1. A curve passes through the points (1, 2), (1.5, 2.4), (2.0, 2.7), (2.5, 2.8), (3, 3), (3.5, 2.6) and (4.0, 2.1). Obtain the area bounded by the curve, the x axis and $x=1$ and $x=4$. Also find the volume of solid of revolution got by revolving this area about the x axis.

Sol $\int_a^b y\,dx$ - Area To find $\int_1^4 y\,dx$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6
x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

$h = 0.5$

$$n = \frac{b-a}{h} = \frac{4-1}{0.5} = \frac{3}{\left(\frac{1}{2}\right)} = 6$$

One third

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[(y_0+y_n)+2(y_2+y_4+\cdots)+4(y_1+y_3+\cdots)\right]$$

$$\int_1^4 y\,dx = \frac{0.5}{3}\left[(y_0+y_6)+2(y_2+y_4)+4(y_1+y_3+y_5)\right]$$

$$= \frac{0.5}{3}\left[(2+2.1)+2(2.7+3)+4(2.4+2.8+2.6)\right]$$

$$= 7.7833 \text{ sq. units.}$$

$$V = \pi \int_a^b y^2 dx$$

$$= \pi \int_1^4 y^2 dx$$

$$\int_1^4 y^2 dx = \frac{h}{3}\left[(y_0^2 + y_6^2) + 2(y_2^2 + y_4^2) + 4(y_1^2 + y_3^2 + y_5^2)\right]$$

$$= \frac{0.5}{3}\left[\underline{\underline{2^2 + 2.1^2}} + 2\underline{\underline{(2.7^2 + 3^2)}} + 4(2.4^2 + 2.8^2 + 2.6^2)\right]$$

$$= \frac{1}{6}\left[8.41 + 32.58 + 81.44\right] = 20.405$$

Volume $V = \pi \int_1^4 y^2 \, dx = \pi (20.405) = 64.13$ cub. units

2. The velocity v of a particle at a distance s from a point on its path is given by the table below:

s in metre	0	10	20	30	40	50	60
v m/sec	47	58	64	65	61	52	38

Estimate the time taken to travel 60 metres by using Simpson's one third rule. Compare your answer with Simpson's three eighth rule.

sol $h = 10$ $\quad n = \frac{b-a}{h} = \frac{60-0}{10} = 6$

Simpson's one third

WKT $V = \frac{ds}{dt} \Rightarrow \frac{1}{V} = \frac{dt}{ds} \Rightarrow \frac{1}{V} ds = dt$

$\int \frac{1}{V} ds = \int dt$ $\quad$ i.e., $t = \int \frac{1}{V} ds$

To find $\int_{0}^{60} \frac{1}{v}\, ds$

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[(y_0+y_n)+2(y_2+y_4+\cdots)+4(y_1+y_3+\cdots)\right]$$

$$= \frac{h}{3}\left[\left(\frac{1}{47}+\frac{1}{38}\right)+2\left(\frac{1}{64}+\frac{1}{61}\right)+4\left(\frac{1}{58}+\frac{1}{65}+\frac{1}{52}\right)\right]$$

$$= \frac{10}{3}\left[\frac{1}{47}+\frac{1}{38}+2\left(\frac{1}{64}+\frac{1}{61}\right)+4\left(\frac{1}{58}+\frac{1}{65}+\frac{1}{52}\right)\right]$$

$$= 1.0635166$$

$$\int_{x_0}^{x_n} y\,dx = \frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4+\cdots)+2(y_3+y_6+\cdots)\right]$$

$$\int_0^{60} \frac{1}{v}\,ds = \frac{30}{8}\left[\left(\frac{1}{47}+\frac{1}{38}\right)+3\left(\frac{1}{58}+\frac{1}{64}+\frac{1}{61}+\frac{1}{52}\right)+2\left(\frac{1}{65}\right)\right]$$

$$= 1.0640655$$

3. A river is 80 metres wide. The depth 'd' in metres at a distance x from one bank

is given by the following table. Calculate the area of cross-section of the river using Simpson's rule.

x	0	10	20	30	40	50	60	70	80
$d=y$	0	4	7	9	12	15	14	8	3
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

Sol: $\text{Area} = \int_a^b y\,dx = \int_0^{80} y\,dx$

$$h = 10 \quad ; \quad n = \frac{b-a}{h} = \frac{80-0}{10} = 8$$

$$\int_0^{40} y\,dx = \frac{h}{3}\left[(y_0+y_8) + 2(y_2+y_4+y_6) + 4(y_1+y_3+y_5+y_7)\right]$$

$$= \frac{10}{3}\left[(0+3) + 2(7+12+14) + 4(4+9+15+8)\right]$$

$$= 710 \text{ sq. units.}$$

4. The table below gives the velocity 'v' of a moving particle at time 't' sec. Find the distance covered by the particle in 12 sec. and also the acceleration at $t=2$ sec.

t	0	2	4	6	8	10	12
v	4	6	16	34	60	94	136

Sol WKT $v = \frac{ds}{dt} \Rightarrow v\,dt = ds \Rightarrow \int v\,dt = \int ds$

i.e., $S = \int v\,dt$

To find $S = \int_0^{12} v\,dt$

$h = 2$

$\frac{b-a}{h} = \frac{12-0}{2} = 6 = n$

$$\int_0^{12} v\,dt = \frac{h}{3}\left[(v_0+v_6)+2(v_2+v_4)+4(v_1+v_3+v_5)\right]$$

$$= \frac{2}{3}\left[(4+136)+2(16+60)+4(6+34+94)\right]$$

$$= 552\ m$$

$$A = \frac{dv}{dt}$$

	t	v	Δv	$\Delta^2 v$	$\Delta^3 v$
t_0	0	v_0 4	Δv_0 2	$\Delta^2 v_0$ 8	0
	2	6	10	8	0
	4	16	18	8	0
	6	34	26	8	0
	8	60	34	8	
	10	94	42		
	12	136			

Newton's forward difference formula

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{2u-1}{2!}\Delta^2 y_0 + \cdots\right] \text{ where}$$

$$u = \frac{x - x_0}{h}.$$

$$\frac{dv}{dt} = \frac{1}{h}\left[\Delta v_0 + \frac{2u-1}{2!}\Delta^2 v_0 + \cdots\right]$$

$$u = \frac{t-t_0}{h} = \frac{2-0}{2} = 1$$

$$= \frac{1}{2}\left[2 + \frac{1}{2}\cdot 8\right]$$

$$= \frac{1}{2}\left[2+4\right] = 6/2 = 3\ m/s^2$$

5. When a train is moving at 30m/sec. Steam is shut off and brakes applied. The speed of the train per second after t sec is given by

Time(t)	0	5	10	15	20	25	30	35	40
Speed(v)	30	24	19.5	16	13.6	11.7	10.0	8.5	7.0
	v_0	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8

Determine the distance moved by the

train in 40 sec.

Sol $V = \frac{ds}{dt} \Rightarrow v\,dt = ds \Rightarrow \int v\,dt = \int ds$ i.e., $S = \int v\,dt$

To find $S = \int_0^{40} v\,dt$

$h = 5$

$n = \frac{b-a}{h} = \frac{40-0}{5} = 8$

Trapezoidal rule

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+\cdots+y_{n-1})\right]$$

$$\int_0^{40} v\,dt = \frac{h}{2}\left[(v_0+v_8) + 2(v_1+v_2+v_3+\cdots+v_7)\right]$$

$$= \frac{5}{2}\left[(30+7)+2(24+19.5+16+13.6+11.7+10.0+8.5)\right]$$

$$= 609$$

$$\int_0^{40} v\,dt = \frac{h}{3}\left[(v_0+v_8)+2(v_2+v_4+v_6)+4(v_1+v_3+v_5+v_7)\right]$$

$$= \frac{5}{3}\left[(30+7)+2(19.5+13.6+10)+4(24+16+11.7+8.5)\right]$$

$$= 606.66$$