Euler's Method, Improved Euler Method, Modified Euler Method

1. **Given** $\frac{dy}{dx}=\frac{y-x}{y+x}$ **with initial condition y = 1 at x = 0; find y(0.1) , y(0.2)** ***h =0.1, y_0 = 1 and x_0=0***

by Euler's method

$$\frac{dy}{dx}=\frac{y-x}{y+x},\ y(0)=1,\ \textit{to find } y(0.1).$$

h =0.1, y_0 = 1 and x_0=0

Euler's formula :

$$y_{n+1}=y_n+hf(x_n,y_n);\quad n=0,1,2,\ldots$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$f(x_0, y_0) = \frac{y_0 - x_0}{y_0 + x_0} = \frac{1}{1} = 1$$

$$y_1 = y(0.1) = 1 + 0.1 = 1.1$$

y(0.2) =y_2 = y_1 + h f(x_1,y_1)

Euler's formula :

$$f(x_1, y_1) = \frac{y_1 - x_1}{y_1 + x_1} = \frac{1.1-0.1}{1.1+0.1} = \frac{1}{1.2} = 0.8333$$

$$y_2 = y(0.2) = 1.1 + 0.1(0.8333) = 0.9996$$

2. Solve the following by Modified Euler method $\frac{dy}{dx}=\log_{10}^{(x+y)}$; y(1) = 2 at x = 1.2 and 1.4.

y_0=2, x_0 = 1, x_1 = 1.2, x_2 = 1.4.

Modified Euler Method:

$$y_1 = y_0 + h\left(f\left(x_0 + \frac{h}{2}, y_0 + \frac{1}{2} h\, f(x_0, y_0) \right) \right)$$

$$y_0 + \frac{1}{2} h\, f(x_0, y_0) = 2 + \frac{1}{2}(0.2) f(x_0, y_0) = 2 + (0.1)\log_{10}(1+2) = 2.04771$$

$$y_1 = 2 + (0.2) f(1.1, 2.04771) = 2 + (0.2)\log_{10}(3.14771) = 2.0995$$

$$y_2 = y_1 + h\left(f\left(x_1 + \frac{h}{2}, y_1 + \frac{1}{2} h\, f(x_1, y_1) \right) \right)$$

$$y_1 + \frac{1}{2} h\, f(x_1, y_1) = 2.0995 + \frac{1}{2}(0.2) f(1.2, 2.0995) = 2.0995 + (0.1)\log_{10}(3.2995) = 2.0995 + (0.1)0.5184 = 2.0995 + 0.05185 = 2.15134$$

$$\therefore y_1 = 2.0995 \quad y_2 = 2.15134$$

3. Using improved Euler method find y at x = 0.1 and y at x = 0.2 given $\frac{dy}{dx} = y - \frac{2x}{y}$

By improved Euler method

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_n + h, y_n + h\, f(x_n, y_n)) \right]$$

$$y_1 = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_0 + h, y_0 + h\, f(x_0, y_0)) \right]$$

$$f(x_0, y_0) = y_0 - \frac{2x_0}{y_0} = 1 - 0 = 1$$

$$f(x_1, y_0 + h\, f(x_0, y_0)) = f(0.1, 1.1) = 1.1 - \frac{2 \times 0.1}{1.1} = 0.91818$$

$$y(0.1) = y_1 = 1 + \frac{0.1}{2}(1 + 0.91818) = 1.095909$$

$$y_2 = y(0.2) = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_1, y_1 + h\, f(x_1, y_1)) \right]$$

$$f(x_1, y_1) = y_1 - \frac{2x_1}{y_1} = 1.095909 - \frac{2 \times 0.1}{1.095909} = 0.913412$$

$$f(x_2, y_1 + h\, f(x_1, y_1)) = f(0.2, 1.095909 + (0.1)(0.913412)) = f(0.2, 1.18732)$$

$$= 1.18732 - \frac{2 \times 0.2}{1.18732} = 0.850427$$

$$\therefore y_2 = 1.095909 + \frac{0.1}{2}(0.913412 + 0.850427) = 1.1841009$$