Spearman Rank Correlation

Rank correlation coefficient is applied to a set of ordinal rank numbers (i.e., 1,2,3,…)

Spearman's Rank correlation coefficient is defined as $\rho = 1 - \frac{6\sum d^2}{N\left(N^2 - 1\right)}$ or

$$\rho = 1 - \frac{6\sum d^2}{\left(N^3 - N\right)}$$

where ρ denotes rank coefficient of correlation and d refers to the ranks between paired items in series. ρ value lies between + 1 and – 1 .

To find rank correlation we have two types of problems

(i) Where actual ranks are given.
(ii) Where ranks are not given.

(i) When ranks are given.
   (a) Find the difference between the two sets of ranks R1 and R2.
   i.e., (R1 – R2) is determined and these differences are denoted by d.
   (b) Square these differences and obtain the total $\sum d^2$.
   (c) Formula for Rank correlation:

$$\rho = 1 - \frac{6\sum d^2}{N\left(N^2 - 1\right)} \quad \text{or} \quad \rho = 1 - \frac{6\sum d^2}{\left(N^3 - N\right)}$$

Problem 1: Two Principals from different schools are asked to rank a group of Post graduate Trained teachers in a school in order of their experience in their field. The ranking of the teachers are as follows:

Post Graduate Trained Teachers	Principal 1	Principal 2
A	10	9
B	2	4
C	1	2
D	4	3
E	3	1
F	6	5
G	5	6
H	8	8
I	7	7
J	9	10

Compare the coefficient of rank correlation and comment on the value.

Post graduate Trained Teachers	Principal 1 R1	Principal 2 R2	d =R1 – R2	d^2
A	10	9	1	1
B	2	4	-2	4
C	1	2	-1	1
D	4	3	1	1
E	3	1	2	4
F	6	5	1	1
G	5	6	-1	1
H	8	8	0	0
I	7	7	0	0
J	9	10	-1	1
				$\Sigma d^2 = 14$

$$\rho = 1 - \frac{6\sum d^2}{N\left(N^2 - 1\right)}$$

$$\rho = 1 - \frac{6 \times 14}{10\left(10^2 - 1\right)} = 1 - \frac{84}{990} = 0.915$$

(ii) When the ranks are not given:

Assign the ranks either by taking the highest value as 1 or the lowest value as 1. Whatever we follow for the first set of variables, the same procedure should be followed for the second set of variables. Then proceed in the way when ranks are given.

Problem 2: Ten candidates obtained the following marks in the two examinations, computer organization and DBMS. Find the rank correlation coefficient to determine whether these results support the suggestion that ability in one subject is associated with ability in the other.

Candidate	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
Computer organization	40	65	61	49	53	42	68	57	58	46
DBMS	51	58	67	55	76	45	69	56	73	63

Computer organization ranks are grouped under R1 and DBMS ranks are assigned under R2.

Candidate	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	
Computer Organization R1	10	2	3	7	6	9	1	5	4	8	
DBMS R2	9	6	4	8	1	10	3	7	2	5	
R1 – R2	1	-4	-1	-1	5	-1	-2	-2	2	3	
$(R1 - R2)^2$	1	16	1	1	25	1	4	4	4	9	66

$$\rho = 1 - \frac{6\sum d^2}{N\left(N^2 - 1\right)} \qquad \rho = 1 - \frac{6 \times 66}{10\left(10^2 - 1\right)} = 0.6$$

Equal Ranks or Tie in Ranks:

When equal ranks are assigned, an adjustment is made in the rank correlation formula. The adjustment is $\frac{1}{12}\left(t^3 - t\right)$ which is added to the value of $\sum d^2$. where t stands for the number of items whose ranks are common. When more than one such group of items with common rank this is added as many of times as the number of such groups. Therefore the formula becomes

$$\rho = 1 - \frac{6\left[\sum d^2 + \frac{\left(t_1^3 - t_1\right)}{12} + \frac{\left(t_2^3 - t_2\right)}{12} + \ldots\ldots\right]}{N^3 - N}$$

Problem 3: Sriram Finance company conducts an aptitude test and reasoning test to eight applicants for the clerical post. Marks obtained by the applicants are tabulated . Compute the coefficient of rank correlation.

Problem 4: The competitors in a photography contest are ranked by three judges in the following order:

Judge A	1	6	5	10	3	2	4	9	7	8
Judge B	3	5	8	4	7	10	2	1	6	9
Judge C	6	4	9	8	1	2	3	10	5	7

Use the rank correlation coefficient to determine which pair of judges has the nearest approach to common tastes in photography.

Let us take ranks given by Judge A as R1 and ranks given by Judge B as R2 and that of Judge C as R3. To find which pair of judges has the nearest approach, Let us find the pair of differences d1 = R1 – R2, d2= R2 – R3, and d3 = R1 – R3.

Rank A R1	Rank **B** R2	Rank C R3	R1 – R2	R2 – R3	R1 – R3	$d1^2 = (R1 – R2)^2$	$d2^2 = (R2 – R3)^2$	$d3^2 = (R1 – R3)^2$
1	3	6	-2	-3	-5	4	9	25
6	5	4	1	1	2	1	1	4
5	8	9	-3	-1	-4	9	1	16
10	4	8	6	-4	2	36	16	4
3	7	1	-4	6	2	16	36	4
2	10	2	-8	8	0	64	64	0
4	2	3	2	-1	1	4	1	1
9	1	10	8	-9	-1	64	81	1
7	6	5	1	1	2	1	1	4
8	9	7	-1	2	1	1	4	1
						200	214	60

$$\rho = 1 - \frac{6\sum d^2}{N\left(N^2 - 1\right)}$$

Rank correlation for Judges A &B

$$\rho_1 = 1 - \frac{6\sum d_1{}^2}{N\left(N^2 - 1\right)} = 1 - \frac{6\times 200}{10\left(10^2 - 1\right)} = -0.212$$

Rank correlation for Judges B&C

$$\rho_2 = 1 - \frac{6\sum d_2{}^2}{N\left(N^2 - 1\right)} = 1 - \frac{6\times 214}{10\left(10^2 - 1\right)} = -0.297$$

Rank correlation for Judges A &C

$$\rho_3 = 1 - \frac{6\sum d_3{}^2}{N\left(N^2 - 1\right)} = 1 - \frac{6\times 60}{10\left(10^2 - 1\right)} = 0.636$$

Since the coefficient of rank correlation is maximum in judgement of A and C , we conclude that the judges A and C have the nearest approach to common taste in photography.