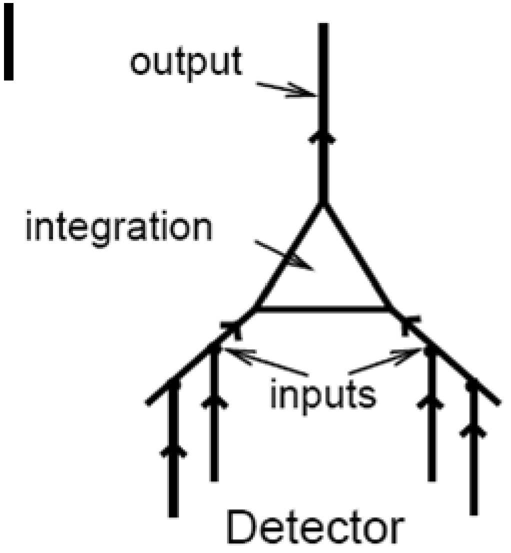
Neurons

- The detector model.
- Biological properties of the neuron – (complex electrochemical systems)
- The computational unit – (simulated neurons – mathematical abstractions of basic functional characteristics)

Acknowledgement: Based on O'Reilly and Munakata's slides

Detector Model



Basic question: what are neurons and what do they do?

1) Each neuron is detecting some set of conditions (e.g. smoke detector). *Representation* is what is detected.

2) Neurons feed on each other's outputs — layers of ever more complicated detectors.

(Things can get very complex in terms of *content*, but each neuron is still carrying out basic detector function).

3) Integration level denotes how well available information (inputs) match what neuron detects

Examples: Place cells in Hippocampus (O'Keefe);

Jennifer Aniston neuron – what respond to?

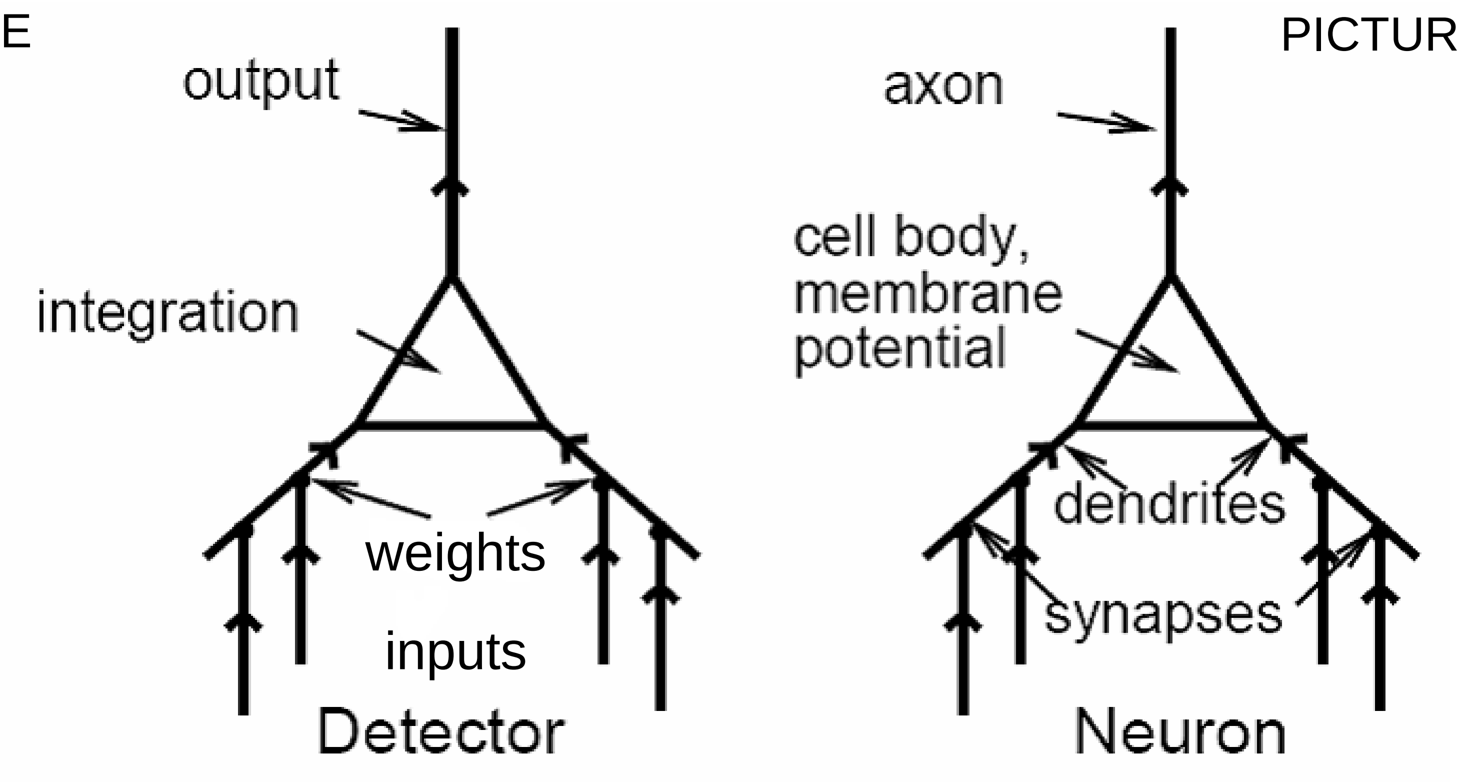
Acknowledgement: Based on O'Reilly and Munakata's slides

localist vs distributed

Understanding Neural Components in Detector Model

CONCEPTUAL PICTURE

BIOLOGICAL PICTURE



Question: what is a membrane potential?

Acknowledgement: Based on O'Reilly and Munakata's slides

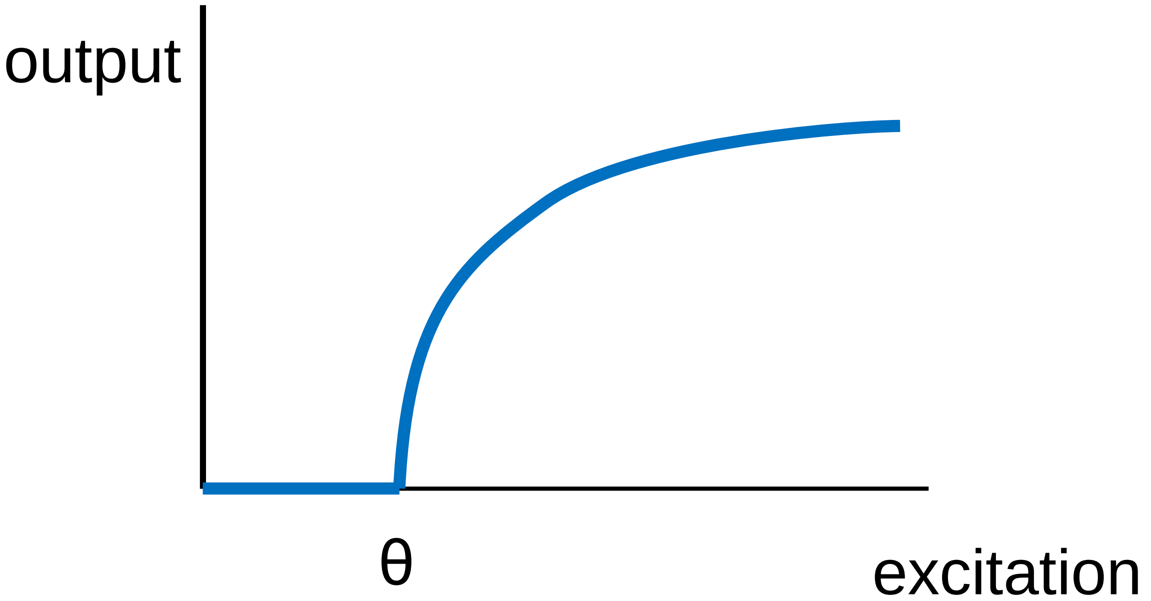
- weights
  - synaptic efficacies / strengths
- integrate
  - combining (usually summing across) weighted inputs
  - yields aggregate measure of degree to which input pattern fits expected one
  - aggregate measure is (biologically) the membrane potential
- different inputs “weigh” more heavily into the detection decision
- **Question: what do negative weights represent?**
- **How could a single negative weight discriminate a whole class?**

Acknowledgement: Based on O’Reilly and Munakata’s slides

- weights characterise what is detected
  - detect patterns of activity across inputs
  - patterns that “fit” weight pattern most accurately yield largest detection response
- threshold on output

Why a threshold?

- **what is nonlinearity?**



Acknowledgement: Based on O’Reilly and Munakata’s slides

Key Concepts (i)

- Channels
  - by which **ions** enter or leave neurons
  - type specific - $\mathbf{Na^+}$, $\mathbf{Cl^-}$, $\mathbf{K^+}$ channels
  - also excitation and inhibition specific
    - (although not a one-to-one relationship)
- Charge: opposites attract, likes repel
- Electrical Potential (voltage)
  - amount of opposite charge that can be attracted to that area
  - difference in concentration of charged atoms

Acknowledgement: Based on O’Reilly and Munakata’s slides

Key Concepts (ii)

- Membrane Potential
  - imbalance in charge across neuron membrane
  - e.g. negative potential - more negatively than positively charged atoms inside neuron
  - in relative terms, **extra cellular space** effectively zero charge
- Voltage gated channels
  - open when membrane potential crosses particular **threshold**

Why a threshold?

Relationship to nonlinearity

Importance of nonlinearity

Acknowledgement: Based on O'Reilly and Munakata's slides

Key Concepts (iii)

- Resting potential
  - **membrane potential** when no inputs
  - steady state in absence of stimulation
  - -70mV due to Na/K pump (electrically primed)
- Action potential (**spike**)
  - electrical pulse / current released down axon when membrane potential crosses a **threshold**, releases **neurotransmitter**
  - Neurotransmitter activates potential via dendritic **synaptic input** channels.
- Resistance (conductance - reciprocal)
  - level of obstruction to ion flow, e.g. opening of channels

Acknowledgement: Based on O'Reilly and Munakata's slides

Neurophysiology

The neuron is a miniature electro-chemical system:

1. Balance of electric and diffusion forces
2. Equilibrium channel potentials.
3. Principal ions – how act together.
4. Integration equations. **Including what will be your favourite equation**
5. The overall membrane equilibrium potential.

Note terminology confusion

What is difference between equilibrium channel potential and equilibrium membrane potential?

Acknowledgement: Based on O'Reilly and Munakata's slides

Balance of Electric and Diffusion Forces

Ions flow into and out of the neuron under the forces of *electricity* and *concentration* gradients (diffusion).

The net result is an electric potential difference between the inside and outside of the cell — **the membrane potential** V_m.

This value represents an integration of the different forces, and an *integration* of the inputs impinging on the neuron.

We will use the *equations* describing this integration in our models.

Acknowledgement: Based on O'Reilly and Munakata's slides

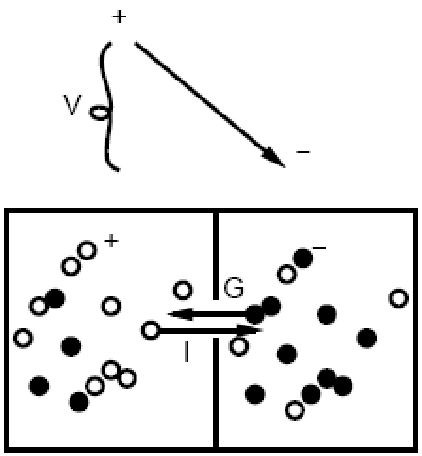
Resistance

Building from ohm's law to thought!

Ions encounter **resistance** when they move.

Neurons have **channels** that limit flow of ions in/out of cell.

What factors may impact rate of flow, i.e. I?



The smaller the channel, the higher the resistance, the greater the potential needed to generate given amount of current (Ohm's law):

$$I = \frac{V}{R}$$

Conductance $G = 1/R$, so $I = VG$

Is there a non-zero voltage here and why?

Acknowledgement: Based on O'Reilly and Munakata's slides

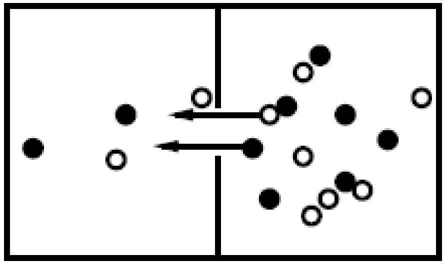
Why not $I = V+G$?

Diffusion

Constant motion causes mixing — evens out distribution.

Unlike electricity, diffusion acts on each ion separately — can't compensate one + ion for another.

Is there an electrical potential here?



FOR EXAMPLE,
○ - Na^+
● - Cl^-

Acknowledgement: Based on O'Reilly and Munakata's slides

Equilibrium Channel Potential

Balance between electricity and diffusion:

E = **Equilibrium** potential = amount of electrical potential needed to counteract diffusion:

$$I = G(V - E)$$

Net electrical potential and Ohm's law

Also called:

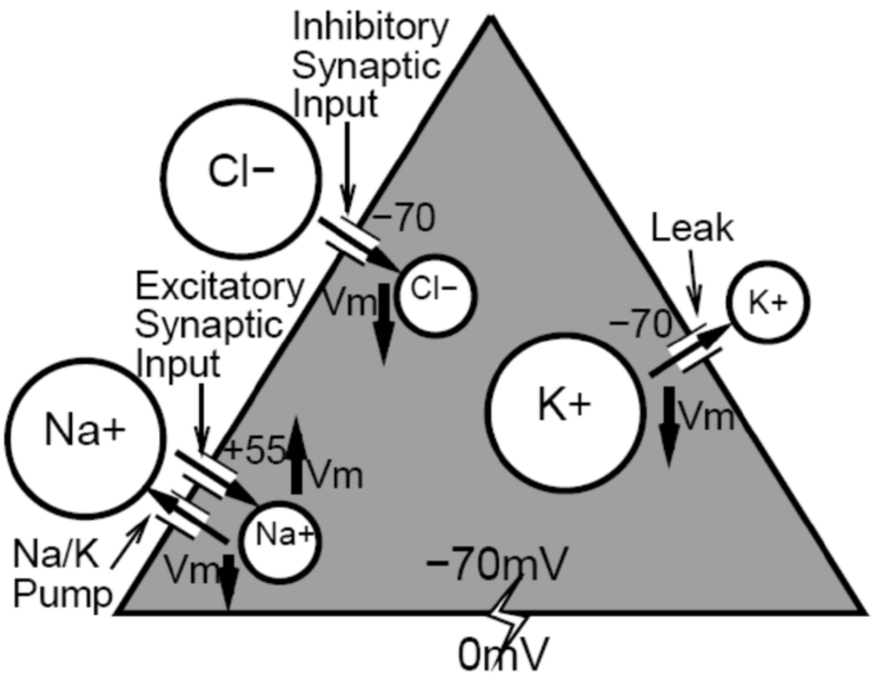
Reversal potential (because current reverses on either side of E)

Driving potential (flow of ions drives potential toward this value)

Each ion channel has its own equilibrium potential

Acknowledgement: Based on O'Reilly and Munakata's slides

The Neuron and its Ions



What state can the Cl- and Na+ channels be in, which K+ cannot?

Everything follows from the sodium pump, which creates the “dynamic tension” (compressing the spring, winding the clock) for subsequent neural action.

Acknowledgement: Based on O’Reilly and Munakata’s slides

Channel Current

Ohm's law:

$$\underbrace{I}_{\text{Current}} = \underbrace{G}_{\text{Conductance}}\underbrace{V}_{\text{Voltage}}$$

Current for an arbitrary channel c:-

$$\underbrace{I_c(t)}_{\text{Channel c current}} = \underbrace{g_c(t)\,\overline{g}_c}_{\text{Conductance}}\underbrace{(V_m(t) - E_c)}_{\text{Net potential}}$$

$g_c(t)$ Fraction of *c* channels open at time *t*

$\overline{g}_c$ Maximum conductance if channel fully open

$V_m(t)$ Membrane potential at time *t*

E_c Equilibrium potential for channel *c*

Which are the constants here?

What about leak conductance: $g_l(t)$

Acknowledgement: Based on O'Reilly and Munakata's slides

Beware of term net – a number of different usages

Net Current:

Three Channel Types

$$I_{net}(t) = g_e(t)\overline{g_e}\,(V_m(t) - E_e) +$$

Excitatory - Na^+ (glutamate)

$$g_i(t)\overline{g_i}\,(V_m(t) - E_i) +$$

Inhibitory - Cl^- (GABA)

$$g_l(t)\overline{g_l}\,(V_m(t) - E_l)$$

Leak - K^+ (always open)

What are the constants here?

Acknowledgement: Based on O'Reilly and Munakata's slides

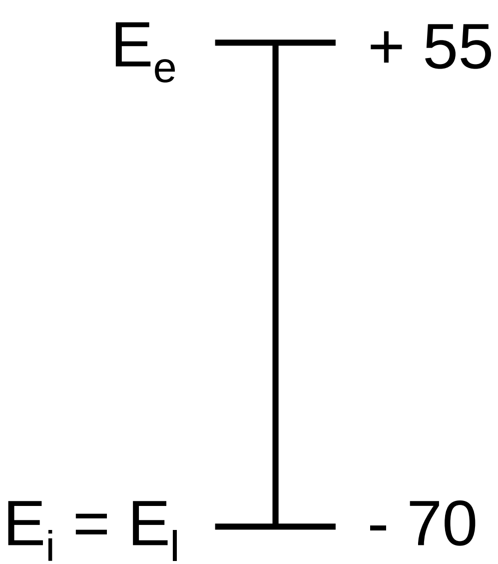
(1) Membrane Potential = previous membrane potential + Time constant x net current

(2)

$$
\begin{aligned}
V_m(t+1) &= V_m(t) + dt_{vm}\, I_{net}(t) \\
&= V_m(t) + dt_{vm} [\\
&\quad g_e(t)\overline{g_e}(E_e - V_m(t)) + \\
&\quad g_i(t)\overline{g_i}(E_i - V_m(t)) + \\
&\quad g_l(t)\overline{g_l}(E_l - V_m(t)) \;]
\end{aligned}
$$

(3) Time constant - dt_{vm} *governs amount of change per time unit*

$$0 \leq dt_{vm} \leq 1$$



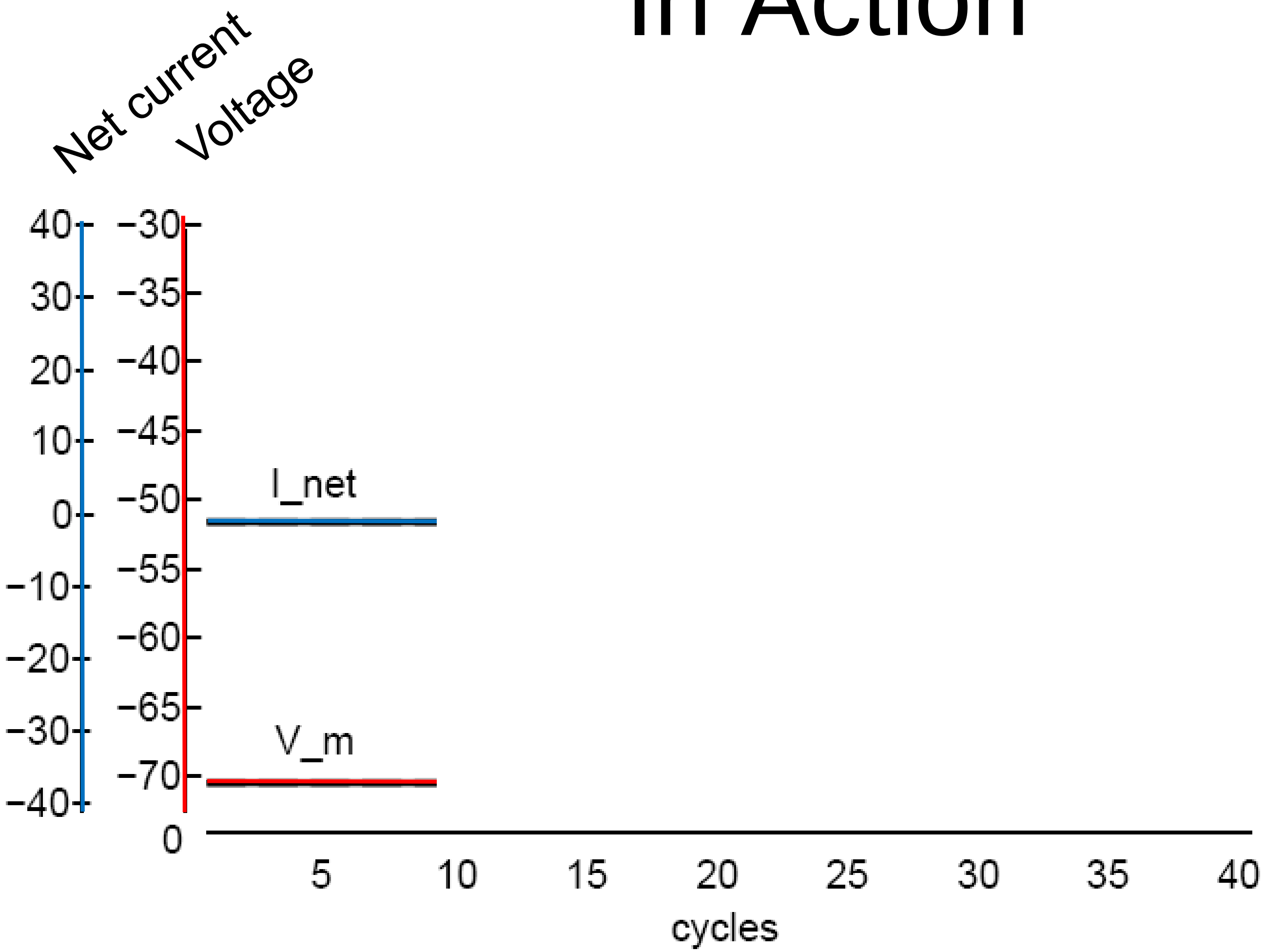
Why is it difficult to excite an excited neuron?

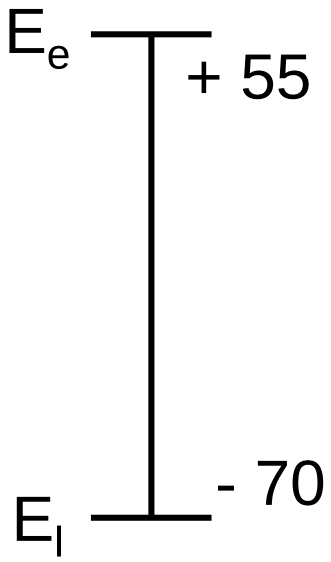
What about inhibiting an inhibited one?

What is true when I_{net} is zero?

Acknowledgement: Based on O'Reilly and Munakata's slides

In Action

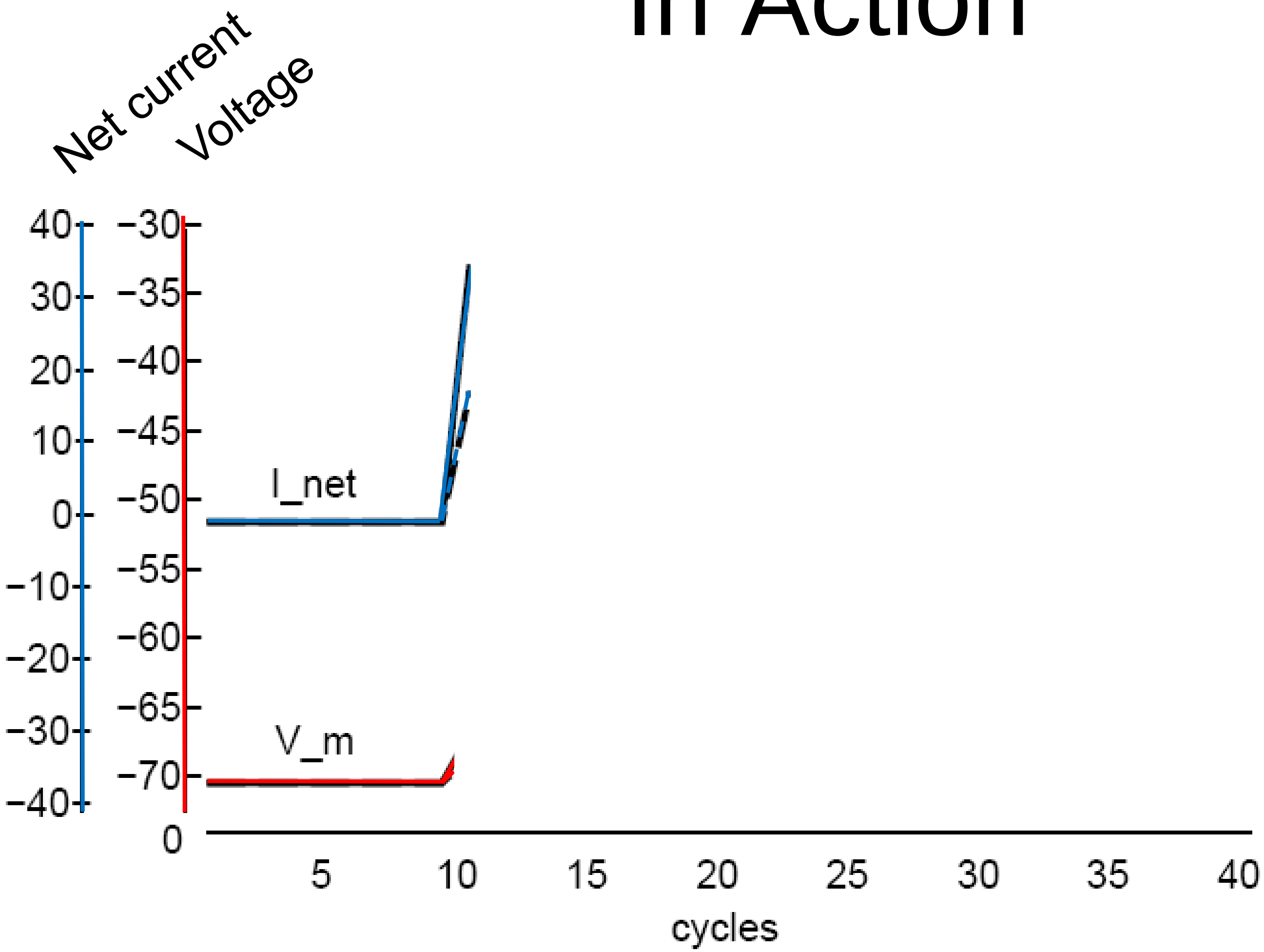


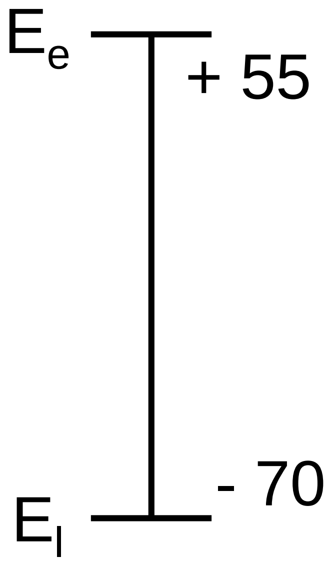


(Two excitatory inputs at time 10 and from then onward, of conductances .4 and .2)

Acknowledgement: Based on O'Reilly and Munakata's slides

In Action





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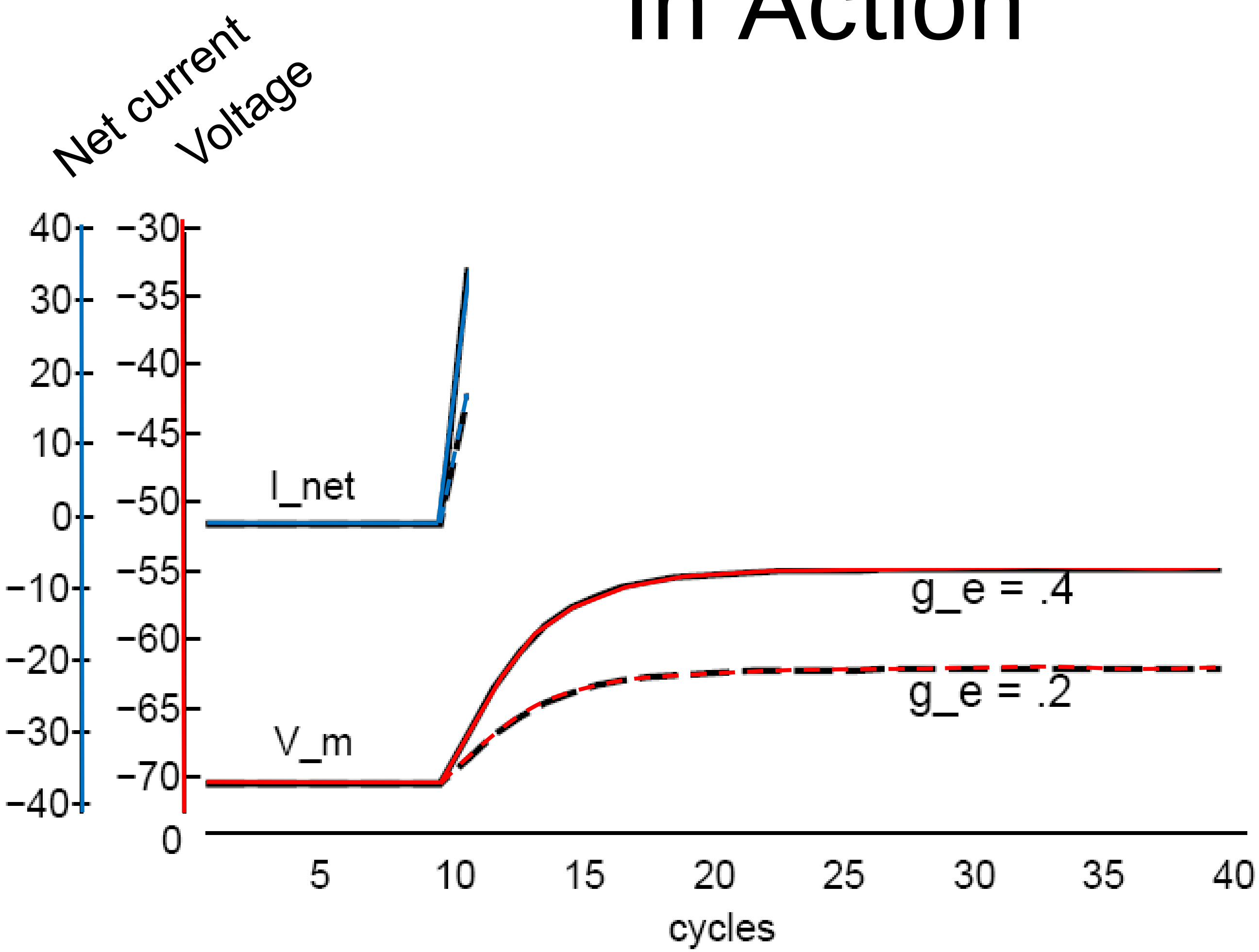

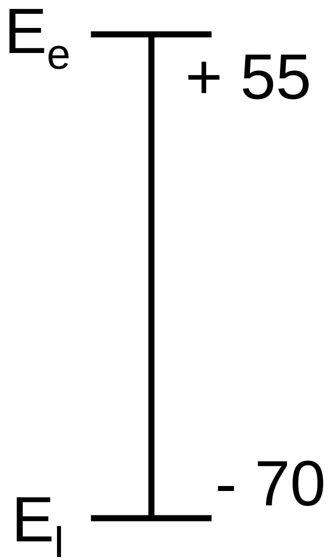

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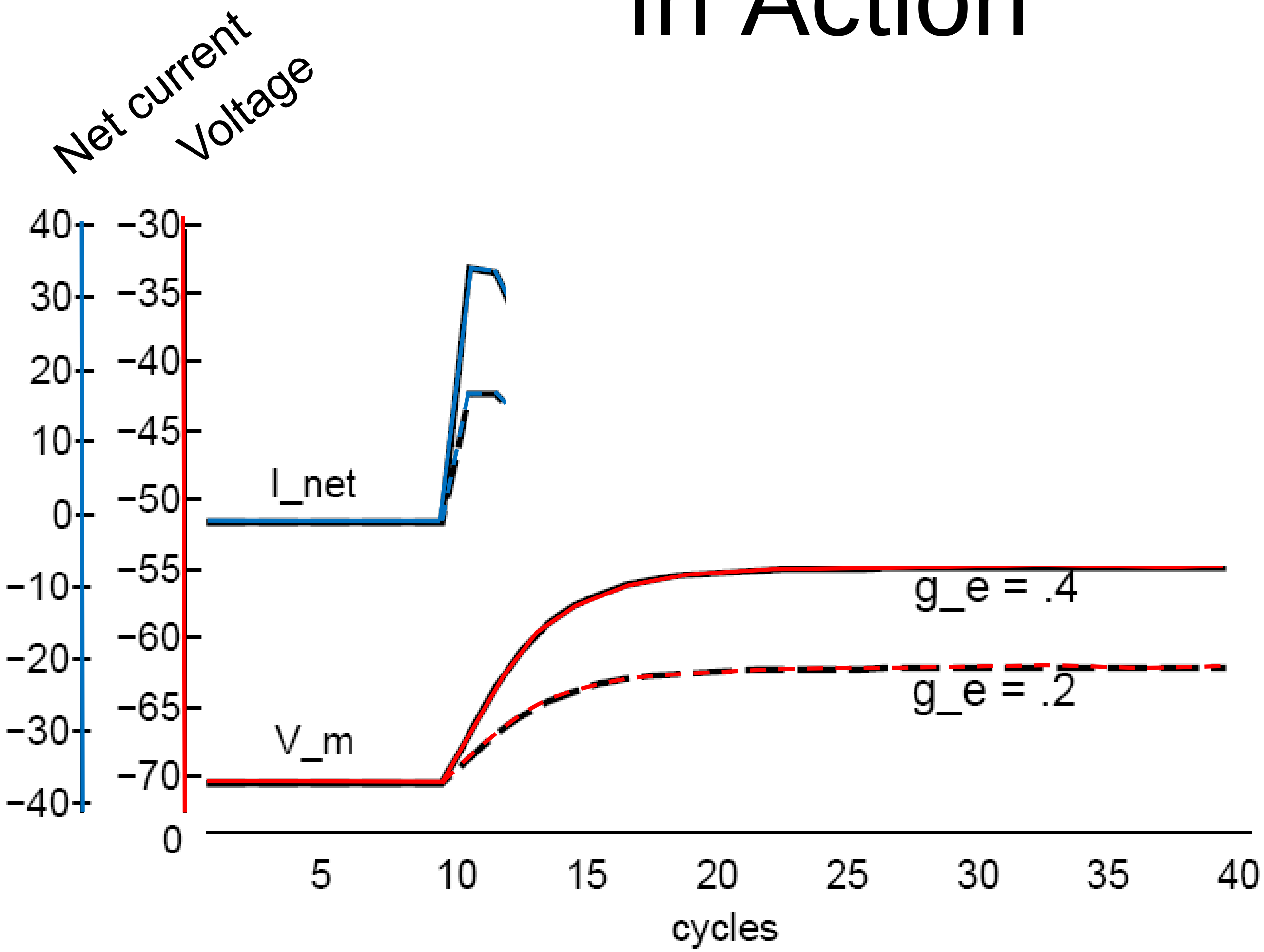

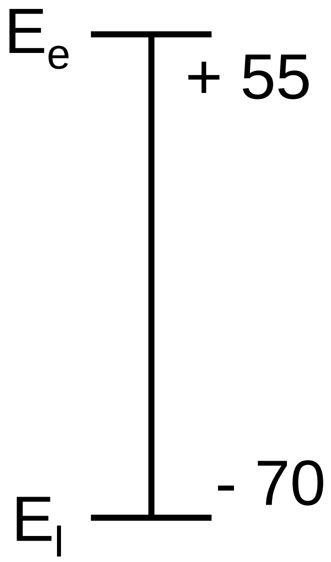

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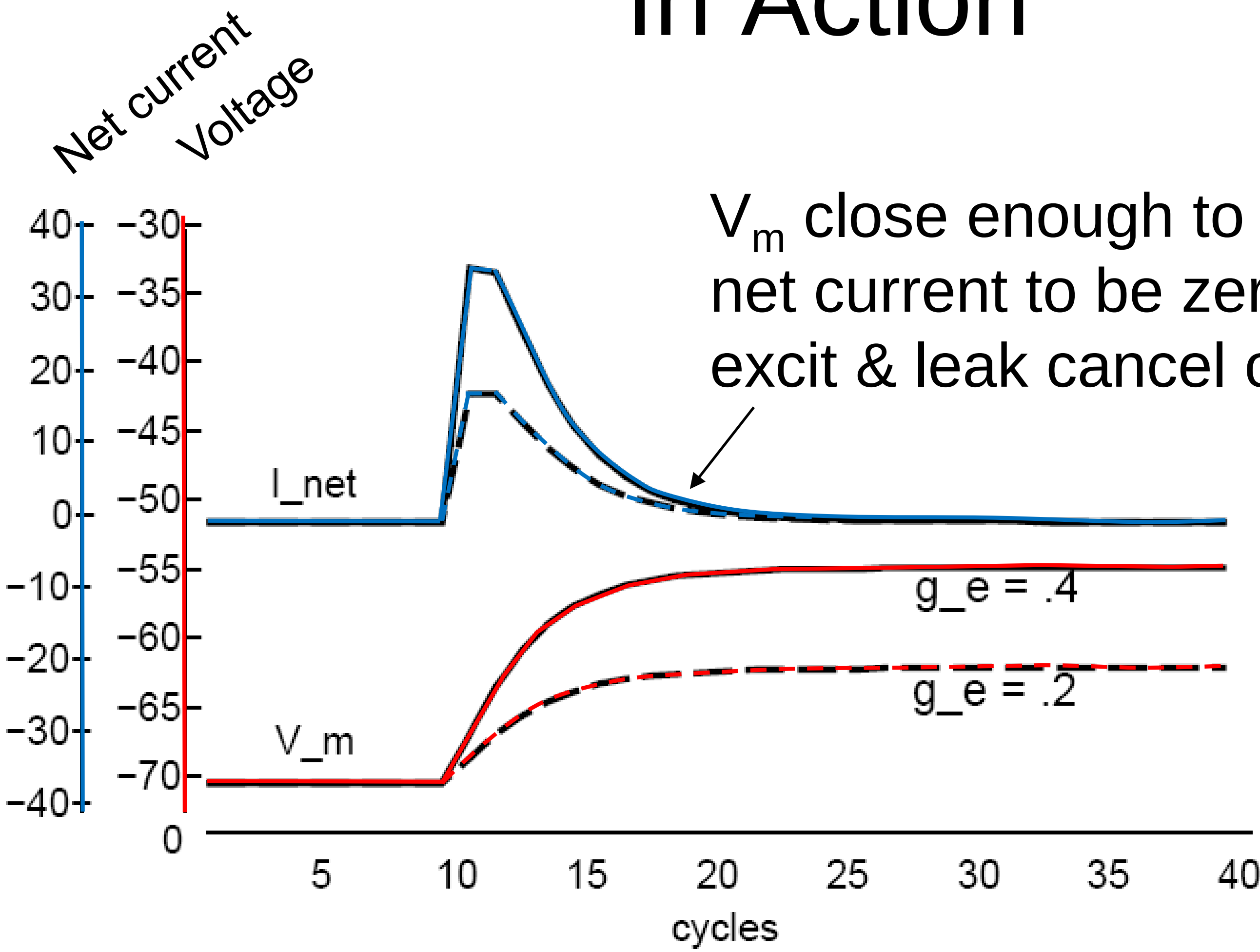
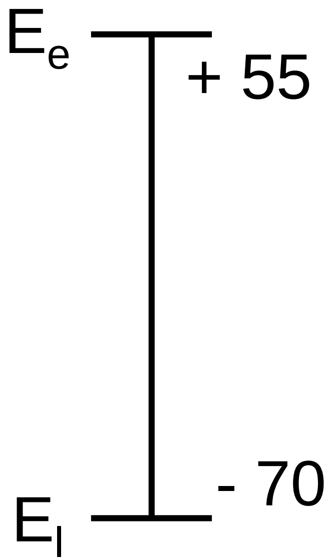


V_m close enough to E_e for net current to be zero, i.e excit & leak cancel out



(Two excitatory inputs at time 10 and from then onward, of conductances .4 and .2)

What happens to V_m and I_net when excitatory input stops?

Acknowledgement: Based on O'Reilly and Munakata's slides

Equilibrium Membrane Potential (i)

- With constant inputs, membrane potential will eventually settle into a new steady state (c.f figure on previous slide)

When will V_m have stabilised?

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- *net* current (I_{net}) will be zero (due to adjustment of V_m). Although individual currents non-zero

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$$0 = g_e(t)\,\overline{g_e}\,(E_e - V_m(t)) + g_i(t)\,\overline{g_i}\,(E_i - V_m(t)) + g_l(t)\,\overline{g_l}\,(E_l - V_m(t))$$

When will V_m have stabilised?

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When will V_m have stabilised?

How calculate resting potential from this equation?

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When will V_m have stabilised?

How calculate resting potential from this equation?

Beware: different to equilibrium channel potential

Acknowledgement: Based on O’Reilly and Munakata’s slides

Equilib Membrane Potential (ii)

If you run V_m update equations with steady inputs, neuron settles to new equilibrium potential.

$$V_m = \frac{g_e \bar{g}_e}{g_e \bar{g}_e + g_i \bar{g}_i + g_l \bar{g}_l} E_e + \frac{g_i \bar{g}_i}{g_e \bar{g}_e + g_i \bar{g}_i + g_l \bar{g}_l} E_i + \frac{g_l \bar{g}_l}{g_e \bar{g}_e + g_i \bar{g}_i + g_l \bar{g}_l} E_l$$

What does sum of 3 fractions equal?

Membrane potential moves towards equilibrium potential for a channel in proportion to the fraction that the conductance for that channel is of the total conductance.

Leak and inhibition counteracts excitation

Alternative formulation:

$$V_m = \frac{g_e \bar{g}_e E_e + g_i \bar{g}_i E_i + g_l \bar{g}_l E_l}{g_e \bar{g}_e + g_i \bar{g}_i + g_l \bar{g}_l}$$

Computational Neurons (Units)

1. Really abstract: The standard sigmoidal function

   What is a sigmoidal?

2. More neuro: The point neuron function.

   The need to abstract

3. Two kinds of outputs: (1) discrete spiking, (2) rate coded.

   Difference between (1) and (2)?

Acknowledgement: Based on O'Reilly and Munakata's slides

ANN vs Point Neurons

- Point neurons explicitly separate input channels, i.e. inhib, excit & leak.
- In ANNs, a single neuron can have both inhibitory and excitatory output links. Not justified by biology - neurons either inhibitory or excitatory
- In ANNs, weights can change sign through learning - biologically implausible.
- In point neurons, net input converted into a membrane potential - V_m, then a further function turns this into an output. In ANNs, done in one step.

Acknowledgement: Based on O'Reilly and Munakata's slides

Classic Model of Neuron, à la ANNs (Artifical Neural Nets)

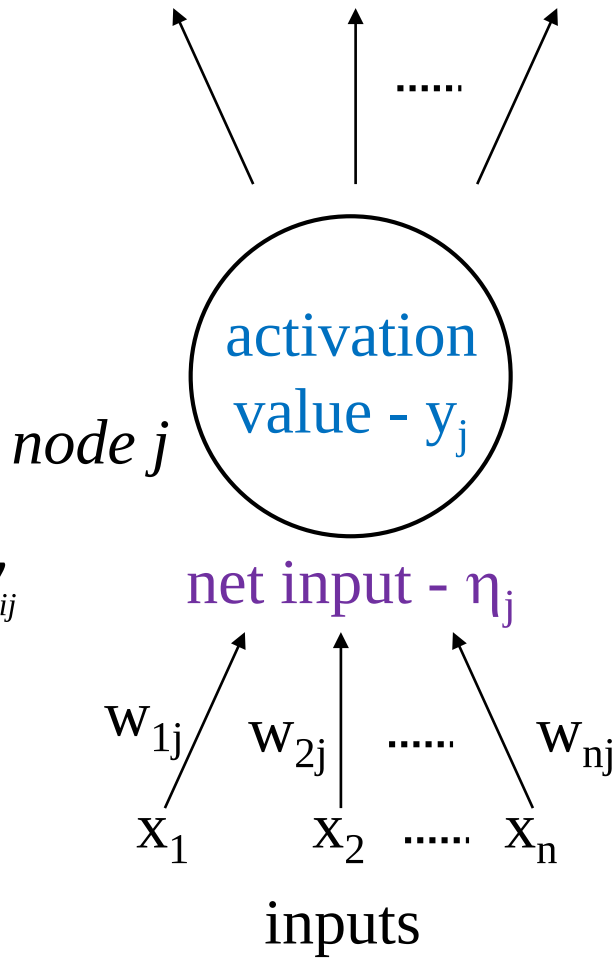
Why called a sigmoidal?

sigmoidal $y_j = \frac{1}{1+e^{-\eta_j}}$

What would result if just outputted weighted sum, i.e. without sigmoidal?

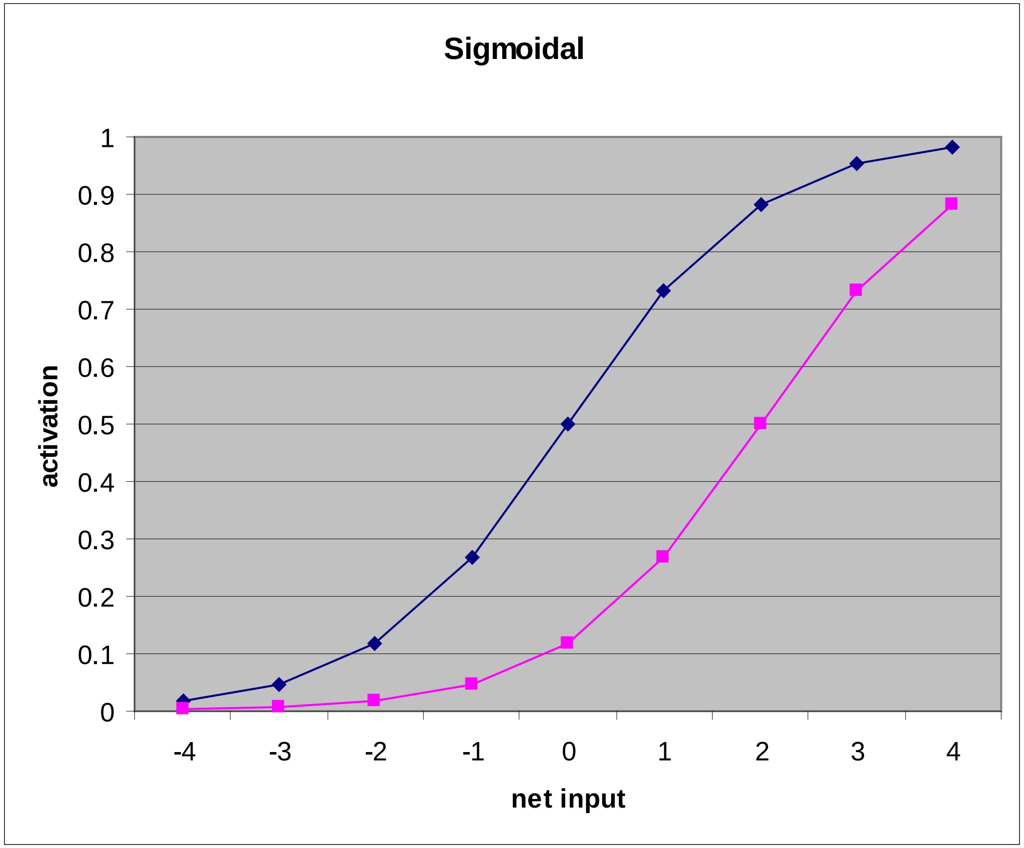
integrate (weighted sum) $\eta_j = \sum_i x_i w_{ij}$

What is the analogue of g_e here?



Acknowledgement: Based on O'Reilly and Munakata's slides

Sigmoidal Activation Function



1) Most responsive around net input of 0 (almost linear)

2) Unresponsive at extreme net input values

3) Thresholds - unresponsive at low net inputs

4) Vary threshold using:

$$y_j = \frac{1}{1+e^{-(\eta_j - c)}}$$

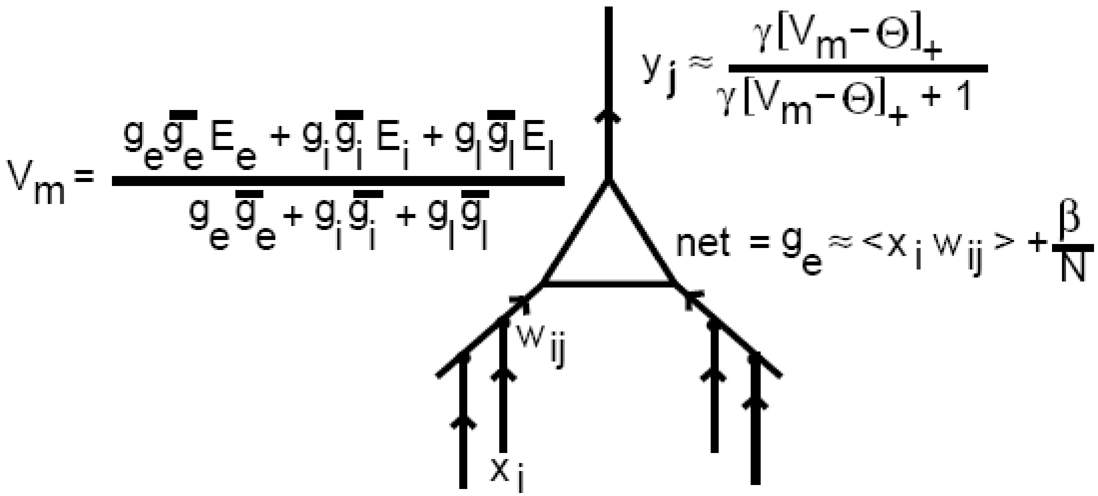
What does saturation mean?

Could a non-saturating system exist?

Why have a threshold?

Acknowledgement: Based on O'Reilly and Munakata's slides

Back to Point Neuron model: Computational Neurons (Units) Overview



- what are Θ, β and γ?
- where is g_i?
- why average in g_e?
- why divide bias by N?
- is net the net current?

1. **Weights** = synaptic efficacy; **weighted input** = $x_i w_{ij}$
2. **Net conductances** (average across all inputs) excitatory (net = $g_e(t)$), inhibitory $g_i(t)$
   * use net and $g_e(t)$ interchangeably.
3. **Integrate** conductances using V_m update equation.
4. Compute output y_j as **spikes** or **rate code**.

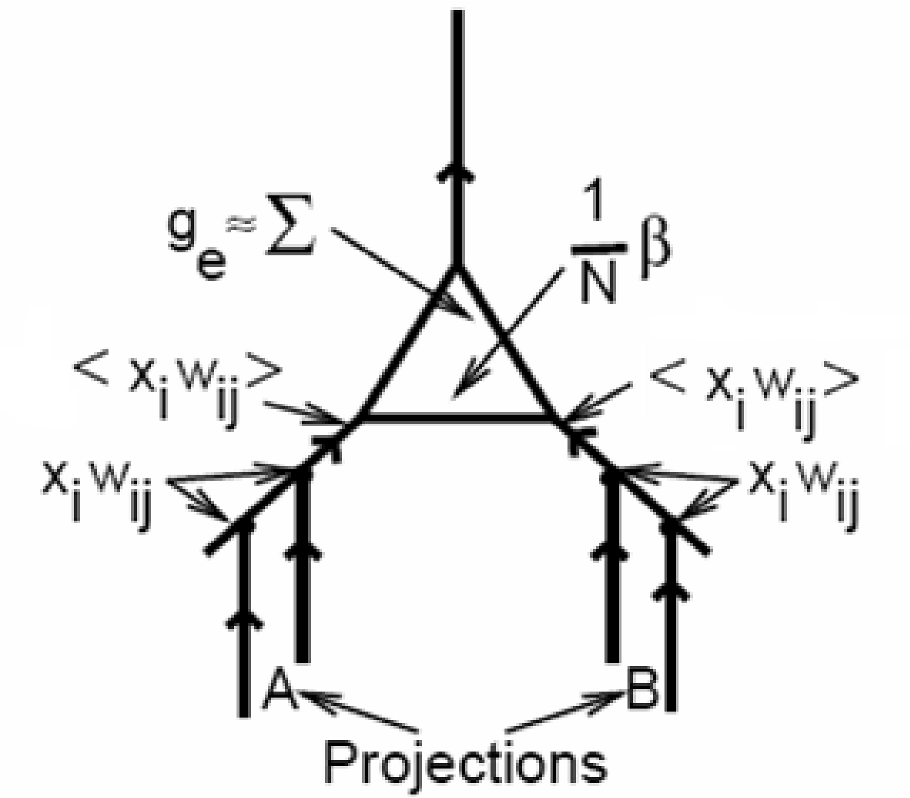
Acknowledgement: Based on O'Reilly and Munakata's slides

Computing Excitatory Conductances

1) **Average weighted inputs** $\langle x_i w_{ij} \rangle = \frac{1}{n} \sum_i x_i w_{ij}$

2) One **projection** per group (layer) of sending units — grouped at different dendritic locations.

3) **Bias weight** β: constant input — baseline difference in excitability between neurons



Acknowledgement: Based on O'Reilly and Munakata's slides

Computing Vm

Parameter	mV	(0-1)
V_{rest}	-70	0.15
$E_l\ (K^+)$	-70	0.15
$E_i\ (Cl^-)$	-70	0.15
Θ	-55	0.25
$E_e\ (Na^+)$	+55	1.00

why not rest at zero?

Normalized used by default.

Can switch between two scales.

Acknowledgement: Based on O'Reilly and Munakata's slides

Output Functions

- Output value as function of Membrane Potential
  - **spiking** - biologically more plausible
  - **rate coding** - approximation
- Spiking - discrete (1 or zero).
- Rate coding - represents the average output of a population of similarly configured spiking neurons
- In modelling, there is always a need for **abstraction**, rate coding is one such abstraction.

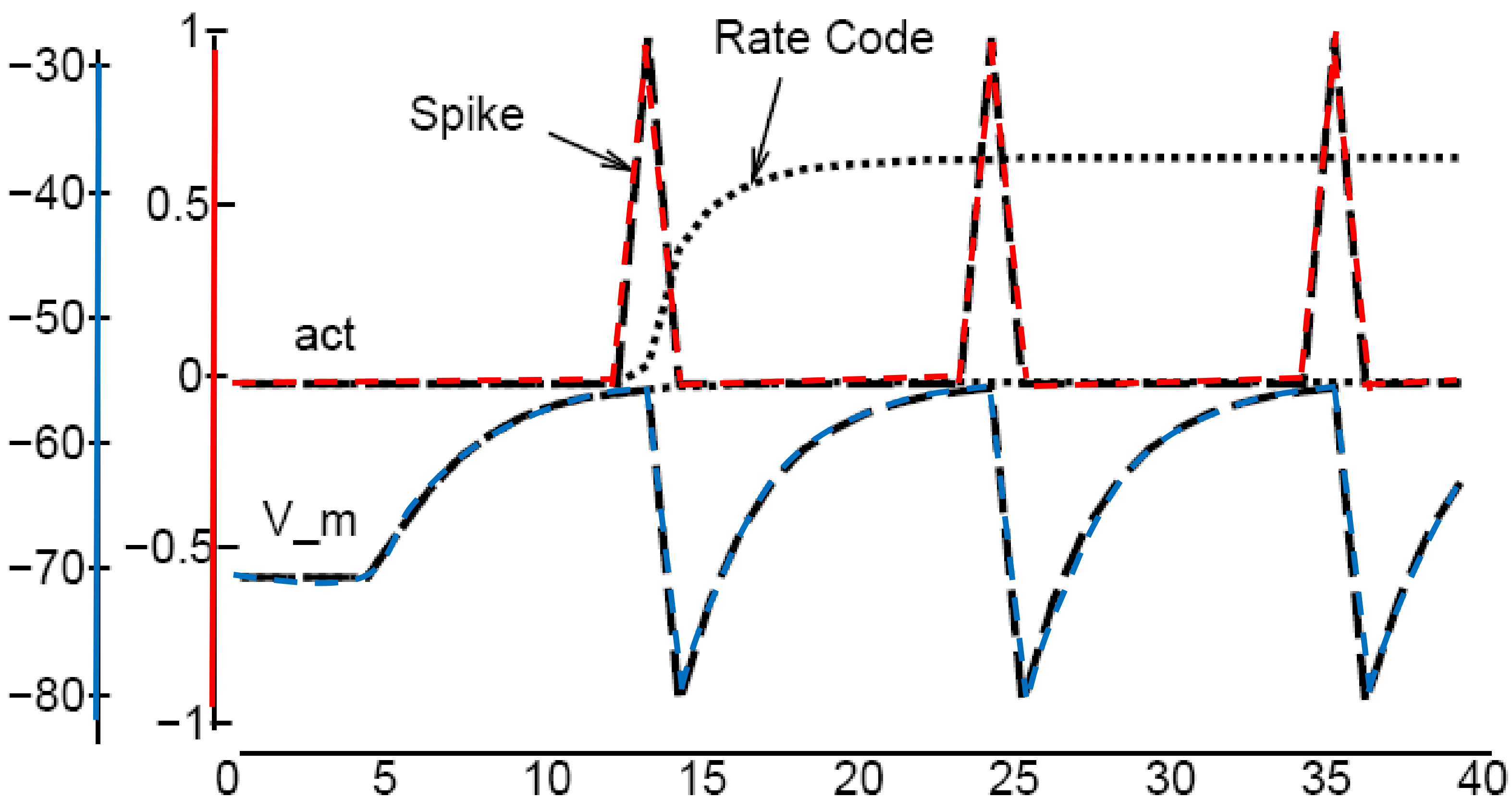
Acknowledgement: Based on O'Reilly and Munakata's slides

Refractory Period

- **suppression of neuron** (below resting levels) following firing (through opening of inhibitory channels)
- conceptually, after a spike, neuron **overshoots** when returning to baseline/ resting
- neurons **unresponsive** for a period following a spike
- **basic firing rate constraints** from here & **saturation** of sigmoidal activation function

Acknowledgement: Based on O'Reilly and Munakata's slides

Thresholded Spike Outputs



Constant excitatory input from time step 5.

In model $y_j = 1$ if $V_m > \Theta$, then reset (also keep track of rate).

What membrane potential spike at?

What happens to rate code if excitation lower or higher?

Acknowledgement: Based on O'Reilly and Munakata's slides

Rate Coded Output

Output is average firing rate value.

One unit = % spikes in population of neurons?

Rate approximated by X-overX-plus-1

$$\frac{x}{x+1}$$

Acknowledgement: Based on O'Reilly and Munakata's slides

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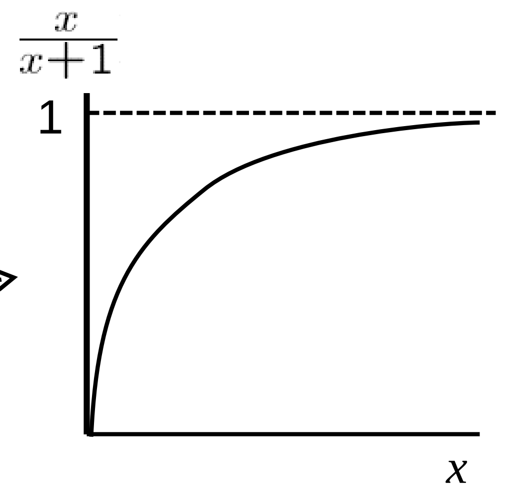

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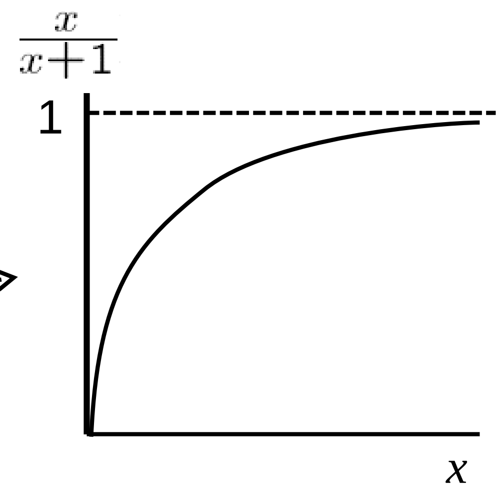

Enforcing a threshold gives,

$$y_j = \frac{\gamma[V_m(t) - \Theta]_+}{\gamma[V_m(t) - \Theta]_+ + 1}$$

$[Z]_+ =$ *if* $Z>0$ *then* Z *else* 0

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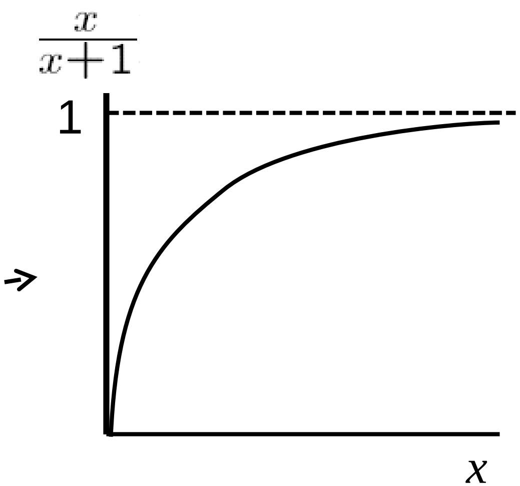

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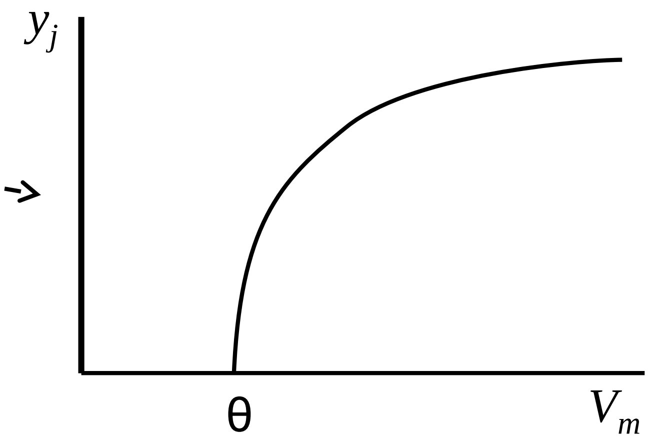


$[Z]_+ =$ if $Z>0$ then Z else 0

What is the role of gamma, γ ?

Acknowledgement: Based on O'Reilly and Munakata's slides

Sigmoidals

X-over-X-plus-1 function,

$$y_j = \frac{\gamma[V_m(t) - \Theta]_+}{\gamma[V_m(t) - \Theta]_+ + 1}$$

which is like a sigmoidal function:

$$y_j \approx \frac{1}{1 + (\gamma[V_m(t) - \Theta]_+)^{-1}}$$

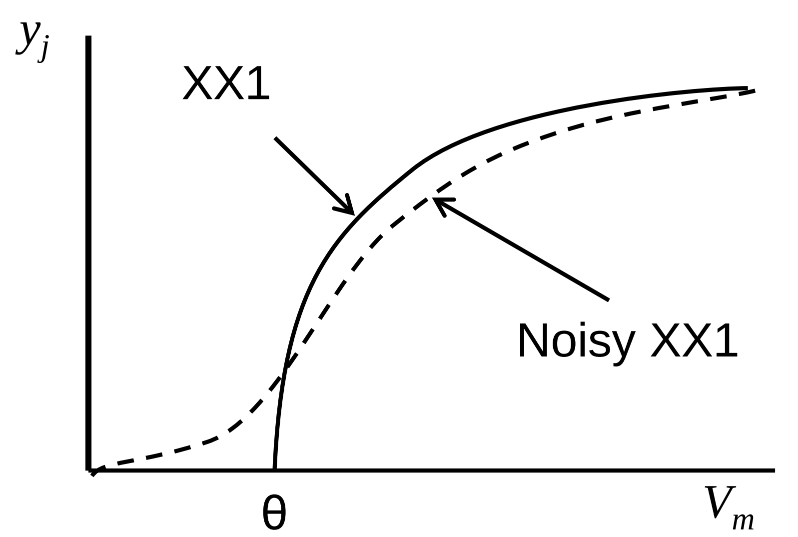
compare to logistic sigmoidal:

$$y_j \approx \frac{1}{1+e^{-\eta_j}}$$

Acknowledgement: Based on O'Reilly and Munakata's slides

Problem with XX1 function

- Even within a single neuron there will be **random variability** in placement of threshold
- Rate code used to model *populations* of neurons
- There will be **variability in threshold placement** across neurons in a population
- Reflect this variability by **convolving** XX1 with normally distributed noise.



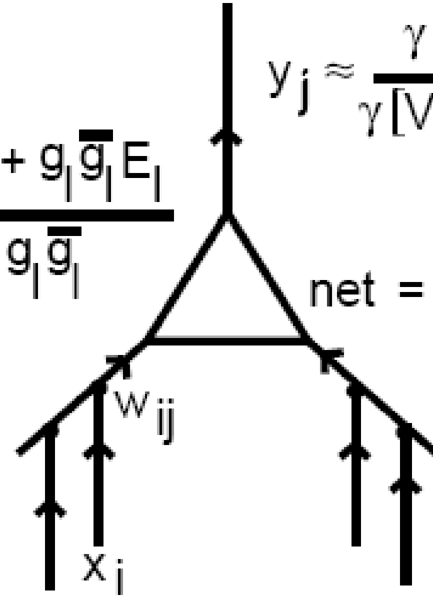
Acknowledgement: Based on O'Reilly and Munakata's slides

Summary

$$V_m = \frac{g_e \bar{g}_e E_e + g_i \bar{g}_i E_i + g_l \bar{g}_l E_l}{g_e \bar{g}_e + g_i \bar{g}_i + g_l \bar{g}_l}$$

$$y_j \approx \frac{\gamma [V_m - \Theta]_+}{\gamma [V_m - \Theta]_+ + 1}$$

$$net = g_e \approx \langle x_i w_{ij} \rangle + \frac{\beta}{N}$$



$$\begin{aligned} V_m(t+1) = V_m(t) + dt_{vm}[\\ & g_e(t)\overline{g_e}(E_e - V_m(t)) + \\ & g_i(t)\overline{g_i}(E_i - V_m(t)) + \\ & g_l(t)\overline{g_l}(E_l - V_m(t))] \end{aligned}$$

Convolving XX1 to get Noisy XX1

Acknowledgement: Based on O'Reilly and Munakata's slides