Simpson's Rule

1. Evaluate $\int_{-3}^{3} x^4 dx$ using Simpson's rule. Verify your result by actual integration.

Sol $\int_{-3}^{3} x^4 dx$ $\quad (a, b) = (-3, 3) \quad b-a = 3-(-3) = 6$

$n = \frac{b-a}{h} \quad$ let $h = 1$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6
x	-3	-2	-1	0	1	2	3
$y=x^4$	81	16	1	0	1	16	81
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Simpson's One-third rule

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[(y_0+y_n)+2(y_2+y_4+\cdots)+4(y_1+y_3+y_5+\cdots)\right]$$

$$\int_{-3}^{3} x^4\,dx = \frac{1}{3}\left[(y_0+y_6)+2(y_2+y_4)+4(y_1+y_3+y_5)\right]$$

$$= \frac{1}{3}\left[(81+81)+2(1+1)+4(16+0+16)\right]$$

$$= 98$$

Simpson's three eighth rule

$$\int_{x_0}^{x_n} y\,dx = \frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4+\cdots)+2(y_3+y_6+\cdots)\right]$$

$$\int_{-3}^{3} x^4\,dx = \frac{3}{8}\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2y_3\right]$$

$$= \frac{3}{8}\left[(81+81)+3(16+1+1+16)+2*0\right] = 99$$

$$\int_{-3}^{3} x^4\,dx = 2\int_{0}^{3} x^4\,dx = 2\left[\frac{x^5}{5}\right]_0^3 = \frac{2}{5}\left[3^5-0\right] = 97.2$$

2. Evaluate $\int_0^1 e^x\,dx$ by Simpson's one-third rule, correct to five decimal places, by proper choice of h.

Sol) $(a, b) = (0, 1) \quad b-a = 1-0 = 1$

$h = 0.25 \qquad n = \frac{b-a}{h} = \frac{1}{1/4} = 4$

	x_0	x_1	x_2	x_3	x_4
x	0	0.25	0.5	0.75	1
$y = e^x$	$e^0 = 1$	$e^{0.25}$	$e^{0.5}$	$e^{0.75}$	e
	y_0	y_1	y_2	y_3	y_4

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[(y_0 + y_4) + 2(y_2) + 4(y_1 + y_3)\right]$$

$$\int_0^1 e^x dx = \frac{0.25}{3}\left[(1+e) + 2e^{0.5} + 4(e^{0.25} + e^{0.75})\right]$$

$$= 1.718283$$

Actual Integration $\int_0^1 e^x dx = \left[e^x\right]_0^1 = e^1 - e^0 = e - 1$

$$= 1.71828183$$

3. The accelerations of a vehicle at nine timing instances from $t=0$ to $t=40$ with an interval of 5 are 40.0, 45.25, 48.5, 51.25

54·35, 59·48, 61·5, 64·3 and 68·7. Find the velocity at $t=40$ using Simpson's rule.

Sol $A = \frac{dv}{dt} \Rightarrow A\,dt = dv$ $\quad \int A\,dt = \int dv$

i.e., $\int A\,dt = v$

To find the velocity at $t=40$. i.e., $\int_0^{40} A\,dt$

Given $h=5$

t	0	5	10	15	20	25	30	35	40
A	40.0	45.25	48.5	51.25	54.35	59.48	61.5	64.3	68.7
	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8

$$n = \frac{b-a}{h} = \frac{40-0}{5} = 8$$

$$\int_0^{40} A\,dt = \frac{h}{3}\left[(A_0 + A_8) + 2(A_2 + A_4 + A_6) + 4(A_1 + A_3 + A_5 + A_7)\right]$$

$$= \frac{5}{3}\left[(40 + 68.7) + 2(48.5 + 54.35 + 61.5) + \right.$$

$4(45.25+51.25+59.48+64.3)] = 2155.14$

4. By dividing the range into ten equal parts, evaluate $\int_0^{\pi} \sin x \, dx$. by simpson's rule. Verify your answer with actual integration.

Sol $\int_0^{\pi} \sin x \, dx$ Given $n=10$ $h = \frac{b-a}{n}$

$$= \frac{\pi - 0}{10} = \frac{\pi}{10}\ ; \quad h = \frac{\pi}{10} \quad ; \ n = 10$$

Simpson's One-third rule

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x	0	$\frac{\pi}{10}$	$2\pi/10$	$3\pi/10$	$4\pi/10$	$5\pi/10$	$6\pi/10$	$7\pi/10$
$y = \sin x$	0	0.3090	0.5878	0.8090	0.9511	1	0.9511	0.8090

x_8	x_9	x_{10}
$\frac{8\pi}{10}$	$9\pi/10$	π
0.5878	0.3090	0

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[(y_0+y_{10}) + 2(y_2+y_4+y_6+y_8) + 4(y_1+y_3+y_5+y_7+y_9)\right]$$

$$\int_0^{\pi} \sin x\,dx = \frac{\pi}{30}\left[(0+0) + 2(0.5878+0.9511+0.9511+0.5878) + 4(0.3090+0.8090+1+0.8090+0.3090)\right]$$

$$= 2.00091$$

$$\int_0^{\pi} \sin x\,dx = \left[-\cos x\right]_0^{\pi} = -\left[\cos\pi - \cos 0\right]$$

$$= -\left[-1-1\right] = -(-2) = 2$$

5. Find the value of $\log_2^{1/3}$ from $\int_0^1 \frac{x^2}{1+x^3}$ using Simpson's one third rule with $h = 0.25$

Sol (a, b) = (0, 1) $h = 0.25$ $n = \frac{b-a}{h} = \frac{1}{0.25} = 4$

x	0	0.25	0.5	0.75	1
$y = \frac{x^2}{1+x^3}$	0	0.061538	0.22222	0.395604	0.5
	y_0	y_1	y_2	y_3	y_4

$$\int_0^1 \frac{x^2}{1+x^3}dx = \frac{h}{3}\left[(y_0+y_4)+2(y_2)+4(y_1+y_3)\right]$$

$$= \frac{0.25}{3}\left[(0+0.5)+2(0.222222)+4(0.061538+0.395604)\right]$$

$$= 0.2310846$$

Let $1+x^3=t$

$$\Rightarrow 3x^2dx = dt$$

$$x^2dx = \frac{dt}{3}$$

$$\int \frac{x^2dx}{1+x^3} = \int \frac{dt}{3t}$$

$$= \frac{1}{3}\int \frac{dt}{t} = \frac{1}{3}\log t$$

$$= \frac{1}{3}\log(1+x^3)$$

$$\int_0^1 \frac{x^2}{1+x^3}\,dx = \frac{1}{3}\left[(\log(1+x^3))\right]_0^1$$

$$= \frac{1}{3}\left[\log(1+1) - \log(1+0)\right]$$

$$= \frac{1}{3}\left[\log 2 - \log 1\right]$$

$$= \frac{1}{3}\left[\log 2 - 0\right] = \frac{1}{3}\log 2$$

$$= \log 2^{1/3}$$

$$\log 2^{1/3} = 0.2310846$$