STAT 440 Homework 12

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1 A

To find the conditional distribution we can utilize the formula for finding a,b fron the lecture notes combined with the given information:

$\bar{Y} = 150$

$\sigma^2 = 20$

$\sigma_0^2 = 40$

$\mu_0 = 180$

$$\mu|\bar{Y} => N(\frac{\frac{\bar{Y}}{\frac{\sigma^2}{n}}}{+\frac{\mu_0}{\sigma_0^2}}\frac{1}{\frac{\sigma^2}{n}} + \frac{1}{\sigma_0^2}, \frac{1}{\frac{1}{\frac{\sigma^2}{n}}+\frac{1}{\sigma_0^2}})$$

After plugging in our values we get the form:

$$\mu|\bar{Y} => N(\frac{\frac{150n}{400}+\frac{180}{1600}}{\frac{n}{400}+\frac{1}{1600}}, \frac{1}{\frac{n}{400}+\frac{1}{1600}})$$

2 B

We need to find $\tilde{Y}|\bar{Y}$, we know that they both belong to the Normal distribution so the conditional distribution here is also a Normal distribution.
We need to integrate the product of $P(\tilde{Y}|\theta) * P(\theta|\bar{Y})$ in terms of θ to get rid of the term

$$\int \frac{1}{20\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{\tilde{Y}-\theta}{20})^2} * \frac{1}{\sqrt{\frac{400}{n}+1600}\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{\theta-\frac{\frac{150n}{400}+\frac{180}{1600}}{\frac{n}{400}+\frac{1}{1600}}}{\sqrt{\frac{400}{n}+1600}})^2} d\theta$$

This integral is too complicated to solve directly but we can use a trick, because we know that it is a Normal Distribution as the answer we can just use the Constant C_1, C_2, C_3 to fill in the blanks:
$C_1 exp(-\tilde{Y}^2 - C_2\tilde{Y})C_3$
with this and some clever manipulation we can get the form:
$exp(-(\tilde{Y} - C_2)^2 C_3$
Knowing this we simply need to find $\mathrm{E}[\tilde{Y}|\bar{Y}]$ and $\mathrm{Var}[\tilde{Y}|\bar{Y}]$
The expectation does not change and is still equal to what we got in section A:
$\mathrm{E}[\tilde{Y}|\bar{Y}] = \frac{\frac{150n}{400}+\frac{180}{1600}}{\frac{n}{400}+\frac{1}{1600}}$
$\mathrm{Var}[\tilde{Y}|\bar{Y}] = \mathrm{E}[\mathrm{Var}(\tilde{Y}|\theta, \bar{Y})|\bar{Y}]$
$= Var(\theta|\bar{Y}) + \sigma^2$
$= \frac{1600}{4n+1} + 400$
$=> N(\frac{\frac{150n}{400}+\frac{180}{1600}}{\frac{n}{400}+\frac{1}{1600}}, \frac{1600}{4n+1} + 400)$

3 C

```
rm(list = ls())
set.seed(440)

n <- 10
mu <- (((150*n)/400)+(180/1600))/((n/400)+(1/1600))
sigma <- 1/((n/400)+(1/1600))

c(mu - 1.96*(sqrt(sigma)),mu + 1.96*sqrt(sigma))
```

```
> mu <- (((150*n)/400)+(180/1600))/((n/400)+(1/1600))
> sigma <- 1/((n/400)+(1/1600))
> c(mu - 1.96*(sqrt(sigma)),mu + 1.96*sqrt(sigma))
[1] 138.4877 162.9757
>
```

4 D

```r
n <- 100
mu <- (((150*n)/400)+(180/1600))/((n/400)+(1/1600))
sigma <- 1/((n/400)+(1/1600))

c(mu - 1.96*(sqrt(sigma)),mu + 1.96*sqrt(sigma))
```

```
>
> n <- 100
> mu <- (((150*n)/400)+(180/1600))/((n/400)+(1/1600))
> sigma <- 1/((n/400)+(1/1600))
> c(mu - 1.96*(sqrt(sigma)),mu + 1.96*sqrt(sigma))
[1] 146.1597 153.9899
>
```