Лаб_10

11.11

Варіант 5

i	0	1	2	3	4
x_i	0,2	0,6	1,1	1,8	2,6
y_i	3,34	4,53	2,75	3,91	3,57

$[0,2; 0,6]; [0,6; 1,1]; [1,1; 1,8]; [1,8; 2,6]$

$h_1 = x_1 - x_0 = 0,6 - 0,2 = 0,4$

$h_2 = x_2 - x_1 = 1,1 - 0,6 = 0,5$

$h_3 = x_3 - x_2 = 1,8 - 1,1 = 0,7$

$h_4 = x_4 - x_3 = 2,6 - 1,8 = 0,8$

$S_1(x) = a_1 + b_1(x - x_0) + c_1(x - x_0) + d_1(x - x_0)^3$

$S_2(x) = a_2 + b_2(x - x_1) + c_2(x - x_1) + d_2(x - x_1)^3$

$S_3(x) = a_3 + b_3(x - x_2) + c_3(x - x_2) + d_3(x - x_2)^3$

$S_4(x) = a_4 + b_4(x - x_3) + c_4(x - x_3) + d_4(x - x_3)^3$

$a_1 = y_0 = 3,34$

$a_2 = y_1 = 4,53$

$a_3 = y_2 = 2,75$

$a_4 = y_3 = 3,91$

i	x_i	h_i	y_i	$\frac{y_i - y_{i-1}}{x_i - x_{i-1}}$	$M_i = 2(h_{i-1} + h_i)$	$k_i = 3\left[\frac{y_i - y_{i-1}}{h_i} - \frac{y_{i-1} - y_{i-2}}{h_{i-1}}\right]$
0	0,2	–	3,34	2,975	–	–
1	0,6	0,4	4,53	-3,56	–	–
2	1,1	0,5	2,75	1,6571	1,8	-19,605
3	1,8	0,7	3,91	-0,425	2,4	15,651
4	2,6	0,8	3,57		3	-6,2464

Где $M_2 = 2(h_{2-1} + h_2) = 2 \cdot (h_1 + h_2) = 2 \cdot (0,4 + 0,5) = 1,8$ (0,9)

$M_3 = 2(h_{3-1} + h_3) = 2 \cdot (h_2 + h_3) = 2 \cdot (0,5 + 0,7) = 2,4$ (1,2)

$M_4 = 2(h_{4-1} + h_4) = 2 \cdot (h_3 + h_4) = 2 \cdot (0,8 + 0,7) = 3$ (1,5)

Где $k_3 = 3\left(\frac{y_3 - y_{3-1}}{h_3} - \frac{y_{3-1} - y_{3-2}}{h_{3-1}}\right) = 3 \cdot \left(\frac{y_3 - y_2}{h_3} - \frac{y_2 - y_1}{h_2}\right) =$ (1,16; -1,78)

$= 3 \cdot \left(\frac{3,91 - 2,75}{0,7} - \frac{2,75 - 4,53}{0,5}\right) = 3 \cdot \left(\frac{1,16}{0,7} - \frac{-1,78}{0,5}\right) = 3 \cdot (1,657 + 3,56) =$ (5,217)

$= 15,651$

$k_2 = 3 \cdot \left(\frac{y_2 - y_{2-1}}{h_2} - \frac{y_{2-1} - y_{2-2}}{h_{2-1}}\right) = 3 \cdot \left(\frac{y_2 - y_1}{h_2} - \frac{y_1 - y_0}{h_1}\right) =$ (-1,78; 1,19)

$= 3 \cdot \left(\frac{2,75 - 4,53}{0,5} - \frac{4,53 - 3,34}{0,4}\right) = 3 \cdot (-3,56 - 2,975) = -19,605$ (-5,535)

$k_4 = 3 \cdot \left(\frac{y_4 - y_{4-1}}{h_4} - \frac{y_{4-1} - y_{4-2}}{h_{4-1}}\right) = 3 \cdot \left(\frac{y_4 - y_3}{h_4} - \frac{y_3 - y_2}{h_3}\right) =$ (-0,34; 1,16)

$= 3 \cdot \left(\frac{3,57 - 3,91}{0,8} - \frac{3,91 - 2,75}{0,7}\right) = 3 \cdot (-0,425 - 1,6571) = -6,2464$ (-2,082)

$\alpha_1 = \beta_1 = 0$ Прямий хід

$\alpha_2 = \frac{k_2}{m_2} = \frac{-19,605}{1,8} = -10,8916$

$\beta_2 = \frac{h_2}{m_2} = \frac{0,5}{1,8} = 0,2777$

$\alpha_3 = \frac{k_3 - h_2\alpha_2}{m_3 - h_2\beta_2} = \frac{15,651 - 0,5\cdot(-10,8916)}{2,4 - 0,5\cdot 0,2777} = \frac{21,0968}{2,26115} =$

$= 9,33$

$\beta_3 = \frac{h_3}{m_3 - h_2\beta_2} = \frac{0,7}{2,4 - 0,5\cdot 0,2777} = \frac{0,7}{2,26115} = 0,3095$

$\alpha_4 = \frac{k_4 - h_3\alpha_3}{m_4 - h_3\beta_3} = \frac{-6,2464 - 0,7\cdot 9,33}{3 - 0,7\cdot 0,3095} = \frac{-12,7774}{2,78335} =$

$= -4,5906$

$\beta_4 = \frac{h_4}{m_4 - h_3\beta_3} = \frac{0,8}{3 - 0,7\cdot 0,3095} = \frac{0,8}{2,78335} = 0,2874$

Зворотний хід

$c_5 = 0$

$c_4 = \alpha_4 - \beta_4 c_5 = \alpha_4 = -4,5906$

$c_3 = \alpha_3 - \beta_3 c_4 = 9,33 - 0,3095\cdot(-4,5906) = 10,75$

$c_2 = \alpha_2 - \beta_2 c_3 = -10,8916 - 0,2777\cdot 10,75 = -13,8768$

$c_1 = \alpha_1 - \beta_1 c_2 = 0$

$$d_i = \frac{c_{i+1} - c_i}{3h_i}$$

$$d_4 = \frac{c_5 - c_4}{3h_4} = \frac{0 - (-4,5906)}{3 \cdot 0,8} = 1,91275$$
$2,4$

$$d_3 = \frac{c_4 - c_3}{3h_3} = \frac{-4,5906 - 10,75}{3 \cdot 0,7} = \frac{-15,3406}{2,1} = -7,3$$

$$d_2 = \frac{c_3 - c_2}{3h_2} = \frac{10,75 - (-13,8768)}{3 \cdot 0,5} = 16,4178$$
$1,5$

$$d_1 = \frac{c_2 - c_1}{3h_1} = \frac{-13,8768 - 0}{3 \cdot 0,4} = -11,564$$
$1,2$

$$b_i = \frac{y_i - y_{i-1}}{h_i} - \frac{(c_{i+1} + 2c_i)h_i}{3}$$

$$b_4 = \frac{y_4 - y_3}{h_4} - \frac{(c_5 + 2c_4)h_4}{3} = \frac{3,57 - 3,91}{0,8} - \frac{(0 + 2 \cdot (-4,5906)) \cdot 0,8}{3} =$$

$$= -0,425 + 2,44832 = 2,02332$$

$$b_3 = \frac{y_3 - y_2}{h_3} - \frac{(c_4 + 2c_3)h_3}{3} = \frac{3,91 - 2,75}{0,7} - \frac{(-4,5906 + 2 \cdot 10,75) \cdot 0,7}{3} =$$

$$= 1,657 - 3,945 = -2,288$$

$$b_2 = \frac{y_2 - y_1}{h_2} - \frac{(c_3 + 2c_2)h_2}{3} = \frac{2,75 - 4,53}{0,5} - \frac{(10,75 + 2 \cdot (-13,8768)) \cdot 0,5}{3} =$$

$$= -3,56 + 2,8339 = -0,7261$$

$$b_1 = \frac{y_1 - y_0}{h_1} - \frac{(c_2 + 2c_1)h_1}{3} = \frac{4,53 - 3,34}{0,4} - \frac{(-13,8768 + 2 \cdot 0) \cdot 0,4}{3} =$$

$$= 2,975 + 1,85 = 4,825$$

$$P_3(x) = \begin{cases} 3,34 + 4,825(x-0,2) + 0\cdot(x-0,2)^2 - 11,564(x-92)^3 \\ 4,53 - 0,7261(x-0,6) - 13,8768\cdot(x-0,6)^2 + 16,4178(x-0,6)^3 \\ 2,75 - 2,288(x-1,1) + 10,75\cdot(x-1,1)^2 - 7,3(x-1,1)^3 \\ 3,91 + 2,02832(x-1,8) - 4,5906\cdot(x-1,8)^2 + 1,91275(x-1,8)^3 \end{cases}$$

i	x	y	$S_i(x)$	$S_{i+1}(x)$	$S_i'(x)$	$S_{i+1}'(x)$	$S_i''(x)$	$S_{i+1}''(x)$
0	0,2	3,34	3,34	—	4,825	—	0	—
1	0,6	4,53	4,53	4,5299	-0,7263	-0,72572	-27,7535	-27,7536
2	1,1	2,75	2,75	2,7499	-2,288	-2,2895	21,5	21,4938
3	1,8	3,91	3,91	3,912	2,023	2,031	-9,1812	-9,16
4	2,6	3,57	—	3,57	—	-1,6491	—	0

$S_i(x)_0 = S_i(0,2) = 3,34 + 4,825(0,2-0,2) + 0\cdot(0,2-0,2)^2 - 11,564(0,2-0,2)^3 = 3,34$

$S_i(x)_1 = S_i(0,6) = 4,53 - 0,7261(0,6-0,6) - 13,8768\cdot(0,6-0,6)^2 + 16,4178(0,6-0,6)^3 = 4,53$

$S_i(x_2) = S_i(1,1) = 2,75 - 2,288(1,1-1,1) + 10,75\cdot(1,1-1,1)^2 - 7,3(1,1-1,1)^3$

$S_i(x)_3 = S_i(1,8) = 3,91 + 2,02332(1,8-1,8) - 4,5906\cdot(1,8-1,8)^2 + 1,91275(1,8-1,8)^3$

$S_{i+1}(x)_1 = S_{i+1}(0,6) = 3,34 + 4,825(0,6-0,2) + 0\cdot(0,6-0,2)^2 - 11,564(0,6-0,2)^3 = 4,5299$

$S_{i+1}(x)_2 = S_{i+1}(1,1) = 4,53 - 0,7261(1,1-0,6) - 13,8768\cdot(1,1-0,6)^2 + 16,4178(1,1-0,6)^3 =$
$= 2,7499$

$S_{i+1}(x)_3 = S_{i+1}(1,8) = 2,75 - 2,288(1,8-1,1) + 10,75\cdot(1,8-1,1)^2 - 7,3(1,8-1,1)^3 = 3,912$

$S_{i+1}(x)_4 = S_{i+1}(2,6) = 3,91 + 2,02332(2,6-1,8) - 4,5906\cdot(2,6-1,8)^2 + 1,91275\times$
$\times(2,6-1,8)^3 = 3,57$

$S_i'(x)_0 = f'(3,34 + 4,825 \cdot (x-0,2) + 0 \cdot (x-0,2)^2 - 11,564 \cdot (x-0,2)^3) =$

$= (4,825 \cdot (x-0,2))' + ((-11,564 \cdot (x-0,2)^3)' = 4,825 \cdot (x-0,2)' -$

$- 11,564 \cdot ((x-0,2)^3)' = 4,825 \cdot x' - 34,692 \cdot (x-0,2)' \cdot (x-0,2)^2 =$

$= 4,825 - 34,692 \cdot x' \cdot (x-0,2)^2 = 4,825 - 34,692 \cdot (x-0,2)^2$

$S_i''(x)_0 = 13,8768 - 69,384 \cdot x$

$S_i'(0,2) = 4,825 - 34,692 \cdot (0,2 - 0,2)^2 = 4,825$

$S_i''(0,2) = 13,8768 - 69,384 \cdot 0,2 = 0$

$S_{i+1}'(0,6) = 4,825 - 34,692 \cdot (0,6 - 0,2)^2 = -0,72572$

$S_i'(x)_1 = (4,53 - 0,7261 \cdot (x-0,6) - 13,8768 \cdot (x-0,6)^2 + 16,4178 \cdot$

$\times (x-0,6)^3)' = (-0,7261 \cdot (x-0,6))' + (-13,8768 \cdot (x-0,6)^2)' +$

$+ 16,4178 \cdot (x-0,6)^3)' = 16,4178 \cdot ((x-0,6)^3)' - 0,7261 \cdot (x-0,6)' -$

$- 13,8768 \cdot ((x-0,6)^2)' = 49,2534 \cdot (x-0,6)' \cdot (x-0,6)^2 - 0,7261 \cdot x' -$

$- 27,7536 \cdot (x-0,6) \cdot (x-0,6)' = 49,2534 \cdot x' \cdot (x-0,6)^2 - 27,7536 \cdot x' \cdot$

$\times (x-0,6) - 0,7261 = 49,2534 \cdot (x-0,6)^2 - 27,7536 \cdot (x-0,6) - 0,7261 =$

$= (x-0,6) \cdot (49,2534 \cdot (x-0,6) - 27,7536) - 0,7261 = x \cdot (49,2534 \cdot x -$

$- 57,30564) - 0,6 \cdot (49,2534 \cdot x - 57,30564) - 0,7261 =$

$= 49,2534 \cdot x^2 - 86,8576 \cdot x + 33,6572$

$S_i''(x)_1 = 98,5068 \cdot x - 86,8576$

$S'_i(0,6) = 49,2534 \cdot x^2 - 86,8576 \cdot x + 33,6572 =$

$= 49,2534 \cdot 0,6^2 - 86,8576 \cdot 0,6 + 33,6572 = -0,7263$

$S''_i(0,6) = 98,5068 \cdot 0,6 - 86,8576 = -27,7535$

$S''_{i+1}(0,6) = 13,8768 - 69,384 \cdot 0,6 = -27,7536$

$S'_{i+1}(1,1) = 49,2534 \cdot 1,1^2 - 86,8576 \cdot 1,1 + 33,6572 =$

$= -2,2895$

$S''_{i+1}(1,1) = 98,5068 \cdot 1,1 - 86,8576 = 21,4998$

$S'_i(x)_2 = (2,75 - 2,288 \cdot (x-1,1) + 10,75 \cdot (x-1,1)^2 - 7,3 \cdot (x-1,1)^3)' =$

$= (-2,288 \cdot (x-1,1))' + (10,75 \cdot (x-1,1)^2)' + (-7,3 \cdot (x-1,1)^3)' =$

$= 10,75 \cdot ((x-1,1)^2)' - 2,288 \cdot (x-1,1)' - 7,3 \cdot ((x-1,1)^3)' =$

$= 21,5 (x-1,1) \cdot (x-1,1)' - 2,288 \cdot x' - 21,9 \cdot (x-1,1)' \cdot (x-1,1)^2 =$

$= 21,5 \cdot x' \cdot (x-1,1) - 21,9 \cdot x' \cdot (x-1,1)^2 - 2,288 = 21,5 \cdot (x-1,1) -$

$- 21,9 \cdot (x-1,1)^2 - 2,288 = (x-1,1) \cdot (21,5 - 21,9 \cdot (x-1,1)) - 2,288 =$

$= x \cdot (45,59 - 21,9 \cdot x) - 1,1 \cdot (45,59 - 21,9 \cdot x) - 2,288 =$

$= 69,68 \cdot x - 21,9 \cdot x^2 - 52,437$

$S''_i(x)_2 = 69,68 - 43,8 \cdot x$

$S'_i(1,1) = 69,68 \cdot 1,1 - 21,9 \cdot 1,1^2 - 52,437 = -2,288$

$S''_i(1,1) = 69,68 - 43,8 \cdot 1,1 = 21,5$

$S'_{i+1}(1,8) = 69,68 \cdot 1,8 - 21,9 \cdot 1,8^2 - 52,437 = 2,031$

$S''_{i+1}(1,8) = 69,68 - 43,8 \cdot 1,8 = -9,16$

$S_i'(x)_3 = (3,91 + 2,02332 \cdot (x-1,8) - 4,5906 \cdot (x-1,8)^2 + 1,91275 \cdot$

$\cdot (x-1,8)^3)' = (2,02332 \cdot (x-1,8))' + (-4,5906 \cdot (x-1,8)^2)' +$

$+ (1,91275 \cdot (x-1,8)^3)' = 2,02332 \cdot (x-1,8)' - 4,5906 \cdot ((x-1,8)^2)' +$

$+ 1,91275 \cdot ((x-1,8)^3)' = 2,02332 \cdot x' - 9,1812 \cdot (x-1,8) \cdot (x-1,8)' + 5,73825$

$\cdot (x-1,8) \cdot (x-1,8)^2 = 5,73825 \cdot x' \cdot (x-1,8)^2 - 9,1812 \cdot x' \cdot (x-1,8) +$

$+ 2,02332 = 5,73825 \cdot (x-1,8)^2 - 9,1812 \cdot (x-1,8) + 2,02332 =$

$= (x-1,8) \cdot (5,73825 \cdot (x-1,8) - 9,1812) + 2,023328 = x \cdot 15,73825x -$

$- 19,51005) - 1,8 \cdot (5,73825 \cdot x - 19,51005) + 2,02332 =$

$= 5,73825 \cdot x^2 - 29,8389 \cdot x + 37,14141$

$S_i''(x)_3 = 11,4765 \cdot x - 29,8389$

$S_i'(1,8) = 5,73825 \cdot 1,8^2 - 29,8389 \cdot 1,8 + 37,14141 = 2,023$

$S_i''(1,8) = 11,4765 \cdot 1,8 - 29,8389 = -9,1812$

$S_{i+1}'(2,6) = 5,73825 \cdot 2,6^2 - 29,8389 \cdot 2,6 + 37,14141 = -1,6491$

$S_{i+1}''(2,6) = 11,4765 \cdot 2,6 - 29,8389 = 0$