Consider $D^2y + a_1Dy + a_2\, y = f(x)$ where $D = d/dx$

Here replace the derivatives by finite difference approximation

i.e., $Dy = y' = (1/h)\,(y_{k+1} - y_k)$ and $D^2y = y'' = (1/h^2)\,(y_{k-1} - 2y_k + y_{k+1})$

1.Solve the differential equation $D^2y - y = x$ with $y(0) = 0$; $y(1) = 0$ with $h = 1/4$

Sol: Given $D^2y - y = x$. Let $x_0 = 0$; $y_0 = 0$. Given $h = 1/4$

$x_1 = x_0 + h = 1/4$; $x_2 = x_1 + h = 2/4$; $x_3 = x_2 + h = 3/4$ and $x_4 = x_3 + h = 1$

Replace the derivatives by finite difference approximation

$16\,(y_{k-1} - 2y_k + y_{k+1}) - y_k = x_k$

$16y_{k-1} - 33y_k + 16y_{k+1} = x_k$

Now put k = 1, 2 and 3 in the above equation, we get

$k = 1$; $16y_0 - 33y_1 + 16y_2 = x_1$

i.e., $0 - 33y_1 + 16y_2 = 1/4$ ________________________________(I)

$k = 2$; $16y_1 - 33y_2 + 16y_3 = x_2$

1.e., $16y_1 - 33y_2 + 16y_3 = 1/2$ _______________________________(II)

$k = 3$; $16y_2 - 33y_3 + 16y_4 = x_3$

i.e., $16y_2 - 33y_3 = 3/4$ (since $y_4 = 0$ given) ____________________(III)

Solving the equations (I), (II) and (III) , using Gauss seidel method, we get $y_1 = -\,0.03488$; $y_2 = -\,0.05632$ and $y_3 = -\,0.05003$.

2.Solve the boundary value problem at $x = 0.5$, $y'' + y + 1 = 0$; $y(0) = y(1) = 0$ with

$h = 1/4$

Sol: Given $D^2y + y + 1 = 0$. Let $x_0 = 0$; $y_0 = 0$. Given $h = 1/4$

$x_1 = x_0 + h = 1/4$; $x_2 = x_1 + h = 2/4$; $x_3 = x_2 + h = 3/4$ and $x_4 = x_3 + h = 1$

Replace the derivatives by finite difference approximation

$16\,(y_{k-1} - 2y_k + y_{k+1}) + y_k + 1 = 0$

$16y_{k-1} - 31y_k + 16y_{k+1} + 1 = 0$

Now put k = 1, 2 and 3 in the above equation, we get

When $k = 1$; $16y_0 - 31y_1 + 16y_2 + 1 = 0$

i.e., $-31y_1 + 16y_2 = -1$ (since $y_0 = 0$ given) ______________(I)

when $k = 2$, $16y_1 - 31y_2 + 16y_3 + 1 = 0$

i.e., $16y_1 - 31y_2 + 16y_3 = -1$ __________________________(II)

when $k= 3$; $16y_2 - 31y_3 + 16y_4 + 1 = 0$

i.e., $16y_2 - 31y_3 = -1$ __________________________(III)

From (I) , $y_1 = (1 + 16y_2) / 31$

From (II), $y_3 = (1 + 16y_2) / 31$

Sub the values of y_1 and y_3 in (II) , we get $y_2 = 0.1403$; i.e., $y(0.5) = 0.1403$

3.Solve $y'' = y$ with $y(0) = 0$ and $y(2) = 3.627$

Sol: Given $D^2y - y = 0$: Let $x_0 = 0$; $y_0 = 0$. Here h value not given. Let us take $h = 1$. Therefore $x_1 = x_0 + h = 1$ and $x_2 = x_1 + h = 2$. Given $y_2 = 3.627$

Replace the derivatives by finite difference approximation

$y_{k+1} - 2y_k + y_{k-1} - y_k = 0$

$y_{k-1} - 3y_k + y_{k+1} = 0$

Put $k = 1$ in the above equation

When $k = 1$; $y_0 - 3y_1 + y_2 = 0$

i.e., $-3y_1 + 3.627 = 0$. This implies $y_1 = 1.209$