

All questions worth 10 points
Must show all work

1. Decide whether the following relation on the set $\{1, 2, 3, 4\}$ is reflexive, whether it is symmetric, and whether it is transitive:

$\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$

$[(1, 1), (4, 4)]$ not in a pairs

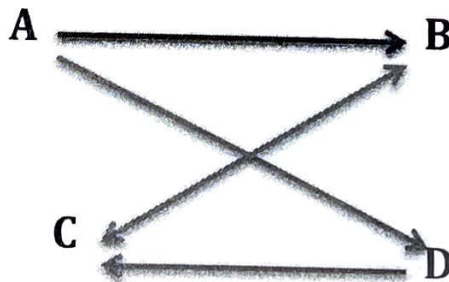
Reflexive: ~~yes. All elements of set in a relation~~
Not because 1 element ~~from a set~~ not in a relation

Symmetric: No because $(2, 4)$ doesn't have pair

a	b	b	c	a	c
2	2	2	2	2	2
2	3	3	2	2	3
3	4	None			
3	2	2	2	3	2
3	3	3	2	3	3
3	4	None			

Yes, because all elements a, c ordered pairs in a relation

2. List the ordered pairs in the relation represented by the directed graph:



$$R = \{(A, B), (A, D), (D, C)\}$$

(B, A)
 (C, D)

3. Show that the relation R consisting of all pairs (x, y) such that x and y are bit strings that agree in their first and third bits is an equivalence relation on the set of all bit strings of length 3 or more.

Reflexive: $((x, y), (x, y)) \in R$

Symmetric: $(x, y) \in R \Rightarrow (y, x) \in R$

Transitive: $(x \in R) \wedge (y \in R) \Rightarrow (x \in R)$

So it's equivalence relation

$x: \square x \Delta$ Reflexive: $(x, x) \in R$; $x R x$

$y: \square x \Delta$ Symmetry: $x R y \Rightarrow (y, x) \in R$

Transitivity: if $x R y$ and $y R z \Rightarrow x R z$

4. Determine whether the following collections of subsets are partitions of $\{1, 2, 3, 4, 5, 6\}$

Justify your responses.

a. $\{1\}, \{2, 3, 6\}, \{4\}, \{5\}$

It does not consist of empty set (\emptyset). And all elements of set are present in partition

b. $\{1, 4, 5\}, \{2, 6\}$

Element of set 3 is not in a partition.

So \rightarrow

5. The intersection graph of a collection of sets A, B, C, \dots has a vertex for each set and has an edge connecting two vertices if their associated sets have a nonempty intersection. Construct the intersection graph for the following collection of sets:

$$A = \{0, 2, 4, 6, 8\}$$

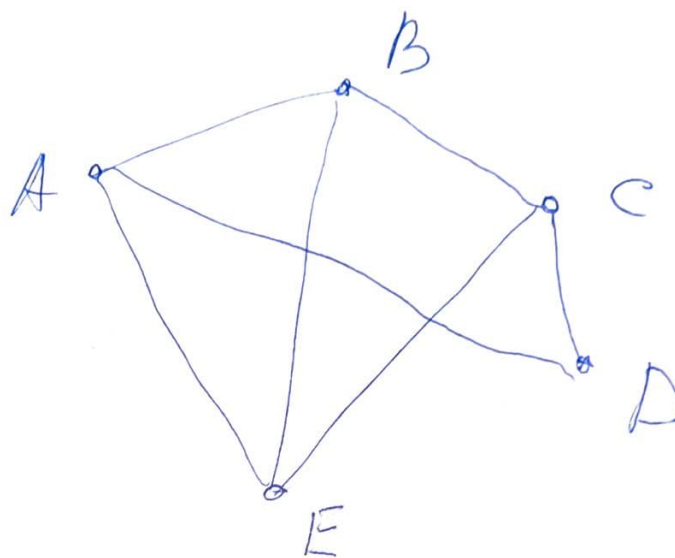
$$B = \{0, 1, 2, 3, 4\}$$

$$C = \{1, 3, 5, 7, 9\}$$

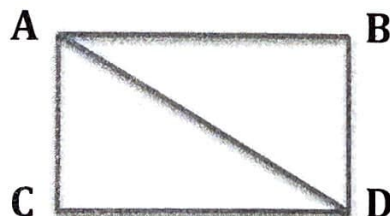
$$D = \{5, 6, 7, 8, 9\}$$

$$E = \{0, 1\}$$

$$A \cup B = \{0, 1, 2, 3, 4, 6, 8\}$$



6. Use an adjacency matrix to represent the following graph:
Note: matrix indices should correspond to alphabetical order



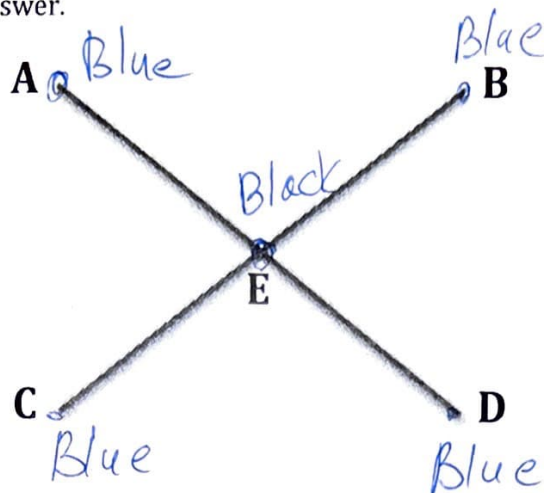
4 by 4
becomes
4 vertices

	A	B	C	D
A	1	1	1	1
B	1	1	1	1
C	1	0	0	1
D	1	1	1	1

which is not a graph

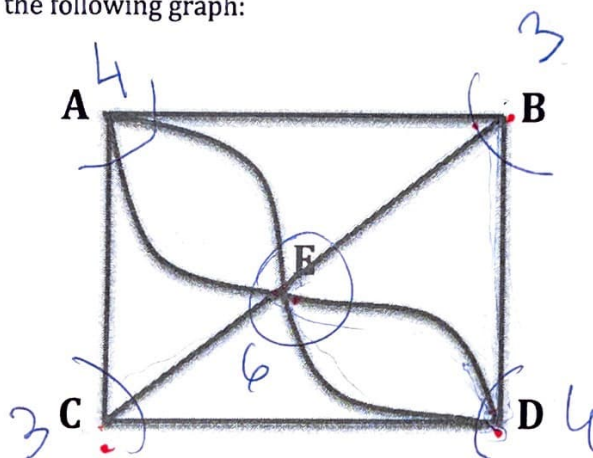
	A	B	C	D
A	0	1	1	1
B	1	0	0	1
C	1	0	0	1
D	1	1	1	1

- 10 7. Determine whether the following graph is bipartite. Recall that a graph is bipartite if and only if it is possible to assign one of two different colors to each vertex of the graph so that no two adjacent vertices are assigned the same color. Justify your answer.



This graph is bipartite because there is not two the same adjacent colors.

- 10 8. Consider the following graph:



- a. Does the graph have an Euler circuit? Why/why not?

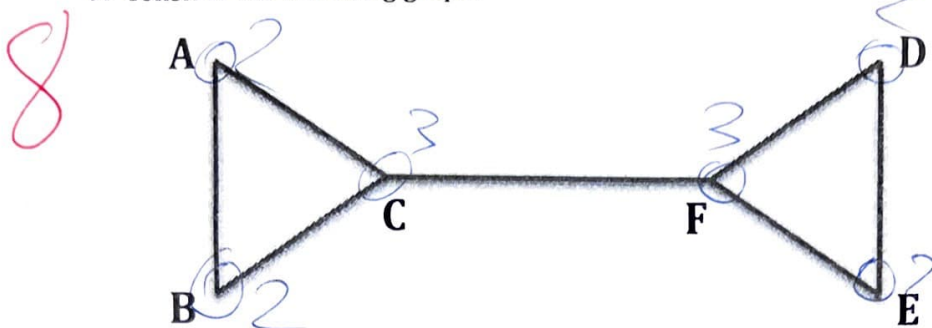
It does not have Euler circuit, because in order to exist it must be even degree on all vertices.

- b. Does the graph have an Euler path? If so, provide such a path.

B A C E A E B D E D C

Path is exist because exactly two vertex have odd degree

9. Consider the following graph:



Path
No circuit

It's not strictly to this condition, because \Rightarrow



a. Does the graph have a Hamilton circuit? Why/why not?

There are 6 vertices ($n \geq 3$) ✓

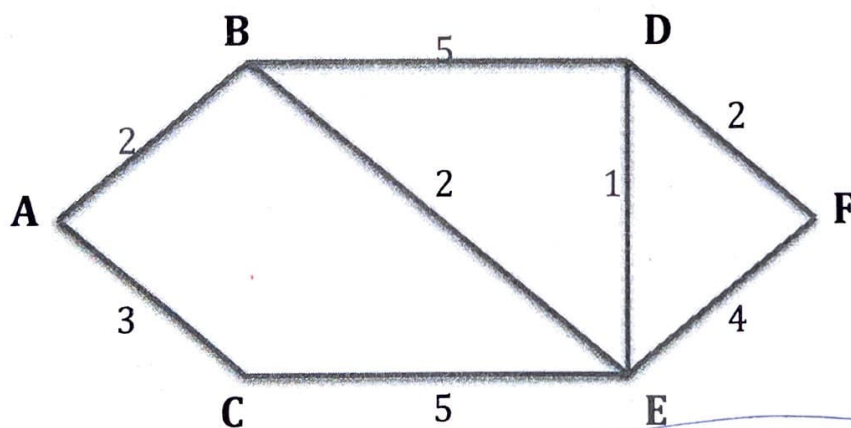
$\frac{6}{2} = 3$ Every vertex should have exactly 3 edges, but not all vertices have 3 edges.

b. Does the graph have a Hamilton path? If so, provide such a path.

Path: A B C F E D

It travel all vertices exactly once

10. Find the shortest path between A and F in the following weighted graph. What is the total length of this path?



A B E D F

$$\{AB\} = 2$$

$$[ABE] = 4$$

$$[ACE] = 8$$

$$[ABED] = 5$$

$$[ABD] = 7$$

$$[ABEDF] = 7$$

$$[ACEF] = 12$$

$$[ABFF] = 8$$