To Solve second order ordinary differential Equation by R-k-Method

1. Find $y(0.1)$ given $y'' + 2xy' - 4y = 0$, $y(0) = 0.2$ and $y'(0) = 0.5$

4th order R-k Method algorithm

$$\frac{d^2y}{dx^2} = f(x, y, z)$$

$$\frac{dy}{dx} = z = f_1(x, y, z) \quad \text{and} \quad \frac{d^2y}{dx^2} = \frac{dz}{dx} = f_2(x, y, z)$$

$$x_0, y_0, y_0' \text{ given}.$$

$k_1 = h f_1(x, y, z)$

$l_1 = h f_2(x, y, z)$

$k_2 = h f_1\left(x + \frac{h}{2}, y + \frac{k_1}{2}, z + \frac{l_1}{2}\right)$

$l_2 = h f_2\left(x + \frac{h}{2}, y + \frac{k_1}{2}, z + \frac{l_1}{2}\right)$

$k_3 = h f_1\left(x + \frac{h}{2}, y + \frac{k_2}{2}, z + \frac{l_2}{2}\right)$

$l_3 = h f_2\left(x + \frac{h}{2}, y + \frac{k_2}{2}, z + \frac{l_2}{2}\right)$

$k_4 = h f_1(x + h, y + y + k_3, z + l_3)$

$l_4 = h f_2(x + h, y + k_3, z + l_3)$

$$\Delta y = \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4 \right)$$

$$y_1 = y_0 + \Delta y .$$

$$y'' + 2xy' - 4y = 0 \quad \text{—①} \qquad \text{initial values } y(0) = 0.2, \; y'(0) = 0.5$$

$$y_0 = y(0) = y(x=0) = 0.2 \qquad y'(0) = 0.5 = y'(x=0) \qquad y(0.1) = ?$$

$$x_0 = 0 \qquad\qquad z_0 = 0.5 \qquad\qquad x_1 = 0.1 \qquad x_0 = 0$$

$$h = 0.1 - 0 = 0.1$$

Consider $\dfrac{dy}{dx} = z = y'.$

$$\therefore y' = z = f_1(x, y, z)$$

$$f_1(x_0, y_0, z_0) = f_1(0, 0.2, 0.5)$$

$$k_1 = 0.1 \, f_1(x_0, y_0, z_0)$$

$$= 0.1 \, f_1(0, 0.2, 0.5)$$

$$= 0.1 \times z_0 = 0.1 \,(0.5)$$

$$= 0.05$$

Rewriting ① $\quad z' = 4y - 2xz$

$$y'' = z' = f_2(x, y, z)$$

$$f_2(x_0, y_0, z_0)^2 \quad f_2(0, 0.2, 0.5)$$

$$\ell_1 = 0.1 \, f_2(x_0, y_0, z_0)$$

$$= 0.1 \, f_2(0, 0.2, 0.5)$$

$$= 0.1 \,(4y_0 - 2x_0 z_0)$$

$$= 0.1 \,(4 \times 0.2 - 2(0)(0.5)) = 0.08$$

$$K_2 = h \, f_1 \left(x_0 + h_{/2}, \, y_0 + ky_{/2}, \, z_0 + ly_{/2}\right)$$

$$= 0.1 \, f_1 \left(0.05, \, 0.2 + 0.025, \, 0.5 + 0.04\right)$$

$$= 0.1 \, f_1 \left(0.05, \, 0.225, \, 0.54\right)$$

$$= 0.1 \times 0.54 = 0.054$$

$$K_3 = h \, f_1 \left(x_0 + h_{/2}, \, y_0 + \frac{K_2}{2}, \, z_0 + \frac{l_2}{2}\right)$$

$$= 0.1 \, f_1 \left(0.05, \, 0.2 + 0.027, \, 0.5 + 0.04\cancel{43}\right)$$

$$= 0.1 \, f_1 \left(0.05, \, 0.227, \, 0.54473\right)$$

$$= 0.1 \left(0.54473\right) = 0.054473$$

$$K_4 = h \, f_1 \left(x_0 + h, \, y_0 + k_3, \, z_0 + l_3\right)$$

$$= 0.1 \, f_1 \left(0.1, \, 0.254473, \, 0.58535\right)$$

$$= 0.1 \times 0.58535 = 0.058535$$

$$y_1 = y_0 + \Delta y.$$

$$l_2 = 0.1 \, f_2 \left(x_0 + h_{/2}, \, y_0 + ky_{/2}, \, z_0 + ly_{/2}\right)$$

$$= 0.1 \, f_2 \left(0.05, \, 0.225, \, 0.54\right)$$

$$= 0.1 \left(4 \times 0.225 - 2 \cdot (0.05)(0.54)\right)$$

$$= 0.1 \left(0.8946\right) = 0.08946$$

$$l_3 = 0.1 \, f_2 \left(0.05, \, 0.227, \, 0.54473\right)$$

$$= 0.1 \left(4 \times 0.227 - 2 \times 0.05 \times 0.54473\right)$$

$$= 0.08535$$

$$l_4 = 0.1 \, f_2 \left(0.1, \, 0.254473, \, 0.58535\right)$$

$$= 0.1 \left(4 \times 0.254473 - 2 \times 0.1 \times 0.58535\right)$$

$$= (0.1)\left(0.9005\right) = 0.09005$$

$$\Delta y = \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

$$= \frac{1}{6} (0.05 + 2(0.054) + 2(0.054473) + 0.058535)$$

$$= 0.05542$$

$$y_1 = y(0.1) = 0.2 + 0.05542$$

$$= 0.25542$$

2) Find $y(0.1)$ given $y'' = y^3$, $y(0) = 10$ and $y'(0) = 5$ by R.K. method.

$$\frac{d^2y}{dx^2} = y^3 \qquad y(x=0) = y(x_0) = y_0 = 10 \qquad x_0 = 0 \qquad x_1 = 0.1$$

$$y'(x=0) = y'(x_0) = y'_0 = 5 \qquad y(x=0.1) = y_1 = y(0.1)$$

$$h = x_1 - x_0 = 0.1$$

Consider $\dfrac{dy}{dx} = z$. & $\dfrac{d^2y}{dx^2} = z' = y^3$

$$\frac{dy}{dx} = z = f_1(x, y, z) \qquad \& \qquad \frac{d^2y}{dx^2} = z' = y^3 = f_2(x, y, z)$$

$$k_1 = h\, f_1(x_0, y_0, z_0) = 0.1\, f_1(0, 10, 5)$$
$$= h \times z_0 = 0.1 \times 5 = 0.5$$

$$k_2 = h\, f_1\left(x_0 + \tfrac{h}{2},\ y_0 + \tfrac{k_1}{2},\ z_0 + \tfrac{l_1}{2}\right)$$
$$= 0.1\, f_1(0.05,\ 10 + 0.25,\ 5 + 50)$$
$$= 0.1\, f_1(0.05,\ 10.25,\ 55)$$
$$= 0.1 \times 55 = 5.5$$

$$l_1 = 0.1\, f_2(0, 10, 5) = 0.1 \times y_0^3 = 0.1 \times 10^3$$
$$= 100$$

$$l_2 = 0.1\, f_2(0.05,\ 10.25,\ 55) = 10.25^3 \times 0.1$$
$$= 107.689$$

$$=$$

$$\frac{107.689}{2}$$
$$= 53.8445$$

$\tfrac{0.5}{2} = 0.25$

$$k_3 = h\, f_1\left(x_0 + h_{/2},\ y_0 + \frac{k_2}{2},\ z_0 + \frac{l_2}{2}\right)$$

$$= 0.1\, f_1\left(0.05,\ 10 + 2.75,\ 5 + 53.8445\right)$$

$$= 0.1\, f_1\left(0.05,\ 12.75,\ 58.8445\right)$$

$$= 0.1 \times 58.8445 = 5.88445$$

$$k_4 = h\, f_1\left(x_0 + h,\ y_0 + k_3,\ z_0 + l_3\right)$$

$$= 0.1\, f_1\left(0.1,\ 10 + 5.88445,\ 5 + 207.267\right)$$

$$= 0.1\, f_1\left(0.1,\ 15.88445,\ 212.267\right)$$

$$= 0.1\left(212.267\right) = 21.2267$$

$$y_1 = y(0.1) = y_0 + \Delta y$$

$$= 10 + 7.4148 = 17.4148$$

$$l_3 = 0.1\, f_2\left(0.05,\ 12.75, \underbrace{58.8445)\cdot 5}_{\overline{2} = 2.75}\right)$$

$$= 0.1\left(12.75^3\right)$$

$$= 207.267$$

$$l_4 = 0.1\, f_2\left(0.1,\ 15.8845,\ 212.267\right)$$

$$= 0.1 \times 15.8845^3$$

$$- .$$

$$\Delta y = \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right) = \frac{1}{6}\left(0.5 + 2(5.5) + 2(5.88445) \right.$$
$$\left. + 21.2267\right)$$

$$= 7.4148$$

3) Find $x(0.1)$ given $\dfrac{d^2x}{dt^2} = t\dfrac{dx}{dt} - 4x$, $x(0) = 3,\ x'(0) = 0$

$x(0) = x(t=0) = x_0 = 3$

$x'(0) = x'(t=0) = x_0' = 0 = z_0$

$t_0 = 0 \qquad t_1 = 0.1$

$h = 0.1$

Consider $\dfrac{dx}{dt} = z = f_1(t, x, z)$ $\dfrac{d^2x}{dt^2} = t\dfrac{dx}{dt} - 4x$

$= tz - 4x = f_2(t, x, z)$

$k_1 = 0.1\, f_1(t_0, x_0, z_0) = 0.1\, f_1(0, 3, 0)$

$= 0.1 \times 0 = 0$

$l_1 = 0.1\, f_2(0, 3, 0)$

$= 0.1(0 \times 0 - 4 \times 3) = 0.1 \times (-12) = -1.2$

$k_2 = 0.1\, f_1\left(t_0 + \tfrac{h}{2},\ x_0 + \tfrac{k_1}{2},\ z_0 + \tfrac{l_1}{2}\right)$

$= 0.1\, f_1(0.05,\ 3 + 0,\ 0 - 0.6)$

$= 0.1\, f_1(0.05,\ 3,\ -0.6)$

$= 0.1(-0.6) = -0.06$

$l_2 = 0.1\, f_2(0.05,\ 3,\ -0.6)$

$= 0.1(-0.05 \times 0.6 - 4 \times 3) = -1.203$

$$k_3 = h\, f_1\left(t_0 + \tfrac{h}{2},\; x_0 + \tfrac{k_2}{2},\; z_0 + \tfrac{l_2}{2}\right)$$

$$= 0.1\, f_1\left(0.05,\; 3 + (-0.03),\; -0.6015\right)$$

$$= 0.1\, f_1\left(0.05,\; 2.97,\; -0.6015\right)$$

$$= -0.06015$$

$$k_4 = 0.1\, f_1\left(t_0 + h,\; x_0 + k_3,\; z_0 + l_3\right)$$

$$= 0.1\, f_1\left(0.1,\; 3 - 0.06015,\; -1.1910\right)$$

$$= 0.1\, f_1\left(0.1,\; 2.94,\; -1.1910\right)$$

$$= 0.1\,(-1.1910) = -0.1910$$

$$l_3 = 0.1\, f_2\left(0.05,\; 2.97,\; -0.6015\right)$$

$$= 0.1\left(0.05 \times (-0.6015) - 4 \times 2.97\right)$$

$$= -1.1910$$

$$l_4 = 0.1\, f_2\left(0.1,\; 2.94,\; -1.1910\right)$$

$$= \quad -$$

3.0000
.0599
$\overline{2.9401}$

3.0000
.6015
$\overline{3985}$

$$\Delta x = \tfrac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right) = \tfrac{1}{6}\left(0 + 2(-0.06) + 2(-0.06015) + (-0.1910)\right)$$

$$= -0.0599$$

$$x(0.1) = x(0) + \Delta x = 3 - 0.0599$$

$$= 2.9401\;//$$