Mixed Frequency TS Models: A Brief Overview

Presenation for the TSE Course 2022

Mixed Frequency Models

Motivation

- The mixed frequency models are usually encountered in macroeconomic context (although MIDAS are used for financial time series al well)
- We want to take an advantage of information contained in high frequency indicators (PMI, Industrial Production, Non-Farm payrolls, Consumer confidence indicators) to forecast some low frequency variable (e.g. GDP)

Mixed Frequency Models

Rough Model Taxonomy

- Sampling at lower frequency - clearly dominated by other methods
- Bridge equations - earliest method, but still frequently used (see e.g. **Götz, T., & Knetsch, T. (2017)**)
- MIDAS models - Relatively simple to estimate (NLS)
- State-space models - Hard to estimate, especially for a larger number of variables (Use of **Kalman filter**)
- Factor models - Exploit information from large datasets **EAI by Bundesbank**
- Combinations of factor models with the approaches above

Bridge Equations

Bridge Model

Uses aggregated values of high frequency explanatory variables x_{it_q} to forecast the low frequency variable y_{t_q}

$$y_{t_q} = \alpha + \sum_{i=1}^{j} \beta_i(L) x_{it_q} + u_{t_q}$$

The order of the lag polynomial k is selected by a information criteria (e.g. BIC) (Foroni, Marcellino, 2013)

MIDAS models

MIDAS models - Overview

1. Actually exploits data sampled at mixed frequency
2. Related to distributed lag models but the modelling philosophy is different
3. Robust with respect to misspecification (compared to state-space models)
4. Simple to estimate and does not disregard additional information in the high frequency indicator (unlike the lower frequency sampling)
5. Introduced by Ghysels et al. (2004), the acronym stands for Mixed-Data Sampling

MIDAS models

Basic MIDAS model

h_m period forecast from MIDAS:

$$y_{t_m+h_m} = \beta_0 + \beta_1 b(L_m;\theta)x^{(m)}_{t_m+w} + \varepsilon_{t_m+h_m} \tag{1}$$

$$b(L_m;\theta) = \sum_{k=0}^{K} c(k;\theta)L_m^k$$

$$c(k;\theta) = \frac{\exp(\theta_1 k + .. + \theta_Q k^Q)}{\sum_k \exp(\theta_1 k + .. + \theta_Q k^Q)}$$

MIDAS models

AR-MIDAS model I

It seems like the following model is a straightforward extension of (1):

$$y_{t_m} = \beta_0 + \lambda y_{t_m - m} + \beta_1 b(L_m; \theta) x^{(m)}_{t_m + w - m} + \varepsilon_{t_m} \tag{2}$$

However:

$$\begin{aligned} (1 - \lambda L_m^m) y_{t_m} &= \beta_0 + \beta_1 b(L_m; \theta) x^{(m)}_{t_m + w - m} + \varepsilon_{t_m} \\ y_{t_m} = (1 - \lambda)^{-1} \beta_0 &+ \beta_1 (1 - \lambda L_m^m)^{-1} b(L_m; \theta) x^{(m)}_{t_m + w - m} + (1 - \lambda L_m^m)^{-1} \varepsilon_{t_m} \end{aligned}$$

The term $(1 - \lambda L_m^m)^{-1} b(L_m; \theta) x^{(m)}_{t_m + w - m}$ causes troubles because there is an interaction between $L^{1/m}$ and L which induces seasonality (Foroni, Marcelliono, 2013)

MIDAS models

AR-MIDAS model II

To resolve this issue we add a common AR factor:

$$y_{t_m} = \beta_0 + \lambda y_{t_m - m} + \beta_1 (1 - \lambda L_m^m) b(L_m; \theta) x_{t_m + w - m}^{(m)} + \varepsilon_{t_m} \qquad (3)$$

Thus multiplication by the inverse term $(1 - \lambda L_m^m)(1 - \lambda L_m^m)^{-1} b(L_m; \theta) x_{t_m + w - m}^{(m)}$ and the problematic interaction is going to vanish.

Estimation of AR-MIDAS I

The estimation proceeds in recursive steps. Each estimate is used as an input for the following one.

Estimation of AR-MIDAS II

The estimation procedure as described in (Foroni, Marcellino, 2013)

1. Estimate the MIDAS without AR component
   $y_{t_m} = \beta_0 + \beta_1 b(L_m;\theta)x^{(m)}_{t_m+w-h_m} + \varepsilon_{t_m}$
2. Use the residuals $\hat{\varepsilon}_{t_m}$ to estimate the init. value of autoregressive coeff. λ (λ_0)
   $\hat{\lambda}_0 = \sum(\hat{\varepsilon}^2_{t_m+w-h_m})^{-1}\sum(\hat{\varepsilon}_{t_m+w-h_m}\hat{\varepsilon}_{t_m})$
3. Construct $y^*_{t_m}$ and $x^*_{t_m}$ as:
   $y^*_{t_m} = y_{t_m} - \hat{\lambda}_0 y_{t_m-h_m}$
   $x^*_{t_m+w-h_m} = x_{t_m+w-h_m} - \hat{\lambda}_0 x_{t_m+w-2h_m}$
4. Use NLS to get $\hat{\theta}_1$ from: $y^*_{t_m} = \beta_0 + \beta_1 b(L_m;\theta)x^{(*m)}_{t_m+w-h_m} + \varepsilon_{t_m}$
5. Repeat to get $\hat{\lambda}_1$ and $\hat{\theta}_1$, ..., $\hat{\lambda}_k \to \hat{\lambda}$ and $\hat{\theta}_k \to \hat{\theta}$, $\{\hat{\lambda};\hat{\theta}\}$ minimise the sum of squared residuals.

MIDAS models

Unrestricted MIDAS (U-MIDAS)

$$c(L^m)\omega(L)y_{t_m} = \delta_1(L)x_{1t_m-1} + \ldots + \delta_N(L)x_{Nt_m-1} + \varepsilon_{t_m} \qquad (4)$$

Where $c(L^m) = 1 - c_1 L^m - \ldots - c_c L^{mc}$ and $\omega(L)$ is an aggregation operator. (4) can be estimated by OLS with. The lag order can differ across variables and can be estimated e.g. b

MIDAS models

Nonlinear Variations of MIDAS

1. Asymmetric MIDAS - Different behaviour with respect to the sign of regressor
2. Smooth Transition MIDAS - Regime transition function (e.g. logistic)
3. Markow-Switching MIDAS - Stochastic regime switching.

Further Variations

1. MIDAS with step functions
2. Multivariate MIDAS
3. Factor MIDAS

Mixed Frequency VARs

Classical Approach I

Consider an example from Foroni and Marcellino (2013), y_{tm} is quarterly GDP and x_{tm} is some related monthly indicator. Suppose that the variables follow bivariate VAR(p)

$$\phi(L_m)\begin{pmatrix} y_{t_m} - \mu_y \\ x_{t_m} - \mu_x \end{pmatrix} = u_{t_m} \tag{5}$$

$$u_{t_m} \sim \mathcal{N}(0, \Sigma)$$

Restrict $p \leq 4$ and define as state space vector s_{t_m}:

$$s_{t_m} = \begin{pmatrix} z_{t_m} \\ \vdots \\ z_{t_m - 4} \end{pmatrix} \quad z_{t_m} = \begin{pmatrix} y_{t_m} - \mu_y \\ x_{t_m} - \mu_x \end{pmatrix}$$

Mixed Frequency VARs

Classical Approach II

The state space representation is then given by:

$$s_{t_m} = Fs_{t_m-1} + Gv_{t_m} \tag{6}$$
$$z_{t_m} = Hs_{t_m} \tag{7}$$

$$F = \begin{bmatrix} \phi_1 \dots \phi_p & 0_{2x2(5-p)} \\ I_8 & 0_{8x2} \end{bmatrix} \quad G = \begin{bmatrix} \Sigma^{1/2} \\ 0_{8x2} \end{bmatrix}$$

H contains the lag polynomial:

$$H(L_m) = \begin{bmatrix} 1/3 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2/3 & 0 \\ 0 & 0 \end{bmatrix} L_m + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} L_m^2 + \begin{bmatrix} 2/3 & 0 \\ 0 & 1 \end{bmatrix} L_m^3 + \begin{bmatrix} 1/3 & 0 \\ 0 & 1 \end{bmatrix} L_m^4$$

References

Example from Czech Economy

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2. Franta, M., Havrlant, D., & Rusnák, M. (2016). Forecasting Czech GDP Using Mixed-Frequency Data Models. *Journal of Business Cycle Research*, 12(2), 165–185. https://doi.org/10.1007/s41549-016-0008-z
3. Götz, T., & Knetsch, T. (2017). Google Data in Bridge Equation Models for German GDP. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.2995421
4. **QuantEcon by Tom Sargent**