Лаб 11

Варіант 5

$f(x) = e^{\sin(5x)}$

Скоротимо $e^{\sin(5x)}$ до $e^{\sin(x)}$

$f'(x) = \left(e^{\sin(x)}\right)' = (\sin(x))' \cdot e^{\sin(x)} = \cos(x) \cdot e^{\sin(x)}$

$f''(x) = f'(\cos(x) \cdot e^{\sin(x)}) = (\cos(x))' \cdot e^{\sin(x)} + \cos(x) \cdot$
$\cdot (e^{\sin(x)})' = \cos(x) \cdot (\sin(x))' \cdot e^{\sin(x)} \cdot (-\sin(x)) \cdot e^{\sin(x)} =$
$= \cos^2(x) \cdot e^{\sin(x)} \cdot (-\sin(x)) \cdot e^{\sin(x)} = e^{\sin(x)} \cdot (\cos^2(x) -$
$- \sin(x))$

$f'''(x) = \left(e^{\sin(x)} \cdot (\cos^2(x) - \sin(x))\right)' = \left(e^{\sin(x)}\right)' \cdot (\cos^2(x) - \sin(x)) +$
$+ e^{\sin(x)} \cdot (\cos^2(x) - \sin(x)) = (\sin(x))' \cdot e^{\sin(x)} \cdot (\cos^2(x) - \sin(x)) +$
$+ e^{\sin(x)} \cdot ((\cos^2(x))' + (-\sin(x))') = \cos(x) \cdot e^{\sin(x)} \cdot (\cos^2(x) - \sin(x)) +$
$+ e^{\sin(x)} \cdot (2 \cdot \cos(x) \cdot (\cos(x))' - (\sin(x))') = \cos(x) \cdot e^{\sin(x)} \cdot (\cos^2(x) - \sin(x)) +$
$+ e^{\sin(x)} \cdot (-2 \cdot \cos(x) \cdot \sin(x) - \cos(x)) = \cos(x) \cdot (e^{\sin(x)} \cdot (\cos^2(x) -$
$- \sin(x)) + e^{\sin(x)} \cdot (-2 \cdot \sin(x) - 1)) = \cos(x) \cdot e^{\sin(x)} \cdot (\cos^2(x) -$
$- 3 \cdot \sin(x) - 1)$

$f^{(4)}(x) = e^{\sin(x)} \cdot (3 \cdot \sin^2(x) + \sin(x) + \cos^2(x) \cdot (\cos^2(x) -$
$- 6 \cdot \sin(x) - 4))$

$$f^{(5)}(x) = \Big(e^{\sin(x)} \cdot \big(3 \cdot \sin^2(x) + \sin(x) + \cos^2(x) \cdot (\cos^2(x) -$$
$$- 6 \cdot \sin(x) - 4)\big)\Big)' = \cos(x) \cdot \big(15 \cdot \sin^2(x) \cdot e^{\sin(x)} +$$
$$+ 15 \cdot \sin(x) \cdot e^{\sin(x)} + \cos^4(x) \cdot e^{\sin(x)} - 10 \cdot \sin(x) \cdot \cos^2(x) \times$$
$$\times e^{\sin(x)} - 10 \cdot \cos^2(x) \cdot e^{\sin(x)} + e^{\sin(x)}\big)$$

$$f(0) = e^{\sin(0)} = 1$$

$$f'(0) = \cos(0) \cdot e^{\sin(0)} = 1$$

$$f''(0) = e^{\sin(0)} \cdot (\cos^2(0) - \sin(0)) = 1$$

$$f'''(0) = \cos(0) \cdot e^{\sin(0)} \cdot (\cos^2(0) - 3 \cdot \sin(0) - 1) = 0$$

$$f^{(4)}(0) = e^{\sin(0)} \cdot \big(3 \cdot \sin^2(0) + \sin(0) + \cos^2(0) \cdot (\cos^2(0) - 6 \cdot \sin(0) -$$
$$- 4)\big) = -3$$

За виразом (3)

$$f(x) \approx f(0) + f'(0)(x-0) + f''(0)\frac{(x-0)^2}{2} =$$

$$= f(x) \approx f(0) + f'(0)(x-0) + f''(0)\frac{(x-0)^2}{2} + f'''(0)\frac{(x-0)^3}{6} +$$
$$+ f^{(4)}(0)\frac{(x-0)^4}{24}$$

Підставляємо:

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$$f(x) \approx 1 + 1\cdot x + 1\cdot \frac{x^2}{2} = 1 + x + \frac{x^2}{2}$$

$$f(x) = 1 + 1\cdot x + 1\cdot \frac{x^2}{2} + 0\cdot \frac{x^3}{6} = 1 + x + \frac{x^2}{2}$$

$$|R_3| \leq \frac{1}{3!}\left|\int_0^1 (t-1)^3\,dt\right| =$$

$$= \frac{1}{6}\left|\frac{(t-1)^4}{4} + C\Big|_0^1\right| = \frac{1}{6}\cdot\left|\frac{(1-1)^4}{4} - \frac{(0-1)^4}{4}\right| =$$

$$= \frac{1}{6}\cdot\left|-\frac{1}{4}\right| = \frac{1}{6}\cdot\frac{1}{4} = \frac{1}{24}$$