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- However, good approximate solutions can often be found efficiently!
- **Techniques** for the design and analysis of approximation algorithms arise from studying specific optimization problems.

Overview

Combinatorial Algorithms

- Introduction (Vertex Cover)
- Set Cover via Greedy
- Shortest Superstring via reduction to SC
- Steiner Tree via MST
- Multiway Cut via Greedy
- k-Center via param. Pruning
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LP-based Algorithms

- introduction to LP-Duality
- Set Cover via LP Rounding
- Set Cover via Primal-Dual Schema
- Maximum Satisfiability
- Scheduling und Extreme Point Solutions
- Steiner Forest via Primal-Dual

VertexCover (card.)

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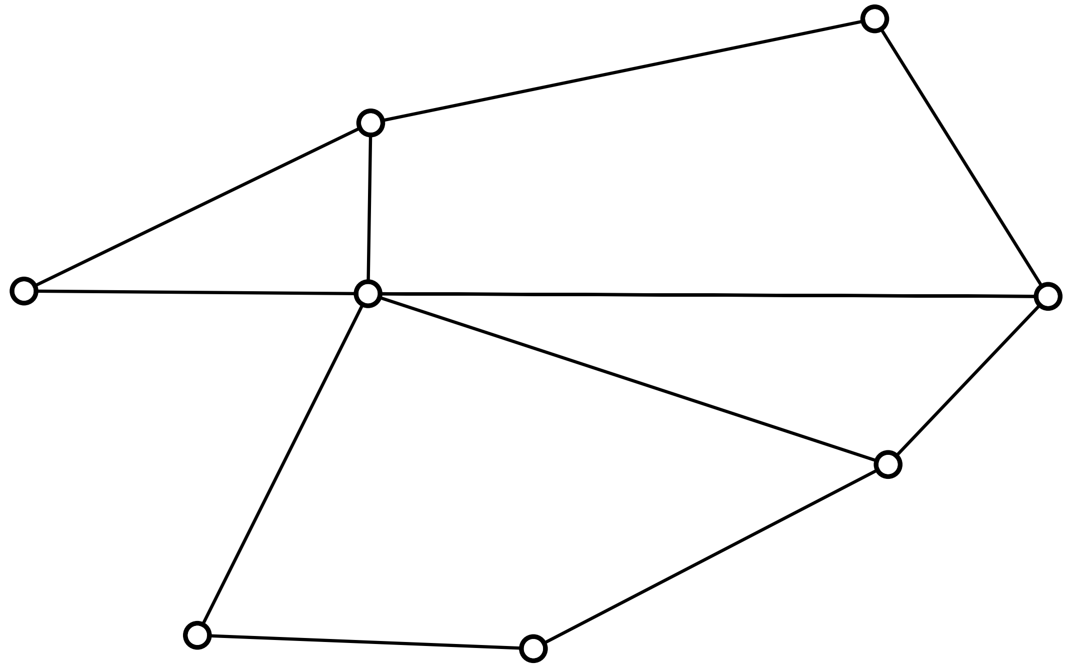

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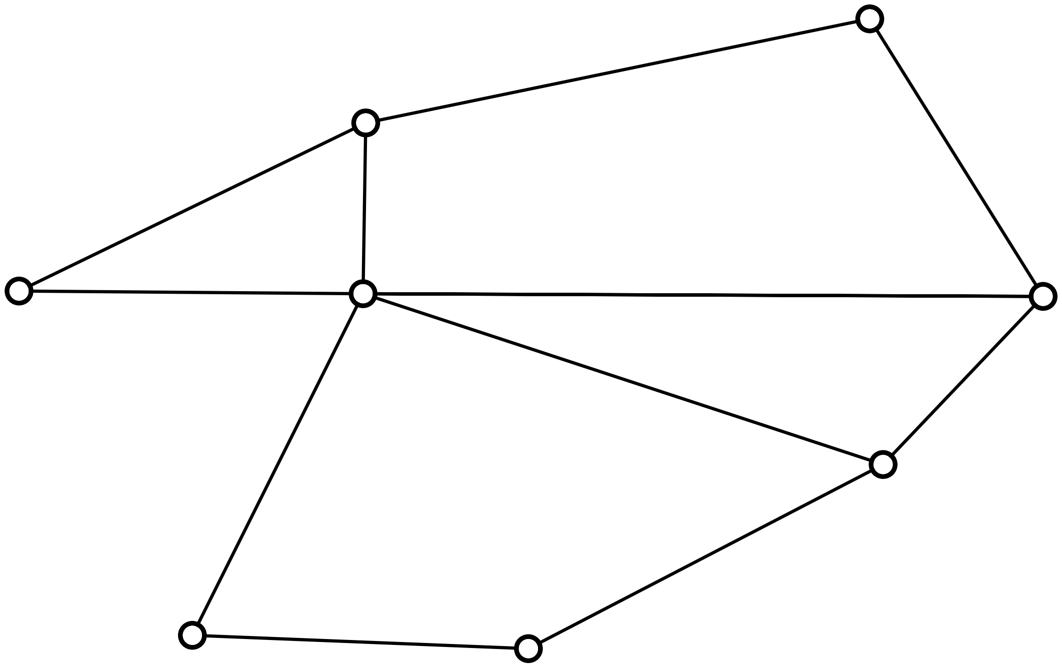

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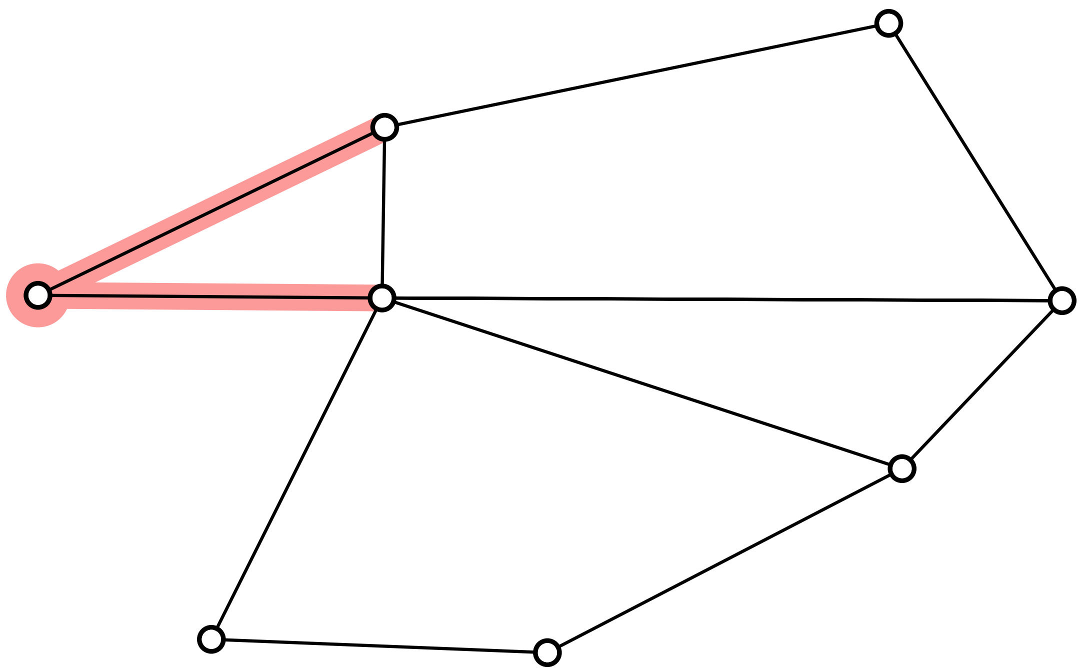

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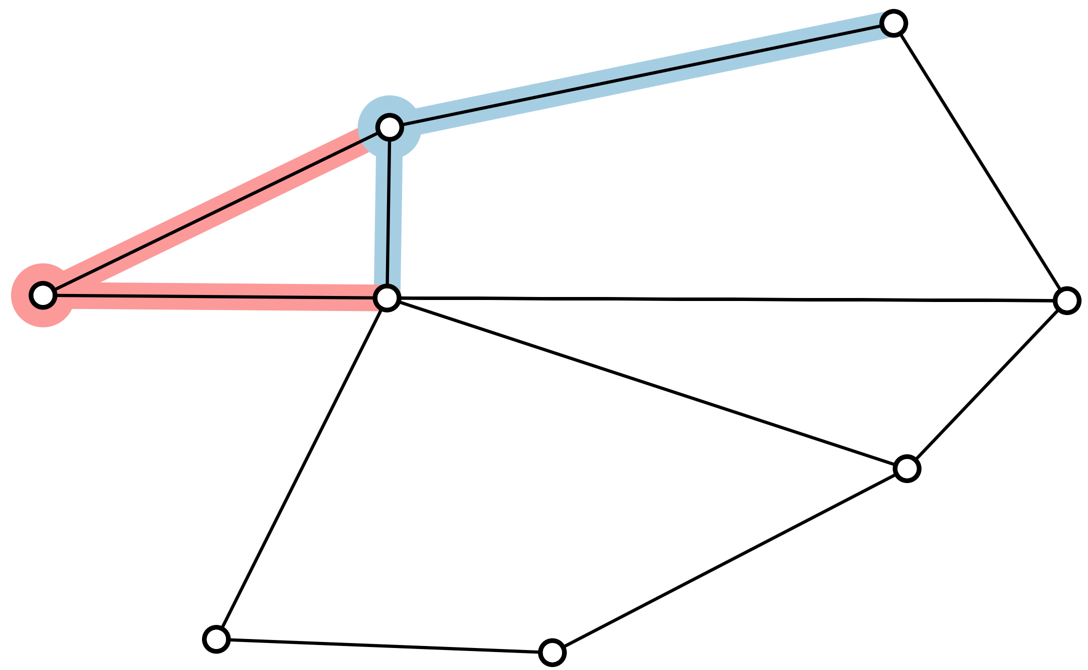

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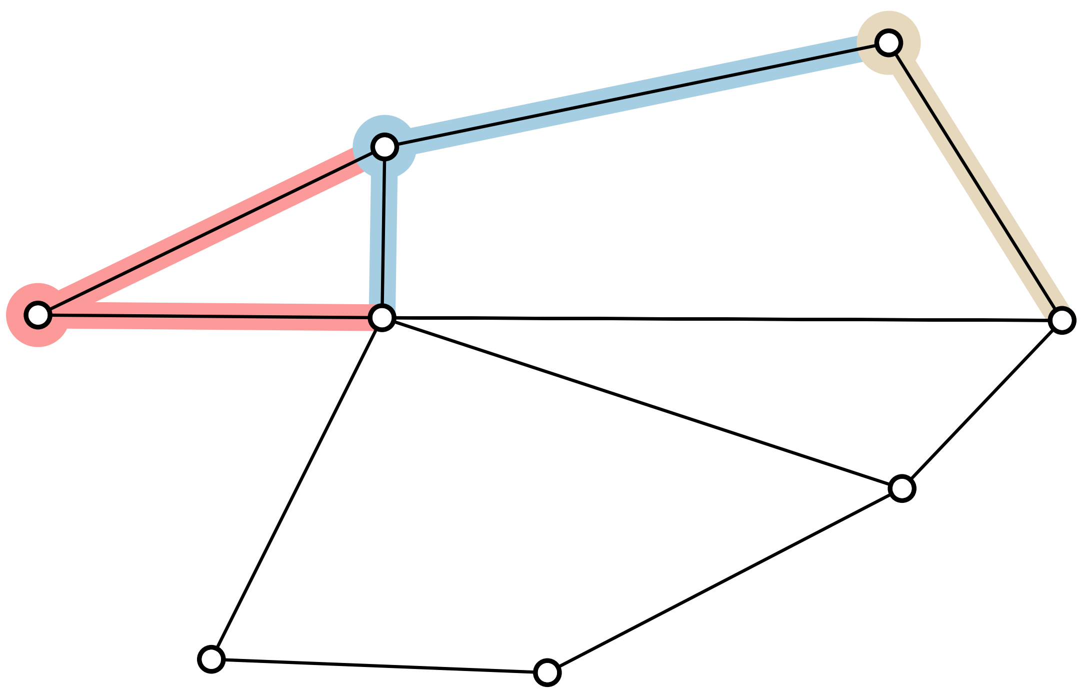

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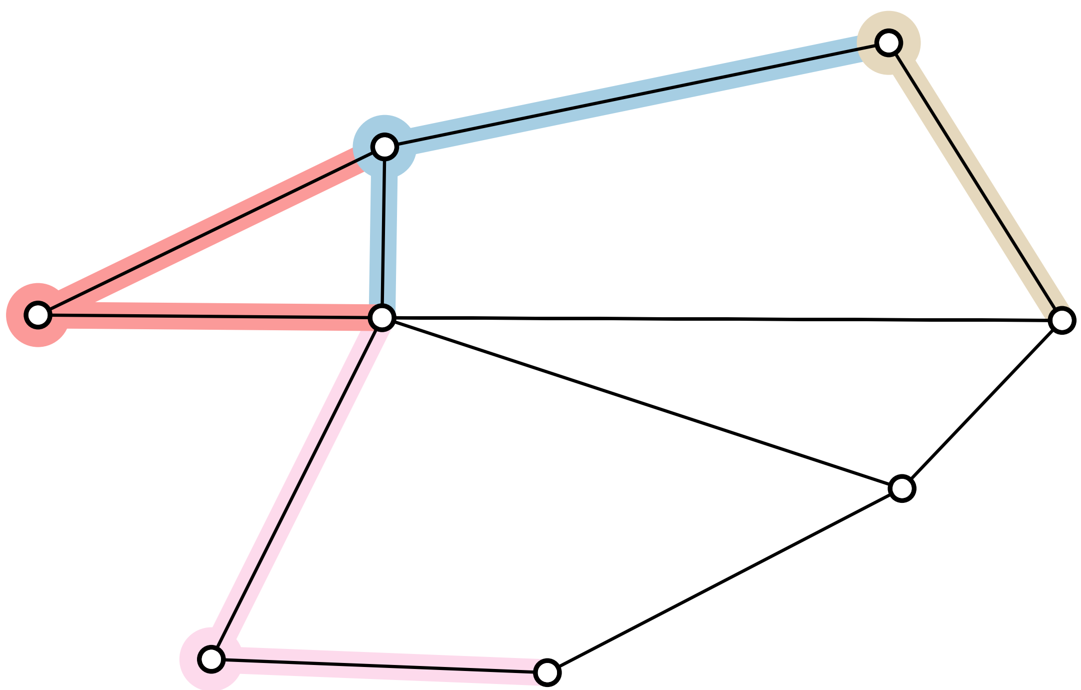

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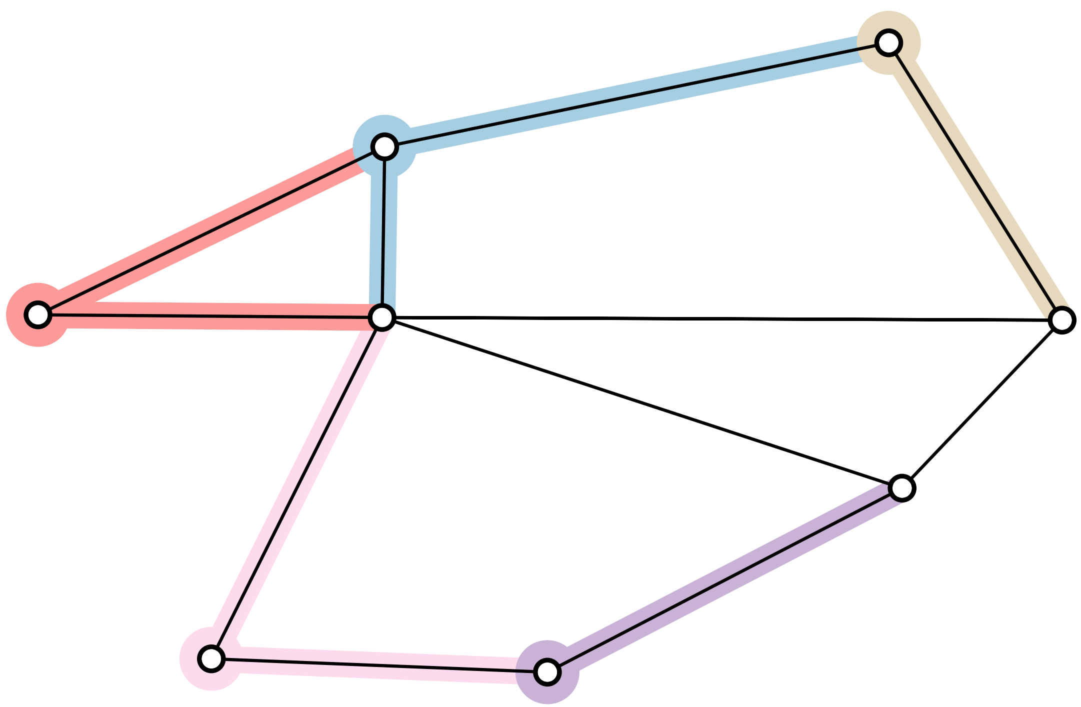

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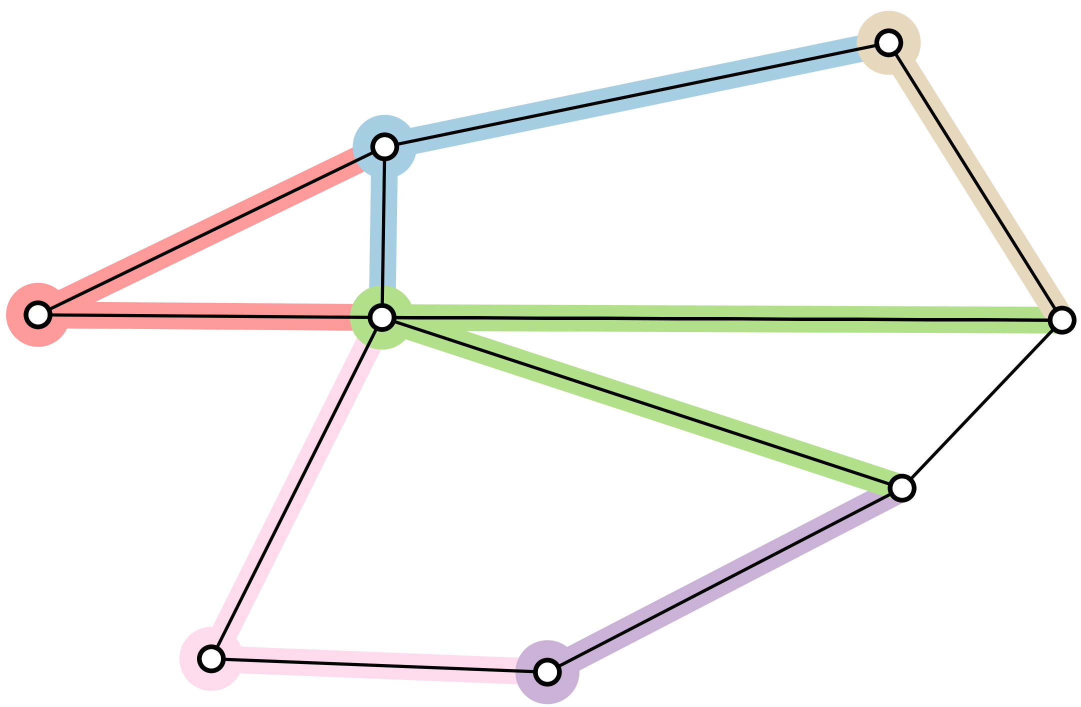

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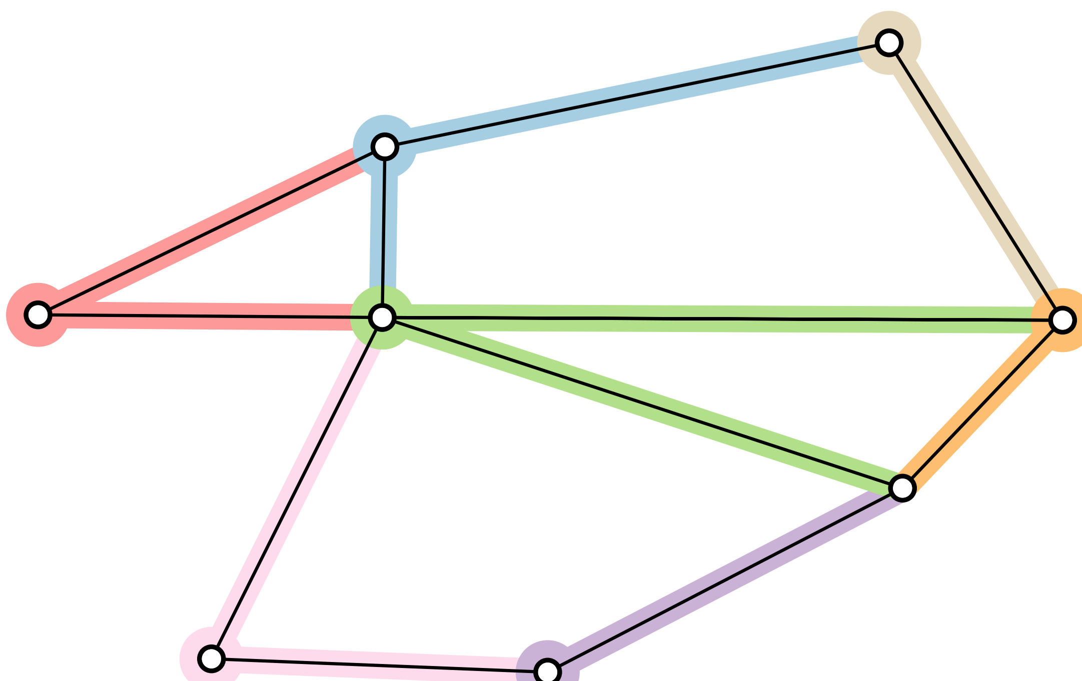

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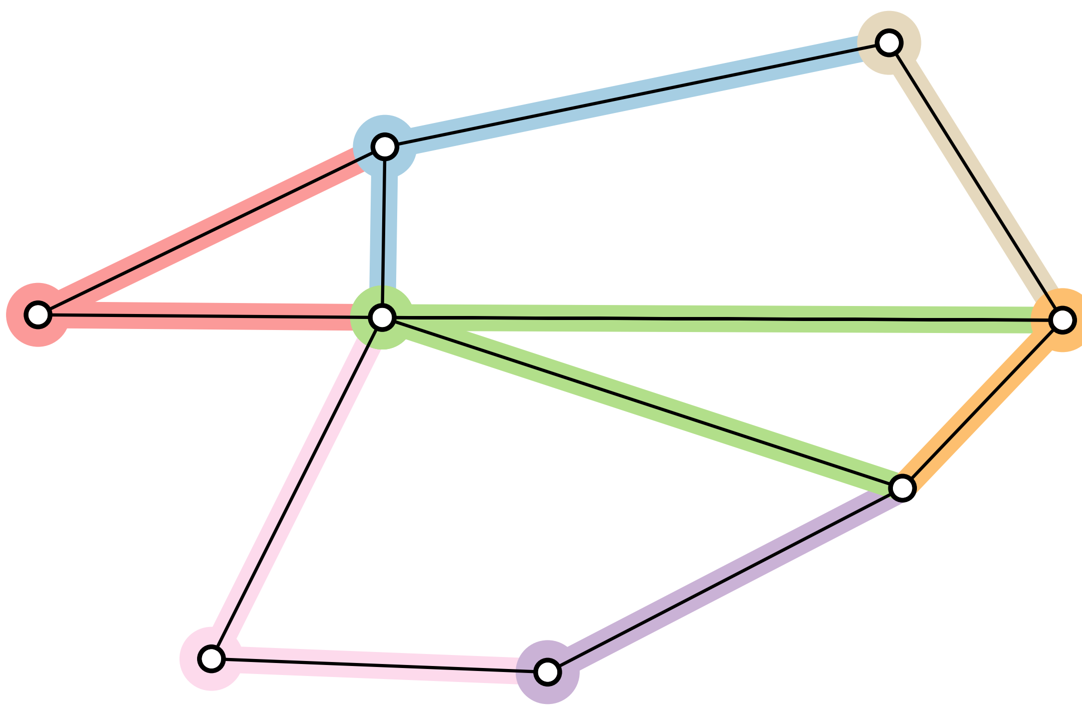

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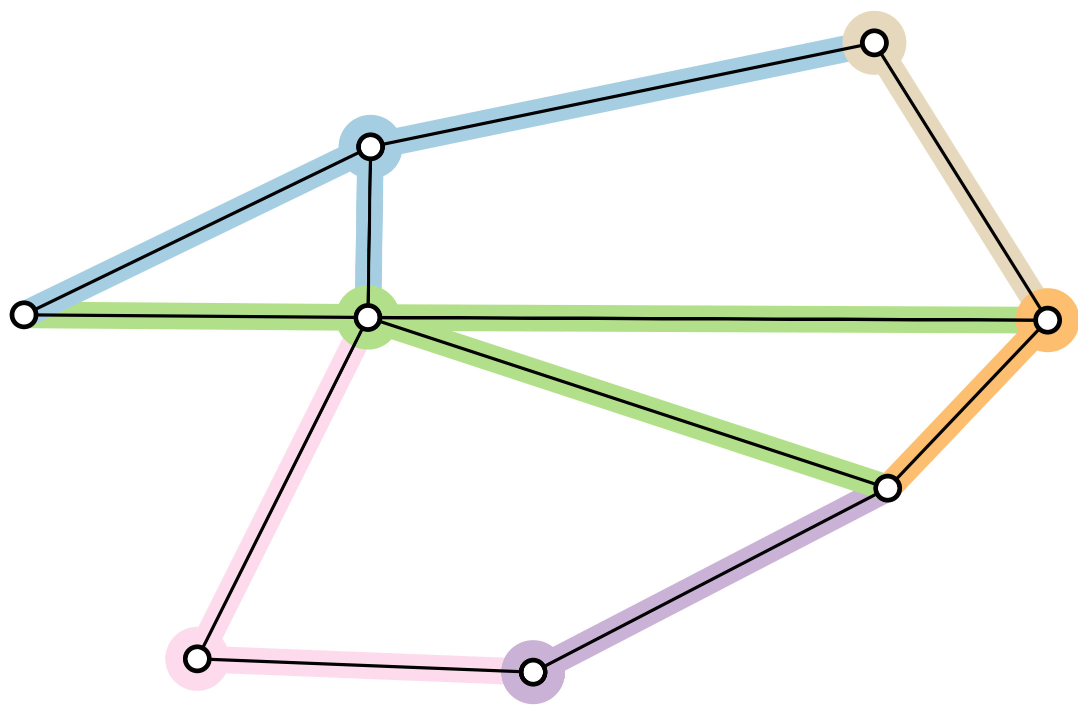

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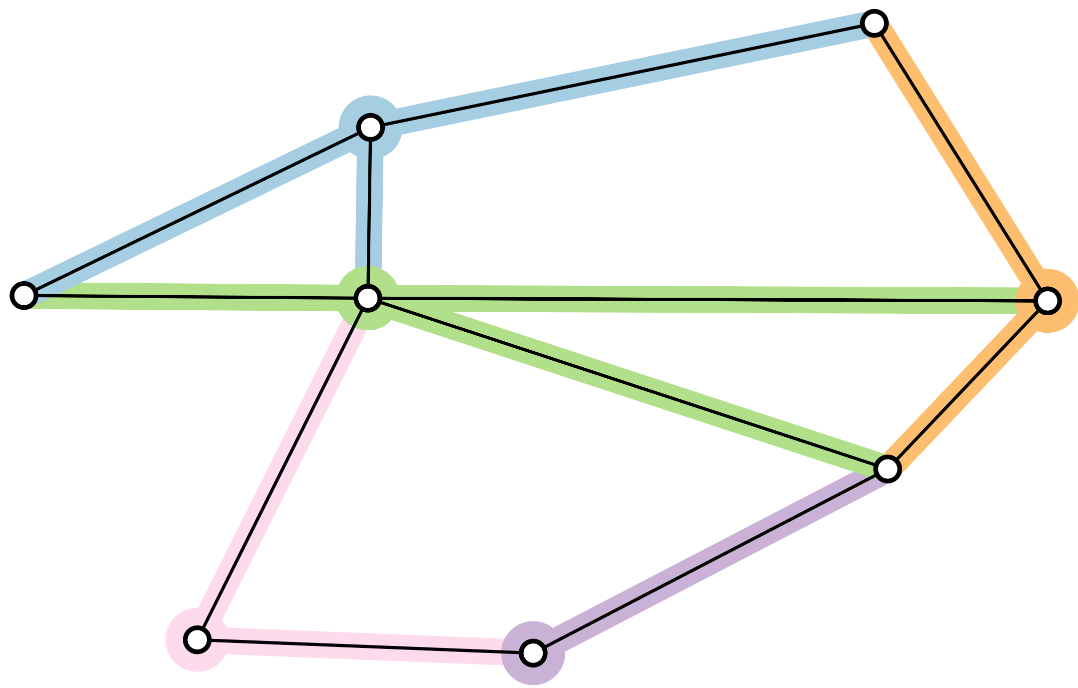

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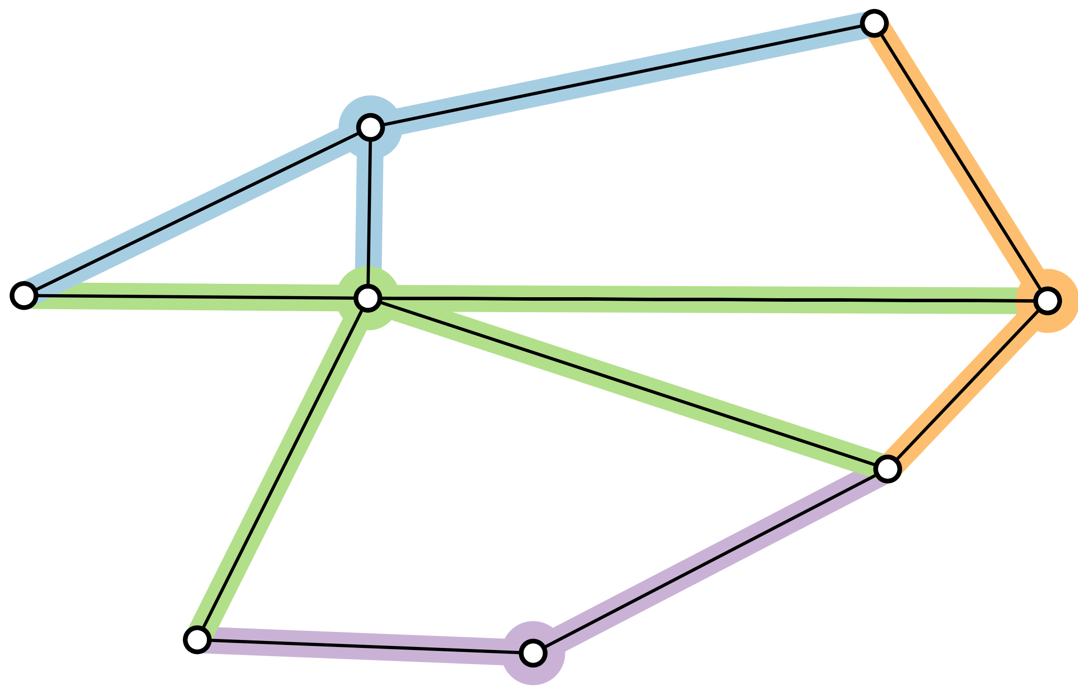

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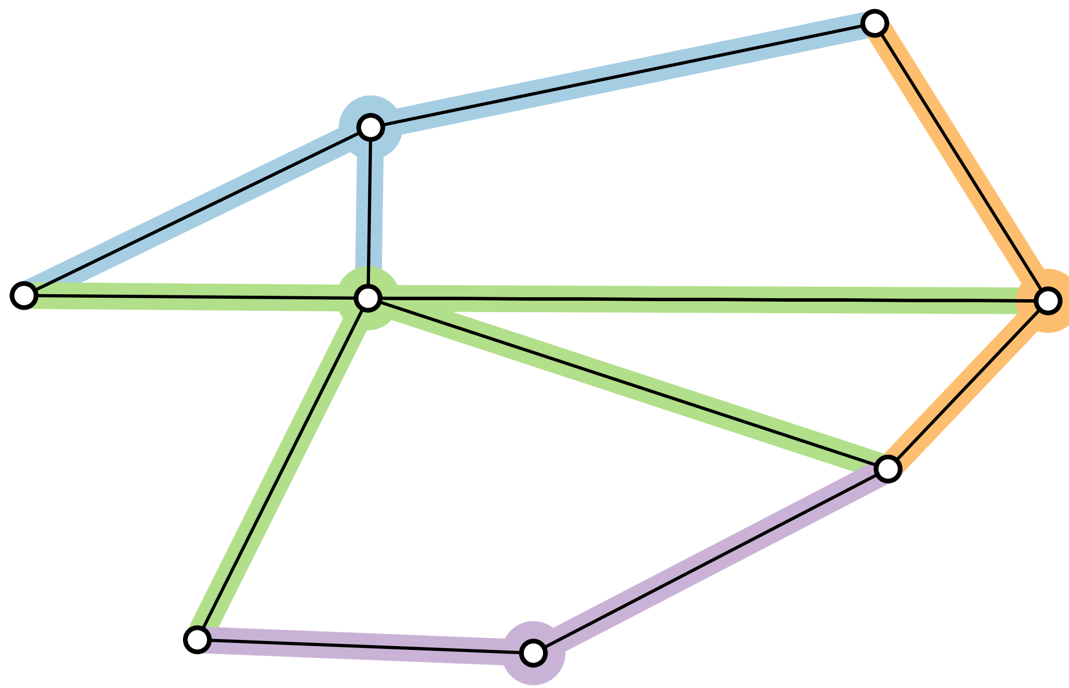

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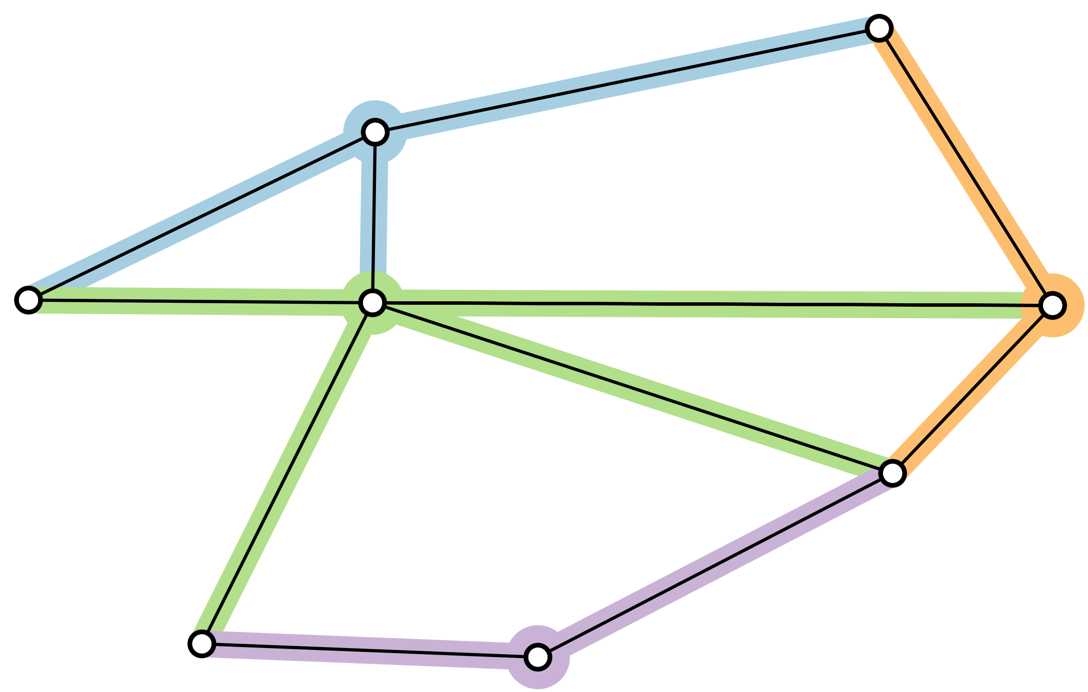

Optimum ($\mathrm{OPT} = 4$) – but in general NP-hard to find :-(

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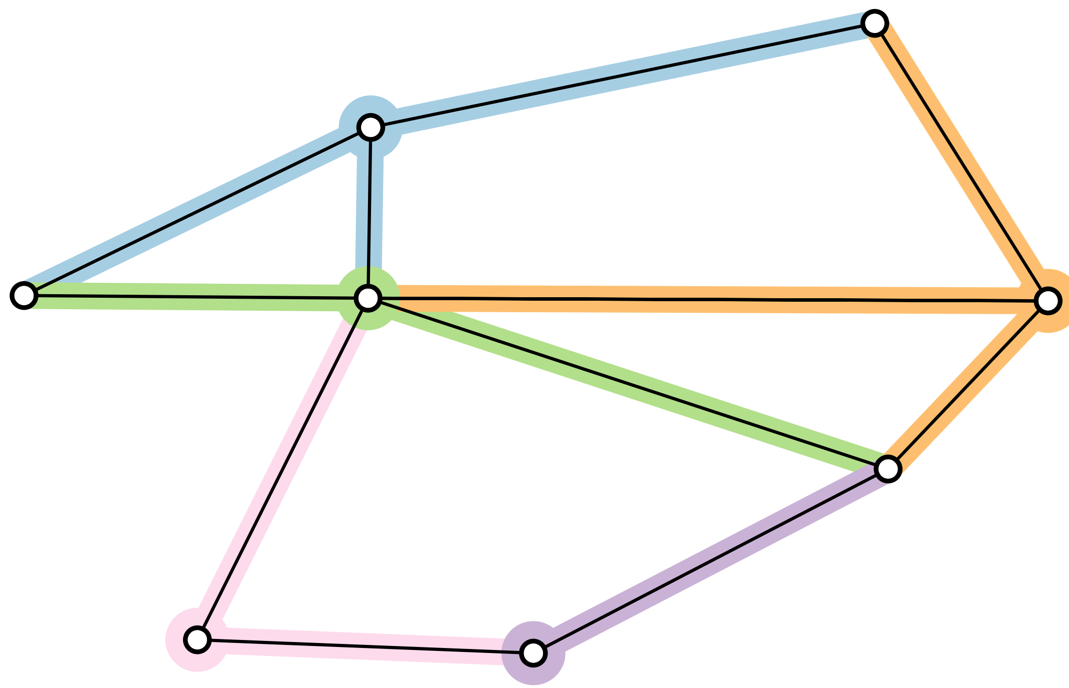

"good" approximate solution (5/4-approximation)

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- Π is either a minimization or maximization problem.

VertexCover: NP-Optimization Problem

Task: Fill in the gaps for $\Pi =$ Vertex Cover.

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The optimal value $\mathrm{obj}_\Pi(I, s^*)$ of the objective function is also denoted by $\mathrm{OPT}_\Pi(I)$ or simply OPT in context.

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Approximation Alg. for VERTEXCOVER

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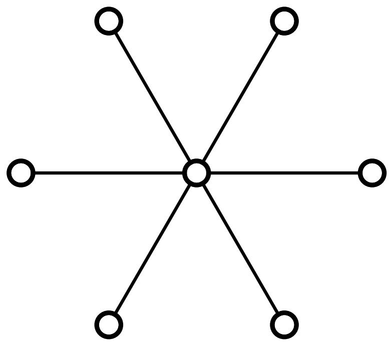

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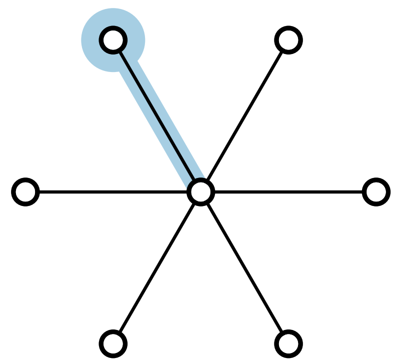

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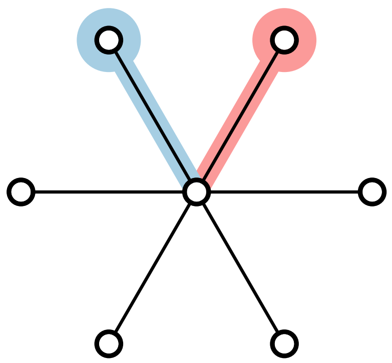

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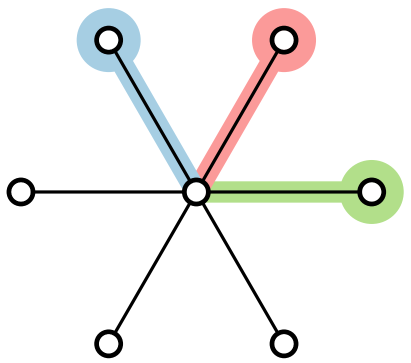

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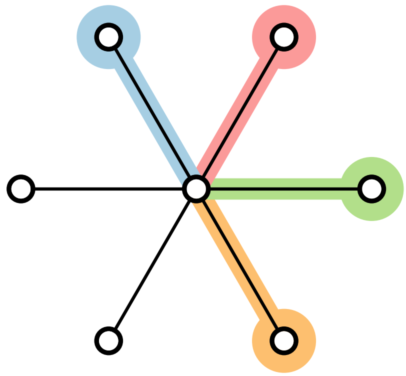

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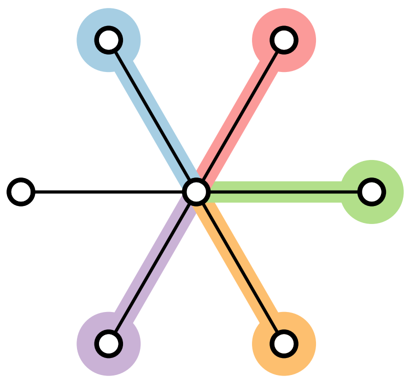

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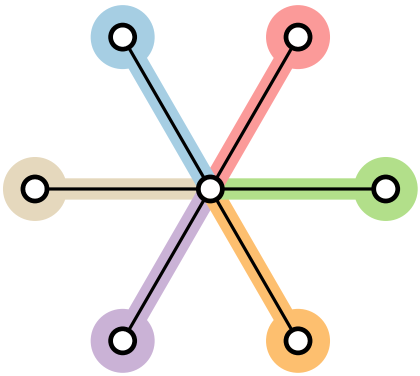

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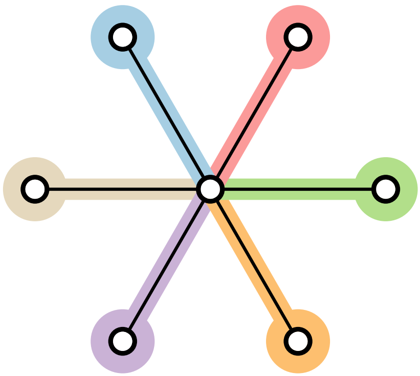

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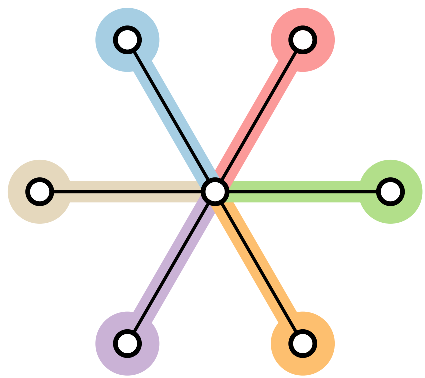

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Problem: How can we estimate $\mathrm{obj}_{\Pi}(I, s) / \mathrm{OPT}$, when it is hard to calculate OPT?

Approximation Alg. for VertexCover

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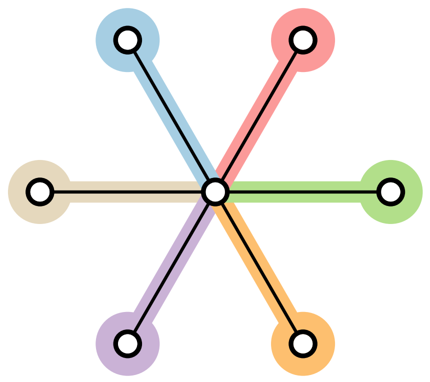
- Vertex-Greedy



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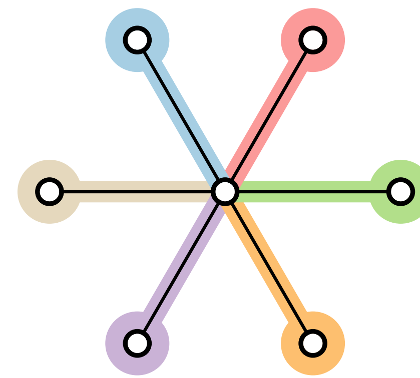

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$$\frac{\mathrm{obj}_\Pi(I, s)}{\mathrm{OPT}} \leq \frac{\mathrm{obj}_\Pi(I, s)}{L}$$

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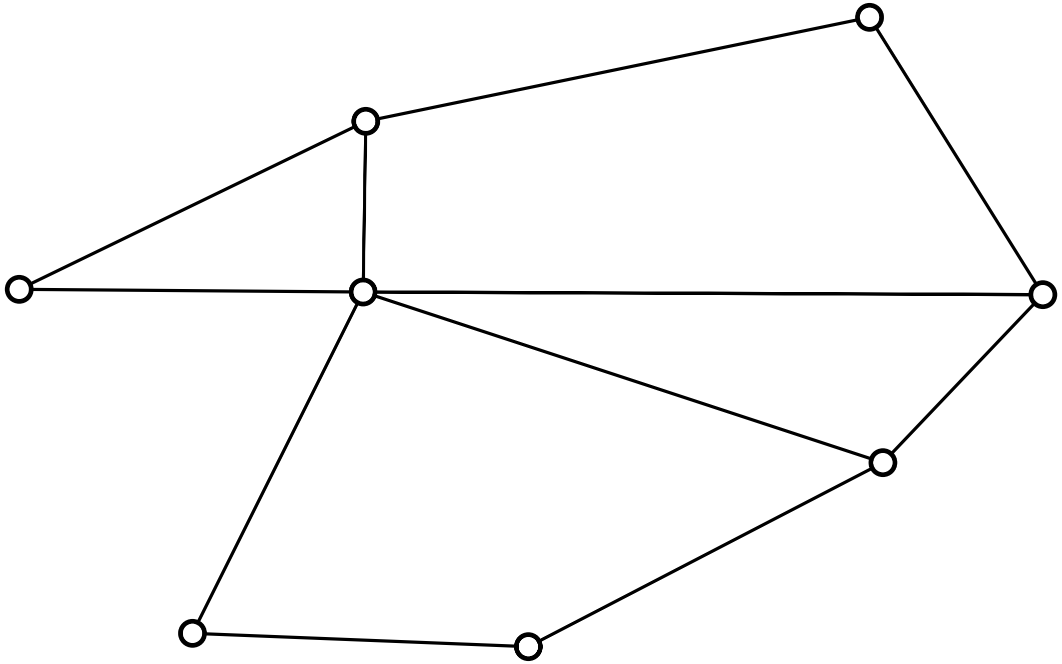

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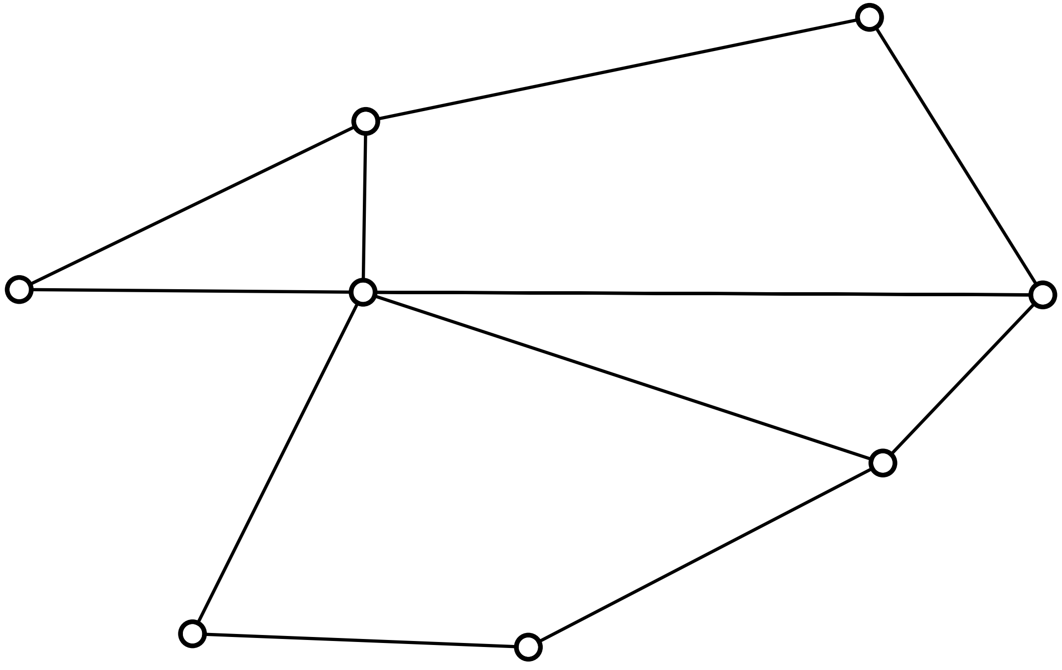

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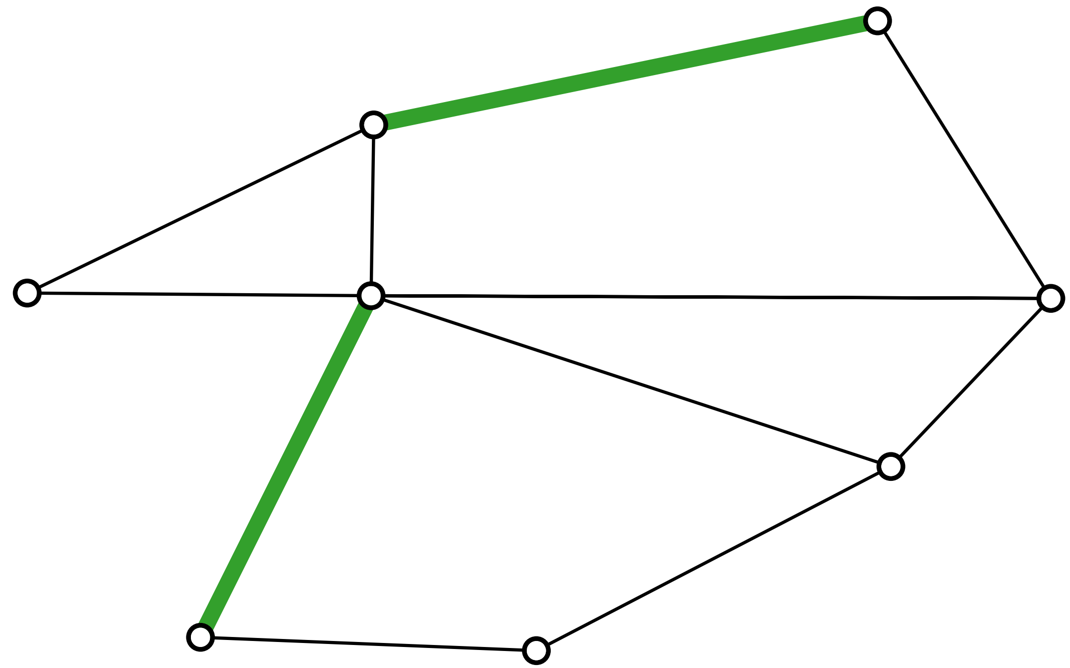

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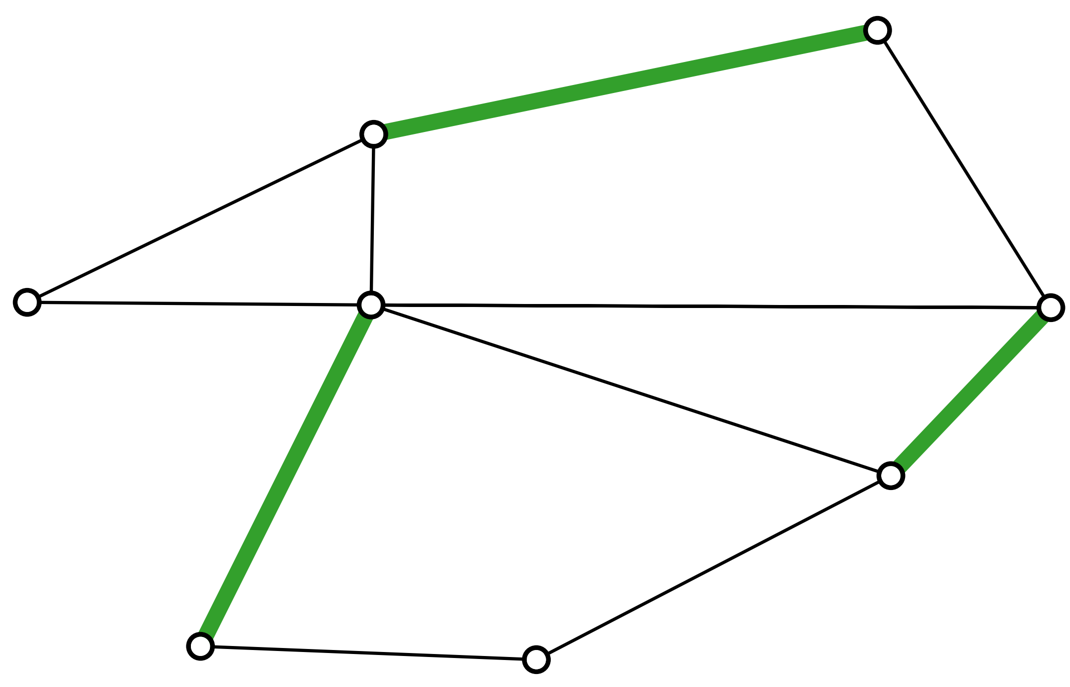

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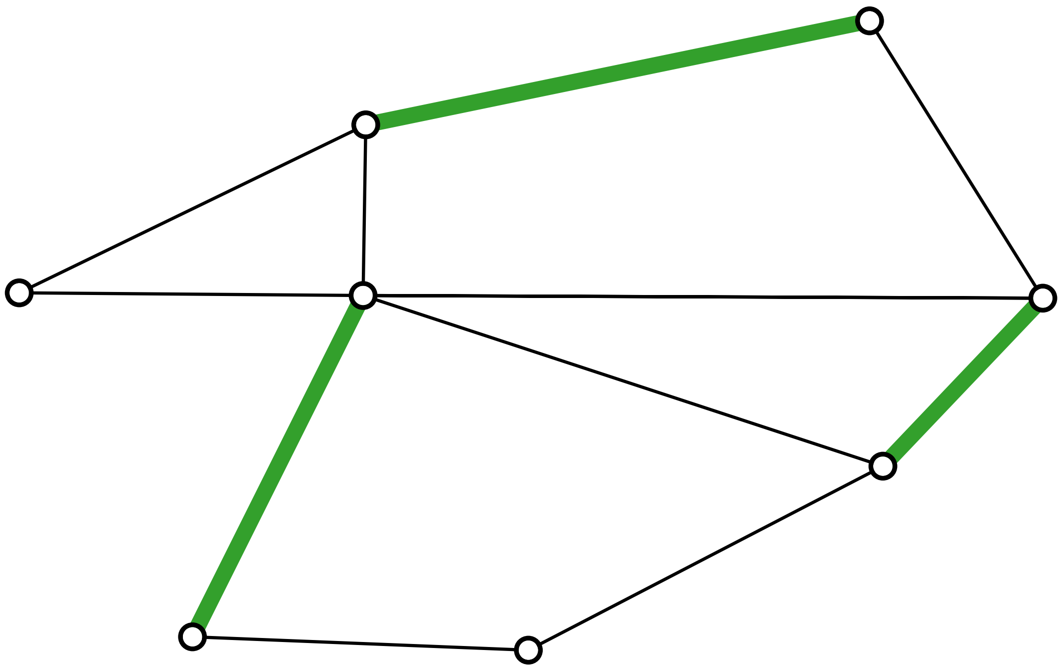

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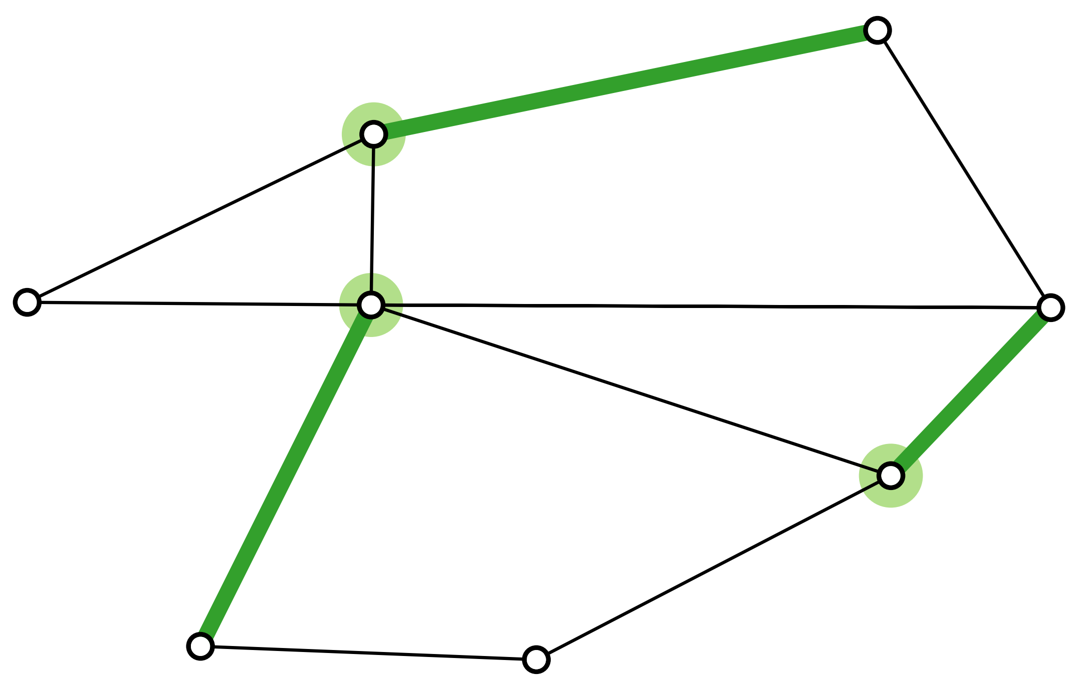
Vertex cover of M



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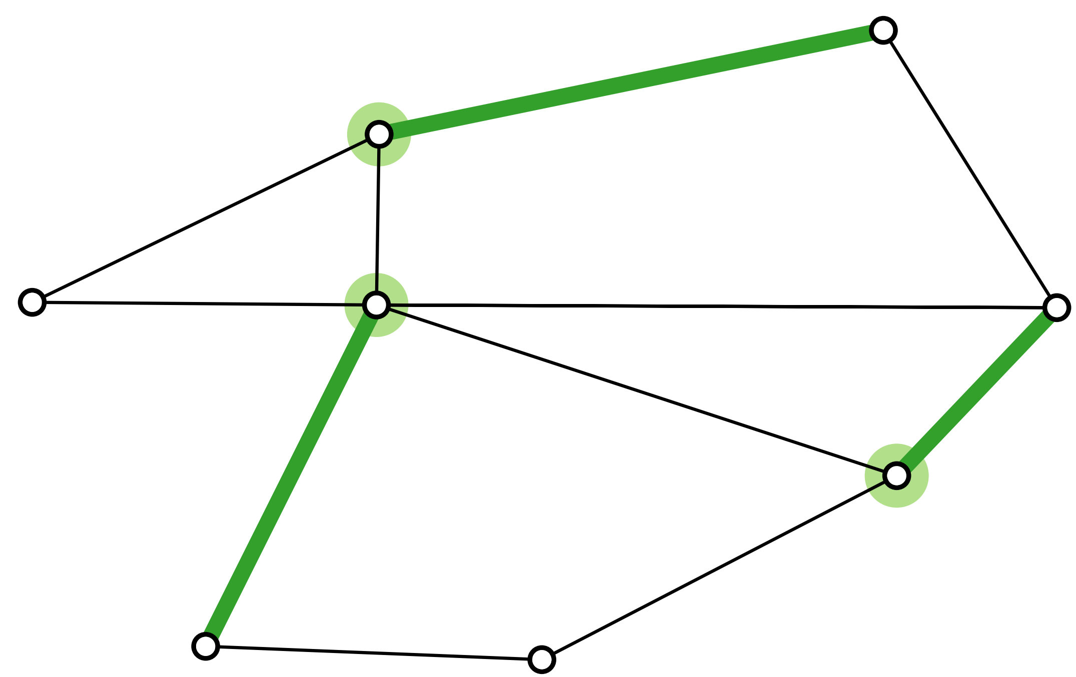

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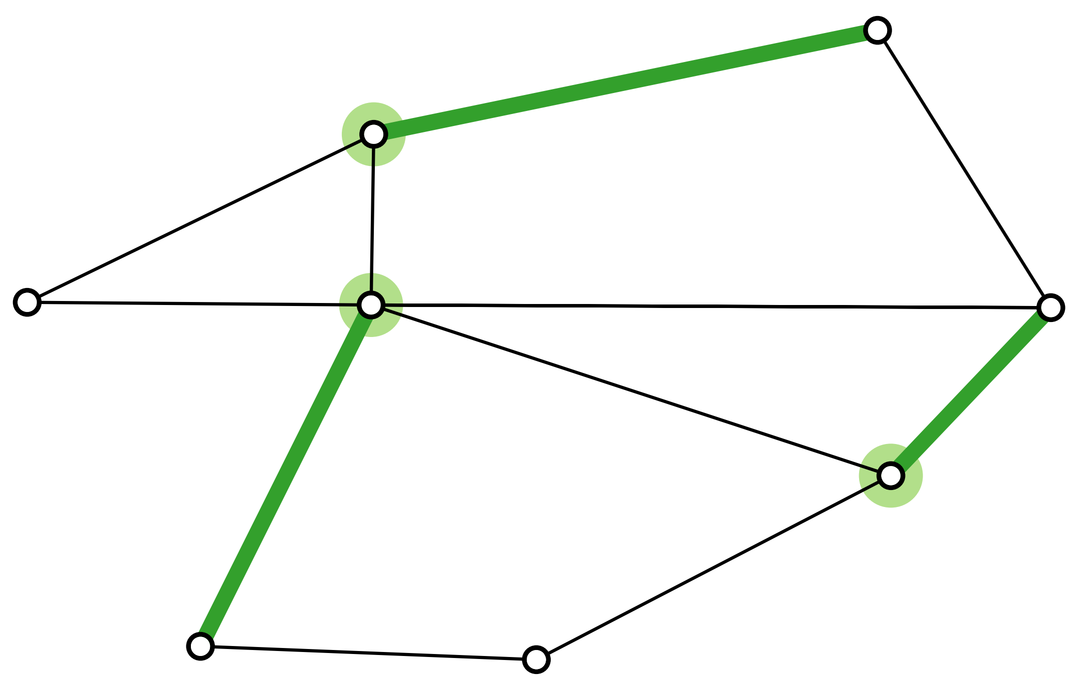

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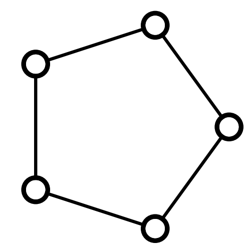

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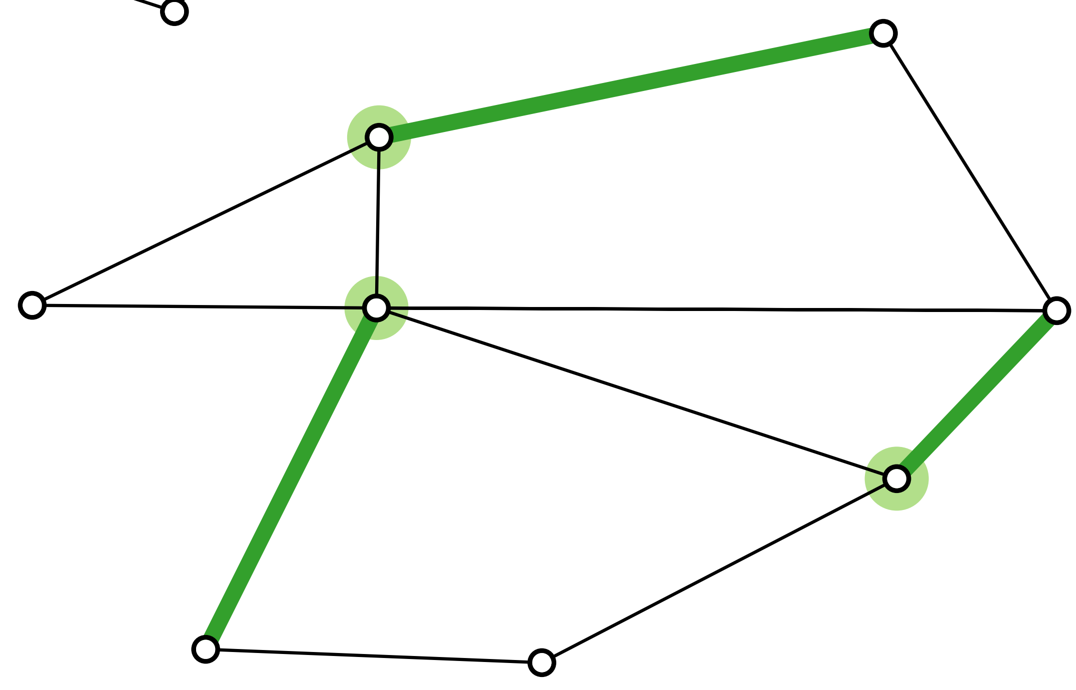

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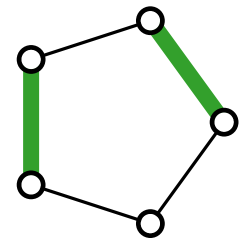

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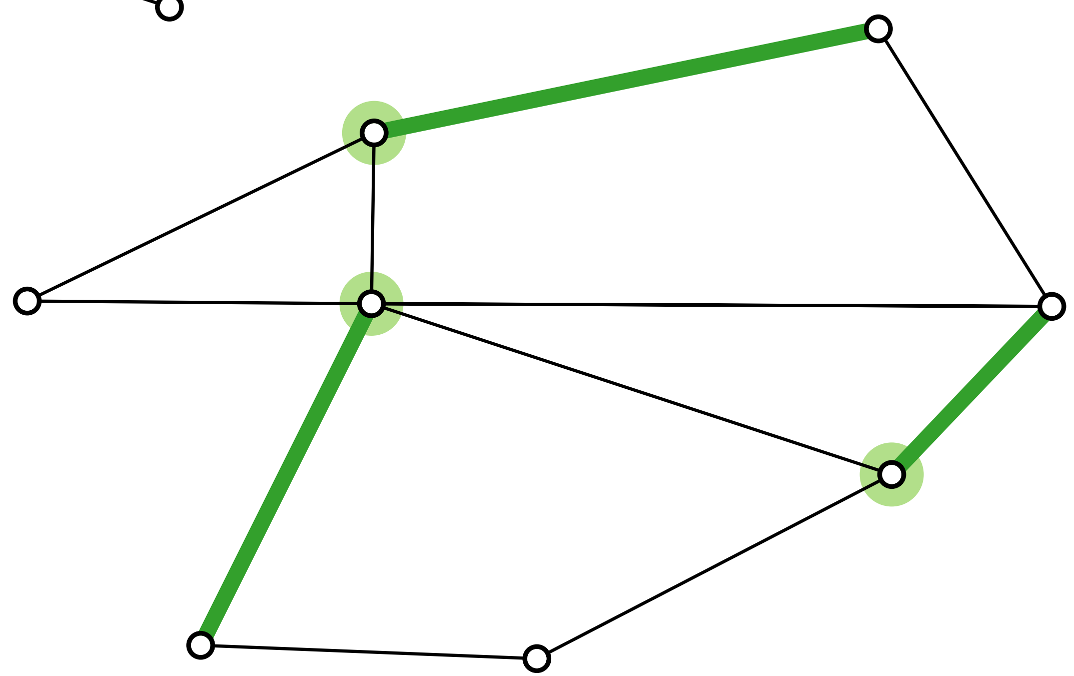

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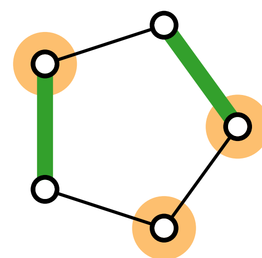

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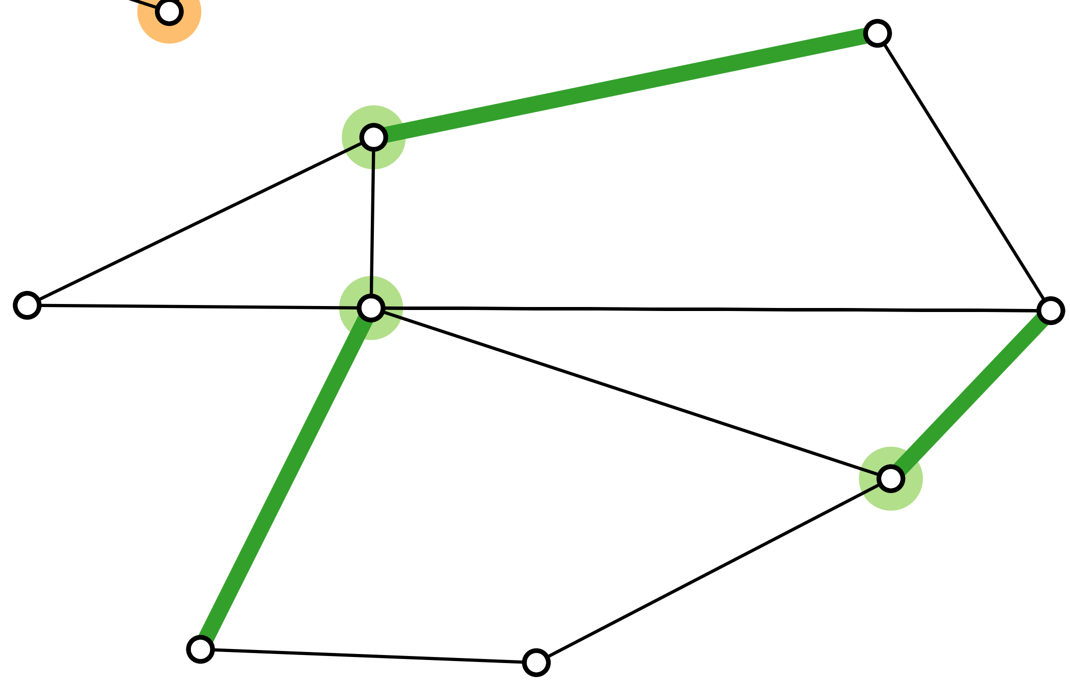

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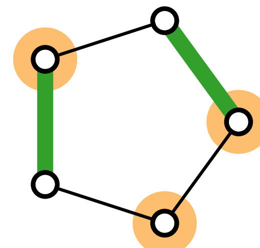

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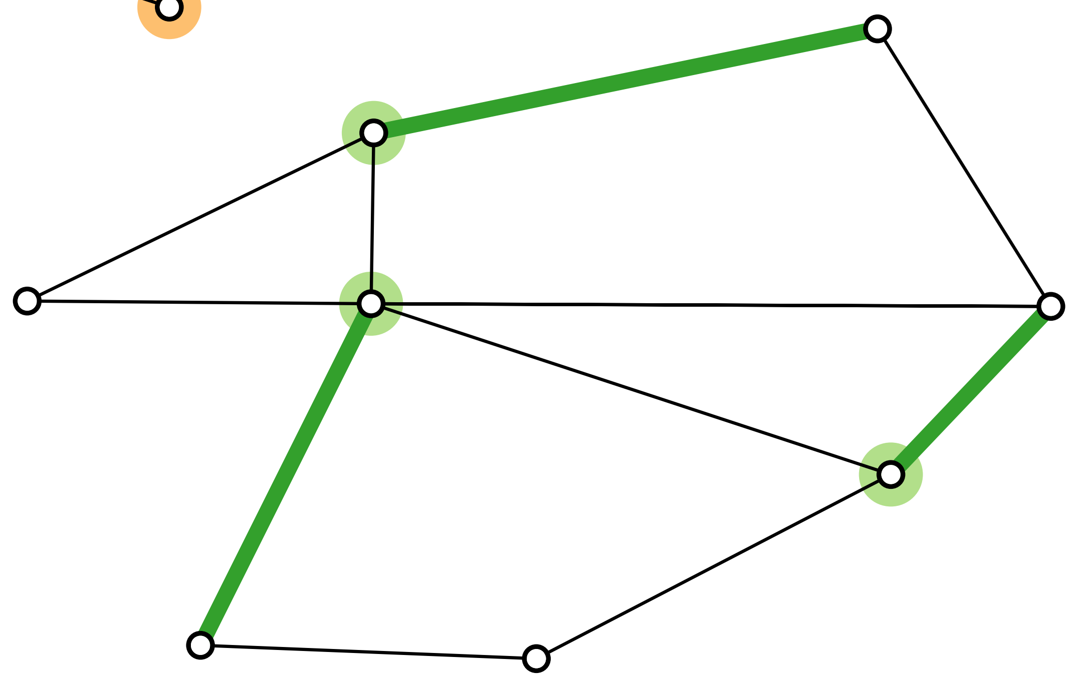

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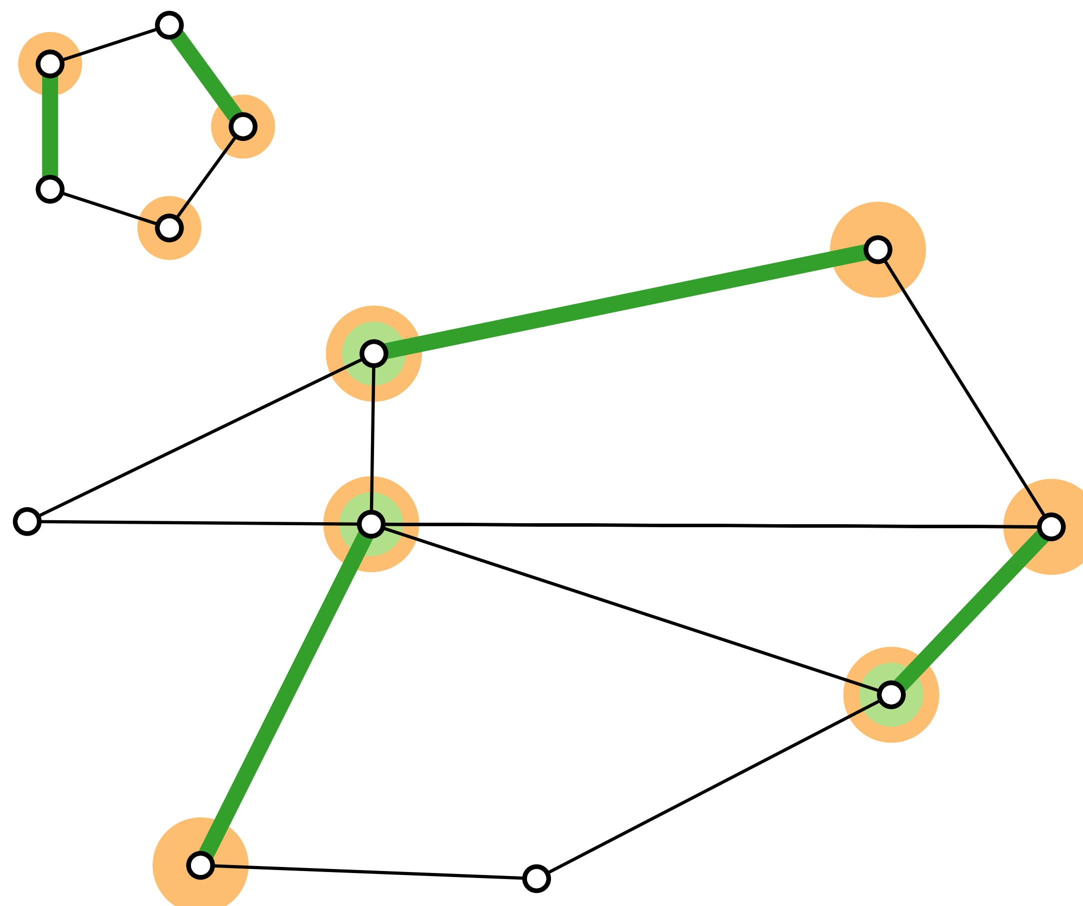

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Vertex cover of E

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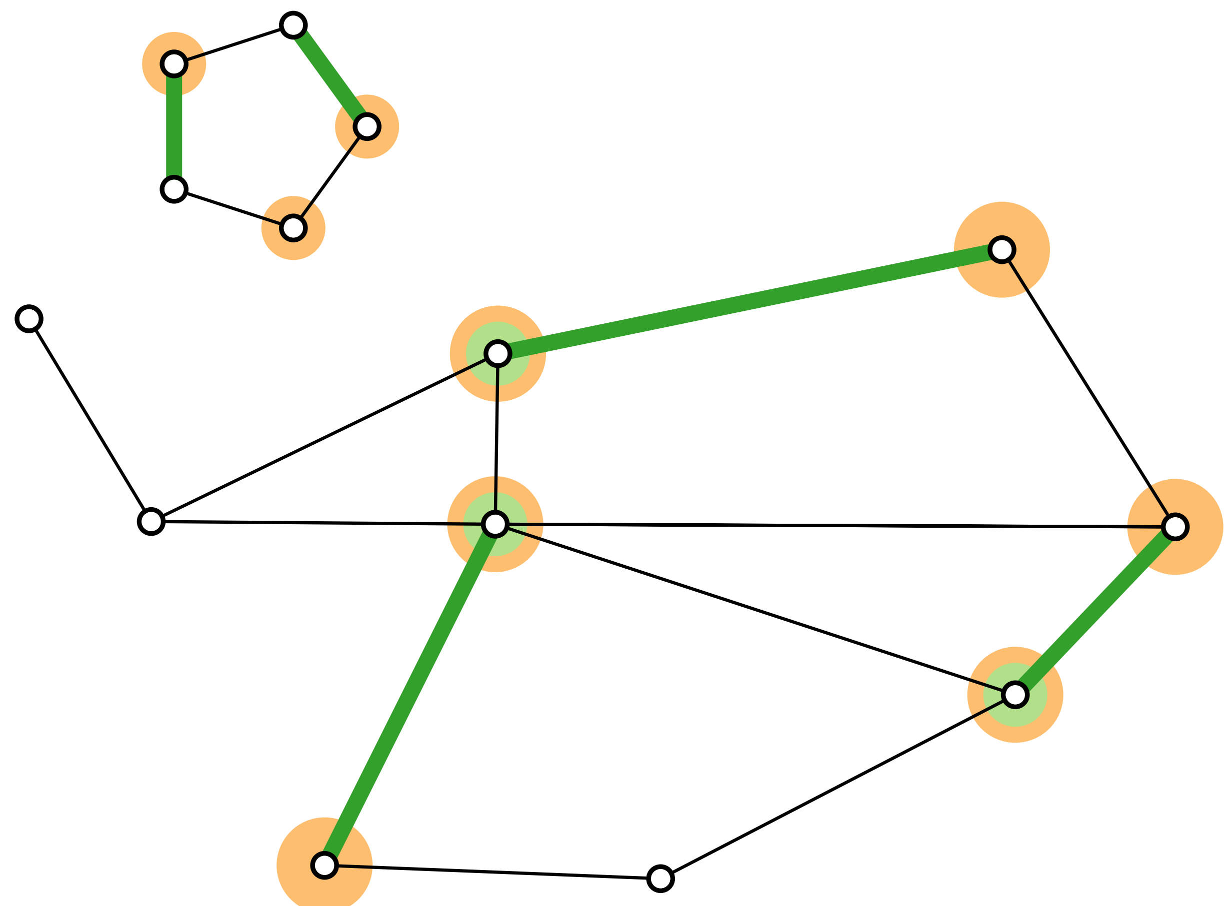

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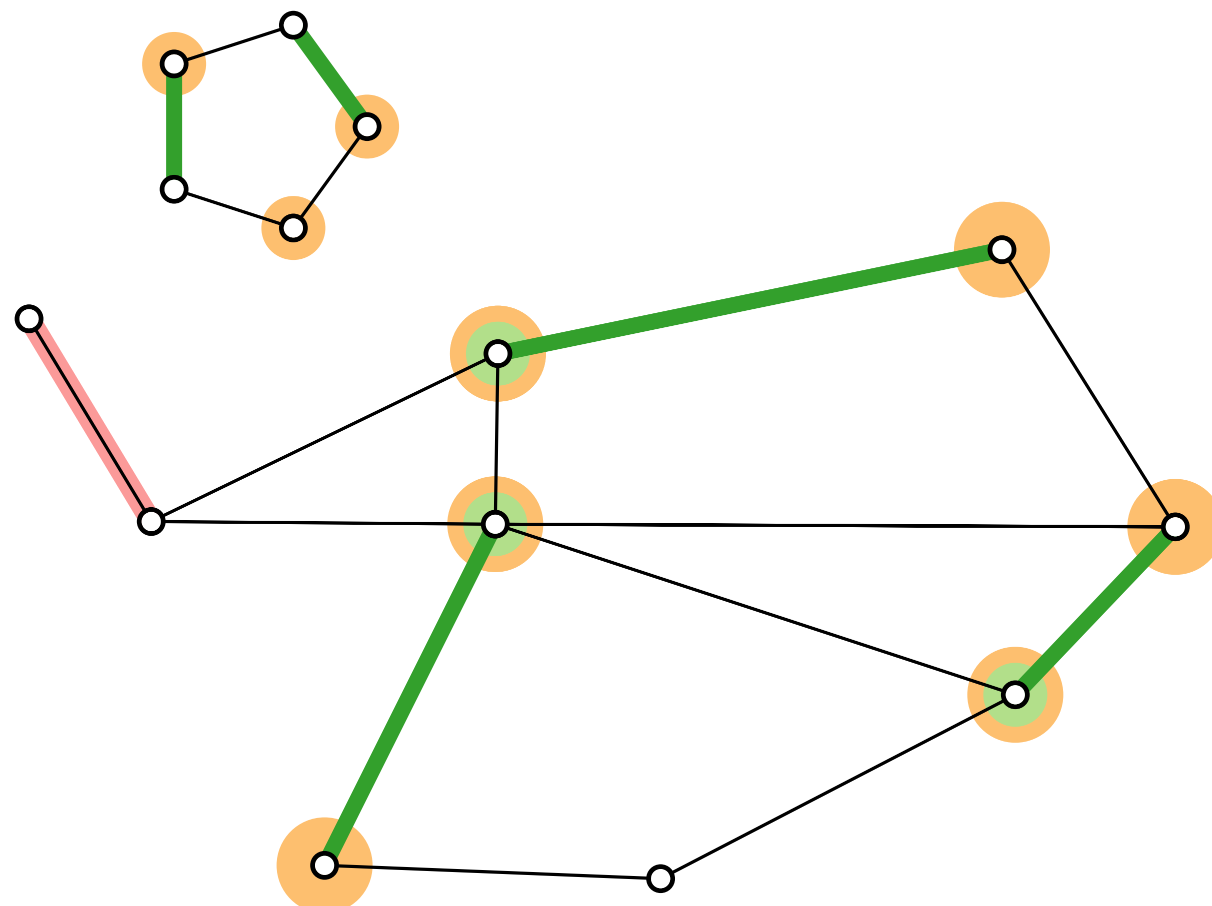

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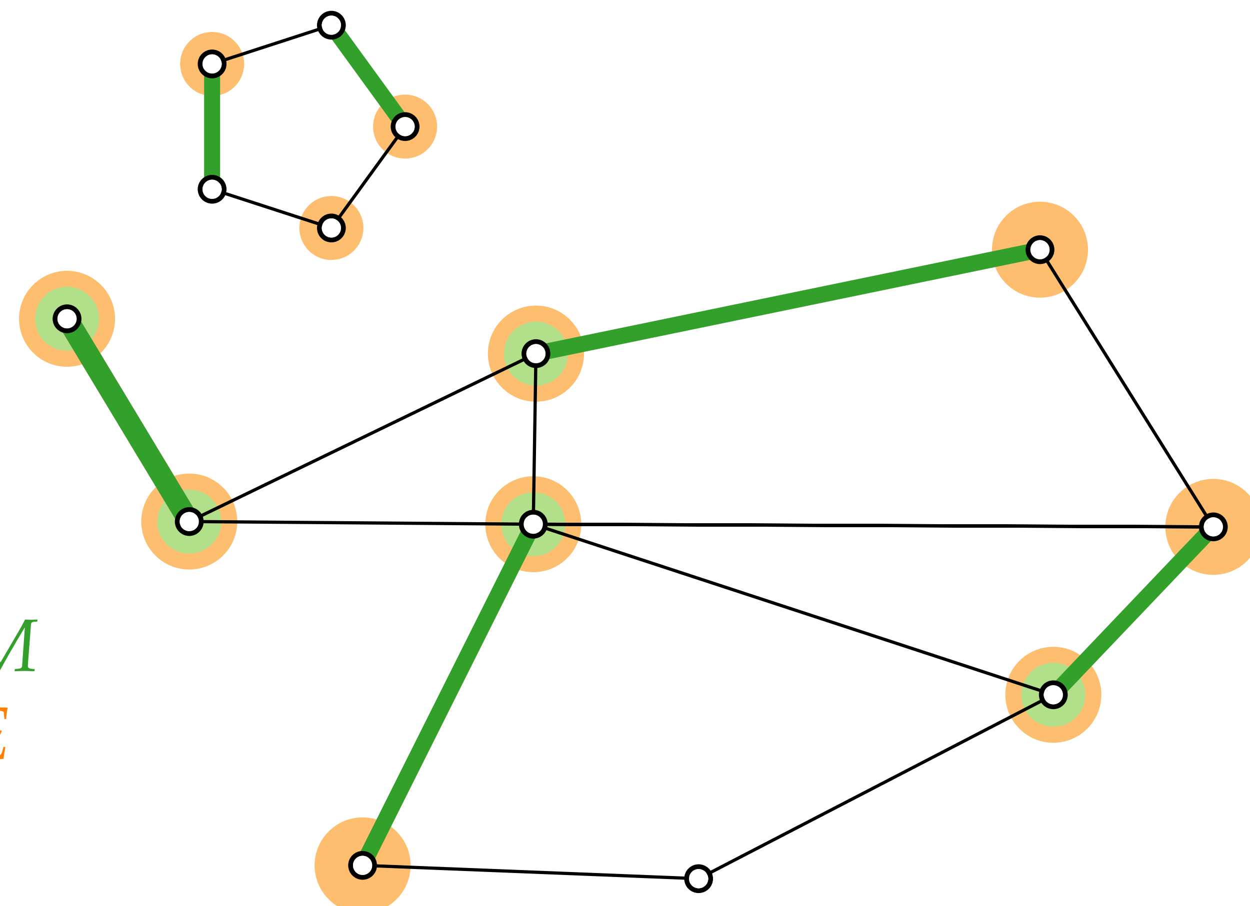

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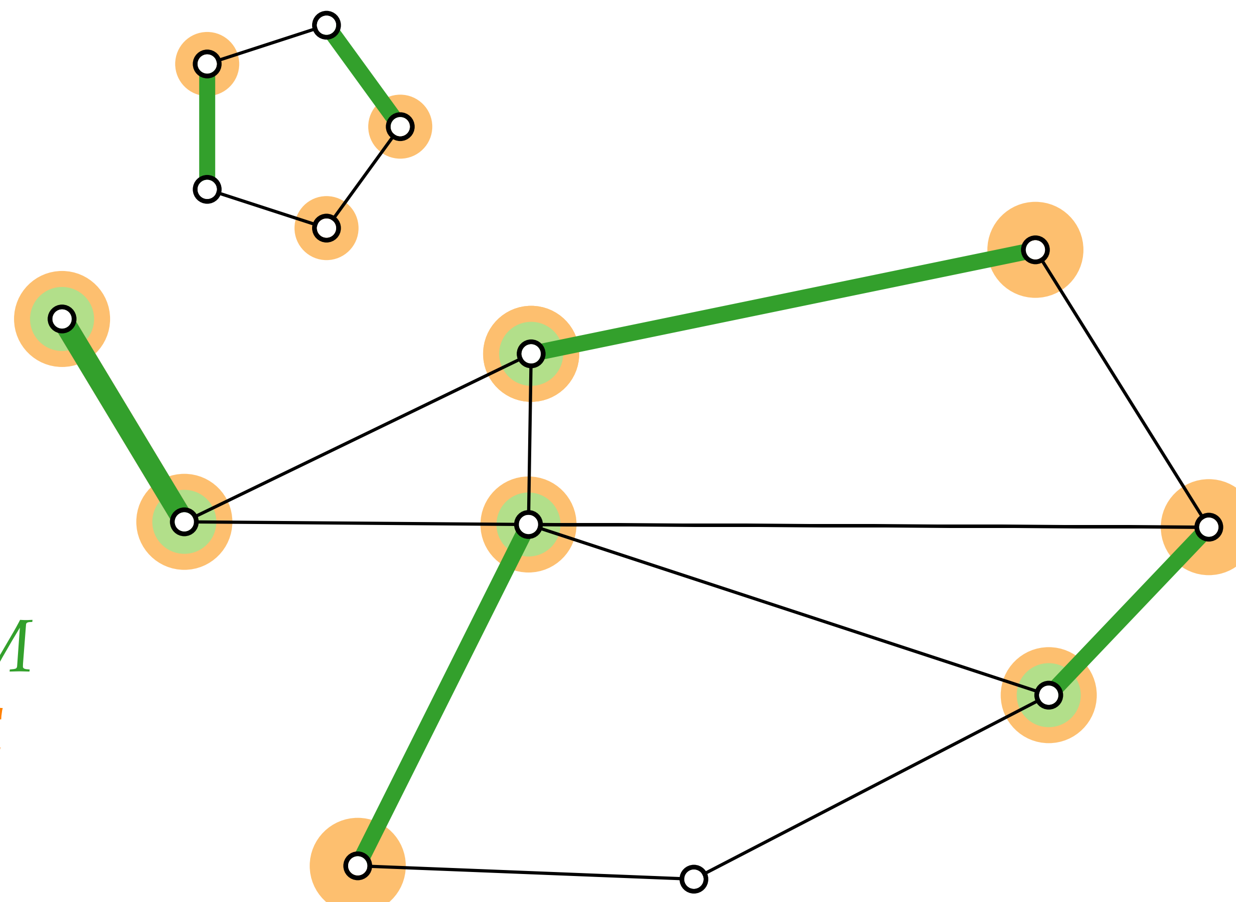

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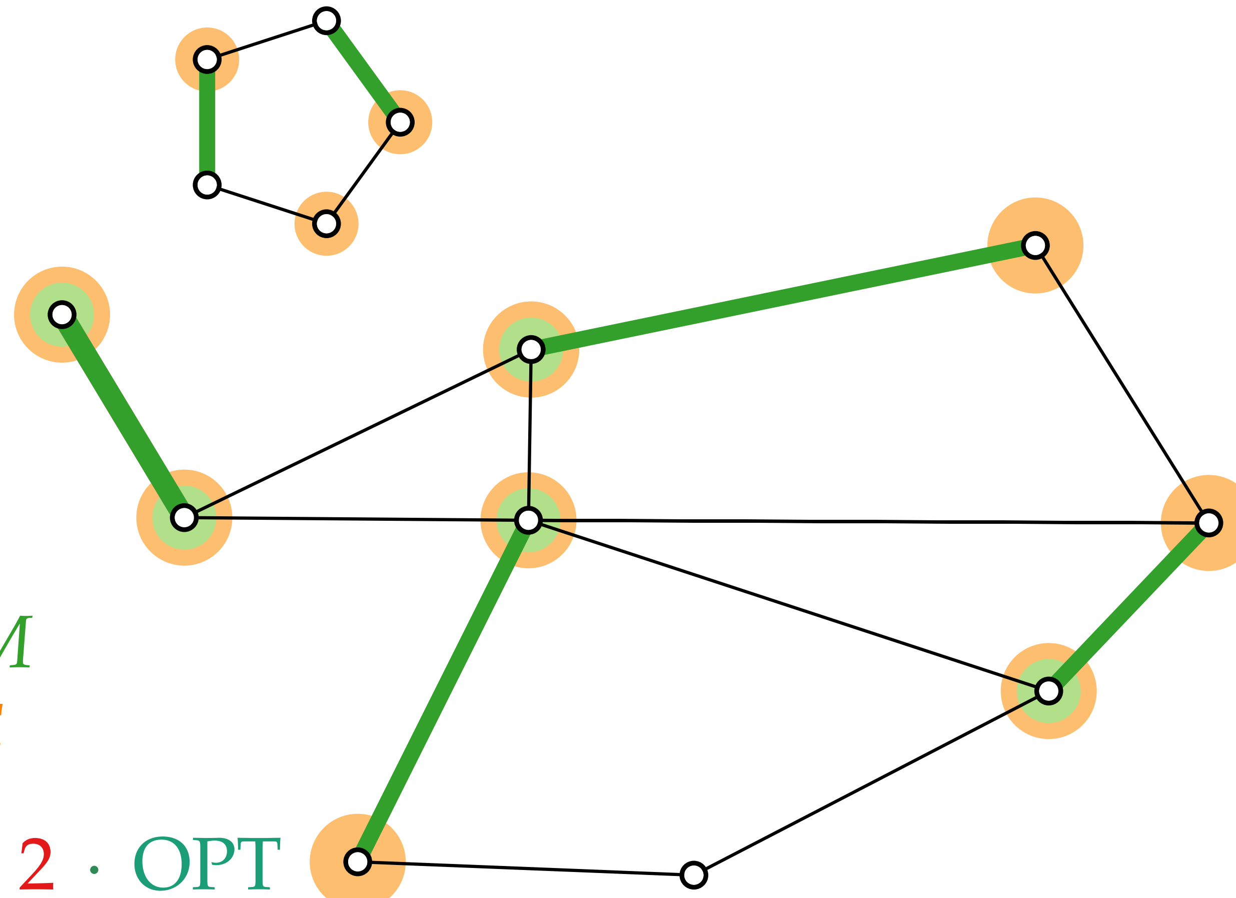

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Approximation Alg. for VertexCover

Algorithm VertexCover(G)

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