MAT2040 Assignment 4 WU, Chenhao 117010285

1. (a). $A\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\\x_6\end{bmatrix}=0 \Rightarrow x=\begin{bmatrix}2x_3-5x_4\\-3x_3-x_4\\x_3\\x_4\\0\\0\end{bmatrix}=x_3\begin{bmatrix}2\\-3\\1\\0\\0\\0\end{bmatrix}+x_4\begin{bmatrix}-5\\-1\\0\\1\\0\\0\end{bmatrix}$. $\left\{\begin{bmatrix}2\\-3\\1\\0\\0\\0\end{bmatrix},\begin{bmatrix}-5\\-1\\0\\1\\0\\0\end{bmatrix}\right\}$ is a set of basis.

(b). $A\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\\x_6\\x_7\end{bmatrix}=0 \Rightarrow x=\begin{bmatrix}-2x_2-5x_4-4x_6-2x_7\\x_2\\-x_4-3x_6\\x_4\\-3x_6+2x_7\\x_6\\x_7\end{bmatrix}=x_2\begin{bmatrix}-2\\1\\0\\0\\0\\0\\0\end{bmatrix}+x_4\begin{bmatrix}-5\\0\\-1\\1\\0\\0\\0\end{bmatrix}+x_6\begin{bmatrix}-4\\0\\-3\\0\\-3\\1\\0\end{bmatrix}+x_7\begin{bmatrix}-2\\0\\0\\0\\2\\0\\1\end{bmatrix}$

(c). $A.\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}=0 \Rightarrow x=x_2\begin{bmatrix}2\\0\\0\\0\\0\end{bmatrix}+x_5\begin{bmatrix}\frac{1}{6}\\0\\\frac{1}{3}\\-\frac{2}{3}\\1\end{bmatrix}$.

2. (a). $2v_1-3v_2+v_3=0$. (b) $2v_1-v_2=0$. (c). $2v_1+v_2=0$

3. (a). $A\xrightarrow{\text{Gaussian-Elimination}}\begin{bmatrix}0&2&-1&0\\0&0&0&7\\0&0&0&0\end{bmatrix}$

$\Rightarrow$ Col(A)'s basis: $\left\{\begin{bmatrix}2\\4\\2\end{bmatrix},\begin{bmatrix}-5\\-3\\-16\end{bmatrix}\right\}$, Row(A) basis: $\left\{\begin{bmatrix}0\\2\\-1\\0\end{bmatrix},\begin{bmatrix}0\\0\\0\\7\end{bmatrix}\right\}$

(b). $A\xrightarrow{\text{Gaussian-Elimination}}\begin{bmatrix}-6&0&3&0&0\\0&-\frac{7}{3}&0&0&0\\0&0&0&-\frac{5}{7}&0\end{bmatrix}$

$\Rightarrow$ Col(A) basis. $\left\{\begin{bmatrix}-6\\2\\-16\end{bmatrix},\begin{bmatrix}-7\\0\\-1\end{bmatrix},\begin{bmatrix}12\\0\\1\end{bmatrix}\right\}$. Row(A) basis. $\left\{\begin{bmatrix}-6\\0\\3\\0\\0\end{bmatrix},\begin{bmatrix}0\\-\frac{7}{3}\\0\\0\\0\end{bmatrix},\begin{bmatrix}0\\0\\0\\-\frac{5}{7}\\0\end{bmatrix}\right\}$

4. (a). $A\xrightarrow{\text{Gaussian-El.}}\begin{bmatrix}1&0&3&5&0&0&-8&0&1\\0&1&-3&4&0&0&-1&4&3\\0&0&0&0&1&0&&6&2\\0&0&0&0&0&\cancel{4}&&*&5\end{bmatrix} \Rightarrow \text{Rank}(A)=4\cancel{3}.\ \dim(\text{Null}(A))=3$

(b). $A\xrightarrow{\text{Gaussian-El.}}\begin{bmatrix}0&1&0&0&0&2\\0&0&1&0&0&1\\0&0&0&1&0&4\\0&0&0&0&1&0\end{bmatrix} \Rightarrow \text{Rank}(A)=4,\ \dim(\text{Null}(A))=2$

5. (a). Row(A) basis: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ \frac{1}{5} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

Col (A^T) basis: $\left\{ \begin{bmatrix} -20 \\ 0 \\ -20 \\ -4 \\ 7 \\ 50 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 5 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -15 \\ 0 \\ -15 \\ -3 \\ 1 \\ 7 \end{bmatrix} \right\}$.

(b). ~~Let~~ $\vec{v} = [r_1, r_2, r_3] \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = [c_1, c_2, c_3] \begin{bmatrix} -20v_1 + 7v_2 + v_3 \\ 5v_1 \\ -15v_1 + v_2 \end{bmatrix}$

6 (c) $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = 0 \Rightarrow x = x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -\frac{1}{5} \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

Basis of Null A: $\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{1}{5} \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

6. (a). rank $A = 4$

(b). rank $A = 5$

(c) $\dim(\mathrm{Col} A) = 1$

(d) 1

(e). 0

7. If A has a rank 2, then $\exists\, \underset{\sim}{c_1}, \underset{\sim}{c_2}$ s.t. c_1 & c_2 are linearly independent and $c_n = a_n c_1 + b_n c_2$. Then, A can be written as $[c_1 \ 0 \ a_3 c_1 \cdots a_n c_1] + [0 \ c_2 \ b_3 c_2 \cdots b_n c_2]$

8 (a). $\begin{bmatrix} 2 & 1 & 5 \\ -6 & 0 & -14 \\ 4 & 17 & 17 \end{bmatrix} \xrightarrow{\text{Gaussian-El.}} \begin{bmatrix} 2 & 1 & 5 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$. $\det(A) = -12$

(b). $\begin{bmatrix} 1 & 3.5 & 3.5 \\ -2 & -4 & -4 \\ 2 & 1 & 5 \end{bmatrix} \xrightarrow{\text{Gaussian-El.}} \begin{bmatrix} 1 & 3.5 & 3.5 \\ 0 & 3 & 3 \\ 0 & 0 & 4 \end{bmatrix}$ $\det(\cancel{A}B) = -12$

(c). $\det(C) = |\det(A)| = 12$.

9. $\det(B) = \det\left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 5 & 2 & -6 & 3 \end{bmatrix}\right) \det(A) = 3\det(A)$

10. $\because \det(A) = \det(-A') = -\det(A^T) = -\det(A)$

$\therefore \det(A) = 0$

$\therefore$ A is not invertible