Predictor-Corrector Methods

- Unless the step sizes are small, Euler's method and Runge-Kutta may not yield precise solutions.
- The Predictor-Corrector Methods iterate several times over the same interval until the solution converges to within an acceptable tolerance.
- Two parts – **Predictor part** and **corrector part**

Predictor –corrector Method –algorithm that proceeds in two steps

Step 1: Prediction step calculates the rough approximation of the dersired quantity.

Step2: The corrector step refines the initial approximation using another Means.

The predictor formula used to predict the value of y at x_{i+1} and the corrector formula is used to correct the error and to improve that value of y_{i+1}.

Milne's method :To solve $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$ and knowing 4 consecutive values $y_{n-3}, y_{n-2}, y_{n-1}$ and y_n we calculate y_{n+1} using the predictor formula

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left(2\,y'_{n-2} - y'_{n-1} + 2\,y'_n\right)$$

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left(2\,f_{n-2} - f_{n-1} + 2\,f_n\right)$$

Use this y_{n+1} in the corrector formula to get y_{n+1} after correction.

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left(y'_{n-1} + 4y'_n + y'_{n+1}\right)$$

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left(f_{n-1} + 4f_n + f_{n+1}\right)$$

That is knowing 4 consecutive values y_4, y_3, y_2 and y_1.

$$y_5^{(p)} = y_1 + \frac{4h}{3}\left(2\,f_2 - f_3 + 2\,f_4\right)$$

$$y_5^{(c)} = y_3 + \frac{h}{3}\left(f_3 + 4f_4 + f_5\right)$$

1. Using Milne's method find *y(4.4)* given $5xy' + y^2 - 2 = 0$ given y(4) = 1, y(4.1)=1.0049, y(4.2)=1.0097 and y(4.3)= 1.0143.

$$y'_2 = \frac{2 - y_2^2}{5\,x_2} = \frac{2 - (1.0049)^2}{5(4.1)} = 0.0493$$

$$y'_3 = \frac{2 - y_3^2}{5\,x_3} = \frac{2 - (1.0097)^2}{5(4.2)} = 0.0467$$

$$y'_4 = \frac{2 - y_4^2}{5\,x_4} = \frac{2 - (1.0143)^2}{5(4.3)} = 0.0452$$

By Milne's predictor formula,

$$y_{5,p} = y_1 + \frac{4h}{3}\left(2\,y'_2 - y'_3 + 2\,y'_4\right)$$

$$= 1 + \frac{4(0.1)}{3}\left(2\,(0.0493) - 0.0467 + 2\,(0.0452)\right)$$

$$= 1.01897$$

$$y_5' = \frac{2-y_5^2}{5x_5} = \frac{2-(1.01897)^2}{5(4..4)} = 0.0437$$

Using

$$y_{5,c} = y_3 + \frac{h}{3}(y_3' + 4y_4' + y_5')$$

$$= 1.0097 + \frac{0.1}{3}[0.0467 + 4(0.0452) + 0.0437]$$

$$y_{5,c} = 1.01874$$

Use this corrected $y_{5,c}$ and find $y'_{5,c}$

$$y_{5,c}' = \frac{2-y_5^2}{5x_5} = \frac{2-(1.01874)^2}{5(4.4)} = 0.043735$$

Again using the corrector formula

$$y_{5,c}^{(2)} = 1.0097 + \frac{0.1}{3}[0.0467 + 4(0.0452) + 0.043735]$$

$$y_{5,c} = 1.01874$$

Since two consecutive values of $y_{5,c}$ are equal, $y_5 = 1.01874$

2. Given $\frac{dy}{dx} = \frac{1}{2}(1+x^2)y^2$ and *y(0)=1,y(0.1)=1.06,y(0.2)=1.12, y(0.3)=1.21*, evaluate y(0.4) by Milne's Predictor corrector Method.

$x_1=0, x_2=0.1, x_3=0.3, x_4=0.4$

$y_1=1, y_2=1.06, y_3=1.12, y_4=1.21$

$$y' = f(x,y) = \frac{1}{2}(1+x^2)y^2$$

$$y_1' = \frac{1}{2}\left(1+x_1^2\right)y_1^2 = \frac{1}{2}(1+0)1^2 = \frac{1}{2}$$

$$y_2' = \frac{1}{2}\left(1+x_2^2\right)y_2^2 = \frac{1}{2}\left(1+(0.1)^2\right)(1.06)^2 = 0.5674$$

$$y_3' = \frac{1}{2}\left(1+x_3^2\right)y_3^2 = \frac{1}{2}\left(1+(0.3)^2\right)(1.12)^2 = 0.6522$$

$$y_4' = \frac{1}{2}\left(1+x_4^2\right)y_4^2 = \frac{1}{2}\left(1+(0.4)^2\right)(1.21)^2 = 0.7979$$

By Milne's method

$$y_5^{(p)} = y_1 + \frac{4h}{3}(2f_2 - f_3 + 2f_4)$$

$$= 1 + \frac{4(0.1)}{3}[2(0.5674) - 0.6522 + 2(0.7979)] = 1.2771$$

$$y_5' = \frac{1}{2}(1+x_5^2)y_5^2$$

$$= \frac{1}{2}(1+0.16)(1.2771)^2 = 0.9460$$

By corrector method,

$$y_{5,c1} = y_3 + \frac{h}{3}\left(y'_3 + 4y'_4 + y'_5\right)$$

$$= 1.12 + \frac{0.1}{3}\left[0.6522 + 4(0.7979) + 0.9460\right]$$

$$y_{5,c1} = 1.2797$$

Now use this on $y_{5,c}$ and $y'_{5,c}$,

$$y_{5,c2} = y_3 + \frac{h}{3}\left(y'_3 + 4y'_4 + y'_{5c1}\right)$$

$$= 1.12 + \frac{0.1}{3}\left[0.6522 + 4(0.7979) + 0.9498\right]$$

$$y_{5,c2} = 1.2798$$

3. Given $y' = 1 - y$, and y(0)=0, find (i) y(0.1) by Euler method Using that value obtain,

   (ii) y(0.2) by modified Euler method (iii) Obtain y(0.3) by improved Euler method and (iv) y(0.4)

   by Milne's method.

   By Euler Method,

$$y_1 = y_0 + h\, f(x_0, y_0) = 0 + (0.1)(1 - 0) = 0.1$$

By modified Euler method:

$$y_2 = y_1 + h\, f\left(x_1 + \frac{h}{2}, y_1 + \frac{h}{2} f(x_1, y_1)\right)$$

$$= 0.1 + (0.1)\left[1 - \left(0.1 + \frac{1}{2}(0.1)\, f(0.1, 0.1\right)\right]$$

$$= 0.1 + 0.1\left[1 - \left(0.1 + \frac{0.1}{2}(1 - 0.1)\right)\right]$$

$$= 0.1855$$

By improved Euler method:

$$y_3 = y_2 + \frac{1}{2}h\left[f(x_2, y_2) + f(x_3, y_2 + h\, f(x_2, y_2))\right]$$

$$= 0.1855 + \frac{0.1}{2}\left[1 - y_2 + 1 - y_2 - hf(x_2, y_2)\right]$$

$$= 0.1855 + \frac{0.1}{2}\left[2 - 2y_2 - h(1 - y_2)\right]$$

$$= 0.1855 + \frac{0.1}{2}(1 - y_2)(2 - h)$$

$$= 0.2629$$

Not knowing y_0, y_1, y_2, y_3 we will find y_4 .

By Milne's method:

$$y_{4,p} = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$$

$$= 0 + \frac{4(0.1)}{3}\left[2(1-y_1) - (1-y_2) + 2(1-y_3)\right]$$

$$= \frac{4(0.1)}{3}\left[3 - 2y_1 + y_2 - 2y_3\right]$$

$$= \frac{0.4}{3}\left[3 - 2(0.1) + 0.1855 - 2(0.2629)\right]$$

$$= 0.3280$$

$$y_4' = 1 - y_4 = 1 - 0.3280 = 0.6720$$

$$y_{4,c} = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_{4,p}'\right]$$

$$= 0.1855 + \frac{0.1}{3}\left[1 - y_2 + 4(1-y_3) + 1 - y_{4,p}\right]$$

$$= 0.1855 + \frac{0.1}{3}\left[6 - y_2 - 4y_3 - y_{4,p}\right]$$

$$= 0.3333$$