Problem 1. The radium in a piece of lead decomposes at a rate which is proportional to the amount present. If 10 percent of the radium decomposes in 200 years, what percent of the original amount of radium will be present in a piece of lead after 1000 years?

Solution: The relationship between the decomposition rate and the amount present can be represented by

$$\frac{dx}{dt} = -kx$$
$$\frac{dx}{x} = -kdt$$

Integration of the equation gives

$$\log x = -kt + c$$
$$x = Ae^{-kt}$$

Substitute the equation with $x = 0.9A$ and $t = 200$, we can obtain

$$0.9A = Ae^{-200k}$$
$$0.9 = e^{-200k}$$
$$k = 0.0005268$$

Hence, after 1000 years

$$x = Ae^{-0.0005268*1000}$$
$$= 59.0492\%A$$

□

Problem 2. Assume that the half life of the radium in a piece of lead is 1600 years. How much radium will be lost in 100 years?

Solution: From problem(1) the relationship can be inherited that

$$x = Ae^{-kt}$$

Substitute the equation with $x = 0.5A$ and $t = 1600$, we can obtain

$$0.5 = e^{-1600k}$$
$$k = 0.0004332$$

Hence after 100 years

$$x = Ae^{-0.0004332*100}$$
$$= 95.7605\%A$$

Therefore the loss will be 4.240% □

Problem 3. The following item appeared in a newspaper. "The expedition used the carbon-14 test to measure the amount of radioactivity still present in the organic material found in the ruins, thereby determining that a town existed there as long ago as 7000 B.C." Using the half-life figure of C-14 as given in the text, determine the approximate percentage of C-14 still present in the organic material at the time of the discovery.

Solution: From the text we know that the half-life of C^{14} is approximate average of 5600 years,

$$\begin{aligned} 0.5 &= e^{-5600k} \\ k &= 0.0001238 \end{aligned}$$

Hence the remaining C^{14} is given by

$$Ae^{-0.0001238*(7000+2019)} = 32.7407\%A$$

□