

Erratum: Pseudo-Hermiticity for a class of nondiagonalizable Hamiltonians [J. Math. Phys. 43, 6343 (2002)]

Ali Mostafazadeh*

Department of Mathematics, Koç University,
Rumelifeneri Yolu, 34450 Sariyer, Istanbul, Turkey

Recently, the authors of [1] used the framework provided in [2] to re-examine the consequences of pseudo-Hermiticity for the class of block-diagonalizable Hamiltonians introduced in [2]. In doing so, they discovered that Theorem 2 of [2] did not hold, as they could find a counter-example. This theorem must be replaced with the following.

Theorem 2: Let H be as in Theorem 1 of Ref. [2]. Then H is pseudo-Hermitian if and only if it is Hermitian with respect to an inner product $\langle\langle \ , \ \rangle\rangle$ that supports a positive-semidefinite basis [3] including the eigenvectors of H . In particular, for every eigenvector ψ of H , $\langle\langle \psi | \psi \rangle\rangle \geq 0$; if the corresponding eigenvalue is real and nondefective (algebraic and geometric multiplicities are equal), $\langle\langle \psi | \psi \rangle\rangle > 0$; otherwise $\langle\langle \psi | \psi \rangle\rangle = 0$.

Proof: As shown in [2], pseudo-Hermiticity of H implies that H is Hermitian with respect to the inner product $\langle\langle \ , \ \rangle\rangle_\eta$ with η given by Eq. (15) of [2] and $\sigma_{\nu_0,a} = 1$. It is not difficult to check that indeed the basis vectors $|\psi_n, a, i\rangle$, constructed in [2], have the property that $\langle\langle \psi_n, a, i | \psi_n, a, i \rangle\rangle \geq 0$, and that $\langle\langle \psi_n, a, i | \psi_n, a, i \rangle\rangle > 0$ only for the cases that $p_{n,a} = 1$ and $E_n \in \mathbb{R}$, i.e., $|\psi_n, a, i = 1\rangle$ is an eigenvector of H with a real eigenvalue. Furthermore, by construction, this basis includes all the eigenvectors of H . The proof of the converse is the same as the one given in [2].

*E-mail address: amostafazadeh@ku.edu.tr

It is important to note that having a positive-semidefinite basis does not imply that the inner product $\langle\langle \ , \ \rangle\rangle_\eta$ is positive-semidefinite (unless the Hamiltonian is diagonalizable and has a real spectrum in which case both the basis and the inner product $\langle\langle \ , \ \rangle\rangle_\eta$ are positive-definite [4].) If the Hamiltonian has defective or complex(-conjugate pair(s) of) eigenvalues, there will always be at least two null vectors with negative [3] linear combinations. Unlike positive vectors, linear combinations of nonnegative vectors need not be nonnegative.

References

- [1] G. Sclarici and L. Solombrino, “On the pseudo-Hermitian nondiagonalizable Hamiltonians,” arXiv: quant-ph/0211161.
- [2] A. Mostafazadeh, J. Math. Phys., **43**, 6343 (2002).
- [3] A vector ϕ is respectively said to be positive, null (zero), negative, if $\langle\langle\phi|\phi\rangle\rangle > 0$, $\langle\langle\phi|\phi\rangle\rangle = 0$, $\langle\langle\phi|\phi\rangle\rangle < 0$. It is said to be nonnegative if $\langle\langle\phi|\phi\rangle\rangle \geq 0$. A basis is called positive-semidefinite if it consists of nonnegative vectors. See, for example, J. Bognar, “Indefinite inner product spaces,” Springer, Berlin, 1974.
- [4] A. Mostafazadeh, J. Math. Phys., **43**, 2814 (2002).