Name of the Programme: M.C.A. (Online Mode)

Name of the Course: NUMERICAL &STATISTICAL ANALYSIS
Topic: Numerical Solution of ODE (Ordinary Differential Equations)

Today we are going to see how the "**ordinary differential equations of first order can be solved numerically**". What is a differential equation? Most of the mathematical models are differential equations and we find differential equations in biological sciences, Physical sciences and Engineering. First we see what is the general first order ordinary differential equation? It is $\frac{dy}{dx} = f(x,y)$ .The solution of this equation at the starting time or starting point is given as (x_0,y_0). The differential equation along with the starting point condition is called the Initial value problem.(IVP). Today we are going to solve the initial value problem numerically. There are various methods for solving such problem. The methods are (i) Euler's method (ii) Improved Euler method (iii) Modified Euler method (iv)Taylor's series method (v) R-k Method(RungeKutta Method) (vi) Picard's method (vii) Predictor- Corrector method. Out of this methods till Picard's method it is a single step method and predictor-corrector method is a multistep method. In single step method we find the approximate solution for the given differential equation say at y_1=y(x_1) given y_0=x_0 .Where as in multi-step method they will provide the values of y_n. We will find the value of y_{n+1} by predictor method. We will use this y_{n+1} value in the corrector formula and once again we refine that y_{n+1} value. But today we are going to see only Taylor's series method.

Taylor's series for first order ordinary differential equation is given by

y'(x) = f(x,y) given (x_0,y_0). Then $y(x_0+h) = y(x_0) + \frac{h}{1!}y'(x_0) + \frac{h^2}{2!}y''(x_0) + \frac{h^3}{3!}y'''(x_0) + \frac{h^4}{4!}y''''(x_0) + \cdots\cdots$

where h = x_{i+1} –x_i in general. Now here our h=x_1-x_0. For example $\frac{dy}{dx} = x + y, \quad y(1) = 1$ .Find y(1.1) In this y(1) = 1 means that x_0 value is zero and y_0 value is 1. And h = 1.1 – 1 = 0.1. And we proceed for find the y(1.1) through Taylor's series. As we increase the order of derivatives of y, the solution converges approximate with the exact solution.

Let us do problems under Taylor's series.

Using Taylor series method find *y(1.1)* given

dy/dx=x+y,y(1)=0.

Solution: Given y'=x+y

y''=1+y'

y'''=y''

y''''=y'''

So x_0 =1 and y_0 = 0

$y_0' = x_0 + y_0 = 1$

$y_0'' = 1 + y_0' = 2;$

$y_0''' = y_0'' = 2;$

$y'''' = y''' = 2$

Therefore $y(1.1) = 0 + \frac{0.1}{1!}1 + \frac{0.1^2}{2!}(2) + \frac{0.1^3}{3!}(2) + \frac{0.1^4}{4!}(2) + \cdots\cdots$

which gives the value of y(1.1).

In the same way we can find the y(1.2) by considering as the previous value and our h=0.1.Using the following formula

$$y_2 = y_1 + \frac{h}{1!}(y_1') + \frac{h^2}{2!}(y_1'') + \frac{h^3}{3!}(y_1''') + \frac{h^4}{4!}(y_1'''') + \cdots\cdots$$

2. Using Taylor's series method find correct to four decimals, the value of *y(0.1),* given

$$\frac{dy}{dx} = x^2 + y^2 \qquad y(0) = 1$$

Here also the same procedure as in the earlier problem where $y_0 = 1$.

And $x_0 = 0$.

$y_0' = x_0^2 + y_0^2 = 1$

$y_0'' = 2x_0 + 2y_0 y_0' = 2;$

$y_0''' = 2 + 2(y_0 y_0'' + y_0'^2) = 8;$

$y_0'''' = 2y_0 y_0''' + 6y_0' y_0'' = 28$

We can use these values in Taylor's series formula and we can find the value of y(0.1) =1.11145.

Taylor's series method can be used to solve second order differential equations and also for solving first order simultaneous differential equations. In our next lecture we can see either Euler's method or improved Euler's method for solving first order ordinary differential equations.