# Nuclear forces from chiral lagrangians

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The method of phenomenological lagrangians is used to derive the consequences of spontaneously broken chiral symmetry for the forces among two or more nucleons.

The forces among nucleons have been studied as much as anything in physics. Much of this work has necessarily been phenomenological: scattering data and deuteron properties are used to determine a two-nucleon interaction, which can then be used as an input to multi-nucleon calculations. As more and more has been learned about the meson spectrum, efforts have been increasingly aimed at calculating the nuclear potential as an expansion in terms of decreasing range arising from the exchange of one or more mesons of various types, but the number of free parameters rises rapidly as more and more meson types are included, especially if one attempts to extend these calculations to forces involving more than two nucleons. This paper applies methods [1] based on the chiral symmetry of quantum chromodynamics to derive an expansion of the potential among any number of low energy nucleons in powers of the nucleon momenta, which is related to but not identical with the expansion in terms of increasing range. It is not clear which expansion will be more useful in dealing with the two-nucleon problem, but the expansion in powers of momenta gives far more specific information about multi-nucleon potentials.

The lagrangian that we shall use in this work will be taken as the most general possible lagrangian involving pions and low-energy nucleons consistent with spontaneously broken chiral symmetry and other known symmetries. It is given by an infinite series of terms with increasing numbers of derivatives and/or nucleon fields, with the dependence of each term on the pion field prescribed by the rules of broken chiral symmetry. Other degrees of freedom, such as heavy vector mesons, $\Delta$'s, and antinucleons, are "integrated out": their contribution is buried in the coefficients of the series of terms in the pion–nucleon lagrangian. We shall also integrate out nucleons with momenta greater than some scale $Q$, which requires that these coefficients in the lagrangian be $Q$-dependent. Later we will consider how to make a judicious choice of $Q$; for the moment, it will be enough to specify that $Q$ is substantially less than $m_\rho$. Any detailed model such as that of Skyrme [2] (also see ref. [3]) that embodies broken chiral symmetry will give results that are consistent with ours, but less general; in particular, we do not specify any particular higher-derivative terms in the lagrangian such as those that are introduced to stabilize skyrmions, but instead we consider all possible terms, with any numbers of derivatives, that are allowed by the symmetries of strong interactions.

Now consider the $S$-matrix for a scattering process with $N$ incoming and $N$ outgoing nucleons, all with momenta no larger than $Q$. The non-relativistic nature of the problem makes it appropriate to apply "old-fashioned" time-ordered perturbation theory: there is an energy denominator for every intermediate state, instead of a propagator for every internal particle line. The energy denominators associated with intermediate states involving just $N$ nucleons are small, of order $Q^2/2m_N$, as compared with $Q$ for the

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energy denominators associated with intermediate states involving one or more pions, and we know from the existence of shallow nuclear bound states that these small energy denominators cause the perturbation series to diverge at low energies. Therefore we will apply effective lagrangian techniques not to the $S$-matrix itself, but rather to the "effective potential", defined as the sum of connected old-fashioned diagrams for the $S$-matrix (or rather the $T$-matrix, since energy conservation is not imposed) *without* $N$-nucleon intermediate states. The full $S$-matrix can be obtained by solving a Lippmann–Schwinger equation (or Schrödinger equation) with this effective potential in place of the interaction hamiltonian, and with *only* $N$-nucleon intermediate states, each nucleon with momentum less than $Q$.

Let us then use our lagrangian to calculate the matrix element of the effective potential between $N$-nucleon states, for which the nucleon momenta are all less than $Q$ but roughly of the same order. Since we are working with a non-renormalizable lagrangian the matrix element will contain ultraviolet divergences of increasing severity as we go to higher and higher order in the perturbation expansion, but these divergences can all be absorbed in a redefinition of the infinite number of constants in the lagrangian. With infinities eliminated in this way, the remaining integrals are effectively cut off at internal momenta of order $Q$. We wish to expand the effective potential in terms of increasing order in $Q$ and $m_\pi$, so let us count powers of these quantities.

An interaction of type $i$ may be characterized by the number $d_i$ of derivatives and $m_\pi$ factors, the number $n_i$ of nucleon fields, and the number $p_i$ of pion fields. Each derivative contributes one power of $Q$ or $m_\pi$ (we use the Dirac equation to express time derivatives of nucleon fields in terms of space derivatives) and each meson field factor contributes $-\frac{1}{2}$ powers of $Q$ or $m_\pi$, arising from the familiar $1/\sqrt{2E}$ factors that accompany meson emission or absorption. Also, because each intermediate state in the *potential* contains at least one pion, each energy denominator makes a contribution no larger than of order $1/Q$ or $1/m_\pi$. Finally, each loop is accompanied with an integral over a three-momentum, and hence contributes three powers of $Q$. Putting this together, we see that a graph with $V_i$ vertices of type $i$, $L$ loops, and $D$ intermediate states, contributes a term with $\nu$ powers of $Q$ or $m_\pi$, where

$$\nu = \sum_i V_i(d_i - \tfrac{1}{2}p_i) - D + 3L \,. \tag{1}$$

The numbers of energy denominators, loops, and vertices of various types as well as the number $I$ of internal lines in a connected graph are related by the following familiar topological identities:

$$D = \sum_i V_i - 1 \,, \tag{2}$$

$$L = I - \sum_i V_i + 1 \,, \tag{3}$$

$$2I + 2N = \sum_i V_i[p_i + n_i] \,. \tag{4}$$

Eq. (1) can thus be put in the form

$$\nu = 2 - N + 2L + \sum_i V_i(d_i + \tfrac{1}{2}n_i - 2) \,. \tag{5}$$

The point of writing the number of powers of $Q$ or $m_\pi$ in this way is that the coefficient of $V_i$ is non-negative for all interactions allowed by chiral symmetry, because each purely pionic interaction has at least two derivatives, and each interaction involving pions and nucleons has at least one derivative. (Strictly speaking, since chiral symmetry is not exact non-derivative interactions are also present, but these are suppressed by factors of $m_\pi^2$, and since we are counting powers of $m_\pi$ along with powers of $Q$ these symmetry-breaking terms effectively have $d_i \geq 2$.) Hence the terms in the potential with the minimum number $2-N$ of powers of $Q$ or $m_\pi$ are tree graphs ($L=0$) with vertices given by just those terms in the effective hamiltonian that have $d_i=2$, $n_i=0$ or $d_i=1$, $n_i=2$ or $d_i=0$, $n_i=4$, but any number of pion fields. These can be obtained from the terms in the lagrangian with the same restrictions on $d_i$ and $n_i$:

$$\begin{aligned}\mathscr{L} = &-\tfrac{1}{2}D^{-2}\,\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - \tfrac{1}{2}D^{-1}m_\pi^2\boldsymbol{\pi}^2 \\ &-\bar{N}[\partial\!\!\!/ + m_N + 2iD^{-1}F_\pi^{-1}\gamma_5 g_A \boldsymbol{t}\cdot\partial\!\!\!/\boldsymbol{\pi} \\ &+2iD^{-1}F_\pi^{-2}\boldsymbol{t}\cdot(\boldsymbol{\pi}\times\partial\!\!\!/\boldsymbol{\pi})]N - (\bar{N}\Gamma_\alpha N)(\bar{N}\Gamma^\alpha N) \,,\end{aligned} \tag{6}$$

where $\Gamma_\alpha$ and $\Gamma^\alpha$ are matrices constrained by Lorentz and isospin invariance; $g_A \simeq 1.25$ and $F_\pi \simeq 195$ MeV are the usual axial coupling constant and pion decay amplitude; and $D \equiv 1 + \boldsymbol{\pi}^2/F_\pi^2$. In the approximation in which we keep only the tree graphs calculated with

this lagrangian, we must also adopt the "static approximation" – that is, we neglect the nucleon kinetic energy in energy denominators, and use zero-momentum Dirac spinors in nucleon matrix elements. The interaction hamiltonian corresponding to the lagrangian (6) then becomes

$$H_{\rm int} = \tfrac{1}{2}(D^2-1)\dot{\boldsymbol{\pi}}^2 + \tfrac{1}{2}(D^{-2}-1)\nabla\boldsymbol{\pi}\cdot\nabla\boldsymbol{\pi} + \tfrac{1}{2}m_\pi^2(D^{-2}-1)\boldsymbol{\pi}^2 + 2F_\pi^{-4}[\bar{N}(\boldsymbol{t}\times\boldsymbol{\pi})N]^2 + \bar{N}[2F_\pi^{-1}g_{\rm A}D^{-1}\boldsymbol{t}\cdot(\boldsymbol{\sigma}\cdot\nabla\boldsymbol{\pi}) + 2F_\pi^{-2}D\boldsymbol{t}\cdot(\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}})]N + C_{\rm S}(\bar{N}N)(\bar{N}N) + C_{\rm T}(\bar{N}\boldsymbol{\sigma}N)(\bar{N}\boldsymbol{\sigma}N)\,, \qquad (7)$$

where $C_{\rm S}$ and $C_{\rm T}$ are unknown constants. [Fermi statistics allows the non-derivative four-fermion interactions involving $\boldsymbol{t}$ to be written as linear combinations of the two last terms in eq. (7).

In two-nucleon states the effective potential derived in this way consists of just a conventional one-pion-exchange term, plus a direct two-nucleon interaction produced by the four-fermion terms in (7). Fourier-transforming this momentum-space effective potential gives a local coordinate-space two-nucleon potential:

$$V_{\text{2-nucleon}} = 2(C_{\rm S} + C_{\rm T}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\delta^3(\boldsymbol{x}_1-\boldsymbol{x}_2) - \left(\frac{2g_{\rm A}}{F_\pi}\right)^2 (\boldsymbol{t}_1\cdot\boldsymbol{t}_2)(\boldsymbol{\sigma}_1\cdot\nabla_1)(\boldsymbol{\sigma}_2\cdot\nabla_2)Y(|\boldsymbol{x}_1-\boldsymbol{x}_2|) - (1'\leftrightarrow 2')\,, \qquad (8)$$

where $Y(r)\equiv\exp(-m_\pi r)/4\pi r$ is the usual Yukawa potential. [Throughout it should be understood that these are local potentials, containing a delta function factor like $\delta^3(\boldsymbol{x}_1'-\boldsymbol{x}_1)$ for each nucleon.]

Using eq. (7) in the tree approximation provides just the first term in an expansion in powers of $Q/M$, where $M$ is some mass scale characteristic of QCD. In particular, the $\boldsymbol{x}$-coordinates in eq. (8) will be smeared out over a volume of order $M^{-3}$ in the full effective potential. Because we have integrated out the ρ and ω vector mesons, which are known to make an important contribution to nucleon–nucleon forces, we may guess that $m_\rho$ can be taken as a representative value for this $M$.

Now let us consider the effects of a two-nucleon effective potential calculated in this way. It is well known that the one-pion exchange potential itself does a good job of accounting for the higher partial waves in nucleon–nucleon scattering. On the other hand, according to current ideas [3] the dominant attraction in low energy s-wave nucleon–nucleon interactions is of range intermediate between the π and ρ Compton wavelengths, and is due to exchange of s-wave isoscalar pion pairs, with the tensor part of one-pion exchange providing a small correction responsible for the fact that the triplet s-wave two-nucleon state is bound and the single state is not. The intermediate range attraction would have to be represented here by the delta-function terms in (8), with $C_{\rm S}$ quite large and negative, and $C_{\rm T}$ much smaller. As already mentioned, constants like these in the effective potential must be given a dependence on the arbitrary momentum scale $Q$ above which nucleons are integrated out, to ensure that final results are $Q$-independent. The perturbative solution of the Lippmann–Schwinger equation for s-wave two-nucleon states is roughly an expansion in powers of $C_{\rm S}Q^3/(Q^2/2m_{\rm N})$, so apart from the small effects of $C_{\rm T}$ and pion exchange, we can conclude that $C_{\rm S}$ scales like $1/Q$. To be specific, in order to have bound or virtual two-nucleon states very near zero binding, it is necessary for the expansion parameter $2m_{\rm N}C_{\rm S}Q$ to have a value near $2\pi^2$. (It is quite mysterious here why $C_{\rm S}$ should have almost precisely the value needed just barely to bind s-wave two-nucleon states, but this is a mystery also in more conventional treatments of nuclear forces.)

Now, it is implicit in the power-counting rules used above that the constants $C_{\rm S}$ and $C_{\rm T}$ should be no larger than roughly of order $2\pi^2/m_\rho^2$, so the consistency of our approach seems to require that $Q$ should be greater than or of the order of $m_\rho^2/2m_{\rm N}$. We also must choose $Q \ll m_\rho$, which makes sense only if we can regard $2m_{\rm N}$ as much larger than $m_\rho$. Of course, there are many unknown dimensionless factors that can enter here, so we cannot really tell whether our analysis is actually self-consistent. However, we note that $m_{\rm N}$ may at least in principle be regarded as an independent parameter which could be arbitrarily large compared with $m_\rho$, since in QCD with $N_{\rm c}$ colors the ratio $m_{\rm N}/m_\rho$ is proportional to $N_{\rm c}$. Even so, with all medium and short-range effects buried in the constants $C_{\rm S}$ and $C_{\rm T}$, it is clear that the terms of lowest order in $Q/m_\rho$ cannot account for the effects usually attributed to a short-ranged repulsive core. For this reason, and also as a check on our approach, it will

be important to calculate the terms in the effective potential of next order in $Q/m_\pi$, which arise from one-loop graphs calculated with the hamiltonian (7), as well as from tree graphs involving higher-order terms in the hamiltonian.

It is in dealing with multi-nucleon potentials that our approach becomes most useful. The diagrams for the effective potential in three-nucleon states are shown in fig. 1. They give the result

$$
\begin{aligned}
V_{\text{3-nucleon}} = & -\frac{1}{2}\left(\frac{2g_A}{F_\pi}\right)^4 (\boldsymbol{\sigma}_1\cdot\nabla_1)(\boldsymbol{\sigma}_2\cdot\nabla_1) \\
& \times(\boldsymbol{\sigma}_2\cdot\nabla_3)(\boldsymbol{\sigma}_3\cdot\nabla_3)(\boldsymbol{t}_1\cdot\boldsymbol{t}_2)(\boldsymbol{t}_2\cdot\boldsymbol{t}_3) \\
& \times[\tilde{Y}(|\boldsymbol{x}_1-\boldsymbol{x}_2|)Y(|\boldsymbol{x}_2-\boldsymbol{x}_3|) \\
& +\tilde{Y}(|\boldsymbol{x}_2-\boldsymbol{x}_3|)Y(|\boldsymbol{x}_1-\boldsymbol{x}_2|)] \\
& -\delta^3(\boldsymbol{x}_1-\boldsymbol{x}_2)\left(\frac{2g_A}{F_\pi}\right)^2 [C_S+C_T(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)] \\
& \times(\boldsymbol{\sigma}_2\cdot\nabla_2)(\boldsymbol{\sigma}_3\cdot\nabla_3)Y(|\boldsymbol{x}_2-\boldsymbol{x}_3|) \\
& \pm\text{permutations}, \qquad (9)
\end{aligned}
$$

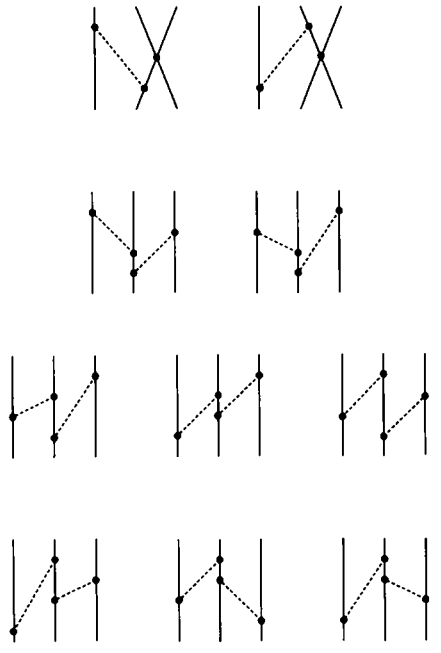

Fig. 1. Diagrams contributing to the three-nucleon effective potential. (Here solid lines are nucleons; dashed lines are pions.) It is necessary to add and subtract terms involving even or odd permutations of initial as well as final nucleons.

where

$$
\tilde{Y}(r) \equiv (2\pi)^{-3}\int \frac{\exp(i\boldsymbol{q}\cdot\boldsymbol{r})\,d^3r}{(q^2+m_\pi^2)^{3/2}}, \qquad (10)
$$

and "$\pm$permutations" indicate that we must add or subtract terms involving even or odd permutations of initial as well as final nucleons. [Eq. (9) does *not* correspond to any sum of Feynmann diagrams, because we have excluded those pieces of Feynman diagrams that correspond to old-fashioned diagrams with three-nucleon intermediate states.] This does not involve any of the non-linear pion interactions in the hamiltonian (7), because the diagrams in fig. 2 all cancel. Thus in leading order, *the three-nucleon effective potential is built up out of two-nucleon interactions, with no intrinsically three-body forces.* On the other hand, the four-nucleon potential does involve the non-linear pion–nucleon couplings as well as the pion self-interaction, through diagrams like those shown in fig. 3. It will be interesting to see whether

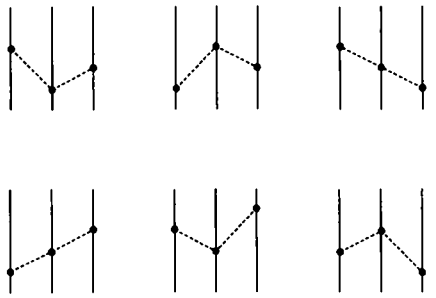

Fig. 2. Diagrams for the three-nucleon effective potential that involve the non-linear pion–nucleon interaction. These diagrams all cancel.

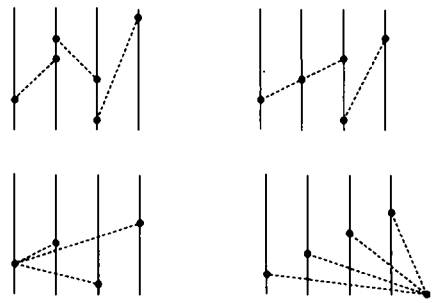

Fig. 3. An assortment of diagrams for the four-nucleon effective potential.

effects of these multi-nucleon effective potentials can be found in nuclear properties.

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## References

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