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# Application of the chiral forces to electroweak processes

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# CONTENTS

<b>1</b>	<b>Plan</b>	<b>2</b>
<b>2</b>	<b>Formalism &amp; numerical methods</b>	<b>5</b>
2.1	Deuteron bound state . . . . .	5
2.2	The Lippman-Schwinger equation . . . . .	7
2.3	Theoretical uncertainties . . . . .	8
<b>3</b>	<b>Pion absorption at the lowest atomic orbital in <math>^2\text{H}</math>, <math>^3\text{H}</math> and <math>^3\text{He}</math></b>	<b>10</b>
<b>4</b>	<b>Results</b>	<b>11</b>
4.1	Deuteron photodisintegration . . . . .	11
	<b>Bibliography</b>	<b>17</b>

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# CHAPTER 1

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## PLAN

- Why we study few nucleon systems
  - Strong interactions (2N and 3N force investigation; QCD, relativistic effects)
  - Electro-magnetic processes (electrons-, photons-induced reactions) (Arenhovel did ...)
  - Weak interactions (neutrons)
- Nuclear forces used in the thesis
  - AV18
  - Chiral (scs, sms; difference between chiral models; regularization problem)
- Currents used in the thesis (regularization of currents to be done)
- Formalism & numerical methods
  - Lippman-Schwinger eq
  - Schrodinger eq for deuteron; wave functions (sms) for deuteron - figures, binding energy
  - Three body: Fadeev eq. for bound (He3, H3) and scattering states
  - Siegert theorem ?
  - Partial wave decomposition, states ( $pq\alpha$ ), Jakobi momenta; operators in PW decomp. (current); Mathematica for PW
  - Theoretical uncertainties: truncation error, cut-off dependency, chiral order dependency
- Results (**find everything what I have calculated: all processes and energies**)
  - H2 photodisintegration
  - He3 and H3 photodisintegration
  - Pion capture

- Summary
- References

## Why we study few nucleon systems

The study of light nuclei for the decades has been serving as an easiest way to study NN systems and forces inside the atom. And convenient way to proceed may be an interaction of atom with other particles: elastic or inelastic scattering. It is possible to construct such an experiments and check if theory works. People take into account that interactions may be caused by different forces and therefore should be described in different ways. It can be either strong, weak or electromagnetic interaction. It depends on the type of particle being scattered and the target which reaction it is.

In order to proper describe the nuclear reactions many factors should be taken into account. First of all, different nuclear forces may act on the participants.

The strong nuclear force appear inside the nuclei and among others bound neutrons and protons together. The description of strong interactions is extremely difficult as it deals not only with nucleon, but with their constituents: quarks and gluons. Quantum Chromodynamics(QCD) is a modern theory describing strong interactions, but it has also its limitations at the moment as it is not reliable at low energies ( $Q^2 \lesssim 1\text{GeV}^2$ ). So other approaches are coming into the scene such as chiral effective theory, lattice calculation and others [1].

Electromagnetic force appears between charged particles like protons and electrons. Also, the force is transferred between charged particles with a photon, so in photon- and electron- scatterings on the nuclei an electromagnetic force is playing an important role. Arenhovel [2] studied electromagnetic process - Deuteron photodisintegration, applying different approaches and comparing the results with experimental data.

The weak force...

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Starting the study of 3- (and more) nucleon systems it was found that 2N force is not enough to describe the system and 3N force was introduced. The first applications of such a force showed that it brings sufficient contribution and cannot be ignored [3]. Whereas the first applications included only early "realistic" potential, the latter investigations only proved this statements [4, 5]. It was also used to construct four-nucleon (4N) bound state [6].

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## Nuclear forces used in the thesis

In order to construct a potential people often use phenomenological or semi-phenomenological approaches. It allows to combine theoretical knowledge about processes and experimental data.

One of such potentials, which was used in current thesis is Argonne V18 (AV18) [7] In order to construct NN force, authors combine analytical electromagnetic and one-pion-exchange parts with phenomenological one, fitting parameters to the Nijmegen partial-wave analysis of  $pp$  and  $np$  data [8]. Authors showed, that AV18 potential delivers good results in the description of nucleon scattering data as well as deuteron properties.

In the early 1990-ies Weinberg [9, 10] introduced an idea of using a most general Lagrangian satisfying assumed symmetry principles and in particular spontaneously bro-

ken chiral symmetry to describe nuclear interactions at low energies. This idea together with effective field theory (EFT) of Quantum chromodynamics (QCD) led to the development of Chiral effective field theory ( $\chi$ EFT) which nowadays has become one of the most advanced approach to describing nuclear reactions at low energies.

For the EFT it is very important to define a quantity, which powers will determine a perturbation order. In the  $\chi$ EFT there are two natural scales: so-called soft scale - the mass of Pion  $Q \sim M_\pi$  and hard scale -  $\Lambda_\chi \sim 1 \text{ GeV}$  (chiral symmetry breaking scale). The ratio between these two scales  $(Q/\Lambda_\chi)^\nu$  is being used as an expansion parameter in  $\chi$ EFT with power  $\nu$ .

Considering so-called irreducible (the diagrams that cannot be split by cutting nucleon lines), Weinberg [9, 10] came to the identity for the powers of such diagrams[11]:

$$\nu_W = 4 - A - 2C + 2L + \sum_i \Delta_i, \quad (1.1)$$

where

$$\Delta_i \equiv d_i + \frac{n_i}{2} - 2 \quad (1.2)$$

In 1.1,  $C$  is a number of pieces which are connected,  $L$  - the number of loops in the graph. In 1.2,  $n_i$  is a number of nucleon field operators,  $d_i$  - the number of insertions (or derivatives) of  $M_\pi$ .

In  $\chi$ EFT the first order is called "leading order" (LO) and it is followed by next-to-leading order (NLO), next-to-next-to-leading order (N2LO) and so on. At the moment, the highest order for which there is a derived term in potential is N4LO. Also some contributions from N5LO are included in the N4LO+ chiral order.

As pointed above, for many-nucleon systems it is important to include not only nucleon-nucleon interaction to the potential, but also a 3- and many- nucleon contributions. In the  $\chi$ EFT 3N force contributes starting from N2LO and 4N force is presented starting from N3LO, so there is a systematic way to include all the forces from simplest diagrams at LO and gradually adding more and more terms. It is also beneficial in the way that one can obtain results using chiral potential at different orders and track which one gives larger or smaller contribution (changes in the final results).

The  $\chi$ EFT may be applied both in coordinate and momentum spaces. Nevertheless in both cases it requires regularization which is cutting low coordinate values in order to avoid infinities (or high momentum values - in momentum space). The value at which the cut is applied (cut-off value) is not fixed and usually calculations are being performed for different cut-off values. The comparison of such results may reveal stronger or weaker dependance and in perfect case one will come up with such a potential, were the cut will not affect results much.

The potential may be transformed from coordinate to momentum space (or vice versa), but it is important at which frame the regularization was performed and what was a regularization function. That's why there are different versions of chiral potential. One is semi-local coordinate space regularized potential (SCS) [12] and another one is similar, but with regularization applied in momentum space (SMS potential) [13].

## Currents

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# CHAPTER 2

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## FORMALISM & NUMERICAL METHODS

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In order to calculate any observable for the Deuteron photodisintegration, one has to find a nuclear matrix elements:

$$N^\mu = \langle \Psi_f \vec{P}_f | \frac{1}{e} J^\mu(0) | \Psi_i \vec{P}_i \rangle = \langle p'(l' s') j' m'_j t' m'_t \vec{P}_f | J^\mu | \phi_d m_d \vec{P}_i \rangle, \quad (2.1)$$

where  $J^\mu$  is a four-vector current operator which acts between initial and final two-nucleon states.

### 2.1 Deuteron bound state

Let's find a deuteron bound state wave function  $\phi_d$ . The time-independent Schrodinger equation for two particles in such case will be:

$$(H_0 + V) | \psi_{12} \rangle = E_d | \psi_{12} \rangle \quad (2.2)$$

$H_0$  is a kinetic energy of the nucleons and  $V$  is a potential. The kinetic energy  $H_0$  can be represented in terms of relative and total momenta of the particles:

$$H_0 = \frac{\vec{p}_1^2}{2m_1} + \frac{\vec{p}_2^2}{2m_2} = \frac{\vec{p}^2}{2\mu} + \frac{\vec{P}^2}{2M}, \quad (2.3)$$

where relative and total momenta are defined as follows:

$$\vec{p} = \frac{(m_1 \vec{p}_1 - m_2 \vec{p}_2)}{m_1 + m_2} \quad (2.4)$$

$$\vec{P} = \vec{p}_1 + \vec{p}_2 \quad (2.5)$$

and  $M = m_1 + m_2$  is a total mass,  $\mu = \frac{m_1 m_2}{M}$  is a relative mass of two nucleons.

We are working in the momentum space, so acting by the momentum operator on the Eq.(2.2) one can obtain two separated equations:

$$\frac{\vec{p}^2}{2\mu} \langle \vec{p} | \Psi_{int} \rangle + \int d\vec{p}' \langle \vec{p} | V | \vec{p}' \rangle \langle \vec{p}' | \Psi_{int} \rangle = (E_d - E_{c.m.}) \langle \vec{p} | \Psi_{int} \rangle \quad (2.6)$$

$$\frac{\vec{P}^2}{2M} \langle \mathcal{P} | \Psi_{c.m.} \rangle = E_{c.m.} \langle \mathcal{P} | \Psi_{c.m.} \rangle \quad (2.7)$$

Eq.(2.6) is basically a Schrodinger equation for one particle with mass  $\mu$  and Eq.(2.7) can be regarded as a Schrodinger equation for particle with mass  $M$  in a free motion. Assuming that deuteron is at rest ( $E_{c.m.} = 0$ ) we can stick to the Eq.(2.6) only. So that:

$$\frac{\vec{p}^2}{2\mu} \langle \vec{p} | \Psi_{int} \rangle + \int d\vec{p}' \langle \vec{p} | V | \vec{p}' \rangle \langle \vec{p}' | \Psi_{int} \rangle = E_d \langle \vec{p} | \Psi_{int} \rangle \quad (2.8)$$

Next we move to the partial-wave representation of the momentum state in the following form:

$$| \vec{p} \rangle = | p\alpha \rangle \equiv | p(ls)jm_j \rangle | tm_t \rangle, \quad (2.9)$$

where we introduce quantum numbers  $l, s, j, t$  as orbital angular momentum, total spin, total angular momentum and total isospin respectively.  $m_j$  and  $m_t$  are isospin and spin projections.

Yet one can introduce simpler states than it is in (2.9).

$$| p(ls)jm_j \rangle = \sum_{m_l} c(ls j; m_l, m_j - m_l, m_j) | plm_l \rangle | s m_j - m_l \rangle \quad (2.10)$$

Also we can decompose spin and isospin states as follows:

$$| sm_s \rangle = \sum_{m_1} c(\frac{1}{2} \frac{1}{2} s; m_1, m_s - m_1, m_s) | \frac{1}{2} m_1 \rangle | \frac{1}{2} m_s - m_1 \rangle \quad (2.11)$$

$$| tm_t \rangle = \sum_{\nu_1} c(\frac{1}{2} \frac{1}{2} t; \nu_1, m_t - \nu_1, m_t) | \frac{1}{2} \nu_1 \rangle | \frac{1}{2} m_t - \nu_1 \rangle \quad (2.12)$$

In Eqs.(2.10) -(2.12),  $c(\dots)$  are Clebsh-Gordon coefficients. Nucleons are spin  $\frac{1}{2}$  particles, and also we treat proton and neutron as the same particle in different isospin states, so that isospin is  $\nu_1 = \frac{1}{2}$  for proton and  $\nu_1 = -\frac{1}{2}$  for neutron.

The states  $| plm_l \rangle$  from Eq.(2.10) are orthogonal, so that

$$\langle p'l'm'_l | plm_l \rangle = \frac{\delta(p - p')}{p^2} \delta_{ll'} \delta_{m_l m'_l} \quad (2.13)$$

and also satisfy the completeness relation:

$$\sum_{l=0}^{\infty} \sum_{m_l=-l}^l \int dp p^2 | plm_l \rangle \langle plm_l | = \mathbb{1} \quad (2.14)$$

These states also fulfill a relation

$$\langle \vec{p}' | plm_l \rangle = \frac{\delta(|\vec{p}'| - p)}{p^2} Y_{lm_l}(\hat{p}'), \quad (2.15)$$

where  $Y_{lm_l}(\hat{p}')$  is a spherical harmonic and ' $\hat{\phantom{x}}$ ' means a unit vector.

If we exchange nucleons 1 and 2 there should be a sign change and this requirement can in mathematical form can be expressed as:

$$(-1)^{l+s+t} = -1 \quad (2.16)$$

Taking into account Eq.(2.16), one can find only one possible case for the deuteron bound state: 2 coupled channels for  $l=0,2$ ;  $s=1$ ;  $j=1$  and  $t = m_t = 0$ . These 2 channels are usually denoted as  ${}^3S_1$  and  ${}^3D_1$  and corresponding wave functions are  $\phi_0(p)$  and  $\phi_2(p)$ .

So with a new basis Eq.(2.8) takes a form:

$$\frac{\vec{p}^2}{2\mu}\phi_l(p) + \sum_{l'=0,2} \int dp' p'^2 \langle plm_l | V | p'l'm'_l \rangle \phi_{l'}(p) = E_d \phi_l(p), \quad (2.17)$$

for  $l = 0, 2$ . Assuming that one has a matrix elements for the potential  $\langle plm_l | V | p'l'm'_l \rangle$ , there is still one complication in the Eq.(2.17) - integration. In order to get rid of the integral I use a Gaussian quadrature method of numerical integration [14]. It allows to replace an integral by the weighted sum:  $\int_a^b f(x)dx = \sum_{i=1}^n \omega_i f(x_i)$  In current work I used 72 points in the interval from 0 to 50 fm. Using this method, Eq.(2.8) becomes

$$\frac{\vec{p}^2}{2\mu}\phi_l(p) + \sum_{l'=0,2} \sum_{j=0}^N \omega_j p_j'^2 \langle p_j l m_l | V | p_j' l' m'_l \rangle \phi_{l'}(p) = E_d \phi_l(p), \quad (2.18)$$

It is possible to solve this equation as an eigenvalue problem  $M\Psi = E_n\Psi$  and find simultaneously wave function values and binding energy  $E_n$ .

## 2.2 The Lippman-Schwinger equation

Let us start from the time-independent formulation of the scattering process. In such a case Hamiltonian will be:

$$H = H_0 + V, \quad (2.19)$$

where  $H_0$  is a kinetic energy operator  $H_0 = \frac{\vec{p}^2}{2m}$ . For a free particle motion,  $V$  will be absent and we will denote an energy eigenstate as  $|\vec{p}\rangle$  - a free particle state. In the case of the scattering process, the eigenstate will differ from  $|\phi\rangle$ , but in case of elastic scattering (which we re interested in) the energy eigenvalue  $E$  should be the same.

So below I write a system of Schrodinger equations for such scattering process:

$$\begin{cases} H_0 |\vec{p}\rangle &= E |\vec{p}\rangle \\ (H_0 + V) |\psi\rangle &= E |\psi\rangle \end{cases} \quad (2.20)$$

I would like to find such a solution to Eq. (2.20), so that  $|\psi\rangle \rightarrow |\vec{p}\rangle$  with  $V \rightarrow 0$  and both  $|\psi\rangle$  and  $|\vec{p}\rangle$  have the same energy eigenvalues  $E$ . As we have scattering process, the energy spectra for both operators  $H_0$  and  $H_0 + V$  are continuous.

From Eq. (2.20) follows that

$$|\psi\rangle = \frac{1}{E - H_0} V |\psi\rangle + |\vec{p}\rangle, \quad (2.21)$$

where  $|\vec{p}\rangle$  was added artificially in order to satisfy a criterion mentioned above and



following the logic from [15]. In addition, it guarantees that application of the operator  $(E - H_0)$  to the (2.21) results in the second equation from the system (2.20).

In order to deal with a singular operator  $\frac{1}{E-H_0}$  in eq.(2.21), the well-known technique is to make such an operator slightly complex by adding small imaginary number to the denominator so Eq.(2.21) becomes

$$|\psi\rangle = G_0(E \pm i\epsilon)V |\psi\rangle + |\vec{p}\rangle, \quad (2.22)$$

where  $G_0$  is a free propagator:

$$G_0(z) = \frac{1}{z - H_0} \quad (2.23)$$

Solution with  $G_0(E - i\epsilon)$  corresponds to the incoming spherical wave, while  $G_0(E + i\epsilon)$  - to the outgoing one. Since we are interested in the scattering process, we will use the (+) sign further.

Eq. (2.22) is known as a Lippman-Schwinger equation (LSE) and using the definition of the transition operator  $t$ :

$$t |\vec{p}\rangle = V |\psi\rangle \quad (2.24)$$

we can rewrite it as

$$|\psi\rangle = (1 + G_0(E + i\epsilon)Vt) |\vec{p}\rangle \quad (2.25)$$

With substitution of Eq. (2.22) into Eq. (2.24) we can find an explicit form of the  $t$  operator:

$$\begin{aligned} t |\vec{p}\rangle &= VG_0(E + i\epsilon)V |\psi\rangle + V |\vec{p}\rangle = \\ &= VG_0(E + i\epsilon)t |\vec{p}\rangle + V |\vec{p}\rangle \end{aligned} \quad (2.26)$$

Getting rid of the initial state  $|\vec{p}\rangle$  in the Eq. (2.26) we can get a LSE for the transition operator in the iterative form:

$$\begin{aligned} t &= V + VG_0Vt = \\ &= V + VG_0V + VG_0VG_0Vt = \dots \end{aligned} \quad (2.27)$$

Using Eq. (2.25) we can write Eq. (2.1) as

$$N^\mu = \langle \phi m_p m_n | (1 + G_0(E + i\epsilon)Vt) \frac{1}{e} J^\mu(0) | \Psi_i \vec{P}_i \rangle \quad (2.28)$$

## 2.3 Theoretical uncertainties

### Truncation error

As it was mention above, each subsequent order of chiral expansion provide us with more and more sophisticated potential. Starling from the leading order (LO) and coming to next N2LO, N3LO and so on we toke into account more Feynmann diagrams and in result potential is able to provide us with more precise predictions for the regarded

process and observable. But the chiral expansion (as any expansion) in principle can be continued to the infinity, improving the resulting series. In practice we are limited by some number of terms, but we would like to find out the uncertainty appearing from cutting off remaining part of the expansion. This uncertainty is called a truncation error and there some ways of estimation its value for the potential of each order of chiral expansion having predictions for some limited number of terms [16, 12, 17, 18].

Let's regard some observable  $X^i(p)$  which is calculated at  $i$ -s order of chiral expansion with expansion parameter  $Q$  ( $i = 0, 2, 3, \dots$ )<sup>1</sup>. Here  $p$  specifies a momentum scale of the current reaction in the center of mass frame (in the case of deuteron photodisintegration it would be a photon's momentum).

If i define a difference between observable at each subsequent orders as:

$$\Delta X^{(2)} = |X^{(2)} - X^{(0)}|, \Delta X^{(i>2)} = |X^{(i)} - X^{(i-1)}|, \quad (2.29)$$

then chiral expansion for  $X$  is:

$$X = X^{(0)} + \Delta X^{(2)} + \Delta X^{(3)} + \dots + \Delta X^{(i)} \quad (2.30)$$

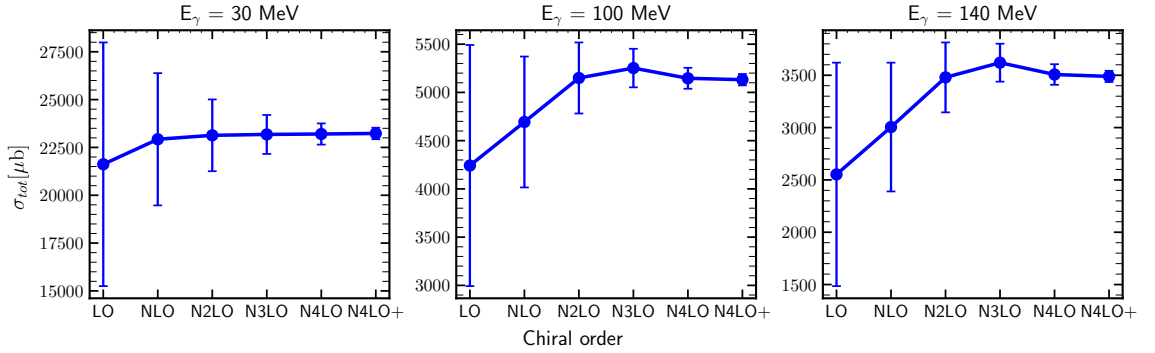


Figure 2.1: Total cross section as a dependance on the chiral order for the  $E_\gamma=100 \text{ MeV}$ . Error bands show an estimated truncation error at each order.

<sup>1</sup>We do not have a first order of expansion because this term is always vanished and NLO corresponds to the second term

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## CHAPTER 3

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PION ABSORPTION AT THE LOWEST  
ATOMIC ORBITAL IN  $^2\text{H}$ ,  $^3\text{H}$  AND  $^3\text{He}$

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# CHAPTER 4

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## RESULTS

### 4.1 Deuteron photodisintegration

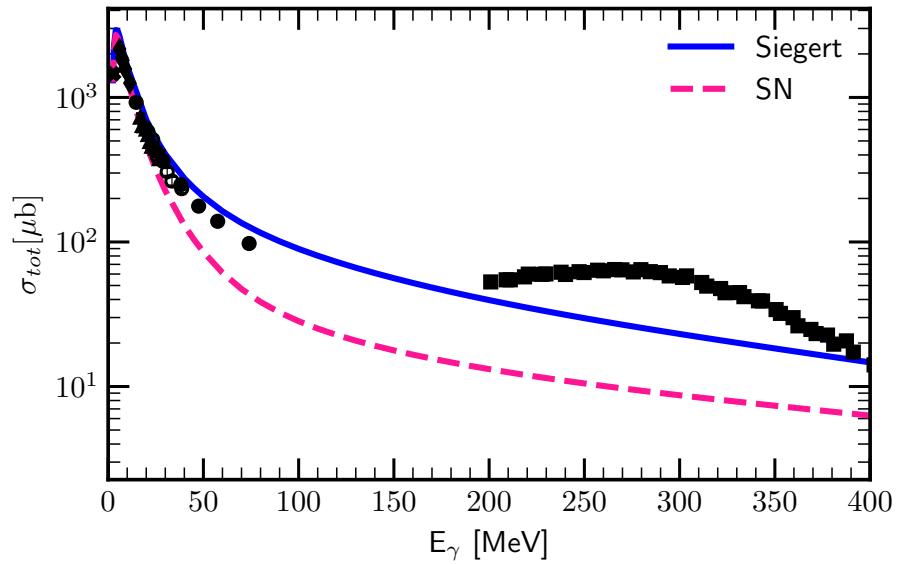


Figure 4.1: Total cross section as a function of the photon's energy  $E_\gamma$ . Solid blue line presents results obtained with SN+Siegert and dashed pink line - with only SN current. The experimental data are from [19] (black filled circles), [20] (empty circles), [21] (squares), [22] (triangles), [23] (cross "X") and [24] (diamonds).

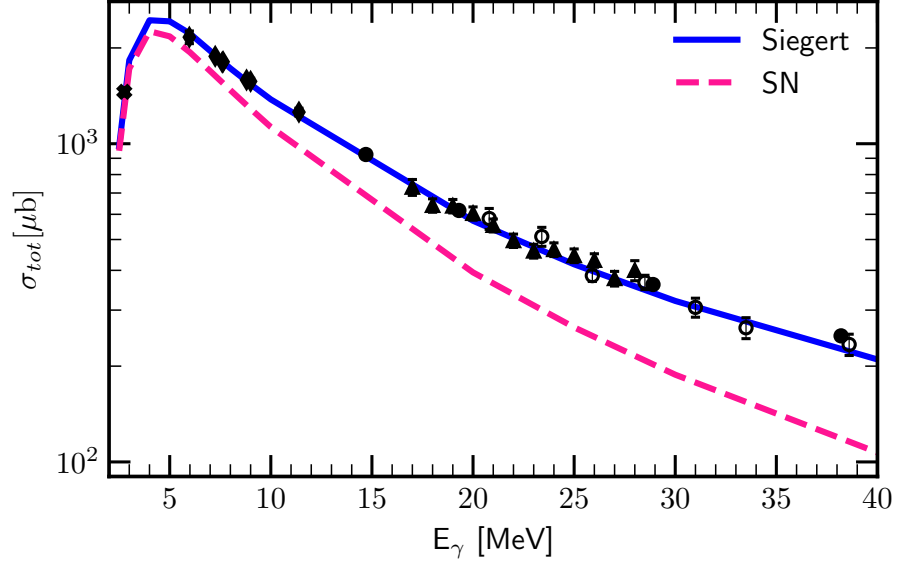


Figure 4.2: The same as on the Fig. 4.1 but for the energy range 2.5 - 40 MeV.

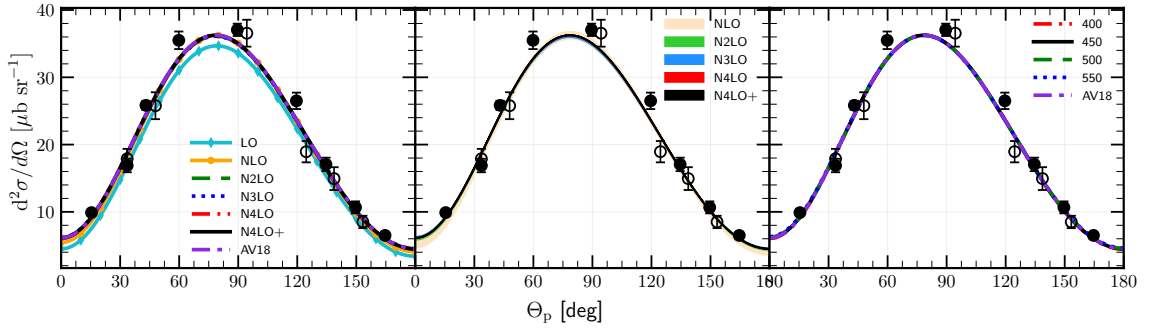


Figure 4.3: Differential cross section as a function of the outgoing proton angle in the center of mass frame for the photon's energy 30 MeV. Left figure presents results obtained using potential with different chiral orders (from LO to N<sup>4</sup>LO+) with cutoff parameter  $\Lambda = 450$  MeV whereas right figure presents a cutoff dependency and chiral potential N<sup>4</sup>LO+ was used in all cases. For the sake of comparison, predictions obtained with AV18 potential are on both figures as well. Data points (filled and empty circles) are from [25].

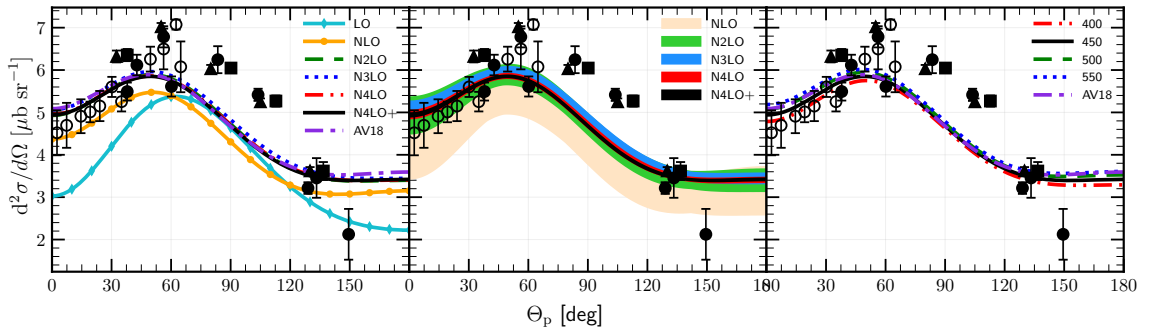


Figure 4.4: The same as on the Fig. 4.3 but for the photon's energy  $E_\gamma = 100$  MeV. All experimental data points (filled and empty circles, squares and triangles) are from [25].

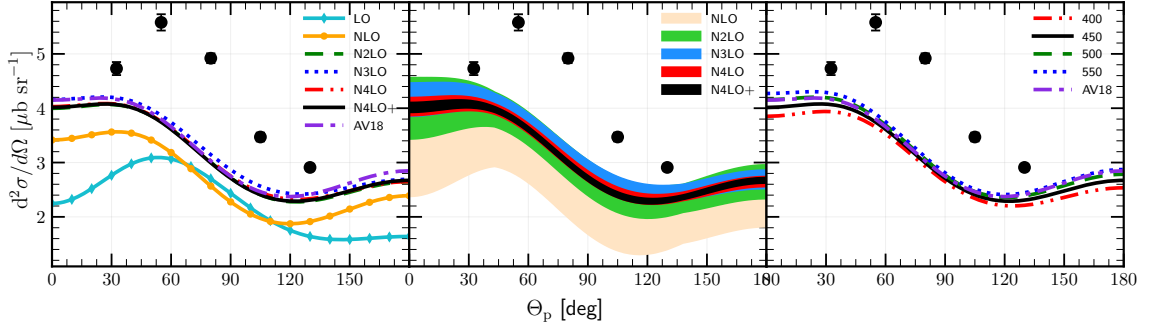


Figure 4.5: The same as on the Fig. 4.3 but for the photon's energy  $E_\gamma=140$  MeV. The data are from [26].

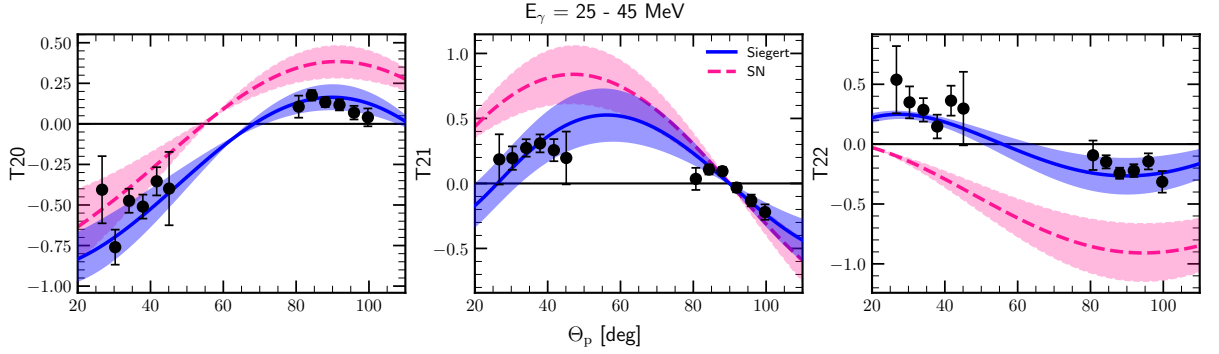


Figure 4.6: Tensor analyzing powers  $T_{20}$ ,  $T_{21}$  and  $T_{22}$  as a functions of the outgoing proton angle  $\theta_p$  (in the center of mass frame). Solid blue line is a mean value of my predictions obtained with a SMS potential at  $N^4\text{LO}+$  chiral order and with  $\Lambda = 450$  MeV at energy values from 25 to 45 MeV and where SN current was used together with Siegert approach. Pink dashed line is similar prediction but with SN only. The corresponding bands show the deviation of predictions in the regarded energy region. Filled circles are experimental data from [27] for the analogous energy span.

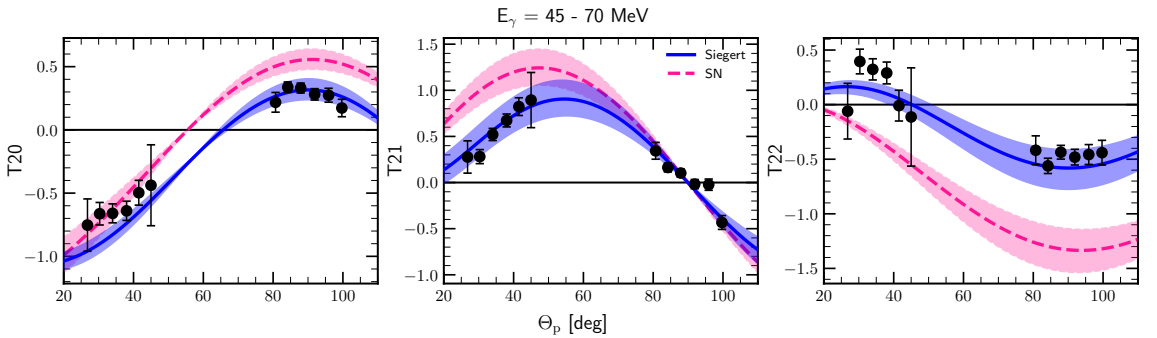


Figure 4.7: The same as on the Fig. 4.6 but for energy bin 45 - 70 MeV

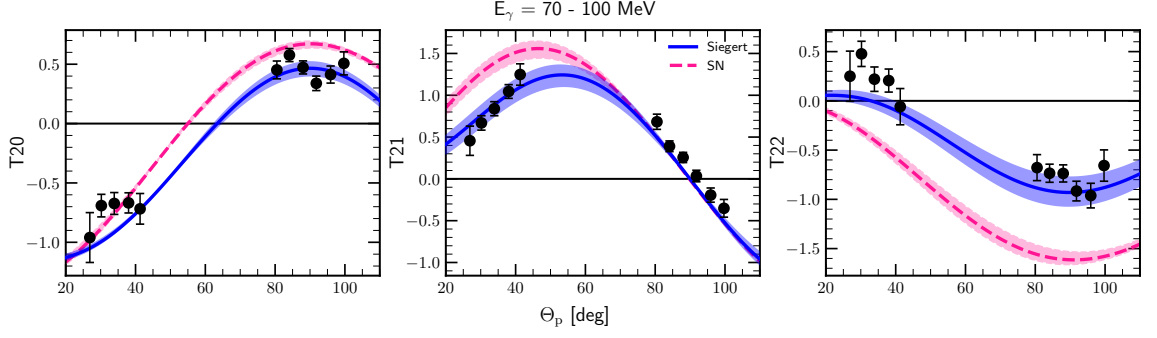


Figure 4.8: The same as on the Fig. 4.6 but for energy bin 70 - 100 MeV

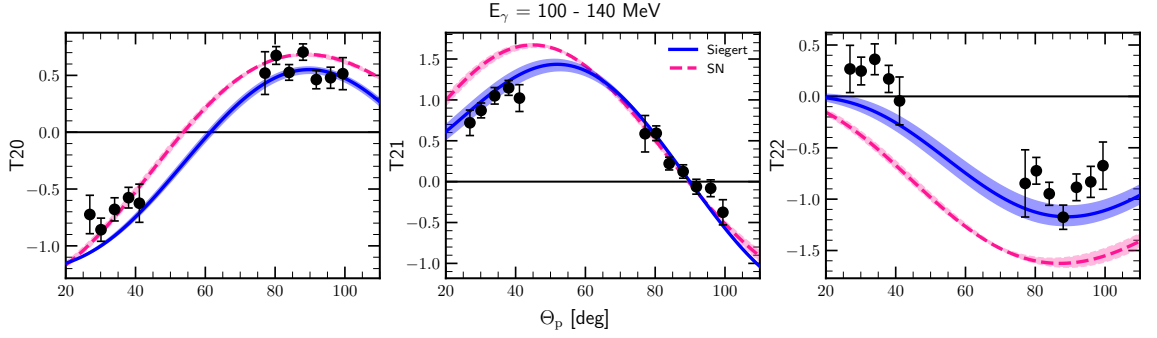


Figure 4.9: The same as on the Fig. 4.6 but for energy bin 100 - 140 MeV

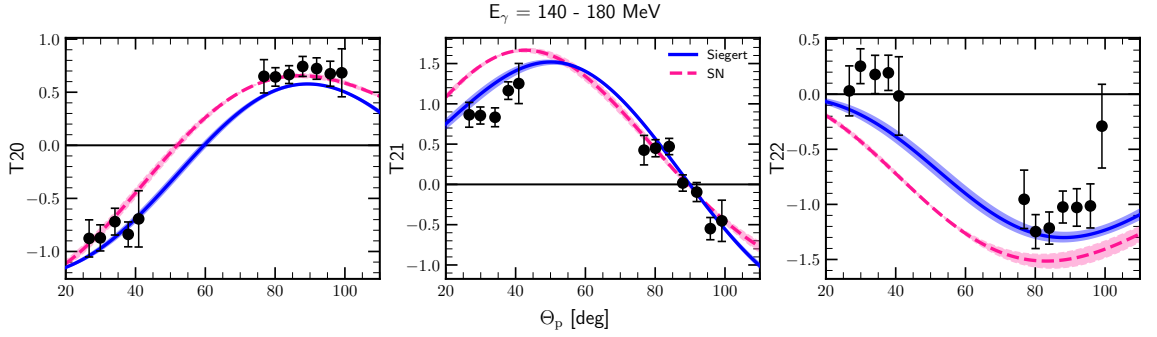


Figure 4.10: The same as on the Fig. 4.6 but for energy bin 140 - 180 MeV

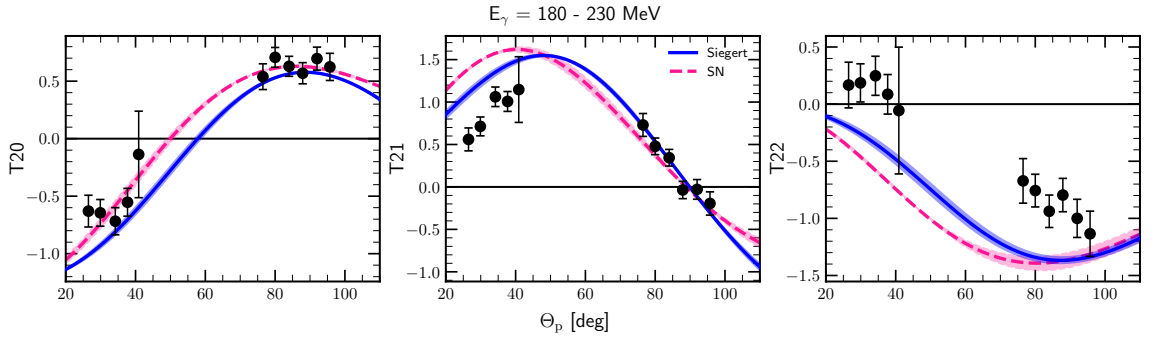


Figure 4.11: The same as on the Fig. 4.6 but for energy bin 180 - 230 MeV

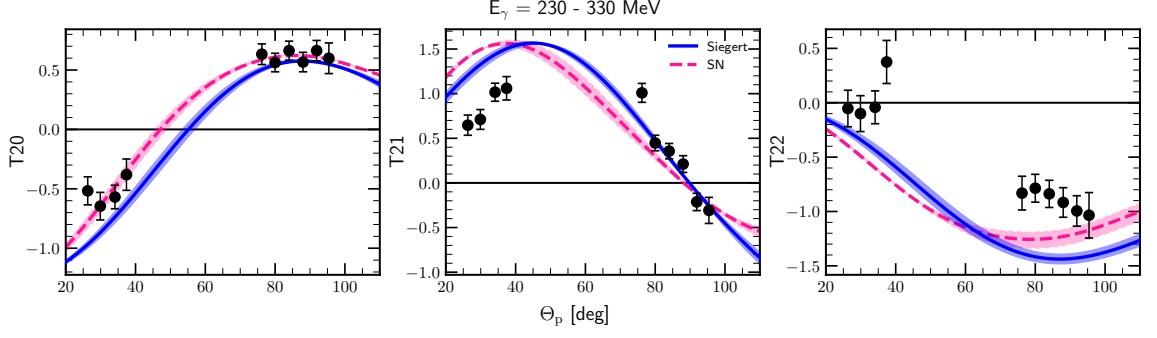


Figure 4.12: The same as on the Fig. 4.6 but for energy bin 230 - 330 MeV

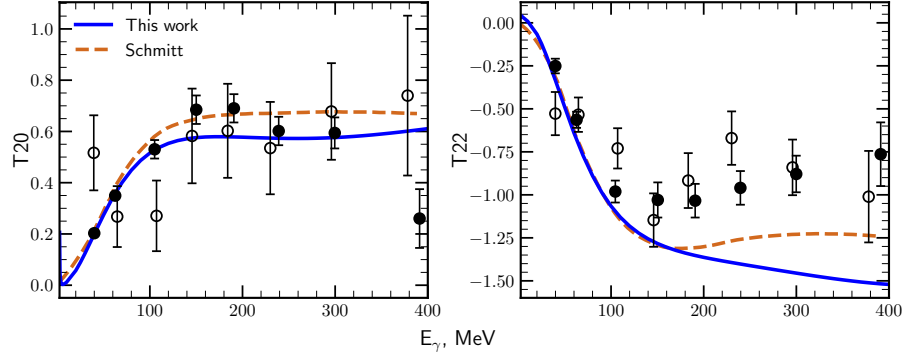


Figure 4.13: Tensor analyzing powers  $T_{20}$  and  $T_{22}$  as a functions of the photon energy  $E_\gamma$  with fixed outgoing proton angle  $\theta_p = 88^\circ$  (in the center of mass frame). My predictions (blue solid line) are obtained with SMS potential at chiral order  $N^4\text{LO}+$  and with cutoff parameter  $\Lambda = 450$  MeV. Dashed brown line presents calculations from [28]. Experimental data is taken from [27] (filled circles) and [29] (empty circles).

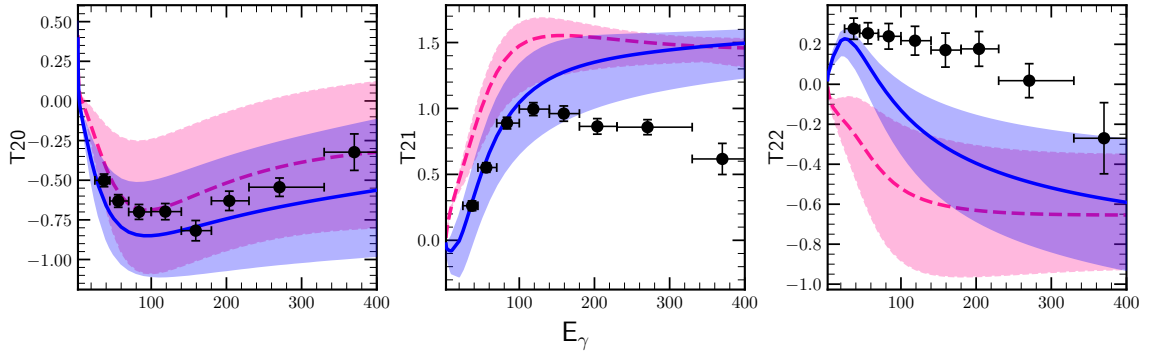


Figure 4.14: Tensor analyzing powers  $T_{20}$ ,  $T_{21}$  and  $T_{22}$  as a functions of the photon's energy within the outgoing proton's angle range  $24^\circ - 48^\circ$  (in the center of mass frame). Solid blue line is a mean value of my predictions obtained with SMS potential at  $N^4\text{LO}+$  chiral order and with  $\Lambda = 450$  MeV at energy values from 25 to 45 MeV within a given angles range and where SN current was used together with Siegert approach. Pink dashed line is similar prediction but with SN only. The corresponding bands show the deviation of predictions in the regarded energy region. Filled circles are experimental data from [27] for the analogous energy span.



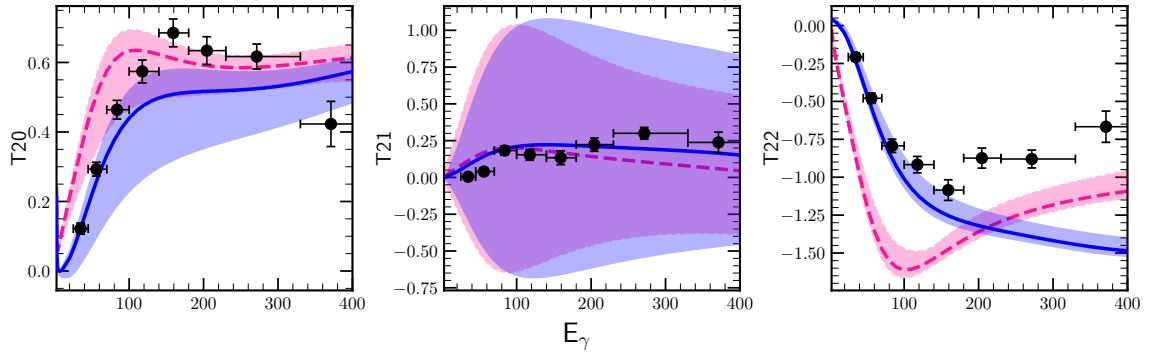


Figure 4.15: The same as on the Fig. 4.14 but for the angles' range  $70^\circ - 102^\circ$ .

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