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Nonlinear model of stock price dynamics with behavioural functions

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Čestné prohlášení

Prohlašuji, že jsem bakalářskou práci na téma Nelineární model dynamiky ceny akcie s behaviorálními funkcemi vypracoval samostatně a veškerou použitou literaturu a další prameny jsem řádně označila uvedl v přiloženém seznamu.

V Praze dne 28. Května, 2021

Declaration of Authorship

The author hereby proclaims that he wrote the bachelor thesis Nonlinear model of stock price dynamics with behavioural functions by himself, under the leadership of his supervisor and included all the relevant resources and literature in the enclosed bibliography.

Prague, May 28th, 2021

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Illichmann Vít

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Abstract

The thesis investigates the relation between inefficient processing of objective probability proposed by the Prospect theory and excess volatility present in stock market prices. The author employs an agent-based model to show the destabilizing influence of the cognitive bias in question with a positive result. For a high level of distortions in probability perception, the model produces chaotic oscillations in price with the fundamental value of shares held fixed. The secondary focus is on the explanation of the origin of the proposed behaviour, which the author attributes to the biological restrictions present in the process of cognitive evolution.

Key Words: Behavioural finance, Agent-based models, Nonlinear dynamics, Stock market, Deterministic chaos

Abstrakt

Tato práce se zabývá vztahem mezi subjektivním vnímáním pravděpodobnosti postulovaným Teorií vyhlídek a nadměrnou volatilitou akciových cen. Autor využívá metodiku agent-based modelů k demonstraci destabilizujícího vlivu subjektivního vnímání na vývoj cen, s pozitivním výsledkem. Vysoká intenzita zkreslení ve vnímání pravděpodobnosti vede k chaotickým oscilacím ceny v rámci navrženého modelu, a to i za předpokladu že fundamentální hodnota akcie zůstává stabilní. Sekundárním cílem práce je vysvětlení původu modelovaných odchylek od racionálního vnímání. Autor dochází k závěru, že toto chování lze vysvětlit jako přirozený důsledek biologických omezení, která se formovala během kognitivní evoluce člověka.

Klíčová slova: Behaviorální finance, Agent-based modely, Nelineární dynamika, Akciové trhy, Deterministický chaos

Contents

1 Introduction

1.1 Thesis Motivation and Objectives

The presence of superfluous volatility in the capital markets was first noted by Schiller (1981). Since then, the phenomenon gained traction as the "excess volatility puzzle". Broadly defined, the suggestive term refers to the existence of unfounded variability in the price of publicly traded shares in the stock market that does not seem to be connected to any change in the expected flow of dividends.

On the theoretical level, the existence of such a phenomenon carries some interesting consequences. For a long time, the asset prices in the financial markets (and in the stock market particularly) had been considered to the best possible estimate of their true value under the efficient market paradigm. The appeal of the efficient markets hypothesis is apparent, especially after one considers the implications of supposed efficiency. If stocks reflect everything as it is, then there is no need to check the financial health of one's investment in terms of accounting and macroeconomic indicators; just look at the stock price and see how they are doing, right? [i]

Put more formally, the efficient market hypothesis (EMH for short) claims that since the market is occupied by rational profit-maximising agents, who utilise every possible strategy leading to the increase in expected profit, each viable option had already been exploited, and the current price thus includes all relevant information available.

Unfortunately, the EMH does not get nearly as much empirical support as would such a formally flawless theory deserve (Yen, Lee, 2008), although the weak version of the efficient market hypothesis enjoys some nonnegligible support of facts (a more up to date view is that markets are weakly efficient and the degree of efficiency is unstable over time (Lim, Brooks, 2011). After all, if the rational market participants exploit all possible profit maximisation methods, why will the excess volatility not get arbitraged out (feasible investment strategy taking advantage of excess volatility is examined, e.g. by Dumas et al. 2005)?

Some answer to the volatility puzzle could perhaps be obtained by adopting a different set of assumptions about investor's behaviour. As hinted before, the conjecture about the market's efficiency relies on the agent's ability to process information at hand rationally. If the traders are not capable to correctly evaluate data related to the traded assets or respond irrationally, then the price could essentially follow any direction no matter what the related fundamentals are.

Regarding the presumed cognitive abilities of the typical market participant, behavioural finance could be thought of as an antithesis to the EMH framework. From the historical perspective, the notion of market efficiency is much older compared to the behavioural paradigm, originating as part of the thesis of Luis Bachelier (Bachelier, 1900).

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[i] This rhetorical question is paraphrasing the point made during **Yale 2009 lectures** on financial theory, it sums ups nicely the sentiment held by many market practitioners regarding the market efficiency.

Later, in the 60s, the EMH was popularised by Samuelson and Fama (Fama, 1965) and gained its modern form, where the price is said to follow a random walk process. In a way, market efficiency captures a much more classical view on the economic interactions, picturing markets as an ideal machine where each cogwheel (investor) is manufactured precisely to fit its purpose (profit maximisation). Abstract from friction (costly information acquisition, delay, et cet.) and the machine works just fine, producing no extra heath (excess price fluctuations).

Compared to such a neat approach to finance, the behavioural framework is messy. In behavioural finance, investors are supposed to suffer from all kinds of innate imperfections. The advantage of this is compliance with empirical observation; studies in experimental psychology consistently fail to confirm the kind of strict rationality so often supposed by economists. In accordance with everyday experience, our decision-making process (regarding problems of economic nature) seems to subject to various psychological biases, especially if the risk is involved. To name at least a few, one can mention, e.g. representativeness heuristic, mental accounting, probability weighting or mental framing (Hirschleifer, 2001).

The asset pricing model that included observed behavioural inefficiencies could be a powerful explanatory tool, promising to elucidate misalignment between the price dynamics proposed by efficient markets excess volatility present in the actual stock prices.

The aim of this thesis is to formulate an agent-based model replicating the excess volatility present in the real-world equity markets. With the emphasis being put on the empirical viability of the strategy assigned to the artificial investor.

The obvious methodological issue related to the adoption of investors psychology into some kind of verifiable model is the manifold character of observed deficiencies in the human decision-making process. In other words, is a positive result derived from some proposed behavioural model proof of the correct choice of agents' preferences? Or it just so happens that our psychology is so rich in this regard that it offers ammunition to justify just any kind of market inefficiency.

The problem can be partially addressed by restricting oneself to the subset of biases that appear to be sufficiently general in the sense that they are observed repeatedly over time and seem to have biological roots. The Prospect Theory of Kahneman and Tversky offers an ideal toolbox in this respect, formulating an alternative framework to the discounted utility theory of Von Neumann while stressing the importance of compliance of the theory with experimental data. As shown later in the thesis, the fundamental concepts of the Prospect theory enjoy empirical support not only in the field of experimental psychology but also in a somewhat more technocratic environment of experimental neurobiology.

Since the thesis proposes an a priori behavioural strategy as a part of the dynamic model, it is desirable to explain its components and fit them into the broader context of supporting facts to show that this is indeed a feasible way of thinking about the human cognitive process. In order to do so, the author argues that proposed inefficiencies are a natural result of the evolutionary history of our species and biological constraints. Several cited scholars hold similar views on the matter (Herold, Netzer (2010); McDermott, Fowler, (2008); Hirschleifer (2001)).

The thesis also contains a brief description of statistical properties of the stock market that appear to contradict the standard efficiency paradigm, which is necessary in order to compare simulation results with real-world data.

Looking beyond its theoretical implications, the presence of excess volatility in stock prices also has a more practical dimension. The stock market constitutes a crucial financing channel for entrepreneurial subjects operating in the real economy. Inefficiencies in the financial sphere can indirectly influence economic growth by affecting firms' access to capital. Conditions responsible for the existence of excess volatility puzzle are thus not only of academic interest, but the question also translates into the regulatory domain.

Past dominance of the efficient market archetype was undoubtedly comfortable for financial corporations involved in speculative trades as it presents a hefty argument against intrusive regulation of the financial market by the government or central banks.

These days, a more cautious approach is adopted in most advanced economies, with a hint of distrust towards financial business in general, reflecting the experience of past market failings. Nevertheless, the efficiency concept still holds undeniable appeal, especially to practitioners and retail investors. Inquiry into behavioural mechanisms and their projection into market dynamics thus represents a valuable area of research.

1.2 Structure

The **Introduction** dealt with the motivation behind the thesis and described its main objectives. The present paragraphs aim to preview the content of the following chapters in order to make their function clear to the reader.

Section 2 contains a review of literature on related topics. Since both behavioural finance and agent-based modelling are growing and very diverse fields, the author restricts himself to works that are either a seminal reference, were used by the author extensively or demonstrate some of the key concepts drawn upon later in the text.

Section 3 opens with some conjectures about the role of investors' psychology and non-rational behaviour in the process of equities' price formation. It continues with an introduction to the basic concepts of the Prospect theory, which are later implemented inside the model.

Section 4 is concerned with some of the statistical properties of stock market prices that are not in line with the market efficiency framework and offer some support for the presence of chaotic dynamics in observed time series. Further, it contains a review of two computational models which recreate some of those characteristics.

Section 5 is all about the model building. The first part sketches a general direction and describes some of the applied methods. The second part deals with the microstructure of the market and the mathematical formulation of the rules governing its dynamics.

Section 6 contains results of the numerical experiment and comparisons of the simulation results with empirical facts observed in the stock market environment. Each subsection contains a notation summary if necessary. Conclusion follows.

2 Literature Rewiev

Regarding the presence of excess volatility in equity markets, the seminal reference is Schiller (1981). Schiller noticed that a portion of the stock price fluctuations could not be tied directly to changes in the expected flow of dividends; such behaviour goes against supposed market efficiency. He offers an explanation of surplus activity by changes of expected real interest rates but remarks that since such a variable is not directly observable, the reasoning is purely academic. The noticing of investment assets to experience rather short-lived and irrational fluctuation in value can be traced back to Keynes, who deems them to have " an absurd effect on the market" (Keynes, 1936, page 153-154). Interestingly enough, Keynes also draws attention to the influence of market psychology in asset valuation. According to him, professional investors spend more time anticipating the overall atmosphere in the market than estimating fundamentals. The fact that he does not spend too much effort on arguing his statement suggests that this was a consensual way of thinking about the market before the EMH took hold.

The literature concerning the excess volatility in equity markets is rich and diverse. In general, there seem to be several distinctive categories of explanations. The first kind attempts to model the phenomenon without the need of discarding rational expectations by including market frictions. The second type is built around the rejection of rationality and explains excess volatility as a consequence of behavioural inefficiencies. A representative example would be the research paper of De Bondt and Thaler (1985) which explains variability in price due to investor overweighting recent information.

The third kind could be thought of as structural explanations as it is concerned with the influence of market microstructure on the price dynamics as much as it is with the investment strategies. A shining example of the structural approach are the recently booming agent-based models, which are explicitly concerned with reflecting an interaction among heterogeneous investors. Agent-based modelling of financial markets is a young but thriving branch of finance; consequently bulk of existing literature is excessive, and the author reviews only a few outstanding examples.

Farmer and Joshi 2002 formulate a model that could be considered a prototype, containing fundamentalist and trend followers performing technical analysis. Exploring the price dynamics of the typified trading strategies, authors successfully recreate excess volatility and volatility clustering present real-world markets.

Giardina and Bouchaud (2003) analyse a rather complex stock market model (with augmenting assumption that agents can also invest in bonds at some risk-free rate). Each market participant is endowed with a fixed number of strategies that are distributed randomly but switched between, based on the agent's past experience.

The model can operate in three distinctive regimes and successfully recreates such features as the formation of bubbles, financial crashes, the influence of agent's wealth or volatility clustering and allows to explore their causes within the model.

Cristelli (2014) focuses directly on implementing psychological biases into agent-based models and carries on a lengthy discussion on the philosophy behind economic complexity and contrasts it with the more traditional economic view. Cristelli argues in his introduction that assumptions of mainstream finance are used to permit the analytical treatment of studied problems rather than to ensure its compliance with empirical research, which is naturally a limiting factor regarding its scientific plausibility.

Agent-based models generally contain some stochastic element; however, models with heterogeneous strategies can also be constructed as purely deterministic with endogenously generated changes in price. Erratic price fluctuations in deterministic models result from deterministic chaos present in models represented by nonlinear or piecewise linear maps if parameters are such that the trajectory is bounded and does not converge to some fixed point (or set of points for periodic trajectories). To the history of applications of the deterministic chaos as a source of explanation of complex price dynamics in financial markets, it is essential to mention the work of Richard Day (1994), which contains an introduction to the application of nonlinear dynamical models to various problems in economics and treatment of deterministic chaos. A significant portion of the book focuses on problems inherent to the analysis of equity markets; such are market mediation or the presence of heterogenous trading strategies.

Day and Huang (1990) propose a more or less basic equity market model where the ecology is occupied by the α and the β investors. The first class of trading strategy represents long-term investors whose approach is based on the sophisticated estimate of traded shares' fundamental value. By extension, α-strategy is a proxy for institutional investors with significant capital endowments who can bear costs of necessary information acquisition and data analysis essential for updating the estimate of fundamental value. The demand of α's is derived using a priori defined nonlinear weighting function and the distortions of current price from the said fundamental value of an investment. β's are traders whose trading is based solely on registered price movements, confusing noise with change in the trend. Exchange is facilitated by a market mediator who adjusts price according to linear pricing policy based on excess market demand given by equation (1).

$$p_{t+1} = p_t + cE(p_t) \tag{1}$$

Where p_t is current price and $E(p_t)$ is excess demand. The authors then proceed to show that if parameter c is sufficiently high, then the resulting system is chaotic, and the price fluctuates aperiodically. Such behaviour can be characterised by stable distribution (Day 1994); the authors show that averages of prices converge to normal distribution over time.

Huang et al. (2010) use a deterministic chaotic model with heterogeneous agents to model a mechanism behind market crashes. The resulting time series are successfully fitted to the time series corresponding to the actual financial crises. Fruitful applications of nonlinear deterministic models (e.g. the work by Huang) hint that there is yet much to be exploited, even if only as to tool to support our economic intuition about the inner mechanics of the complex economic process.

The foundational hypothesis in behavioural finance is the Prospect theory, which originated in the research paper by Kahneman and Twersky (1979), presenting all of its essential theoretical concepts and empirical facts responsible for their development. Behavioural finance draw upon postulates of the Prospect theory extensively while adding new concepts relevant for finance specifically. Seth and Chowdary (2017) discuss the relation of Prospect theory to the field of behavioural finance in detail. Since it is often criticised that psychological experiments are conducted involving students or the general public as test subjects, the authors recreate experiments of Kahneman and Tversky on individuals well trained in probability and statistics. The results have shown that training reduces the observed bias but does not fundamentally eliminate it. Concerning other topics in psychology exploited extensively by the thesis, Hirshleifer (2001) offers an excellent in-depth survey on the implications of cognitive biases identified in psychological experiments for the asset pricing models used by economists. The research in question is rather extensive, containing an overview of elementary results of related topics in psychology and their link to finance. The general tone is rather critical to the rational paradigm utilised by mainstream economics, deeming it to be just the simplest element of a much larger set of possibilities and concluding that much of the experimental findings could directly be exploited as part of asset pricing models in the future.

3 Behavioural Preferences

3.1 Rationality of Choice, Market Efficiency and Risk Perception

The concept of rational investor had for long been a dominant paradigm in finance. Although a toolkit for theoretical analysis of human behaviour is more diverse these days, supposed rationality still implicitly rests in the core of many frequently applied concepts.

The efficient market hypothesis claims that all available information is already included in the current price because any misalignment, allowing for excess profit, would get quickly arbitraged out (Makiel, 2003). Several kinds of efficiency hypothesis exist, varying in the strength of their premises. Nevertheless, any market efficiency, in general, requires participants to correctly interpret available information (related to the traded asset) and convert them into a price signal without bias.

That is, of course, true only partially markets do not need every investor to be well behaved to deliver efficient results. What they do need, though, is for the deviations from rationality to be distributed randomly in population so that they average out on a macro scale. Requirements put on every single individual are thus not too restrictive.

Efficient markets are traditionally associated with the notion of prices following the random walk process. While efficiency does not necessarily require the random walk and vice versa (Peters, 1991), both concepts are somewhat tied together. The main reason being that the random walk's subsequent outcome depends purely on the previous state, and, as such, the price would present an up to date reflection of information available to market participants. In other words, the future price cannot be inferred using its past states since history is already fully incorporated in past prices and carries no implications for the future (weak efficiency).

Semi-strong efficiency extends the concept by including all publicly available info into the current price. More modern works hint that degree of efficiency in equity markets most likely changes over time. From a practical standpoint, rational asset pricing by an investor is maybe the shakiest component of EMH assumptions. Hirshleifer (2001) presents a robust survey of psychological biases related to the asset pricing process. He concludes that human perception is, in this regard far, from perfect, and should efforts in the modelling of the financial markets be successful, they need to adopt a more nuanced framework first. He also adds that observed biases are universal to a certain degree, and deviations from rational behaviour are thus not likely to be random.

However, if the valuation process is biased (that is biased even on average), then the mere absence of abnormal returns does not imply efficiency (if everyone makes the same mistake by default, it is still a mistake). Dragotă et al. (2006) makes a similar point in the introduction.

Without rationality, prices could eventually go in any direction, no matter what the related fundamentals have to say. That is, of course, not the case; prices track their fundamental values in the long term. However, stock prices are subject to excess volatility and persistent misalignment (Westerhoff, Schmitt, 2011). The author attributes mispricing to agents' failure to process received price signals correctly; this speculation is developed in more detail and accompanied by supporting results of the model simulation. The working hypothesis is that the violation of strict rationality regarding the interpretation and response to observed price signals then manifests itself as a persistent misalignment of share prices with respect to their fundamental value, and results in fluctuation of the price that could be partially responsible for the excess volatility described by empirical studies.

To set some solid empirical foundations for the proposed hypothesis, one has first shown that considered biases are sufficiently uniform across the population in order for them to be relevant on a macro scale. The purpose of the following paragraphs is to argue that, although agents participating in equity markets can assess incoming price signals in a reasonably accurate manner, they are nevertheless subject to universal cognitive biases arising from biological restrictions and bottom-up architecture of behavioural patterns build through the process of gradual evolution.

McDermott et al. (2007) point out that the human cognitive process is undoubtedly rational in a certain sense but does not necessarily fully adhere to rationality defined normatively in the language of economic theory. While it is generally plausible to approximate individual decision-making process as maximising the infinite sum of discounted expected payoffs. It has been shown in multiple empirical studies that in real-world situations, decision outcome depends highly on framing. If confronted with wager posed in terms of possible gains, subjects tend to be significantly more risk-averse than if a bet is posed in terms of loss reduction, indicating that risk aversion level is not something inherent to the individual but changes with the subject's perception of reality. The same agent might be more risk-sensitive when hedging himself against possible loss, while risk-taking when performing similar transaction following the profit motive.

Further, human brains do not seem to stick to decisions based purely on expected payoff even when possible outcomes and probabilities are explicitly stated. Small probabilities are overweighted comparatively to those medium and large, with certainty being significantly overvalued against large probabilities (Tversky, Kahneman, 1979). The latter is usually referred to as the common ratio effect; it sums up the empirical observation that if offered two simple prospects, agents tend to prefer the option with lower expected value while the probability of a positive outcome is close to one, but revert their preferences in favour of riskier offer if both probabilities are reduced by some common ratio.

Theoretical explanations of the thought process behind the common ratio effect is a nonlinear transformation of objective probabilities into subjective decision weights, called a probability weighting function (PWF for short). The concrete functional form of the weighting function was derived (among others) by Drazen Prelec (1998) and is based on preferences observed empirically in laboratory experiments.

One of the more recent studies conducted by Takemura and Murakami (2016) assessed the preferences of fifty students of psychology. Participants were presented with simple wagers generated through a computer program in random order and reported preferred options. The authors then attempted to fit several functional forms of PWF. The functional form proposed by Prelec (1998) performed best (although few other models fitted satisfyingly well).

3.2 Excursion into Prospect Theory

In order to comfortably handle described psychological deviations from rational behaviour, one needs to contain them through mathematical formulation. Prospect theory is designed to deal with empirically observed discrepancies in behavioural patterns. Offering a widely accepted formal equivalent to discounted utility framework, the Prospect theory draws on two main concepts, the value function, which encapsulates the differences in the subject's response to gain/loss framing and the probability weighting function, which reflects inconsistency in the perception of low and high probabilities.

The Value function is basically utility function equivalent built with empirically observed results in mind. Usually denoted as $V(x)$ where x is payoff related to given prospect, value function is mapping $V(x) : \mathbb{R} \to \mathbb{R}$ with several general properties:

(i) $V(x) \leq 0$ for $x \leq 0$ and $V(x) > 0$ for $x > 0$

(ii) $V'(x) > 0 \ \forall x \in \mathbb{R}$ and $V'(x)_{x<0} > V'(x)_{x>0}$

(iii) $V''(x) > 0$ for $x \leq 0$ and $V''(x) < 0$ for $x > 0$

A concrete example of the Value function is plotted in Figure 1. Since its asymmetrical properties concerning gains and losses are relatively in line with common sense, I will not present further evidence supporting its plausibility. Kahneman and Tversky (1979) contain in length discussion on the connection between behavioural experiments and the shape of the Value function.

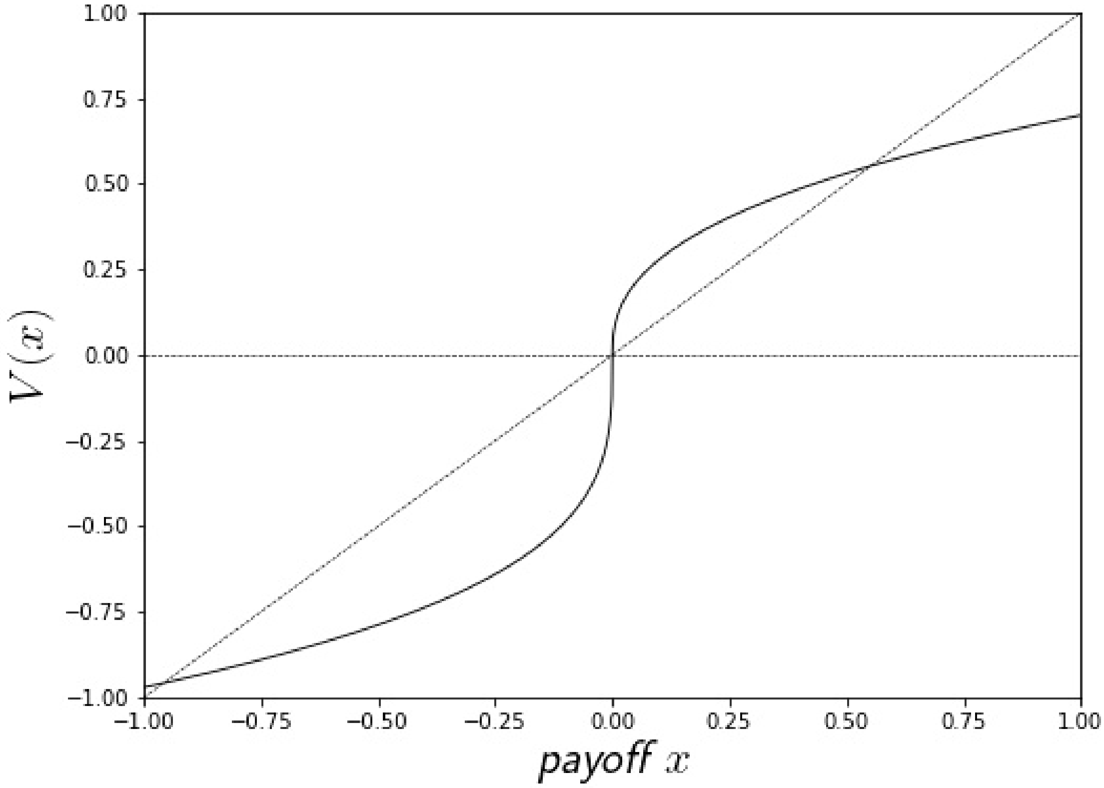

Figure 1: example of the Value function

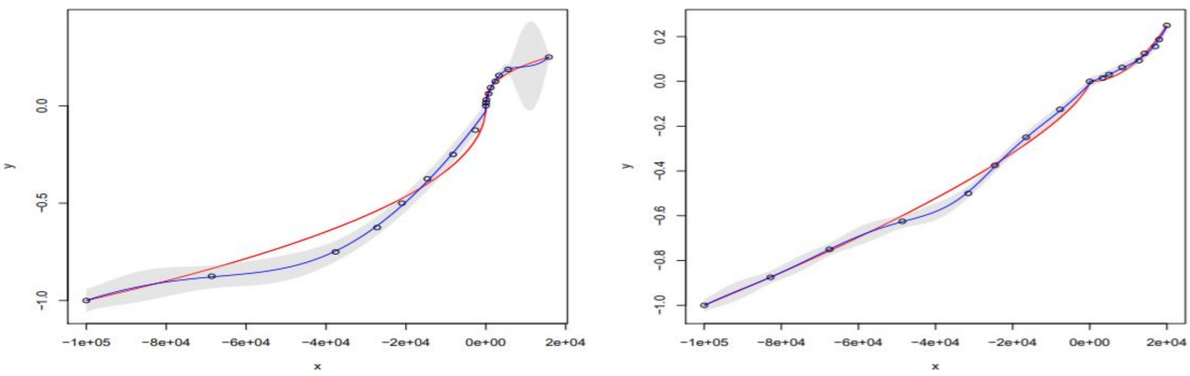

Figure 2: Nonparametric estimate of utility function of two individuals (Gu et al. 2018, page 18)

Figure 2 shows empirically fitted utility functions of two individuals. The smooth red line is the least-squares fit of power function, while the blue line represents a nonparametric estimate that authors show to be superior to less sensitive parametric methods. The grey area is 95 % confidence band of the estimated relationship (Gu et al., 2018). Apparent asymmetry between gains and losses can be observed in both cases seems to be in line with preferences proposed by the Prospect theory; otherwise, estimates suggest that not considering shared characteristics of general nature (e.g. that both relationships are not decreasing) utility functions might be significantly different for each person. The existence of significant individual differences naturally poses further difficulties for describing preferences through generalised relationships.

In contrast with the intuitive concept of the Value function, probability weighting is somewhat more puzzling. Individuals with higher sensitivity to potential threats have an apparent advantage over those who are less cautious. But what could be possible advantages of distorted probability perception?

Firstly we have to consider an option that probability weighting is simply the best attainable optimum concerning natural constraints faced by the evolutionary process. Herold and Netzer (2011) refer to probability weighting function as: "Evolutionary Second-best", arguing that the allele allowing its owner to assess risks accurately might, on the one hand, increase the reproductive fitness of its bearer but at the same time be too costly in terms of energy and consequently did not offer a real competitive advantage. The second argument is that a mutation facilitating the correct perception of objective probability is too specific and therefore rare and as such is quickly diluted by the process of sexual recombination.

An example of the PWF is given in Figure 3. Hereby applied functional form of the probability weighting function is given by equation (2) and originated in Prelec (1998) as mentioned before.

$$\omega(P) = e^{-(-ln(P))^{\alpha}} \quad P \in [0,1] \tag{2}$$

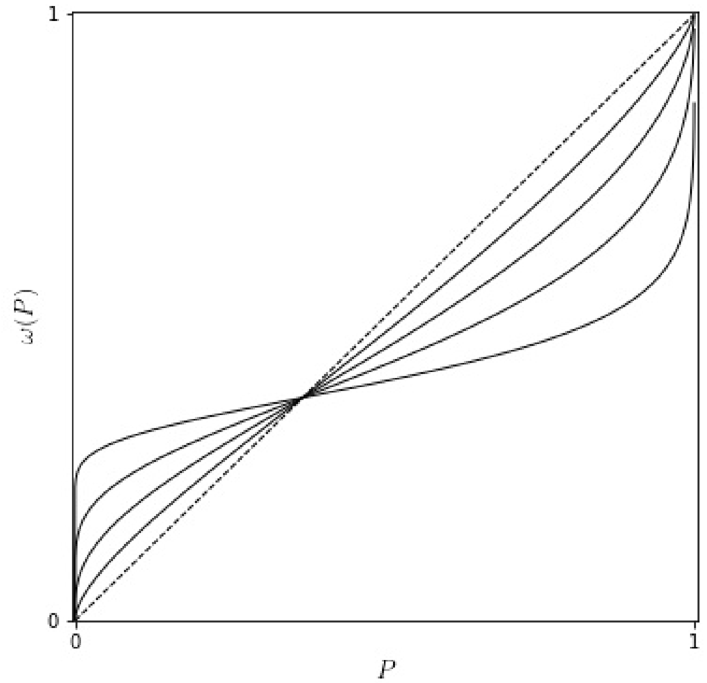

Figure 3: Probability weighting function, $\alpha = 0.2, 0.4, 0.6, 0.8$

The fixed point of PWF represents a point of reference of an individual. The existence of a transformation relationship with a reference point makes perfect sense from an evolutionary viewpoint. Numerical probabilities related to different decision outcomes are rarely available in the natural environment of most species. Consequently, agents have to draw on individual experience and recover the probability distribution of the expected payoffs from past subjective observations. A single individual is unlikely to possess a reliable collection of data related to the specific task at hand and will have to scrape together memories of weakly similar events to estimate underlying distribution. Should we be optimistic and assume such an estimate is unbiased, a small sample effect still causes it to be floating inside a large confidence band. Under described circumstances, probability weighting follows "better safe than sorry" logic, correcting for the possibility that very low estimated probability is in reality much more significant than expected (and vice versa). Weighting algorithm then squeezes expected chances in category "smaller than reference point" and "larger than reference point" and treat them differently as insurance against possible catastrophic error. However, how does this cautionary tactic explain the undervaluation of probabilities above the reference point? Firstly, it may function as a safety measure against pursuing prospects with too much optimism since every action is associated with cost paid in units of wasted energy, e.g. failure to catch a pray might influence future chances of success because each failed attempt reduces the hunter's stock of disposable calories. Secondly, dangers with high expected probability have been most likely dealt with in the past and solved successfully and therefore are not viewed as threatening.

Along with the psychological experiments implying the existence of a nonlinear probability weighting relationship (e.g. already mentioned Takemura, Murakami, 2016), neurobiological studies confirm the presence of such subjective transformation as well.

The experiment conducted by Berns et al. (2008) was based on measuring response in different areas of the brain to offered prospects. Subjects were presented with an option of receiving the electrical shock of a given magnitude and with given probability in several different settings. Shocks were previously calibrated according to their sensitivity. The authors comment that fMRI[ii] measurements' results favour the existence of an S-shaped nonlinear relationship converting objective probabilities into decision weights.

They also point out that bias occurred even if subjects were not allowed to chose but were merely presented with a probability of such event, suggesting that weighting of probabilities is a matter of perception and is not necessarily connected to the decision making process exclusively.

That shows weighting cannot be unlearned; inputs are already deformed before entering the decision-making process. The experiment also confirmed the notion that the value of presented prospects (the magnitude of shock in this case) and their subjective probability weights are assessed separately, although there appear to be interdependencies in some regions of the brain. That is a valuable result supporting the plausibility of separate evaluation of Value function and related probability weight.

A sample of measured neural response in different parts of the brain is showed in the following figure. It is worth noting that the presented areas displayed comparatively most substantial bias. Both are associated with visual perception, which is the most common channel for us to receive information trough.

[ii]Functional magnetic resonance imaging is a method of measuring the brain activity based on changes in blood flow.

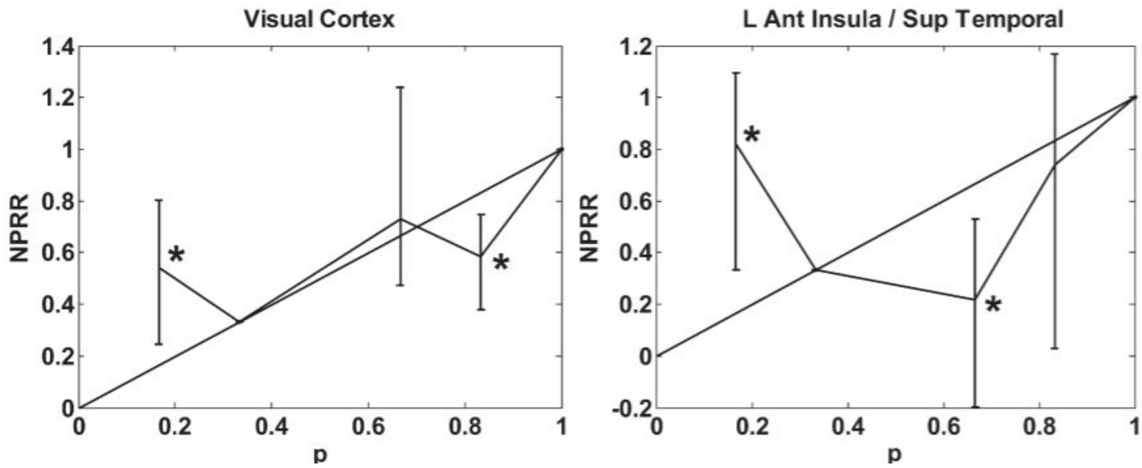

Figure 4: Visualised results the fMRI measurements, showing the relationship between an objective probability and its image after transformation in a given area of the brain, selected results correspond to parts associated with visual perception (x-axis is the objective the probability, Berns et al., 2008).

Viewed purely through the lenses of mathematically formulated rationality of the discounted utility paradigm, many features inherent to our decision making could be dismissed as simply wrong or inefficient.

Nevertheless, we have to consider that wiring of our brains formed in the period where life was vastly different from that which we experience in modern society. Making fight or flight decisions and forming social bonds necessary for survival was the most common task modern humans had faced for the prevailing part of their existence. Most choices of economic nature encountered by modern people were framed by binary outcomes of either survival or death. Failure to come up with a sufficiently quick solution could result in severe penalties in terms of malnutrition, social exclusion or wounds.

Under such circumstances, the development of cognitive shortcuts optimising between utility increase and 'computational time' seems entirely rational. Optimal decision-making rule that considers all possible outcomes of the given situation will lose evolutionary competition against the slightly suboptimal policy in each specific setting but generally leads to utility increase and is easy to implement.

It is also necessary to keep in mind that for any variety of behavioural pattern to become a prevalent strategy in the whole population, not only it has to increase the fitness of its bearer relatively to that of other competitors, but each its precursor also have to offer at least slight advantage.

Fact, that behavioural strategies do not form through the process of large 'jumps' but rather as a gradual sequence of small discrete changes imposes further restrictions on how strictly rational cognitive process can be. Strategy constructed by evolutionary process necessarily reflects not only the current environment but also residual information about conditions in the past (in contrast to a policy determined by direct optimisation concerning the conditions relevant only in the current period). That implies that contemporary wiring of our mind might carry residues of past structures that not necessarily contribute to the efficiency of the decision-making framework.

Consider all the anatomical structures in the human body that are (in theory) redundant or could be replaced by something more efficient but remain included because they were necessary for the past.

The author have already touched slightly constraints posed by the need for flexibility and decision making speed. Unlike artificial algorithms that can always be manufactured precisely for the task they are designed to perform, our mind has to handle a wide array of exercises, ranging from fine motor skills needed for handwriting to handling mathematical formulas. The need for universality reduces performance in highly specialised tasks. Data analysis necessary for forming a successful investing strategy falls precisely into the category of mental endeavours that are extremely niche. One's mental capacity also limits individual capabilities to collect and process information. As a result, investment decisions are formed based on utilising only a subset of available data.

We do not reason by strictly considering each possibility but instead classify possible outcomes into more or less similar classes heuristically, using some suitable (but arbitrary) metric. This habit is also closely tied to the need of generating a sufficiently fast response because some time limit usually frames real-life cognitive tasks.

Such in-built reasoning flaws can vary depending on cultural background, the time period, and education. Without any doubt, professional training and experience can largely override biological wiring and bring the individual closer to rational being in a strictly economic sense. However, as shown by fMRI measurements, bias is already involved in our perception, which to a large extent disqualifies the possibility to reduce it by experience. If cognitive biases were unique and random in each person, individual effects would cancel out if pooled together via the market. Unfortunately, there is no reason to think such inclinations are randomly distributed in the population; on the contrary, thanks to shared evolutionary history and the fact that individuals are subject to the same biological restrictions, shared systemic biases are very likely to exist.

4 Statistical Properties of Stock Markets - Long Memory and Deterministic Chaos

4.1 Anomalies in the Price Behaviour

The presence of excess volatility in the stock market implies that the real-world markets are not strictly efficient and respond inaccurately to external stimuli. Over the years of empirical research, a noticeable collection of other facts contradicting the reigning efficiency paradigm accumulated, supporting the impression that the supposed efficiency is a gross oversimplification of the actual process.

Peters (1991) points out that distribution of short term (namely 5-day) log-returns of S&P 500 significantly deviate from the normal, with a large portion of mass concentrated in its tails, and is much more similar to distributions from the family of stable Paretian proposed by Benoit Mandelbrot that are characteristic for processes with cycles, frequent rallies and crashes. Thus, the market seems to favour extreme changes in price more than would correspond to the hypothesis about random walk with normally distributed changes (log-normally in this case). Mandelbrot himself also notes that such a shape is often associated with processes that are heavily infested by background noise to the point where recovery of the causal structure of the process itself is nearly impossible; as he further adds, this seems to be an unfortunate reality of empirical endeavour in economic research in general (Mandelbrot, 1963).

The shape of the probability density is not a threat to the efficiency per se. The extreme changes might represent an accurate response to a highly volatile flow of external information. However, the fact that the kind of clues that the investors usually follow (e.g. earnings, ROE, et cet.) tend to be more than stable for the companies of S&P 500 calibre suggest that might as well be structural reasons causing returns to react in said manner.

Peters offers delay in response to new information as a plausible explanation of "fat tails". Instead of news being incorporated into price signals in real-time, as suggested by EMH, the delayed impact of news accumulates over time and then hits price as a significant shock. Non-normality of returns' distribution might seriously weaken the validity of usual statistical measures (e.g. correlation, Peters, (1991)), which is unfortunate since correlation is a favourite measure among practitioners. Peters also estimated Hurst exponent of USA stock price time series, showing that its value is significantly larger than would correspond to random walk process, implying that time series have long memory ($\approx$ 48 months). Since irregular crashes, rallies, and historical dependence (memory) are typical features of nonlinear dynamical systems in the chaotic regime; the author views these properties as proof that time series in equity markets are indeed induced by a chaotic process (further evidence is presented, for example, fractal-like self-similarity of stock returns concerning different time scales). Unfortunately, chaotic time series with a small stochastic element is hard to distinguish from linear series with a significant stochastic compound; while there is a large bulk of literature presenting evidence on underlying nonlinearity lot of studies reports inconclusive results when it comes to a direct proof of a presence of the deterministic chaos.

To identify deterministic chaos in the underlying causal structure with definite conclusiveness proofs to be quite a complicated task. Some of the up to date results are review in the paragraphs below. However, it has to be noted that research in this area delivers mixed results. The author speculates that one reason responsible could be the character of statistical tools available to researchers. Generally speaking, related techniques, e.g. the reconstruction of the phase space, require some external parameter to be set by the researcher himself (embedding dimension in this case), thus leaving room for variability in empirical results. Since the application of related techniques is very much a separate field calling for specialised knowledge, the thesis focuses on the results themselves, omitting discussion on the plausibility of methods in question. The following studies were partially chosen for their novelty, which allows them to reflect on the preceding research results and partly because they employ a varied range of methods instead of betting on a single approach.

Perez (2020) carries on a comprehensive review regarding the conclusions of preceding studies. Six reviewed papers support the presence of deterministic nonlinear chaos; on the other hand, most studies fail to prove its significance, with the majority supporting existence of nonlinear dependence of some kind. The author himself carries out complex testing using several stock indices among other time series and fails to support deterministic chaos hypothesis except in the time series of Nikkei stock.

Another modern parer reporting on the issue is work by BenSaïda, Litimi (2013). Research tests the hypothesis of chaotic components in various financial time series (including equity markets prices).

Estimated values of the Lyapunov exponent support the significance of deterministic chaos in all cases. BenSaïda and Litimi attribute high variance of results in the available literature and frequent ambiguous conclusions to inappropriate application of the used tests and emphasise the importance of distinguishing between the chaotic and the stochastic components of data in question. Further, they conduct a simulation to check their test's ability to detect nonlinear chaos in artificial data generated by a known process. Simulation is successful (that is, applied methods are able to detect differences), with the lowest rate of correct classification being 95 %. The use of the simulation to cross-check the validity of the employed statistic makes these results stand out from other papers surveyed by the author, making a solid case in favour of deterministic chaos in stock price time series.

Considering the high variance of results in the contemporary literature, the question of whenever the deterministic chaos is or is not part of the data generating process appears to be an unresolved puzzle still.

Should we think about its possible causes, deterministic chaos is often a feature of dynamical systems composed of interacting agents who apply nonlinear decision rules (e.g. models in Day, 1994). A Decision-making strategy built along the lines of the behavioural framework could thus make a promising candidate should one look for causal explanation. Results reviewed in the preceding section showed that nonlinear transformation is clearly present regarding the perception of probabilities. Hurst exponents estimated by Peters (1991) speak against widespread conviction that the past does not matter in the markets. Historical dependence poses a severe issue for all variants of EMH since it implies that the path of the price depends on itself, and it is not a mere reflection of the current information dispersed among investors.

Research paper by Schmitt and Westerhoff (2017) show that the distribution of distortions of the S&P 500 index from its fundamental value (in terms of log differences) is bimodal with a drop around zero, suggesting that the index spends relatively more time in disequilibrium than close to its fair value. The authors then show that it is possible to replicate similar data by several different agent-based models. If EMH was to hold, distortions concerning fundamental value would most likely be unimodal with modus in zero. Observed bimodality seems to imply that S&P 500 is more frequently mispriced than not, showing that interactions among traders are more likely to cause misalignments than to arbitrage out the differences. Systematic mispricing is hard to explain under the efficiency paradigm. However, relaxing assumptions about the degree of rationality of the average market participant could resolve the dilemma. If the perception of probability is warped, then the mispricing seems to be much more plausible.

4.2 Alternative Explanations of the Observed Patterns

Behavioural finance represents a specific approach according to which the causal explanations to economic puzzles can be derived from the cognitive deficiencies of the economic actors. A similar strategy is also utilised later in this thesis.
Another distinct sort of explanations attributes observed inefficiencies to the complex microstructure of the stock market (and financial markets in general). Participating actors are not necessarily irrational themselves, but the resulting structure produces unexpected behaviour due to its high level of complexity. Should one summarise this approach in one sentence, the modelling philosophy says that the whole is greater than the sum of its parts, or more precisely, that the whole behaves much differently than one could deduce by researching its components.

Here, the cause for the complex behaviour of the equity market derives from its complicated inner structure composed of numerous interacting elements. Sanders, Farmer, Galla (2018) suggest that financial markets could be thought of as a p-player game where participants play multiple rounds. The objective of each one of them is to maximise his payoffs. Agents are able to learn and adapt their strategies based on past results. Authors explore the stability of such systems concerning various parameters and conclude that they tend to display chaotic behaviour for a large number of players, the tendency to exhibit chaos is also inversely proportional to the length of players' memory (The more accurately agents assess past, the more probable is the existence of stable points). Researches also point out that changes in payoffs display volatility clustering similar to financial time series (levels of variability are autocorrelated over time).

In the previously mentioned paper (Schmitt and Westerhoff, 2017), the authors resolve the observed bimodality of price distortion by simulating several agent-based models with chaotic dynamics, showing that they can result in distributions similar to the bimodal distribution of price misalignments described before. They simultaneously attempt to reproduce such characteristics through the linear ARMA process, with a conclusion that the linear model failed to match observed characteristics with statistical significance.

Under the complexity framework, organised markets are no longer necessarily the best way of resource allocation since the large systems with heterogenous elements produce inefficiencies by design. In the behavioural markets, the excess volatility can be controlled at least in principle by accepting measures that reduce the likelihood of high impact "bad" individual choice.

Although any results obtained from computational models cannot with finality resolve empirical issues, the above-reviewed results showed that games with many players are prone to instability and chaotic dynamics. Similar results can arise from interaction among heterogeneous trading strategies. A structural explanation like those reviewed seems to be appealing, mainly since they are able to reflect the complex microstructure of the markets and the heterogeneity of their participants, which are essential characteristics of financial markets that fail to be reflected by the more mainstream approach, which often assumes that it is possible to aggregate individuals into the representative agent.

5 Modelling Market with Behavioural Preferences

As hinted by its title, this section is concerned with constructing a model that could demonstrate a causal relation between inefficient decision-making and excess volatility puzzle.

The preceding sections have introduced some facts concerning individual behaviour when facing risky choices and offered a set of tools allowing economist to reflect irrationality as a feature of a mathematical model. So far, the thesis only speculated on the possible connections between the volatility of equity markets and psychological imperfections. To have a legitimate research hypothesis, one has to propose a specific mechanism by which the proposed cause channels into the observed effects. In other words, how does the subjective weighing of probabilities translates into price fluctuations and mispricing? It may seem reasonably plausible that the incorrect scale of preception in market participants causes assets to be incorrectly priced. However, the exact influence of probability weighting on the price dynamics can not be determined trivially. As shown by the computational models reviewed, not only the individual strategy itself but also the organisational structure of the market and interactions among market participants themselves affect the resulting price formation process. The microstructure of the model proposed is minimal in the organisational sense, containing only the market maker and trader. For a more complex model with a large number of free parameters, it could hard to attribute simulation results to some specific cause. The intuition behind the model is the following; at the start of the trading market maker makes an initial offer of the price per share, the fundamental value of the asset in question is publicly known (i.e. available to both subjects). The trader can either decide to buy or sell depending on which option he deems more profitable in accordance with his predetermined strategy. Market maker collects the orders and quotes a new price based on the excess demand (or supply).

The question to answer is following: Under the condition that the trader displays strong behavioural bias, will the price settle on some equilibrium level, or will the inbuild deficiency generate persistent fluctuations? A brief sketch of the modelling strategy could be summarised in trough following bullet points:

1. Adapt the main concepts of the Prospect theory, the value function and probability weighting function into the representative trading strategy.

2. Show pronounced probability weighting can cause excess volatility and chaotic oscillations of the price.

3. Simulate distribution of price misalignment and show that it is bimodal with the lowest density in the close neighbourhood of zero.

4. Explore statistical properties of short term returns and contrast them with empirical data measured on a comparable time scale.

The central philosophy behind the model creation is not to build a descriptive model that satisfyingly matches all the quantitative characteristics of observed data.

That would be impossible since the real-world markets are infinitely more complex. The focus is to propose a mechanism illustrating how could a certain type of behaviour in individual agents cause some of the statistical characteristics observed in the resulting price. However, it is still a crude approximation at best.

Because of the competitive nature of financial markets, successful practitioners must learn and adapt new strategies to keep up with the quickly changing environment, meaning that the data generating process is probably changing over time.

Because the objective of analysing the model is resolved via means of numerical simulation, part of the subsequent text is concerned with an overview of applied computational tools. Then follows a formal mathematical description of the representative trading strategy and the process used by the market maker for price adjustment. Numerical results are discussed separately.

5.1 Techniques Used for Model Building and Analysis

The present subsection contains a short description of the tools applied in the process of model creation and its analysis. Less common techniques are reviewed in detail and accompanied by a simplified "recipe" in pseudocode and a verbal description of the algorithm. Full python code accompanied by a crude commentary is also available on the authors online drive. But the need to constantly define parameters for visualisation tools in order to produce a clean image and need to comply with other technical requirements obscures the actual algorithm behind it. Common statistical tools are applied with sufficient context in the section directly concerned with the analysis of results but without any further discussion (e.g. standard deviation).

5.1.1 Agent-Based Modelling

Financial markets are essentially a system composed of multiple interacting elements, mostly trying to fulfil the shared goal of maximising their profits. That might not seem to be a groundbreaking statement, but as the author going to argue, complex patterns of aggregate behaviour caused by simple interactions on the individual level are the property inherent to economic systems that get systematically underappreciated by a more traditional optimising approach that utilises representative agents with perfect rational foresight.

Before starting the discussion, let it be clarified what is precisely meant by the agent-based model in the context of this work. Defined widely, the adjective "agent-based" refers to the diverse class of computational models, sharing their bottom-up architecture as the unifying characteristic. Instead of focusing on macro aspects of some specific object of interest, e.g. geometry of growing monocrystal and describing its quantitative properties as a function of concentration and surrounding temperature, we create artificial molecules and ascribe them with simple rules that will govern their interactions under the condition that surrounding environment has such and such properties. Then we assess if the resulting properties of the simulated chemical match with observed crystals.

Such vague description naturally covers a very diverse class of computational tools, from evolving spatial simulations such as Conway's Game of Life to more conformist models produced by simulating a system of equations. For the sake of this thesis, I will consider models of markets composed of agents with heterogeneous strategies expressed as functions where price formation is essentially disequilibrium, following excess demand according to some predetermined rule. This type of models was extensively studied by Day (1994).

Preceding metaphor borrowed from physics might seem out of touch while discussing economic applications, but such comparison is intentional. The traditional approach to modelling prices in the financial markets by enforcing rationality and impeccable foresight is not unlike modelling an ideal gas, where particles' behaviour follows a common trend if subjected to external pressure. Individual particles are allowed to deviate, but since such deviations are random and independent of each other, on the macro-scale individual effects cancel out, and the behaviour of the system as a whole appears to be following clearly defined law. The problem with such an approach in economics is twofold. Firstly systems studied by economics are infinitely smaller in terms of the number of independent units they are composed of; secondly, there is no evidence that agents participating in the market deviate randomly from the guidelines of rational choice.

On the contrary, investors' decision-making process seems to be subject to the rich pallet of internal systematic biases (Hirschleifer, 2001). In an environment where agents operate according to strategies that have a character of responding to external stimuli based on local rules; or based on a specific niche they occupy inside the system, it seems only natural that many features observed on the macro-level could be in principal traced back to the microstructure of the systems. In other words, observed fluctuations of some state variable on aggregate level, e.g. price, is not the result of markets being constantly subjected to significant stochastic shocks but is instead a direct implication of nonrandom interactions among its participants. Implicitly, a model that would attempt to explain such a process, simply through changes of external conditions, would be destined to fail since observed behaviour is an intrinsic property of such structure. That is, observed fluctuations of some state variable are not caused by random external shock but are a result of individual units interacting among themselves.

Of course, there is no doubt that external shocks of essentially random character play an important role in setting so complicated as financial markets where the price of certain assets can be tied through proxies to hardly predictable external conditions. An example could be the value of company shares tied to the price of mineral resources, which is in turn connected to climatic and political conditions of the supplier region. Changes in stock price induced by rapid local climate changes could indeed be considered a random external factor. The same is true for much simpler causal chains of events influencing prices of globally traded assets.

The focus of agent-based models is not essentially the path of the price itself but rather the process by which the market converts external signals into the current price.

That is, do markets act as well behaved machine that accepts real-world inputs in the form of news, financial statements and commodity prices and converts them into the fair price that makes further profit impossible? Or, does the trading process amplify noise in incoming signals?

Man-made tools are usually constructed in the bottom-down fashion, consider, for example, the clockwork. Each component in the machine is constructed with the function of a device as a whole in mind. As an implication, each piece has the most efficient properties to ensure the proper functioning of the clockwork itself, considering restriction posed by technology and laws of physics.

To put it more abstractly, each component of the device carries inside an implicit map of the structure it is part of (supposing the purpose of said device is known to us) because there is a single most efficient way to construct a machine performing such function while including said component. Contrary to centrally organised architecture, societal structures are created following different principles. Let's say that the most elementary case of society is the gathering of single-cell organisms in an amoeba-like structure. Individual cells absorb nutrients from the outside environment and metabolise them by harvesting energy from photons.

Comparing such type of organisation to the previously mentioned clockwork, several differences are apparent. Firstly, each individual in cellular society can function meaningfully without the rest, unlike an artificial machine where using a single component to measure time is unthinkable. Secondly, the optimal function of clockwork is ensured by design. For the cellular society, it is not impossible to follow an optimal path that maximises the amount of absorbed nutrients and sunlight. But such optimality is not facilitated by design; each movement is not the result of cells working together to ensure maximisation of aggregate consumption. Instead, singular cells maximise their metabolic payoffs, and a higher level of organisation is an accidental byproduct. In such a setting, the path of the as a whole amoeba might be very close to optimal for most of the time but might regularly deviate as a result of individual cells acting as independent units. For example, an uneven concentration of nutrients could produce a temporary whirl inside the amoeba, caused by agents following their best possible paths and other cells in their immediate neighbourhood following the temporary trend.

Consequently, such an imaginary proto society's behaviour could be quite successfully modelled by a system of equations without reflecting the inner structure; the model would have reasonable predictive power with occasional deviations that would be attributed to randomness. However, since the proposed phenomenon breaks down to the collection of individual actions, the above-described approach would be destined to fail regularly, especially on a shorter time scale (large gaps between measurements would allow to maintain the illusion of moving along optimal as they would not reflect short-lived fluctuations). In an extreme case, the multicellular organism could collapse as a result of different parts responding to their individual needs; such infrequent but significant catastrophes would naturally puzzle a hypothetical observer, occurring once a while without apparent external cause.

Events of similar character would be unexplainable under the aggregate approach. Economic systems often combine elements of centrally organised structures with self-organisation arising from shared interests (e.g. need to exchange goods and services). Similarly to described proto society economic system might display a deceptive appearance of efficiency on a larger time scale while showing complex fluctuations if measured in shorter time intervals. Effects of the underlying microstructure could be more prominent in times of economic crises when the need of agents to protect their assets' value would lead them to cease cooperation and cause the market collapse. Alternatively, the formation of semi-autonomous interest groups could lead to the destabilising system as a whole.

Social structures are built mainly around cooperative principles, causing them to deliver more or less optimal results to each participant. However, faced with external shocks, they may amplify destructive effects due to their heterogeneous internal structure. Agent-based modelling is a promising tool in this context, offering a technique for examing dynamics of structures operating in a similar manner.

Their application could elucidate mechanisms by which stock markets produce excess volatility, frequently forms short term bubbles and how crises might be amplified due to the interconnectedness of financial institution. In the context of this work, it is applied with the intention to explain how shared behavioural characteristics embedded inside the trading strategy of stock market participants influence short-term price stability and its eventual convergence to assets' fundamental value.

5.1.2 Computational Tools and Environment

Software Environment Code[iii] for simulating the dynamical system representing the market was written in Python language on the open-source platform Jupyter Notebook. The Jupyter Notebook is now a widespread tool used frequently for scientific computing since its birth in 2014. **Further specifications are available on the Jupyter official website.**

Iterating Functions Similarly to Matlab and other higher-level languages, Python offers its user an option to define custom functions. For the generation of the model time series, it is necessary to iterate a recursive relationship of the form:

$$x_{t+1} = f(x_t)$$

The following pseudocode statement encompasses the general logic behind the algorithm. And reads as following: For initial condition x_0 picked from the domain of $f(\cdot)$ iterate function f T-times and return the vector of all iterates $\mathbf{x}$. In other words, the code returns time series of length T generated according to process $f(\cdot)$.

1. $x_0 = c,\ c \in D(f)$

[iii] The complete code is accessible on the online drive trough this link.

2. **for index** $t = 1, 2, .., T$ **:**

3. $x_{t+1} = f(x_t)$

4. **return** $\mathbf{x} = (f(x_0), f^2(x_0), .., f^T(x_0))$

Bifurcation Diagram serves as visualisation device for the asymptotic properties of systems with complex dynamics. It allows us to gain insight into systems' behaviour with respect to changes in the parameter of choice through a relatively simple algorithm. Denoting the block of code in the preceding paragraph by $\mathbf{A}(x_0, T, f)$ it is possible to express architecture of bifurcation diagram in analogous fashion.

1. $\mathbf{p} = (p_0, p_1, .., p_n),\ p_i - p_{i-1} = h\ \forall i$

2. **for index** $i = 1, 2, .., n$:

3. $\mathbf{B_i} = \mathbf{A}(x_0, T, f_{p_i})$

---

4. **for index** $i = 1, 2, .., n$:

5. **for index** $j = T - n_0, T - n_0 + 1, ..., T$:

6. **plot**$(p_i, B_{i,j})$

First, some parameter of interest p in function f is chosen, then code creates an increasing sequence of its successive values, with a step of size h between them. Then matrix B is constructed, each its row B_i contains time series of length T as described in the previous paragraph, generated conditional that parameter $p = p_i$. Then we plot the last n_0 elements in each row into the $x - y$ plane, with the parameter p on the x-axes and iterates of f on y-axis.

Interpretation is straightforward; the Bifurcation diagram can show if the system approaches steady state for some fixed value of the parameter p or if it oscillates in aperiodic orbit.

Steady-state is represented by a single point in the diagram since, after a large number of iterations, successive values are very close to each other. An example of the bifurcation diagram is shown later in the text, where the model is examined.

Cobweb Plots are used to visualise orbits of one dimensional dynamical systems. Since verbal description would hardly capture their usefulness author presents cobweb plots of the well known logistic model as an example (Figure 5). Logistic model is given by:

$$p_{t+1} = rp_t(1 - p_t)$$

Where r is a real parameter. For low values of r system converges to steady state or periodic orbit. When r becomes close to 4, the system starts to exhibit chaotic behaviour.

Sample of the orbit in chaotic regime can is visualised the fourth subplot. Orbit starts at initial condition $p_0 = 0.01$ and travels trough the plot erratically with no sign of convergence whatsoever.

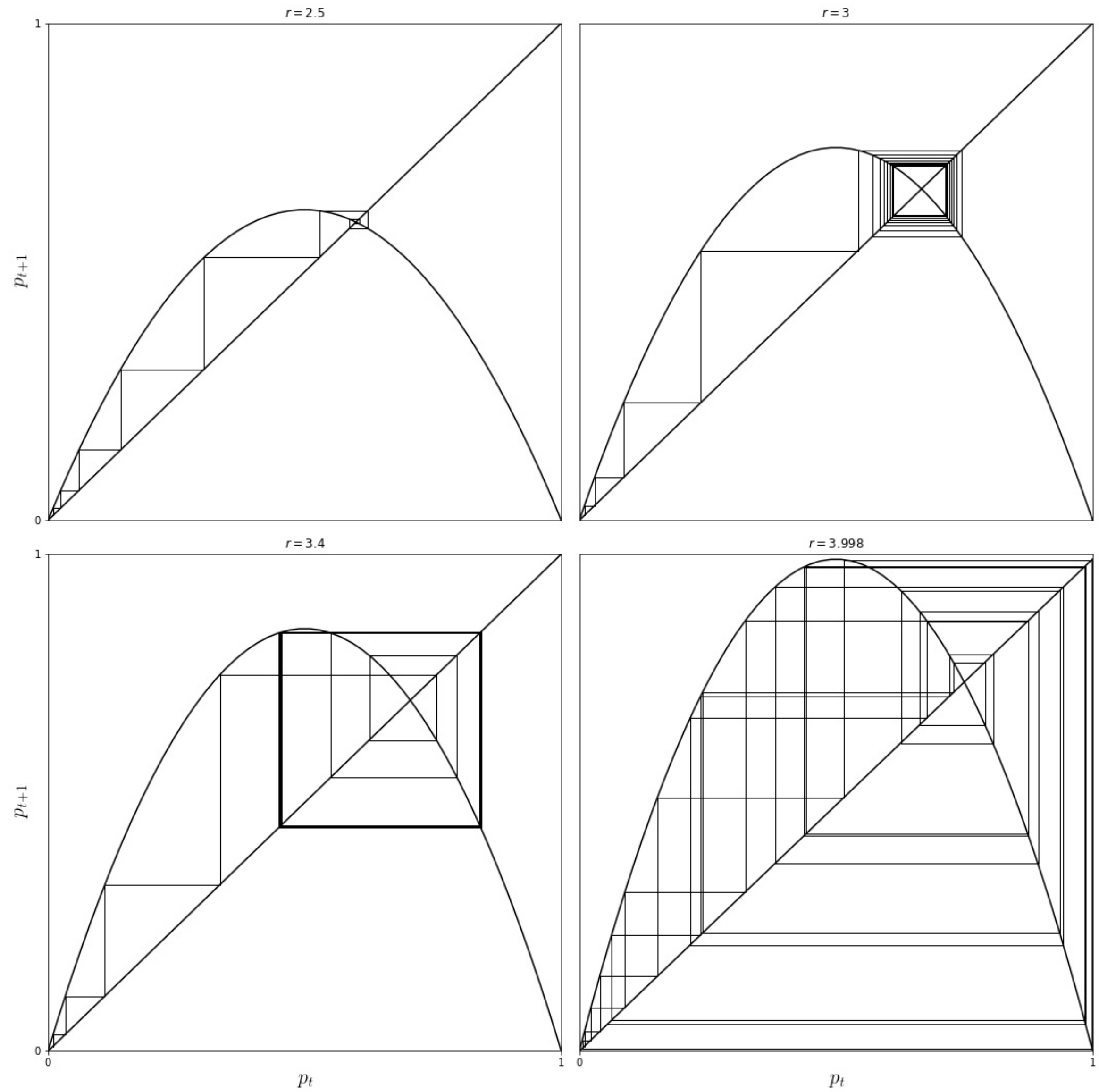

Figure 5: Cobweb plots of the logistic equation, Source: Author.

The purpose of cobweb plots is precisely to visualise such behaviour. If the map defining the dynamical system is in a chaotic regime, then cobweb plots present a useful way of seeing if the fluctuations concentrate around certain points.

Suppose we are interested in the properties of orbit dynamical system defined by the recursive relationship:

$$x_{t+1} = f(x_t)$$

The coding of the cobweb plot goes along following lines:

1. $x = c,\ c \in D(f)$
2. **for** $i = 1, 2, ..., n$ **:**
3. $y = f(x)$
4. **plot**$([x, x], [x, y])$
5. **plot**$([x, y], [y, y])$
6. $x = y$

First-line uploads initial into local variable x. The second line then starts the iterative cycle with n repetitions. Code then defines variable y as a function f of the initial condition and plots line connecting point $[x, x]$ on 45-degree line and point $[x, y]$, that is first coordinate is x and second its image under mapping f. The last step overwrites the initial condition stored in x and sets its image as the new starting point. The process described is then repeated n-times.

5.1.3 Simulating Random Variables

Draws from Probability Distributions For simulation of the stochastic variables the author uses in-build functions of the **SciPy** and **NumPy** libraries. Which enable the coder to draw a set of numbers of the predefined size out of the probability distribution of choice with an arbitrary choice of parameters.

5.2 Market Microstructure

5.2.1 Trading Strategy

The proposed representative strategy is qualitatively based on α-investors described by Day and Huang (1990). As briefly mentioned in the introduction, the α's represent value investors, who expect the stock price to track its fundamental value in the long term. Thus, their demand is a function of price misalignment regarding their estimate of a traded asset's underlying value. Let us denote fundamental value by u from now on.

The estimation method of u is based on the evaluation of micro and macroeconomic fundamentals related to traded shares. It does not explicitly enter into the model in the form of the estimation process itself; rather, u enters the model as a real parameter independent of nominal price movements.

The fact that u is supposed to be stable in time indicates that we deal with dynamics on a very short time scale. Define observed price distortion as:

$$d(p,u) = |p - u| \tag{3}$$

With the p denoting the price.

If $d(p,u) > 0$ value traders expect future price correction. If market price surpasses fundamental value, traders consider stock to be overvalued and hedge themselves against expected drop in price by selling off shares and profit on perceived misaligment between purchasing and selling price. Similarly, if stock is undervalued α- traders issue buying orders because they view traded shares as undervalued and likely to increase in value (Day, Huang, 1990).

Value function according to prospect theory preferences is then given by:

$$V(p) = \begin{cases} \rho_{\mathcal{G}}(u-p)^{\beta} & \text{if } p < u \\ -\rho_{\mathcal{L}}(p-u)^{\beta} & \text{if } p > u \end{cases} \tag{4}$$

With $\rho_{\mathcal{L}} = \rho_{\mathcal{G}}$. As there is no strict distinction between the area of loss and profit.
A graph of the applied value function is shown in Figure 3.
Value function is positive in area of expected gains and negative if trader expects future drop in market price.

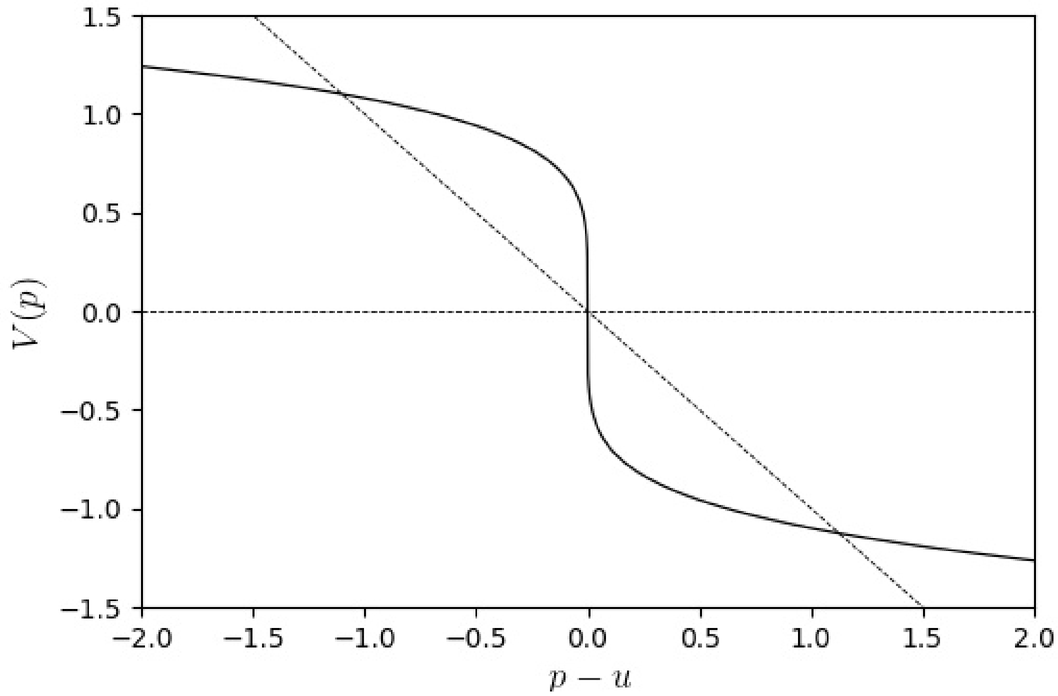

Figure 6: Value function of α-strategy

From a more practical point of view, the relation between $\rho_{\mathcal{L}}$ and $\rho_{\mathcal{G}}$ depends purely on traders personal framing if he already owns traded stock and considers the act of selling the overpriced asset as harm reduction then he could be significantly more risk-taking compared to a case when he thinks of the selling to be an act of profit increase (he bought under the perceived value and sells for a higher price). In the latter case, both parameters could be equal as the difference in perception between profit maximisation and loss reduction disappears.

Agents participating in stock trading typically have some innate expectations about the magnitude of price movements. Here is supposed that fundamentalists anticipate price to fluctuate inside some closed interval $I = [u - m_0, u + m_1]$. Interval I can be in principle arbitrarily large and asymmetric, reflecting the agent's uneven expectations but for simplicity, let's consider I to be symmetrically centred around u with $m = m_0 = m_1$.

The probability of expected price correction is proportional to the magnitude of deviation and can be expressed by.

$$P(p) = \frac{d(p, u)}{d(u, m + u)} \tag{5}$$

P is converted into decision weights by a nonlinear weighting function derived by Prelec (1998) given by equation (2). The strength of the agent's bias is expressed by the parameter α, as α decreases the weighting function $\omega(\cdot)$ expresses comparatively stronger bias. Combining together Value function with probability weighting we can now derive traders's demand.

Let $\delta_v(\cdot)$ be demand function associated with value trading strategy.

$$\delta_v(p) = \begin{cases} \rho_{\mathcal{G}} m^{\beta} & \text{if } p < u - m \\ V(p)\omega(P) & \text{if } p \in [u - m, u + m] \\ -\rho_{\mathcal{L}} m^{\beta} & \text{if } p > u + m \end{cases} \tag{6}$$

The constant value of the demand function outside the anticipated interval I represents transaction limits. Practically, the transaction limit is usually determined by some institutional requirement. In this setting, it represents rather investors' unwillingness to risk further resources when the price leaves expected boundaries.

5.2.2 Price Adjustment Process

As briefly mentioned, trading is facilitated by the market mediator/market maker. Mediator collects incoming orders and cuts off excess demand by applying pricing policy.

Consider marked populated by n traders with individual order functions $\delta_i(p)$, where $i = 1, 2, ..., n$ and p denotes asset price per unit. A positive value of $\delta_i(.)$ is interpreted as an order to buy; similarly, $\delta_i(.)$ being negative means that i-th market agent is selling its assets.

Total net market demand is then given by the sum of individual orders as:

$$D(p) = \sum_{i}^{n} \delta_i(p) \tag{7}$$

If selling orders overweight buying ones $D(p)$ is less than zero, representing supply $S(p)$ instead.

$$S(p) = -D(p)$$

Let u be the fundamental value of asset traded (determined, for example, as the sum of discounted expected cash flows from investment). Suppose that u is based on publicly available information. Mediator pairs together supply with demand by adjusting price p. Resulting price of next period p_{t+1} is based on the deviation of current price p_t from u, the current total net demand $D(p_t)$ and u. Assuming price adjustment process is linear. It is possible to write it down as:

$$p_{t+1} = u + (1 + \mu_0)(p_t - u) + (1 + \mu_1)D(p_t) \quad \mu_0, \mu_1 \in \mathbb{R} \text{ are parameters} \tag{8}$$

Since direct influence of market mediation is of only mild interest for our purposes, lets further assume that: $\mu_0 = \mu_1 = \mu$.

$$p_{t+1} = u + (1 + \mu)(p_t - u + D(p_t)) \tag{9}$$

Changes in u only shift the function along the 45 degree line, and do not influence the stability of fixed points, ,consequently, the price dynamics is invariant with respect to u. We can thus conveniently chose $u = 0$. Equation (9) defines a one-dimensional dynamical system. The mathematical expression on the right-hand side maps the price in the current period into the price of the period following.

The right-hand side represents the impact of the trading process on the future price, thus it is referred to as the market impact function and denoted by $\theta(p_t)$ in the following text.

6 Running a Simulation

The properties of the proposed model are assessed via simulation. The price stability concerning model parameters setting and distributional properties of resulting time series and their correspondence with the empirical observations are the central subject of interest.

Numerical values of the parameters as set for each part of the simulation are shown in the description of each figure respectively. **The notation regarding model outputs reads as follows:**

1. $R_{XX}(x)$...autocorrelation function of variable x
2. σ_x... rolling standard deviation of variable x
3. p_t... price at period t
4. $r(x)^{log}_{t,t+n}$... n-period log returns on variable x
5. $\theta(\cdot)$... market impact function
6. u...fundamental value of traded shares
7. t... period index, $t = 0, 1, 2, .., T$
8. $m, u, \alpha, \beta, \rho_{\mathcal{G}}, \rho_{\mathcal{L}}$ [iv]...parameters of the trading strategy (equations 4,5,6)
9. μ...parameter of the price adjustment process

6.1 Interpreation of Parameters

The model contains three main categories of parameters. Those which specify the properties of the trading strategy and the parameters μ_0 and μ_1, which drive the intensity of the market maker's response to the deviation of price away from the fundamental value and excess demand, respectively, the last exogenous parameter u represents the fundamental value itself. Since the strategy of the market maker does not include any explicit behavioural elements author assumes that $\mu_0 = \mu_1 = \mu$, as stated in the model building section already.

The character of the investor's strategy is controlled by several real parameters. As mentioned before, the value function in the Prospect theory is characterised by profit/loss asymmetry rooted in the framing effect. Here is no strict distinction between profit chasing and diversion of losses. If the trader sells while the price is above fundamental value, he views such action as the realisation of profit since the assets were bought while below their fundamental value. Conversely, buying in the bear market is considered "a purchase with discount" because the trader expects the price to rise in the future. Thus there is no explicit reason to incorporate asymmetry inside the value function, and it is possible to make another simplifying assumption that $\rho_{\mathcal{L}} = \rho_{\mathcal{G}} = \rho$.

Parameter β is supposed to be in the range $\beta \in (0, 1)$. β controls for risk aversion, considered range implies that risk-seeking behaviour is not allowed. The probability weighting function expresses the intensity of probability weighting bias; specifically, it is regulated by shifting the parameter α, with $\alpha \in (0, 1)$. Since the influence of the

[iv]For the purpose of model simulation the author makes an assumption that $\rho_{\mathcal{G}}, \rho_{\mathcal{L}} = \rho$

probability weighting bias is in the centre of the author's interest, much of the following computational analysis is devoted to exploring its effect on the price dynamics. Thus the simulations are run under the assumption that said bias is high (that is, α is low, $\alpha \approx 0.1$) with one exemption.

At last, the parameter m regulates the expected range of price changes, causing the price to move in a broader range if increased but without any principal change in dynamics. Overview of parameters with their respective setting is showed in the following table.

Parameter	Interpretation	Range of chaotic behaviour
α	intensity of probability weighting	$(0.1, 0.37)$
β	risk aversion	$(0.1, 0.37) \cup (0.8, 1)$
μ	magnitude of price adjustment	$(0.6, 0.26)$
ρ	framing parameter	$(1, 2)$
u	fundamental value	-
m	width of expected interval	-
	Default setting of parameters	
Parameter	Value	
α	0.115	
β	0.102	
μ	2	
ρ	1.1	
m	1	
u	0	

Table 1: Table of model parameters

It is necessary to note that the above intervals are only approximate because the system enters periodic windows characterised by regular dynamics even in a chaotic region and are computed conditional other properties held fixed on default values. The parameter u causes the model to shift along the 45 -line and does not influence resulting dynamics qualitatively.

6.2 Influence of Probability Weighting

To analyse the impact of the probability weighting bias on the dynamics of the price adjustment process, the author employs the bifurcation diagram as described earlier. A bifurcation diagram is a visualisation tool commonly used to assess the stability properties of nonlinear systems. The diagram's base principle is quite simple; a computer algorithm generates long time series conditional that a particular parameter of interest has some predetermined value. The last n elements of the resulting vector are then cut off and plotted in the $x-y$ plane where the y coordinate is the value of the observed state variable (in our case deviation of the unit price of share from u), and the x axis are values of the parameter. If the system approaches a steady-state after some number of iterations, then the last n points approximately coincide and look like a dot in the $x - y$ plain; aperiodic oscillations, which are characteristic for the chaotic regime, translate into a bifurcation diagram as a collection of distinct points above the given value of the parameter on the x-axis.

Intuitively probability weighting has destabilising influence in the capital market environment because it works against the agent's ability to interpret signals given by price movements and chose a response of the correct magnitude. Small fluctuations above fundamental value are processed as signals to sell in scale slightly more prominent than would be consistent with their actual size, and more significant deviations are instead treated with caution in case they would signal an overall change in long term trend.

Properties of probability weighting function and empirical its empirical plausibility had been discussed in the preceding sections and are therefore hereby omitted.

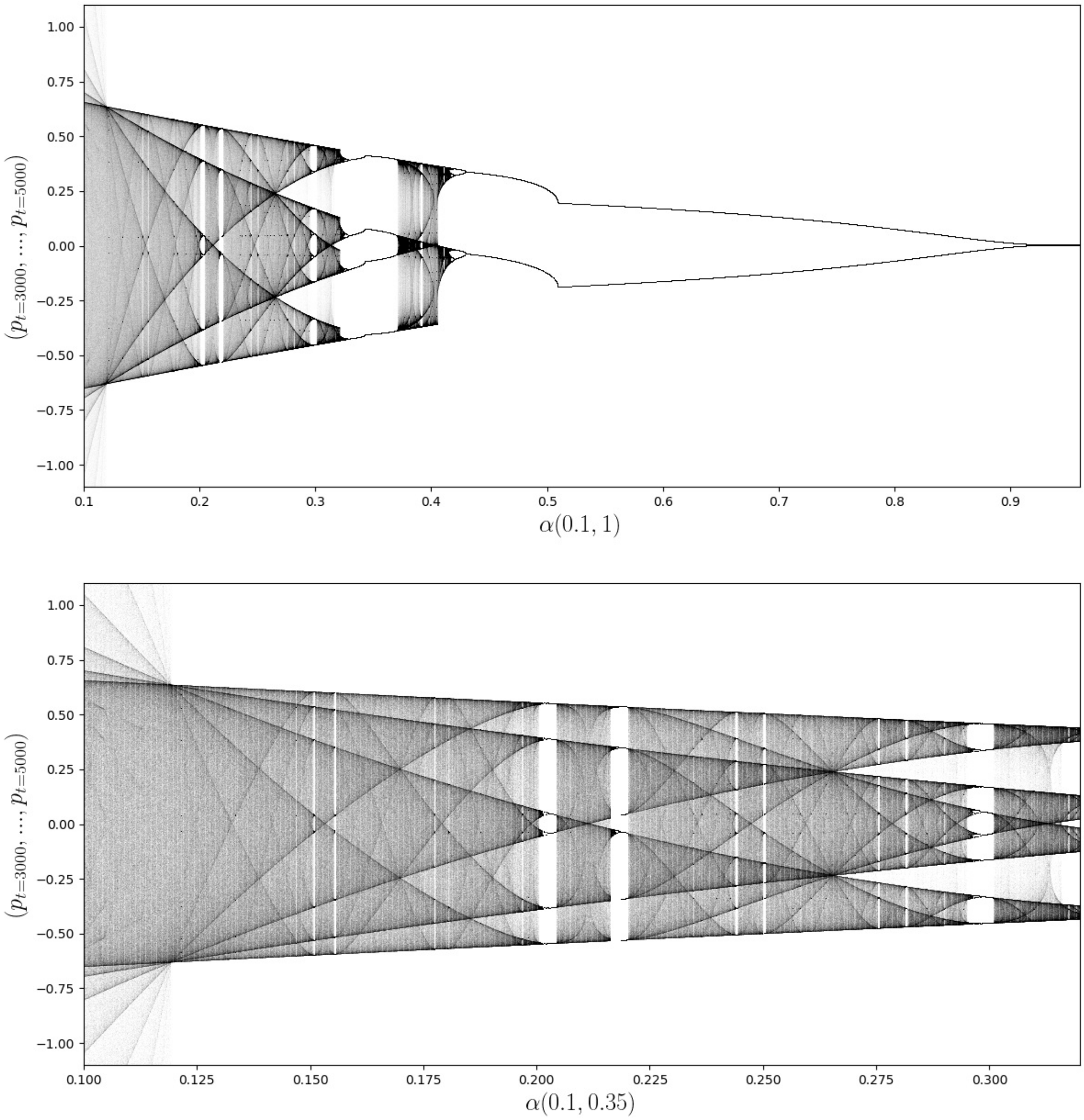

Figure 7: The bifurcation diagram of the model with respect to probability weighting parameter α, the lower subplot zooms into the left hand area of the diagram; x**-axis:** parameter α in the prob. weighting funciton, y**-axis:** 3000th to 5000th iterate of the market impact function $\theta(\cdot)$; **Parameter setting:** $u = 0,\ m = 1,\ \mu = 2,\ \beta = 0.102,$ $\rho = 1.1$; **Source:** Author, model output

In the bifurcation diagram of the model (Figure 7), it is possible to observe how the price formation process responds to change in α. Price in the diagram is expressed in terms of deviations from the fundamental value.

For large values of the parameter, the market is efficient in the sense that distortion converges to zero and thus, price sets on fundamental value. However, as the α decreases and the set of points approached by price branches out into periodic oscillations and for very low α ($\alpha \approx 0.27$), price travels erratically on a bounded interval.

As a bias towards misinterpretation of pricing signal received from market maker increases, price movements become increasingly more volatile without any related change in fundamental value. Biased perception of probability thus generates excess volatility in the model as hypothesised.

To determine distributional properties of the simulated price deviations in chaotic regime the author uses histogram of their relative densities.

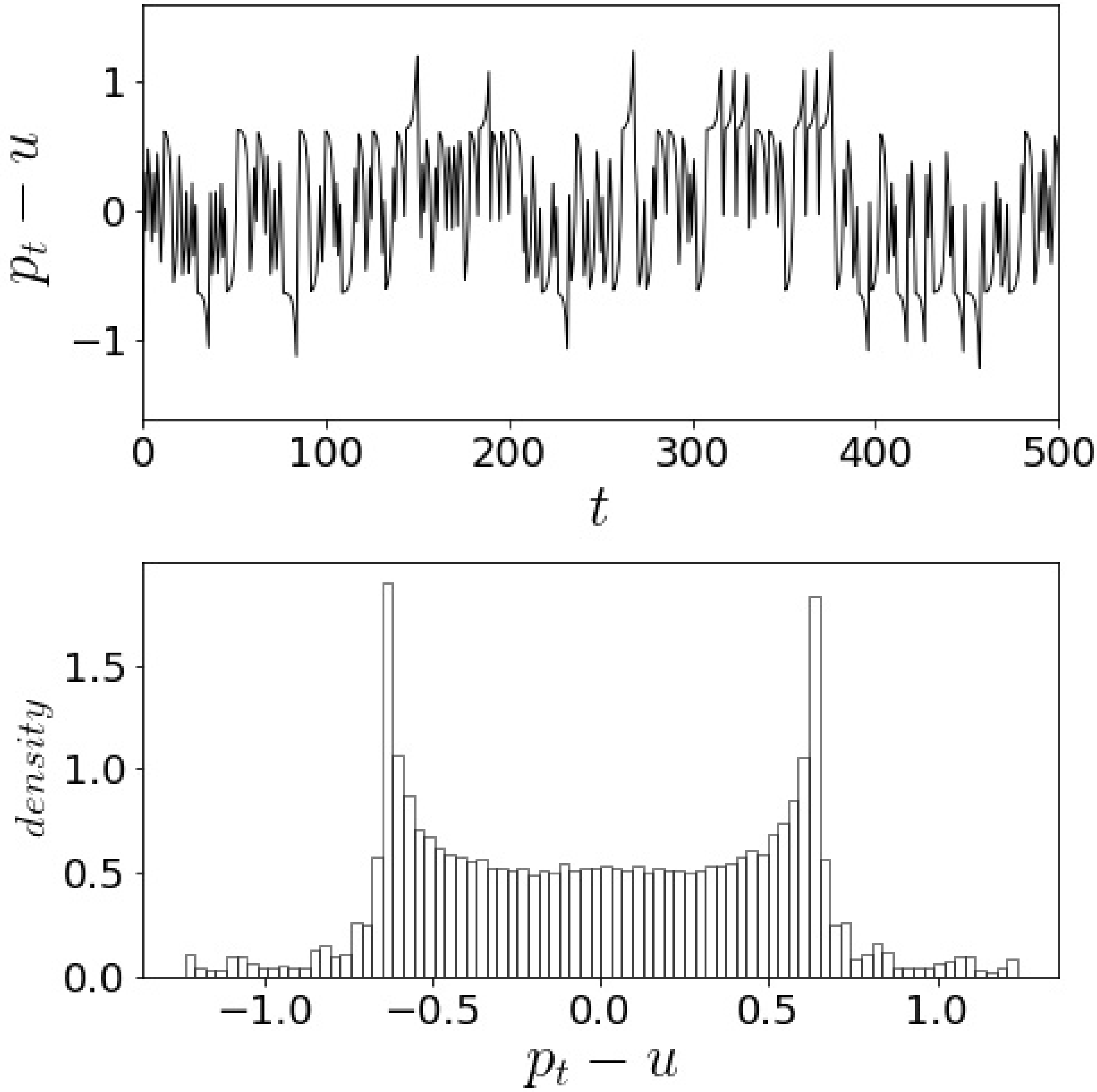

Figure 8: Time series of price misalignments and its distribution; **Upper subplot:** x-axis: periods, y-axis: price in terms of misalignments; **Lower subplot**: histogram of relative densities of the misalignment values, simulated sample contains 100000 values; **Parameter setting:** $u = 0,\ m = 1,\ \mu = 2,\ \alpha = 0.115,\ \beta = 0.102,\ \rho = 1.1$; **Source:** Author, model output

Figure 8 shows a histogram of the time series of generated prices in the chaotic regime ($\alpha = 0.1$). The behaviour of price is highly unpredictable, and the portraited market is prone to frequent jumps. The resulting distribution of the process is bimodal and centred around zero; this shows that significant distortions of price are relatively more common than small mispricing close to fundamental value. Thus the market spends relatively more time in the bear or bull regime than in the equilibrium, which qualitatively corresponds with the previously mentioned property observed in the S & P index by Schmitt and Westerhoff (2017).

Such results hint that the probability weighting effectively pushes the price away from the traded asset's fundamental value. Intuitively this does not seem too surprising because if price travels close to its fundamental value, traders reaction is disproportionately more significant than would correspond to unbiased behaviour and causes the price to diverge away into a bull or bear regime. Descriptive statistic of associated time series are summarised in the following table.

Mean	Variance	Skewness	Kurtosis
0	0.262	$5.052 \cdot 10^{-4}$	0.137

Table 2: Summary of statistics associated with the time series of deviations from fundamental value.

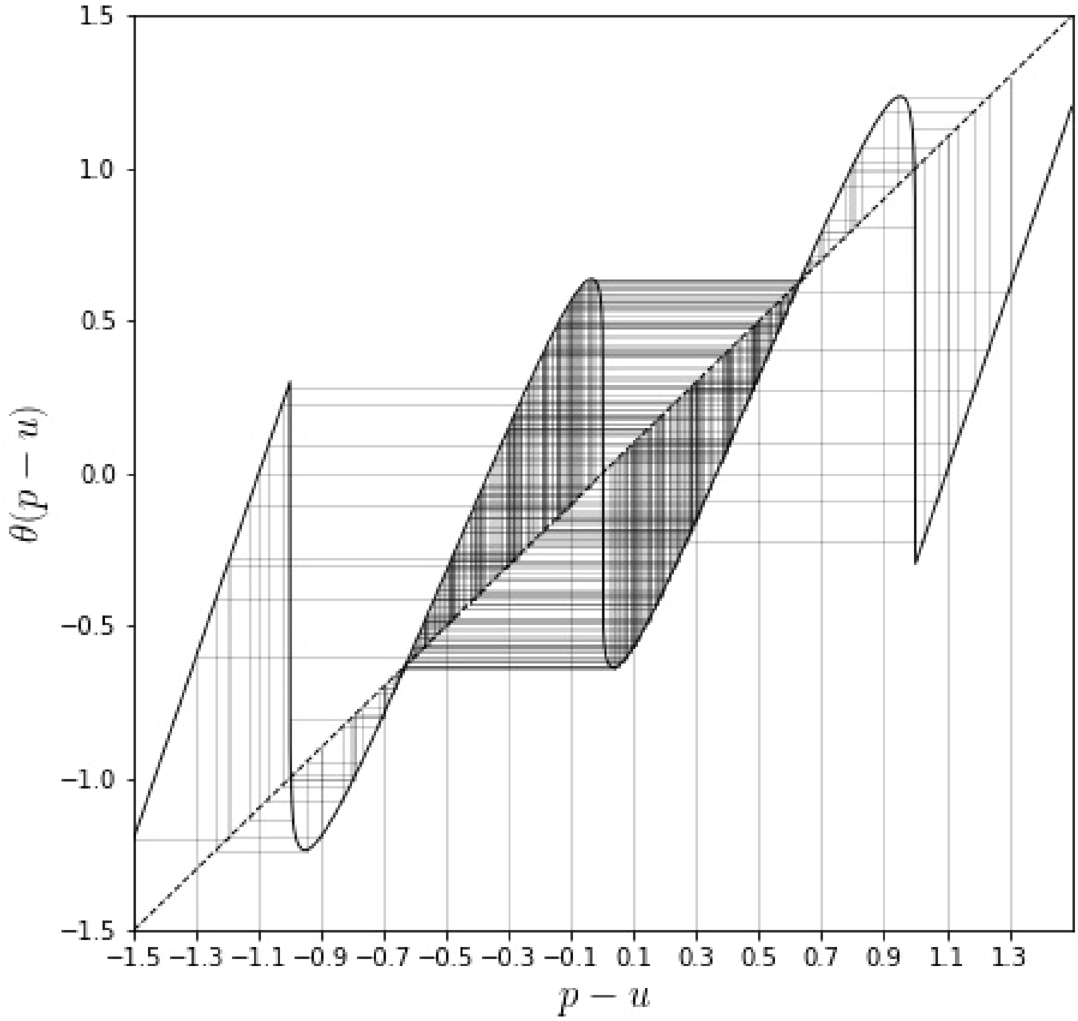

Figure 9: Cobweb plot of the market impact function shows orbits for a fifteen distinct initial conditions; **x-axis:** price misalignment $p-u$, **y-axis:** value of the market impact function $\theta(p-u)$; **Parameter setting:**$u=0, m=1, \mu=2, \alpha=0.115, \beta=0.102, \rho=1.1$; **Source:** Author, model output

The cobweb plot in Figure 9 shows orbits computed for 15 distinct initial conditions; for any initial price from the interval anticipated by the trader, the resulting orbits are attracted closer to the fixed point in zero. The higher density of visualised trajectories around in the centre of the diagram suggests that the trading process primarily generates small to medium-sized price adjustments.

Thin tales in the histogram hint that price is occasionally kicked out of the inner trapping set, causing sudden crash or rally in the market (prominent peaks and dips, occurring with lower frequency, be also seen in the sample time series in Figure 8).

It is worth noting that the model is not suited to simulate long-term properties of prices in equity markets. On a larger time scale, price indeed tends to track the fundamental value as it shifts over time. Modelled psychological biases influence the price on a micro time-scale where the individual transactions have a considerable impact. Thus fundamentals can be thought of as constants. Similarly, the trader's proposed behaviour is meant to account for biases dominant in situations that require quick decisions with limited information and are rooted in heuristically formed expectations about the price's future behaviour.

The purpose is to show that volatility is still present even without any stochastic element present inside the system. Here the oscillations are endogenous and form independent of external changes.

Considered perception bias is naturally not a dominant force regarding more longer-term behaviour when agents have more time to correct their expectations and adjust their strategy accordingly. Nevertheless, such behaviour might still shape the overall character of a price adjustment process in the capital markets and explain why stock prices are more likely to be misaligned than to revert to their fundamental value as predicted by the efficient market hypothesis.

6.3 Properties of Short-Term Returns

Speaking in a stylised fashion, the stock market can be characterised by quickly decaying autocorrelation of the high-frequency returns and their non-normal leptokurtic distribution. In theory, each period in the model simulation corresponds to a single exchange between a market maker and the trader; comparable empirical data should thus be measured on a corresponding time scale.

Figure 10 shows several empirical distributions of the stock log-returns computed on the level of individual trades. They all display kurtosis significantly higher than would be in line with normally distributed data (Gu, Chen et al. 2008).

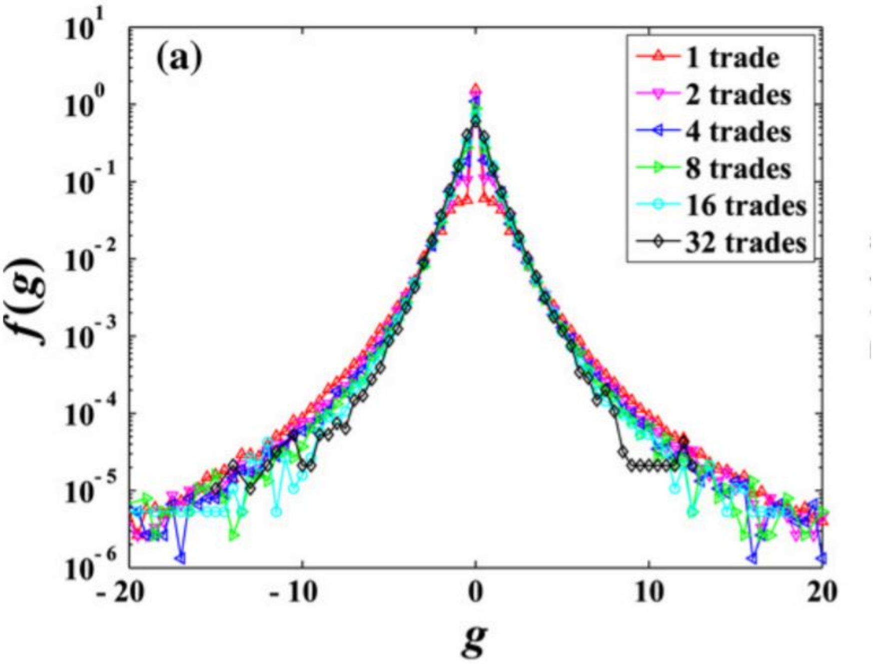

Figure 10: Distributions of high frequency returns from the Chinese stock market; **Source:** Gu, Chen et al. (2008)

The distribution of simulated ten-period log returns is shown in Figure 11, along with the sample time series of the returns. Simulated values of the normal distribution with equal first two central moments are plotted by a dashed line for comparison. Similarly to empirical returns, the simulated values have density with a high peak that decays exponentially, clearly much faster than that of normal probability density. Statistical characteristics are showed in Table 3 below.

Mean	Variance	Skewness	Kurtosis
0	$5.017 \cdot 10^{-3}$	$9.684 \cdot 10^{-7}$	$6.364 \cdot 10^{-5}$

Table 3: Summary of statistics associated with the time series of log returns.

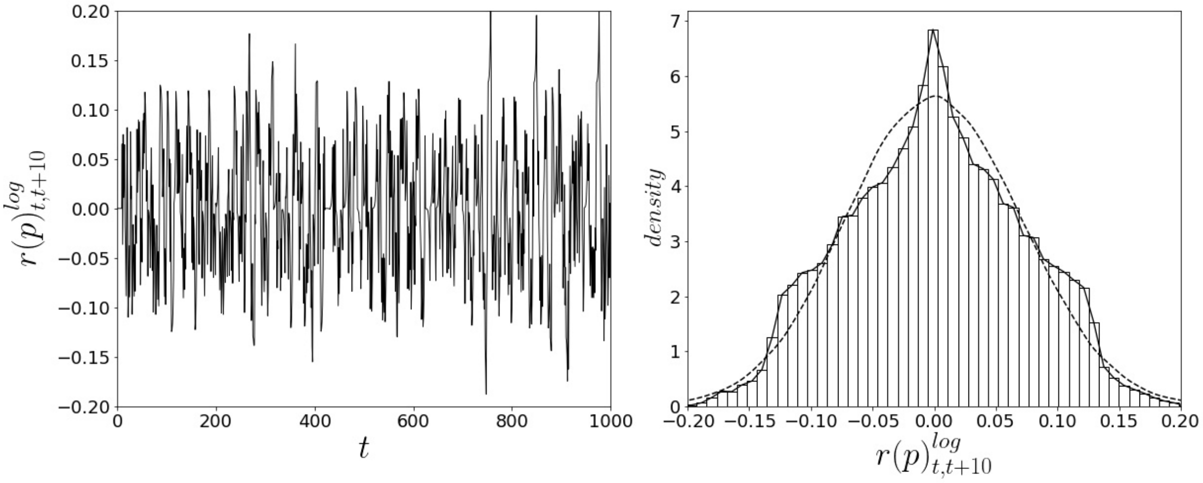

Figure 11: **Left subplot:** The time series of simulated 10-period log-returns, **x-axis:** periods t, **y-axis:** 10-period log-returns $r(p)^{log}_{t,t+10}$; **Right subplot:** The histogram of log-returns with relative densities. Sample size $N = 10000$. Black line corresponds to the same histogram fitted by a polygon. Dashed line represents the estimated probability density of draws from normal distribution with a matching mean and variance and serves for a comparison; **Parameter setting:** $u = 10, m = 1$, $\mu = 2$, $\alpha = 0.115$, $\beta = 0.102$, $\rho = 1.1$; **Source:** Author, model output

Compare related cumulative distribution functions in Figure 12.

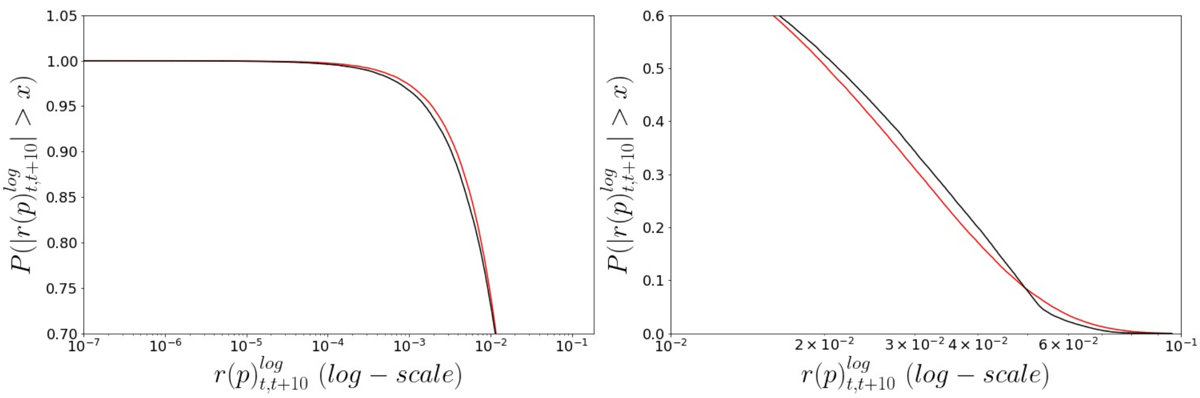

Figure 12: Cumulative distribution of simulated normal returns (red line) compared with the cumulative distribution of returns from the model (black line), the graph is broken into two parts; **Parameter values are equal to those in Figure 11**; **Source:** Author, model output

It is generally accepted that long term autocorrelation is almost nonexistent in equity markets (Cont, 2005). But on a micro time scale on intraday trading, the autocorrelation of returns is a well-documented phenomenon.

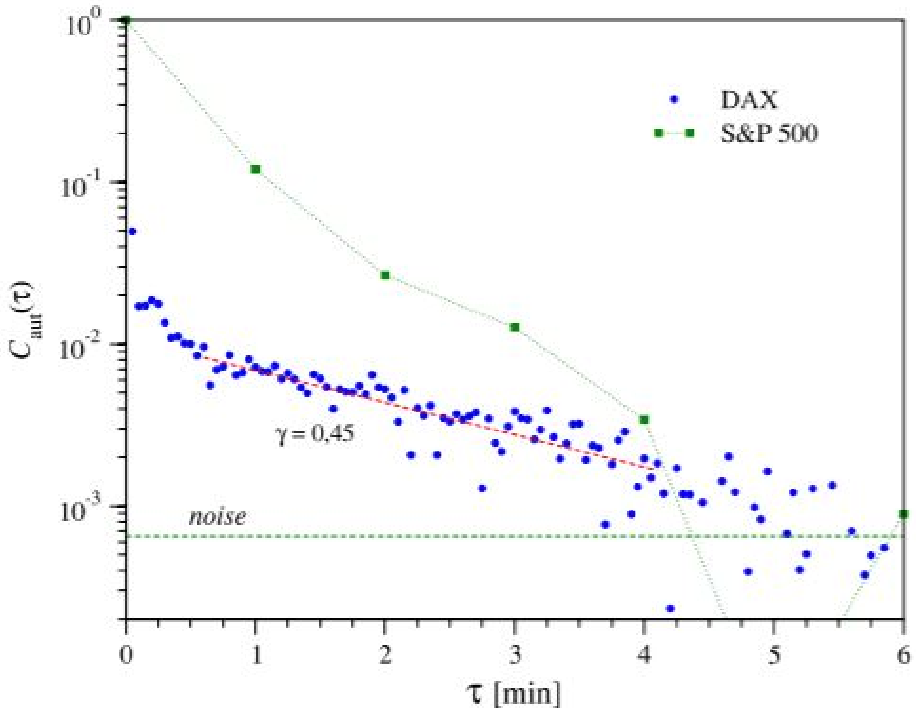

Figure 13: Autocorrelation in stock indices measured on micro-time scale, empirical data; **Source:** Kwapień, Drożdż, 2011, page 178)

Figure 13 demonstrates autocorrelation in stock indices that quickly dissolves into statistically insignificant noise in a few minutes. A similar pattern can be seen in Figure 14, which shows the autocorrelation of simulated returns to be positive for a small number of lags (left subplot, black line). As in the empirical data above, it quickly drops and becomes close to zero for a large number of shifts.

Model returns also display a certain degree of volatility clustering (measured by the autocorrelation of rolling standard deviation), but volatility clustering is generally explained by the presence of trend followers in agent-based models relying on the market microstructure, which are not included inside the model at hand. Further, the volatility clustering in returns is usually more prominent on a larger time scale than is considered here.

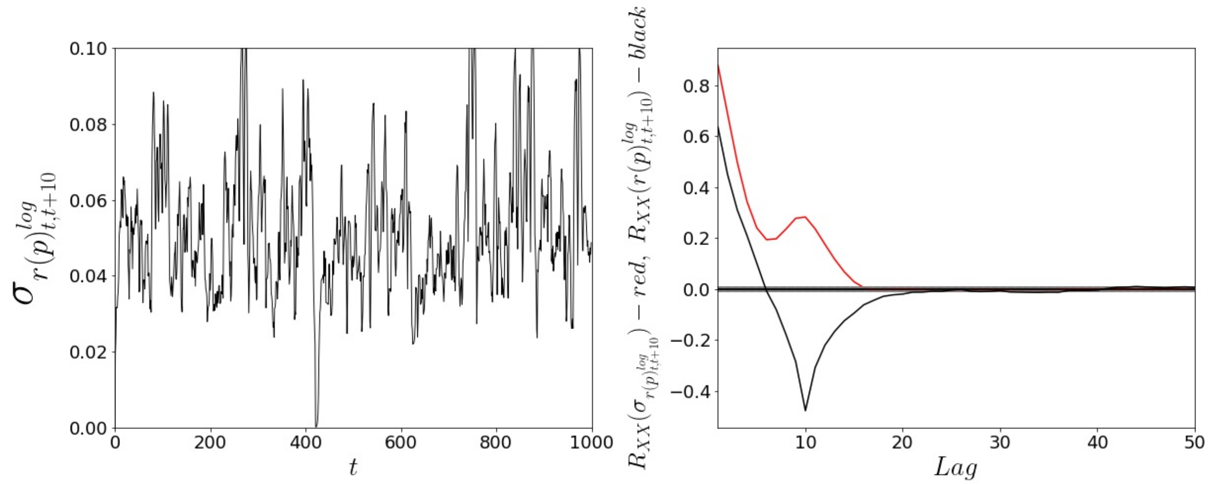

Figure 14: **Left subplot:** Rolling 10-period standard deviation of simulated returns; **Right subplot:** Autocorrelation function of the 10-period log-returns (black line), Autocorrelation function of the rolling standard deviation of the log-returns used as measure of volatility clustering (red line); **Source:** Author, model output

6.4 Conclusion to Model Analysis

Nonlinear probability weighting appears to induce chaotic oscillation in price, thus creating excess volatility even with external factors held fixed (as represented through constant fundamental value). Fluctuations of price not related to new information about the traded shares' value can be observed in real-world equity markets (Schiller, 1981). Evidence on the presence of deterministic chaos in stock price dynamics has been reviewed previously in the text; despite the significant share of inconclusive and negative studies, BenSaïda, Litimi (2013) claim that other works tend to test for chaotic behaviour without properly adjusting their data and consequently are not able to detect the presence of the chaotic element. Considering that the decision-making rule is likely to be nonlinear due to inbuilt bias present on the neurobiological level, chaos inducing behaviour of agents seems to be a plausible option.

In the model, excess volatility disappears, and price converges to its fundamental value if trader behaves rationally. Vice versa, the price trajectory becomes more complex as a response to more prone probability weighting; this can be seen in the bifurcation diagram. The distribution of returns is qualitatively similar to empirical distributions measured on a micro time-scale, in the sense that it also displays a sharp peak with density decreasing much more rapidly in the close neighbourhood of zero than would correspond to the normal distribution.

Returns possess a short term autocorrelation pattern comparable with that present in empirical data. In conclusion, the model is able to recreate stylised empirical properties of price dynamics in equity markets. The primary focus is to explain persistent mispricing and excess volatility universally present in equity markets. Described properties of returns are of secondary interest and thus can be viewed more as a curious additional feature than an attempt to explain the behaviour of the actual stock returns fully. In real-world markets, the whole spectrum of strategies is present in a dynamic environment restricted by numerous regulations and technical constraints. A simple model cannot thus successfully explain empirical properties in sufficient detail; it is merely a tool to boost our intuition about the processes standing behind seemingly random fluctuations.

7 Conclusion

The thesis discusses the connection between excess volatility present in equity market prices, as first noted in the seminal paper by Schiller (1981) and the decision making biases observed in behavioural experiments. Since Schiller, the "excess volatility puzzle" became a well-known riddle among academics and market practitioners (Wang, Ma, 2014). As mentioned in the introduction, financial markets and stock market particularly present crucial financing channel for the private sector and, as such, indirectly facilitate economic growth. Thus market volatility is not merely a problem of an academic character but carries implications for regulatory policy (Scott, Mathieson, 1990).

Superfluous price fluctuations are, as a form of inefficiency, antithetical to the efficient market paradigm. The author attempts to argue that the notion of rational investor, which rests at the core of EHM assumptions, is highly unrealistic and contrary to many empirical observations made by behavioural sciences. Namely, results from psychological and neurobiological studies are reviewed, showing that risk perception and valuation of expected payoffs do not conform to expected utility theory. The thesis also offers an explanation for the presence of the observed anomalies through the evolutionary framework. The connection between inefficiencies in decision making and human evolutionary history is a notion supported by McDermont, Fowler & Smirnov (2008) and by Herold and Netzer (2010). The later paper explains the evolution of heuristic probability processing as a result of optimisation under biological constraints.

The author then utilises concepts of behavioural Prospect theory as a mean of incorporating described behaviour into a computational model.

Subsequent analysis of the models' dynamic properties shows that pronounced probability weighting leads to chaotic oscillations of the price. That fulfils the central goal of the thesis, which was to propose a mechanism generating excess price volatility based on empirical behavioural patterns. Further, the distribution of price misalignments demonstrates that market price prefers bull and bear markets relative to its fundamental value, which is a vital characteristic observed in S & P 500 index by Schmitt and Westerhoff (2017). The author also examines model returns and compares them with characteristics of actual stock returns measured on a micro time-scale.

While the model is relatively simple with a trivial market maker-trader microstructure, it can successfully recreate a mechanism generating excess price volatility, thus meeting its main objective. Behavioural theory standing behind the formulation of trading strategy is firmly rooted in empirical results, as confirmed by several experimental studies reviewed. To summarise, the thesis finds evidence for conjecture about the relation between the excess volatility puzzle and psychological biases influencing traders' asset pricing strategy, supporting the hypothesis by literature on one side and by the model results on the other. The natural weakness of the approach used is that it does not reflect the complexity of the stock market in its current state but merely illustrates how could particular behaviour lead to global instability. The model does not consider the possible stabilising influence of other classes of trading agents or impact of learning, institutional restrictions and stochastic shifts in the underlying value of the traded shares.

As a result, displayed effects can only be thought of as determining force in short time windows since traders' expectations would change in time and with them, the corresponding model parameters. Nevertheless, the author believes that the proposed explanation is of value because it illustrates the possible connection between experimental results of psychology on an individual level and statistical properties of the stock price on the aggregate level, which frequently overlooked aspect of the real-world market.

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