CS-35
All questions worth 10 points
Must show all work

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1. Prove that if $a|b$, then $a|bc$ for all integers c. Assume $a \neq 0$.

if a/b, $b = a \cdot K$, $bc = j \cdot a$

$a(\underbrace{jK}_{int}) = bj$

$bc = \frac{a}{bc}$?

Only if $c = j$

$a/b \Rightarrow b = a \cdot K \ (\cdot c)$

$bc = a(Kc)$

$\Rightarrow a/bc$

10

2. Demonstrate why 13 and 33 are congruent, mod 5. In other words, show that $13 \equiv 33 \pmod 5$.

13 % 5 = 3 remainder
33 % 5 = 3 remainder
According to the theorem if they are both have the same remainder, they are congruent

5

3. Prove by induction that the product of any 2 consecutive positive integers is even. The base case, induction hypothesis, and induction step should be clearly labeled.

$n \cdot (n+1) = 2n$

Base Case: $n=1$: $1 \cdot (1+1) = 2 \cdot 1$ True

Induction Hypothesis Step

$n = k > 0$

P_k: $k \cdot (k+1) = 2k$

cannot be same variable

Induction Step

not true

P_{k+1}: $\boxed{(k+1)((k+1)+1) = 2(k+1)}$

$k^2 + 3k + 2 = 2(k+1)$ not true

$P_k = k^2 + k$

$~~2k~~ + 2k + 2 = 2(k+1)$

$2(k+1) = 2(k+1)$ True

0

4. Give a recursive algorithm (pseudocode is fine) for finding the sum of the first n positive integers.

$n + (n+1)$ $n > 0$, $k > 0$

if $n > 0$?

$k = n + (n+1)$

if $n = 1$

return 1 ← return 1 because of Base case

else return $n + \text{sum}(n-1)$

$n(n+1) = 2 \cdot \cancel{k}\, m$

Base : $n = 1$

$1(2) = 2$ ✓

IH if $k(k+1) = 2p$

Show

$(k+1)((k+1)+1) = 2f$

$(k+1)(k+2)$

$k(k+1) + 2(k+1)$

$2p + 2(k+1)$

$2\underbrace{(p+k+1)}_{int}$

even

5. Prove by induction that the sum of the first n positive even integers is $n(n+1)$. In other words, prove that $2+4+6+\ldots+2n = n(n+1)$ holds $\forall n \geq 1$. The base case, induction hypothesis, and induction step should be clearly labeled.

Base Case: $n = 1$; $2 \cdot 1 = 1(1+1)$ True

Induction Hypothesis step

$n = k$

$P_k : 2 + 4 + 6 + \ldots + 2k = k(k+1)$

Induction Step

$P_{k+1} : \underbrace{2 + 4 + 6 + \ldots + 2k}$ $+ 2k+1 =$

be careful!

$= (k+1)((k+1)+1) =$

$= (k+1)(k+2)$

$k(k+1) + 2k+1 = (k+1)(k+2)$

$k^2 + 3k + 2 = k^2 + 3k + 2$ True

or factor $(k+1)(k+2) = k+1)(k+2$

$(k+1)(k+2) = k^2 + 2k + 1k + 2 = k^2 + 3k + 2$

1

6. Prove by induction that $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^n}=\frac{2^n-1}{2^n}$ for all positive values of n. The base case, induction hypothesis, and induction step should be clearly labeled.

Base Case: $n=1$ $\frac{1}{2}=\frac{1}{2}$ True

Induction Hypothesis Step

$n=k>0$

$P_k: \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^k}=\frac{2^k-1}{2^k}$

Induction Step

$P_{k+1}: \underbrace{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\frac{1}{2^k}}_{P_k}+\frac{1}{2^{k+1}}=\frac{2^{k+1}-1}{2^{k+1}}$

$\frac{2^k-1}{2k}+\frac{1}{2^{k+1}}=\frac{2^{k+1}-1}{2^{k+1}}$

trick

$\frac{(2^k-1)(2^{k+1})+2k}{2k\cdot 2^{k+1}}=\frac{2^{k+1}-1}{2^{k+1}}$

what is this?

trick: take common diminado

2^k need to multiple by $(\cdot 2)$ in order to get 2^{k+1}

$$\frac{2}{2}\left(\frac{2^k-1}{2^k}\right) \quad \frac{1}{2^{k+1}}$$

$$\frac{2^{k+1}-2}{2^{k+1}} + \frac{1}{2^{k+1}}$$

$$\frac{2^{k+1}-1}{2^{k+1}}$$

0

7. Find $f(2)$, $f(3)$, $f(4)$, $f(5)$, and $f(6)$ if f is defined recursively by $f(0)=f(1)=1$ and for $n \geq 1$, $f(n+1)=f(n)-f(n-1)$.

$f(2) = f(1) - f(1-1) =$

$f(3) = f(2) - f(2-1) =$

$f(4) = f(3) - f(3-1) =$

$f(5) = f(4) - f(4-1) =$

$f(6) = f(5) - f(5-1) =$

$f(2) = f(1) - f(0)$
$= 1 - 1 = 0$

$f(3) = f2 - f1 =$
$= 0 - 1$
$= -1$

$f(4) = f(3) - f(2)$
$= -1 - 0$
$= -1$

$f5 = f4 - f3 =$
$-1 - -1$
$= 0$

$f(6) = f5 - f4$
$= 0 - (-1)$
$= 1$

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8. A multiple-choice test contains 4 questions. Each question has 5 possible answers. How many ways can the student answer the questions on the test, assuming the student answers every question (no blanks).

5 5 5 5 = 5^4 5 ?

$= \boxed{}$

10

9. Find the following:

a. ${}_5P_2$ (or $P(5,2)$, as it may appear in the textbook)

$$P(5,2) = \frac{5!}{(5-2)!} = \frac{5!}{3!} = 5 \cdot 4 = 20$$

b. ${}_7C_5$ (or $C(7,5)$, as it may appear in the textbook)

$$C(7,5) = \frac{7!}{(7-5)!\,5!} = \frac{7!}{2!\,\cancel{5!}} = \frac{7 \cdot \cancel{6}^{3}}{\cancel{2}} = 21$$

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10. How many different strings can be formed by the letters in DOGGONE, using all the letters?

DOGGONE

DOGNE

OG

$$\frac{7!}{1!\,2!\,2!\,1!\,1!} = \boxed{}$$