

STAT 440 Homework 12

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1 A

To find the conditional distribution we can utilize the formula for finding a,b from the lecture notes combined with the given information:

$$\bar{Y} = 150$$

$$\sigma^2 = 20$$

$$\sigma_0^2 = 40$$

$$\mu_0 = 180$$

$$\mu|\bar{Y} \Rightarrow N\left(\frac{\frac{\bar{Y}}{\sigma^2}}{\frac{\mu_0}{\sigma_0^2} + \frac{1}{\sigma^2}}, \frac{1}{\frac{1}{\sigma_0^2} + \frac{1}{\sigma^2}}\right)$$

After plugging in our values we get the form:

$$\mu|\bar{Y} \Rightarrow N\left(\frac{\frac{150n}{400} + \frac{180}{1600}}{\frac{n}{400} + \frac{1}{1600}}, \frac{1}{\frac{n}{400} + \frac{1}{1600}}\right)$$

2 B

We need to find $\tilde{Y}|\bar{Y}$, we know that they both belong to the Normal distribution so the conditional distribution here is also a Normal distribution.

We need to integrate the product of $P(\tilde{Y}|\theta) * P(\theta|\bar{Y})$ in terms of θ to get rid of the term

$$\int \frac{1}{20\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{\tilde{Y}-\theta}{20})^2} * \frac{1}{\sqrt{\frac{400}{n}+1600}\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{\theta - \frac{\frac{150n}{400} + \frac{180}{1600}}{\frac{n}{400} + \frac{1}{1600}})^2} d\theta$$

This integral is too complicated to solve directly but we can use a trick, because we know that it is a Normal Distribution as the answer we can just use the Constant C_1, C_2, C_3 to fill in the blanks:

$$C_1 \exp(-\tilde{Y}^2 - C_2 \tilde{Y}) C_3$$

with this and some clever manipulation we can get the form:

$$\exp(-(\tilde{Y} - C_2)^2) C_3$$

Knowing this we simply need to find $E[\tilde{Y}|\bar{Y}]$ and $\text{Var}[\tilde{Y}|\bar{Y}]$

The expectation does not change and is still equal to what we got in section A:

$$E[\tilde{Y}|\bar{Y}] = \frac{\frac{150n}{400} + \frac{180}{1600}}{\frac{n}{400} + \frac{1}{1600}}$$

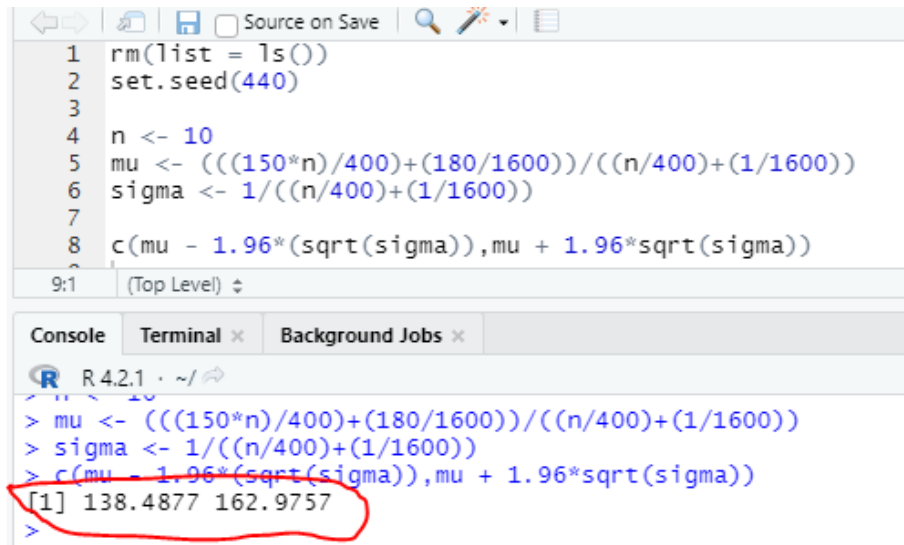
$$\text{Var}[\tilde{Y}|\bar{Y}] = E[\text{Var}(\tilde{Y}|\theta, \bar{Y})|\bar{Y}]$$

$$= \text{Var}(\theta|\bar{Y}) + \sigma^2$$

$$= \frac{1600}{4n+1} + 400$$

$$\Rightarrow N\left(\frac{\frac{150n}{400} + \frac{180}{1600}}{\frac{n}{400} + \frac{1}{1600}}, \frac{1600}{4n+1} + 400\right)$$

3 C



The image shows a screenshot of an R script editor and its console. The script editor has a toolbar at the top with icons for undo, redo, save, source on save, search, and help. The script contains the following code:

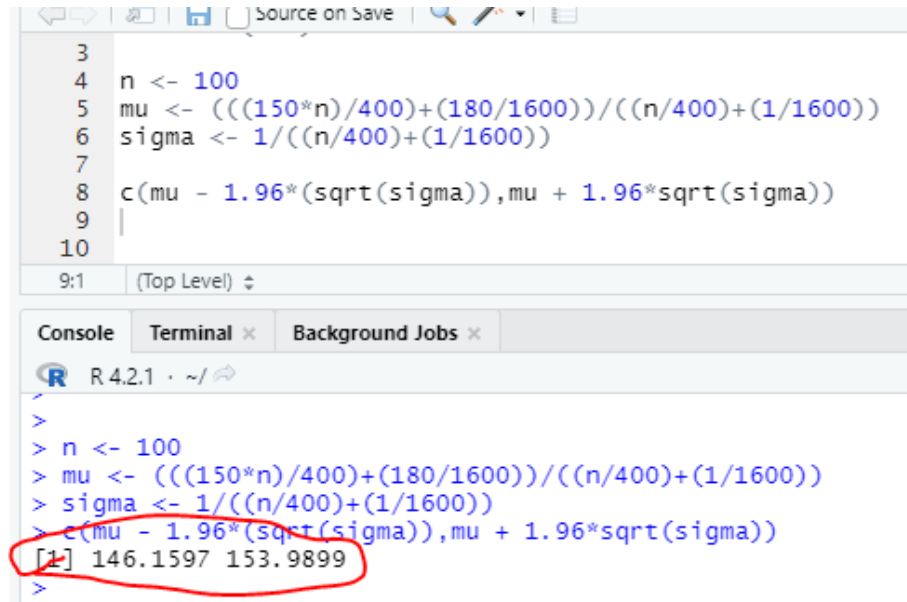
```
1 rm(list = ls())
2 set.seed(440)
3
4 n <- 10
5 mu <- (((150*n)/400)+(180/1600))/((n/400)+(1/1600))
6 sigma <- 1/((n/400)+(1/1600))
7
8 c(mu - 1.96*(sqrt(sigma)),mu + 1.96*sqrt(sigma))
```

The console shows the execution of the script, with the following output:

```
> mu <- (((150*n)/400)+(180/1600))/((n/400)+(1/1600))
> sigma <- 1/((n/400)+(1/1600))
> c(mu - 1.96*(sqrt(sigma)),mu + 1.96*sqrt(sigma))
[1] 138.4877 162.9757
>
```

The output line `[1] 138.4877 162.9757` is circled in red.

4 D



The image shows a screenshot of an R script editor and its console. The script editor at the top contains the following code:

```
3
4 n <- 100
5 mu <- (((150*n)/400)+(180/1600))/((n/400)+(1/1600))
6 sigma <- 1/((n/400)+(1/1600))
7
8 c(mu - 1.96*sqrt(sigma),mu + 1.96*sqrt(sigma))
9
10
```

Below the script editor is the console window, which shows the execution of the code. The output of the final line is circled in red:

```
>
> n <- 100
> mu <- (((150*n)/400)+(180/1600))/((n/400)+(1/1600))
> sigma <- 1/((n/400)+(1/1600))
> c(mu - 1.96*sqrt(sigma),mu + 1.96*sqrt(sigma))
[1] 146.1597 153.9899
>
```