STAT 440 Homework 12

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1 A

First Coin

Prior = 0.5

Likelihood = 0.5

Evidence = $\frac{5}{8}$ Using Bayes theorem:

$\frac{0.5*0.5}{\frac{5}{8}} = 0.4$

Second Coin

Prior = 0.5

Likelihood = 0.75

Evidence = $\frac{5}{8}$ Using Bayes theorem:

$\frac{0.5*0.75}{\frac{5}{8}} = 0.125$

2 B

P-value $= \sum_{i=3}^{3} i heads in 3 tosses | \theta = 0.5 = 0.125$

3 C

Because there is equal chance of any number between 0 and 1, we will logically use the standard uniform distribution as our prior distribution for theta: Unif(0,1)

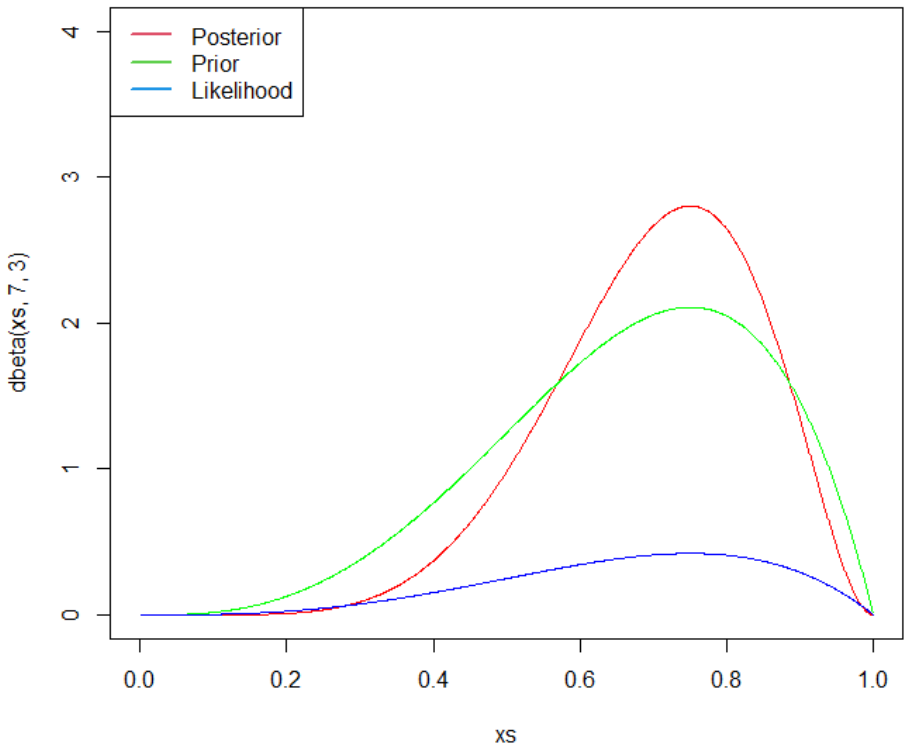
4 D

Our data is 3 tosses which all came up heads
$x = [H, H, H]$
Prior = Unif(0,1)
Given our data we get the likelihood = θ^3
as that is the probability of 3 heads in a row
We ignore the denominator of the Bayes theorem equation because it has nothing to do with our parameter:
and the Unif(0,1) is simply just 1, so our posterior probability is just θ^3

5 E

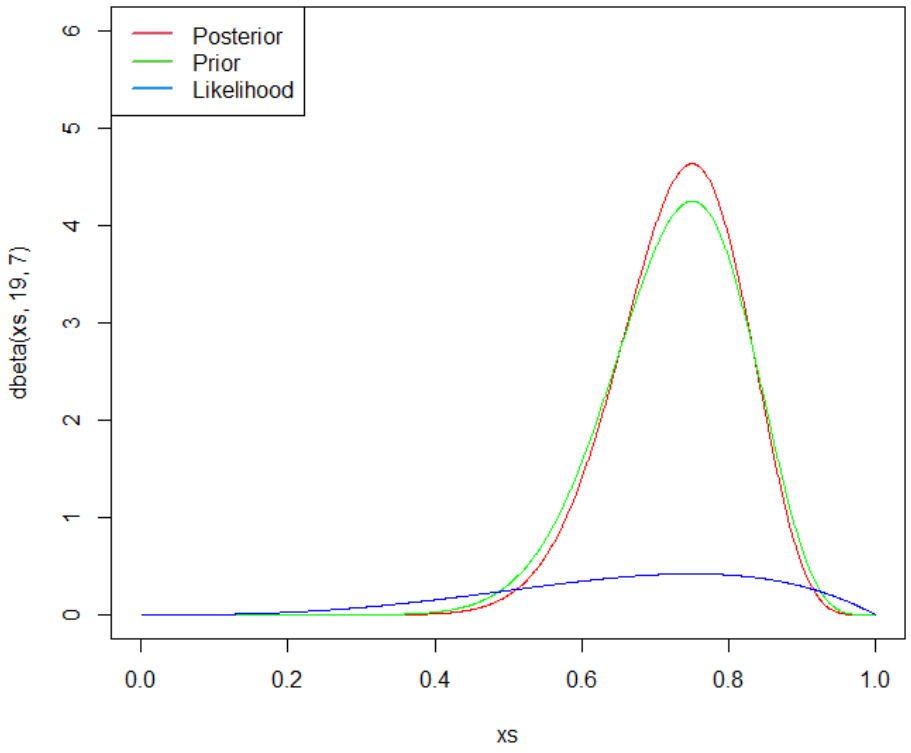
m = number of heads
n = number of tosses
given: Beta(α, β)
likelihood: rbinom(m,n,xs)
Posterior: Beta($m + \alpha, n - m + \beta$)

```r
# define x range for plotting
xs = seq(.001, .999, .001)

set.seed(440)
xs = seq(.001, .999, .001)
#prior 1
plot(xs,dbeta(xs,7,3),type="l",col="red",ylim=c(0,4),xlim=c(0,1))
lines(xs,dbeta(xs,4,2),col="green")
lines(xs,dbinom(3,4,xs),col = "blue")
legend(x = "topleft",          # Position
       legend = c("Posterior", "Prior", "Likelihood"),  # Legend texts
       col = c( 2, 3, 4),           # Line colors
       lwd = 2)                 # Line width

```



Observations for Beta(4,2) of this result make sense. The posterior is indeed between the prior and likelihood.

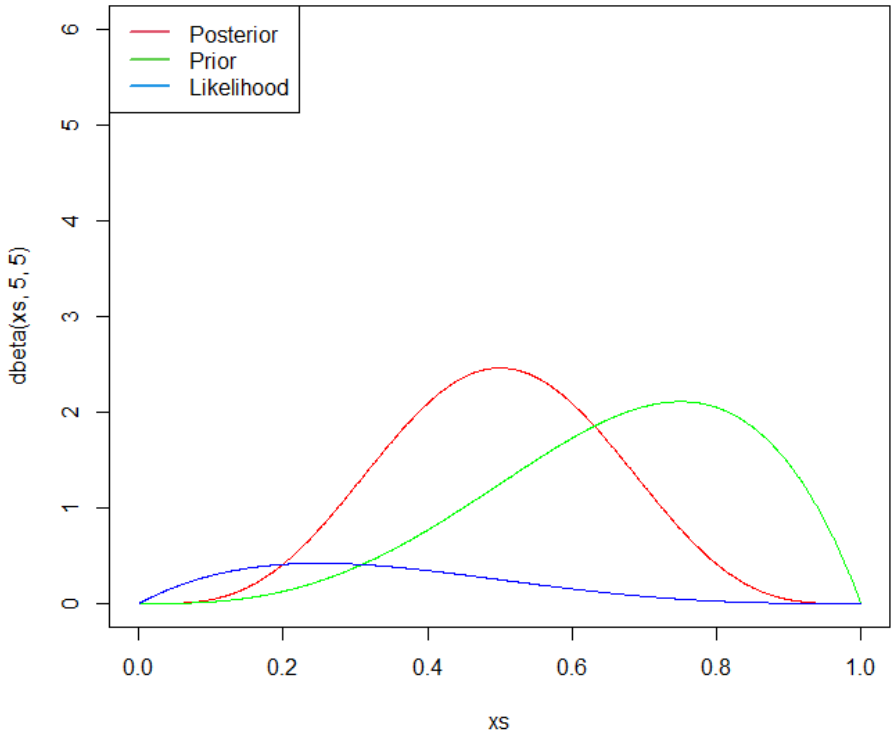
```r
#prior 2
set.seed(440)
plot(xs,dbeta(xs,19,7),type="l",col="red",ylim=c(0,6),xlim=c(0,1))
lines(xs,dbeta(xs,16,6),col="green")
lines(xs,dbinom(3,4,xs),col = "blue")
legend(x = "topleft",          # Position
       legend = c("Posterior", "Prior", "Likelihood"),  # Legend texts
       col = c( 2, 3, 4),           # Line colors
       lwd = 2)                 # Line width

```



Observations for Beta(16,6) of this result make sense. The posterior is indeed between the prior and likelihood.

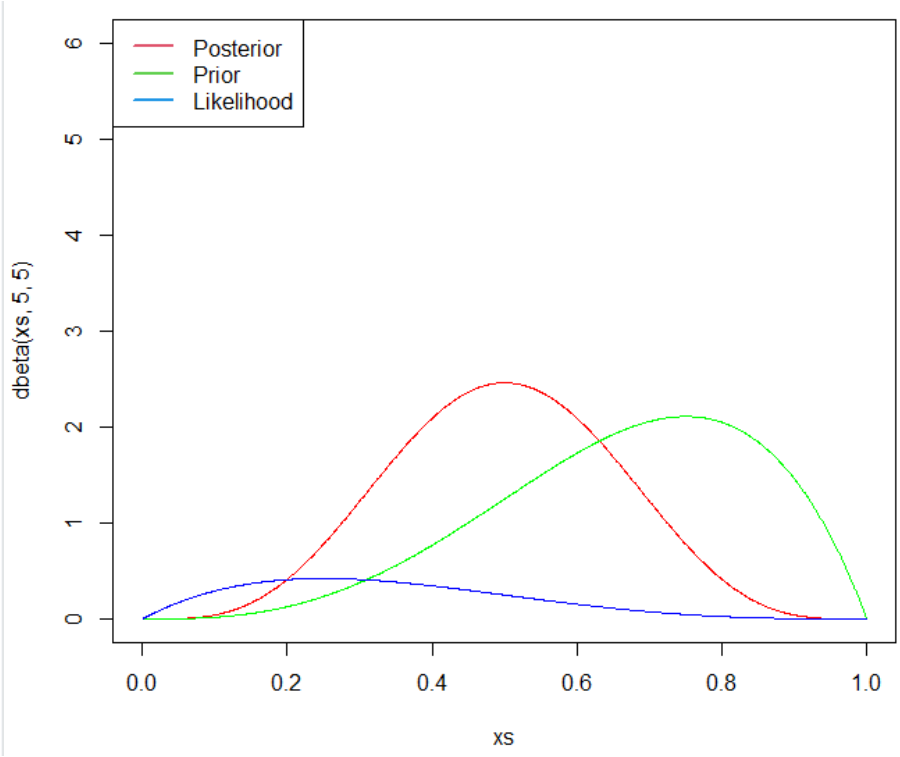
6 F

```
#part 1
set.seed(440)
plot(xs,dbeta(xs,5,5),type="l",col="red",ylim=c(0,6),xlim=c(0,1))
lines(xs,dbeta(xs,4,2),col="green")
lines(xs,dbinom(1,4,xs),col = "blue")
legend(x = "topleft",          # Position
       legend = c("Posterior", "Prior", "Likelihood"),  # Legend texts
       col = c( 2, 3, 4),           # Line colors
       lwd = 2)                 # Line width

```



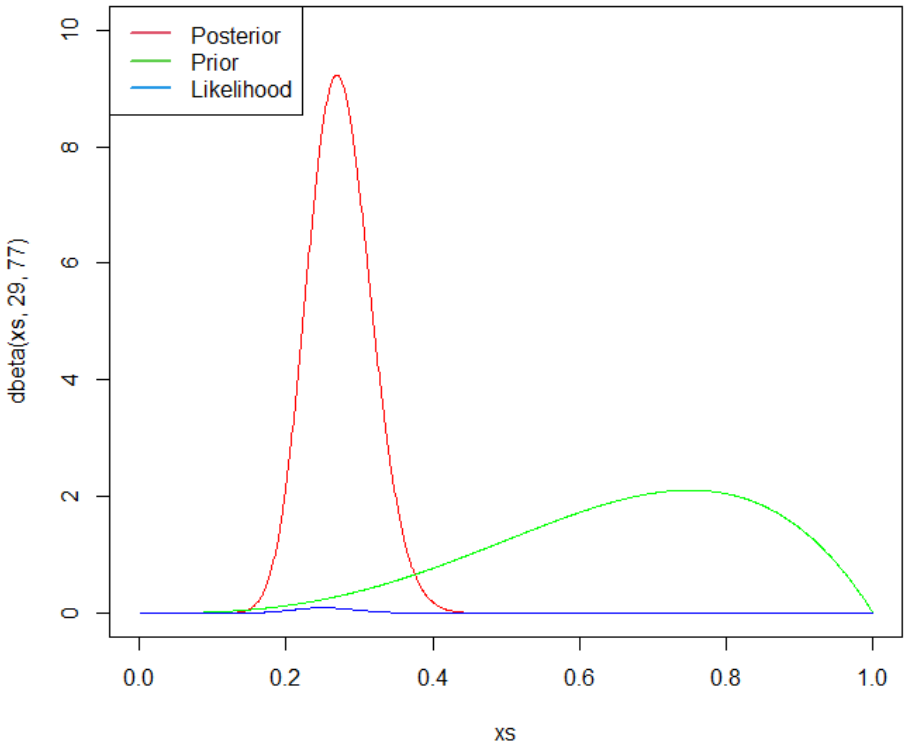
Observations for 4 tosses with this result make sense. The posterior is indeed between the prior and likelihood.

```r
#part 2
set.seed(440)
plot(xs,dbeta(xs,5,5),type="l",col="red",ylim=c(0,6),xlim=c(0,1))
lines(xs,dbeta(xs,4,2),col="green")
lines(xs,dbinom(1,4,xs),col = "blue")
legend(x = "topleft",          # Position
       legend = c("Posterior", "Prior", "Likelihood"),  # Legend texts
       col = c( 2, 3, 4),           # Line colors
       lwd = 2)                 # Line width

```



Observations for 20 tosses with this result make sense. The posterior is indeed between the prior and likelihood.

```r
#part 3
set.seed(440)
plot(xs,dbeta(xs,29,77),type="l",col="red",ylim=c(0,10),xlim=c(0,1))
lines(xs,dbeta(xs,4,2),col="green")
lines(xs,dbinom(25,100,xs),col = "blue")
legend(x = "topleft",          # Position
       legend = c("Posterior", "Prior", "Likelihood"),  # Legend texts
       col = c( 2, 3, 4),           # Line colors
       lwd = 2)                 # Line width

```



Observations for 100 tosses is a little strange but from what I can tell the posterior is still inbetween the prior and likelihood.