2.5 HOMOGENEOUS LINEAR EQUATIONS

An equation of the form $x^n \frac{d^n y}{dx^n} + P_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \ldots\ldots + P_n y = \phi(x)$

where $P_1, P_2, \ldots\ldots, P_n$ are real constants and $\varphi(x)$ is a function of x is called a homogeneous linear equation or Euler–Cauchy's linear equation of order n.

The equation in the operator form is $(x^n D^n + P_1 x^{n-1} D^{n-1} + \ldots + P_n) y = \phi(x)$

where $\frac{d}{dx} \equiv D$. Cauchy's linear differential equation can be transformed into a linear equation with constant coefficients by the change of independent variable with the substitution

$$x = e^z \text{ (or) } z = \log x, \; x > 0 \qquad \ldots(1)$$

$$\therefore \frac{dz}{dx} = \frac{1}{x}. \quad \text{Now} \quad \frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = \frac{1}{x}\frac{dy}{dz}$$

or
$$x \cdot \frac{dy}{dx} = \frac{dy}{dx} \qquad \ldots(2)$$

Again $\frac{d^2 y}{dx^2} = \frac{d}{dx}\left(\frac{1}{x}\frac{dy}{dz}\right) = -\frac{1}{x^2}\frac{dy}{dz} + \frac{1}{x}\frac{d}{dx}\left(\frac{dy}{dz}\right)$

$$= \frac{-1}{x^2}\frac{dy}{dz} + \frac{1}{x}\frac{d}{dz}\left(\frac{dy}{dz}\right)\left(\frac{dz}{dx}\right) = \frac{-1}{x^2}\frac{dy}{dz} + \frac{1}{x^2}\frac{d^2 y}{dz^2}$$

i.e., $$x^2\frac{d^2y}{dx^2}=\frac{d^2y}{dz^2}-\frac{dy}{dz} \quad ...(3)$$

Similarly, we can prove that

$$x^3\frac{d^3y}{dx^3}=\frac{d^3y}{dz^3}-3\frac{d^2y}{dz^2}+2\frac{dy}{dz} \quad ...(4)$$

Let us denote $\frac{d}{dx}\equiv D$ and $\frac{d}{dz}\equiv\theta$. Then (2), (3), (4) can be written as $xD=\theta$, $x^2D^2=\theta(\theta-1), x^3D^3=\theta(\theta-1)(\theta-2)$ etc.

This will be illustrated through the following examples.

Using these in (1), the Euler–Cauchy equation reduces to a differential equation with constant coefficients where y is dependent variable and z is independent variable. By methods discussed earlier, the equation can be solved and we get the required solution by putting $z=\log x$ in this solution.

SOLVED EXAMPLES

Example 1 : Solve $(x^2D^2-4xD+6)y=x^2$

(2) $(x^2D^2-3xD+4)y=2x^2$

$y=(c_1z+c_2)e^{2z}+z^2e^{2z}$

$\Rightarrow y=(c_1\log x+c_2)x^2+(\log x)^2x^2$

Solution : Given equation is $(x^2D^2-4xD+6)y=x^2$ ...(1)

This is a homogeneous differential equation.

Let $x=e^z$ or $\log x=z$ ...(2)

Let $\frac{d}{dx}=D$ and $\frac{d}{dz}=\theta$. Then we have $xD=\theta$ and $x^2D^2=\theta(\theta-1)$

Substituting in (1), we get

$$[\theta(\theta-1)-4\theta+6]y=e^{2z} \quad ...(3)$$

This is a differential equation with constant coefficients.

A.E. is $m^2-5m+6=0$. The roots are $m=3$ and $m=2$ which are real and different.

$\therefore$ C.F. is $y_c=c_1e^{3z}+c_2e^{2z}$

$$\text{P.I.}=y_p=\frac{e^{2z}}{(\theta-3)(\theta-2)}=\frac{e^{2z}}{(2-3)}\frac{z}{1!}=-ze^{2z}$$

$\therefore$ General solution is $y=y_c+y_p\Rightarrow y=c_1e^{3z}+c_2e^{2z}-ze^{2z}$

$$\Rightarrow y=c_1x^3+c_2x^2-(\log x)x^2$$

Example 2 : Solve $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=\log x$ [JNTU 1995, 2006 (Set No.4)]

Solution : Given equation in the operator form is

$$(x^2D^2-xD+1)y=\log x\text{, where } D\equiv\frac{d}{dx} \quad ...(1)$$

Let $x=e^z$ so that $z=\log x$ ...(2)

Denoting $\theta=d/dz$, $xD=\theta, x^2D^2=\theta(\theta-1)$, so that (1) becomes

$$[\theta(\theta-1)-\theta+1]y=z$$

i.e. $(\theta^2-2\theta+1)y=z$ ...(3)

This is a linear differential equation with constant coefficients.

A.E. is $m^2-2m+1=0 \Rightarrow (m-1)^2=0$. The roots are $m=1, 1$ real and repeated.

$\therefore$ C.F. $= y_c=(c_1+c_2z)e^z$

$$\text{P.I.} = y_p=\frac{z}{(\theta-1)^2}=(1-\theta)^{-2}(z)=(1+2\theta+....)z=z+2$$

$\therefore$ General solution of (1) is $y=y_c+y_p$

i.e. $y=(c_1+c_2z)e^z+(z+2)=(c_1+c_2\log x)x+\log x+2$

Example 3 : Solve $x^3\dfrac{d^3y}{dx^3}+2x^2\dfrac{d^2y}{dx^2}+2y=10\left(x+\dfrac{1}{x}\right)$ **[JNTU 2007S (Set No. 2)]**

Solution : Given equation in the operator form is

$$(x^3D^3+2x^2D^2+2)y=10\left(x+\frac{1}{x}\right) \text{ where } \frac{d}{dx}\equiv D. \qquad ...(1)$$

Let $x=e^z$ or $\log x=z$ ...(2)

Denote $\dfrac{d}{dz}\equiv\theta$. Then we have

$$x^3D^3=\theta(\theta-1)(\theta-2) \qquad ...(3)$$

and $x^2D^2=\theta(\theta-1)$ ...(4)

Substituting in (1), we get

$$\left[\theta(\theta-1)(\theta-2)+2\theta(\theta-1)+2\right]y=10(e^z+e^{-z})$$

i.e. $(\theta^3-\theta^2+2)y=10(e^z+e^{-z})$ ...(5)

This is a linear differential equation with constant coefficients.

Let $f(\theta)=\theta^3-\theta^2+2$. Then A.E. is $f(m)=0$ *i.e.,* $m^3-m^2+2=0$

$(m+1)(m^2-2m+2)=0$. The roots are $m=-1, 1+i, 1-i$. One root is real and two roots are complex conjugate numbers.

$\therefore$ C.F. $= y_c=c_1e^{-z}+e^z(c_2\cos z+c_3\sin z)$ and P.I. $=\dfrac{10(e^z+e^{-z})}{\theta^3-\theta^2+2}=y_{p_1}+y_{p_2}$

Now $y_{p_1}=\text{PI}_1=10\left(\dfrac{e^z}{\theta^3-\theta^2+2}\right)$ [Put $\theta=1$]

$$=10\left(\frac{e^z}{1-1+2}\right)=5e^z$$

and $y_{p_2}=\text{PI}_2=10\left(\dfrac{e^{-z}}{\theta^3-\theta^2+2}\right)=10\dfrac{(e^{-z})}{(\theta+1)(\theta^2-2\theta+2)}$

$$=\frac{10(e^{-z})}{5}z=2ze^{-z}$$

The general solution is $y=y_c+y_{p_1}+y_{p_2}$

(*i.e.*) $y=c_1e^{-z}+e^z(c_2\cos z+c_3\sin z)+5e^z+2ze^{-z}$

or $y=c_1x^{-1}+x(c_1\cos\log x+c_2\sin\log x)+5x+2\log x\left(\dfrac{1}{x}\right)$

2.6 LEGENDRE'S LINEAR EQUATION

An equation of the form

$$(a+bx)^n \frac{d^n y}{dx^n} + P_1 (a+bx)^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \ldots\ldots + P_n y = Q(x)$$

where $P_1, P_2, \ldots, P_n$ are real constants and $Q(x)$ is a function of x is called Legendre's linear equation.

This can be solved by the substitution $a + bx = e^z$ or $z = \log(a + bx)$ and $\theta \equiv \frac{d}{dz}$

SOLVED EXAMPLES

Example 1 : Solve $(x+1)^2 \dfrac{d^2y}{dx^2} - 3(x+1)\dfrac{dy}{dx} + 4y = x^2 + x + 1$

Solution : Given equation in the operator form is

$$\left[(x+1)^2 D^2 - 3(x+1)D + 4\right]y = x^2 + x + 1 \qquad \text{...(1)}$$

This is a Legendre's differential equation.

Let $x + 1 = u$ so that $x = u - 1$, $\dfrac{du}{dx} = 1$.

Now $\quad \dfrac{dy}{dx} = \dfrac{dy}{du}\cdot\dfrac{du}{dx} = \dfrac{dy}{du}$

Then given equation becomes $u^2 \dfrac{d^2y}{du^2} - 3u\dfrac{dy}{du} + 4y = (u-1)^2 + (u-1) + 1$

$$\Rightarrow \quad u^2 \frac{d^2y}{du^2} - 3u\frac{dy}{du} + 4y = u^2 - u + 1 \qquad \text{...(2)}$$

Let $u = e^z$ so that $z = \log u$ ...(3)

Denote $\dfrac{d}{dz} = \theta$. Then $u^2 \dfrac{d^2y}{du^2} = \theta(\theta-1)y$ ...(4)

and $u\dfrac{dy}{du} = \theta y$ ...(5)

Substituting in (2), we get $[\theta(\theta-1) - 3\theta + 4]\ y = e^{2z} - e^z + 1$

$$\Rightarrow \quad (\theta^2 - 4\theta + 4)y = e^{2z} - e^z + 1 \qquad \text{...(6)}$$

A.E. is $m^2 - 4m + 4 = 0 \Rightarrow (m-2)^2 = 0$. The roots are $m = 2, 2$ which are real and equal.

$\therefore \quad$ C.F. $= y_c = (c_1 + c_2 z)e^{2z}$

$$\text{PI}_1 = y_{p_1} = \frac{e^{2z}}{(\theta-2)^2} = \frac{z^2}{2}e^{2z}$$

$$\text{PI}_2 = y_{p_2} = \frac{-e^z}{(\theta-2)^2} = \frac{-e^z}{1}$$

$$\text{PI}_3 = y_{p_3} = \frac{1}{(\theta-2)^2} = \frac{1}{(-2)^2} = \frac{1}{4}$$

General solution is $y = y_c + y_{p_1} + y_{p_2} + y_{p_3}$

i.e. $\quad y = (c_1 + c_2 z)e^{2z} + \dfrac{z^2}{2}e^{2z} - e^z + \dfrac{1}{4}$

$$= (c_1 + c_2 \log u)u^2 + \frac{(\log u)^2}{2}u^2 - u + \frac{1}{4}$$

or $\quad y = [c_1 + c_2 \log(x+1)](x+1)^2 + \dfrac{1}{2}\,[(\log(x+1)]^2(x+1)^2 - (x+1) + \dfrac{1}{4}$

Example 2 : Solve the D.E., $(2x-1)^3\dfrac{d^3y}{dx^3}+(2x-1)\dfrac{dy}{dx}-2y=x.$ [JNTU 2007 (Set No. 3)]

Solution : Given Equation is

$$(2x-1)^3\frac{d^3y}{dx^3}+(2x-1)\frac{dy}{dx}-2y=x \qquad \text{...(1)}$$

...(2)

Put $2x-1=t$

$$\Rightarrow \quad 2=\frac{dt}{dx} \therefore \frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}=2\frac{dy}{dt} \qquad \text{...(3)}$$

Similarly $\dfrac{d^2y}{dx^2} = 2^2\dfrac{d^2y}{dt^2}$ ...(4)

$$\frac{d^3y}{dx^3}=2^3\frac{d^3y}{dt^3} \qquad \text{...(5)}$$

Substituting these in (1), we get

$$8t^3\frac{d^3y}{dt^3}+2t\frac{dy}{dt}-2y=\frac{t+1}{2} \qquad \text{...(6)}$$

Let $z=\log t \Rightarrow t=e^z$, and $\dfrac{d}{dz}=\theta$. Then $t\dfrac{d}{dt}=\theta, t^3\dfrac{d^3}{dt^3}=\theta(\theta-1)(\theta-2)$

Now (6) becomes

$$[8\,\theta(\theta-1)(\theta-2)+2\,\theta-2]y=\frac{e^z+1}{2}$$

$$\Rightarrow (8\theta^3-24\theta^2+18\theta-2)\ y=\frac{e^z+1}{2}$$

A.E. is $8m^3-24m^2+18m-2=0$

$\therefore \quad m=1$ and $m=1\pm\sqrt{3}/2$

C.F. is $y_c=c_1e^z+c_2e^{(1+\sqrt{3}/2)z}+c_3e^{(1-\sqrt{3}/2)z}$

Now $y_{p_1}=\dfrac{e^z}{2(8\theta^3-24\theta^2+18\theta-2)}=\dfrac{e^z}{2(\theta-1)(8\theta^2-16\theta+2)}$

$$=\frac{e^z\cdot z}{2(8-16+2)}=-\frac{ze^z}{12} \quad [\text{Putting } \theta=1]$$

and $y_{p_2}=\dfrac{1/2}{8\theta^3-24\theta^4+18\theta-2}=\dfrac{1/2}{-2}=-\dfrac{1}{4}$

$\therefore$ General solution is

$$y=y_c+y_{p_1}+y_{p_2}=c_1e^z+c_2e^{(1+\sqrt{3}/2)z}+c_3e^{(1-\sqrt{3}/2)z}+\frac{ze^z}{-12}-\frac{1}{4}, \text{ where } z=\log(t)=\log(2$$

i.e. $y = c_1u(x) + c_2v(x) + A(x)u(x) + B(x)v(x)$.

Working Rule :

To solve $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$ by the method of variation of parameters, follow these steps:

1. Reduce the given equation to the standard form, if necessary.
2. Find the solution of

   $$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$$

   and let the solution be

   C.F. $= y_c = c_1u(x) + c_2v(x)$
3. Take P.I. $= y_p = Au + Bv$, where A and B are functions of x
4. Find $W(u, v) = u\frac{dv}{dx} - v\frac{du}{dx}$
5. Find A and B using

   $$A = -\int \frac{vRdx}{W(u,v)} = -\int \frac{vR\,dx}{u\frac{dv}{dx} - v\frac{du}{dx}}, \quad B = \int \frac{uR\,dx}{u\frac{dv}{dx} - v\frac{du}{dx}} = \int \frac{uRdx}{W(u,v)}$$
6. Write the general solution of the given equation as

   $y = y_c + y_p$

   i.e., $y = c_1u(x) + c_2v(x) + A(x)u(x) + B(x)v(x)$, where c_1 and c_2 are constants.

SOLVED EXAMPLES

Example 1 : Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} + y = \text{cosec}\, x$

[JNTU 1995S, 1998, 1999, 2007S (Set No. 4)]

Solution : Given equation in the operator form is $(D^2 + 1)y = \text{cosec}\, x$ ...(1)

A.E. is $m^2 + 1 = 0 \Rightarrow m = \pm i$. The roots are complex conjugate numbers.

$\therefore$ C.F is $y_c = c_1 \cos x + c_2 \sin x$ ...(2)

Let $y_p = A\cos x + B\sin x$ be P.I. of (1).

Here $u = \cos x$, $v = \sin x$

$$\therefore \quad u\frac{dv}{dx}-v\frac{du}{dx}=\cos^2 x+\sin^2 x=1$$

A and B are given by

$$A=-\int \frac{vR}{\frac{udv}{dx}-\frac{vdu}{dx}}dx=-\int \frac{\sin x \operatorname{cosec} x}{1}dx=-\int dx=-x$$

$$B=\int \frac{uR}{\frac{udv}{dx}-\frac{vdu}{dx}}dx=\int \cos x.\operatorname{cosec} x dx=\int \cot x dx=\log|\sin x|$$

$\therefore \quad y_p=-x\cos x+\sin(x)\log|\sin x|$

$\therefore$ The general solution is given by $y=y_c+y_p$

(i.e.) $\quad y=c_1\cos x+c_2\sin x-x\cos x+\sin x\log(\sin x)$

Example 2 : Solve $(D^2+a^2)y=\tan ax$, by the method of variation of parameters.

[JNTU (ATP) June 2009 (Set No.4)]

Solution : Given equation is $(D^2+a^2)y=\tan ax$ ...(1)

A.E. is $m^2+a^2=0 \Rightarrow m=\pm ai$. The roots are complex conjugate numbers.

$\therefore \quad y_c=\text{C.F.}=c_1\cos ax+c_2\sin ax.$

Let $y_p=\text{P.I.}=A\cos ax+B\sin ax$

Let $u=\cos ax$ and $v=\sin ax$, and $R=\tan ax$

Now, $u\frac{dv}{dx}-v\frac{du}{dx}=a$

A and B are given by

$$A=-\int \frac{vR}{u\frac{dv}{dx}-v\frac{du}{dx}}dx=-\int \frac{\sin ax.\tan ax}{a}dx$$

$$=-\frac{1}{a}\int \frac{\sin^2 ax}{\cos ax}dx=-\frac{1}{a}\int \frac{1-\cos^2 ax}{\cos ax}dx=-\frac{1}{a}\left[\int \sec ax dx-\int \cos ax\, dx\right]$$

or $\quad A=-\frac{1}{a^2}\log|\sec ax+\tan ax|+\frac{1}{a^2}\sin ax$

$$B=\int \frac{uR\,dx}{u\frac{dv}{dx}-v\frac{du}{dx}}=\int \frac{\cos ax\tan ax}{a}dx=\frac{1}{a}\int \sin ax dx=-\frac{1}{a^2}\cos ax$$

The general solution is given by $y=y_c+y_p$

i.e., $y=c_1\cos ax+c_2\sin ax+\left[-\frac{1}{a^2}\log|\sec ax+\tan ax|+\frac{1}{a^2}\sin ax\right]\cos ax-\frac{1}{a^2}\cos ax.\sin ax$

which can be written as $y=c_1\cos ax+c_2\sin ax-\frac{\cos ax}{a^2}\log|\sec ax+\tan ax|$

Example 3 : Solve $(D^2+1)y=\cos x$ by the method of variation of parameters.

[JNTU 2003S (Set No

Solution : Given equation is

$(D^2+1)y=\cos x$ ...(1)

Here $\quad P=0, \quad Q=1, \quad R=\cos x$

Auxiliary equation is $m^2 + 1 = 0$

$\therefore$ Complementary Function is $y_c = c_1 \cos x + c_2 \sin x$

Let the Particular Integral of (1) be

$$y_p = A \cos x + B \sin x \quad \text{...(2)}$$

where A and B are functions of x.

Let $u = \cos x$ and $v = \sin x$.

$$\therefore \quad u\frac{dv}{dx} - v\frac{du}{dx} = \cos x (\cos x) - \sin x (-\sin x) = \cos^2 x + \sin^2 x = 1$$

A and B are given by

$$A = -\int \frac{vR}{u\frac{dv}{dx} - v\frac{du}{dx}} dx = -\int \frac{\sin x \cos x \, dx}{1} = -\frac{1}{2}\int \sin 2x \, dx$$

$$= -\frac{1}{2}\left(-\frac{\cos 2x}{2}\right) = \frac{1}{4} \cos 2x$$

and $$B = \int \frac{uR}{u\frac{dv}{dx} - v\frac{du}{dx}} dx = \int \frac{\cos x(\cos x)}{1} dx = \int \cos^2 x \, dx$$

$$= \frac{1}{2}\int (1 + \cos 2x)\, dx = \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right)$$

$$\therefore y_p = \frac{1}{4} \cos 2x \,.\, \cos x + \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) \sin x, \text{ [by (2)]}$$

$\therefore$ General solution of (1) is

$$y = y_c + y_p$$

i.e. $$y = c_1 \cos x + c_2 \sin x + \frac{1}{4} \cos x \,.\, \cos 2x + \frac{1}{2} \sin x (x + \sin x \cos x).$$

Example 4 : Solve $\frac{d^2y}{dx^2} + y = x \cos x$ by the method of variation of parameters

<u>Solution :</u> Given equation in the operator form is

$$(D^2 + 1)y = x \cos x \quad \text{...(1)}$$

A.E. is $m^2 + 1 = 0 \Rightarrow m = \pm i$. The roots are complex conjugate numbers.

C.F. $= y_c = c_1 \cos x + c_2 \sin x$. Let $u = \cos x$, $v = \sin x$, $R = x \cos x$

Then $u\frac{dv}{dx} - v\frac{du}{dx} = \cos^2 x + \sin^2 x = 1$

Let $y_p = \text{P.I.} = A \cos x + B \sin x$, where A and B are functions of x.

A is given by

$$A = -\int \frac{vR}{u\frac{dv}{dx} - v\frac{du}{dx}} dx = -\int \frac{\sin x . x \cos x}{1} dx = -\frac{1}{2}\int x \sin 2x \, dx$$

$$= -\frac{1}{2}\left[-\frac{x}{2}\cos 2x + \frac{1}{2}\int \cos 2x dx\right] = \frac{x}{4}\cos 2x - \frac{1}{8}\sin 2x.$$

$$B = \int \frac{uR\, dx}{u\frac{dv}{dx} - v\frac{du}{dx}} = \int \frac{\cos x . x \cos x}{1} dx = \int \frac{x(1+\cos 2x)}{2} dx$$

$$= \frac{x^2}{4} + \frac{x \sin 2x}{4} + \frac{\cos 2x}{8} \text{ (Integrating by parts)}$$

The general solution is given by $y = y_c + y_p$

i.e. $$y = c_1 \cos x + c_2 \sin x + \left(\frac{x}{4}\cos 2x - \frac{1}{8}\sin 2x\right)\cos x + \left[\frac{x^2}{4} + \frac{x\sin 2x}{4} + \frac{\cos 2x}{8}\right]\sin x$$

Example 5: Solve by the method of variation of parameters $(D^2 - 2D)y = e^x \sin x$ [JNTU 2002S (Set No. 1)]

Solution: Given equation is $(D^2 - 2D)y = e^x \sin x$ ...(1)

AE is $m^2 - 2m = 0 \Rightarrow m(m-2) = 0 \Rightarrow m = 0$ or $m = 2$

The roots are real and different $\therefore$ C.F. $= y_c = c_1 + c_2 e^{2x}$

Let $y_p = A(x) + B(x).e^{2x}$, where A and B are functions of x

Let $u = 1$, $v = e^{2x}$ and $R = e^x \sin x$

Then $u\frac{dv}{dx} - v\frac{du}{dx} = 1(2e^{2x}) - e^{2x}(0) = 2e^{2x}$

A and B are given by

$$A = -\int \frac{vR\, dx}{u\frac{dv}{dx} - v\frac{du}{dx}} = -\int \frac{e^{2x}e^x \sin x}{2e^{2x}} dx$$

$$= \frac{-1}{2}\int e^x \sin x dx = \frac{-1}{4}e^x(\sin x - \cos x)$$

$$B = \int \frac{uR}{u\frac{dv}{dx} - v\frac{du}{dx}} dx = \int \frac{e^x \sin x}{2e^{2x}} dx = \frac{1}{2}\int e^{-x}\sin x dx = -\frac{e^{-x}}{4}(\sin x + \cos x)$$

$\therefore$ $y_p = A + Be^{2x}$

$$= \frac{-1}{4}e^x(\sin x - \cos x) - \frac{e^{-x}}{4}(\sin x + \cos x)e^{2x} = \frac{1}{2}e^x \sin x.$$

$\therefore$ The general solution of (1) is $y = y_c + y_p$

i.e. $y = c_1 + c_2 e^{2x} - \frac{1}{2}e^x \sin x.$

Example 6: Solve $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$

Solution: Given equation in the operator form is $(D^2 - 1)y = \frac{2}{1+e^x}$ ...(1)

AE is $m^2 - 1 = 0 \Rightarrow m = \pm 1$. The roots are real and different.