Assignment 2

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1) Differential Equations

$$\dot{u} = -S_{long} - 0.5u + 10\theta$$
$$\dot{\theta} = 2S_{col} - u + \theta$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} u \\ \theta \end{bmatrix}$$
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \dot{u} \\ \dot{\theta} \end{bmatrix}$$

State variables x_1, x_2

$$x_1 = u, \quad x_2 = \theta$$

$$\dot{x}_1 = \dot{u} = -S_{long} - 0.5u + 10\theta$$
$$\dot{x}_2 = \dot{\theta} = 2S_{col} - u + \theta$$
$$\dot{x} = Ax + Bu$$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -0.5 & 10 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -S_{long} \\ 0 \end{bmatrix}$$

$$y = Cx = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

2.

$$S = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A = \begin{bmatrix} X_u & X_a & X_\theta & 0 \\ Z_u & Z_a & Z_\theta & Z_q \\ 0 & 0 & 0 & 1 \\ M_u & M_a & M_\theta & M_q \end{bmatrix}$$

$$SI - A = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix} - \begin{bmatrix} X_u & X_a & X_\theta & 0 \\ Z_u & Z_a & Z_\theta & Z_q \\ 0 & 0 & 0 & 1 \\ M_u & M_a & M_\theta & M_q \end{bmatrix}$$

$$= \begin{bmatrix} s-X_u & -X_a & -X_\theta & 0 \\ -Z_u & s-Z_a & -Z_\theta & -Z_q \\ 0 & 0 & 0 & -1 \\ -M_u & -M_a & -M_\theta & s-M_q \end{bmatrix}$$

$$[SI-A] = (s-X_u)\times \det(M_{s.X_u}) - (-X_a)\times \det(M_{X_a}) + (-X_\theta)\times \det(M_{-X_\theta})$$

$$\Big[(s-X_u)(s-Z_a)\big((s-M_q)-(M_\theta)\big) + Z_\theta(-M_a)\Big] -$$

$$(-X_a)\big(s((s-M_q)s-(-1)(-M_\theta))-(-X_\theta)(-M_\theta)\big) +$$

$$(-X_\theta)\big(s((s-M_q)s-(-1)(-M_\theta))-(-X_a)(-M_\theta)\big)$$

3) for creating a model of an aircraft, having a uniform and constant distribution of mass is needed and plays a major role but not mandatory.

Uniform distribution of weight has its own benefits

1) Ease of Modelling :- A constant mass distribution allows for easier integration in the equation.

2) Stability :- Even mass distributions helps in maintaining the stability of aircraft especially during load changes.

3) Control System :- A very more advanced and complex control system is needed in case of uneven mass distribution.

4) Real life parameters :- When the model is taken into real life the uniform mass distribution of weight will help in conceiving fuel during the flight.

5) Accuracy :- This also accounts for more accuracy of the load.

So to conclude uniform and constant mass distribution for aircraft is an obligation but not mandatory.

4) Dynamic Mode decomposition is a data driven technique used for analysing the dynamics of complex systems based on observed data. Unlike traditional modelling approaches that rely on explicit mathematical models.

This works by

1. Data driven technique: 1. DMD analyses time series data collection from the system. This data is from measurements from sensors, simulations, or any other sources.
   - DMD identifies spatial and temporal patterns within the data
2. Mode decomposition: It decomposes the data into a set of modes that represent coherent spatial and temporal patterns or structures present in data.

→ DMD is model free, captures Complex Behaviour and Reduced ~~orbitalt~~ Modelling.

5)

a	b	c	p(a,b,c)
0	0	0	0.192
0	0	1	0.144
0	1	0	0.048
0	1	1	0.216
1	0	0	0.192
1	0	1	0.064
1	1	0	0.048
1	1	1	0.096

$p(a=0) = 0.192 + 0.144 + 0.048 + 0.216$

$= 0.6$

$p(c=0) = 0.192 + 0.048 + 0.192 + 0.048$

$= 0.4$

$$p(c|a) = \frac{p(a=0, c=0)}{p(a=0)} = \frac{0.192 + 0.048}{0.6} = \frac{0.24}{0.6} = 0.4$$

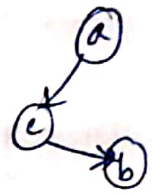
$$p(b|a) : \frac{p(b=0, c=0)}{p(c=0)} = \frac{0.192 + 0.192}{0.4} = \frac{0.384}{0.4}$$

$$p(a)\, p(c|a)\, p(b|a) = 0.6 \times \frac{0.24}{0.6} \times \frac{0.384}{0.4} = 0.192$$

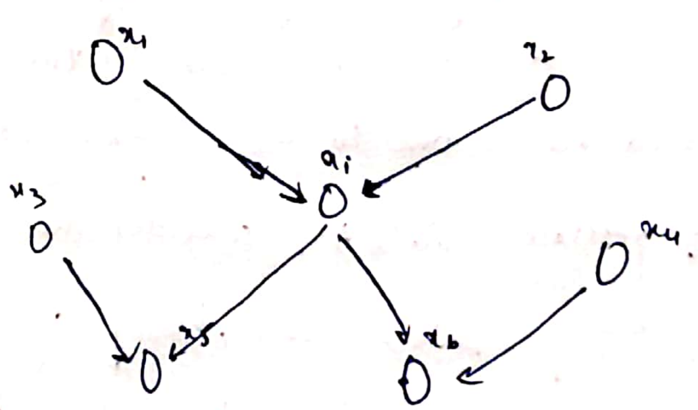
and $p(a=0, b=0, c=0) = 0.192$

$\therefore p(a,b,c) = p(a)\, p(c|a)\, p(b|c)$ Hence proved

$$p(a,b,c) = p(b,c|a)\, p(a)$$
$$= p(c|a)\, p(b|c)\, p(a)$$



6) The Markov blanket of node x_i comprises the set of parents, children, and coparents of the node. It has the property that the conditional distribution of x_i, conditioned on all the remaining variables in graph.



Markov blanket ϕ of node x_i consists of 3 types of nodes ϕ_1, ϕ_2, ϕ_3

$\phi_1 = \{x_1, x_2\}$, $\phi_2 = \{x_r, x_c\}$, $\phi_3 = \{x_3, x_4\}$

The d-seperation criterion is used to show that path from node x_i to arbitary node $\hat{x} \in \phi$ are blocked. The arrows from $\hat{x}$ can only connect to Target nod x_2 via 2 types of node ϕ_1, ϕ_2. If $\hat{x}$ connects to Target x_i via same node $x^* \in \phi_1$ then arrows meet head to tail or tail to tail at node x^* as the link from a parent the path from $\hat{x}$ to x_i gets blocked through node x^* from ϕ_3. In final case $\hat{x}$ to x_i via $x^* \in \phi_2$ this path doesnt go through any node from ϕ_3. The arrows cannot meet head to head at node v so they need to meet head to tail or tail to tail at node $x^* \in \phi_2 \subseteq \phi$.

resulting path blocked.

7) A Matrix A with size $M \times M$ can be constructed represent an undirected graph by having entries equal to 1 or 0 if there is a node i to node j. The matrix A is symmetric and has a one to one mapping. To count the number of possible matrix A, consider the criteria of each entry being either 0 or 1, being symmetric, and having all diagnol entries equal to 0. Free variables on the lower triangles of matrix are $(M-1)$ and $(M-2)$

$$(M-1) + (M-2) + \cdots + 0 = \frac{M(M-1)}{2}$$

Each value of these free variables have 2 choice 1 or 0.
Therefore total number of such matrix has $2^{\frac{M(M-1)}{2}}$

for $M = 3 \longrightarrow 8$ possible undirected graphs