20/1/16
Wednesday

Module 4 : Probability Distribution

Syllabus :

Concept of random variables - probability distribution - bernoulli's trial - discrete distribution - binomial distribution, it's mean and variance - fitting of binomial distribution - poisson distribution as a limiting case of binomial distribution, it's mean and variance - fitting of poisson distribution - continuos distribution - uniform distribution - exponential distribution, its mean and variance - normal distribution - standard normal curve and its properties.

↳ Random variable:

It is a variable associated with the outcome of a random experiment and it depends on chance. They are denoted by capital letters, usually X, Y, Z etc.

↳ Discrete and continuos variables:

A discrete random variable is one which can assume isolated values such as 0, 1, 2, 3, etc.

eg: The no. of heads in 3 tosses of a coin. The random variable can assume the values 0, 1, 2, 3.

A continous random variable is one which can assume any value with in an interval.

eg: weights of a group of individuals.

Discrete probability distribution:

Let x such that $x_1, x_2, \ldots x_n$ with probabilities $p(x_1), p(x_2) \ldots p(x_n)$ where $\sum p(x_i) = 1$ and $p(x_i) \geq 0$ for all i, then:

$$X: \; x_1, \; x_2, \; \ldots, \; x_n$$
$$P(X): \; p(x_1), \; p(x_2), \; \ldots, \; p(x_n)$$

is called the discrete probability distribution for x and it defines how a total probability of one is distributed over several values of x.

Mean and variance of random variables:

$$X: \; x_1, \; x_2, \; \ldots, \; x_n$$
$$\cancel{P(X)}: \; \cancel{p(x_1)}, \; \cancel{p(x_2)}, \; \ldots, \; \cancel{p(x_n)}$$
$$P(X): \; P_1, \; P_2, \; \ldots, \; P_n$$

be a discrete probability distribution.

then, Mean $= \mu = \dfrac{\sum x_i p_i}{\sum p_i} = \sum x_i p_i$ bcoz $\sum p_i =$

$$\text{Variance} = \sigma^2 = \sum x_i^2 p_i - \mu^2$$

Standard deviation:

$$S.D = \sigma = \sqrt[+]{\text{variance}}$$

Mathematical expectation:

$E(x)$ expectation of x $= \begin{cases} \sum x_i p_i & \text{for discrete R.V - random variables} \\ \int x_i f_i \, dx & \text{for continuos R.V} \end{cases}$

OR.

$$E(x) = \begin{cases} \sum x f(x) & \text{for discrete R.V} \\ \int x f(x) & \text{for continuos R.V} \end{cases}$$

Note:

Continious:

$$E(x) = \int x f(x) \, dx$$
$$E(x^2) = \int x^2 f(x) \, dx.$$

$$E(X(X-1)) = \int x(x-1) f(x)\, dx.$$

$$\boxed{\begin{aligned} E(X) &= \text{Mean} = \mu \\ V(X) &= E(X^2) - [E(X)]^2. \end{aligned}}$$

Mean =	probable no.
=	expected no
=	average
=	mathematical expectation.

6/16 wdays

<u>Cumulative probability distribution</u>:

If X is a discrete or continuous random variable then (probability that $X \le x$) $P[X \le x]$ is called cumulative distribution of x and denoted by $F(x)$.

If X is discrete $F(x) = \sum_j P_j$ where $x_j \le x$.

If X is continuous, $F(x) = P[X \le x]$

$$= \int_{-\infty}^{x} f(x)\,dx.$$

Q. Find the mean and variance of the R.V with probability distribution fn;

X:	0	1	2	3
$P(x)$:	$\frac{8}{27}$	$\frac{12}{27}$	$\frac{6}{27}$	$\frac{1}{27}$.

ans. Mean $= \sum x_i p_i$

$$= 0\times\frac{8}{27} + 1\times\frac{12}{27} + 2\times\frac{6}{27} + 3\times\frac{1}{27}.$$

$$= \frac{12}{27} + \frac{12}{27} + \frac{3}{27}$$

$$= \frac{27}{27} = \underline{\underline{1}}$$

Variance $= \sum x_i^2 p_i - \mu^2$

$$\sum x_i^2 p_i = 0\times\frac{8}{27} + 1\times\frac{12}{27} + 2^2\times\frac{6}{27} + 3^2\times\frac{1}{27}$$

$$= \frac{12}{27} + \frac{24}{27} + \frac{9}{27}$$

$$= \frac{45}{27} = \underline{\underline{\frac{5}{3}}}$$

$$\sigma^2 = \frac{5}{3} - 1 = \underline{\underline{\frac{2}{3}}}$$

Q If the density (distribution) fn of X is

$$f(x) = \begin{cases} c\,e^{-2x} & 0 < x < \infty \\ 0 & x < 0 \end{cases}$$

Find the value of c, mean and variance and $P(X > 2)$ and cumulative distribution fn?

ans: we know that total probability = 1

i.e. $\int_0^{\infty} f(x)\,dx = 1$

$\int_0^{\infty} c\,e^{-2x}\,dx = 1$

$c\int_0^{\infty} e^{-2x}\,dx = 1$

$c \times \left[\frac{e^{-2x}}{-2}\right]_0^{\infty} = 1$

$\frac{c}{-2} \times \left[e^{-2\times\infty} - e^{0}\right] = 1$

$\frac{c}{-2} \times [0 - 1] = 1$

$\frac{c}{2} = 1$

$\underline{c = 2}$

So we can write $f(x)$ as,

$$f(x) = \begin{cases} 2x\,e^{-2x} & 0 < x < \infty \\ 0 & x < 0 \end{cases}$$

Mean $= \mu = E(X) = \int_0^{\infty} x f(x)\,dx$

$= \int_0^{\infty} x \cdot 2 \cdot e^{-2x}\,dx$

$= 2\int_0^{\infty} x \cdot e^{-2x}\,dx$

$= 2\left[(x) \times \left(\frac{e^{-2x}}{-2}\right) - (1)\left(\frac{e^{-2x}}{-2 \times -2}\right)\right]_0^{\infty}$

$= 2\left[\frac{xe^{-2x}}{-2} - \frac{e^{-2x}}{4}\right]_0^{\infty}$

$= 2\left[\frac{\infty e^{-\infty}}{-2} - \frac{e^{-\infty}}{4} - 0 + \frac{1}{4}e^{0}\right]$

$= \frac{2}{4} = \underline{\underline{\frac{1}{2}}}$

Variance $= \sigma^2 = E(X^2) - [E(X)]^2$

$E(X^2) = \int x^2 f(x)\,dx$

$= \int_0^{\infty} x^2 \cdot 2e^{-2x}\,dx.$

$$= 2\int_0^\infty x^2 . e^{-2x}\, dx.$$

$$= 2\left[(x^2)\left(\frac{e^{-2x}}{-2}\right) - (2x)\left(\frac{e^{-2x}}{4}\right) + (2)\left(\frac{e^{-2x}}{-8}\right)\right]_0^\infty$$

$$= 2\left[\frac{x^2 e^{-2x}}{-2} - \frac{2x e^{-2x}}{4} + \frac{2e^{-2x}}{-8}\right]_0^\infty$$

$$= 2\left[e^{-\infty} - e^{-\infty} + e^{-\infty} + 0 + 0 + \frac{2}{8}\right]$$

$$= \frac{4}{8} = \underline{\underline{\frac{1}{2}}}$$

$$\sigma^2 = E(X^2) - (E(X))^2$$

$$= \frac{1}{2} - \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{2} - \frac{1}{4}$$

$$= \underline{\underline{\frac{1}{4}}}$$

$$P(X > 2) = \int_2^\infty f(x)\, dx$$

$$= \int_2^\infty 2.e^{-2x}\, dx$$

$$= 2 \times \left[\frac{e^{-2x}}{-2}\right]_2^\infty$$

$$= 2 \times \left[\frac{-e^{-4}}{-2}\right]$$

$$= e^{-4} = \underline{\underline{\frac{1}{e^4}}}$$

Cumulative distribut^n fn $= P[X \le x]$

$$= \int_{-\infty}^{x} f(x)\, dx$$

$$= \int_0^x f(x)\, dx$$

$$= \int_0^x 2.e^{-2x}\, dx$$

$$= 2\left[\frac{e^{-2x}}{-2}\right]_0^x$$

$$= \frac{2}{-2}\left[e^{-2x} - e^0\right]$$

$$= -1\left[e^{-2x} - 1\right]$$

$$= \underline{\underline{1 - e^{-2x}}}$$

Q. A R.V X has the following probability distribution

x:	0	1	2	3	4	5	6	7	8
$P(x)$:	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$.

1) Determine the value of a.

2) Find $P(X<3)$, $P(X\geq 3)$, $P(2\leq X<5)$.

3) What is the smallest value of x for which $P[X\leq x] > 0.5$.

ans: 1) Total probability = 1

ie $a+3a+5a+7a+9a+11a+13a+15a+17a=1$

$81a=1$

$a=\frac{1}{81}$

2) $P[X<3] = P[X=0]+P[X=1]+P[X=2]$

$= a+3a+5a.$

$= 9a$

$= 9\times\frac{1}{81}$

$= \frac{1}{9}$

$P[X\geq 3] = 1-P[X<3]$

$= 1-\frac{1}{9}$

$= 8/9$

$P[2\leq X<5] = P[X=2]+P[X=3]+P[X=4]$

$= 5a+7a+9a.$

$= 21a$

$= 21\times\frac{1}{81}$

$= \frac{7}{27}$

3) $P[X=0] = a = \frac{1}{81} < 0.5$

$P[X=0,1] = a+3a = 4a = \frac{4}{81} = 0.04 < 0.5$

$P[X=0,1,2] = a+3a+5a = 9a = \frac{9}{81} = 0.1 < 0.5$

$P[X=0,1,2,3] = a+3a+5a+7a = 16a = \frac{16}{81} = 0.19 < 0$

$P[X=0,1,2,3,4] = a+3a+5a+7a+9a = 25a = \frac{25}{81} = 0$

$P[X=0,1,2,3,4,5] = a+3a+5a+7a+9a+11a$

$= 36a = \frac{36}{81} = 0.4 < 0.5$

$P[X=0,1,2,3,4,5,6] = a+3a+5a+7a+9a+11a+1$

$= 49a = \frac{49}{81} = 0.6 > 0.5$

$P[X\leq 6] > 0.5$; $x=6$

1.10 Q. A R.V x has the following probability distribut^n.

x :	0	1	2	3	4	5	6	7.
$P(x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

1) Find the value of k.

2) $P(x<6)$, $P(x \geq 6)$ $P(3<x\leq 6)$

3) Find the minimum value of x, so that $P[X \leq x] > \frac{1}{2}$.

ans: 1) Total probability $= 1$

ie $0+k+2k+2k+3k+k^2+2k^2+7k^2+k = 1$

$9k+10k^2 = 1$

$10k^2+9k-1=0$

$k = 1/10, -1$

$\therefore \underline{\underline{k = 1/10}}$ (+ve value)

2) $P(x<6) = P[X=0, X=1, X=2, X=3, X=4, X=5]$

$= 0+k+2k+2k+3k+k^2$

$= 8k+k^2$

$= 8\times 1/10 + (1/10)^2$

$= \frac{81}{100} = \underline{\underline{0.81}}$

$P(x\geq 6) = 1-P(x<6)$

$= 1-0.81$

$= \underline{\underline{0.19}}$

$P(3<x\leq 6) = P(X=4)+P(X=5)+P(X=6)$

$= 3k+k^2+2k^2$

$= 3k+3k^2$

$= 3(k+k^2)$

$= 3\left(1/10 + 1/10^2\right)$

$= 3\times 11/100$

$= \frac{33}{100} = \underline{\underline{0.33}}$

3) $P(X=0) = 0 < 1/2$.

$P(X=0,1) = k = 1/10 < 1/2$

$P(X=0,1,2) = k+2k = 3k = \frac{3}{10} < 1/2$

$P(X=0,1,2,3) = 3k+2k = 5k = \frac{5}{10} = \frac{1}{2} = 1/2$.

$P(X=0,1,2,3,4) = 5k+3k = 8k = \frac{8}{10} > 1/2$.

$P(X\leq 4) > 1/2$, $x=4$

5/1/16 Monday Q

UQ

Find $E(X)$

x:	0	1	2	3
$P(x)$:	0.1	0.2	0.4	0.3

ans: Mean $= E(X) = \cancel{0\times 0}\ \sum x_i p_i$

$= 0\times 0.1 + 1\times 0.2 + 2\times 0.4 + 3\times 0.3$

$= \underline{\underline{1.9}}$

Q The probability mass fn of X, the no. of mistakes per page in a book is as follows:

x:	0	1	2	3	4
$P(x)$:	0.33	0.41	0.20	0.05	0.01

Find the expected no. of mistakes per page in a book.

ans: $\sum x_i p_i = 0\times 0.33 + 1\times 0.41 + 2\times 0.20 + 3\times 0.05 + 4\times 0.01$

$= \underline{\underline{1}}$

UQ 12 marks

Q. Find the variance of $f(x) = \begin{cases} \frac{1}{16}(x+3)^2 & -3\leq x\leq \\ \frac{1}{16}(6-2x^2) & -1\leq x\leq \\ \frac{(3-x)^2}{16} & 1\leq x\leq 3 \end{cases}$

ans: $V(x) = E(x^2) - [E(x)]^2$

$E[X] = \mu = \int_{-3}^{3} x f(x)\, dx$

$= \int_{-3}^{-1} x \frac{1}{16}(x+3)^2\, dx + \int_{-1}^{1} \frac{x}{16}(6-2x^2)\, dx$

$+ \int_{1}^{3} \frac{x(3-x)^2}{16}\, dx$

$= \frac{1}{16}\left[(x)\frac{(x+3)^3}{3} - (1)\frac{(x+3)^4}{3\times 4}\right]_{-3}^{-1} +$

$\cancel{\frac{1}{16}\int_{-1}^{1}(6x - 2x^3)\, dx}^{\,0}$ odd fn $+ \frac{1}{16}\left[(x)\frac{(3-x)^3}{3\times -1}\right.$

$\left.(1)\frac{(3-x)^4}{-3\times 4\times -1}\right]_1^3$

$= \frac{1}{16}\left\{\left[(-1)\frac{2^3}{3} - \frac{2^4}{12}\right] - \left[0-0\right]\right\} +$

$$\frac{1}{16}\left\{[0-0]-\left[\frac{2^3}{-3}-\frac{2^4}{12}\right]\right\}$$

$$= \underline{\underline{0}}$$

$$E(x^2) = \int_{-3}^{3} x^2 f(x)\,dx$$

$$= \int_{-3}^{-1} \frac{x^2}{16}(x+3)^2\,dx + \int_{-1}^{1} \frac{x^2}{16}(6-2x^2)\,dx$$

$$+ \int_{1}^{3} \frac{x^2}{16}(3-x)^2\,dx$$

$$= \frac{1}{16}\int_{-3}^{-1} x^2(x+3)^2\,dx + \frac{1}{16}\int_{-1}^{1} x^2(6-2x^2)\,dx$$

$$+ \frac{1}{16}\int_{1}^{3} x^2(3-x)^2\,dx.$$

$$= \frac{1}{16}\left[(x^2)\frac{(x+3)^3}{3} - (2x)\frac{(x+3)^4}{3\times4} + \right.$$

$$\left.(2)\frac{(x+3)^5}{12\times5}\right]_{-3}^{-1} +$$

$$\frac{1}{16}\int_{-1}^{1} 6x^2 - 2x^4\,dx +$$

$$\frac{1}{16}\left[(x^2)\frac{(3-x)^3}{3\times(-1)} - (2x)\frac{(3-x)^4}{3\times4\times(-1)^2} + \right.$$

$$\left.(2)\frac{(3-x)^5}{12\times5\times-1}\right]_{1}^{3}$$

$$= \frac{1}{16}\left\{\left[(+1)\frac{2^3}{3} + 2\times\frac{2^4}{12} + (2)\times\frac{2^5}{60}\right] - \right.$$

$$\left.[9\times0 - 0 - 0]\right\} + \frac{2}{16}\int_{-1}^{1} 3x^2 - x^4\,dx.\ \text{(even.)}$$

$$+ \frac{1}{16}\left[0 - \left(\frac{2^3}{-3} - 2\frac{\times2^4}{+12} + 2\frac{\times2^5}{-60}\right)\right]$$

$$= \frac{1}{16}\left[\frac{+8}{3} + \frac{32}{12} + \frac{64}{60}\right] + \frac{2}{16}\times2\int_{0}^{1}\left[3x^2 - x^4\right]dx$$

$$+ \frac{1}{16}\left[\frac{8}{3} + \frac{32}{12} + \frac{64}{60}\right]$$

$$= \frac{1}{16}\times\frac{32}{5} + \frac{4}{16}\left[\frac{3x^3}{3} - \frac{x^5}{5}\right]_0^1 + \frac{1}{16}\times\frac{32}{5}$$

$$= \frac{2}{5} + \frac{4}{16}\left[\left(1-\frac{1}{5}\right) - 0\right] + \frac{2}{5}$$

$$= \frac{2}{5} + \frac{4}{16}\times\frac{4}{5} + \frac{2}{5}$$

$$= \underline{\underline{1}}$$

$$V(x) = E(x^2) - [E(x)]^2$$
$$= 0 - 1$$
$$= \underline{1}$$

4/2/16 Thursday

Binomial law of probability:

Consider a random experiment with following properties:

1) Total no. of trials is a finite no (say n).
2) Each trial has 2 outcomes usually called success (s) and the failure (f).
3) All trials are independent.
4) ~~The~~ Probability of a success is the same for each trial.

Let p denote the probability that an outcome is a success. ie $P(S) = p$.

then, the probability that an outcome is a failure is $P(f) = 1 - p = q$ so that $p + q = 1$.

Since all trials are independent, the probability that, there are x success's is $P(\underbrace{S, S, S \ldots S}_{x \text{ times}}) = \underbrace{p \cdot p \cdot \ldots p}_{x \text{ times}} = p^x$

The probability of $n-x$ failures is

$$P(\underbrace{f, f, \ldots}_{n-x \text{ times}}) = \underbrace{q \ldots q}_{n-x \text{ times}} = q^{n-x}.$$

Out of n outcomes, x successes can be obtained in nC_x different ways $\left[nC_x = \frac{n!}{x!(n-x)!} \right]$

$\therefore$ probability of x successes and $n-x$ failures in n trails is $\boxed{f(x) = nC_x \, p^x q^{n-x}}$ where $x = 0, 1, 2 \ldots n$. It is denoted by $b(x : n, p)$ where n and p are called parameter's of the binomial distribution.

Note:

$$(q+p)^n = nC_0 q^n p^0 + nC_1 q^{n-1} p^1 + nC_2 q^{n-2} p^2 + \ldots + nC_n q^0 p^n.$$
$$= \sum_{x=0}^{n} nC_x \, q^{n-x} p^x. = \underline{1}$$

UQ

Mean of the binomial solut^n.

If $X \sim b(x:n,p)$ then $f(x) = nC_x p^x q^{n-x}$

where $x = 0,1,2 \ldots n$.

$$\text{Mean} = E(x) = \sum x \cdot f(x)$$

$$= \sum_{x=0}^{n} x \cdot nC_x p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x \cdot \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \frac{x \cdot n!}{x(x-1)!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n(n-1)! \, p \cdot p^{x-1} q^{n-x}}{(x-1)!(n-x)!}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)! \, p^{x-1} q^{(n-1)-(x-1)}}{(x-1)![(n-1)-(x-1)]!}$$

[put $x-1 = y$, $n-1 = m$.]

$$= np \sum_{y=0}^{m} \frac{m! \, p^y q^{m-y}}{y!(m-y)!}$$

when $x=1$, $y = x-1 = 1-1 = 0$ when $x=n$, $y = n-1 = m$

[Mean = np]

$$\text{Mean} = np \sum_{y=0}^{m} mC_y p^y q^{m-y}$$

$$= np (p+q)^m$$

$$= np$$

$$\left\{ \begin{array}{l} nC_x = \dfrac{n!}{x!(n-x)!} \\ mC_y = \dfrac{m!}{y!(m-y)!} \end{array} \right.$$

5/2/16 Friday

UQ

Variance of binomial distribution: [σ^2]

$$\boxed{\sigma^2 = npq}$$

Proof:

$X \sim b(x:n,p)$ then $f(x) = nC_x p^x q^{n-x}$

where $x = 0,1,2 \ldots n$.

Variance $= \sigma^2 = E(x^2) - (E(x))^2$

Consider $x^2 = x^2 - x + x$

$= x(x-1) + x$

$E(x^2) = E(x(x-1)) + E(x)$

$$E(x(x-1)) = \sum x(x-1) f(x) \quad \left[E x^n = \sum x^n f(x)\right]$$

$$= \sum_{x=0}^{n} x(x-1) \cdot {}^nC_x \, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$\left[x! = x(x-1)(x-2)! \right.$$

$$\text{eg: } 6! = 6.5.4.3.2.1$$

$$\left. = 6.5.4! \right]$$

$$= \sum_{x=0}^{n} \cancel{x(x-1)} \frac{n!\, p^x q^{n-x}}{\cancel{x(x-1)}(x-2)!(n-x)!}$$

$$= \sum_{x=2}^{n} \frac{n(n-1)(n-2)!\, p^2 p^{x-2} q^{[(n-2)-(x-2)]}}{(x-2)!\,[(n-2)-(x-2)]!}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!\, p^{x-2} q^{(n-2)-(x-2)}}{(x-2)!\,[(n-2)-(x-2)]!}$$

put $x-2=y$

$n-2=m$

when $x=2$,

$2-2=y=0$

when $x=n$,

$n-2=y=m$

$$= n(n-1)p^2 \sum_{y=0}^{m} \frac{m!\, p^y q^{m-y}}{y!(m-y)!}$$

$$= n(n-1)p^2 \sum_{y=0}^{m} {}^mC_y \, p^y q^{n-y}$$

$$= n(n-1)p^2 (q+p)^m$$

$$= n(n-1)p^2$$

$$E(x^2) = E(x(x-1)) + E(x)$$

$$= n(n-1)p^2 + np$$

$$= n^2p^2 - np^2 + np$$

$$V(x) = E(x^2) - (E(x))^2$$

$$= n^2p^2 - np^2 - np - (np)^2$$

$$= np - np^2$$

$$= np(1-p)$$

$$= npq \qquad (\because 1-p=q)$$

Fitting of a distribution:

Fitting a probability distributⁿ means estimatin the approximate values of the unknown parameters involved in the distributⁿ and writing down the corresponding probability distribution and theoretical frequencies.

Fitting a binomial distribution:

For this we have to estimate the values of n and p. Let $x_0, x_1, x_2 \dots x_n$ be the sample values with the observed frequency $O_0, O_1, O_2 \ldots O_n$.

Let $N = O_0 + O_1 + O_2 \cdots + O_n$

Estimation of n:

$n =$ the maximum value that the random variable can take.

Estimation of p:

we know that $\bar{X} = \mu = np$

also $\bar{x} = \dfrac{\sum x_i O_i}{N}$

$\therefore \boxed{p = \dfrac{\bar{x}}{n}}$

Fitting the distribut^n:

If $X \sim b(x : n, p)$ then.

$f(x) = nC_x p^x q^{n-x}$.. $x = 0, 1, 2 \ldots n$.

The theoretical frequencies $E_0, E_1 \ldots E_n$, is given by:

$E_0 = f(0) N$

$E_1 = f(1) \cdot N$

$E_2 = f(2) \cdot N$

$\vdots$

$E_n = f(n) \cdot N$.

finally check whether $\sum O_i = \sum E_i$.

Q Fit a binomial distribution to the following data

x:	0	1	2	3	4	5
f:	38	144	342	287	164	25

ans: $X \sim b(x : n, p)$ then $f(x) = nC_x p^x q^{n-x}$

$x = 0, 1, 2 \ldots n$.

1.) Estimation of n:

$n = 5$ (maximum value of x)

2) Estimation of p:

$\bar{x} = \dfrac{\sum x_i O_i}{\sum O_i}$

$N = \sum O_i = 38 + 144 + 342 + 287 + 164 + 25$
$= \underline{1000}$

$\sum x_i O_i = 0 \times 38 + 1 \times 144 + 2 \times 342 + 3 \times 287 + 4 \times 164 + 5 \times 25$

$= 0 + 144 + 684 + 851 + 656 + 125$

$= \underline{\underline{2470}}$

$$\bar{X} = \frac{2470}{1000}$$

$= \underline{\underline{2.47}}$

$\bar{x} = np$

$2.47 = 5.p$

$$\therefore p = \frac{2.47}{5}$$

$= 0.494$

$$= \frac{247}{500}$$

$$q = 1 - p = \frac{253}{500}$$

x	$f(x) = 5C_x\,(.497)^x\,(.506)^{5-x}$	$E_i = Nf(i)$
0.	$f(0) = 0.03317$	33.17
1.	$f(1) = 0.16290$	162.9
2.	$f(2) = 0.32$	320
3.	$f(3) = 0.31431$	314.31
4.	$f(4) = 0.15436$	154.36
5.	$f(5) = 0.03032$	30.32

$\Sigma E_i = 1015.06$

15/2/16
Monday

Poisson Distributⁿ:

In a binomial distribution, if the no. of trials is extremely large and probability of success is very small, then binomial tends to poisson distribution. pdf (probability distributⁿ Fn) of poisson distribution is given by;

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2 \ldots x.$$

and it denotes $x \sim P(x:\lambda)$ where λ is the parameter.

→ Binomial approximatⁿ of a poisson distributⁿ

[poisson distributⁿ is a limiting case of binomial distribut^n]:

The binomial distribut^n tends to poisson ~~has~~ as $n \to \infty$, $p \to 0$ and $np = \lambda$.

proof:

If $X \sim b(x; n, p)$, $f(x) = {}^nC_x p^x q^{n-x}$.

$$= \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \frac{1}{x!} \frac{[\cancel{1 \cdot 2 \cdot 3 \cdot 4 \cdots (n-x-1)(n-x)}(n-x+1)\cdots(n-1)n]}{[\cancel{1 \cdot 2 \cdot 3 \cdots (n-x)}]} p^x (1-p)^{n-x}$$

$$= \frac{1}{x!} n(n-1)\cdots(n-(x-1))\, p^x (1-p)^n (1-p)^{-x}$$

$$= \frac{1}{x!} n\, n(1-1/n)\, n(1-2/n) \cdots n(1-\frac{x-1}{n})\, p^x (1-p)^n (1-p)^{-x}$$

$$= \frac{1}{x!} n\, n^{x-1} (1-1/n)(1-2/n)\cdots(1-\frac{x-1}{n})\, p^x (1-p)^n (1-p)^{-x}$$

$$= \frac{1}{x!} n^x p^x (1-1/n)(1-2/n)\cdots(1-\frac{x-1}{n})(1-p)^n (1-p)^{-x}.$$

put $n \to \infty$, & $np = \lambda$, ie $p = \lambda/n$.

$$= \frac{1}{x!} \lim_{n\to\infty} \lambda^x (1-1/n)(1-2/n)\cdots(1-\frac{x-1}{n})(1-\lambda/n)^n (1-\lambda/n)^{-x}$$

$$= \frac{\lambda^x}{x!} \lim_{n\to\infty} \left[(1-1/n)(1-2/n)\cdots(1-\frac{x-1}{n})\right] \lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n} \xrightarrow{} e^{-\lambda}$$

$$\lim_{n\to\infty} \left(1-\lambda/n\right)^{-x} \to 1$$

$$= \frac{\lambda^x}{x!} [\underbrace{1 \cdot 1 \cdots 1}_{(x-1)\text{ times}}]\, e^{-\lambda} \cdot 1$$

ie $f(x) = \dfrac{e^{-\lambda} \lambda^x}{x!}$.

Mean and variance of poisson distribut^n:

$\boxed{\text{Mean} = \lambda}$

If $x \sim p(x; \lambda)$ then $f(x) = \dfrac{e^{-\lambda} \lambda^x}{x!}$ $[x = 0, 1, \ldots$

$$\text{Mean} = \mu = E(x) = \sum x f(x)$$

$$= \sum_{x=0}^{\infty} x \cdot \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{\cancel{x} \cdot \lambda^x}{\cancel{x}(x-1)!}$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!}$$

$$= e^{-\lambda}\left[\frac{\lambda^{1}}{0!} + \frac{\lambda^{2}}{1!} + \frac{\lambda^{3}}{2!} + \cdots\right]$$

$$= e^{-\lambda}\left[\lambda \; 1 + \frac{\lambda}{1!} + \frac{\lambda^{2}}{2!} + \cdots\right]$$

$$= e^{-\lambda} \cdot \lambda \cdot e^{\lambda}$$

$$\underline{\underline{= \lambda}}$$

→ **Variance**:

$\sigma^2 \cdot V.x. = E(x^2) - (E(x))^2$

20|1|16
Wednesday

Module 1: Fourier Series

In many engineering problems it is convinient to express a function in a series of sines and cosines in the form:

$$f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x \dots + b_1 \sin x + b_2 \sin 2x + \dots$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

Euler formulae of Fourier (Euler) constants:

The fourier series for the fn $f(x)$ defined in the interval $c < x < c+2\pi$ is given by:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

where $a_0 = \frac{1}{\pi} \int_c^{c+2\pi} f(x)\,dx$

$$a_n = \frac{1}{\pi} \int_c^{c+2\pi} f(x) \cos nx \, dx.$$

$$b_n = \frac{1}{\pi} \int_c^{c+2\pi} f(x) \sin nx \, dx.$$

where $a_0, a_n, b_n;\ n = 1, 2, 3$ etc are fourier consts.

→ <u>when $c = 0$, $0 < x < 2\pi$</u> :

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx.$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx.$$

→ <u>when $c = -\pi$, $-\pi < x < \pi$</u> :

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx.$$

<u>case 1 : $f(x)$ is even $[f(-x) = f(x)]$</u>

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx. = \underline{\underline{\frac{2}{\pi}\int_0^{\pi} f(x)\,dx.}}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx \qquad \begin{bmatrix}\cos nx \text{ is even} \\ \cos(-\theta) = \cos\theta\end{bmatrix}$$

$$= \underline{\underline{\frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx.}}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx \qquad \begin{bmatrix}\sin(-\theta) = -\sin\theta \\ \therefore \sin x \text{ is odd}\end{bmatrix}$$

$$= \underline{\underline{0}}$$

then, $\underline{\underline{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx}}$

<u>case 2 : $f(x)$ is odd $[f(-x) = -f(x)]$</u>

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \underline{\underline{0}}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx \qquad \begin{bmatrix}f(x) \text{ odd} \\ \cos nx \text{ even}\end{bmatrix}$$

$$= \underline{\underline{0}}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx. \qquad \begin{bmatrix}f(x) \text{ odd} \\ \sin nx \text{ odd}\end{bmatrix}$$