1. The population of a town is as follows:

Year x:	1941	1951	1961	1971	1981	1991
Popu (lakhs) y:	20	24	29	36	46	51

Estimate the population increase during the period 1946 to 1976

Sol : Here the values of x are equally spaced. x = 1946 is nearer to the beginning value of the table. So first form the forward difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1941	20					
		4				
1951	24		1			
		5		1		
1961	29		2		0	
		7		1		-9
1971	36		3		-9	
		10		-8		
1981	46		-5			
		5				
1991	51					

Here x_0 = 1941 ; x = 1946 and h = 10 $u = (x - x_0)/h$

u = (1946 – 1941) / 10 = 0.5

Newton's forward difference formula for interpolation is

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + ...$$

In the table, the uppermost diagonal values are the forward differences of y_0.

y(1946) = 20 + (0.5 * 4) / 1! + (0.5 *(0.5 – 1) * 1) / 2! + (0.5 * (0.5 -1) * (0.5 -2) * 1)/3!
+ 0 + (0.5 * (0.5 -1) * (0.5 -2) * (0.5 – 3) * -9)/4!
= 20 + 2 – 0.125 + 0.0625 – 0.24609 = 21.69

Next, x= 1976 is nearer to the end value of the table. So use Newton's backward difference formula for interpolation

$$y(x) = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + ...$$

$v = (x - x_n) / h$. Here x_n = 1991 ; x = 1976 and hence v = (1976 – 1991) / 10 = - 1.5

In the table, the lowermost diagonal values are the backward differences of y_n.

y(1976) = 51 + (-1.5 * 5) / 1! + (-1.5 * (-1.5 + 1) * -5) / 2! +
(-1.5 * (-1.5 + 1) * (-1.5 + 2) * -8) / 3! +
(-1.5 * (-1.5 + 1) * (-1.5 + 2) * (-1.5 + 3) * -9) / 4! +
(-1.5 * (-1.5 + 1) * (-1.5 + 2) * (-1.5 + 3) * (-1.5 + 4) * -9) / 5!
= 51 – 7.5 - 1.875 – 0.5 – 0.2109375 – 0.10546875 = 40.8085938

Therefore, increase in population during the period = 40.809 – 21.69 = 19.119 lakhs

2. From the following data, find θ at x = 43 and x = 94

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

Also express θ in terms of x.

Sol : Here the values of x are equally spaced. x = 43 is nearer to the beginning value of the table. So first form the forward difference table

x	θ	Δθ	Δ^2θ	Δ^3θ	Δ^4θ	Δ^5θ
40	184					
		20				
50	204		2			
		22		0		
60	226		2		0	
		24		0		0
70	250		2		0	
		26		0		
80	276		2			
		28				
90	304					

Here x_0 = 40 ; x = 43 and h = 10 u = $(x - x_0)/h$

u = (43 – 40) / 10 = 0.3

Newton's forward difference formula for interpolation is

$$\theta(x) = \theta_0 + \frac{u}{1!}\Delta\theta_0 + \frac{u(u-1)}{2!}\Delta^2\theta_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3\theta_0 + \ldots\ldots\ldots$$

In the table, the uppermost diagonal values are the forward differences of θ_0.

θ(x = 43) = 184 + (0.3 * 20) / 1! + (0.3 * (0.3 – 1) * 2) / 2! + 0 + 0 + 0

= 184 + 6.0 – 0.21 = 189.79

Next, to find the value of θ when x = 94

x = 94 is nearer to the end value of the table. x = 94 is outside the given interval. But here we use Newton's backward difference formula for interpolation

$$\theta(x) = \theta_n + \frac{v}{1!}\nabla\theta_n + \frac{v(v+1)}{2!}\nabla^2\theta_n + \frac{v(v+1)(v+2)}{3!}\nabla^3\theta_n + \ldots\ldots\ldots$$

v = $(x - x_n)$ / h . Here x_n = 90; x = 94 and hence v = (94 – 90) / 10 = 0.4

In the table, the lowermost diagonal values are the backward differences of θ_n.

θ(x = 94) = 304 + (0.4 * 28) / 1! + (0.4 * (0.4 + 1) * 2) / 2! + 0 + 0 + 0

= 304 + 11.2 + 0.56 = 315.76

Now to find θ in terms of x. Here we use either Newton's forward or backward difference formula for interpolation. Suppose , if Newton's forward difference formula for interpolation is used, then u can be taken as (x – 40) / 10

$$\theta(x) = \theta_0 + \frac{u}{1!}\Delta\theta_0 + \frac{u(u-1)}{2!}\Delta^2\theta_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3\theta_0 + \ldots\ldots\ldots$$

= 184 + ((x – 40) * 20) / 10 + ((x – 40) * (x – 50) * 2) / (2 * 100) + 0 + 0

$= 184 + 2x – 80 + (1 / 100) * (x^2 – 90x + 2000) = 0.01x^2 + 1.1x + 124$

3. From the data given below, find the number of students whose weight is between 60 and 70

Weight in lbs	0 – 40	40 – 60	60 – 80	80 – 100	100 – 120
No. of students	250	120	100	70	50

Sol :

Weight x	No of students y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
Below 40	250				
		120			
Below 60	370		-20		
		100		-10	
Below 80	470		-30		20
		70		10	
Below 100	540		-20		
		50			
Below 120	590				

Let us calculate the number of students whose weight is less than 70. We will use forward difference formula

$u = (x – x_0) / h = (70 – 40) / 20 = 1.5$

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots\ldots\ldots\ldots$$

y(70) = 250 + (1.5 * 120) / 1! + (1.5 * 0.5 * - 20) / 2! + (1.5 * 0.5 * -0.5 * -10) / 3! + (1.5 * 0.5 * - 0.5 * -1.5 * 20) / 4!

= 250 + 180 – 7.5 + 0.625 + 0.46875 = 423.59 = 424(approximately)

Number of students whose weight is between 60 and 70 is y(70) – y(60)

= 424 – 370 = 54

4. From the following table, find the value of tan45˚15’

x°	45	46	47	48	49	50
$\tan x^\circ$	1.00000	1.03553	1.07237	1.11061	1.15037	1.19175

Sol: We use forward interpolation formula ; also $h = 1^\circ$

$u = (x – x_0) / h = (45˚15’ - 45˚) / 1˚ = 0.25$ (u is dimensionless)

x	y = tan x˚	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
45˚	1.00000	0.03553	0.00131	0.00009	0.00003	-0.00005

46°	1.03553	0.03684	0.00140	0.00012	-0.00002	
47°	1.07237	0.03824	0.00152	0.00010		
48°	1.11061	0.03976	0.00162			
49°	1.15037	0.04138				
50°	1.19175					

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots\ldots\ldots$$

= 1.00000 + (0.03553) * (.25) + (1/4) * (-3/4) * 0.00131 * (1/2!) +
(1/4) * (-3/4) * (-7/4) * 0.00009 * (1/3!) +
(1/4) * (-3/4) * (-7/4) * (-11/4) * 0.00003 * (1/4!) +
(1/4) * (-3/4) * (-7/4) * (-11/4) * (-15/4) * -0.00005 * (1/5!)
= 1.0000 + 0.0088825 – 0.0001228 + 0.0000049 + … = 1.00876

5. The following table gives the values of the probability integral $f(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-x^2}dx$ for certain values of x. Find the value of the integral when x = 0.5437

x	0.51	0.52	0.53	0.54	0.55	0.56	0.57
y = f(x)	0.5292437	0.5378987	0.5464641	0.5549392	0.5633232	0.5716157	0.5798158

Sol: Here x = 0.5437 is nearer to the middle value of the table . We take the origin x_0 = 0.54 and x = 0.5437, h = 0.01.
Hence, u = (x – x_0) / h = (0.5437 – 0.54) / 0.01 = 0.37

x	u	y	Δy	Δ^2y	Δ^4y	Δ^5y
0.51	-3	0.5292437	0.0086550	-0.0000896		
					-0.0000007	
0.52	-2	0.5378987	0.0085654	-0.0000903		0.0
					-0.0000007	
0.53	-1	0.5464641	0.0084751	-0.0000910		0.0
					-0.0000007	
0.54	0	0.5549392	0.0083841	-0.0000917		0.0000001
					-0.0000006	
0.55	1	0.5633232	0.0082924	-0.0000923		

0.56	2	0.5716157	0.0082001			
0.57	3	0.5798158				

$$y(x) = y_0 + \frac{u}{1!}\left[\frac{\Delta y_0 + \Delta y_{-1}}{2}\right] + \frac{u^2}{2!}\Delta^2 y_{-1} + \frac{u(u^2-1^2)}{3!}\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right] + \frac{u^2(u^2-1^2)}{4!}\Delta^4 y_{-2} + ...$$

y(0.5437) = 0.5549392 + 0.37 * ((0.0083841 + 0.0084751) / 2) + (0.37^2 * - 0.0000910) /2

+ ((0.37 * (0.37^2 – 1) / 6)) * (- 0.0000007)

= 0.5549392 + 0.003118952 – 0.00000623 + 0.00000004 = 0.55805196