

STAT 440 Homework 12

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1 A

First Coin

Prior = 0.5

Likelihood = 0.5

Evidence = $\frac{5}{8}$ Using Bayes theorem:

$$\frac{0.5 * 0.5}{\frac{5}{8}} = 0.4$$

Second Coin

Prior = 0.5

Likelihood = 0.75

Evidence = $\frac{5}{8}$ Using Bayes theorem:

$$\frac{0.5 * 0.75}{\frac{5}{8}} = 0.125$$

2 B

$$\text{P-value} = \sum_{i=3}^3 i \text{heads in } 3 \text{ tosses} | \theta = 0.5 = 0.125$$

3 C

Because there is equal chance of any number between 0 and 1, we will logically use the standard uniform distribution as our prior distribution for theta:
 $\text{Unif}(0,1)$

4 D

Our data is 3 tosses which all came up heads

$x = [H, H, H]$

Prior = Unif(0,1)

Given our data we get the likelihood = θ^3

as that is the probability of 3 heads in a row

We ignore the denominator of the Bayes theorem equation because it has nothing to do with our parameter:

and the Unif(0,1) is simply just 1, so our posterior probability is just θ^3

5 E

m = number of heads

n = number of tosses

given: $\text{Beta}(\alpha, \beta)$

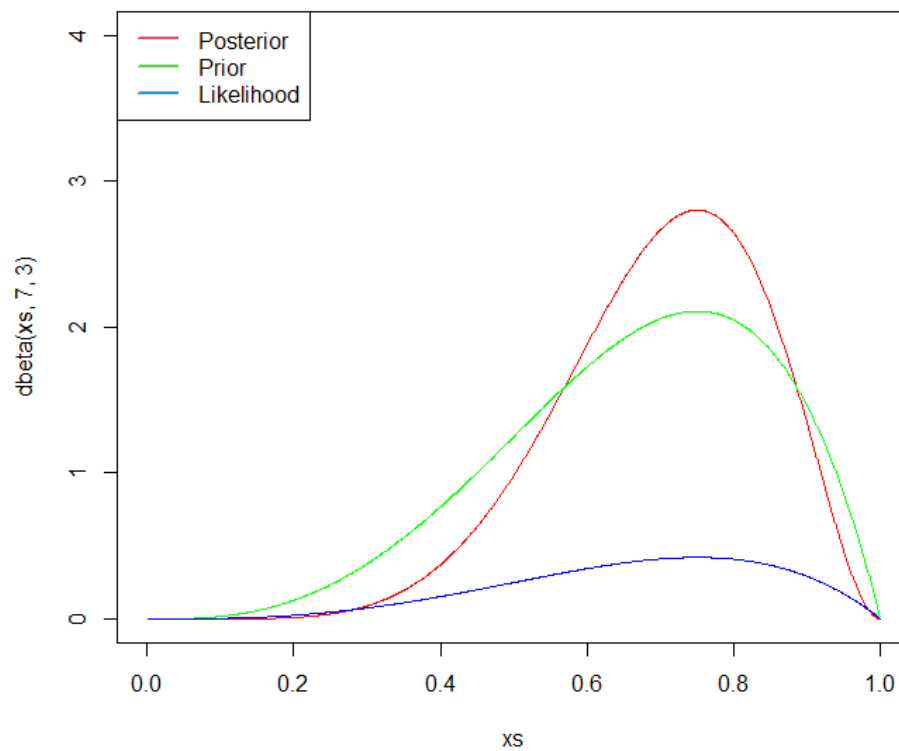
likelihood: $\text{rbinom}(m, n, x)$

Posterior: $\text{Beta}(m + \alpha, n - m + \beta)$

```

7 # define x range for plotting
8 xs = seq(.001, .999, .001)
9 |
10 set.seed(440)
11 xs = seq(.001, .999, .001)
12 #prior 1
13 plot(xs,dbeta(xs,7,3),type="l",col="red",ylim=c(0,4),xlim=c(0,1))
14 lines(xs,dbeta(xs,4,2),col="green")
15 lines(xs,dbinom(3,4,xs),col = "blue")
16 legend(x = "topleft", # Position
17       legend = c("Posterior", "Prior", "Likelihood"), # Legend texts
18       col = c( 2, 3, 4), # Line colors
19       lwd = 2) # Line width
20

```

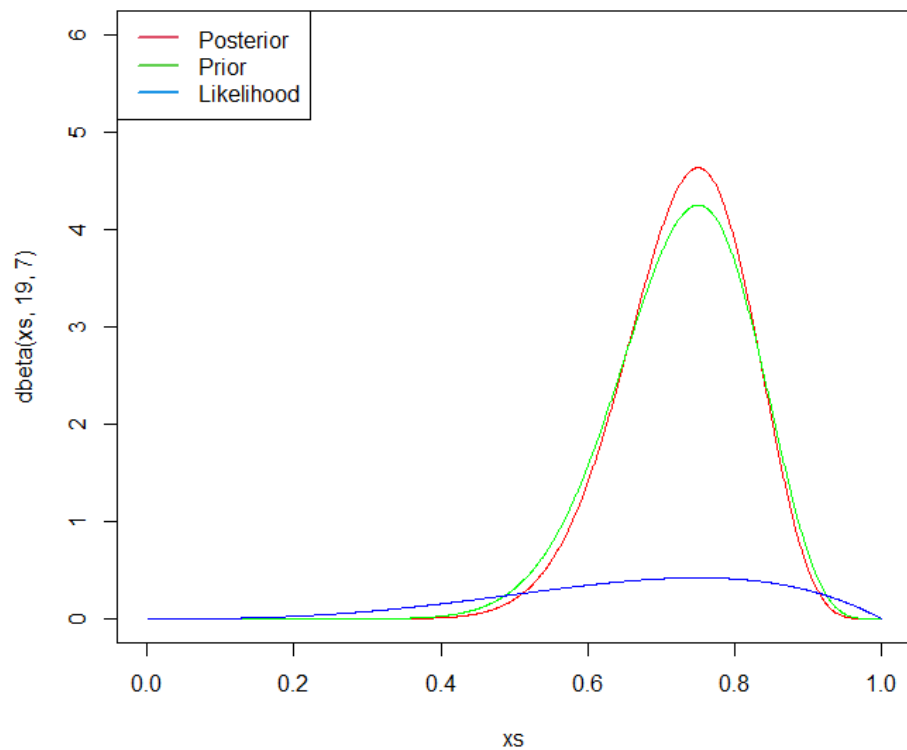


Observations for Beta(4,2) of this result make sense. The posterior is indeed between the prior and likelihood.

```

21 #prior 2
22 set.seed(440)
23 plot(xs,dbeta(xs,19,7),type="l",col="red",ylim=c(0,6),xlim=c(0,1))
24 lines(xs,dbeta(xs,16,6),col="green")
25 lines(xs,dbinom(3,4,xs),col="blue")
26 legend(x = "topleft", # Position
27       legend = c("Posterior", "Prior", "Likelihood"), # Legend texts
28       col = c(2, 3, 4), # Line colors
29       lwd = 2) # Line width
30

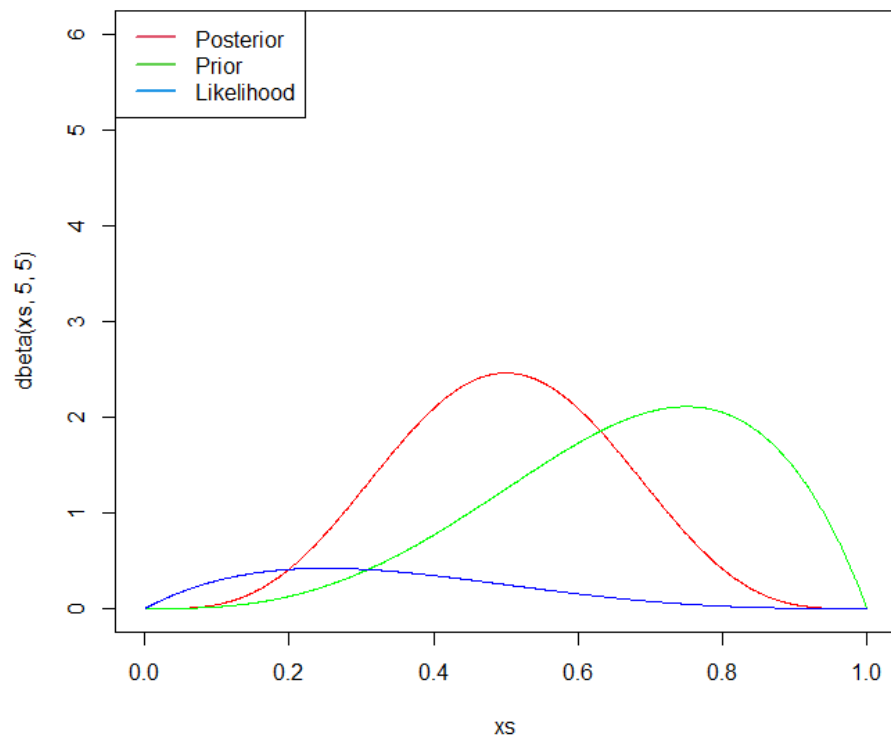
```



Observations for Beta(16,6) of this result make sense. The posterior is indeed between the prior and likelihood.

6 F

```
32 #part 1
33 set.seed(440)
34 plot(xs,dbeta(xs,5,5),type="l",col="red",ylim=c(0,6),xlim=c(0,1))
35 lines(xs,dbeta(xs,4,2),col="green")
36 lines(xs,dbinom(1,4,xs),col = "blue")
37 legend(x = "topleft",      # Position
38       legend = c("Posterior", "Prior", "Likelihood"), # Legend texts
39       col = c( 2, 3, 4),    # Line colors
40       lwd = 2)              # Line width
41
```

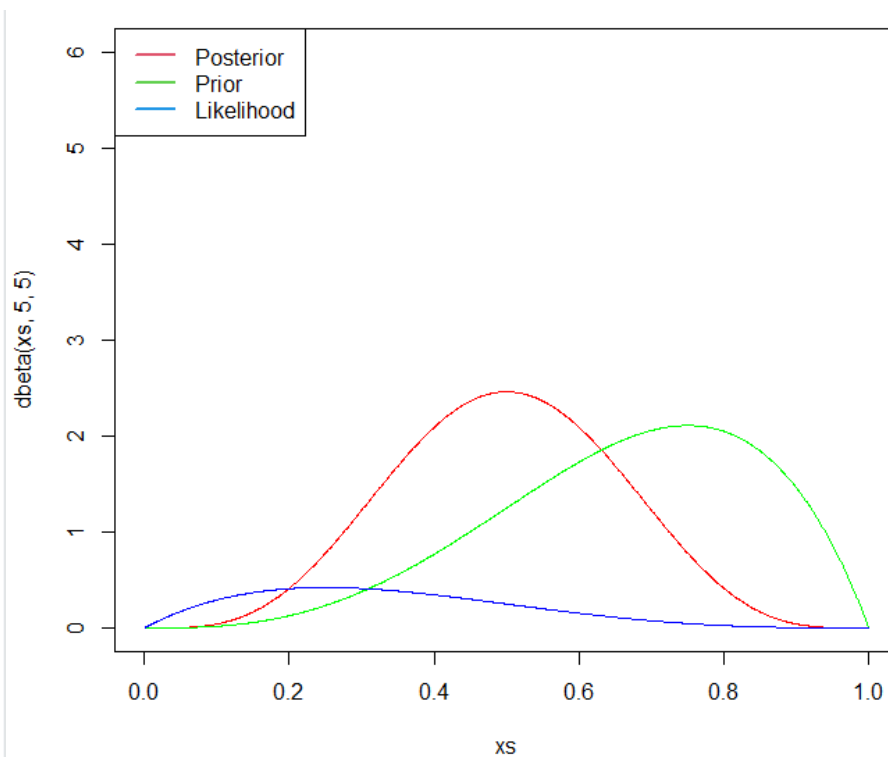


Observations for 4 tosses with this result make sense. The posterior is indeed between the prior and likelihood.


```

44 #part 2
45 set.seed(440)
46 plot(xs,dbeta(xs,5,5),type="l",col="red",ylim=c(0,6),xlim=c(0,1))
47 lines(xs,dbeta(xs,4,2),col="green")
48 lines(xs,dbinom(1,4,xs),col = "blue")
49 legend(x = "topleft", # Position
50       legend = c("Posterior", "Prior", "Likelihood"), # Legend texts
51       col = c( 2, 3, 4), # Line colors
52       lwd = 2) # Line width
53
54

```

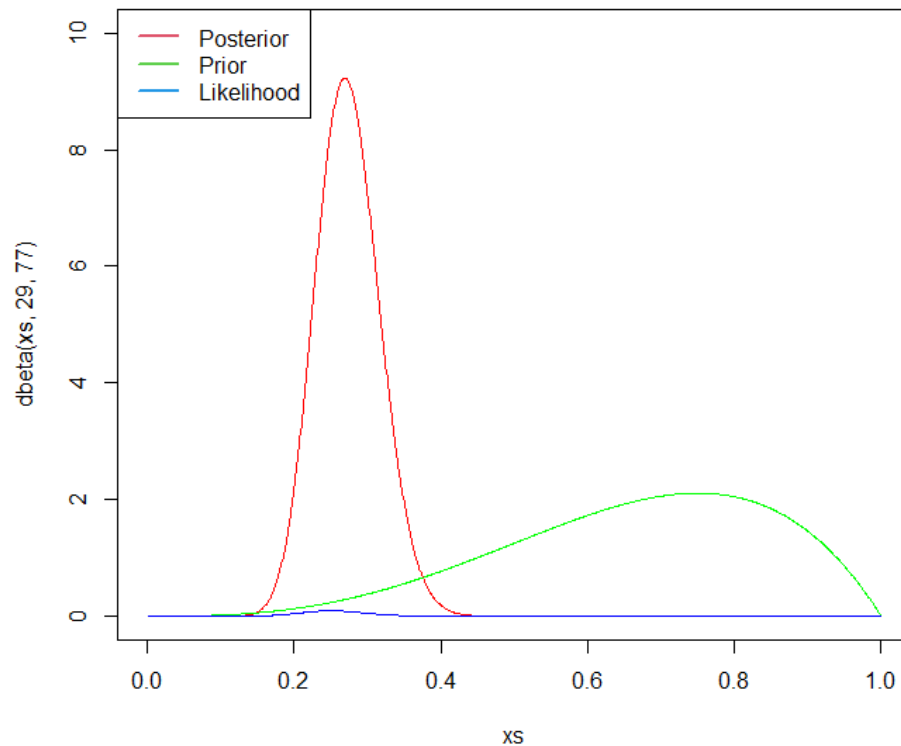


Observations for 20 tosses with this result make sense. The posterior is indeed between the prior and likelihood.

```

55 #part 3
56 set.seed(440)
57 plot(xs,dbeta(xs,29,77),type="l",col="red",ylim=c(0,10),xlim=c(0,1))
58 lines(xs,dbeta(xs,4,2),col="green")
59 lines(xs,dbinom(25,100,xs),col = "blue")
60 legend(x = "topleft", # Position
61       legend = c("Posterior", "Prior", "Likelihood"), # Legend texts
62       col = c( 2, 3, 4), # Line colors
63       lwd = 2) # Line width
64

```



Observations for 100 tosses is a little strange but from what I can tell the posterior is still inbetween the prior and likelihood.