6. Задача 9.6 вариант 38

$$f(x) = x_1 x_2 x_3 \to extr \tag{1}$$
$$x_1 + x_2 + x_3 \leq 5 \tag{2}$$
$$x_1 x_2 + x_2 x_3 + x_1 x_3 = 9 \tag{3}$$

Решение

$$F(\lambda_0, x_1, x_2, x_3, \lambda, \mu) = \lambda_0 x_1 x_2 x_3 + \lambda(x_1 + x_2 + x_3 - 5) + \mu(x_1 x_2 + x_2 x_3 + x_1 x_3 - 9)$$

$$\begin{cases} \lambda_0 x_2 x_3 + \lambda + \mu(x_2 + x_3) = 0 \\ \lambda_0 x_1 x_3 + \lambda + \mu(x_1 + x_3) = 0 \\ \lambda_0 x_1 x_2 + \lambda + \mu(x_1 + x_2) = 0 \\ \lambda(x_1 + x_2 + x_3 - 5) = 0 \\ x_1 x_2 + x_2 x_3 + x_1 x_3 = 9 \end{cases}$$

1. $\lambda_0 = 0$

$$\begin{cases} \lambda + \mu(x_2 + x_3) = 0 \\ \lambda + \mu(x_1 + x_3) = 0 \\ \lambda + \mu(x_1 + x_2) = 0 \\ \lambda(x_1 + x_2 + x_3 - 5) = 0 \\ x_1 x_2 + x_2 x_3 + x_1 x_3 = 9 \end{cases}$$

   Решения системы:

   1) $(x1, x2, x3, \lambda, \mu) = (-\sqrt{3}, -\sqrt{3}, -\sqrt{3}, 0, 0)$
   2) $(x1, x2, x3, \lambda, \mu) = (\sqrt{3}, \sqrt{3}, \sqrt{3}, 0, 0)$
      не подходит по (2)

2. $\lambda_0 = 1$

$$\begin{cases} x_2 x_3 + \lambda + \mu(x_2 + x_3) = 0 \\ x_1 x_3 + \lambda + \mu(x_1 + x_3) = 0 \\ x_1 x_2 + \lambda + \mu(x_1 + x_2) = 0 \\ \lambda(x_1 + x_2 + x_3 - 5) = 0 \\ x_1 x_2 + x_2 x_3 + x_1 x_3 = 9 \end{cases}$$

   Решения системы:

1) $(x1, x2, x3, \lambda, \mu) = (-\sqrt{3}, -\sqrt{3}, -\sqrt{3}, 0, 0)$

2) $(x1, x2, x3, \lambda, \mu) = (\sqrt{3}, \sqrt{3}, \sqrt{3}, 0, 0)$

не подходит по (2)

$$\frac{\partial^2 F}{x^2} = \begin{pmatrix} 0 & x3+\mu & x2+\mu \\ x3+\mu & 0 & x1+\mu \\ x2+\mu & x1+\mu & 0 \end{pmatrix}$$

$$\frac{\partial^T g}{x} l = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} = l_1 + l_2 + l_3$$

$$\frac{\partial^T h}{x} l = \begin{pmatrix} x_2 + x_3 & x_1 + x_3 & x_1 + x_2 \end{pmatrix} \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} = (x_2 + x_3)l_1 + (x_1 + x_3)l_2 + (x_1 + x_2)l_3$$

1) $a = (x1, x2, x3) = (-\sqrt{3}, -\sqrt{3}, -\sqrt{3})$

$$\frac{\partial^2 F(a)}{x^2} = \begin{pmatrix} 0 & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & 0 & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & 0 \end{pmatrix}$$

$$\frac{\partial^T h}{x} l = -2\sqrt{3}l_1 - 2\sqrt{3}l_2 - 2\sqrt{3}l_3 = 0$$

$$l_1 + l_2 + l_3 = 0 \tag{4}$$

$$l^T \frac{\partial^2 F(a)}{x^2} l = l^T \begin{pmatrix} 0 & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & 0 & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & 0 \end{pmatrix} l = \begin{pmatrix} -\frac{\sqrt{3}}{2}l_2 - \frac{\sqrt{3}}{2}l_3 \\ -\frac{\sqrt{3}}{2}l_1 - \frac{\sqrt{3}}{2}l_3 \\ -\frac{\sqrt{3}}{2}l_1 - \frac{\sqrt{3}}{2}l_2 \end{pmatrix}^T \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} =$$

$$= -\frac{\sqrt{3}}{2} \begin{pmatrix} l_2 + l_3 \\ l_1 + l_3 \\ l_1 + l_2 \end{pmatrix}^T \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} = [(4)] = -\frac{\sqrt{3}}{2} \begin{pmatrix} -l_1 \\ -l_2 \\ -l_3 \end{pmatrix}^T \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} = \frac{\sqrt{3}}{2}(l_1^2 + l_2^2 + l_3^2) > 0$$

Значит a — лок. min

Ответ: min — $(-\sqrt{3}, -\sqrt{3}, -\sqrt{3})$, max нет