]$$

$$= \frac{2}{n^2\pi} \left[(-1)^n - 1 \right]$$

$$a_n = \left. \begin{matrix} 0 \\ \frac{-4}{n^2\pi} \end{matrix} \right\} \begin{matrix} \text{when } n \text{ is even.} \\ \text{when } n \text{ is odd} \end{matrix}$$

$$b_n = \frac{1}{l} \int_0^{2l} f(x) \cdot \sin \frac{n\pi x}{l} \, dx.$$

$$= \frac{1}{1} \left[\int_0^1 \pi x \cdot \sin \frac{n\pi x}{1} \, dx + \int_1^2 \pi(2-x) \cdot \frac{\sin n\pi x}{1} dx \right]$$

$$= \pi \int_0^1 x \sin n\pi x \, dx + \pi \int_1^2 (2-x) \sin n\pi x \, dx$$

$$= \pi \left[x \cdot \frac{-\cos n\pi x}{n\pi} - 1 \cdot \cancel{\frac{-\sin n\pi x}{n^2\pi^2}} \right]_0^1$$

$$+ \pi \left[-2 \cdot \frac{\cos n\pi x}{n\pi} - \left\{ x \cdot \frac{-\cos n\pi x}{n\pi} - - \cancel{\frac{\sin n\pi x}{n^2\pi^2}} \right\} \right]_1^2$$

$$= \pi \left(-\frac{1}{n\pi} \left\{ 1 \times \cos n\pi - 0 \times \cos 0 \right\} \right]$$

$$+ \pi \left[\frac{-2}{n\pi} \left\{ \cos 2n\pi - \cos n\pi \right\} + \frac{1}{n\pi} \left\{ 2\cos 2n\pi - \cos n\pi \right\} \right]$$

$$= \cancel{\pi \left[\frac{-((-1)^n - 0)}{n\pi} \right] + \pi \left[\frac{2\{(-1)^{2n} - (-1)^n\}}{n\pi} \right.} +$$

$$\cancel{\frac{1}{n\pi} \left\{ 2 (-1)^{2n} - (-1)^n \right\}]}$$

$$= \cancel{\frac{\pi}{n\pi} \left[-(-1)^n + 1 - 2 + 2(-1)^n + 2 - 2(-1)^n \right]}$$

$$= \cancel{\frac{\pi}{n\pi} \times (-1)^n}$$

$$= \cancel{\frac{-(-1)^n}{n} - \frac{2}{n}\left((-1)^{2n} - (-1)^n\right)} +$$

$$\cancel{\frac{2}{n}\left((-1)^{2n} - (-1)^n\right)}$$

$$= -\frac{\cos n\pi}{n} - \cancel{\frac{2\cos 2n\pi}{n}} + \cancel{\frac{2\cos 2n\pi}{n}}$$

$$- \left\{ -\frac{2\cos n\pi}{n} + \frac{\cos n\pi}{n} \right\}$$

$$= -\frac{\cos n\pi}{n} + \frac{2\cos n\pi}{n} - \frac{\cos n\pi}{n}$$

$$= \frac{2\cos n\pi}{n} - \frac{2\cos n\pi}{n}$$

$$= \underline{\underline{0}}$$

$$\therefore f(x) = \frac{\pi}{2} - \frac{4}{\pi} \left(\frac{\cos \pi x}{1^2} + \frac{\cos 3\pi x}{3^2} + \frac{\cos 5\pi x}{5^2} + \cdots \right)$$

Put $x = 0$,

$f(0) = \pi \times 0 = \underline{0}$

$$0 = \frac{\pi}{2} - \frac{4}{\pi}\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right)$$

$$\therefore \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi}{2} \times \frac{\pi}{4}$$

$$= \frac{\pi^2}{8}$$

Half Range Series.

If $f(x)$ is defined in the interval $0<x<l$ it can be expanded as a fourier series of period $2l$.

1. Half range Cosine series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$$

where,

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx.$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cdot\cos\frac{n\pi x}{l}\,dx.$$

2. Half range Sine series

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}.$$

where, $b_n = \frac{2}{l}\int_0^l f(x)\cdot\sin\frac{n\pi x}{l}\,dx.$

12. Expand $f(x) = x$ in $0<x<2$ as a half range cosine series and a half range sine series.

Half range cosine series

$l = 2.$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx.$$

$$= \frac{2}{2}\cdot\int_0^2 x\,dx.$$

$$= \left[\frac{x^2}{2}\right]_0^2$$

$$= \frac{1}{2}(4-0) = \underline{\underline{2}}$$

$$a_n = \frac{2}{l}\int_0^{l} f(x) \cos\frac{n\pi x}{l}\, dx.$$

$$= \frac{2}{2} \cdot \int_0^2 x \cdot \cos\frac{n\pi x}{2}\, dx.$$

$$= \left[x \times \frac{2}{n\pi} \sin\frac{n\pi x}{2} - 1 \cdot -\frac{4}{n^2\pi^2} \cdot \cos\frac{n\pi x}{2} \right]_0^2$$

$$= \frac{2}{n\pi}\left[2 \cdot \sin n\pi - 0\right] + \frac{4}{n^2\pi^2}\left[\cos n\pi - \cos 0\right]$$

$$= \frac{2}{n\pi} \times 0 + \frac{4}{n^2\pi^2}\left[(-1)^n - 1\right]$$

$$= \frac{4}{n^2\pi^2}\left[(-1)^n - 1\right] \begin{cases} 0 & n = \text{even} \\ -\dfrac{8}{n^2\pi^2} & n = \text{odd} \end{cases}$$

$$\therefore\ x = 1 + \frac{4}{\pi^2}\sum_{n=1}^{\infty} \frac{\left((-1)^n - 1\right)}{n^2} \cos\frac{n\pi x}{2}$$

$$x = 1 - \frac{8}{\pi^2}\left[\frac{\cos\frac{\pi x}{2}}{1^2} + \frac{\cos\frac{3\pi x}{2}}{3^2} + \frac{\cos\frac{5\pi x}{2}}{5^2} + \cdots\right]$$

Half range sine series

$$b_n = \frac{2}{l}\int_0^{l} f(x) \cdot \sin\frac{n\pi x}{l}\, dx$$

$$= \frac{2}{2} \cdot \int_0^2 x \cdot \sin\frac{n\pi x}{2}\, dx.$$

$$= \left[x \times \frac{2}{n\pi} - \cos\frac{n\pi x}{2} + 1 \cdot \frac{4}{n^2\pi^2} \sin\frac{n\pi x}{2} \right]_0^2$$

$$= \frac{-2}{n\pi}\left[2 \cdot \cos n\pi - 0\right] + \frac{4}{n^2\pi^2}\left[\sin n\pi - 0\right]$$

$$= -\frac{2}{n\pi} \cdot 2 \cdot (-1)^n.$$

$$= -\frac{4}{n\pi} \cdot (-1)^n.$$

$$\therefore\ x = \frac{-4}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin\frac{n\pi x}{2}.$$

13. Expand $x \sin x$ as a cosine series in $0 < x < \pi$ hence show that $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} - \cdots = \frac{\pi - 2}{4}$

$$a_0 = \frac{2}{\pi} \cdot \int_0^{\pi} f(x)\, dx$$

$$= \frac{2}{\pi} \cdot \int_0^{\pi} x \sin x\, dx.$$

$$= \frac{2}{\pi} \cdot \left[x \cdot -\cos x - 1 \cdot -\sin x \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[-\pi \cdot \cos\pi - 0 \right]$$

$$= \frac{2}{\cancel{\pi}} \cancel{\times} + \cancel{\pi}$$

$$= \underline{\underline{2}}$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} x \sin x \cos nx\, dx$$

$$= \frac{1}{\pi} \int_0^{\pi} x \,(2 \cos nx \sin x)\, dx$$

$$= \frac{1}{\pi} \int_0^{\pi} x \left(\sin(n+1)x - \sin(n-1)x \right) dx.$$

$$= \cancel{\frac{1}{\pi} \left[x \left\{ \frac{-\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1} \right\} \right.}$$

$$= \frac{1}{\pi} \left[x \left\{ \frac{-\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1} \right\} \right.$$

$$\left. - 1 \cdot \left\{ - \frac{\sin(n+1)x}{(n+1)^2} - \frac{\sin(n-1)x}{(n-1)^2} \right\} \right]_0^{\pi}$$

$$= \frac{1}{\pi} \left[-\pi \frac{\cos(n+1)\pi}{n+1} + \pi \frac{\cos(n-1)\pi}{n-1} \right]$$

$$= -\frac{(-1)^{n+1}}{n+1} + \frac{(-1)^{n-1}}{n-1}$$

$$= (-1)^{n-1} \left[\frac{1}{n-1} - \frac{1}{n+1} \right]$$

$$= \frac{2 \cdot (-1)^{n-1}}{(n-1)(n+1)} = \underline{\underline{\frac{2 \cdot (-1)^{n-1}}{n^2 - 1}}} \text{ when } n \neq 1$$

When $n = 1$, we have

$$a_1 = \frac{2}{\pi} \int_0^{\pi} x \sin x \cos x\, dx.$$

$$= \frac{1}{\pi} \int_0^{\pi} x \sin 2x\, dx$$

$$= \frac{1}{\pi} \left[x \left(\frac{-\cos 2x}{2} \right) - 1 \cdot \left(\frac{\sin 2x}{2} \right) \right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[-\frac{\pi \cdot \cos 2\pi}{2}\right] = \frac{1}{2}$$

$$\therefore x \sin x = 1 - \frac{1}{2}\cos x - 2\left(\frac{\cos 2x}{1.3} - \frac{\cos 3x}{2.4} + \frac{\cos 4x}{35} \dots\right]$$

Put $x = \pi/2$,

$$\Rightarrow \frac{\pi}{2} = 1 - 2\left[-\frac{1}{1.3} + \frac{1}{3.5} - \frac{1}{5.7} - \dots\right]$$

$$\Rightarrow 1 + \frac{2}{1.3} - \frac{2}{3.5} + \frac{2}{5.7} \dots = \pi/2$$

$$\Rightarrow 2\left(\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} \dots\right) = \frac{\pi}{2} - 1$$

$$= \frac{\pi - 2}{2}$$

$$\therefore \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots = \frac{\pi - 2}{4}$$

14\. Obtain the half range cosine series for

$$f(x) = kx \quad , \quad 0 < x < \frac{l}{2}.$$

$$= k(l - x), \quad l/2 < x < l.$$

and deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{8}$.

Half range cosine series in $0 < x < l$ is,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}. \qquad l = l.$$

$$l^2 - \frac{l^2}{4}$$

$$\frac{3l^2}{4}$$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$$

$$= \frac{2}{l}\left[\int_0^{l/2} kx\,dx + \int_{l/2}^{l} k(l-x)\,dx\right]$$

$$= \frac{2}{l}\left\{k\left[\frac{x^2}{2}\right]_0^{l/2} + kl\left[x\right]_{l/2}^{l} - k\left[\frac{x^2}{2}\right]_{l/2}^{l}\right\}$$

$$= \frac{2}{l}\left[\frac{k}{2}\cdot\frac{l^2}{4} + kl \times \frac{l}{2} - \frac{k}{2}\times\frac{3l^2}{4}\right]$$

$$= \frac{2}{l}\left[\frac{kl^2}{8} + \frac{kl^2}{2} - \frac{3kl^2}{8}\right]$$

$$= \frac{2}{\cancel{l}} \times K\cancel{l}^2 \left[\frac{1}{8} + \frac{1}{2} - \frac{3}{8} \right]$$

$$= 2Kl \times \frac{1}{4}$$

$$= \frac{Kl}{2}$$

$$\frac{1}{8} + \frac{1}{2} - \frac{3}{8}$$

$$\frac{1}{8} - \frac{3}{8} + \frac{1}{2}$$

$$-\frac{2}{8} + \frac{1}{2}$$

$$-\frac{1}{4} + \frac{1}{2}$$

$$= -\frac{1}{4} + \frac{2}{4}$$

$$= \frac{1}{4}$$

$$a_n = \frac{2}{l} \int_0^l f(x) \cdot \cos \frac{n\pi x}{l} \, dx$$

$$= \frac{2}{l} \left[\int_0^{l/2} kx \cdot \cos \frac{n\pi x}{l} \, dx + \int_{l/2}^{l} k(l-x) \cdot \cos \frac{n\pi x}{l} \, dx \right]$$

$$= \frac{2k}{l} \left[\left[x \cdot \frac{l}{n\pi} \sin \frac{n\pi x}{l} - 1 \cdot \frac{l^2}{n^2\pi^2} - \cos \frac{n\pi x}{l} \right]_0^{l/2} + \left[\frac{l \times l \cdot \sin}{n\pi} \frac{n\pi x}{l} - \left\{ x \cdot \frac{l}{n\pi} \sin \frac{n\pi x}{l} - 1 \cdot \frac{l^2}{n^2\pi^2} - \cos \frac{n\pi x}{l} \right\} \right]_{l/2}^{l} \right]$$

$$= \frac{2k}{l} \left\{ \frac{l}{2} \times \frac{l}{n\pi} \sin \frac{n\pi}{2} + \frac{l^2}{n^2\pi^2} \left(\cos \frac{n\pi}{2} - \cos 0 \right) + \left[\frac{l}{n\pi} \cdot \cancel{l}\left(0 - \sin \frac{n\pi}{2}\right) - \frac{l}{n\pi} \left[0 - \frac{l}{2} \cdot \sin \frac{n\pi}{2} \right] - \frac{l^2}{n^2\pi^2} \left[\cos n\pi - \cos n\frac{\pi}{2} \right] \right] \right.$$

$$= \frac{2k}{l} \left[\frac{l^2}{2n\pi} \cdot \sin \frac{n\pi}{2} + \frac{l^2}{n^2\pi^2} (\cos n\pi/2 - 1) - \frac{l^2}{n\pi} \sin n\pi/2 + \frac{l^2}{2n\pi} \sin n\pi/2 - \frac{l^2}{n^2\pi^2} (\cos n\pi - \cos n\pi/2) \right]$$

$$= \frac{2k}{l} \left[\sin n\pi/2 \cdot \left(\frac{l^2}{2n\pi} - \frac{l^2}{n\pi} + \frac{l^2}{2n\pi} \right) + \frac{l^2}{n^2\pi^2} \cos n\pi/2 - \frac{l^2}{n^2\pi^2} - \frac{l^2}{n^2\pi^2} \cos n\pi + \frac{l^2}{n^2\pi^2} \cos n\pi/2 \right]$$

$$= \frac{2k}{l}\left[\frac{2l^2}{n^2\pi^2}\cos n\pi/2 - \frac{l^2}{n^2\pi^2} - \frac{l^2}{n^2\pi^2}\cos n\pi\right]$$

$$= \frac{2k}{l} \times \frac{l^2}{n^2\pi^2}\left[2\cos n\pi/2 - 1 - \cos n\pi\right]$$

$$= \frac{2kl}{n^2\pi^2}\left[2\cos n\pi/2 - 1 - \cos n\pi\right]$$

when n is odd

$\cos n\pi/2 = 0, \quad \cos n\pi = -1$

$$\therefore a_n = \frac{2kl}{n^2\pi^2}\left[-1 - -1\right] = 0$$

$$\therefore a_1 = a_3 = a_5 = \cdots = 0$$

when n is even

$$a_2 = \frac{2kl}{2^2\pi^2}\left[2\cos\pi - 1 - \cos 2\pi\right]$$

$$= \frac{2kl}{2^2\pi^2}\left[-2-1-1\right]$$

$$= \frac{-8kl}{2^2\pi^2}$$

$$a_4 = \frac{2kl}{4^2\pi^2}\left[2\cos 2\pi - 1 - \cos 4\pi\right]$$

$$= \frac{2kl}{4^2\pi^2}\left(2-1-1\right)$$

$$= 0$$

$$a_6 = \frac{2kl}{6^2\pi^2}\left[2\cos 3\pi - 1 - \cos 6\pi\right]$$

$$= \frac{2kl}{6^2\pi^2}\left[-2-1-1\right]$$

$$= \frac{-8kl}{6^2\pi^2}$$

$$\therefore f(x) = \frac{kl}{4} - \frac{8kl}{\pi^2}\left[\frac{1}{2^2}\cdot\cos\frac{2\pi x}{l} + \frac{1}{6^2}\cdot\cos\frac{6\pi x}{l} + \cdots\right]$$

Put $x = l$, $f(l) = 0$

$$0 = \frac{kl}{4} - \frac{8kl}{\pi^2}\left[\frac{1}{2}\cos 2\pi + \frac{1}{6}\cos 6\pi + \cdots\right]$$

$$\frac{8kl}{\pi^2}\left[\frac{1}{2^2} + \frac{1}{6^2} + \cdots\right] = \frac{kl}{4}$$

$$\frac{1}{2^2}+\frac{1}{6^2}+\cdots = \frac{\cancel{1}}{4}\times\frac{\pi^2}{8\cancel{1}}$$

$$\frac{1}{1^2 2^2}+\frac{1}{2^2 3^2}+\cdots = \frac{\pi^2}{32}$$

$$\therefore \frac{1}{2^2}\left[\frac{1}{1^2}+\frac{1}{3^2}+\cdots\right]=\frac{\pi^2}{32}$$

$$\therefore \frac{1}{1^2}+\frac{1}{3^2}+\cdots = \frac{\pi^2}{\cancel{32}\,8}\times\cancel{4} = \frac{\pi^2}{8}$$

15. Expand $f(x)=e^x$ as a half-range sine series in $0<x<1$

$$f(x)=\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$l=1$

$$\therefore b_n = \frac{2}{l}\int_0^l f(x)\cdot\sin\frac{n\pi x}{l}\,dx$$

$$= 2\cdot\int_0^1 e^x\cdot\sin n\pi x\,dx.$$

$$= \cancel{2\left[e^x\cdot\frac{-\cos n\pi x}{n\pi}\right.}$$

$$\int e^{ax}\sin(bx+c)\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin(bx+c) - b\cos(bx+c)\right]$$

$a=1$, $b=n\pi$, $c=0$.

$$b_n = 2\cdot\left[\frac{e^x}{1+n^2\pi^2}\left\{\sin n\pi x - n\pi\cdot\cos n\pi x\right\}\right]_0^1$$

$$= \frac{2}{1+n^2\pi^2}\left(e^1\left(\cancel{\sin n\pi - n\pi}\right) - n\pi\left[e^1\cdot\cos n\pi - e^0\cdot\cos 0\right]\right)$$

$$= \frac{2}{1+n^2\pi^2}\left[-n\pi\left[e\cdot(-1)^n - 1\right]\right]$$

$$= \frac{2}{1+n^2\pi}\left[n\pi - n\pi\cdot e(-1)^n\right]$$

$$= \frac{2n\pi}{1+n^2\pi^2}\left[1-e\cdot(-1)^n\right]$$

$$\therefore f(x)=e^x=\sum_{n=1}^{\infty}\frac{2n\pi}{1+n^2\pi^2}\left[1-e(-1)^n\right]\sin n\pi x$$

16. Expand $f(x) = \cos x$ as a half-range sine series in $0 < x < \pi$.

$$f(x) = \sum_{n=1}^{\infty} b_n \cdot \sin nx$$

$$\therefore b_n = \frac{2}{\pi} \cdot \int_0^{\pi} f(x) \cdot \sin nx\,dx$$

$$= \frac{2}{\pi} \int_0^{\pi} \cos x \cdot \sin nx\,dx$$

$$= \frac{1}{\pi} \cdot \int_0^{\pi} 2 \sin nx \cdot \cos x\,dx$$

$$= \frac{1}{\pi} \int_0^{\pi} \left(\sin(n+1)x + \sin(n-1)x \right) dx$$

$$= \frac{1}{\pi} \left[\frac{-\cos(n+1)x}{n+1} - \frac{\cos(n-1)x}{n-1} \right]_0^{\pi}$$

$$= \frac{1}{\pi} \left[-\frac{1}{n+1} \left(\cos(n+1)\pi - \cos 0 \right) - \frac{1}{n-1} \left(\cos(n-1)\pi - \cos 0 \right) \right]$$

$$= \frac{1}{\pi} \left[-\frac{1}{n+1} \left((-1)^{n+1} - 1 \right) - \frac{1}{n-1} \left((-1)^{n-1} - 1 \right) \right]$$

$$\frac{1}{n+1} + \frac{1}{n-1}$$

$$= \frac{n-1+n+1}{n^2-1^2}$$

$$= \frac{2n}{n^2-1}$$

$$= \frac{1}{\pi} \left[-\frac{(-1)^{n+1}}{n+1} + \frac{1}{n+1} - \frac{(-1)^{n-1}}{n-1} + \frac{1}{n-1} \right]$$

$$= \frac{1}{\pi} \left[(-1)^n \left\{ \frac{1}{n+1} + \frac{1}{n-1} \right\} + \frac{1}{n+1} + \frac{1}{n-1} \right]$$

$$= \frac{1}{\pi} \left[(-1)^n + 1 \right] \left[\frac{1}{n+1} + \frac{1}{n-1} \right]$$

$$= \frac{2n}{\pi(n^2-1)} \left((-1)^n + 1 \right)$$

$= 0$ if n is odd

$\dfrac{4n}{\pi(n^2-1)}$ if n is even

$$\therefore \cos x = \sum_{n=1}^{\infty} \frac{2n}{\pi(n^2-1)} \left((-1)^n + 1 \right) \sin nx$$

$$\cos x = \frac{4}{\pi} \left[\frac{2 \sin 2x}{3} + \frac{4 \sin 4x}{15} + \frac{6 \sin 6x}{35} + \cdots \right]$$

17. Show that the constant c can be expanded as an infinite series $\frac{4c}{\pi}\left[\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \cdots\right]$ in $0 < x < \pi$.

$f(x) = c$ as a half range sine series is,

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

where, $b_n = \frac{2}{\pi}\int_0^{\pi} f(x) \cdot \sin nx \, dx$.

$$= \frac{2}{\pi}\int_0^{\pi} c \cdot \sin nx \, dx$$

$$= \frac{2c}{\pi}\left[-\frac{\cos nx}{n}\right]_0^{\pi}$$

$$= \frac{-2c}{n\pi}\left[\cos n\pi - \cos 0\right]$$

$$= -\frac{2c}{n\pi}\left[(-1)^n - 1\right]$$

$= 0$ when n is even

$\frac{4c}{n\pi}$ when n is odd

$$\therefore c = \sum_{n=1}^{\infty} -\frac{2cn}{\pi}\left[(-1)^n - 1\right] \sin nx.$$

$$= \frac{4c}{\pi}\left[1 \cdot \sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \cdots\right]$$

ie, $c = \frac{4c}{\pi}\left[\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \cdots\right]$

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18. Expand $f(x) = \frac{1}{4} - x$, $0 < x < \frac{1}{2}$

$= x - \frac{3}{4}$, $\frac{1}{2} < x < 1$

as a half range sine series.

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}.$$

$l = 1$

$$b_n = \frac{2}{l}\int_0^{l} f(x) \cdot \sin\frac{n\pi x}{l} \, dx$$

$$= 2 \cdot \int_0^{1} f(x) \cdot \sin n\pi x \, dx.$$

$$= 2\left[\int_0^{1/2}\left(\frac{1}{4}-x\right)\sin n\pi x\,dx + \int_{1/2}^{1}\left(x-\frac{3}{4}\right)\sin n\pi x\,dx\right]$$

$$= 2.\left[\left[\frac{1}{4}\cdot\frac{-\cos n\pi x}{n\pi} - \left\{x\cdot\frac{-\cos n\pi x}{n\pi} - 1\cdot\frac{-\sin n\pi x}{n^2\pi^2}\right]_0^{1/2} + \left[x\cdot\frac{-\cos n\pi x}{n\pi} - 1\cdot\frac{-\sin n\pi x}{n^2\pi^2} - \frac{3}{4}\frac{-\cos n\pi x}{n\pi}\right]_{1/2}^{1}\right]$$

$$= 2\left[\left[-\frac{1}{4n\pi}\cos n\pi x + \frac{1}{n\pi}x\cos n\pi x - \frac{1}{n^2\pi^2}\sin n\pi x\right]_0^{1/2} + \left[-\frac{1}{n\pi}x\cdot\cos n\pi x + \frac{1}{n^2\pi^2}\sin n\pi x + \frac{3}{4n\pi}\cos n\pi x\right]_{1/2}^{1}\right]$$

$$= 2\left[-\frac{1}{4n\pi}\left(\cos\frac{n\pi}{2} - \cos 0\right) + \frac{1}{n\pi}\cdot\frac{1}{2}\cdot\cos\frac{n\pi}{2} - \frac{1}{n^2\pi^2}\sin\frac{n\pi}{2} - \frac{1}{n\pi}\left(\cos n\pi - \frac{1}{2}\cos\frac{n\pi}{2}\right) + \frac{1}{n^2\pi^2}\left(0 - \sin\frac{n\pi}{2}\right) + \frac{3}{4n\pi}\left(\cos n\pi - \cos\frac{n\pi}{2}\right)\right]$$

$$= 2\left[-\frac{1}{4n\pi}\left(\cos n\pi/2 - 1\right) + \frac{1}{2n\pi}\cos\frac{n\pi}{2} - \frac{1}{n^2\pi^2}\sin\frac{n\pi}{2} - \frac{1}{n\pi}\left[(-1)^n\right] + \frac{1}{2n\pi}\cos\frac{n\pi}{2} - \frac{1}{n^2\pi^2}\sin\frac{n\pi}{2} + \frac{3\cdot(-1)^n}{4n\pi} - \frac{3}{4n\pi}\cdot\cos n\pi/2\right]$$

$$= 2\left[\frac{1}{n\pi}\cos n\pi/2\left(-\frac{1}{4}+\frac{1}{2}+\frac{1}{2}-\frac{3}{4}\right) - \frac{2}{n^2\pi^2}\cdot\sin\frac{n\pi}{2} + \frac{(-1)^n}{n\pi}\left(\frac{3}{4}-1\right) + \frac{1}{4n\pi}\right]$$

$$= 2\left[0 - \frac{2}{n^2\pi^2}\sin n\pi/2 - \frac{(-1)^n}{4n\pi} + \frac{1}{4n\pi}\right]$$

$$= 2\left[\frac{1}{4n\pi}\left[1-(-1)^n\right] - \frac{2}{n^2\pi^2}\sin n\pi/2\right]$$

$$= \frac{1-(-1)^n}{2n\pi} - \frac{4}{n^2\pi^2}\sin n\pi/2$$

$$b_n = \frac{1-(-1)^n}{2n\pi} - \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}$$

$$\therefore f(x) = \sum_{n=1}^{\infty}\left(\frac{1-(-1)^n}{2n\pi} - \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}\right)\sin n\pi x$$

Parseval's Theorem for fourier constants

Let $f(x)$ be a function defined on the interval $c \le x \le c+2l$. and if $f(x)$ equals,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cdot \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

is the fourier series of $f(x)$ then

$$\frac{1}{l}\int_c^{c+2l}[f(x)]^2\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}(a_n^2+b_n^2)$$

Particular cases

① If $c=0$, then $0 \le x \le 2l$.

then theorem is,

$$\frac{1}{l}\int_0^{2l}[f(x)]^2\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}(a_n^2+b_n^2).$$

② If $c=-l$. then $-l \le x \le l$.

then theorem is,

$$\frac{1}{l}\int_{-l}^{l}[f(x)]^2\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}(a_n^2+b_n^2)$$

1) if $f(x)$ even. $\quad f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$

$$\frac{2}{l}\int_0^{l}[f(x)]^2\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2$$

2) if $f(x)$ odd $\quad f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$

$$\frac{2}{l}\int_0^{l}[f(x)]^2\,dx = \sum_{n=1}^{\infty} b_n^2.$$

③ if $f(x)$ is a sine series in $0 \le x \le l$

$$\frac{2}{l}\int_0^{l}[f(x)]^2\,dx = \sum_{n=1}^{\infty} b_n^2.$$

(4) if $f(x)$ is half range cosine series in $0 \le x \le l$.

$$\frac{2}{l}\int_0^l [f(x)]^2\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2$$

Root Mean Square value of a function

$$\left[[f(x)]_{rms}\right]^2 = \frac{1}{l}\int_c^{c+2l} [f(x)]^2\,dx.$$

18. ~~Show that the fourier series for~~

In all particular cases the LHS of Parseval's theorem is the rms^2.

19. Show that fourier series for x in $0 \le x \le l$ is $x = \frac{2l}{\pi}\left[\sin\frac{\pi x}{l} - \frac{1}{2}\sin\frac{2\pi x}{l} + \frac{1}{3}\sin\frac{3\pi x}{l} \ldots\ldots\right]$

Hence, deduce that,

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\ldots = \frac{\pi^2}{6}.$$

Since the interval is $0 < x < l$ the required expansion is half-range sine series.

$$\therefore f(x) = \sum_{n=1}^{\infty} b_n \cdot \sin\frac{n\pi x}{l}$$

where,
$$b_n = \frac{2}{l}\int_0^l f(x)\cdot \sin\frac{n\pi x}{l}\,dx.$$

$$= \frac{2}{l}\cdot\int_0^l x\cdot \sin\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{l}\left[x\cdot l\frac{-\cos\frac{n\pi x}{l}}{n\pi} - \frac{l^2}{n^2\pi^2}\cdot\frac{\sin n\pi x}{l}\right]_0^l$$

$$= \frac{2}{l}\left[\frac{-l}{n\pi}\left[l\cos n\pi - 0\right] + \frac{l^2}{n^2\pi^2}\left[\sin n\pi - \sin 0\right]\right]$$

$$= \frac{2}{l}\cdot\left[-\frac{l}{n\pi}\cdot l\cdot(-1)^n\right]$$

$$= \frac{-2l\,(-1)^n}{n\pi}$$

$$\therefore x = \sum_{n=1}^{\infty} \frac{-2l\cdot(-1)^n}{n\pi} \sin\frac{n\pi x}{l}$$

$$= \cancel{\frac{2l}{n}\left[\sin 2\pi x\right.}$$

$$= \cancel{\frac{2l}{n}} \left[\cancel{\sin}\, n\pi x\right.$$

$$= \frac{2l}{\pi}\left[\sin\frac{\pi x}{l} - \frac{1}{2}\cdot \sin\frac{2\pi x}{l} + \frac{1}{3}\sin\frac{3\pi x}{l}\right]$$

$$x = \frac{2l}{\pi}\left(\sin\frac{\pi x}{l} - \frac{1}{2}\sin\frac{2\pi x}{l} + \frac{1}{3}\sin\frac{3\pi x}{l} \cdots\right)$$

By Parseval's theorem,

$$\frac{2}{l}\int_0^l [f(x)]^2\,dx = \sum_{n=1}^{\infty} b_n^2.$$

$$\frac{2}{l}\int_0^l x^2\,dx = \sum_{n=1}^{\infty}\left(\frac{-2l\cdot(-1)^n}{n\pi}\right)^2$$

ie, $$\frac{2}{l}\cdot\left[\frac{x^3}{3}\right]_0^l = \sum_{n=1}^{\infty}\frac{4l^2}{n^2\pi^2}$$

$$\frac{2}{l}\left[\frac{l^3}{3}-0\right] = \frac{4l^2}{\pi^2}\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots\right]$$

$$\frac{2}{\cancel{l}}\times\frac{l^{\cancel{3}2}}{3} = \frac{4l^2}{\pi^2}\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots\right]$$

$$\frac{2l^2}{3} = \frac{4l^2}{\pi^2}\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots\right]$$

$$\therefore \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots = \frac{2l^2}{3}\times\frac{\pi^2}{4l^2}$$

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots = \frac{\pi^2}{6}$$

20. Find the fourier series for $y = x^2$ in $-\pi \le x < \pi$ and show that $\dfrac{\pi^4}{90} = \dfrac{1}{1^4} + \dfrac{1}{2^4} + \dfrac{1}{3^4} + \cdots$

21. Obtain the half range ~~cos~~ sine and cosine series to represent, $f(x) = x - x^2$ in $0 \le x \le 1$ and deduce,

$$\frac{1}{1^6} + \frac{1}{3^6} + \frac{1}{5^6} + \cdots \quad \text{and} \quad \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots$$

Ans:

20. $f(x) = x^2$.

$f(x)$ is even function.

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx.$$

$$a_0 = \frac{2}{\pi} \cdot \int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi} \cdot \int_0^{\pi} x^2\,dx$$

$$= \frac{2}{\pi} \cdot \left(\frac{x^3}{3}\right)_0^{\pi}$$

$$= \frac{2}{3\pi} \cdot \left[\pi^3 - 0\right].$$

$$= \frac{2}{3\cancel{\pi}} \times \pi^{\cancel{3}2} = \underline{\underline{\frac{2\pi^2}{3}}}$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx\,dx$$

$$= \frac{2}{\pi} \cdot \int_0^{\pi} x^2 \cos nx\,dx$$

$$= \frac{2}{\pi} \cdot \left[x^2 \cdot \cancelto{0}{\frac{\sin nx}{n}} - 2x \cdot \frac{-\cos nx}{n^2} + 2 \cdot \cancelto{0}{\frac{-\sin nx}{n^3}} \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[\frac{2}{n^2} \left(\pi \cdot \cos n\pi - 0 \right) \right]$$

$$= \frac{4}{n^2 \cancel{\pi}} \times \cancel{\pi} \times (-1)^n$$

$$= \underline{\underline{\frac{4}{n^2} \cdot (-1)^n}} \quad \text{where } n \ne 0.$$

$$\therefore f(x) = x^2 = \frac{\pi^2}{3} - 4\cos x + \cos 2x - \frac{4}{9}\cos 3x + \cdots$$

By Parseval's theorem,

$$\frac{2}{\pi} \int_0^{\pi} \left(f(x)\right)^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2$$

$$\cancel{\frac{2}{\pi} \int_0^{\pi} x^4\,dx = \frac{2\pi^4}{9} + 16\Big(}$$

$$\frac{2}{\pi} \cdot \int_0^{\pi} x^4 \, dx = \frac{4\pi^4}{9 \times 2} + \sum_{n=1}^{\infty} \left(\frac{4 \cdot (-1)^n}{n^2} \right)^2$$

$$\frac{2}{\pi} \cdot \left[\frac{x^5}{5} \right]_0^{\pi} = \frac{2\pi^4}{9} + 16 \left[\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots \right]$$

$$\frac{2}{\cancel{\pi}} \times \frac{\pi^{\cancel{5}4}}{5} = \frac{2\pi^4}{9} + 16 \left[\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots \right]$$

$$\frac{2\pi^4}{5} - \frac{2\pi^4}{9} = 16 \left[\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots \right]$$

$\frac{1}{5} - \frac{1}{9} = \frac{4}{45}$

$$2\pi^4 \times \frac{4}{45} = 16 \left[\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots \right]$$

$$\therefore \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots = \frac{8\pi^4}{45 \times \cancel{16}\, 2}$$

$$\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots = \frac{\pi^4}{90}$$

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Mariya Varghese

18

Harmonic Analysis

24. (3)

The process of finding fourier series, when $f(x)$ is given in tabular form is known as harmonic analysis.

If N is the no. of ordinates given, and $2l$ is the period of the function or the interval is $\boxed{0 \le x \le 2l}$.

Then,

$$a_0 = \frac{2}{N} \Sigma y$$

$$a_n = \frac{2}{N} \Sigma y \cdot \cos\frac{n\pi x}{l}$$

$$b_n = \frac{2}{N} \Sigma y \cdot \sin\frac{n\pi x}{l}.$$

$$f(x) = \frac{a_0}{2} + \underbrace{\left(a_1 \cos\frac{\pi x}{l} + b_1 \sin\frac{\pi x}{l}\right)}_{1^{st} \text{ harmonic}} + \underbrace{\left(a_2 \cos\frac{2\pi x}{l} + b_2 \sin\frac{2\pi x}{l}\right)}_{2^{nd} \text{ harmonic}} + \cdots \underbrace{\cdots}_{n^{th} \text{ harmoni}}$$

$$\text{Amplitude of } n^{th} \text{ harmonic} = \sqrt{a_n^2 + b_n^2}$$

22. Analyse harmonically and express y as a fourier series upto 3rd harmonic.

Given,

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y	1	1.4	1.9	1.7	1.5	1.2	1

When $x = 0$, $y = 1$ and $x = 2\pi$, $y = 1$

$\therefore$ period $= 2l = 2\pi$.

$2l = 2\pi \implies l = \pi$

$N = 6$

x	y	$\cos x$	$\sin x$	$\cos 2x$	$\sin 2x$	$\cos 3x$	$\sin 3x$
0	1	1	0	1	0	1	0
$\pi/3$	1.4	0.5	0.866	−0.5	0.866	−1	0
$2\pi/3$	1.9	−0.5	0.866	−0.5	−0.866	1	0
π	1.7	−1	0	1	0	−1	0
$4\pi/3$	1.5	−0.5	−0.866	−0.5	0.866	1	0
$5\pi/3$	1.2	0.5	−0.866	−0.5	−0.866	−1	0

$$a_0 = \frac{2}{N} \cdot \sum y$$

$$= \frac{2}{6}\left[1 + 1.4 + 1.9 + 1.7 + 1.5 + 1.2\right]$$

$$= \underline{\underline{2.9}}$$

$$a_1 = \frac{2}{N} \cdot \sum y \cdot \cos x.$$

$$= \frac{2}{6}\left[1 + 0.7 + 0.95 - 1.7 - 0.75 + 0.6\right]$$

$$= \underline{\underline{-0.366}}$$

$$b_1 = \frac{2}{N} \sum y \sin x$$

$$= \frac{2}{6}\left[0 + 1.2124 + 1.6454 + 0 - 1.299 - 1.0392\right]$$

$$= \underline{\underline{0.1732}}$$

$$a_2 = \frac{2}{N} \cdot \Sigma y \cdot \cos 2x$$

$$= \frac{2}{6}\left[1 - 0.7 - 0.95 + 1.7 - 0.75 - 0.6\right]$$

$$= \underline{\underline{-0.1}}$$

$$b_2 = \frac{2}{N} \cdot \Sigma y \cdot \sin 2x$$

$$= \frac{2}{6}\left[0 + 1.2124 - 1.6454 0 + 1.299 - 1.0392\right]$$

$$= \underline{\underline{-0.06}}$$

$$a_3 = \frac{2}{N} \cdot \Sigma y \cos 3x$$

$$= \frac{2}{6}\left[1 - 1.4 + 1.9 - 1.7 + 1.5 - 1.2\right]$$

$$= \underline{\underline{0.03}}$$

$$b_3 = \frac{2}{N} \Sigma y \sin 3x$$

$$= \frac{2}{6}\left[0 + 0 + 0 + 0 + 0 + 0\right]$$

$$= \underline{\underline{0}}$$

$$f(x) = \frac{2.9}{2} + (-0.37 \cos x + 0.17 \sin x) + (-0.1 \cos 2x + -0.06 \sin 2x) + (0.03 \cos 3x + 0).$$

23\. The following values of y if the displacement in inches of a certain machine part for the rotation x of a fly be expand by in the form of a fourier series.

x :	0	$\pi/6$	$2\pi/6$	$3\pi/6$	$4\pi/6$	$5\pi/6$	π
y :	0	9.2	14.4	17.8	17.3	11.7	0

$N = 6$

$2l = \pi$

$\therefore l = \frac{\pi}{2}$

$\cos \frac{n\pi x}{l} = \cos \frac{n\pi x}{\pi/2}$

$= \cos 2nx$

$$\cos \frac{n\pi x}{l} = \cos \frac{n\pi x}{\pi/2} = \cos 2nx$$

x	y	$\cos 2x$	$\sin 2x$	$\cos 4x$	$\sin 4x$
0	0	1	0	1	0
$\pi/6$	9.2	0.5	0.866	−0.5	0.866
$2\pi/6$	14.4	−0.5	0.866	−0.5	−0.866
$3\pi/6$	17.8	−1	0	1	0
$4\pi/6$	17.3	−0.5	−0.866	−0.5	0.866
$5\pi/6$	11.7	0.5	−0.866	−0.5	−0.866
π	0	1	0	1	0

$$a_0 = \frac{2}{N} \Sigma y$$

$$= \frac{2}{6}\left[0+9.2+14.4+17.8+17.3+11.7+0\right]$$

$$= 23.46$$

$$a_1 = \frac{2}{N} \cdot \Sigma y \cos 2x$$

$$= \frac{2}{6} \cdot \left[0 + 4.6 - 7.2 - 17.8 - 8.65 + 5.85\right]$$

$$a_1 = -7.73$$

$$b_1 = \frac{2}{N} \cdot \Sigma y \sin 2x .$$

$$= \frac{2}{6}\left[0 + 7.9672 + 12.4704 + 0 - 14.9818 - 10.1322\right]$$

$$= -1.558$$

$$a_2 = \frac{2}{N} \Sigma y . \cos 4x$$

$$= \frac{2}{6}\left[0 - 4.6 - 7.2 + 17.8 - 8.65 - 5.85\right]$$

$$= -2.83$$

$$b_2 = \frac{2}{N} \Sigma y \sin 4x$$

$$= \frac{2}{6}\left[0 + 7.9672 - 12.4704 + 0 + 14.9818 - 10.1322\right]$$

$$= 0.115$$

$$\therefore f(x) = \frac{a_0}{2} + a_1 \cos \frac{\pi x}{l} + b_1 \sin \frac{\pi x}{l} + a_2 \cos \frac{2\pi x}{l} + \ldots$$

$$f(x) = \frac{23.46}{2} - 7.73 \cos 2x - 1.558 \sin 2x - 2.83 . \cos 4x + 0.115 \sin 4x$$

24. Obtain the constant term and the coefficients of the 1^{st} sine and cosine terms of the fourier expansion of y given in the following table.

x :	0	1	2	3	4	5	6
y :	9	18	24	28	26	20	9

$N = 6$

$2l = 6 \Rightarrow l = 3$

$$\cos\frac{n\pi x}{l} = \cos\frac{n\pi x}{3}$$

x	y	$\cos\frac{\pi x}{3}$	$\sin\frac{\pi x}{3}$	~~$\cos\frac{2\pi x}{3}$~~	~~$\sin\frac{2\pi x}{3}$~~
0	9	1	0		
1	18	0.5	0.866		
2	24	-0.5	0.866		
3.	28	-1	0		
4	26	-0.5	-0.866		
5	20	0.5	-0.866		
6	9	1	0		

$$a_0 = \frac{2}{N}\cdot \Sigma y$$

$$= \frac{2}{6}\left[9+18+24+28+26+20\right]$$

$$= 41.66$$

$$a_1 = \frac{2}{N}\cdot \Sigma y \cos\frac{n\pi x}{l}$$

$$= \frac{2}{6}\left[9+9-12-28-13+10\right]$$

$$= -8.33$$

$$b_1 = \frac{2}{N}\cdot \Sigma y \sin\frac{n\pi x}{l}$$

$$= \frac{2}{6}\cdot\left[0+15.588+20.784+0-22.516-17.32\right]$$

$$= -1.15$$

$$\therefore f(x) = \frac{41.66}{2} + -8.33\cos\frac{\pi x}{3} - 1.15\cdot\sin\frac{\pi x}{3}$$

25 The following table gives variations of periodic current over a period.

t (sec):	0	$\frac{T}{6}$	$\frac{2T}{6}$	$\frac{3T}{6}$	$\frac{4T}{6}$	$\frac{5T}{6}$	T
A (amp):	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

~~Res~~. Show by numerical analysis that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of 1^{st} harmonic.

Direct current part $= \frac{a_0}{2}$ const term in F.S

Amplitude of 1^{st} harmonic $= \sqrt{a_1^2 + b_1^2}$

$N = 6$.

$2l = T$.

$\therefore l = \frac{T}{2}$

$$\cos \frac{n\pi x}{l} = \cos \frac{n\pi x}{T/2} = \cos \frac{2n\pi x}{T}$$

t	A	$\cos \frac{2\pi t}{T}$	$\sin \frac{2\pi t}{T}$
0	1.98	1	0
T/6	1.30	0.5	0.866
2T/6	1.05	-0.5	0.866
3T/6	1.30	-1	0
4T/6	-0.88	-0.5	-0.866
5T/6	-0.25	0.5	-0.866

$$a_0 = \frac{2}{N} \cdot \Sigma A$$

$$= \frac{2}{6} \left[1.98 + 1.30 + 1.05 + 1.30 - 0.88 - 0.25 \right]$$

$$= \underline{\underline{1.50}}$$

$$a_1 = \frac{2}{N} \Sigma A \cdot \cos \frac{2\pi t}{T}$$

$$= \frac{2}{6} \left[1.98 + 0.65 - 0.525 - 1.30 + 0.44 - 0.125 \right]$$

$$= \underline{\underline{0.373}}$$

$$b_1 = \frac{2}{N} \cdot \Sigma A \sin \frac{2\pi t}{T}$$

$$= \frac{2}{6}\left[0 + 1.1258 + 0.9093 + 0 + 0.76208 + 0.2165 \right]$$

$$= \underline{1.005}$$

$$\therefore f(x) = A = \frac{1.50}{2} + 0.373 \cos \frac{2\pi t}{T} + 1.005 \sin \frac{2\pi t}{T} + \ldots$$

Direct current part $= \frac{a_0}{2}$

$$= \frac{1.50}{2}$$

$$= \underline{0.75 \text{ amp}}$$

Amplitude of 1st harmonic $= \sqrt{a_1^2 + b_1^2}$

$$= \sqrt{(0.373)^2 + (1.005)^2}$$

$$= \underline{1.072}$$

26. obtain the 1st 3 coefficients in the fourier cosine series from the following data.

u.Q

x :	0	1	2	3	4	5
y :	4	8	15	7	6	2

Here, it is a fourier cosine series so its range is half range.

$N = 6$

$\therefore l = 6$

$\cos \frac{n\pi x}{l} = \cos \frac{n\pi x}{6}$

x	y	$\cos \frac{\pi x}{6}$	~~$\sin \frac{\pi x}{6}$~~	$\cos \frac{2\pi x}{6}$	~~$\sin \frac{2\pi x}{6}$~~	$\cos \frac{3\pi x}{6}$
0	4	1	~~0~~	1	~~0~~	1
1	8	0.866	~~0.5~~	0.5		0
2	15	0.5	~~0.866~~	-0.5		-1
3	7	0		-1		0
4	6	-0.5	~~0.866~~	-0.5		1
5	2	-0.866	~~0.5~~	0.5		0

$$a_0 = \frac{2}{N} \Sigma y$$

$$= \frac{2}{6}\left[4 + 8 + 15 + 7 + 6 + 2 \right]$$

$$= \frac{42 \times 2}{6}$$

$$= \underline{\underline{14}}$$

$$a_1 = \frac{2}{N} \cdot \Sigma y \cos \frac{\pi x}{6}$$

$$= \frac{2}{6}\left[4 + 6.928 + \cancel{7.50} + 0 - 3 - 1.732\right]$$

$$= \cancel{2.315} \quad 4.565$$

$$a_2 = \frac{2}{N} \cdot \Sigma y \cos \frac{\pi x}{3}$$

$$= \frac{2}{6}\left[4 + 4 - 7.5 - 7 - 3 + 1\right]$$

$$= \underline{\underline{-2.833}}$$

$$a_3 = \frac{2}{N} \cdot \Sigma y \cos \frac{\pi x}{2}$$

$$= \frac{2}{6}\left[4 + 0 - 15 + 0 + 6 + 0\right]$$

$$= \underline{\underline{-1.666}}$$

$$f(x) = \frac{a_0}{2} + a_1 \cos \frac{\pi x}{l} + a_2 \cos \frac{2\pi x}{l} + a_3 \cos \frac{3\pi x}{l}$$

$$f(x) = 7 + \overset{4.565}{\cancel{2.315}} \cos \frac{\pi x}{6} - 2.833 \cos \frac{2\pi x}{3} - 1.666 \cos \frac{\pi x}{2}$$