



COMPUTER ORGANIZATION AND ARCHITETCURE

Dr. K. Geetha

Senior Assistant Professor, CSE, SOC

Outline

- Number systems
- Number systems conversion
- Representing numbers
 - Unsigned magnitude
 - Signed magnitude
 - 1's complement
 - 2's complement

Number System

A number system of **base**, or **radix**, **r** is a system that uses **r** distinct symbols .

Numbers are represented by a **string of digit** .

A number **N** in base or radix **b** can be written as: **N = I . F**

$$(N)_b = d_{n-1} d_{n-2} \text{ — — — — } d_1 d_0 . d_{-1} d_{-2} \text{ — — — — } d_{-m}$$

In the above, **d_{n-1} to d_0 is integer** part referred as **I** , then follows a radix point, and then **d_{-1} to d_{-m} is fractional** part referred as **F** .

d_{n-1} = Most significant bit (MSB) , **d_{-m} = Least significant bit (LSB)**

Number Systems

□ Decimal number system $r = 10$

■ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

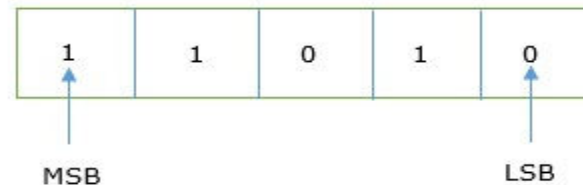
| | | | | | |
|--------|--------|--------|--------|--------|--------|
| 10^5 | 10^4 | 10^3 | 10^2 | 10^1 | 10^0 |
|--------|--------|--------|--------|--------|--------|

□ Binary number system $r = 2$

■ 0, 1

| | | | | | |
|-------|-------|-------|-------|-------|-------|
| 2^5 | 2^4 | 2^3 | 2^2 | 2^1 | 2^0 |
|-------|-------|-------|-------|-------|-------|

Each binary digit is also called a **bit**. Rightmost digit is **least significant bit (LSB)** leftmost digit is called **most significant bit (MSB)**.



Number Systems contd...

□ Octal number system $r = 8$

■ 0, 1, 2, 3, 4, 5, 6, 7

| | | | | | |
|-------|-------|-------|-------|-------|-------|
| 8^5 | 8^4 | 8^3 | 8^2 | 8^1 | 8^0 |
|-------|-------|-------|-------|-------|-------|

□ Hexadecimal number system $r = 16$

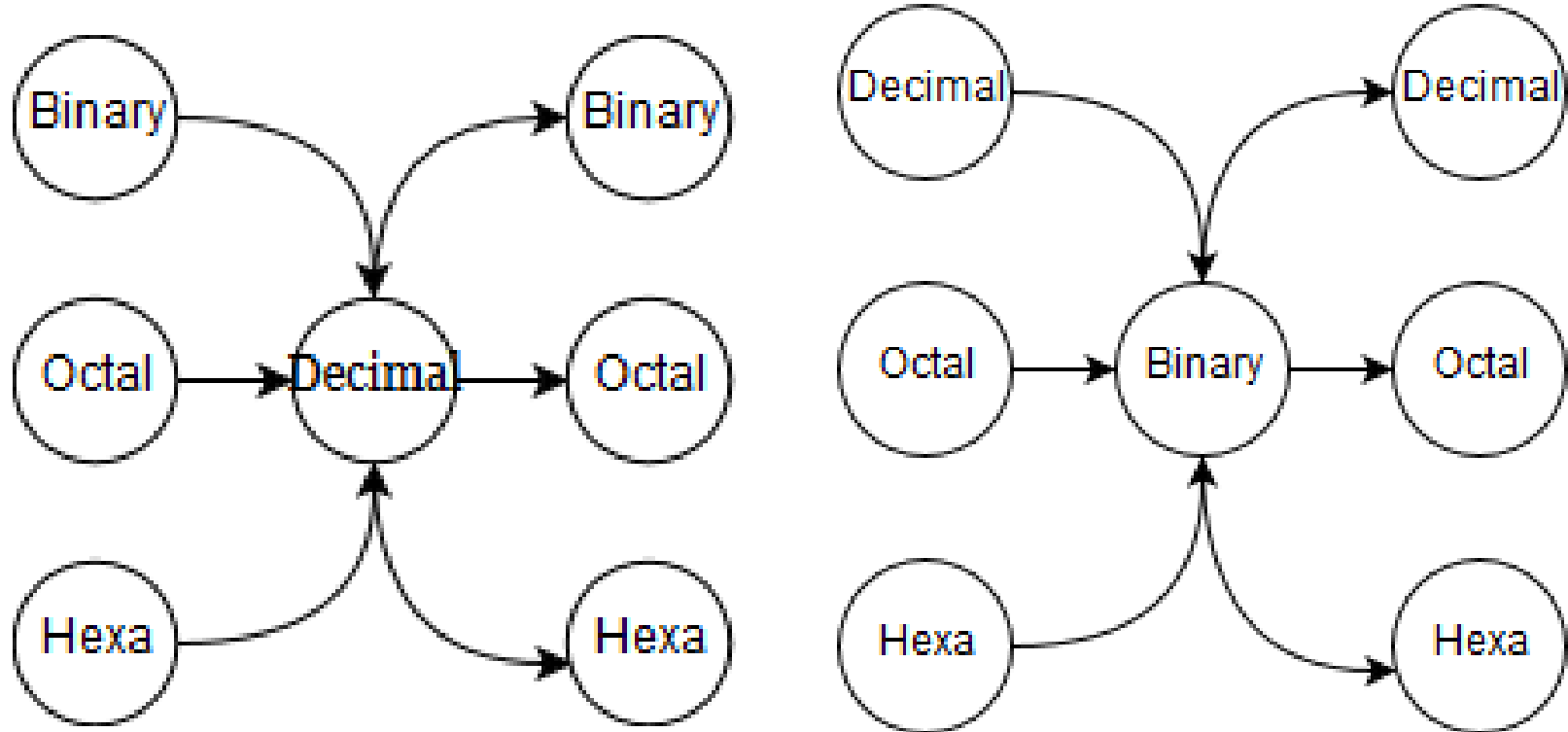
■ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

| | | | | | |
|--------|--------|--------|--------|--------|--------|
| 16^5 | 16^4 | 16^3 | 16^2 | 16^1 | 16^0 |
|--------|--------|--------|--------|--------|--------|

Number Systems relationship

| HEXADECIMAL | DECIMAL | OCTAL | BINARY | System | Radix | Symbols |
|-------------|---------|-------|--------|------------|-------|------------------------------------|
| 0 | 0 | 0 | 0000 | Binary - B | 2 | 0,1 |
| 1 | 1 | 1 | 0001 | Octal - O | 8 | 0,1,2,3,4,5,6,7 |
| 2 | 2 | 2 | 0010 | Decimal- D | 10 | 0,1,2,3,4,5,6,7,8,9 |
| 3 | 3 | 3 | 0011 | Hexa - H | 16 | 0,1,2,3,4,5,6,7,8,9 A,B,C,D,E,F |
| 4 | 4 | 4 | 0100 | | | |
| 5 | 5 | 5 | 0101 | | | |
| 6 | 6 | 6 | 0110 | | | |
| 7 | 7 | 7 | 0111 | | | |
| 8 | 8 | 10 | 1000 | | | |
| 9 | 9 | 11 | 1001 | | | |
| A | 10 | 12 | 1010 | | | |
| B | 11 | 13 | 1011 | | | |
| C | 12 | 14 | 1100 | | | |
| D | 13 | 15 | 1101 | | | |
| E | 14 | 16 | 1110 | | | |
| F | 15 | 17 | 1111 | | | |

Number System Conversion



Number System Conversion

I. Conversion from **decimal** to any **base r = 2,8,16** (Integer part)

1. Divide **I** by **r**, collect the quotient **q** and remainders **rem**
2. Repeat step 1 with **I = q** until **q** becomes 0
3. Write the **rem** from **bottom to top** to provide the integer equivalent of the result.

$$\begin{array}{rcl}
 162 / 2 & = & 81 \text{ rem } 0 \\
 81 / 2 & = & 40 \text{ rem } 1 \\
 40 / 2 & = & 20 \text{ rem } 0 \\
 20 / 2 & = & 10 \text{ rem } 0 \\
 10 / 2 & = & 5 \text{ rem } 0 \\
 5 / 2 & = & 2 \text{ rem } 1 \\
 2 / 2 & = & 1 \text{ rem } 0 \\
 1 / 2 & = & 0 \text{ rem } 1
 \end{array}$$



Example: 162.375: So, $(162.375)_{10} = (10100010.011)_2$

Number System Conversion

I. Conversion from **decimal** to any **base r = 2.8.16** Fraction part

Example: 162.375: So, $(162.375)_{10} = (10100010.011)_2$

1. Multiply **F** by **r** and find the product
2. Repeat step 1 with **F** part of the product until any of the following is satisfied
 1. $F = 0$
 2. F recurs again
 3. Repeat for **p** times where **P** refers to precision in terms of no. of digits
3. Write the **I** part of the product from **top to bottom** to provide the fraction equivalent of the result

$$\begin{aligned} 0.375 \times 2 &= 0.750 \\ 0.750 \times 2 &= 1.500 \\ 0.500 \times 2 &= 1.000 \end{aligned}$$



Number System Conversion

Decimal to Octal $(152.512)_{10} = (?)_8$

| 8 | 152 | Remainder |
|---|-----|-----------|
| 8 | 19 | 0 LSB |
| | 2 | 3 |
| | | 2 MSB |

$$0.512 \times 8 = 4.104 \quad 4$$

$$0.104 \times 8 = 0.832 \quad 0$$

$$0.832 \times 8 = 6.656 \quad 6$$

$$0.656 \times 8 = 5.248 \quad 5$$

$$0.248 \times 8 = 1.984 \quad 1$$

$$(0.512)_{10} = (0.40651...)_8$$

Complete answer is $(152.512)_{10} = (230.40651...)_8$

Number System Conversion

Decimal to Hexa $(2607.565)_{10} = (?)_{16}$

| 16 | 2607 | Remainder |
|----|------|-----------|
| 16 | 162 | 15 LSB |
| | 10 | 2 |
| | | 10 MSB |

$$(2607)_{10} = (A2F)_{16}$$

$$\begin{array}{ll}
 0.565 \times 16 = 9.04 & 9 \\
 0.04 \times 16 = 0.64 & 0 \\
 0.64 \times 16 = 10.24 & 10 = A \\
 0.24 \times 16 = 3.84 & 3 \\
 0.84 \times 16 = 13.44 & 13 = D \\
 0.44 \times 16 = 7.04 & 7 \\
 0.04 \times 16 = 0.64 & 0
 \end{array}$$

$$(0.565)_{10} = (0.90A3D70...)_{16}$$

Complete answer is $(2607.565)_{10} = (A2F.90A3D70...)_{16}$

Binary Number System

★ Base = 2

- 2 digits { 0, 1 }, called *binary digits* or “*bits*”

★ Weights

- Weight = $(Base)^{Position}$

| | | | | | |
|----------|----------|----------|---|----------|----------|
| 4 | 2 | 1 | | 1/2 | 1/4 |
| 1 | 0 | 1 | • | 0 | 1 |
| 2 | 1 | 0 | | -1 | -2 |

$$1 * 2^2 + 0 * 2^1 + 1 * 2^0 + 0 * 2^{-1} + 1 * 2^{-2}$$
$$=(5.25)_{10}$$
$$(101.01)_2$$

★ Magnitude

- Sum of “*Bit x Weight*”

★ Formal Notation

★ Groups of bits

4 bits = *Nibble*

1 0 1 1

8 bits = *Byte*

1 1 0 0 0 1 0 1

Octal Number System

★ Base = 8

- 8 digits { 0, 1, 2, 3, 4, 5, 6, 7 }

★ Weights

- Weight = $(Base)^{Position}$

★ Magnitude

- Sum of “*Digit x Weight*”

★ Formal Notation

| | | | | | |
|----------|----------|----------|---|----------|----------|
| 64 | 8 | 1 | | 1/8 | 1/64 |
| 5 | 1 | 2 | • | 7 | 4 |
| 2 | 1 | 0 | | -1 | -2 |

$$5 \cdot 8^2 + 1 \cdot 8^1 + 2 \cdot 8^0 + 7 \cdot 8^{-1} + 4 \cdot 8^{-2}$$
$$=(330.9375)_{10}$$
$$(512.74)_8$$

Decimal Number System

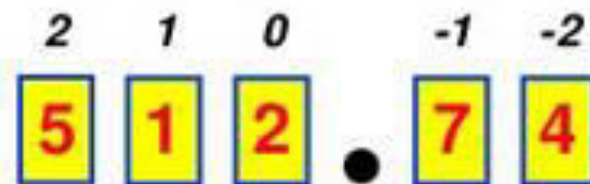
★ Base (also called radix) = 10

- 10 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }



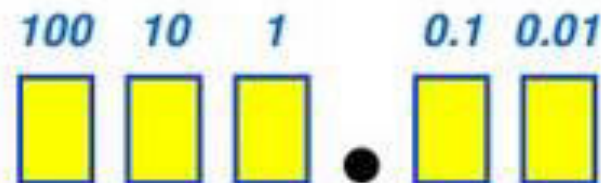
★ Digit Position

- Integer & fraction



★ Digit Weight

- Weight = $(Base)^{Position}$



★ Magnitude

- Sum of “Digit x Weight”

500 10 2 0.7 0.04

$$d_2 \cdot B^2 + d_1 \cdot B^1 + d_0 \cdot B^0 + d_{-1} \cdot B^{-1} + d_{-2} \cdot B^{-2}$$

★ Formal Notation

(512.74)₁₀

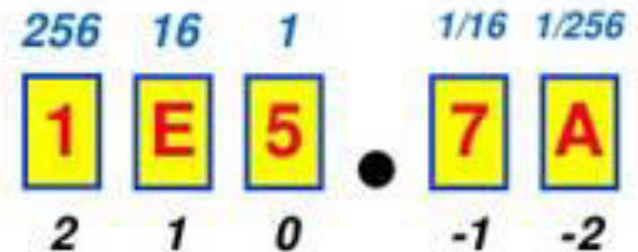
Hexa Decimal Number System

★ Base = 16

- 16 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F }

★ Weights

- Weight = $(Base)^{Position}$



★ Magnitude

- Sum of “Digit x Weight”

$$1 * 16^2 + 14 * 16^1 + 5 * 16^0 + 7 * 16^{-1} + 10 * 16^{-2} \\ = (485.4765625)_{10}$$

★ Formal Notation

$$(1E5.7A)_{16}$$

Powers of 2

| n | 2^n |
|----------|-----------------------------|
| 0 | $2^0=1$ |
| 1 | $2^1=2$ |
| 2 | $2^2=4$ |
| 3 | $2^3=8$ |
| 4 | $2^4=16$ |
| 5 | $2^5=32$ |
| 6 | $2^6=64$ |
| 7 | $2^7=128$ |



| n | 2^n |
|-----------|---------------------------------|
| 8 | $2^8=256$ |
| 9 | $2^9=512$ |
| 10 | $2^{10}=1024$ |
| 11 | $2^{11}=2048$ |
| 12 | $2^{12}=4096$ |
| 20 | $2^{20}=1M$ |
| 30 | $2^{30}=1G$ |
| 40 | $2^{40}=1T$ |

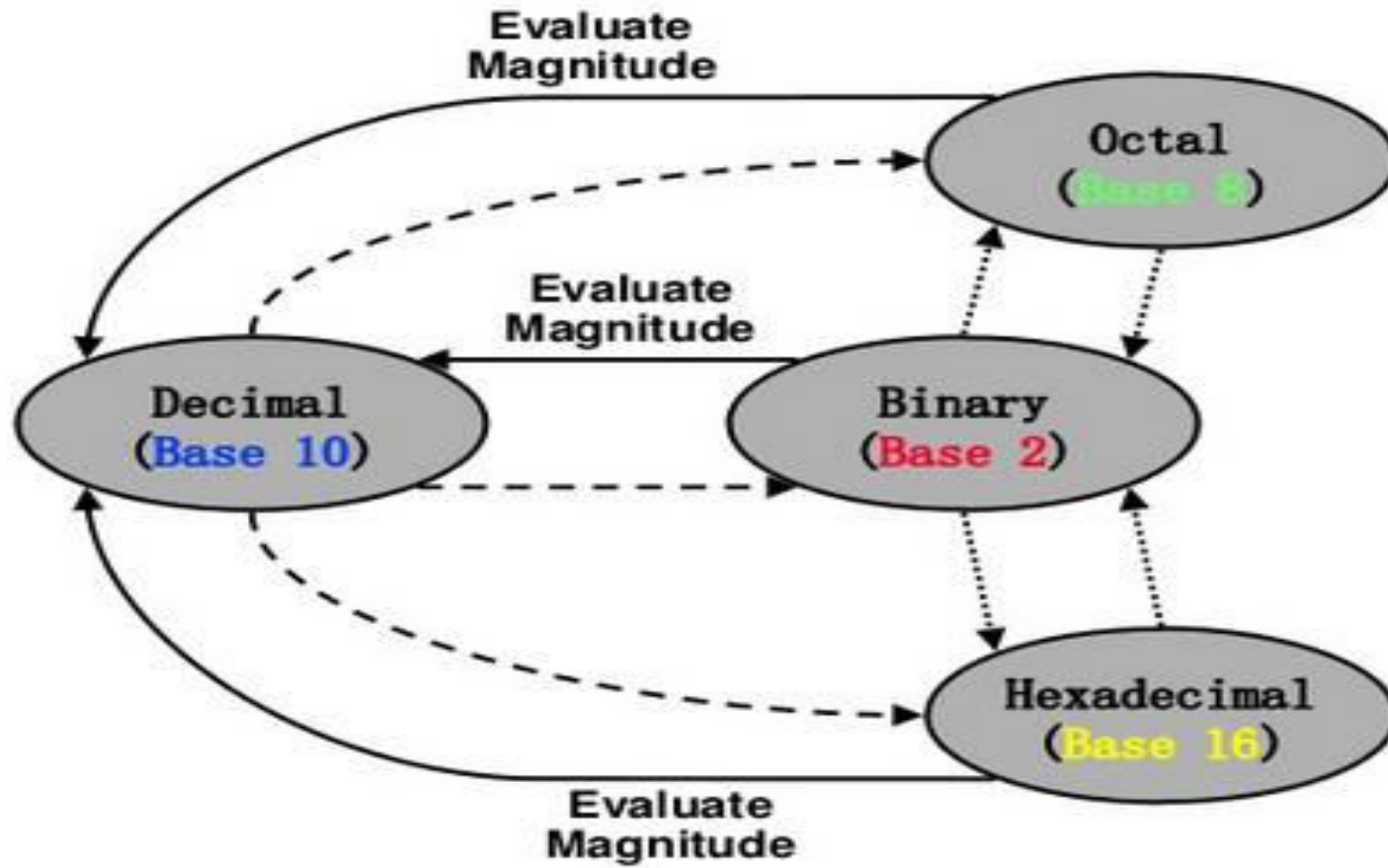
Kilo

Mega

Giga

Tera

Number base Conversions



Number System Conversion

I. Conversion from any **base r = 2,8,16 to decimal**

A number N in base or radix b can be written as: **$N = I . F$**

$$(N)_b = d_{n-1} d_{n-2} \text{ — — — — } d_1 d_0 . d_{-1} d_{-2} \text{ — — — — } d_{-m}$$

$$I = (d_{n-1} * r^{n-1}) + (d_{n-2} * r^{n-2}) + (d_{n-3} * r^{n-3}) + \dots + (d_1 * r^1) + (d_0 * r^0)$$

$$F = (d_{-1} * r^{-1}) + (d_{-2} * r^{-2}) + \dots + (d_{-m} * r^{-m})$$

Number System Conversion

examples....

Binary to Decimal conversion

$$(1101.01)_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 0 \times 2^{(-1)} + 1 \times 2^{(-2)} = (13.25)_{10}$$

Octal to Decimal conversion

$$(431.2)_8 = 4 \times 8^2 + 3 \times 8^1 + 1 \times 8^0 + 2 \times 8^{(-1)} = (281.25)_{10}$$

Hexadecimal to Decimal conversion

$$(6E9.D8)_{16} = 6 \times 16^2 + 14 \times 16^1 + 9 \times 16^0 + 13 \times 16^{(-1)} + 8 \times 16^{(-2)} =$$

$$(1769.84375)_{10}$$

Number System Conversion

Binary to Octal Conversion ($2^1 \rightarrow 2^3$)

step 1a: Split the Integer part of given binary number into groups of 3 bits from right (LSB).

Step 1 b: Split the fraction part of given binary number into groups of 3 bits from left (MSB)

step 2: Add 0s to the left side in Integer part and , add 0s to the right side in the fraction for lack of 3 bits.

step 3: Find the Octal equivalent for each group in both integer and fraction portion

step 4: Form the each group Octal number together in the same order.

Number System Conversion

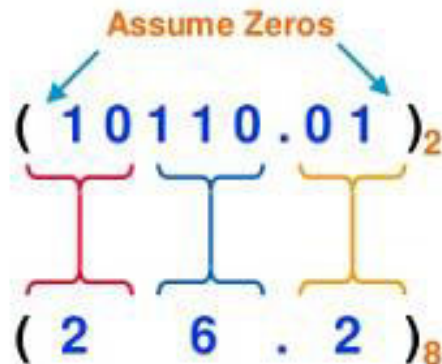
Solved Example:

Binary – Octal Conversion

★ $8 = 2^3$

★ Each group of 3 bits represents an octal digit

Example:



| Octal | Binary |
|-------|--------|
| 0 | 0 0 0 |
| 1 | 0 0 1 |
| 2 | 0 1 0 |
| 3 | 0 1 1 |
| 4 | 1 0 0 |
| 5 | 1 0 1 |
| 6 | 1 1 0 |
| 7 | 1 1 1 |

Works **both** ways (*Binary to Octal & Octal to Binary*)

Number System Conversion

Binary to Hexa Conversion ($2^1 \rightarrow 2^4$)

step 1a: Split the Integer part of given binary number into groups of 4 bits from right (LSB).

Step 1 b: Split the fraction part of given binary number into groups of 4 bits from left (MSB)

step 2: Add 0s to the left side in Integer part and , add 0s to the right side in the fraction for lack of 4 bits.

step 3: Find the Hexa equivalent for each group in both integer and fraction portion

step 4: Form the each group Hexa number together in the same order.

Number System Conversion

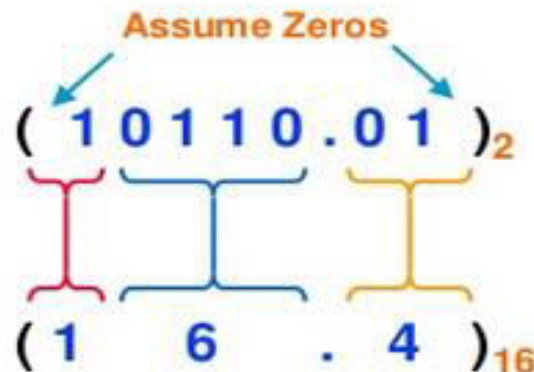
Solved Example:

Binary – Hexadecimal Conversion

★ $16 = 2^4$

★ Each group of 4 bits represents a hexadecimal digit

Example:



| Hex | Binary |
|-----|--------|
| 0 | 0000 |
| 1 | 0001 |
| 2 | 0010 |
| 3 | 0011 |
| 4 | 0100 |
| 5 | 0101 |
| 6 | 0110 |
| 7 | 0111 |
| 8 | 1000 |
| 9 | 1001 |
| A | 1010 |
| B | 1011 |
| C | 1100 |
| D | 1101 |
| E | 1110 |
| F | 1111 |

Works **both** ways (*Binary to Hex & Hex to Binary*)

Number System Chart

| Decimal | Binary | Octal | Hex |
|---------|--------|-------|-----|
| 00 | 0000 | 00 | 0 |
| 01 | 0001 | 01 | 1 |
| 02 | 0010 | 02 | 2 |
| 03 | 0011 | 03 | 3 |
| 04 | 0100 | 04 | 4 |
| 05 | 0101 | 05 | 5 |
| 06 | 0110 | 06 | 6 |
| 07 | 0111 | 07 | 7 |
| 08 | 1000 | 10 | 8 |
| 09 | 1001 | 11 | 9 |
| 10 | 1010 | 12 | A |
| 11 | 1011 | 13 | B |
| 12 | 1100 | 14 | C |
| 13 | 1101 | 15 | D |
| 14 | 1110 | 16 | E |
| 15 | 1111 | 17 | F |

1's Complement

★ 1's Complement (*Diminished Radix Complement*)

- All '0's become '1's
- All '1's become '0's

Example $(10110000)_2$

$\Rightarrow (01001111)_2$

If you add a number and its 1's complement ...

$$\begin{array}{r} 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0 \\ +\ 0\ 1\ 0\ 0\ 1\ 1\ 1\ 1 \\ \hline 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1 \end{array}$$

2's Complement

★ 2's Complement (*Radix Complement*)

OR

- Take 1's complement then add 1
- Toggle all bits to the left of the first '1' from the right

Example:

Number: 1 0 1 1 0 0 0 0

1 0 1 1 0 0 0 0

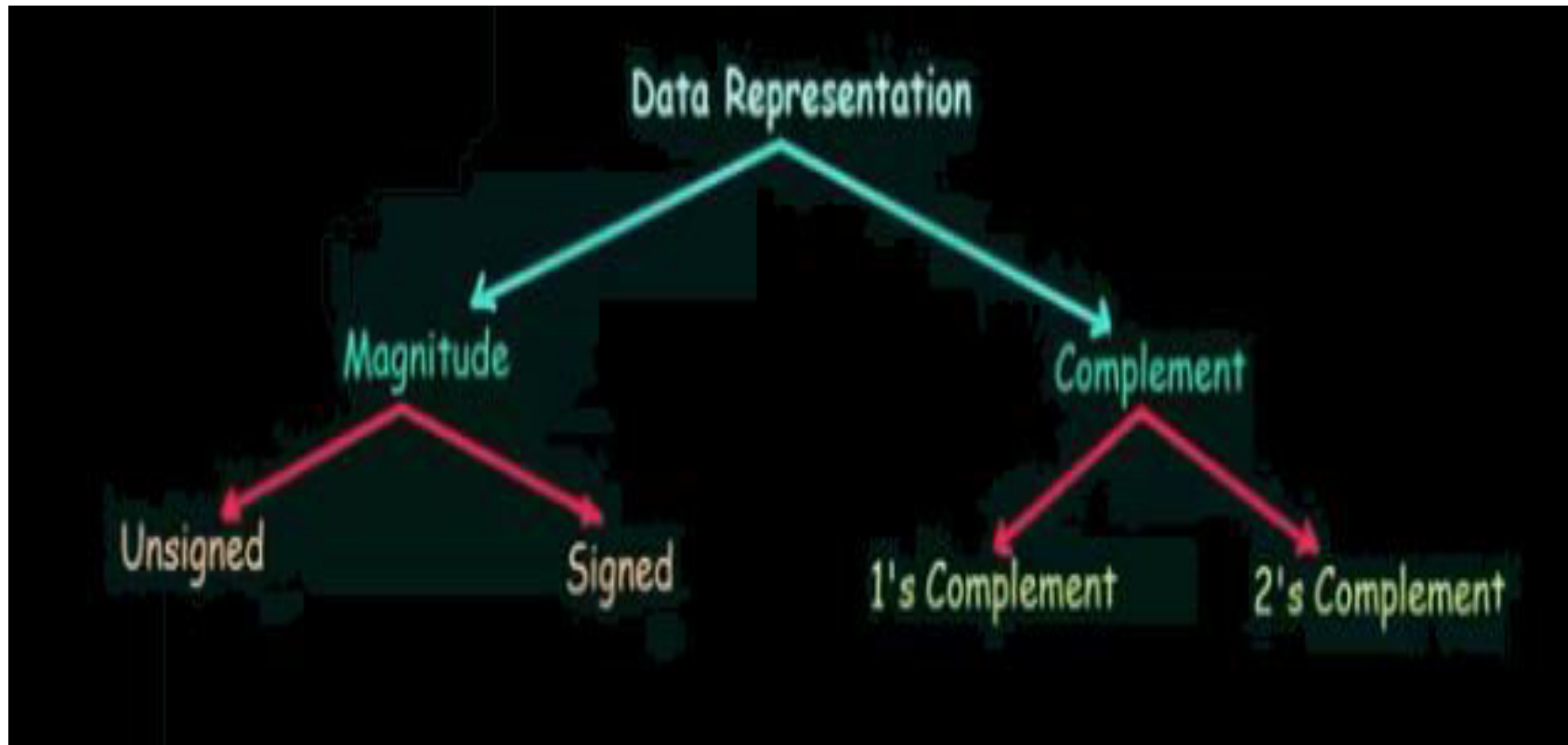
1's Comp.: 0 1 0 0 1 1 1 1

+ 1

0 1 0 1 0 0 0 0

0 1 0 1 0 0 0 0

Types of representation



Unsigned – magnitude rep.

An n -bit pattern can represent 2^n distinct integers.

Range 0 to $(2^n)-1$, as tabulated below

Can represent only +ve nos.

| n | Minimum | Maximum |
|----|---------|---|
| 8 | 0 | $(2^8)-1$ (=255) |
| 16 | 0 | $(2^{16})-1$ (=65,535) |
| 32 | 0 | $(2^{32})-1$ (=4,294,967,295) (9+ digits) |
| 64 | 0 | $(2^{64})-1$ (=18,446,744,073,709,551,615) (19+ digits) |

Negative numbers

★ Computers Represent Information in '0's and '1's

- '+' and '-' signs have to be represented in '0's and '1's

★ 3 Systems

- Signed Magnitude
- 1's Complement
- 2's Complement

All three use the *left-most bit* to represent the sign:

♦ '0' \Rightarrow positive

♦ '1' \Rightarrow negative

Signed magnitude representation

★ Magnitude is magnitude, *does not change with sign*

S **Magnitude (Binary)**

$$(+3)_{10} \Rightarrow (0011)_2$$

$$(-3)_{10} \Rightarrow (1011)_2$$

Sign Magnitude

★ Can't include the *sign bit* in 'Addition'

$$0011 \Rightarrow (+3)_{10}$$

$$+ 1011 \Rightarrow (-3)_{10}$$

—————

$$1110 \Rightarrow (-6)_{10}$$

1's Complement representation

★ Positive numbers are represented in “Binary”

0 Magnitude (Binary)

★ Negative numbers are represented in “1's Comp.”

1 Code (1's Comp.)

$$(+3)_{10} \Rightarrow (0\ 011)_2$$

$$(-3)_{10} \Rightarrow (1\ 100)_2$$

★ There are 2 representations for ‘0’

$$(+0)_{10} \Rightarrow (0\ 000)_2$$

$$(-0)_{10} \Rightarrow (1\ 111)_2$$

1's Complement range

★ 4-Bit Representation

$2^4 = 16$ Combinations

$$-7 \leq \text{Number} \leq +7$$

$$-2^3 + 1 \leq \text{Number} \leq +2^3 - 1$$

★ n-Bit Representation

$$-2^{n-1} + 1 \leq \text{Number} \leq +2^{n-1} - 1$$

| Decimal | 1's Comp. |
|---------|-----------|
| +7 | 0 1 1 1 |
| +6 | 0 1 1 0 |
| +5 | 0 1 0 1 |
| +4 | 0 1 0 0 |
| +3 | 0 0 1 1 |
| +2 | 0 0 1 0 |
| +1 | 0 0 0 1 |
| +0 | 0 0 0 0 |
| -0 | 1 1 1 1 |
| -1 | 1 1 1 0 |
| -2 | 1 1 0 1 |
| -3 | 1 1 0 0 |
| -4 | 1 0 1 1 |
| -5 | 1 0 1 0 |
| -6 | 1 0 0 1 |
| -7 | 1 0 0 0 |

2's Complement representation

★ Positive numbers are represented in “Binary”

0 Magnitude (Binary)

★ Negative numbers are represented in “2's Comp.”

1 Code (2's Comp.)

$$(+3)_{10} \Rightarrow (0\ 011)_2$$

$$(-3)_{10} \Rightarrow (1\ 101)_2$$

★ There is 1 representation for '0' 1's Comp. 1 1 1 1

$$(+0)_{10} \Rightarrow (0\ 000)_2$$

$$(-0)_{10} \Rightarrow (0\ 000)_2$$

$$\begin{array}{r} + \quad 1 \\ 1\ 0000 \end{array}$$

2's Complement range

★ 4-Bit Representation

$2^4 = 16$ Combinations

$$-8 \leq \text{Number} \leq +7$$

$$-2^3 \leq \text{Number} \leq +2^3 - 1$$

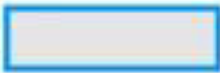






★ n-Bit Representation

$$-2^{n-1} \leq \text{Number} \leq +2^{n-1} - 1$$

| Decimal | 2's Comp. |
|---------|-----------|
| +7 | 0 1 1 1 |
| +6 | 0 1 1 0 |
| +5 | 0 1 0 1 |
| +4 | 0 1 0 0 |
| +3 | 0 0 1 1 |
| +2 | 0 0 1 0 |
| +1 | 0 0 0 1 |
| +0 | 0 0 0 0 |
| -1 | 1 1 1 1 |
| -2 | 1 1 1 0 |
| -3 | 1 1 0 1 |
| -4 | 1 1 0 0 |
| -5 | 1 0 1 1 |
| -6 | 1 0 1 0 |
| -7 | 1 0 0 1 |
| -8 | 1 0 0 0 |

All types of representation

★ 4-Bit Example

| | Unsigned Binary | Signed Magnitude | 1's Comp. | 2's Comp. |
|----------|---|---|--|--|
| Range | $0 \leq N \leq 15$ | $-7 \leq N \leq +7$ | $-7 \leq N \leq +7$ | $-8 \leq N \leq +7$ |
| Positive |  Binary |  Binary |  Binary |  Binary |
| Negative | X |  Binary |  1's Comp. |  2's Comp. |

Binary arithmetic

Binary addition:-

| A | B | Sum | Carry |
|---|---|-----|-------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

Binary subtraction:-

| A | B | Difference | Borrow |
|---|---|------------|--------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |

Binary Multiplication:-

| A | B | Output |
|---|---|--------|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |


Binary Division:-

| A | B | Output |
|----------------------------------|---|--------|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| Division by zero is meaning less | | |

1's complement addition

With one's complement addition,
the carry bit is “carried around”
and added to the sum.

- Example: Using one's complement binary arithmetic, find the sum of 48 and - 19


$$\begin{array}{r} 11 \\ 00110000 \\ 11101100 \\ \hline 00011100 \\ + 1 \\ \hline 00011101 \end{array}$$

We note that 19 in binary is 00010011,
so -19 in one's complement is: 11101100.

2's complement addition

$$\begin{aligned} 48 &= 00110000 & 19 &= 00010011 \\ -19 &= 1\ 1101101 & 2'scomp(19) \end{aligned}$$

$$\begin{array}{r} 48 = 00110000 \\ -19 = 1\ 1101101 \\ \hline 1\ 00011101 \\ \hline \end{array}$$

Discard the end around carry 1
result is 00011101 = + 29

2's complement addition contd..

Case 1: Two positive numbers

+29 ---- 0 001 1101 (Augend)

+19 ---- 0 001 0011 (Addend)

0 011 0000 (Sum = +48)

Case 2: Positive augend & negative addend

+39 ---- 0 010 0111 (Augend)

- 22 ---- 1 110 1010 (Addend)-2's comp.

1 0 001 0001 (Sum = +17)



Discarded

2's complement addition contd..

Case 3: Positive addend & negative augend

- 47 ---- 1 101 0001 (Augend)

+29 ---- 0 001 1101 (Addend)

1 110 1110 (Sum = -18)-2's comp

Case 4: Two negative numbers

-32 ---- 1 110 0000 (Augend)

-44 ---- 1 101 0100 (Addend)

1 1 011 0100 (Sum = -76)-2's comp

↓
discarded

Binary Subtraction – 1's complement

★ Change “*Subtraction*” to “*Addition*”

★ If “*Carry*” = 1
then add it to the
LSB, and the result
is positive
(in *Binary*)

★ If “*Carry*” = 0
then the result
is negative
(in *1's Comp.*)

$$\begin{array}{r}
 (5)_{10} - (1)_{10} \\
 (+5)_{10} + (-1)_{10} \\
 \begin{array}{r}
 0101 \\
 + 1110 \\
 \hline
 10011 \\
 \hline
 0100 \\
 \hline
 +4
 \end{array}
 \end{array}$$

Note: In the original image, a blue arrow points from the carry '1' in the result '10011' back to the LSB of the first number '0101'. A red arrow points from the carry '1' in the result '10011' to the LSB of the second number '1110'.

$$\begin{array}{r}
 (5)_{10} - (6)_{10} \\
 (+5)_{10} + (-6)_{10} \\
 \begin{array}{r}
 0101 \\
 + 1001 \\
 \hline
 \cancel{0}1110 \\
 \hline
 1110 \\
 \hline
 -1
 \end{array}
 \end{array}$$

Note: In the original image, the carry '0' in the result '01110' is crossed out with a blue 'X'.

Binary Subtraction – 2's complement

★ Change “*Subtraction*” to “*Addition*”

★ If “*Carry*” = 1
ignore it, and the
result is positive
(in *Binary*)

★ If “*Carry*” = 0
then the result
is negative
(in *2's Comp.*)

$$\begin{array}{r}
 (5)_{10} - (1)_{10} \\
 (+5)_{10} + (-1)_{10} \\
 \begin{array}{r}
 0101 \\
 + 1111 \\
 \hline
 \boxed{1}0100 \\
 \underbrace{}_{+4}
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 (5)_{10} - (6)_{10} \\
 (+5)_{10} + (-6)_{10} \\
 \begin{array}{r}
 0101 \\
 + 1010 \\
 \hline
 \boxed{0}1111 \\
 \underbrace{}_{-1}
 \end{array}
 \end{array}$$

2's complement Subtraction

Case 1: Two positive numbers

$$\begin{array}{r} +28 \text{ ---- } 0 \ 001 \ 1100 \text{ (Minuend)} \\ +19 \text{ ---- } 1 \ 110 \ 1101 \text{ (Subtrahend)-2's comp} \\ \hline 1 \ 000 \ 1001 \text{ (Sum = +9)} \\ \downarrow \\ \text{discarded} \end{array}$$

Case 2: Positive no. & smaller Negative no.

$$\begin{array}{r} +39 \text{ ---- } 0 \ 010 \ 0111 \text{ (Minuend)} \\ -21 \text{ ---- } 0 \ 001 \ 0101 \text{ (Subtrahend)-2's comp} \\ \hline 0 \ 011 \ 1100 \text{ (Sum = +60)} \end{array}$$

2's complement subtraction contd..

Case 3: Positive No. & larger Negative No.

$$\begin{array}{r} +19 \text{ ---- } 0 \quad 001 \quad 0011 \text{ (Minuend)} \\ -43 \text{ ---- } 0 \quad 010 \quad 1011 \text{ (Subtrahend)-2's comp} \\ \hline 0 \quad 011 \quad 1110 \text{ (Sum = +62)} \end{array}$$

Case 4: Two negative numbers

$$\begin{array}{r} -57 \text{ ---- } 1 \quad 100 \quad 0111 \text{ (Minuend)} \\ -33 \text{ ---- } 0 \quad 010 \quad 0001 \text{ (Subtrahend)-2's comp} \\ \hline 1 \quad 110 \quad 1000 \text{ (Sum = -24)} \end{array}$$

Binary Arithmetic Problems

Add the following binary numbers:

1. $(1001)_2$ and $(0101)_2$
2. $(101.01)_2$ and $(1101.10)_2$

Subtract the following binary numbers:

1. $(0110)_2$ from $(1010)_2$
2. $(01011)_2$ from $(11011)_2$

Binary Arithmetic Problems

Solve the following binary multiplication

- | | |
|------------------------------|--------------------------------------|
| 1. $(101)_2$ and $(11)_2$ | 1. $5 * 3 = 15 = 1111$ |
| 2. $(1011)_2$ and $(1001)_2$ | 2. $11 * 9 = 99 = \mathbf{01100011}$ |

Binary Arithmetic Problems

Solve the following division

1. (11001) by (101)

2. (110000) by (100)

1. $25 / 5 = 5$, 101 2. $48 / 4 = 12$, **01100**

Binary Codes

★ Group of n bits

- Up to 2^n combinations
- Each *combination* represents *an element* of information

★ Binary Coded Decimal (BCD)

- Each Decimal Digit is represented by 4 bits
- (0 – 9) \Rightarrow Valid combinations
- (10 – 15) \Rightarrow Invalid combinations

| Decimal | BCD |
|---------|---------|
| 0 | 0 0 0 0 |
| 1 | 0 0 0 1 |
| 2 | 0 0 1 0 |
| 3 | 0 0 1 1 |
| 4 | 0 1 0 0 |
| 5 | 0 1 0 1 |
| 6 | 0 1 1 0 |
| 7 | 0 1 1 1 |
| 8 | 1 0 0 0 |
| 9 | 1 0 0 1 |

BCD addition

★ One decimal digit + one decimal digit

- If the result is 1 decimal digit (≤ 9), then it is a simple binary addition

Example:

$$\begin{array}{r} 5 \\ + 3 \\ \hline \end{array} \quad \begin{array}{r} 0101 \\ + 0011 \\ \hline \end{array}$$

8 \longleftrightarrow 1000

- If the result is two decimal digits (≥ 10), then binary addition gives invalid combinations

Example:

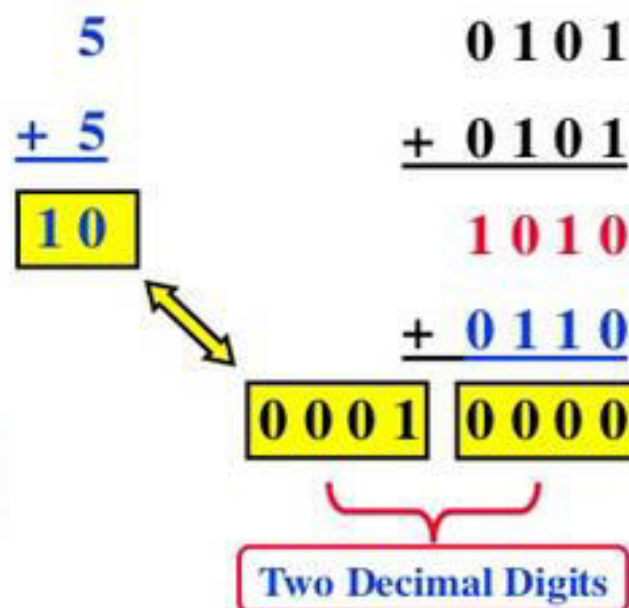
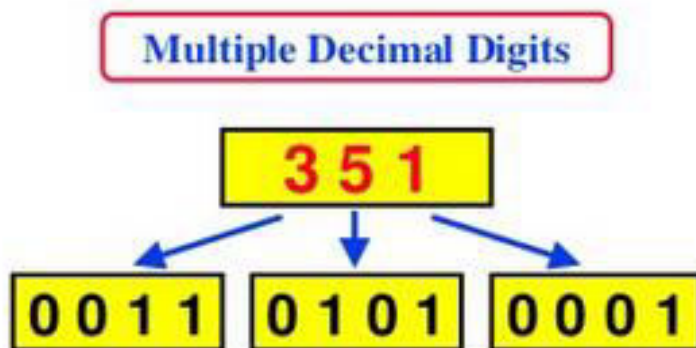
$$\begin{array}{r} 5 \\ + 5 \\ \hline \end{array} \quad \begin{array}{r} 0101 \\ + 0101 \\ \hline \end{array}$$

0001 0000 \longleftrightarrow 10 1010

Binary Coded addition

BCD Addition

★ If the binary result is greater than 9, correct the result by adding 6

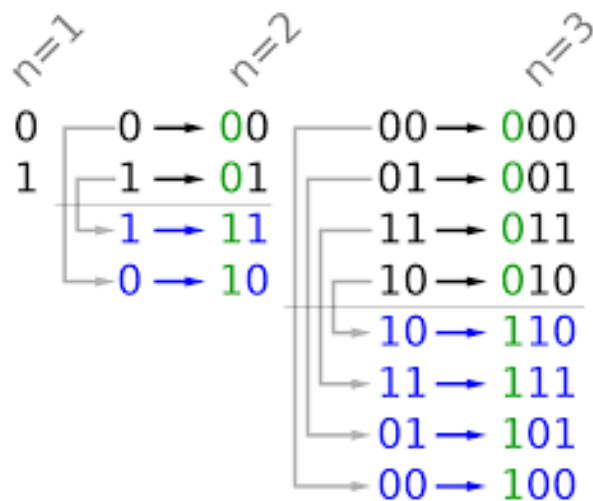


Reflected code / Unweighted code

Gray Code

★ One bit changes from one code to the next code

★ Different than Binary



| Decimal | Gray | Binary |
|---------|------|--------|
| 00 | 0000 | 0000 |
| 01 | 0001 | 0001 |
| 02 | 0011 | 0010 |
| 03 | 0010 | 0011 |
| 04 | 0110 | 0100 |
| 05 | 0111 | 0101 |
| 06 | 0101 | 0110 |
| 07 | 0100 | 0111 |
| 08 | 1100 | 1000 |
| 09 | 1101 | 1001 |
| 10 | 1111 | 1010 |
| 11 | 1110 | 1011 |
| 12 | 1010 | 1100 |
| 13 | 1011 | 1101 |
| 14 | 1001 | 1110 |
| 15 | 1000 | 1111 |

Character Representation (Cont.)

- ❑ With a single byte (8-bits) 256 characters can be represented
- ❑ Standards
 - ASCII – American Standard Code for Information Interchange
 - EBCDIC – Extended Binary-Coded Decimal Interchange Code
 - Unicode

ASCII Code

- ❑ De facto world-wide standard
- ❑ Used to represent
 - Upper & lower-case Latin letters
 - Numbers
 - Punctuations
 - Control characters
- ❑ There are 128 standard ASCII codes
 - Can be represented by a 7 digit binary number
 - ❑ 000 0000 through 111 1111
 - Plus parity bit

ASCII code

American Standard Code for Information Interchange

| Info | 7-bit Code |
|------|------------|
| A | 1000001 |
| B | 1000010 |
| ⋮ | ⋮ |
| Z | 1011010 |
| | |
| a | 1100001 |
| b | 1100010 |
| ⋮ | ⋮ |
| z | 1111010 |
| | |
| @ | 1000000 |
| ? | 0111111 |
| + | 0101011 |
| | |

ASCII Table

| ASCII | Hex | Symbol |
|-------|-----|--------|
| 0 | 0 | NUL |
| 1 | 1 | SOH |
| 2 | 2 | STX |
| 3 | 3 | ETX |
| 4 | 4 | EOT |
| 5 | 5 | ENQ |
| 6 | 6 | ACK |
| 7 | 7 | BEL |
| 8 | 8 | BS |
| 9 | 9 | TAB |
| 10 | A | LF |
| 11 | B | VT |
| 12 | C | FF |
| 13 | D | CR |
| 14 | E | SO |
| 15 | F | SI |

| ASCII | Hex | Symbol |
|-------|-----|---------|
| 32 | 20 | (space) |
| 33 | 21 | ! |
| 34 | 22 | " |
| 35 | 23 | # |
| 36 | 24 | \$ |
| 37 | 25 | % |
| 38 | 26 | & |
| 39 | 27 | ' |
| 40 | 28 | (|
| 41 | 29 |) |
| 42 | 2A | * |
| 43 | 2B | + |
| 44 | 2C | , |
| 45 | 2D | - |
| 46 | 2E | . |
| 47 | 2F | / |

| ASCII | Hex | Symbol |
|-------|-----|--------|
| 48 | 30 | 0 |
| 49 | 31 | 1 |
| 50 | 32 | 2 |
| 51 | 33 | 3 |
| 52 | 34 | 4 |
| 53 | 35 | 5 |
| 54 | 36 | 6 |
| 55 | 37 | 7 |
| 56 | 38 | 8 |
| 57 | 39 | 9 |
| 58 | 3A | : |
| 59 | 3B | ; |
| 60 | 3C | < |
| 61 | 3D | = |
| 62 | 3E | > |
| 63 | 3F | ? |

ASCII Table (Cont.)

| ASCII | Hex | Symbol | ASCII | Hex | Symbol | ASCII | Hex | Symbol |
|-------|-----|--------|-------|-----|--------|-------|-----|--------|
| 64 | 40 | @ | 80 | 50 | P | 96 | 60 | ` |
| 65 | 41 | A | 81 | 51 | Q | 97 | 61 | a |
| 66 | 42 | B | 82 | 52 | R | 98 | 62 | b |
| 67 | 43 | C | 83 | 53 | S | 99 | 63 | c |
| 68 | 44 | D | 84 | 54 | T | 100 | 64 | d |
| 69 | 45 | E | 85 | 55 | U | 101 | 65 | e |
| 70 | 46 | F | 86 | 56 | V | 102 | 66 | f |
| 71 | 47 | G | 87 | 57 | W | 103 | 67 | g |
| 72 | 48 | H | 88 | 58 | X | 104 | 68 | h |
| 73 | 49 | I | 89 | 59 | Y | 105 | 69 | i |
| 74 | 4A | J | 90 | 5A | Z | 106 | 6A | j |
| 75 | 4B | K | 91 | 5B | [| 107 | 6B | k |
| 76 | 4C | L | 92 | 5C | \ | 108 | 6C | l |
| 77 | 4D | M | 93 | 5D |] | 109 | 6D | m |
| 78 | 4E | N | 94 | 5E | ^ | 110 | 6E | n |
| 79 | 4F | O | 95 | 5F | _ | 111 | 6F | o |

Unicode

- ❑ Designed to overcome limitation of number of characters
- ❑ Assigns unique character codes to characters in a wide range of languages
- ❑ 65,536 (2^{16}) distinct Unicode characters

*Unicode provides a unique number for every character,
no matter what the platform,
no matter what the program,
no matter what the language*

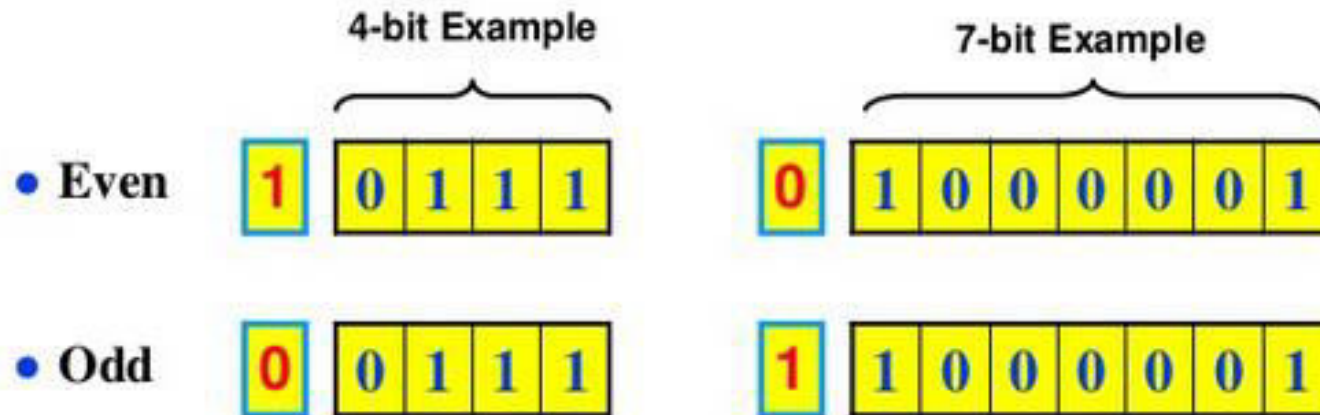
Unicode Goals

- Universal – Should be the only character set ever needed
- Semantics – All characters must have well defined semantics
- Unicode Transformation Format (UTF) is available as 8,16,32 and are referred as
- UTF – 8, UTF – 16, UTF – 32

Error detecting codes

★ Parity

One **bit** added to a group of bits to make the total number of '**1**'s (including the parity bit) *even* or *odd*



★ Good for checking single-bit errors