CS 224n Assignment #2: Word2Vec and Dependency Parsing

Due Date: April 18th, Thursday, 4:30 PM PST.

In this assignment, you will review the mathematics behind Word2Vec and build a neural dependency parser using PyTorch. For a review of the fundamentals of PyTorch, please check out the PyTorch review session on Canvas. In Part 1, you will explore the partial derivatives involved in training a Word2vec model using the naive softmax loss. In Part 2, you will learn about two general neural network techniques (Adam Optimization and Dropout). In Part 3, you will implement and train a dependency parser using the techniques from Part 2, before analyzing a few erroneous dependency parses.

If you are using LaTeX, you can use `\ifans{}` to type your solutions.

Please tag the questions correctly on Gradescope, otherwise the TAs will take points off if you don't tag questions.

1. Understanding word2vec (15 points)

Recall that the key insight behind `word2vec` is that *'a word is known by the company it keeps'*. Concretely, consider a 'center' word c surrounded before and after by a context of a certain length. We term words in this contextual window 'outside words' (O). For example, in Figure 1, the context window length is 2, the center word c is 'banking', and the outside words are 'turning', 'into', 'crises', and 'as':

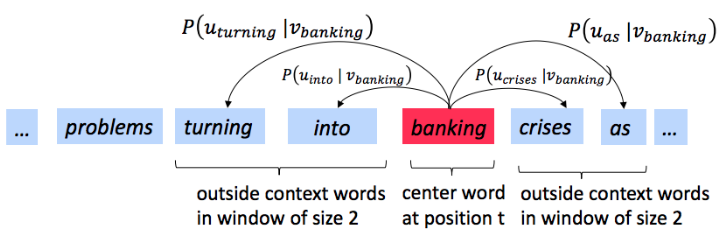


Figure 1: The word2vec skip-gram prediction model with window size 2

Skip-gram `word2vec` aims to learn the probability distribution $P(O|C)$. Specifically, given a specific word o and a specific word c, we want to predict $P(O = o|C = c)$: the probability that word o is an 'outside' word for c (i.e., that it falls within the contextual window of c). We model this probability by taking the softmax function over a series of vector dot-products:

$$P(O = o \mid C = c) = \frac{\exp(\mathbf{u}_o^\top \mathbf{v}_c)}{\sum_{w \in \text{Vocab}} \exp(\mathbf{u}_w^\top \mathbf{v}_c)} \tag{1}$$

For each word, we learn vectors u and v, where $\mathbf{u}_o$ is the 'outside' vector representing outside word o, and $\mathbf{v}_c$ is the 'center' vector representing center word c. We store these parameters in two matrices, $\mathbf{U}$ and $\mathbf{V}$. The columns of $\mathbf{U}$ are all the 'outside' vectors $\mathbf{u}_w$; the columns of $\mathbf{V}$ are all of the 'center' vectors $\mathbf{v}_w$. Both $\mathbf{U}$ and $\mathbf{V}$ contain a vector for every $w \in$ Vocabulary.[1]

Recall from lectures that, for a single pair of words c and o, the loss is given by:

$$\mathbf{J}_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U}) = -\log P(O = o|C = c). \tag{2}$$

[1] Assume that every word in our vocabulary is matched to an integer number k. Bolded lowercase letters represent vectors. $\mathbf{u}_k$ is both the k^{th} column of $\mathbf{U}$ and the 'outside' word vector for the word indexed by k. $\mathbf{v}_k$ is both the k^{th} column of $\mathbf{V}$ and the 'center' word vector for the word indexed by k. **In order to simplify notation we shall interchangeably use k to refer to word k and the index of word k.**

We can view this loss as the cross-entropy[2] between the true distribution $\mathbf{y}$ and the predicted distribution $\hat{\mathbf{y}}$, for a particular center word c and a particular outside word o. Here, both $\mathbf{y}$ and $\hat{\mathbf{y}}$ are vectors with length equal to the number of words in the vocabulary. Furthermore, the k^{th} entry in these vectors indicates the conditional probability of the k^{th} word being an 'outside word' for the given c. The true empirical distribution $\mathbf{y}$ is a one-hot vector with a 1 for the true outside word o, and 0 everywhere else, for this particular example of center word c and outside word o.[3] The predicted distribution $\hat{\mathbf{y}}$ is the probability distribution $P(O|C = c)$ given by our model in equation (1).

Note: Throughout this homework, when computing derivatives, please use the method reviewed during the lecture (i.e. no Taylor Series Approximations).

[2] The **cross-entropy loss** between the true (discrete) probability distribution p and another distribution q is $-\sum_i p_i \log(q_i)$.
[3] Note that the true conditional probability distribution of context words for the entire training dataset would not be one-hot.

(a) (2 points) Prove that the naive-softmax loss (Equation 2) is the same as the cross-entropy loss between $\mathbf{y}$ and $\hat{\mathbf{y}}$, i.e. (note that $\mathbf{y}$ (true distribution), $\hat{\mathbf{y}}$ (predicted distribution) are vectors and $\hat{\mathbf{y}}_o$ is a scalar):

$$-\sum_{w \in \text{Vocab}} \mathbf{y}_w \log(\hat{\mathbf{y}}_w) = -\log(\hat{\mathbf{y}}_o). \tag{3}$$

Your answer should be one line. You may describe your answer in words.

Answer: The true empirical distribution (i.e., the ground truth) $\mathbf{y}$ is a one-hot vector where $\mathbf{y}_w = 1$ when $w = o$ and $\mathbf{y}_w = 0$ when $w \neq o$. Mathematically,

$$\mathbf{y}_w = \begin{cases} 1 \text{ if } w = o \\ 0 \text{ if } w \neq o \end{cases}$$

As such, considering the cross-entropy loss $J_{cross-entropy}$,

$$J_{cross-entropy}(\mathbf{y}, \hat{\mathbf{y}}) = -\sum_{w \in Vocab} \mathbf{y}_w \log(\hat{\mathbf{y}}_w)$$

$$= -(y_1 \log(\hat{y}_1) + \cdots + y_o \log(\hat{y}_o) + \cdots + y_{|V|} \log(\hat{y}_{|V|}))$$

$$= -y_o \log(\hat{y}_o)$$

$$= -\log(\hat{y}_o)$$

$$= -\log(P(O = o \mid C = c))$$

$$= J_{naive-softmax}(\mathbf{v}_c, o, \mathbf{U})$$

Thus, the naive-softmax loss $J_{naive-softmax}$ given in Equation (2) is the same as the cross-entropy loss $J_{cross-entropy}$.

(b) (6 points) i. Compute the partial derivative of $\mathbf{J}_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})$ with respect to $\mathbf{v}_c$. *Please write your answer in terms of* $\mathbf{y}$, $\hat{\mathbf{y}}$, $\mathbf{U}$, *and show your work to receive full credit.*

- **Note**: Your final answers for the partial derivative should follow the shape convention: the partial derivative of any function $f(x)$ with respect to x should have the **same shape** as x.[4]
- Please provide your answers for the partial derivative in vectorized form. For example, when we ask you to write your answers in terms of $\mathbf{y}$, $\hat{\mathbf{y}}$, and $\mathbf{U}$, you may not refer to specific elements of these terms in your final answer (such as $\mathbf{y}_1$, $\mathbf{y}_2$, $\ldots$).

Answer:

$$\frac{\partial J_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})}{\partial \mathbf{v}_c} = \frac{\partial}{\partial \mathbf{v}_c}(-\log(\hat{y}_o))$$
$$= \frac{\partial}{\partial \mathbf{v}_c}\left(-\log\left(\frac{\exp(\mathbf{u}_o^T \mathbf{v}_c)}{\sum_{w \in Vocab} \exp(\mathbf{u}_w^T \mathbf{v}_c)}\right)\right)$$

[4] This allows us to efficiently minimize a function using gradient descent without worrying about reshaping or dimension mismatching. While following the shape convention, we're guaranteed that $\theta := \theta - \alpha \frac{\partial J(\theta)}{\partial \theta}$ is a well-defined update rule.

Applying the divisive property of logarithms, i.e., $\log\left(\frac{x}{y}\right) = \log(x) - \log(y)$,

$$\frac{\partial J_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})}{\partial \mathbf{v}_c} = \frac{\partial}{\partial \mathbf{v}_c}\left[-\left(\log(\exp(\mathbf{u}_o^T\mathbf{v}_c)) - \log\sum_{w\in Vocab}\exp(\mathbf{u}_w^T\mathbf{v}_c)\right)\right]$$

$$= \frac{\partial}{\partial \mathbf{v}_c}\left[-\log(\exp(\mathbf{u}_o^T\mathbf{v}_c)) + \log\sum_{w\in Vocab}\exp(\mathbf{u}_w^T\mathbf{v}_c)\right]$$

$$= \frac{\partial}{\partial \mathbf{v}_c}\left[-\mathbf{u}_o^T\mathbf{v}_c + \log\sum_{w\in Vocab}\exp(\mathbf{u}_w^T\mathbf{v}_c)\right]$$

$$= -\frac{\partial}{\partial \mathbf{v}_c}(\mathbf{u}_o^T\mathbf{v}_c) + \frac{\partial}{\partial \mathbf{v}_c}\left[\log\sum_{w\in Vocab}\exp(\mathbf{u}_w^T\mathbf{v}_c)\right]$$

Since $\frac{\partial \mathbf{u}^T\mathbf{x}}{\partial \mathbf{x}} = \mathbf{u}$, we have $\frac{\partial}{\partial \mathbf{v}_c}(\mathbf{u}_o^T\mathbf{v}_c) = \mathbf{u}_o$,

$$\frac{\partial J_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})}{\partial \mathbf{v}_c} = -\mathbf{u}_o + \frac{\partial}{\partial \mathbf{v}_c}\left[\log\sum_{w\in Vocab}\exp(\mathbf{u}_w^T\mathbf{v}_c)\right]$$

Using the chain rule of derivatives on log,

$$\frac{\partial J_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})}{\partial \mathbf{v}_c} = -\mathbf{u}_o + \frac{1}{\sum_{w\in Vocab}\exp(\mathbf{u}_w^T\mathbf{v}_c)}\sum_{w\in Vocab}\frac{\partial}{\partial \mathbf{v}_c}\left[\exp(\mathbf{u}_w^T\mathbf{v}_c)\right]$$

Since $\frac{\partial \exp(\mathbf{u}^T\mathbf{x})}{\partial \mathbf{x}} = \exp(\mathbf{u}^T\mathbf{x})\cdot\mathbf{u}$ as per the chain rule,

$$\frac{\partial J_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})}{\partial \mathbf{v}_c} = -\mathbf{u}_o + \frac{1}{\sum_{w\in Vocab}\exp(\mathbf{u}_w^T\mathbf{v}_c)}\sum_{w\in Vocab}\exp(\mathbf{u}_w^T\mathbf{v}_c)\cdot\mathbf{u}_w$$

Since $\frac{\exp(\mathbf{u}_w^T\mathbf{v}_c)}{\sum_{w'\in Vocab}\exp(\mathbf{u}_{w'}^T\mathbf{v}_c)}$ is the conditional probability distribution $\hat{y}_w$ per word2vec,

$$\frac{\partial J_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})}{\partial \mathbf{v}_c} = -\mathbf{u}_o + \sum_{w\in Vocab}\hat{y}_w\mathbf{u}_w$$

Since $y_w = \begin{cases}1 \text{ if } w = o\\ 0 \text{ if } w \neq o\end{cases}$, we can write $\mathbf{u}_o^T\mathbf{v}_c$ in the above equation as $\sum_{w\in Vocab} y_w\mathbf{u}_w$. As such,

$$\frac{\partial J_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})}{\partial \mathbf{v}_c} = -\sum_{w\in Vocab} y_w\mathbf{u}_w + \sum_{w\in Vocab}\hat{y}_w\mathbf{u}_w$$

$$= \sum_{w\in Vocab}\left[-y_w\mathbf{u}_w + \hat{y}_w\mathbf{u}_w\right]$$

$$= \sum_{w\in Vocab}\mathbf{u}_w(-y_w + \hat{y}_w)$$

$$= \sum_{w\in Vocab}\mathbf{u}_w(\hat{y}_w - y_w)$$

Note that y is a 1-hot vector with a 1 at word o. Vectorizing the above equation in terms of y, $\hat{y}$, and U,

$$\frac{\partial J_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})}{\partial \mathbf{v}_c} = U(\hat{y} - y)$$

ii. When is the gradient you computed equal to zero?
Answer: The gradient is zero when the predicted distribution $\hat{\mathbf{y}}$ equals the true distribution $\mathbf{y}$, meaning $\hat{\mathbf{y}} = \mathbf{y}$ for all w.

iii. The gradient you found is the difference between the two terms. Provide an interpretation of how each of these terms improves the word vector when this gradient is subtracted from the word vector v_c.
Answer: The term $\hat{\mathbf{y}}_w \mathbf{u}_w$ represents the model's prediction, and $\mathbf{y}_w \mathbf{u}_w$ represents the true distribution. Subtracting the gradient $\mathbf{U}(\hat{\mathbf{y}} - \mathbf{y})$ from $\mathbf{v}_c$ adjusts the center word vector to make the model's predictions more accurate by reducing the error between the predicted and true distributions.

(c) (1 point) In many downstream applications using word embeddings, L2 normalized vectors (e.g. $\mathbf{u}/||\mathbf{u}||_2$ where $||\mathbf{u}||_2 = \sqrt{\sum_i u_i^2}$) are used instead of their raw forms (e.g. $\mathbf{u}$). Let's consider a hypothetical downstream task of binary classification of phrases as being positive or negative, where you decide the sign based on the sum of individual embeddings of the words. When would L2 normalization take away useful information for the downstream task? When would it not?

Hint: Consider the case where $\mathbf{u}_x = \alpha \mathbf{u}_y$ for some words $x \neq y$ and some scalar α. When α is positive, what will be the value of normalized $\mathbf{u}_x$ and normalized $\mathbf{u}_y$? How might $\mathbf{u}_x$ and $\mathbf{u}_y$ be related for such a normalization to affect or not affect the resulting classification?

Answer:

L2 normalization would take away useful information when the magnitude of the vectors carries important information (e.g., frequency or importance). If $\mathbf{u}_x = \alpha \mathbf{u}_y$, after normalization, $\mathbf{u}'_x$ and $\mathbf{u}'_y$ would be identical, losing the information about their relative magnitudes. This could affect the classification if the decision relies on the magnitude of the embeddings.

However, if the task only relies on the direction of the vectors (i.e., their orientation in the vector space), normalization would not affect the resulting classification. In this case, the normalized vectors $\mathbf{u}'_x$ and $\mathbf{u}'_y$ still provide the necessary directional information for making the decision.

(d) (5 points) Compute the partial derivatives of $\mathbf{J}_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})$ with respect to each of the 'outside' word vectors, $\mathbf{u}_w$'s. There will be two cases: when $w = o$, the true 'outside' word vector, and $w \neq o$, for all other words. Please write your answer in terms of $\mathbf{y}$, $\hat{\mathbf{y}}$, and $\mathbf{v}_c$. In this subpart, you may use specific elements within these terms as well (such as $\mathbf{y}_1$, $\mathbf{y}_2$, $\ldots$). Note that $\mathbf{u}_w$ is a vector while $\mathbf{y}_1, \mathbf{y}_2, \ldots$ are scalars. Show your work to receive full credit.

Answer:

$$\frac{\partial J_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})}{\partial \mathbf{u}_w} = \frac{\partial}{\partial \mathbf{u}_w}(-\log(\hat{y}_o))$$
$$= \frac{\partial}{\partial \mathbf{u}_w}\left(-\log\left(\frac{\exp(\mathbf{u}_o^T \mathbf{v}_c)}{\sum_{w \in Vocab} \exp(\mathbf{u}_w^T \mathbf{v}_c)}\right)\right)$$

Applying the divisive property of logarithms, i.e., $\log\left(\frac{x}{y}\right) = \log(x) - \log(y)$,

$$\frac{\partial J_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})}{\partial \mathbf{u}_w} = \frac{\partial}{\partial \mathbf{u}_w}\left[-\left(\log(\exp(\mathbf{u}_o^T \mathbf{v}_c)) - \log \sum_{w \in Vocab} \exp(\mathbf{u}_w^T \mathbf{v}_c)\right)\right]$$
$$= \frac{\partial}{\partial \mathbf{u}_w}\left[-\log(\exp(\mathbf{u}_o^T \mathbf{v}_c)) + \log \sum_{w \in Vocab} \exp(\mathbf{u}_w^T \mathbf{v}_c)\right]$$
$$= \frac{\partial}{\partial \mathbf{u}_w}\left[-\mathbf{u}_o^T \mathbf{v}_c + \log \sum_{w \in Vocab} \exp(\mathbf{u}_w^T \mathbf{v}_c)\right]$$

$$= -\frac{\partial}{\partial \mathbf{u}_w}(\mathbf{u}_o^T \mathbf{v}_c) + \frac{\partial}{\partial \mathbf{u}_w}\left[\log \sum_{w \in Vocab} \exp(\mathbf{u}_w^T \mathbf{v}_c)\right]$$

Since $\frac{\partial \mathbf{u}^T \mathbf{x}}{\partial \mathbf{x}} = \mathbf{u}$, we have $\frac{\partial}{\partial \mathbf{u}_w}(\mathbf{u}_o^T \mathbf{v}_c) = \mathbf{v}_c$,

$$\frac{\partial J_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})}{\partial \mathbf{u}_w} = -\mathbf{v}_c + \frac{\partial}{\partial \mathbf{u}_w}\left[\log \sum_{w \in Vocab} \exp(\mathbf{u}_w^T \mathbf{v}_c)\right]$$

Using the chain rule of derivatives on log,

$$\frac{\partial J_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})}{\partial \mathbf{u}_w} = -\mathbf{v}_c + \frac{1}{\sum_{w \in Vocab} \exp(\mathbf{u}_w^T \mathbf{v}_c)} \sum_{w \in Vocab} \frac{\partial}{\partial \mathbf{u}_w}\left[\exp(\mathbf{u}_w^T \mathbf{v}_c)\right]$$

Since $\frac{\partial \exp(\mathbf{u}^T \mathbf{x})}{\partial \mathbf{x}} = \exp(\mathbf{u}^T \mathbf{x}) \cdot \mathbf{u}$ as per the chain rule,

$$\frac{\partial J_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})}{\partial \mathbf{u}_w} = -\mathbf{v}_c + \frac{1}{\sum_{w \in Vocab} \exp(\mathbf{u}_w^T \mathbf{v}_c)} \sum_{w \in Vocab} \exp(\mathbf{u}_w^T \mathbf{v}_c) \cdot \mathbf{v}_c$$

Since $\frac{\exp(\mathbf{u}_w^T \mathbf{v}_c)}{\sum_{w' \in Vocab} \exp(\mathbf{u}_{w'}^T \mathbf{v}_c)}$ is the conditional probability distribution $\hat{y}_w$ per word2vec,

$$\frac{\partial J_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})}{\partial \mathbf{u}_w} = -\mathbf{v}_c + \sum_{w \in Vocab} \hat{y}_w \mathbf{v}_c$$

Since $y_w = \begin{cases} 1 \text{ if } w = o \\ 0 \text{ if } w \neq o \end{cases}$, we can write $\mathbf{u}_o^T \mathbf{v}_c$ in the above equation as $\sum_{w \in Vocab} y_w \mathbf{u}_w$. As such,

$$\begin{aligned}
\frac{\partial J_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})}{\partial \mathbf{u}_w} &= -\sum_{w \in Vocab} y_w \mathbf{v}_c + \sum_{w \in Vocab} \hat{y}_w \mathbf{v}_c \\
&= \sum_{w \in Vocab} \left[-y_w \mathbf{v}_c + \hat{y}_w \mathbf{v}_c\right] \\
&= \sum_{w \in Vocab} \mathbf{v}_c(-y_w + \hat{y}_w) \\
&= \sum_{w \in Vocab} \mathbf{v}_c(\hat{y}_w - y_w)
\end{aligned}$$

Since $\sum_{w \in Vocab} \hat{y}_w = 1$, we can rewrite $\sum_{w \in Vocab} y_w \mathbf{u}_w$ in the above equation as $\sum_{w \in Vocab} y_w \mathbf{u}_w$. As such,

$$\begin{aligned}
\frac{\partial J_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})}{\partial \mathbf{u}_w} &= -y_w \mathbf{v}_c + \sum_{w \in Vocab} \hat{y}_w \mathbf{v}_c \\
&= \sum_{w \in Vocab} \left[-y_w \mathbf{v}_c + \hat{y}_w \mathbf{v}_c\right] \\
&= \sum_{w \in Vocab} \mathbf{v}_c(-y_w + \hat{y}_w) \\
&= \sum_{w \in Vocab} \mathbf{v}_c(\hat{y}_w - y_w)
\end{aligned}$$

Note that y is a 1-hot vector with a 1 at word o. Vectorizing the above equation in terms of y, $\hat{y}$, and U,

$$\frac{\partial J_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})}{\partial \mathbf{u}_w} = U(\hat{y} - y)$$

(e) (1 point) Write down the partial derivative of $\mathbf{J}_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})$ with respect to $\mathbf{U}$. Please break down your answer in terms of the column vectors $\frac{\partial \mathbf{J}(\mathbf{v}_c,o,\mathbf{U})}{\partial \mathbf{u}_1}$, $\frac{\partial \mathbf{J}(\mathbf{v}_c,o,\mathbf{U})}{\partial \mathbf{u}_2}$, $\cdots$, $\frac{\partial \mathbf{J}(\mathbf{v}_c,o,\mathbf{U})}{\partial \mathbf{u}_{|\text{Vocab}|}}$. No derivations are necessary, just an answer in the form of a matrix.

Answer:

The derivative of a scalar y by a matrix A is given by,

$$\frac{\partial y}{\partial A_{m\times n}} = \begin{bmatrix} \frac{\partial y}{\partial A_{11}} & \cdots & \frac{\partial y}{\partial A_{1n}} \\ \vdots & \ddots & \vdots \\ \frac{\partial y}{\partial A_{m1}} & \cdots & \frac{\partial y}{\partial A_{mn}} \end{bmatrix}$$

Given $\mathbf{u}_w$ represents the vector for 'outside' word w, the derivative of $J_{\text{naive-softmax}}$ (which is a scalar) by U (which is a matrix) is,

$$\frac{\partial J_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})}{\partial \mathbf{U}} = \begin{bmatrix} \frac{\partial J}{\partial \mathbf{u}_1} & \frac{\partial J}{\partial \mathbf{u}_2} & \cdots & \frac{\partial J}{\partial \mathbf{u}_{|Vocab|}} \end{bmatrix}$$

where,

$$\frac{\partial J}{\partial \mathbf{u}_w} = \begin{cases} \mathbf{v}_c(\hat{y}_w - 1) \text{ if } w = o \\ \mathbf{v}_c \hat{y}_w \text{ if } w \neq o \end{cases}$$

2. Machine Learning & Neural Networks (8 points)

(a) (4 points) Adam Optimizer

Recall the standard Stochastic Gradient Descent update rule:

$$\boldsymbol{\theta}_{t+1} \leftarrow \boldsymbol{\theta}_t - \alpha \nabla_{\boldsymbol{\theta}_t} J_{\text{minibatch}}(\boldsymbol{\theta}_t)$$

where $t+1$ is the current timestep, $\boldsymbol{\theta}$ is a vector containing all of the model parameters, ($\boldsymbol{\theta}_t$ is the model parameter at time step t, and $\boldsymbol{\theta}_{t+1}$ is the model parameter at time step $t+1$), J is the loss function, $\nabla_{\boldsymbol{\theta}} J_{\text{minibatch}}(\boldsymbol{\theta})$ is the gradient of the loss function with respect to the parameters on a minibatch of data, and α is the learning rate. Adam Optimization[5] uses a more sophisticated update rule with two additional steps.[6]

i. (2 points) First, Adam uses a trick called *momentum* by keeping track of $\mathbf{m}$, a rolling average of the gradients:

$$\begin{aligned}
\mathbf{m}_{t+1} &\leftarrow \beta_1 \mathbf{m}_t + (1-\beta_1)\nabla_{\boldsymbol{\theta}_t} J_{\text{minibatch}}(\boldsymbol{\theta}_t) \\
\boldsymbol{\theta}_{t+1} &\leftarrow \boldsymbol{\theta}_t - \alpha \mathbf{m}_{t+1}
\end{aligned}$$

where β_1 is a hyperparameter between 0 and 1 (often set to 0.9). Briefly explain in 2–4 sentences (you don't need to prove mathematically, just give an intuition) how using $\mathbf{m}$ stops the updates from varying as much and why this low variance may be helpful to learning, overall.

Answer: The momentum trick ensures that the updates made to network parameters are not "drastic" in some sense. The value of β is typically set to 0.9 or 0.95. What this implies is that, roughly 95% of the updates to θ_{t+1} is still contributed by $\mathbf{m}_t$, with just 5% coming from the gradient update. This ensures that, in case of an unusually large gradient value, θ_{t+1} is not significantly affected by the step. This can be thought of as a "smoothing" effect, hence reducing variance.

A reduction in variance of the updates, in turn, ensure that learning is more stable. The steps taken are "smoothened" and we do not see the gradients ricochet back and forth near minimas.

ii. (2 points) Adam extends the idea of *momentum* with the trick of *adaptive learning rates* by keeping track of $\mathbf{v}$, a rolling average of the magnitudes of the gradients:

$$\begin{aligned}
\mathbf{m}_{t+1} &\leftarrow \beta_1 \mathbf{m}_t + (1-\beta_1)\nabla_{\boldsymbol{\theta}_t} J_{\text{minibatch}}(\boldsymbol{\theta}_t) \\
\mathbf{v}_{t+1} &\leftarrow \beta_2 \mathbf{v}_t + (1-\beta_2)(\nabla_{\boldsymbol{\theta}_t} J_{\text{minibatch}}(\boldsymbol{\theta}_t) \odot \nabla_{\boldsymbol{\theta}_t} J_{\text{minibatch}}(\boldsymbol{\theta}_t)) \\
\boldsymbol{\theta}_{t+1} &\leftarrow \boldsymbol{\theta}_t - \alpha \mathbf{m}_{t+1} / \sqrt{\mathbf{v}_{t+1}}
\end{aligned}$$

where $\odot$ and $/$ denote elementwise multiplication and division (so $\mathbf{z} \odot \mathbf{z}$ is elementwise squaring) and β_2 is a hyperparameter between 0 and 1 (often set to 0.99). Since Adam divides the update by $\sqrt{\mathbf{v}}$, which of the model parameters will get larger updates? Why might this help with learning?

Answer: The Adam optimization algorithm combines the concepts of momentum and adaptive learning rates to achieve efficient and stable learning.

The momentum term $\mathbf{m}_{t+1}$ is a running average of the gradients, which helps smooth out the updates by reducing the variance caused by noisy gradients. The adaptive learning rate is controlled by $\mathbf{v}_{t+1}$, which is a running average of the squared gradients. This adaptive rate ensures that each parameter has its learning rate adjusted based on the magnitude of its gradients.

---

[5] Kingma and Ba, 2015, `https://arxiv.org/pdf/1412.6980.pdf`

[6] The actual Adam update uses a few additional tricks that are less important, but we won't worry about them here. If you want to learn more about it, you can take a look at: `http://cs231n.github.io/neural-networks-3/#sgd`

Since Adam divides the update by $\sqrt{\mathbf{v}_{t+1}}$, parameters with smaller values in $\mathbf{v}_{t+1}$ (i.e., parameters with smaller average squared gradients) will receive larger updates. This means that parameters that have not been changing much (due to consistently small gradients) will be updated more significantly, allowing them to catch up and potentially explore more beneficial directions.

This mechanism helps in learning by providing stability and efficiency:

- **Stability:** Parameters with large gradient magnitudes (which can cause unstable updates) will have their updates scaled down, preventing drastic changes.
- **Efficiency:** Parameters with small gradient magnitudes (which might need larger updates to move out of flat regions) will receive larger updates, accelerating convergence.

Overall, the combination of momentum and adaptive learning rates in Adam leads to a more robust optimization process, capable of handling the varying scales of different parameters and improving the overall learning dynamics.

(b) (4 points) Dropout[7] is a regularization technique. During training, dropout randomly sets units in the hidden layer $\mathbf{h}$ to zero with probability p_{drop} (dropping different units each minibatch), and then multiplies $\mathbf{h}$ by a constant γ. We can write this as:

$$\mathbf{h}_{\text{drop}} = \gamma \mathbf{d} \odot \mathbf{h}$$

where $\mathbf{d} \in \{0,1\}^{D_h}$ (D_h is the size of $\mathbf{h}$) is a mask vector where each entry is 0 with probability p_{drop} and 1 with probability $(1 - p_{\text{drop}})$. γ is chosen such that the expected value of $\mathbf{h}_{\text{drop}}$ is $\mathbf{h}$:

$$\mathbb{E}_{p_{\text{drop}}}[\mathbf{h}_{\text{drop}}]_i = h_i$$

for all $i \in \{1, \ldots, D_h\}$.

i. (2 points) What must γ equal in terms of p_{drop}? Briefly justify your answer or show your math derivation using the equations given above.

Answer:

i. To find γ such that $\mathbb{E}_{p_{\text{drop}}}[\mathbf{h}_{\text{drop}}]_i = h_i$, we start with the expectation:

$$\mathbb{E}[h_{\text{drop},i}] = \mathbb{E}[\gamma d_i h_i] = \gamma h_i \mathbb{E}[d_i].$$

Since d_i is 0 with probability p_{drop} and 1 with probability $(1 - p_{\text{drop}})$, we have:

$$\mathbb{E}[d_i] = 0 \cdot p_{\text{drop}} + 1 \cdot (1 - p_{\text{drop}}) = 1 - p_{\text{drop}}.$$

Thus,

$$\mathbb{E}[h_{\text{drop},i}] = \gamma h_i (1 - p_{\text{drop}}).$$

To ensure $\mathbb{E}[h_{\text{drop},i}] = h_i$, we set:

$$\gamma h_i (1 - p_{\text{drop}}) = h_i.$$

Solving for γ, we get:

$$\gamma = \frac{1}{1 - p_{\text{drop}}}.$$

[7]Srivastava et al., 2014, `https://www.cs.toronto.edu/~hinton/absps/JMLRdropout.pdf`

ii. (2 points) Why should dropout be applied during training? Why should dropout **NOT** be applied during evaluation? **Hint:** it may help to look at the dropout paper linked.

Answer

During training, dropout helps prevent overfitting by randomly setting a subset of the neurons' outputs to zero in each forward pass. This forces the network to learn more robust and redundant representations, preventing co-adaptation of neurons and introducing noise that acts as a regularizer.

During evaluation, however, we want to use the full capacity of the network. Applying dropout during evaluation would reduce the network's effective capacity, leading to suboptimal predictions. Instead, we use the scaling factor $\gamma = \frac{1}{1-p_{\text{drop}}}$ during training to ensure the expected output is consistent. Thus, during evaluation, the network uses all of its learned representations without any random dropping of neurons.

3. Neural Transition-Based Dependency Parsing (54 points)

In this section, you'll be implementing a neural-network based dependency parser with the goal of maximizing performance on the UAS (Unlabeled Attachment Score) metric.

Before you begin, please follow the README to install all the needed dependencies for the assignment. We will be using PyTorch 2.1.2 from `https://pytorch.org/get-started/locally/` with the `CUDA` option set to `None`, and the tqdm package – which produces progress bar visualizations throughout your training process. The official PyTorch website is a great resource that includes tutorials for understanding PyTorch's Tensor library and neural networks.

A dependency parser analyzes the grammatical structure of a sentence, establishing relationships between *head* words, and words which modify those heads. There are multiple types of dependency parsers, including transition-based parsers, graph-based parsers, and feature-based parsers. Your implementation will be a *transition-based* parser, which incrementally builds up a parse one step at a time. At every step it maintains a *partial parse*, which is represented as follows:

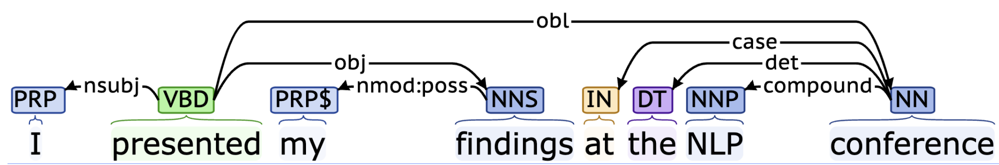
- A *stack* of words that are currently being processed.
- A *buffer* of words yet to be processed.
- A list of *dependencies* predicted by the parser.

Initially, the stack only contains ROOT, the dependencies list is empty, and the buffer contains all words of the sentence in order. At each step, the parser applies a *transition* to the partial parse until its buffer is empty and the stack size is 1. The following transitions can be applied:

- `SHIFT`: removes the first word from the buffer and pushes it onto the stack.
- `LEFT-ARC`: marks the second (second most recently added) item on the stack as a dependent of the first item and removes the second item from the stack, adding a *first_word* → *second_word* dependency to the dependency list.
- `RIGHT-ARC`: marks the first (most recently added) item on the stack as a dependent of the second item and removes the first item from the stack, adding a *second_word* → *first_word* dependency to the dependency list.

On each step, your parser will decide among the three transitions using a neural network classifier.

(a) (4 points) Go through the sequence of transitions needed for parsing the sentence *"I presented my findings at the NLP conference"*. The dependency tree for the sentence is shown below. At each step, give the configuration of the stack and buffer, as well as what transition was applied this step and what new dependency was added (if any). The first three steps are provided below as an example.



Stack	Buffer	New dependency	Transition
[ROOT]	[I, presented, my, findings, at, the, NLP, conference]		Initial Configuration
[ROOT, I]	[presented, my, findings, at, the, NLP, conference]		`SHIFT`
[ROOT, I, presented]	[my, findings, at, the, NLP, conference]		`SHIFT`
[ROOT, presented]	[my, findings, at, the, NLP, conference]	presented→I	`LEFT-ARC`

Answer:

Stack	Buffer	New dependency	Transition
[ROOT]	[I, presented, my, findings, at, the, NLP, conference]		Initial Configuration
[ROOT, I]	[presented, my, findings, at, the, NLP, conference]		`SHIFT`
[ROOT, I, presented]	[my, findings, at, the, NLP, conference]		`SHIFT`
[ROOT, presented]	[my, findings, at, the, NLP, conference]	presented→I (nsubj)	`LEFT-ARC`
[ROOT, presented, my]	[findings, at, the, NLP, conference]		`SHIFT`
[ROOT, presented, my, findings]	[at, the, NLP, conference]		`SHIFT`
[ROOT, presented, findings]	[at, the, NLP, conference]	findings→my (nmod:poss)	`LEFT-ARC`
[ROOT, presented]	[at, the, NLP, conference]	presented→findings (obj)	`RIGHT-ARC`
[ROOT, presented, at]	[the, NLP, conference]		`SHIFT`
[ROOT, presented, at, the]	[NLP, conference]		`SHIFT`
[ROOT, presented, at, the, NLP]	[conference]		`SHIFT`
[ROOT, presented, at, the, NLP, conference]	[]		`SHIFT`
[ROOT, presented, at, the, conference]	[]	conference→NLP (compound)	`LEFTT-ARC`
[ROOT, presented, at, conference]	[]	conference→the (det)	`LEFT-ARC`
[ROOT, presented, conference]	[]	conference→at (case)	`LEFT-ARC`
[ROOT, presented, conference]	[]	presented→conference (obl)	`RIGHT-ARC`
[ROOT, presented]	[]	ROOT→presented (root)	`RIGHT-ARC`
[ROOT]	[]		`DONE`

(b) (2 points) A sentence containing n words will be parsed in how many steps (in terms of n)? Briefly explain in 1–2 sentences why.

Answer:

A sentence containing n words will be parsed in $(2n - 1)$ steps. This is because there will be n shift transitions and $n - 1$ dependency transitions

(c) (6 points) Implement the `__init__` and `parse_step` functions in the `PartialParse` class in `parser_transitions.py`. This implements the transition mechanics your parser will use. You can run basic (non-exhaustive) tests by running `python parser_transitions.py part_c`.

(d) (8 points) Our network will predict which transition should be applied next to a partial parse. We could use it to parse a single sentence by applying predicted transitions until the parse is complete. However, neural networks run much more efficiently when making predictions about *batches* of data at a time (i.e., predicting the next transition for any different partial parses simultaneously). We can parse sentences in minibatches with the following algorithm.

Algorithm 1 Minibatch Dependency Parsing

Input: `sentences`, a list of sentences to be parsed and `model`, our model that makes parse decisions

Initialize `partial_parses` as a list of PartialParses, one for each sentence in `sentences`
Initialize `unfinished_parses` as a shallow copy of `partial_parses`
while `unfinished_parses` is not empty **do**
  Take the first `batch_size` parses in `unfinished_parses` as a minibatch
  Use the `model` to predict the next transition for each partial parse in the minibatch
  Perform a parse step on each partial parse in the minibatch with its predicted transition
  Remove the completed (empty buffer and stack of size 1) parses from `unfinished_parses`
end while

Return: The `dependencies` for each (now completed) parse in `partial_parses`.

Implement this algorithm in the `minibatch_parse` function in `parser_transitions.py`. You can run basic (non-exhaustive) tests by running `python parser_transitions.py part_d`.

Note: You will need `minibatch_parse` to be correctly implemented to evaluate the model you will build in part (e). However, you do not need it to train the model, so you should be able to complete most of part (e) even if `minibatch_parse` is not implemented yet.

(e) (20 points) We are now going to train a neural network to predict, given the state of the stack, buffer, and dependencies, which transition should be applied next.

First, the model extracts a feature vector representing the current state. We will be using the feature set presented in the original neural dependency parsing paper: *A Fast and Accurate Dependency Parser using Neural Networks*.[8] The function extracting these features has been implemented for you in `utils/parser_utils.py`. This feature vector consists of a list of tokens (e.g., the last word in the stack, first word in the buffer, dependent of the second-to-last word in the stack if there is one, etc.). They can be represented as a list of integers $\mathbf{w} = [w_1, w_2, \ldots, w_m]$ where m is the number of features and each $0 \leq w_i < |V|$ is the index of a token in the vocabulary ($|V|$ is the vocabulary size). Then our network looks up an embedding for each word and concatenates them into a single input vector:

$$\mathbf{x} = [\mathbf{E}_{w_1}, ..., \mathbf{E}_{w_m}] \in \mathbb{R}^{dm}$$

[8] Chen and Manning, 2014, https://nlp.stanford.edu/pubs/emnlp2014-depparser.pdf

where $\mathbf{E} \in \mathbb{R}^{|V|\times d}$ is an embedding matrix with each row $\mathbf{E}_w$ as the vector for a particular word w with dimension d. We then compute our prediction as:

$$\begin{aligned}
\mathbf{h} &= \mathrm{ReLU}(\mathbf{xW} + \mathbf{b}_1) \\
\mathbf{l} &= \mathbf{hU} + \mathbf{b}_2 \\
\hat{\mathbf{y}} &= \mathrm{softmax}(l)
\end{aligned}$$

where $\mathbf{h}$ is referred to as the hidden layer, $\mathbf{l}$ is referred to as the logits, $\hat{\mathbf{y}}$ is referred to as the predictions, and $\mathrm{ReLU}(z) = \max(z, 0))$. We will train the model to minimize cross-entropy loss:

$$J(\theta) = CE(\mathbf{y}, \hat{\mathbf{y}}) = -\sum_{j=1}^{3} \mathbf{y}_j \log \hat{\mathbf{y}}_j$$

where $\mathbf{y}_j$ denotes the jth element of $\mathbf{y}$. To compute the loss for the training set, we average this $J(\theta)$ across all training examples.

i. Compute the derivative of $\mathbf{h} = \mathrm{ReLU}(\mathbf{xW} + \mathbf{b}_1)$ with respect to $\mathbf{x}$. For simplicity, you only need to show the derivative $\frac{\partial h_i}{\partial x_j}$ for some index i and j. You may ignore the case where the derivative is not defined at 0.

Answer:

$$h = \mathrm{ReLU}(xW + b_1)$$

where $\mathrm{ReLU}(z) = \max(z, 0)$.

We need to compute the derivative of h with respect to x, specifically $\frac{\partial h_k}{\partial x_i}$ for some indices k and i.

1. Compute the intermediate result z:

$$z = xW + b_1$$

2. Apply the ReLU function:

$$h_k = \mathrm{ReLU}(z_k) = \max(z_k, 0)$$

3. Determine the derivative:

$$\frac{\partial h_k}{\partial z_k} = \begin{cases} 1 & \text{if } z_k > 0 \\ 0 & \text{if } z_k \leq 0 \end{cases}$$

4. Chain rule application:

$$\frac{\partial h_k}{\partial x_i} = \frac{\partial h_k}{\partial z_k} \cdot \frac{\partial z_k}{\partial x_i}$$

Since $z = xW + b_1$:

$$z_k = \sum_j x_j W_{jk} + b_{1k}$$

Thus:

$$\frac{\partial z_k}{\partial x_i} = W_{ik}$$

So:

$$\frac{\partial h_k}{\partial x_i} = \frac{\partial \mathrm{ReLU}(z_k)}{\partial z_k} \cdot W_{ik} = \begin{cases} W_{ik} & \text{if } z_k > 0 \\ 0 & \text{if } z_k \leq 0 \end{cases}$$

Therefore, the derivative is:

$$\frac{\partial h_k}{\partial x_i} = \begin{cases} W_{ik} & \text{if } (xW + b_1)_k > 0 \\ 0 & \text{if } (xW + b_1)_k \leq 0 \end{cases}$$

ii. Recall in part 1b, we computed the partial derivative of $\mathbf{J}_{\text{naive-softmax}}(\mathbf{v}_c, o, \mathbf{U})$. Likewise, please compute the partial derivative of $J(\theta)$ with respect to the ith entry of $\mathbf{l}$, which is denoted as $\mathbf{l}_i$. Specifically, compute $\frac{\partial CE(\mathbf{y},\hat{\mathbf{y}})}{\partial \mathbf{l}_i}$, assuming that $\mathbf{l} \in \mathbb{R}^3$, $\hat{\mathbf{y}} \in \mathbb{R}^3$, $\mathbf{y} \in \mathbb{R}^3$, and the true label is c.
Hints: You may recall from part 1a, $\frac{\partial CE(\mathbf{y},\hat{\mathbf{y}})}{\partial \mathbf{l}_i} = \sum_j \frac{\partial CE(\mathbf{y},\hat{\mathbf{y}})}{\partial \hat{\mathbf{y}}_j} \frac{\partial \hat{\mathbf{y}}_j}{\partial \mathbf{l}_i}$, and $\frac{\partial CE(\mathbf{y},\hat{\mathbf{y}})}{\partial \hat{\mathbf{y}}_j} = 0$ if $j \neq c$.
Answer:
Given:

$$J(\theta) = CE(y, \hat{y}) = -\sum_{j=1}^{3} y_j \log \hat{y}_j$$

where $\hat{y} = \text{softmax}(l)$.
The softmax function is defined as:

$$\hat{y}_j = \frac{e^{l_j}}{\sum_{k=1}^{3} e^{l_k}}$$

We need to compute the partial derivative of $J(\theta)$ with respect to the i-th entry of $\mathbf{l}$, denoted as $\mathbf{l}_i$.
To compute $\frac{\partial CE(\mathbf{y},\hat{\mathbf{y}})}{\partial \mathbf{l}_i}$:
1. **Compute the partial derivative of $\hat{y}_j$ with respect to l_i:**

$$\frac{\partial \hat{y}_j}{\partial l_i} = \hat{y}_j(\delta_{ij} - \hat{y}_i)$$

where δ_{ij} is the Kronecker delta function, which is 1 if $i = j$ and 0 otherwise.
2. **Compute the partial derivative of $J(\theta)$ with respect to l_i:**

$$\frac{\partial J}{\partial l_i} = -\sum_{j=1}^{3} y_j \frac{\partial \log \hat{y}_j}{\partial l_i}$$

Since $\frac{\partial \log \hat{y}_j}{\partial l_i} = \frac{1}{\hat{y}_j} \frac{\partial \hat{y}_j}{\partial l_i}$:

$$\frac{\partial J}{\partial l_i} = -\sum_{j=1}^{3} y_j \frac{1}{\hat{y}_j} \hat{y}_j(\delta_{ij} - \hat{y}_i)$$

$$\frac{\partial J}{\partial l_i} = -\sum_{j=1}^{3} y_j(\delta_{ij} - \hat{y}_i)$$

Because $\frac{\partial CE(\mathbf{y},\hat{\mathbf{y}})}{\partial \hat{\mathbf{y}}_j} = 0$ if $j \neq c$ and assuming y is a one-hot vector where $y_c = 1$ and $y_j = 0$ for $j \neq c$, we get:

$$\frac{\partial J}{\partial l_i} = -y_i(1 - \hat{y}_i)$$

Since y is a one-hot vector, only the term where $i = c$ (the true label) will contribute, and for all other i, $y_i = 0$:

$$\frac{\partial J}{\partial l_i} = \begin{cases} \hat{y}_i - 1 & \text{if } i = c \\ \hat{y}_i & \text{if } i \neq c \end{cases}$$

So, the final expression for $\frac{\partial CE(\mathbf{y},\hat{\mathbf{y}})}{\partial \mathbf{l}_i}$ is:

$$\frac{\partial J}{\partial l_i} = \hat{y}_i - y_i$$

$$\boxed{\frac{\partial J}{\partial l_i} = \hat{y}_i - y_i}$$

iii. We will use UAS score as our evaluation metric. UAS refers to Unlabeled Attachment Score, which is computed as the ratio between number of correctly predicted dependencies and the number of total dependencies despite of the relations (our model doesn't predict this).

In `parser_model.py` you will find skeleton code to implement this simple neural network using PyTorch. Complete the `__init__`, `embedding_lookup` and `forward` functions to implement the model. Then complete the `train_for_epoch` and `train` functions within the `run.py` file.

Finally execute `python run.py` to train your model and compute predictions on test data from Penn Treebank (annotated with Universal Dependencies).

Note:

- For this assignment, you are asked to implement Linear layer and Embedding layer. Please **DO NOT** use **torch.nn.Linear** or **torch.nn.Embedding** module in your code, otherwise you will receive deductions for this problem.
- Please follow the naming requirements in our TODO if there are any, e.g. if there are explicit requirements about variable names you have to follow them in order to receive full credits. You are free to declare other variable names if not explicitly required.

Hints:

- Each of the variables you are asked to declare (`self.embed_to_hidden_weight`, `self.embed_to_hidden_bias`, `self.hidden_to_logits_weight`, `self.hidden_to_logits_bias`) corresponds to one of the variables above ($\mathbf{W}$, $\mathbf{b}_1$, $\mathbf{U}$, $\mathbf{b}_2$).
- It may help to work backwards in the algorithm (start from $\hat{\mathbf{y}}$) and keep track of the matrix/vector sizes.
- Once you have implemented `embedding_lookup (e)` or `forward (f)` you can call `python parser_model.py` with flag `-e` or `-f` or both to run sanity checks with each function. These sanity checks are fairly basic and passing them doesn't mean your code is bug free.
- When debugging, you can add a debug flag: `python run.py -d`. This will cause the code to run over a small subset of the data, so that training the model won't take as long. Make sure to remove the `-d` flag to run the full model once you are done debugging.
- When running with debug mode, you should be able to get a loss smaller than 0.2 and a UAS larger than 65 on the dev set (although in rare cases your results may be lower, there is some randomness when training).
- It should take up to **15 minutes** to train the model on the entire training dataset, i.e., when debug mode is disabled.
- When debug mode is disabled, you should be able to get a loss smaller than 0.08 on the train set and an Unlabeled Attachment Score larger than 87 on the dev set. For comparison, the model in the original neural dependency parsing paper gets 92.5 UAS. If you want, you can tweak the hyperparameters for your model (hidden layer size, hyperparameters for Adam, number of epochs, etc.) to improve the performance (but you are not required to do so).

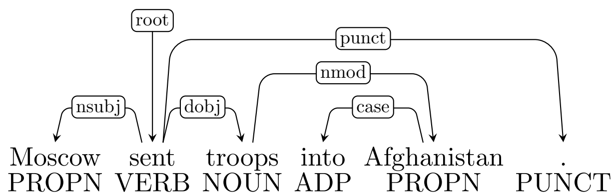
Deliverables:

- Working implementation of the transition mechanics that the neural dependency parser uses in `parser_transitions.py`.
- Working implementation of minibatch dependency parsing in `parser_transitions.py`.
- Working implementation of the neural dependency parser in `parser_model.py`. (We'll look at and run this code for grading).

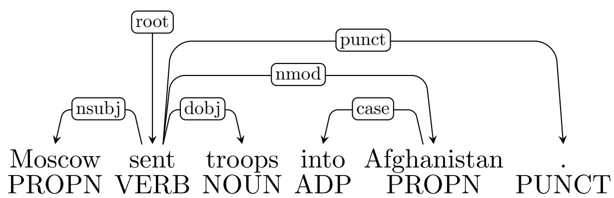
- Working implementation of the functions for training in `run.py`. (We'll look at and run this code for grading).
- **Report the best UAS your model achieves on the dev set and the UAS it achieves on the test set in your written submission**.

Answer:
Best Dev UAS: **87.51**
Test UAS: **87.84**

(f) (12 points) We'd like to look at example dependency parses and understand where parsers like ours might be wrong. For example, in this sentence:



the dependency of the phrase *into Afghanistan* is wrong, because the phrase should modify *sent* (as in *sent into Afghanistan*) not *troops* (because *troops into Afghanistan* doesn't make sense, unless there are somehow weirdly some troops that stan Afghanistan). Here is the correct parse:



More generally, here are four types of parsing error:

- **Prepositional Phrase Attachment Error**: In the example above, the phrase *into Afghanistan* is a prepositional phrase[9]. A Prepositional Phrase Attachment Error is when a prepositional phrase is attached to the wrong head word (in this example, *troops* is the wrong head word and *sent* is the correct head word). More examples of prepositional phrases include *with a rock*, *before midnight* and *under the carpet*.
- **Verb Phrase Attachment Error**: In the sentence *Leaving the store unattended, I went outside to watch the parade*, the phrase *leaving the store unattended* is a verb phrase[10]. A Verb Phrase Attachment Error is when a verb phrase is attached to the wrong head word (in this example, the correct head word is *went*).
- **Modifier Attachment Error**: In the sentence *I am extremely short*, the adverb *extremely* is a modifier of the adjective *short*. A Modifier Attachment Error is when a modifier is attached to the wrong head word (in this example, the correct head word is *short*).
- **Coordination Attachment Error**: In the sentence *Would you like brown rice or garlic naan?*, the phrases *brown rice* and *garlic naan* are both conjuncts and the word *or* is the coordinating conjunction. The second conjunct (here *garlic naan*) should be attached to the first conjunct (here *brown rice*). A Coordination Attachment Error is when the second conjunct is attached to the wrong head word (in this example, the correct head word is *rice*). Other coordinating conjunctions include *and*, *but* and *so*.

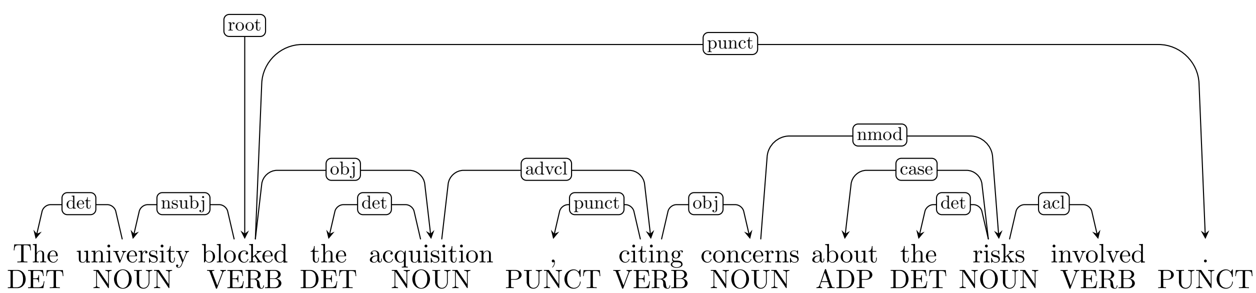
[9] For examples of prepositional phrases, see: https://www.grammarly.com/blog/prepositional-phrase/
[10] For examples of verb phrases, see: https://examples.yourdictionary.com/verb-phrase-examples.html

In this question are four sentences with dependency parses obtained from a parser. Each sentence has one error type, and there is one example of each of the four types above. For each sentence, state the type of error, the incorrect dependency, and the correct dependency. While each sentence should have a unique error type, there may be multiple possible correct dependencies for some of the sentences. To demonstrate: for the example above, you would write:

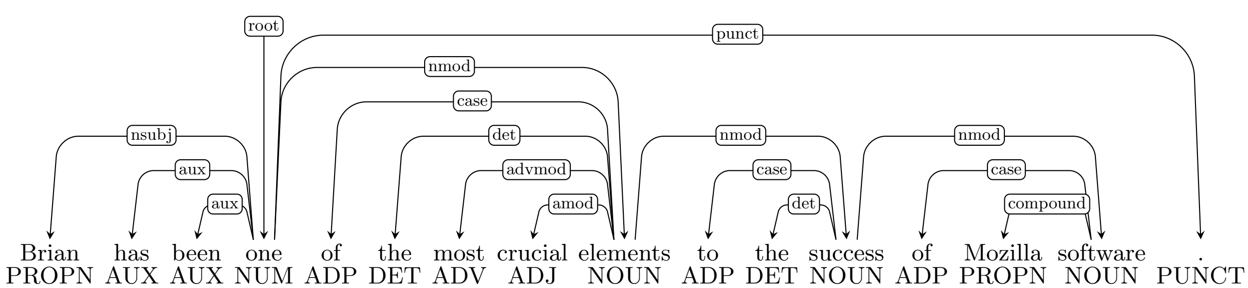
- **Error type**: Prepositional Phrase Attachment Error
- **Incorrect dependency**: troops → Afghanistan
- **Correct dependency**: sent → Afghanistan

***Note**: There are lots of details and conventions for dependency annotation. If you want to learn more about them, you can look at the UD website: `http://universaldependencies.org`[11] or the short introductory slides at: `http://people.cs.georgetown.edu/nschneid/p/UD-for-English.pdf`. Note that you **do not** need to know all these details in order to do this question. In each of these cases, we are asking about the attachment of phrases and it should be sufficient to see if they are modifying the correct head. In particular, you **do not** need to look at the labels on the the dependency edges – it suffices to just look at the edges themselves.*

i.

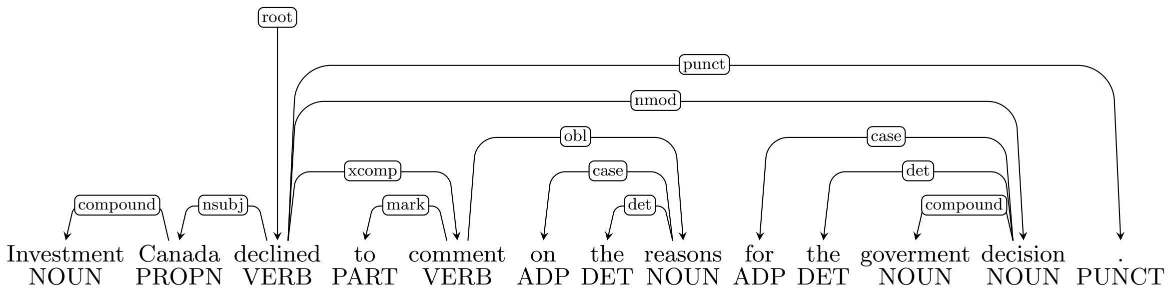


ii.

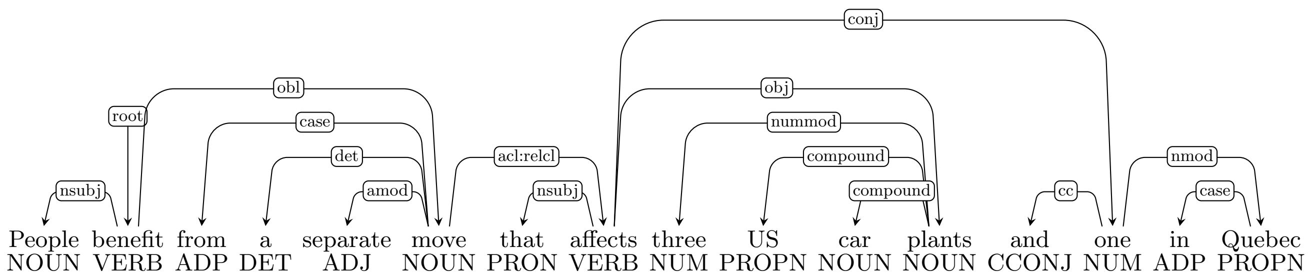


iii.

[11] But note that in the assignment we are actually using UDv1, see: `http://universaldependencies.org/docsv1/`



iv.



Answer:

No.	Error Type	Incorrect Dependency	Correct Dependency
i	Adverbial Attachment Error	citing → blocked	citing → acquisition
ii	Noun Phrase Attachment Error	elements → has	elements → one
iii	Prepositional Phrase Attachment Error	reasons → comment	reasons → declined
iv	Conjunction Attachment Error	affects → move	affects → plants

Table 1: Error types, incorrect dependencies, and correct dependencies for each sentence

(g) (2 points) Recall in part (e), the parser uses features which includes words and their part-of-speech (POS) tags. Explain the benefit of using part-of-speech tags as features in the parser?

Answer: Using part-of-speech (POS) tags as features in a dependency parser provides several benefits:

- **Disambiguation:** POS tags help resolve ambiguities by indicating the grammatical role of words that can function as multiple parts of speech (e.g., "book" as a noun or verb).
- **Improved Accuracy:** POS tags provide syntactic information that aids in determining correct dependencies, improving parsing accuracy.
- **Contextual Understanding:** They help the parser understand the context and relationships between words, such as subject-verb and adjective-noun pairs.
- **Efficiency:** Leveraging pre-processed POS tags can make parsing more efficient by reducing computational complexity.

Submission Instructions

You shall submit this assignment on GradeScope as two submissions – one for "Assignment 2 [coding]" and another for 'Assignment 2 [written]":

1. Run the `collect_submission.sh` script to produce your `assignment2.zip` file.
2. Upload your `assignment2.zip` file to GradeScope to "Assignment 2 [coding]".
3. Upload your written solutions to GradeScope to "Assignment 2 [written]".