MAT3007 Assignment 4

Due in class (12pm), Oct 24th, Wednesday

Problem 1 (25pts). Consider the following table of food and its nutritional values:

	Protein, g	Carbohydrates, g	Calories	Cost
Bread	4	8	130	2
Milk	6	12	120	3.5
Fish	20	0	150	8
Potato	1	30	70	1.5

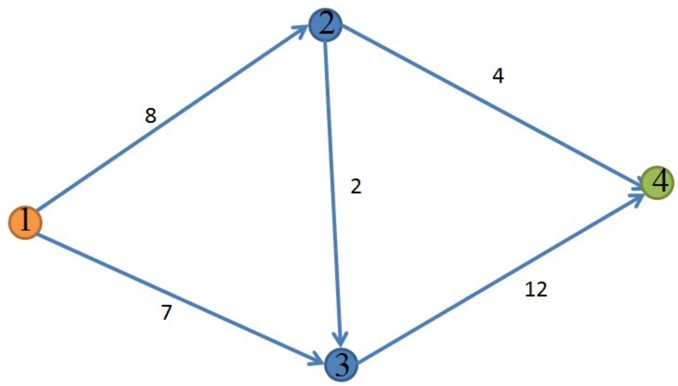
The ideal intake for an adult is at least 25 grams of protein, 40 grams of carbohydrates, and 400 calories per day. The problem is to find the least costly way to achieve those amounts of nutrition by using the four types of food shown in the table.

1. Formulate this problem as a linear optimization problem (specify the meaning of each decision variable and constraint).
2. Solve it using MATLAB, find the optimal solution and the optimal value.
3. Formulate the dual problem. Interpret the dual problem. (Hint: Suppose a pharmaceutical company produces each of the nutrients in pill form and sells them each for a certain price.)
4. Use MATLAB to solve the dual problem. Find the optimal solution and the optimal value.

Problem 2 (30pts). Write down the dual of the nurse scheduling problem (see lecture slides 3) and interpret the dual problem. (Hint: Suppose there is a firm offering nursing service for hospitals. Interpret the ith dual variable as the price this firm charges for offering a service on day i.)

Problem 3 (35pts). Consider the max flow problem on the graph below with the orange node being the source node and the green node being the terminal node (the number on each edge is its capacity, see the lecture slides). Do the following based on the lecture slides.

1. Formulate it as a linear program and solve it using MATLAB.
2. Formulate the dual of this problem and solve it using MATLAB.
3. Find the corresponding maximum flow and minimum cut of the solution.



Problem 4 (10pts). Use linear program duality to show that exactly one of the following systems has a solution

1. $A\boldsymbol{x} \leq \boldsymbol{b}$
2. $\boldsymbol{y}^T A = 0$, $\boldsymbol{b}^T\boldsymbol{y} < 0$, $\boldsymbol{y} \geq 0$

Hint: You can first show that it can't both have solutions. Then you show that if the second one is infeasible, the first one must be feasible.

Problem 5 (bonus, 10pts). Suppose M is a square matrix such that $M = -M^T$, for example,

$$M = \begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & -4 \\ -2 & 4 & 0 \end{pmatrix}.$$

Consider the following optimization problem:

$$\begin{array}{rl} \text{minimize}_{\boldsymbol{x}} & \boldsymbol{c}^T\boldsymbol{x} \\ \text{subject to} & M\boldsymbol{x} \geq -\boldsymbol{c} \\ & \boldsymbol{x} \geq 0 \end{array}$$

1. Show that the dual problem of it is equivalent to the primal problem.
2. Argue that the problem has optimal solution if and only if there is a feasible solution,