Must show all work

pq = 4 rows; pqr = 8 rows; when → then F with something will be true

5

1. Construct a truth table for the following compound propositions:

2. ⇔ means equivalance so checking one side with other they should match up means its true TT=T FF=F

a. $(p \vee q) \wedge r$

p	q	$p \vee q$	$(p \vee q) \wedge r$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

8 rows required

p	q	r	$p \vee q$	$(p \vee q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

b. $(p \wedge q) \vee \neg r$

p	q	$p \wedge q$	$(p \wedge q) \vee \neg r$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

8

2. Show that $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$ are logically equivalent.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

$p \rightarrow q \Leftrightarrow \equiv \neg p \vee q$

$p \rightarrow r \Leftrightarrow \equiv \neg p \vee r$

$(\neg p \vee q) \vee (\neg p \vee r)$

$\equiv \neg p \vee (q \vee r)$

$\equiv p \rightarrow (q \vee r) \Leftrightarrow \equiv \neg p \vee (q \vee r)$

$\therefore \neg p \vee (q \vee r) \equiv \neg p \vee (q \vee r)$

same statement

or thruth statement in implication FT = T || TT=T

FT ... T || TF=F

You only have to show only one case when it does not work

3. Determine the truth value of each of these statements if the domain for all variables consists of all integers.

a. $\forall n(n^2 \geq n)$ if ~~[illegible]~~ 0 will work out

square of negative will be positive

so ~~there~~ is ~~to~~ always True

b. $\exists n(n^2 < 0)$ when n is a negative int.

find only case

0 not work

− not work

+ not work

False

4. Let $L(x, y)$ be the statement "**x** loves **y**", where the domain for both *x* and *y* consists of all people in the world. Use quantifiers to express each of the following statements:

a. "there is somebody whom everybody loves"

∃ ~~[illegible] (somebody) (who loves everybody)~~

use x, y, L(x,y)

Not the same statement

b. "nobody loves everybody"

~~∀ everybody (loves nobody)~~

$\forall x \exists y \neg L(x, y)$

everybody has someone whom do esnot love

same work

$\neg\exists x \forall y L(x, y)$

$\exists x \forall y L(x, y)$

n can be any integers → You ca

There is somebody whom everybody loves

$\exists\, y\ \forall x\ L(x,y)$

there is y (somebody) whom everyone loves

It is not the same: $\forall x\ \exists y\ L(x,y)$
$\exists x\ \forall y\ L(x,y)$

Can't use n for both

5. Use a direct proof to show that the product of two odd integers is odd.

2

if $x = 2n + 1$

and $y = 2n + 1$

where $n > 0$

then $z = x \cdot y$ is odd.

How?

you gonna use algebra

0

6. Prove that if n is an integer and $3n+2$ is odd, then n is odd. Use the technique of contraposition (the idea that $p \rightarrow q$ may be proven by showing that $\neg q \rightarrow \neg p$).

if $3n + 2$ is odd $\rightarrow$ n is odd

counterpositive

if n is even then $3n+2$ is even.

if n is even $\Rightarrow$ $n = 2p$, $p \in \mathbb{Z}$

$3n+2 = 3(2p)+2 =$ just smth else

factored

$= 2(3p+1)$

just int

Even

$= 2m;\ m = 3p+1$

odd int $= (2p+1)$ $\quad p\ ^{+}Z$

$(2p+1)(2q+1)$ $\quad q\ ^{+}Z$

you can <u>not</u> use same variable because when u multiple them will be one (p and q) has to be distinct

$4pq + 2p + 2q + 1$

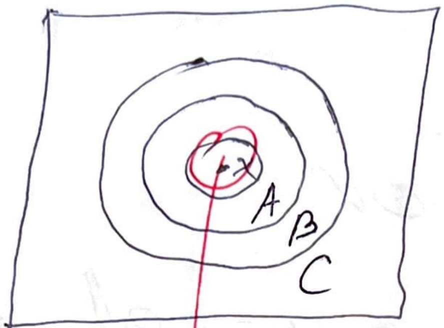
factor out

$2(\underbrace{2pq + p + q}_{\text{integer}}) + 1$

$2 \quad n \quad + 1$ odd

10

7. Use a Venn diagram to illustrate the relationship $A \subseteq B$ and $B \subseteq C$.



~~There is element of x inside of subset A therefore is subset of B. and ∴ C.~~

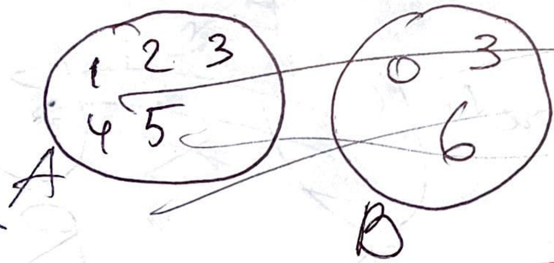
Do not put dot
dot will indicate
proper subset.

B/c correctly with dot

0

8. Let $A = \{1,2,3,4,5\}$ and $B = \{0,3,6\}$. Find the following. Use proper set notation.

a. $A \cup B$



$A \not\subset B$

$\cancel{B \not\subset A}$

b. $A \cap B$

$\cancel{A \not\subset B}$

$\cancel{B \not\subset A}$

$A \cup B = \{0, 1, 2, 3, 4, 5, 6\}$

union is collection of all elements but not repeat)

$A \cap B = \{3\}$ is one common in both because = we have to do twice (verse versa)

9. Show that if A and B are sets, then $A - B = A \cap \bar{B}$.

Show $A - B = A \cap \bar{B}$

Let $x \in A - B$

then $x \in A$ and $x \notin B$

$\to x \in A$ and $x \in \bar{B}$

$\to x \in A \cap \bar{B}$

And $A \cap \bar{B} \subseteq A - B$

$x \in A \cap \bar{B}$

$x \in A \wedge x \in \bar{B}$

$\to x \notin B \quad x \in A - B$

10. Determine whether each of the following functions is a bijection (one-to-one and onto) from $\boldsymbol{R}$ to $\boldsymbol{R}$. *Hint:* a graph may be helpful.

a. $f(x) = -3x + 4$

$x_1 \neq x_2 \quad f(x_1) \neq f(x_2)$ counterpositiv

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$f(x_1) = -3x_1 + 4 = -3x_2 + 4$

$-3x_1 = -3x_2$

$x_1 = x_2$

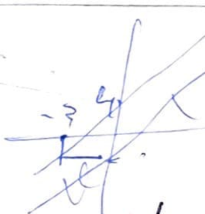
$f(x) \to y$

$y \to f x$

$y = -3x + 4$

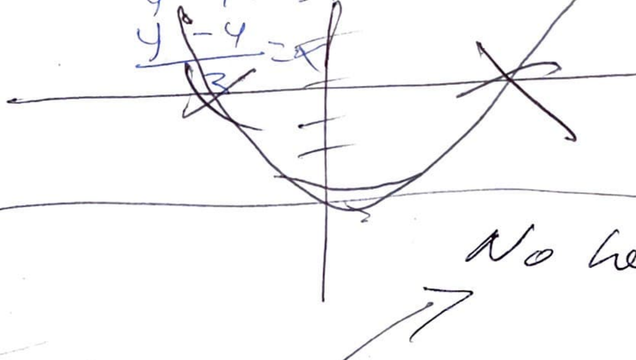
$y - 4 = -3x$

$\frac{y - 4}{-3} = x$



it is two x for one y

b. $f(x) = x^2 + 1$



No here

if is not one to one so it's no bijection

and not onto because nothing below x

yes it is bijection

y is range, and it cover all y

for any choice of x is only one y.

$fx = -3x + 4$

one to one

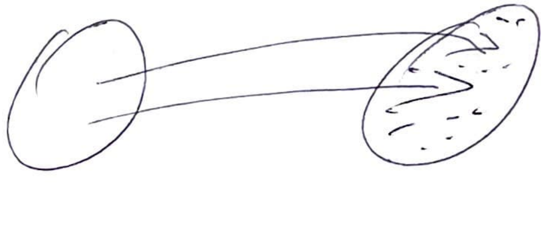
not ref

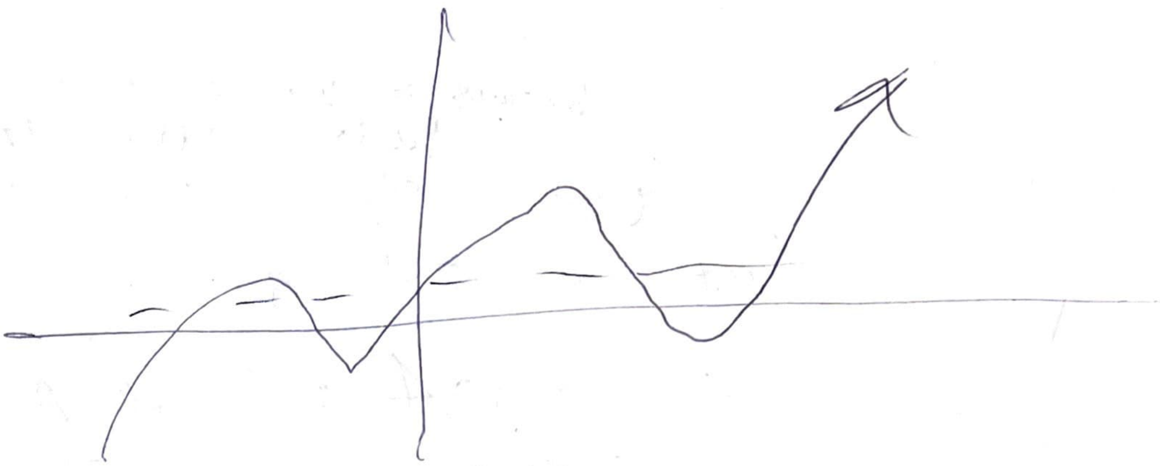
D R D R

pointing to unique member

onto function

cover entire range (trace back)





it is cover all y

not in example here

$$y = \pm \sqrt{x}$$

square root