HW3 - Time Series Econometrics

Vít Illichmann

Problem 1

Plot of the Data

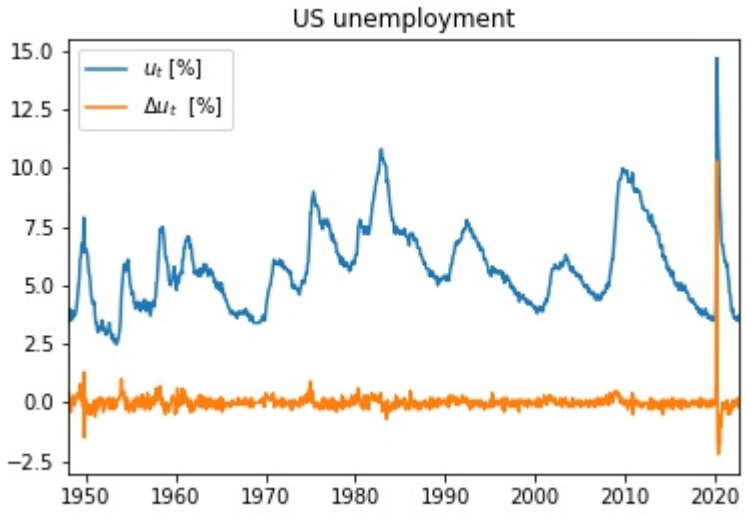

Figure 1: US unemployment (u_t) and its first differences Δu_t, monthly time series

Problem 2

From the original sample 600 observations has been selected for the model estimation of the autoregressive model. See results reported in the table bellow. Based on the BIC diagnostic the **AR**(5) process has been selected as a model for Δu_t. Coefficients were estimated by the homoscedastic OLS matrix estimator.

OLS Regression Results

Variable	β_i	SE
Δu_{t-1}	0.008	0.0414
Δu_{t-2}	0.228	0.0416
Δu_{t-3}	0.147	0.0417
Δu_{t-4}	0.081	0.0412
Δu_{t-5}	0.081	0.0413
Diagnostic	Value	Sample Size
BIC	1988.36	594

Problem 3

Estimated ACF

Visual diagnostic of the regression residual ACF does not seem to show significant autocorrelation. For lags 10 and 12 there appears to be serial correlation, slightly above the significance level but this could be caused by cycles in unemployment that were not fully removed by differencing.

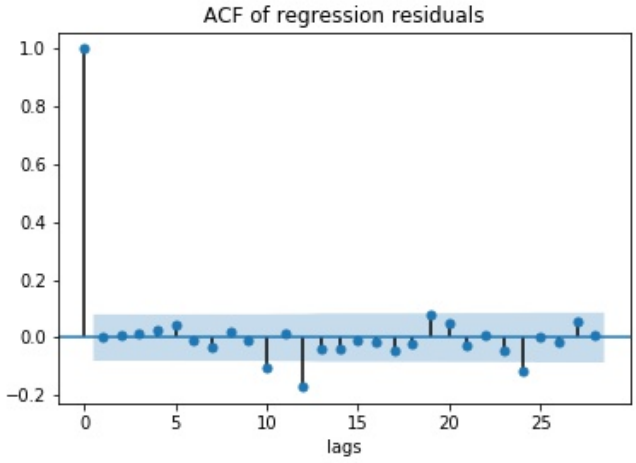

Figure 2: Estimated ACF of regression residuals

Ljung-Box Test

Reference for Ljung-Box test formula

Ljung-Box test appears to detect serial autocorrelation in the regression residua. However increasing the lag-order any further does not improve the performance of the model with respect to the Ljung-Box Test.

Diagnostic	Value	α	$q_{\chi^2}^{1-\alpha}$
Ljung-Box Test	598.8	0.05	18.30

Problem 4

Impulse-Response Functions of the Model

Administering a unit shock into the estimated model in $t = 0$ yields following response of the variables.

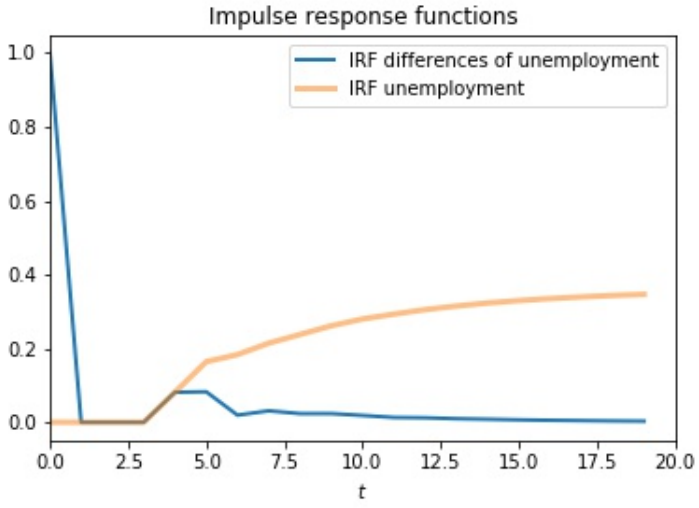

Figure 3: Computed impulse-response functions for the unemployment and its differences respectively.

Problem 5

Forecast

The multi-period forecast appears to track general short term trend of unemployment. However this approach to forecasting would perform poorly in the long run due to the cyclical nature of the unemployment and structural changes in the economy. For a one period forecast the **AR**(5) could serve quite well if the seasonal factors are taken into the account.Adding a **MA** component into the model could somewhat improve the ability of the model to reflect influence of the shocks in past periods.

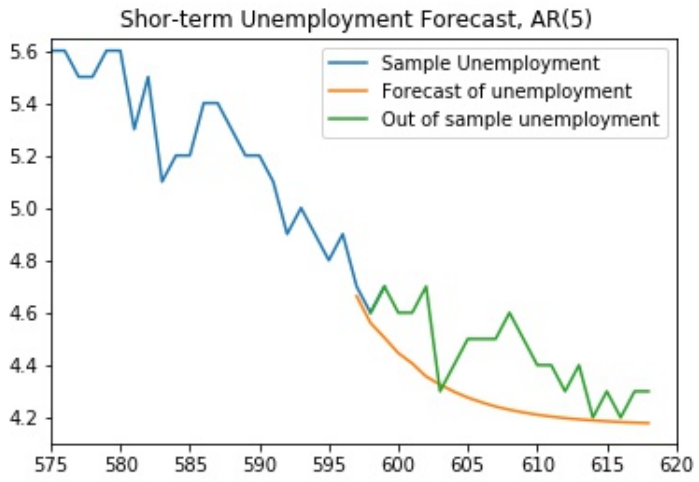

Figure 4: 10-period forecast of the unemployment time series with the out of sample realisation for comparison.

Problem 6

Augmented Dicky-Fuller Test

Testing the unit root by the ADF test amounts to estimating regression:

$$u_t = c + \beta u_{t-1} + \sum_{k=1}^{h} \delta_k \Delta u_{t-k}$$

H_0 for this test assumes $\beta = 1$. The number of lagged differences h was set to 10 for this particular test. Deterministic time trend was omitted after visual inspection of the data. See the results reported in the table bellow.

Diagnostic	Value	α	q^{α}_{DF}
ADF Test	0.983	0.01	-3.434

Based on the ADF test result the unemployment series has a unit root with high level of statistical significance.

References

1. *Time Series Analysis.* James D. Hamilton Princeton University Press. Wilmott 2003, no. 6 (November 2003): 92–92. https://doi.org/10.1002/wilm.42820030622.

2. Wikipedia contributors, *Ljung–Box test*,**link**,(accessed October 28, 2022).