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Standard Test Method for Determining Transmissivity of Nonleaky Confined Aquifers by the Theis Recovery Method[1]

This standard is issued under the fixed designation D 5269; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This test method covers an analytical procedure for determining the transmissivity of a confined aquifer. This test method is used to analyze data from the recovery of water levels following pumping or injection of water to or from a control well at a constant rate.

1.2 The analytical procedure given in this test method is used in conjunction with the field procedure in Test Method D 4050.

1.3 *Limitations*—The valid use of the Theis recovery method is limited to determination of transmissivities for aquifers in hydrogeologic settings with reasonable correspondence to the assumptions of the Theis theory (see 5.1).

1.4 The values stated in SI units are to be regarded as standard.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*

D 653 Terminology Relating to Soil, Rock and Contained Fluids[2]

D 4043 Guide for Selection of Aquifer-Test Method in Determining Hydraulic Properties by Well Techniques[2]

D 4050 Test Method (Field Procedure) for Withdrawal and Injection Well Tests for Determining Hydraulic Properties of Aquifer Systems[2]

D 4105 Test Method (Analytical Procedure) for Determining Transmissivity and Storage Coefficient of Nonleaky Confined Aquifers by the Modified Theis Nonequilibrium Method[2]

D 4106 Test Method (Analytical Procedure) for Determining Transmissivity and Storage Coefficient of Nonleaky Confined Aquifers by the Theis Nonequilibrium Method[2]

D 4750 Test Method for Determining Subsurface Liquid Levels in a Borehole or Monitoring Well (Observation Well)[2]

3. Terminology

3.1 *Definitions:*

3.1.1 *aquifer, confined*—an aquifer bounded above and below by confining beds and in which the static head is above the top of the aquifer.

3.1.2 *confining bed*—a hydrogeologic unit of less permeable material bounding one or more aquifers.

3.1.3 *control well*—a well by which the aquifer is stressed, for example, by pumping, injection, or change of head.

3.1.4 *drawdown*—vertical distance the static head is lowered due to the removal of water.

3.1.5 *hydraulic conductivity (field aquifer tests)*—the volume of water at the existing kinematic viscosity that will move in a unit time under unit hydraulic gradient through a unit area measured at right angles to the direction of flow.

3.1.6 *observation well*—a well open to all or part of an aquifer.

3.1.7 *piezometer*—a device used to measure head at a point in the subsurface.

3.1.8 *residual drawdown*—The difference between the projected prepumping water-level trend and the water level in a well or piezometer after pumping or injection has stopped.

3.1.9 *specific storage*—the volume of water released from or taken into storage per unit volume of the porous medium per unit change in head.

3.1.10 *step-drawdown test*—a test in which a control well is pumped at constant rates in "steps" of increasing discharge. Each step is approximately equal in duration, although the last step may be prolonged.

3.1.11 *storage coefficient*—the volume of water an aquifer releases from or takes into storage per unit surface area of the aquifer per unit change in head. For a confined aquifer it is equal to the product of specific storage and aquifer thickness. For an unconfined aquifer, the storage coefficient is approximately equal to the specific yield.

3.1.12 *transmissivity*—the volume of water of the prevailing kinematic viscosity transmitted in a unit time through a unit width of the aquifer under a unit hydraulic gradient.

3.2 *Symbols:*Symbols and Dimensions:

3.2.1 b [L]—aquifer thickness.

3.2.2 K [LT^{-1}]—hydraulic conductivity.

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[1] This test method is under the jurisdiction of ASTM Committee D-18 on Soil and Rock and is the direct responsibility of Subcommittee D18.21 on Ground Water and Vadose Zone Investigations.

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[2] *Annual Book of ASTM Standards*, Vol 04.08.

3.2.2.1 *Discussion*—The use of the symbol K for the term hydraulic conductivity is the predominant usage in groundwater literature by hydrogeologists, whereas the symbol k is commonly used for this term in rock mechanics and soil science.

3.2.3 K_r—hydraulic conductivity in the plane of the aquifer, radially from the control well.

3.2.4 K_z—hydraulic conductivity in the vertical direction.

3.2.5 *ln*—natural logarithm.

3.2.6 $\log_{10}$—logarithm to the base 10.

3.2.7 Q [L^3T^{-1}]—discharge.

3.2.8 r [L]—radial distance from control well.

3.2.9 r_c [L]—equivalent inside radius of control well.

3.2.10 S [nd]—storage coefficient.

3.2.11 s [L]—drawdown.

3.2.12 s_c [L]—drawdown corrected for the effects of reduction in saturated thickness.

3.2.13 S_y [nd]—specific yield.

3.2.14 s' *[L]*—residual drawdown.

3.2.15 $\Delta s'$ [L]—change in residual drawdown over one log cycle of t/t'.

3.2.16 T [L^2T^{-1}]—transmissivity.

3.2.17 t [T]—time since pumping or injection began.

3.2.18 t' [T]—time since pumping or injection stopped.

3.2.19 u—dimensionless parameter, equal to $r^2S/4Tt$.

3.2.20 u'—dimensionless parameter, equal to $r^2S/4Tt'$.

4. Summary of Test Method

4.1 This test method describes an analytical procedure for determining transmissivity using data collected during the recovery phase of a withdrawal or injection well test. The field test (see Test Method D 4050) requires pumping or injecting a control well that is open to the entire thickness of a confined aquifer at a constant rate for a specified period. The water-levels in the control well, observation wells, or piezometers are measured after pumping is stopped and used to calculate the transmissivity of the aquifer using the procedures in this test method. Alternatively, this test method can be performed by injecting water into the control well at a constant rate. With some modification, this test method can also be used to analyze the residual drawdown following a step test. This test method is used by plotting residual drawdown against either a function of time or a function of time and discharge and determining the slope of a straight line fitted to the points.

4.2 *Solution*—The solution given by Theis **(1)**[3] can be expressed as follows:

$$s = \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-y}}{y}\,dy \qquad (1)$$

and:

$$u = \frac{r^2S}{4Tt} \qquad (2)$$

4.3 At a control well, observation well, or piezometer, for large values of time, t, and small values of radius, r, the Theis equation reduces, as shown by Cooper and Jacob **(2)** and Jacob **(3)** to the following:

$$s' = \frac{Q}{4\pi T}\,ln(t/t') \qquad (3)$$

where:
t = the time after pumping began and
t' = the time after pumping ceases. From which it can be shown that:

$$T = \frac{2.3Q}{4\pi\Delta s'} \qquad (4)$$

where:
$\Delta s'$ = the measured or projected residual drawdown over one $\log_{10}$ cycle of t/t'.

4.4 A similar analysis (see 4.3) may also be used for a step-drawdown test in which a well is pumped at a constant rate for an initial period, and then the pumping rate is increased through several new constant rates in a series of steps. Harrill **(4)** shows that:

$$s' = \frac{2.3\Delta Q_1}{4\pi T}\left(\log_{10}\frac{t_1}{t'}\right) + \frac{2.3\Delta Q_2}{4\pi T}\left(\log_{10}\frac{t_2}{t'}\right) + \ldots \frac{2.3\Delta Q_n}{4\pi T}\left(\log_{10}\frac{t_n}{t'}\right) \qquad (5)$$

where:
$t_1, t_2, \ldots t_n$ = the elapsed times since either pumping was begun or the discharge rate was increased,
$Q_1, Q_2, \ldots Q_n$ = the well discharge rates, and
$\Delta Q_1, \Delta Q_2 \ldots \Delta Q_n$ = the incremental increases in discharge.

Eq 5 can be rewritten as follows:

$$T = \frac{2.3Q_n}{4\pi s'}\log_{10}f(t, Q) \qquad (6)$$

where:

$$f(t, Q) = \frac{t_1^{\Delta Q_1/Q_n} t_2^{\Delta Q_2/Q_n} t_3^{\Delta Q_3/Q_n} \ldots t_n^{\Delta Q_n/Q_n}}{t'} \qquad (7)$$

and:

$$T = \frac{2.3Q_n}{4\pi\Delta s'_h} \qquad (8)$$

where:
$\Delta s'_h$ = the residual drawdown over one log cycle of the expression $f(t, Q)$ in Eq 6.

Eq 8 can also be used to analyze the residual drawdown following a test in which discharge varies significantly, so long as the discharge can be generalized as a series of constant-discharge steps.

5. Significance and Use

5.1 *Assumptions*:

5.1.1 The well discharges at a constant rate, Q, or at steps of constant rate $Q_1, Q_2 \ldots Q_n$.

5.1.2 Well is of infinitesimal diameter and is open through the full thickness of the aquifer.

5.1.3 The nonleaky aquifer is homogeneous, isotropic, and areally extensive.

5.1.4 Discharge from the well is derived exclusively from storage in the aquifer.

[3] The boldface numbers given in parentheses refer to a list of references at the end of the text.

5.1.5 The geometry of the assumed aquifer and well are shown in Fig. 1.

5.2 *Implications of Assumptions*:

5.2.1 Implicit in the assumptions are the conditions of radial flow. Vertical flow components are induced by a control well that partially penetrates the aquifer, that is, not open to the aquifer through the full thickness of the aquifer. If vertical flow components are significant, the nearest partially penetrating observation well should be located at a distance, *r*, beyond which vertical flow components are negligible. See 5.2.1 of Test Method D 4106 for assistance in determining the minimum distance to partially penetrating observation wells and piezometers.

5.2.2 The Theis method assumes the control well is of infinitesimal diameter. The storage in the control well may adversely affect drawdown measurements obtained in the early part of the test. See 5.2.2 of Test Method D 4106 for assistance in determining the duration of the effects of well-bore storage on drawdown.

5.2.3 *Application of Theis Recovery Method for Unconfined Aquifers*:

5.2.3.1 Although the assumptions are applicable to artesian or confined conditions, the Theis solution may be applied to unconfined aquifers if (*A*) drawdown is small compared with the saturated thickness of the aquifer or if the drawdown is corrected for reduction in thickness of the aquifer and (*B*) the effects of delayed gravity yield are small. See 5.2.3 of Test Method D 4106 for guidance in treating reduction in saturated thickness and delayed gravity drainage in unconfined aquifers.

6. Apparatus

6.1 Analysis of data by this test method from the field procedure given in Test Method D 4050 requires that the control well and observation wells meet the requirements specified in the following subsections.

6.2 *Construction of Control Well*—Install the control well in the aquifer and equip with a pump capable of discharging water from the well at a constant rate, or several steps at constant rate, for the duration of the test. Preferably, the control well should be open throughout the full thickness of the aquifer. If the control well partially penetrates the aquifer, take special precautions in the placement or design of observation wells (see 5.2.1).

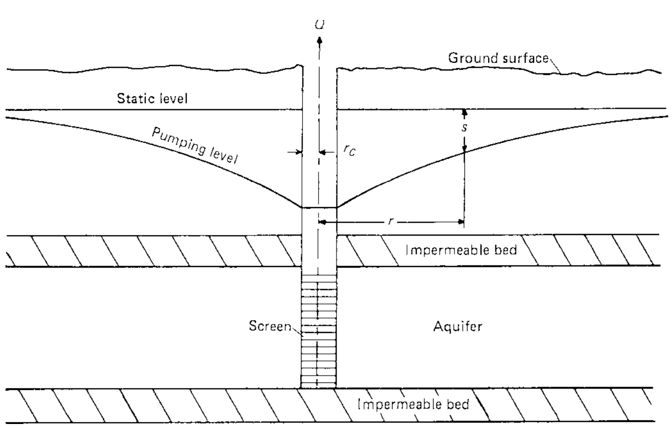

FIG. 1 Cross Section Through a Discharging Well in a Nonleaky Aquifer

6.3 *Construction of Observation Wells and Piezometers*—Construct one or more observation wells or piezometers at a distance from the control well. Observation wells may be open through all or part of the thickness of the aquifer.

6.4 *Location of Observation Wells and Piezometers*—Wells may be located at any distance from the control well within the area of influence of pumping. However, if vertical flow components are significant and if piezometers or partially penetrating observation wells are used, locate them at a distance beyond the effect of vertical flow components. If the aquifer is unconfined, constraints are imposed on the distance to partially penetrating observation wells and the validity of early time measurements (see 5.2.1).

7. Procedure

7.1 The overall procedure consists of conducting the field procedure for withdrawal or injection well tests (described in Test Method D 4050) and analysis of the field data, which is addressed in Section 8.

8. Calculation and Interpretation of Results

8.1 The Theis recovery method gives satisfactory results when properly used. However, the method is valid only for small values of *u*, that is:

for confined aquifers:

$$u' = \frac{r^2 S}{4Tt'} \tag{9}$$

or for unconfined aquifers:

$$u' = \frac{r^2 S_y}{4Tt'} \tag{10}$$

NOTE 1—The limiting value for *u* of less than 0.01 may be excessively restrictive in some applications. The errors for small values of *u*, from Kruseman and De Ridder **(5)** are:

Error less than, %	1	2	5	10
For *u* smaller than	0.03	0.05	0.1	0.15

8.1.1 This test method allows only the calculation of transmissivity, *T*, not storage coefficient, *S*, or specific yield, S_y. Therefore, to determine whether the assumption in Eq 9 or Eq 10 has been violated it is necessary to estimate a value for storage coefficient for confined aquifers or specific yield for unconfined aquifers. If data are available during the pumping period, the storage may be computed using the procedures in Test Method D 4105. Storage coefficients can be estimated as about $3 \times 10^{-5}b$, where *b* is aquifer thickness in meters. Whereas the specific yield of unconfined aquifers averages about 0.2 according to Lohman **(6)**. After calculating *T*, substitute the appropriate values into Eq 9 or Eq 10 and solve for u'. It is not adequate to simply note that the data described a straight line on semi-log graph paper.

8.2 Plot either residual drawdown, s', or water level, on the arithmetic axis of semilogarithmic graph paper versus either t/t' (for recovery from a constant-discharge test) (see Fig. 2) or $f(t, Q)$ (for recovery from a step-drawdown test) (see Fig. 3) on the logarithmic axis. Fit a straight line to the linear part of the data plot, usually at smaller values of t/t'. Extend the straight line to intercept the $t/t' = 1$ axis. At $t/t' = 1$, residual drawdown should be approximately equal to zero, or if water levels were plotted, the intercept should be equal to the prepumping

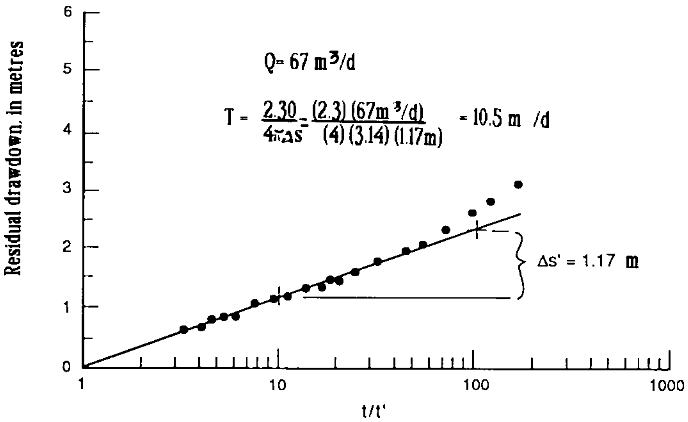

FIG. 2 Example Analysis Using the Theis Recovery Method

water levels corrected for prepumping water-level trends. Substitute the values for $\Delta s'$ or $\Delta s'_h$ in Eq 4 or Eq 8 and solve for transmissivity. Check that all values of t' for the points used in defining the straight line meet the criterion that $u' < 0.01$ (Eq 9 and Eq 10), as described in 8.1.

9. Report

9.1 Prepare a report including the information described below. The report of the analysis will include information from the field testing procedure.

9.1.1 *Introduction*—The introductory section is intended to present the scope and purpose of the Theis recovery method for determining transmissivity in a confined nonleaky aquifer. Summarize the field hydrogeologic conditions and the field equipment and instrumentation including the construction of the control well and observation wells and piezometers, the method of measurement of discharge and water levels, and the duration of the test and pumping rates. Discuss rationale for selecting the Theis recovery method.

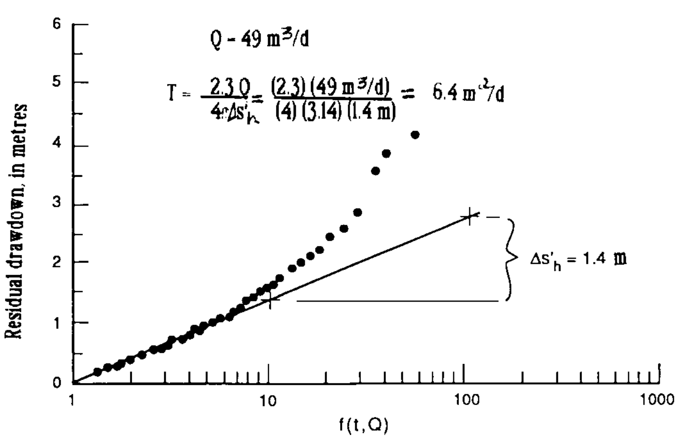

FIG. 3 Example Analysis Using Harrill's Method

9.1.2 *Hydrogeologic Setting*—Review the information available on the hydrogeology of the site. Include driller's logs and geologist's description of drill cuttings. Interpret and describe the hydrogeology of the site as it pertains to the selection of this method for conducting and analyzing an aquifer test. Compare the hydrogeologic characteristics of the site as it conforms and differs from the assumptions in the solution to the aquifer test method.

9.1.3 *Scope of Aquifer Test*:

9.1.3.1 *Equipment*—Report the field installation and equipment for the aquifer test, including the construction, diameter, depth of screened interval, and location of control well and pumping equipment, and the construction, diameter, depth, and screened interval of observation wells or piezometers.

9.1.3.2 *Instrumentation*—Report the field instrumentation for observing water levels, pumping rate, barometric changes, and other environmental conditions pertinent to the test. Include a list of measuring devices used during the test, the manufacturer's name, model number, and basic specifications for each major item, and the name and date of the last calibration, if applicable.

9.1.3.3 *Testing Procedures*—State the steps taken in conducting pretest, drawdown, and recovery phases of the test. Include the frequency of measurements of discharge rate, water level in observation wells, and other environmental data recorded during the testing procedure.

9.1.4 *Presentation of Interpretation of Test Results*:

9.1.4.1 *Data*—Present tables of data collected during the test. Show methods of adjusting water levels for barometric changes and calculation of drawdown and residual drawdown.

9.1.4.2 *Data Plots*—Present data plots used in analysis of the data. Show data plots with straight line segments and intercepts of the $t/t' = 1$ axis.

9.1.4.3 Evaluate qualitatively the overall accuracy of the test on the basis of the adequacy of instrumentation and observations of stress end response, and the conformance of the hydrogeologic conditions and the performance of the test to the assumptions of the test method (see 5.1) and the implications of the assumptions (see 5.2).

10. Precision and Bias

10.1 It is not practicable to specify the precision of this test method because the response of aquifer systems during aquifer tests is dependent upon ambient system stresses. No statement can be made about bias because no true reference values exist.

11. Keywords

11.1 aquifers; aquifer tests; confined aquifers; control wells; ground water; hydraulic properties; observation wells; step tests; transmissivity; unconfined aquifers

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(**2**) Cooper, H. H., and Jacob, C. E., "A Generalized Graphical Method for Evaluating Formation Constants and Summarizing Well-Field History," *Trans. Amer. Geophys. Union*, Vol 27, 1946, pp. 526–534.

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(**4**) Harrill, J. R., "Determining Transmissivity from Water-Level Recovery of a Step-Drawdown Test," *U.S. Geological Survey Professional Paper 700-C*, 1970.

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