Application of the chiral forces to elctroweak processes

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Why we study few nucleon systems

The study of light nuclei for the decades has been serving as an easiest way to study NN systems and forces inside the atom. And convenient way to proceed may be an interaction of atom with other particles: elastic or inelastic scattering. It is possible to construct such an experiments and check if theory works. People take into account that interactions may be caused by different forces and therefore should be described in different ways. It can be either strong, weak or electromagnetic interaction. It depends on the type of particle being scattered and the target which reaction it is.

In order to proper describe the nuclear reactions many factors should be taken into account. First of all, different nuclear forces may act on the participants.

The strong nuclear force appear inside the nuclei and among others bound neutrons and protons together. The description of strong interactions is extremely difficult as it deals not only with nucleon, but with their constituents: quarks and gluons. Quantum Chromodynamics(QCD) is a modern theory describing strong interactions, but it has also its limitations at the moment as it is not reliable at low energies ($Q^2 \lesssim 1 GeV^2$). So other approaches are coming into the scene such as chiral effective theory, lattice calculation and others [?].

Electromagnetic force appears between charged particles like protons and electrons. Also, the force is transferred between charged particles with a photon, so in photon- and electron- scatterings on the nuclei an electromagnetic force is playing an important role. Arenhovel [?] studied electromagnetic process - Deuteron photodisintegration, applying different approaches and comparing the results with experimental data.

The weak force...

...

Starting the study of 3- (and more) nucleon systems it was found that 2N force is not enough to describe the system and 3N force was introduced. The first applications of such a force showed that it brings sufficient contribution and cannot be ignored [?]. Whereas the first applications included only early "realistic" potential, the latter investigations only proved this statements [?, ?]. It was also used to construct four-nucleon (4N) bound state [?].

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Nuclear forces used in the thesis

In order to construct a potential people often use phenomenological or semi-phenomenological approaches. It allows to combine theoretical knowledge about processes and experimental data.

One of such potentials, which was used in current thesis is Argonne V18 (AV18) [?] In order to construct NN force, authors combine analytical electromagnetic and one-pion-exchange parts with phenomenological one, fitting parameters to the Nijmegen partial-wave analysis of pp and np data [?]. Authors showed, that AV18 potential delivers good results in the description of nucleon scattering data as well as deuteron properties.

In the early 1990-ies Weinberg [?, ?] introduced an idea of using a most general Lagrangian satisfying assumed symmetry principles and in particular spontaneously broken

chiral symmetry to describe nuclear interactions at low energies. This idea together with effective field theory (EFT) of Quantum chromodynamics (QCD) led to the development of Chiral effective field theory (χEFT) which nowadays has become one of the most advanced approach to describing nuclear reactions at low energies.

For the EFT it is very important to define a quantity, which powers will determine a perturbation order. In the χEFT there are two natural scales: so-called soft scale - the mass of Pion $Q \sim M_\pi$ and hard scale - $\Lambda_\chi \sim 1 \ GeV$ (chiral symmetry breaking scale). The ratio between these two scales $(Q/\Lambda_\chi)^\nu$ is being used as an expansion parameter in χEFT with power ν.

Considering so-called irreducible (the diagrams that cannot be split by cutting nucleon lines), Weinberg [?, ?] came to the identity for the powers of such diagrams[?]:

$$\nu_W = 4 - A - 2C + 2L + \sum_i \Delta_i, \tag{1.1}$$

where

$$\Delta_i \equiv d_i + \frac{n_i}{2} - 2 \tag{1.2}$$

In 1.1, C is a number of pieces which are connected, L - the number of loops in the graph. In 1.2, n_i is a number of nucleon field operators, d_i - the number of insertions (or derivatives) of M_π.

In χEFT the first order is called "leading order" (LO) and it is followed by next-to-leading order (NLO), next-to-next-to-leading order (N2LO) and so on. At the moment, the highest order for which there is a derived term in potential is N4LO. Also some contributions from N5LO are included in the N4LO+ chiral order.

As pointed above, for many-nucleon systems it is important to include not only nucleon-nucleon interaction to the potential, but also a 3- and many- nucleon contributions. In the χEFT 3N force contributes starting from N2LO and 4N force is presented starting from N3LO, so there is a systematic way to include all the forces from simplest diagrams at LO and gradually adding more and more terms. It is also beneficial in the way that one can obtain results using chiral potential at different orders and track which one gives larger or smaller contribution (changes in the final results).

The χEFT may be applied both in coordinate and momentum spaces. Nevertheless in both cases it requires regularization which is cutting low coordinate values in order to avoid infinities (or high momentum values - in momentum space). The value at which the cut is applied (cut-off value) is not fixed and usually calculations are being performed for different cut-off values. The comparison of such results may reveal stronger or weaker dependance and in perfect case one will come up with such a potential, were the cut will not affect results much.

The potential may be transformed from coordinate to momentum space (or vice versa), but it is important at which frame the regularization was performed and what was a regularization function. That's why there are different versions of chiral potential. One is semi-local coordinate space regularized potential (SCS) [?] and another one is similar, but with regularization applied in momentum space (SMS potential) [?].

Currents

Formalism & numerical methods

In order to calculate any observable for the Deuteron photodisintegration, one has to find a nuclear matrix elements:

$$N^\mu = \langle \Psi_f \vec{P}_f \mid \frac{1}{e} J^\mu(0) \mid \Psi_i \vec{P}_i \rangle = \langle p'(l's')j'm_j't'm_t'\vec{P}_f \mid J^\mu \mid \phi_d m_d \vec{P}_i \rangle, \tag{1.3}$$

where J^μ is a four-vector current operator which acts between initial and final two-nucleon states.

Deuteron bound state

Let's find a deuteron bound state wave function ϕ_d. The time-independent Schrodinger equation for two particles in such case will be:

$$(H_0 + V) \mid \psi_{12} \rangle = E_d \mid \psi_{12} \rangle \tag{1.4}$$

H_0 is a kinetic energy of the nucleons and V is a potential. The kinetic energy H_0 can be represented in terms of relative and total momenta of the particles:

$$H_0 = \frac{\vec{p}_1^2}{2m_1} + \frac{\vec{p}_2^2}{2m_2} = \frac{\vec{p}^2}{2\mu} + \frac{\vec{\mathcal{P}}^2}{2M}, \tag{1.5}$$

where relative and total momenta are defined as follows:

$$\vec{p} = \frac{(m_1\vec{p}_1 - m_2\vec{p}_2)}{m_1 + m_2} \tag{1.6}$$

$$\vec{\mathcal{P}} = \vec{p}_1 + \vec{p}_2 \tag{1.7}$$

and $M = m_1 + m_2$ is a total mass, $\mu = \frac{m_1 m_2}{M}$ is a relative mass of two nucleons.

We are working in the momentum space, so acting by the momentum operator on the Eq.1.4 one can obtain two separated equations:

$$\frac{\vec{p}^2}{2\mu}\langle \vec{p} \mid \Psi_{int} \rangle + \int d\vec{p}''\langle \vec{p} \mid V \mid \vec{p}'' \rangle\langle \vec{p}'' \mid \Psi_{int} \rangle = (E_d - E_{c.m.})\langle \vec{p} \mid \Psi_{int} \rangle \tag{1.8}$$

$$\frac{\vec{\mathcal{P}}^2}{2M}\langle \mathcal{P} \mid \Psi_{c.m.} \rangle = E_{c.m.}\langle \mathcal{P} \mid \Psi_{c.m.} \rangle \tag{1.9}$$

Eq.1.8 is basically a Schrodinger equation for one particle with mass μ and Eq.1.9 can be regarded as a Schrodinger equation for particle with mass M in a free motion. Assuming that deuteron is at rest ($E_{c.m.} = 0$) we can stick to the Eq.1.8 only. So that:

$$\frac{\vec{p}^2}{2\mu}\langle \vec{p} \mid \Psi_{int} \rangle + \int d\vec{p}'\langle \vec{p} \mid V \mid \vec{p}' \rangle\langle \vec{p}' \mid \Psi_{int} \rangle = E_d\langle \vec{p} \mid \Psi_{int} \rangle \tag{1.10}$$

Next we move to the partial-wave representation of the momentum state in the following form:

$$\mid \vec{p} \rangle = \mid p\alpha \rangle \equiv \mid p(ls)jm_j \rangle \mid tm_t \rangle, \tag{1.11}$$

where we introduce quantum numbers l, s ,j ,t as orbital angular momentum, total spin, total angular momentum and total isospin respectively. m_j and m_t are isospin and spin projections.

Yet one can introduce simpler states than it is in 1.11.

$$| p(ls)jm_j \rangle = \sum_{m_l} c(lsj; m_l, m_j - m_l, m_j) | plm_l \rangle | s\ m_j - m_l \rangle \tag{1.12}$$

Also we can decompose spin and isospin states as follows:

$$| sm_s \rangle = \sum_{m_1} c(\frac{1}{2}\frac{1}{2}s; m_1, m_s - m_1, m_s) | \frac{1}{2}m_1 \rangle | \frac{1}{2}m_s - m_1 \rangle \tag{1.13}$$

$$| tm_t \rangle = \sum_{\nu_1} c(\frac{1}{2}\frac{1}{2}t; \nu_1, m_t - \nu_1, m_t) | \frac{1}{2}\nu_1 \rangle | \frac{1}{2}m_t - \nu_1 \rangle \tag{1.14}$$

In Eqs.1.12 -1.14, $c(...)$ are Clebsh-Gordon coefficients. Nucleons are spin $\frac{1}{2}$ particles, and also we treat proton and neutron as the same particle in different isospin states, so that isospin is $\nu_1 = \frac{1}{2}$ for proton and $\nu_1 = -\frac{1}{2}$ for neutron.

The states $| plm_l \rangle$ from Eq.1.12 are orthogonal, so that

$$\langle p'l'm_l' | plm_l \rangle = \frac{\delta(p - p')}{p^2} \delta_{ll'} \delta_{m_l m_l'} \tag{1.15}$$

and also satisfy the completeness relation:

$$\sum_{l=0}^{\infty} \sum_{m_l=-l}^{l} \int dp p^2 | plm_l \rangle \langle plm_l | = \mathbb{1} \tag{1.16}$$

These states also fulfill a relation

$$\langle \vec{p}' | plm_l \rangle = \frac{\delta(|\vec{p}'| - p)}{p^2} Y_{lm_l}(\hat{p}'), \tag{1.17}$$

where $Y_{lm_l}(\hat{p}')$ is a spherical harmonic and 'hat' means a unit vector.

If we exchange nucleons 1 and 2 there should be a sign change and this requirement can in mathematical form can be expressed as:

$$(-1)^{l+s+t} = -1 \tag{1.18}$$

Taking into account Eq.1.18, one can find only one possible case for the deuteron bound state: 2 coupled channels for l=0,2; s=1; j=1and $t = m_t = 0$. These 2 channels are usually denoted as $^3S_1$ and $^3D_1$ and corresponding wave functions are $\phi_0(p)$ and $\phi_2(p)$.

So with a new basis Eq.1.10 takes a form:

$$\frac{\vec{p}^2}{2\mu} \phi_l(p) + \sum_{l'=0,2} \int dp' p'^2 \langle plm_l | V | p'l'm_l' \rangle \phi_{l'}(p) = E_d \phi_l(p), \tag{1.19}$$

for $l = 0, 2$. Assuming that one has a matrix elements for the potential $\langle plm_l | V | p'l'm_l' \rangle$, there is still one complication in the Eq.1.19 - integration. In order to get rid of the integral I use a Gaussian quadrature method of numerical integration [?]. It allows to replace an integral by the weighted sum: $\int_a^b f(x)dx = \sum_{i=1}^n \omega_i f(x_i)$ In current work I used 72 points in the interval from 0 to $50 fm$. Using this method, Eq.1.10 becomes

$$\frac{\vec{p}^2}{2\mu}\phi_l(p) + \sum_{l'=0,2}\sum_{j=0}^{N}\omega_j p_j'^2 \langle p_j l m_l \mid V \mid p_j' l' m_l'\rangle \phi_{l'}(p) = E_d\phi_l(p), \qquad (1.20)$$

It is possible to solve this equation as an eigenvalue problem and find simultaneously wave function values and binding energy E_n.