AMERICAN SOCIETY FOR TESTING AND MATERIALS
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Standard Practice for Calculating Thermal Diffusivity of Rocks[1]

1. Scope

1.1 This practice involves calculation of the thermal diffusivity from measured values of the mass density, thermal conductivity, and specific heat at constant pressure. It is applicable for any materials where these data can be determined. The temperature range covered by this practice is 20 to 300°C.

NOTE 1—The diffusivity, as determined by this practice, is intended to be a volume average value, with the averaging volume being $\geq 2 \times 10^{-5}$ m^3 (20 cm^3). This requirement necessitates the use of specimens with volumes greater than the minimum averaging volume and precludes use of flash methods of measuring thermal diffusivity, such as the laser pulse technique.

NOTE 2—This practice is closely linked to the overall test procedure used in obtaining the primary data on density, specific heat, and conductivity. It cannot be used as a "stand alone" practice because the thermal diffusivity values calculated by this practice are dependent on the nature of the primary data base. The practice furnishes general guidelines but cannot be considered to be all-inclusive.

1.2 The practice is intended to apply to isotropic samples; that is, samples in which the thermal transport properties do not depend on the direction of heat flow. If the thermal conductivity depends on the direction of heat flow, then the diffusivity derived by this practice must be associated with the same direction as that utilized in the conductivity measurement.

1.3 The thermal conductivity, specific heat, and mass density measurements must be made with specimens that are as near identical in composition and water content as possible.

1.4 The generally inhomogeneous nature of geologic formations precludes the unique specification of a thermal diffusivity characterizing an entire rock formation. Geologic media are highly variable in character, and it is impossible to specify a practice for diffusivity determination that will be suitable for all possible cases. Some of the most important limitations arise from the following factors:

1.4.1 *Variable Mineralogy*—If the mineralogy of the formation under study is highly variable over distances on the same order as the size of the sample from which the conductivity, specific heat, and density specimens are cut, then the calculated diffusivity for a given set of specimens will be dependent on the precise locations from which these specimens were obtained.

1.4.2 *Variable Porosity*—The thermal properties of porous rock are highly dependent on the amount and nature of the porosity. A spatially varying porosity introduces problems of a nature similar to those encountered with a spatially varying composition. In addition, the character of the porosity may preclude complete dehydration by oven drying.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*

C 177 Test Method for Steady-State Thermal Transmission Properties by Means of the Guarded Hot Plate[2]

C 351 Test Method for Mean Specific Heat of Thermal Insulation[2]

C 518 Test Method for Steady-State Thermal Transmission Properties by Means of the Heat Flow Meter[2]

C 642 Test Method for Specific Gravity, Absorption, and Voids in Hardened Concrete[3]

D 2766 Test Method for Specific Heat of Liquids and Solids[4]

D 4611 Test Method for Specific Heat of Rock and Soil[5]

3. Terminology

3.1 *Parameter Definitions:*

3.1.1 *mass density*—the mass of the sample per unit volume of sample, $\rho(\mathrm{kg/m^3})$.

3.1.2 *instantaneous specific heat*—the rate of change of specimen enthalpy per unit mass, h, with respect to temperature, T, at constant pressure,

$$p, c_p = (\delta h/\delta T)_p$$

3.1.3 *thermal conductivity*—the constant of proportionality, k, relating the vector heat flux, $\rightarrow \Phi$ expressed in watts per square metre, to the temperature gradient, ∇T, $\rightarrow \Phi = -k\nabla T$. The thermal conductivity may be a function of the direction of $\rightarrow \Phi$ and the temperature, T. The units of k are $W/m - K$.

[1] This practice is under the jurisdiction of ASTM Committee D-18 on Soil and Rock and is the direct responsibility of Subcommittee D18.12 on Rock Mechanics.

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[2] *Annual Book of ASTM Standards*, Vol 04.06.

[3] *Annual Book of ASTM Standards*, Vol 04.02.

[4] *Annual Book of ASTM Standards*, Vol 05.02.

[5] *Annual Book of ASTM Standards*, Vol 04.08.

3.1.4 *thermal diffusivity*—the thermal diffusivity, α, is a derived parameter. It is related to ρ, c_p, and k by the relation,

$$\alpha = k/\rho c_p$$

The units of α are m^2/s.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *sample*—a sample is a large piece of rock from which the specimens used in the k, ρ, and c_p measurements are obtained. Usually the samples are obtained in the form of cores from a drilling operation.

3.2.2 *specimens*—the specimens are pieces cut from the sample for the k, ρ, and c_p measurements. Their sizes and shapes are governed by the applicable ASTM standards listed in 2.1.

4. Summary of Practice

4.1 The thermal diffusivity is determined from the equation in 3.1.4. The data for k and c_p must be available over the temperature range of interest. For density, ρ, a single measurement at room temperature may be used because the density is approximately constant over the 20 to 300°C temperature range covered by this practice.

4.2 The measurements of k, ρ, and c_p are to be performed using the test methods in Section 6.

5. Significance and Use

5.1 The thermal diffusivity is a parameter that arises in the solution of transient heat conduction problems. It generally characterizes the rate at which a heat pulse will diffuse through a solid material.

5.2 The number of parameters required for solution of a transient heat conduction problem depends on both the geometry and imposed boundary conditions. In a few special cases, only the thermal diffusivity of the material is required. In most cases, separate values of k, ρ, and c_p are required in addition to α. This practice provides a consistent set of parameters for numerical or analytical heat conduction calculations related to heat transport through rocks.

5.3 In order to use this practice for determination of the thermal diffusivity, all of the required parameters (k, ρ, c_p) must be determined under as near identical specimen conditions as possible.

5.4 The diffusivity determined by this practice can only be used to analyze heat transport in rock under thermal conditions identical to those existing for the k, ρ, and c_p measurements.

6. Procedure

6.1 *General*:

6.1.1 Obtain the data for k, ρ, and c_p as a function of temperature using the appropriate ASTM standard for the given test as qualified below. If possible, make all measurements with specimens obtained from the same general location in the sample in order to ensure that the specimens are as near identical in composition and morphology as possible.

6.1.2 To minimize water content variation among specimens, vacuum dry all specimens at 105°C until mass loss is constant to ±0.5 %.

6.2 *Parameter Test Procedures*:

6.2.1 Measure the specimen density in one of the two following ways:

6.2.1.1 Determine the mass of the specimen, m_s, on an analytical balance and the volume, V_s, by measurement of sample dimensions or by water displacement (immersion). If the volume is measured by immersion, the specimen must be encapsulated in a waterproof flexible container of negligible volume compared to the specimen volume. Record the density, as follows,

$$\rho = m_s/V_s \tag{1}$$

where:

m_s = specimen mass, and
V_s = specimen volume.

Also estimate the accuracy of the ρ determination from the uncertainties associated with the m_s and V_s measurements.

6.2.1.2 Measure the specimen specific gravity using Test Method C 642. In situations where the measurement is to be made at temperatures near or above the boiling point of water, a suitable oil working fluid may be substituted for water in this procedure. Determine the density, ρ, by multiplying the specific gravity by the density of the working fluid at the immersion temperature.

6.2.2 Measure the specimen specific heat using Test Method D 4611.

6.2.3 Measure the specimen conductivity using Test Method C 518 or Test Method C 177.

7. Calculations

7.1 *General*—The following method of calculation is recommended for deriving the temperature dependent diffusivity, $\alpha(T)$, from data from k, ρ, and c_p.

NOTE 3—The recommended data analysis technique is not intended to preclude the use of other methods of data analysis which may be more suitable in certain cases. It does provide a method by which a consistent set of temperature dependent parameters may be derived from the primary data base, and also a method by which the uncertainties in each parameter may be estimated. The results of the calculations for the temperature dependent parameters will be in a form which is useful for most thermal analysis computer programs.

7.2 *Description of the Method*—The parameter data for an associated set of specimens will usually be in the form of tables giving the measured parameter value versus the measurement temperature. Each parameter should be fit to an equation of the following form:

$$\gamma(T) = \sum_{n=o}^{N} c_n (T - T_o)^n \tag{2}$$

where:

γ = parameter (k, ρ, or, c_p),
T_o = 293 K,
T = temperature (K), and
N = maximum power used in the fit.

7.2.1 The fit shall be performed using ordinary least squares techniques.[6] The value of N should be as small as possible, consistent with obtaining a reasonable fit to the data. The result of this fit will be a set of coefficients, (c_n), and the estimated standard error in the parameter, given by the following equation:

[6] Beck, J. V., and Arnold, K. J. *Parameter Estimation in Engineering and Science*, John Wiley, NY, 1977, pp. 234–237.

$$s_\gamma \{\sum_{i=1}^{M} [_\gamma(T_i) - {}_{\gamma i}(T_i)]^2\}^{1/2} \tag{3}$$

where:

M = number of temperatures at which γ is measured, and
γ_i = measured value of the parameter at temperature T_i.

NOTE 4—In the case of density, a measurement is usually available only at room temperature, T_o. In this situation, take $\rho(T) = \rho(T_o)$, corresponding to $c_o = \rho(T_o)$, $c_n = 0$ for $n > 0$. The associated error, s_ρ, is the estimated error for the single measurement.

7.3 Calculate the diffusivity from the curve fit relation determined in 7.1 and 7.2 as follows:

$$\alpha(T) = k(T)/[\rho(T)c_p(T)]$$

$$\text{or } \alpha(T) = k(T)/[\rho(T_o)c_p(T)]$$

7.4 Fit $\alpha(T)$ to an equation of the form of Eq 2, and calculate s_α from (Eq 3).

7.5 Estimate the error in α, $\delta\alpha$, caused by measurement errors in k, ρ, and c_p from the equation,

$$(\delta\alpha/\alpha)^2 = (\delta k/k)^2 + (\delta\rho/\rho)^2 + (\delta c_p/c_p)^2 \tag{4}$$

7.5.1 The relative parameter errors; $\delta k/k$, $\delta\rho/\rho$, and $\delta c_p/c_p$, are determined or estimated, or both, in each of the separate parameter measurements where the appropriate ASTM procedures are used.

8. Report

8.1 The report shall contain the following information:

8.1.1 Data base used for the diffusivity calculation giving k, ρ, and c_p versus the temperatures at which each parameter was measured.

8.1.2 The methods used to obtain the data and any deviations from ASTM procedures in these methods. If one or more of the parameters was determined from literature values in lieu of direct measurement, then a complete reference should be given and, in addition, the following information should be quoted directly from the cited references:

8.1.2.1 Raw parameter versus temperature data unless the parameter was determined from a recommended curve.

8.1.2.2 The estimated relative error in the reported value, $\delta\gamma/\gamma$, where $\gamma = k$, ρ, or c_p, and the method of determining this error, if reported.

8.1.3 The method used to derive the diffusivity from the data in 8.1.2.1. If the recommended method in 7.2 to 7.4 is not followed, a complete description of the alternate method used should be given. In the case where the recommended method is employed, the following calculated results should be reported:

8.1.3.1 The coefficients, c_n; $n = 0, 1, ..., N$ characterizing the fit of each parameter as a function of temperature, Eq 2, and the estimated standard error for each fit, s_γ, from Eq 3.

8.1.3.2 The coefficients characterizing $\alpha(T)$ from 7.4 and the estimated standard error of the fit s_α.

8.1.3.3 The estimated relative error in α, as found in 7.5.

8.1.4 Sample identification and characterization information.

8.1.4.1 Identification of block or core sample from which the specimens were cut, including geographic location and depth from which the sample was obtained.

8.1.4.2 Qualitative description of sample mineralogy, morphology, isotropy.

8.1.4.3 Sample dimensions.

8.1.4.4 Dimensions of specimens used in each parameter measurement and location relative to the sample from which each specimen was taken.

8.1.4.5 Specimen porosity, if measured, and method of determination.

8.1.4.6 Specimen residual saturation, if measured, and method of determination.

9. Keywords

9.1 density; heating tests-specific heat; enthalpy; isotropic content; porosity; rock; temperature tests; thermal analysis-diffusivity