Assignment 2

Hand-in Evaluation Deadline: 5:00 pm, 14th October
In-class Evaluation: L1: 2:40 pm - 2:50 pm, 18th October
L2: 9:40 am - 9:50 am, 18th October
L3: 2:40 pm - 2:50 pm, 17th October
L4: 4:40 pm - 4:50 pm, 18th October

1. For $a, b, c, d \in \mathbb{R}$, let

$$A = \begin{bmatrix} 1 & 0 & a & b \\ 0 & 1 & c & d \end{bmatrix}, \qquad B = \begin{bmatrix} -a & -b \\ -c & -d \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Compute AB and BA.

2. a) Determine the value(s) of λ for which the matrix

$$\begin{bmatrix} 1 & \lambda & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

is invertible.
b) For those values found in part (a) find the inverse of A.

3. If possible, find the inverse of the following matrices.
(a)

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}$$

(b)

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{bmatrix}$$

4. One may raise a square matrix to any nonnegative integer power multiplying it by itself repeatedly in the same way as for ordinary numbers. That is,
$A^0 = I$.
$A^1 = A$.
$A^k = \underbrace{AA\cdots A}_{\text{k times}}$.

Suppose

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

(a) Calculate A^2, A^3, A^4.
(b) What is A^k for general k?

5. For any same size matrices A and B with $A = A^T$ and $B = B^T$, which of the following matrices are symmetric?
a) $A^2 - B^2$
b) $(A + B)(A - B)$
c) ABA
d) $ABAB$

6. Suppose A is a square matrix of size n, B is an invertible matrix of size n. Let $C = B^{-1}AB$. Show that

$$C^3 = B^{-1}A^3B.$$

7. True or false(with a counter example if false and a reason if true).
a) A 4×4 matrix with a row of zeros is not invertible.
b) A matrix with 1's down the main diagonal is invertible.
c) If A is invertible, then A^{-1} is invertible.
d) If A^T is invertible, then A is invertible.
e) If A and B are row equivalent, then either A and B are both invertible, or A and B are both not invertible.

8. Suppose $A = BC$ where B is invertible. Show that any sequence of row operations that reduces B to I also reduces A to C.

9. (a) What is $E_1E_2E_3 \cdots E_8$ for the matrices below? (Hint: think before you start doing messy calculations!) (Explain what you did.)

$$E_1 = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$E_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix}, \quad E_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad E_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 5 & 0 & 1 \end{bmatrix},$$

$$E_7 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad E_8 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{bmatrix}.$$

(b) What is the inverse of the matrix you just calculated? (Hint: again, think before you start doing messy calculations!) (Explain what you did.)

10. Define a symmetric matrix A of the following form. Find four conditions on a, b, c, d such that A is non-singular. Moreover, compute LU decomposition of A.

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$

11. Solve the following system of equations using an LU decomposition.

$$\begin{cases} 3x_1 + 3x_2 + x_3 - 4x_4 = 5 \\ 3x_1 + 5x_2 - x_3 - 3x_4 = 5 \\ -9x_1 - 3x_2 - 4x_3 + 16x_4 = -5 \\ 15x_1 + 13x_2 - 8x_3 - 21x_4 = -5 \end{cases}$$