EIE2050
Digital Logic and Systems

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Boolean Algebra and Logic Simplification

Karnaugh Maps

- Karnaugh maps: basics
- Mappig SOP expression to Karnaugh map
- Karnaugh map simplification of SOP expression
- Mapping directly from truth table
- “Don’t Care” conditions
- Mapping POS expression to Karnaugh map
- Karnaugh map simplification for POS expression
- Conversion between POS and SOP expressions

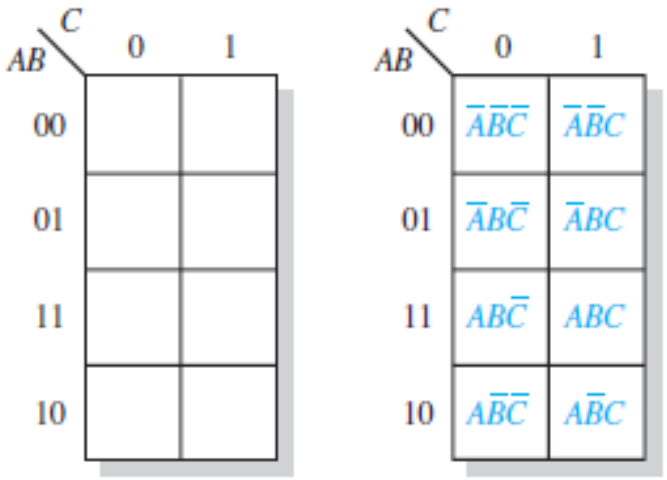
Reading Material: Chapter 4 of Textbook:
Textbook: *Digital Fundamentals (global edition, 11th edition)*, by Thomas Floyd, Pearson 2015.
The examples used in the lecture are based on the textbook.

Boolean Algebra and Logic Simplification

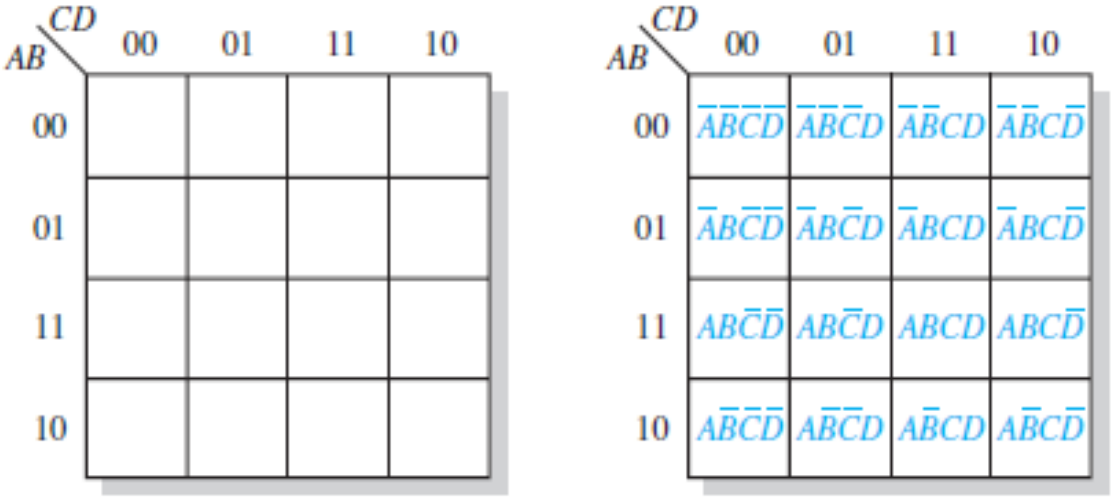
- Karnaugh map
  - Is similar to a truth table but arranged in a different way
  - Usually for expressions with 2, 3, 4 or 5 variables
  - Is an array of 2^n cells, where n is the number of variables
  - Useful for simplification of logic expressions

Boolean Algebra and Logic Simplification

- 3-variable
  Karnaugh map



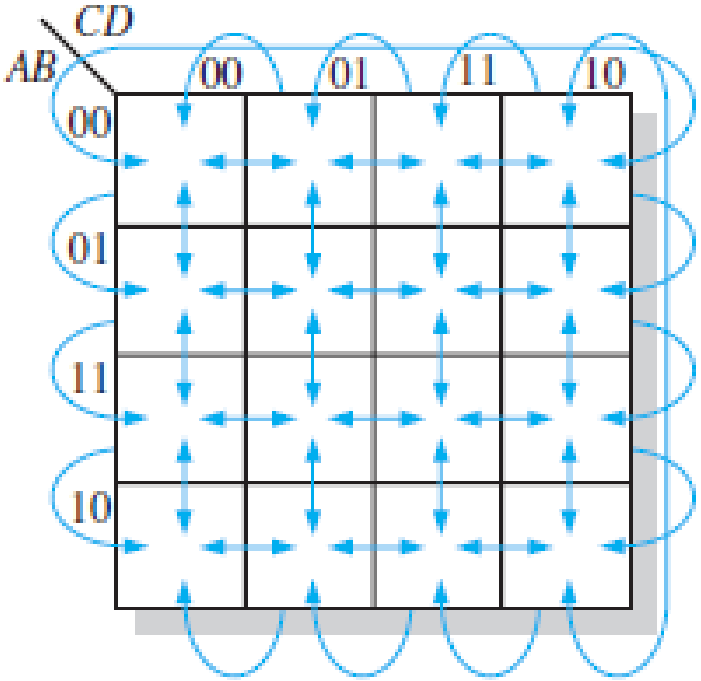
- 4-variable
  Karnaugh map



Boolean Algebra and Logic Simplification

- Adjacency of cells in Karnaugh map

  - defined by a single variable change

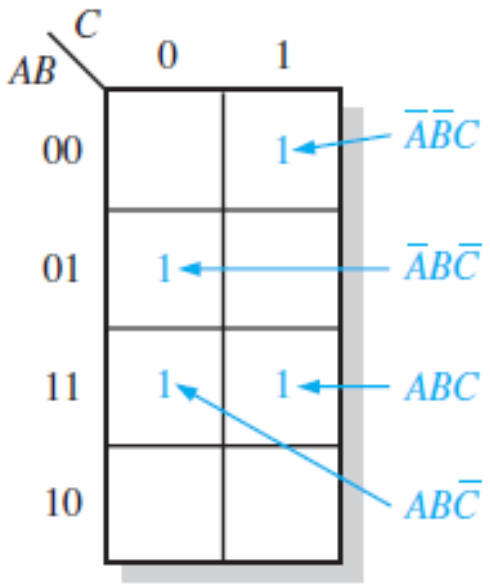


Boolean Algebra and Logic Simplification

- Karnaugh map

  - mapping a standard SOP expression:

$$\overline{A}\,\overline{B}C + \overline{A}B\overline{C} + AB\overline{C} + ABC$$
$$0\,0\,1 \quad 0\,1\,0 \quad 1\,1\,0 \quad 1\,1\,1$$

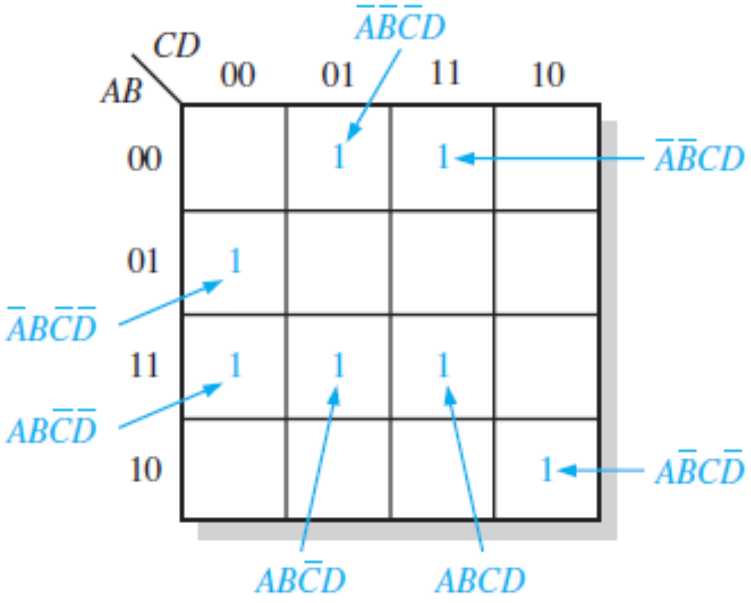


Boolean Algebra and Logic Simplification

- Karnaugh map

  - mapping a standard SOP expression:

$$\underset{0011}{\overline{A}\,\overline{B}CD} + \underset{0100}{\overline{A}B\overline{C}\,\overline{D}} + \underset{1101}{AB\overline{C}D} + \underset{1111}{ABCD} + \underset{1100}{AB\overline{C}\,\overline{D}} + \underset{0001}{\overline{A}\,\overline{B}\,\overline{C}D} + \underset{1010}{A\overline{B}C\overline{D}}$$

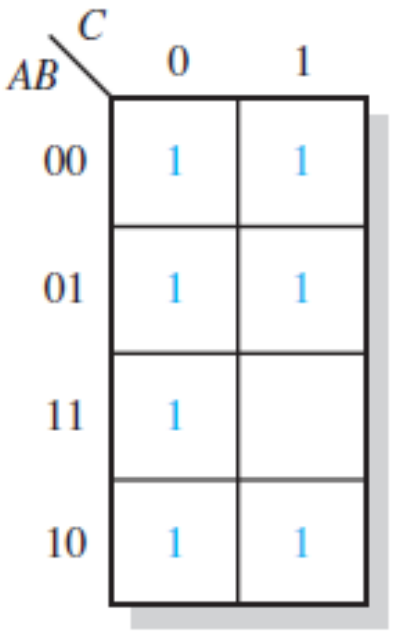


Boolean Algebra and Logic Simplification

- Karnaugh map

  - mapping a non-standard SOP expression:

$\overline{A}$	$+ A\overline{B}$	$+ AB\overline{C}$
000	100	110
001	101	
010		
011		

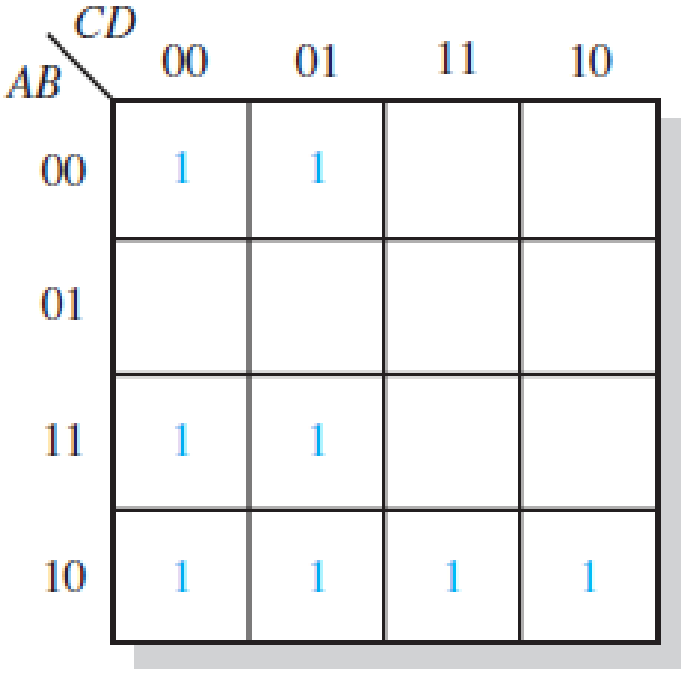


Boolean Algebra and Logic Simplification

- Karnaugh map

  - mapping a non-standard SOP expression :

$\overline{B}\overline{C}$	+	$A\overline{B}$	+	$AB\overline{C}$	+	$A\overline{B}C\overline{D}$	+	$\overline{A}\overline{B}\overline{C}D$	+	$A\overline{B}CD$
0000		1000		1100		1010		0001		1011
0001		1001		1101						
1000		1010								
1001		1011								



Boolean Algebra and Logic Simplification

- Karnaugh map

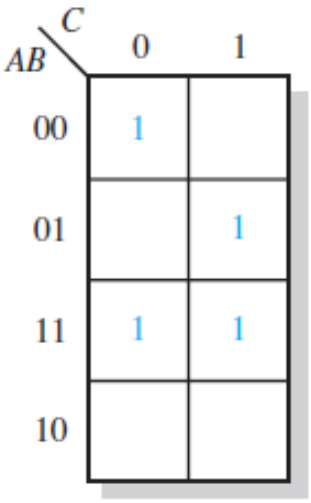
  - simplification of SOP expression :

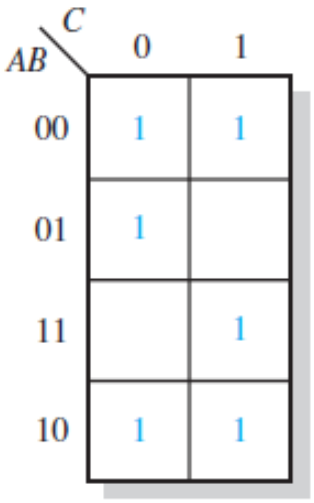
    - group the cells containing 1’s
    - adjacent cells are grouped together
    - each group contains 1, 2, 4, 8, or 16 cells
    - largest possible number of cells in each group
    - every 1 must belong to a group

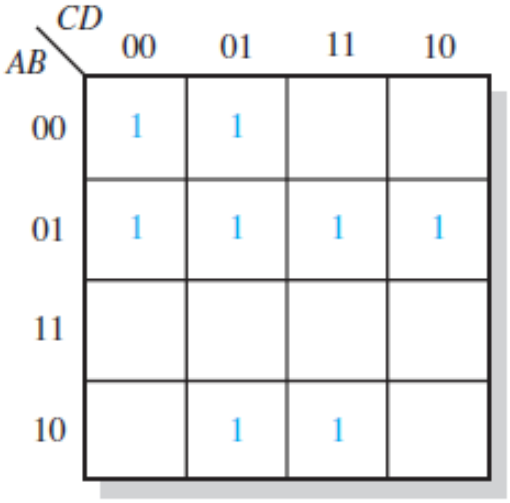
Boolean Algebra and Logic Simplification

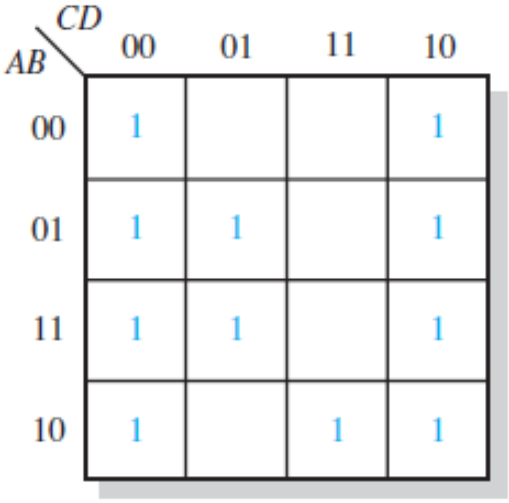
- Karnaugh map

  - simplification of SOP expression :





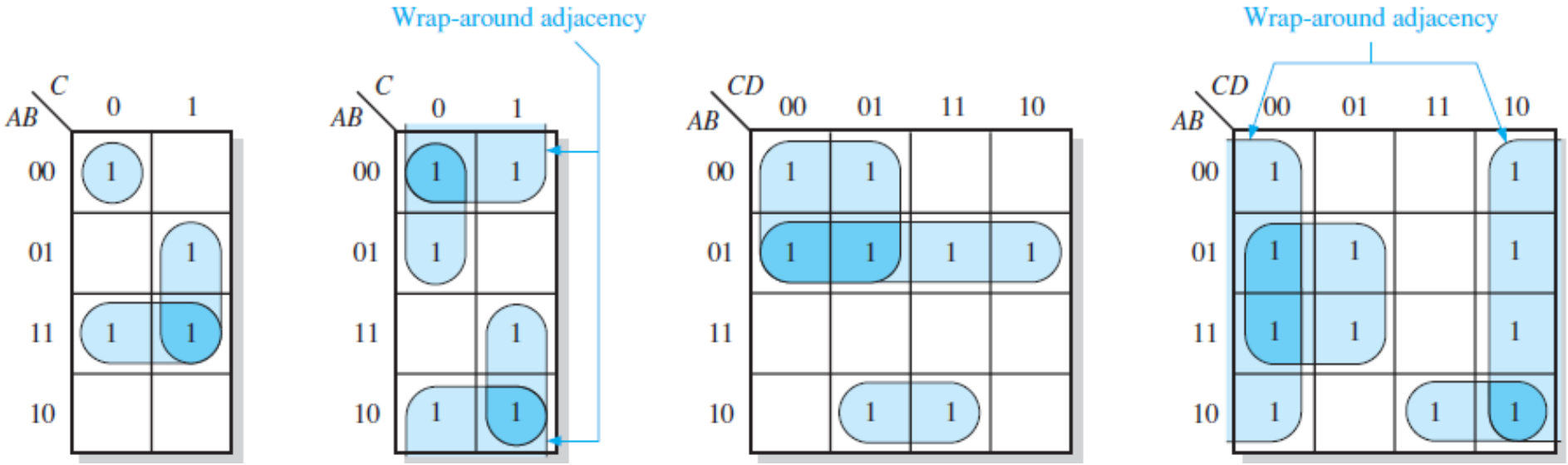




Boolean Algebra and Logic Simplification

- Karnaugh map

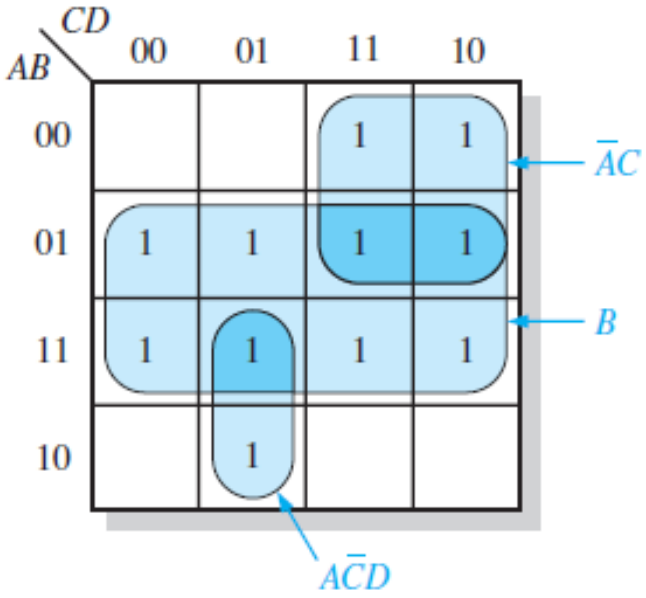
  - simplification of SOP expression:



Boolean Algebra and Logic Simplification

- Karnaugh map

  - determine minimum SOP expression from map :

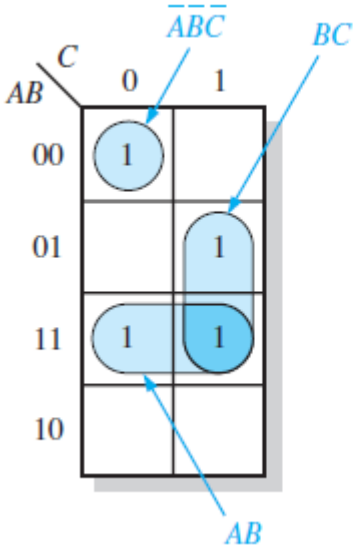


- final result: minimum SOP expression:

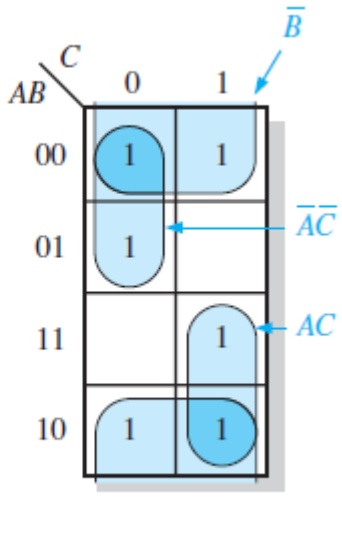
$$B + \bar{A}C + A\bar{C}D$$

Boolean Algebra and Logic Simplification

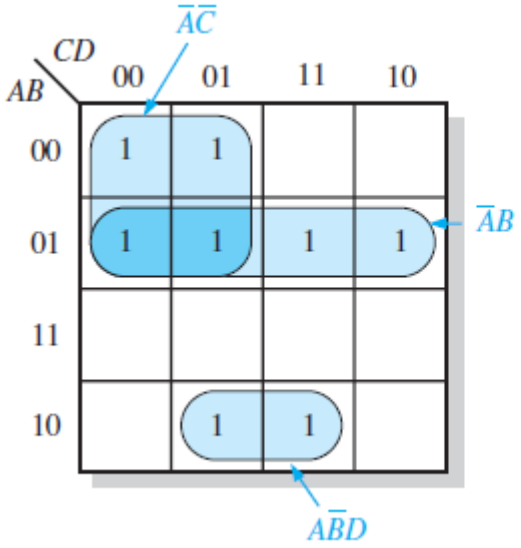
- Karnaugh map – determine minimum SOP expression



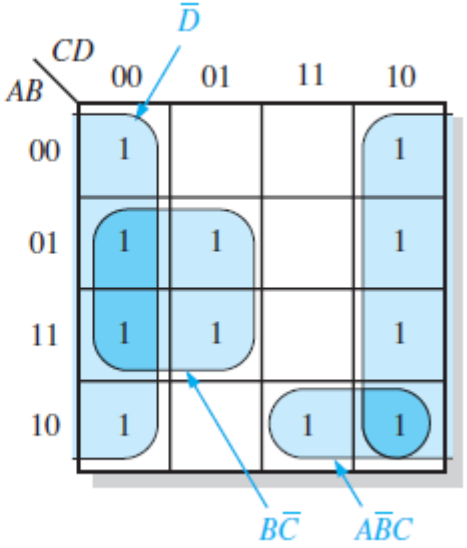
$AB + BC + \overline{A}\,\overline{B}\,\overline{C}$



$\overline{B} + \overline{A}\,\overline{C} + AC$



$\overline{A}B + \overline{A}\,\overline{C} + A\overline{B}D$



$\overline{D} + A\overline{B}C + B\overline{C}$

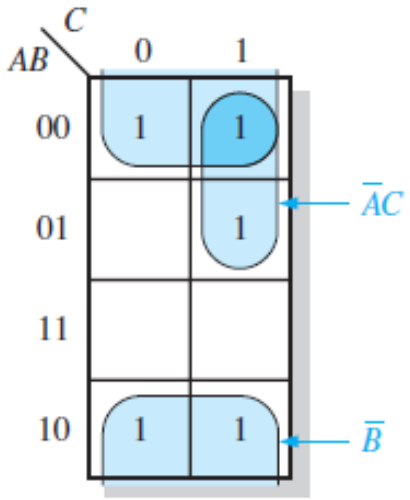
3-variable case:
group of 1 cell ->3 variable product term
group of 2 cells->2 variable product term
group of 4 cells->1 variable term

4-variable case:
group of 1 cell -> 4 variable product term
group of 2 cells -> 3 variable product term
group of 4 cells -> 2 variable product term
group of 8 cells -> 1 variable term

Boolean Algebra and Logic Simplification

- Use Karnaugh map to minimize SOP expression:

$$A\overline{B}C + \overline{A}BC + \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C}$$



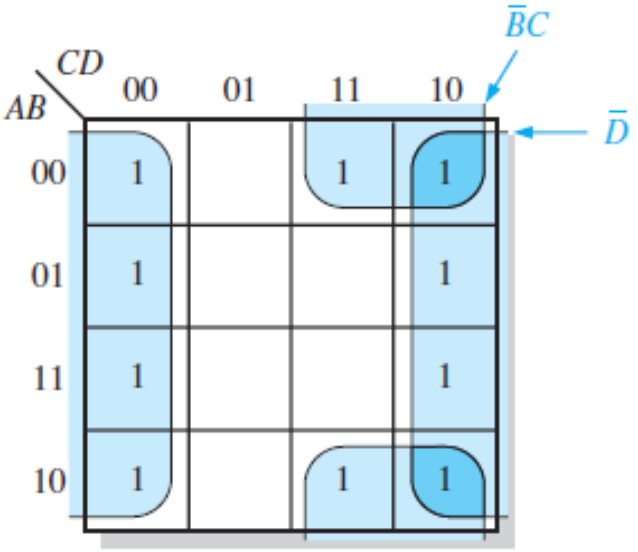
Final result: minimum SOP expression:

$$\overline{B} + \overline{A}C$$

Boolean Algebra and Logic Simplification

- Use Karnaugh map to minimize SOP expression:

$$\overline{B}\,\overline{C}\,\overline{D} + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + A\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}BC\overline{D} + ABC\overline{D} + A\overline{B}C\overline{D}$$



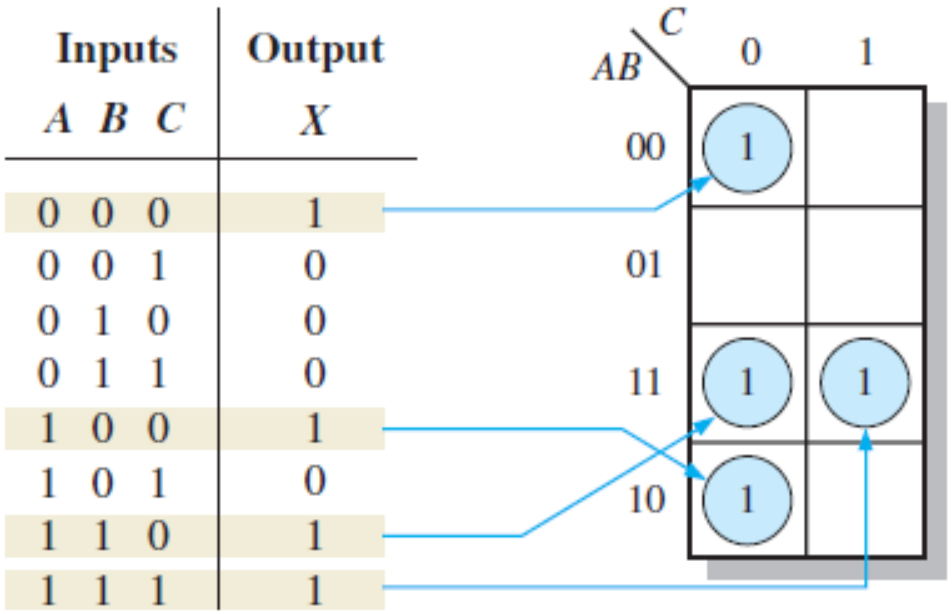
Final result: minimum SOP expression:

$$\overline{D} + \overline{B}C$$

Boolean Algebra and Logic Simplification

- Karnaugh map

  - mapping directly from truth table:



Inputs			Output
A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

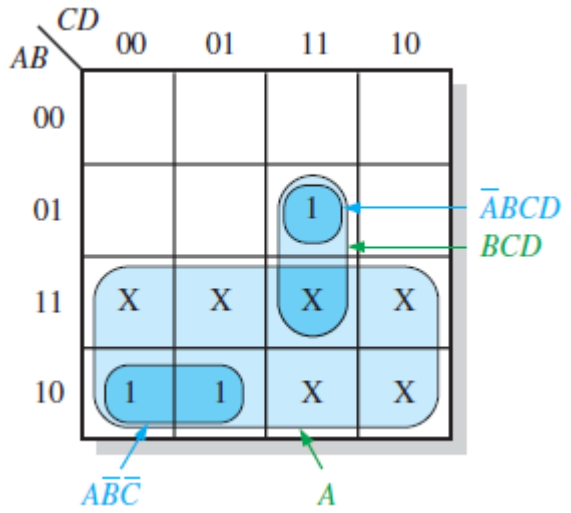
Boolean Algebra and Logic Simplification

- Karnaugh map

  - “don’t care” conditions:

Inputs				Output
A	*B*	*C*	*D*	*Y*
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Don’t cares

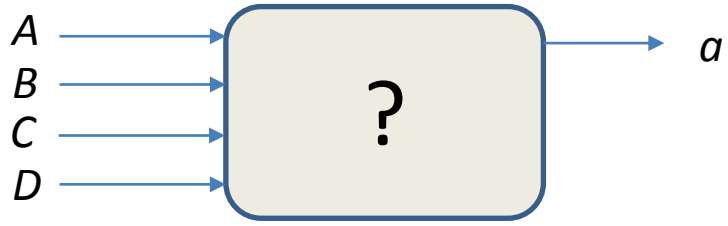


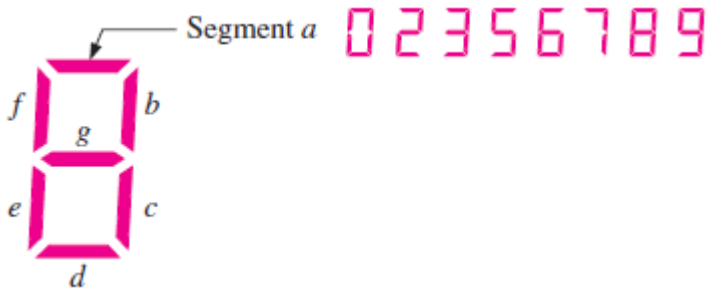
Without “don’t care”: $Y = A\bar{B}\bar{C} + \bar{A}BCD$

With “don’t care”: $Y = A + BCD$

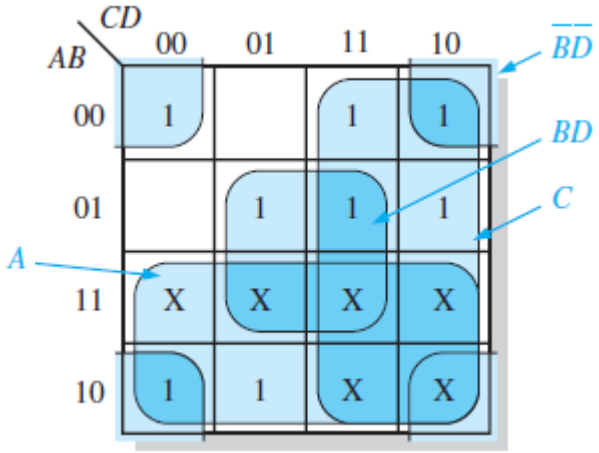
Boolean Algebra and Logic Simplification

Application of Karnaugh map: 7-segment display





$$a = \overline{A}\,\overline{B}\,C\,\overline{D} + \overline{A}\,\overline{B}\,C\,D + \overline{A}\,B\,C\,D + A\,\overline{B}\,C\,\overline{D} + \overline{A}\,B\,C\,\overline{D} + A\,\overline{B}\,C\,D + A\,\overline{B}\,\overline{C}\,\overline{D} + A\,\overline{B}\,\overline{C}\,D$$

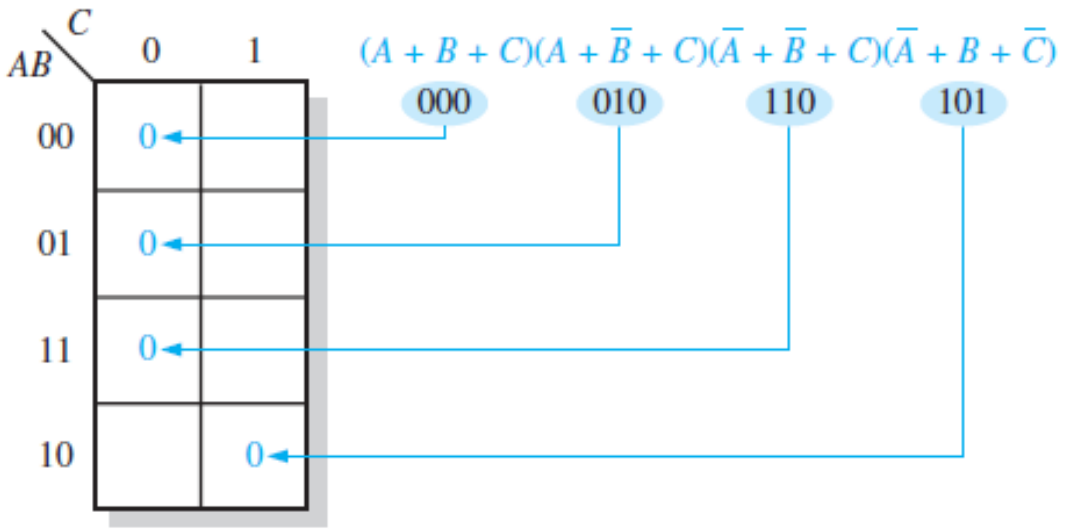


Minimum SOP expression: $a = A + C + BD + \overline{B}\,\overline{D}$

Boolean Algebra and Logic Simplification

- Karnaugh map:

  - mapping a standard POS expression :

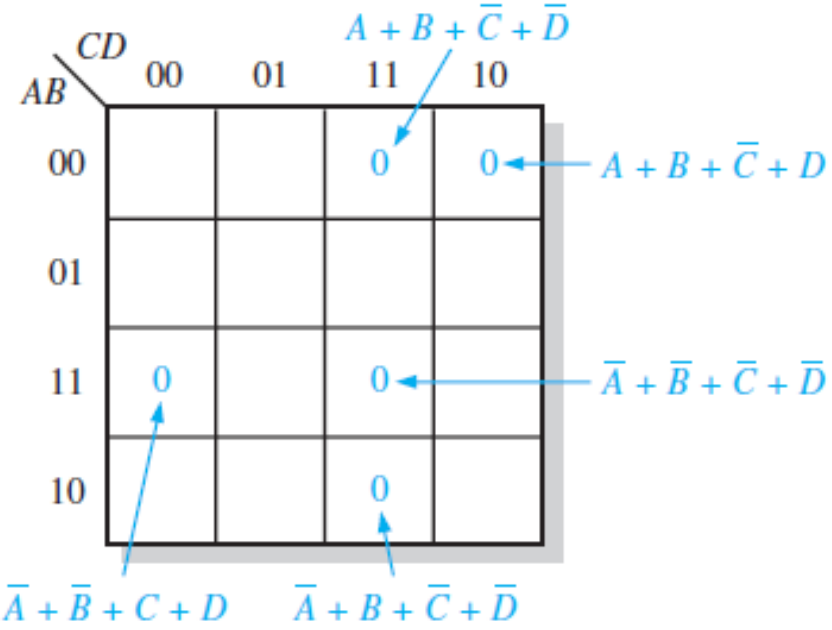


Boolean Algebra and Logic Simplification

- Karnaugh map:

  - mapping a standard POS expression :

$$\underset{1100}{(\overline{A}+\overline{B}+C+D)}\underset{1011}{(\overline{A}+B+\overline{C}+\overline{D})}\underset{0010}{(A+B+\overline{C}+D)}\underset{1111}{(\overline{A}+\overline{B}+\overline{C}+\overline{D})}\underset{0011}{(A+B+\overline{C}+\overline{D})}$$

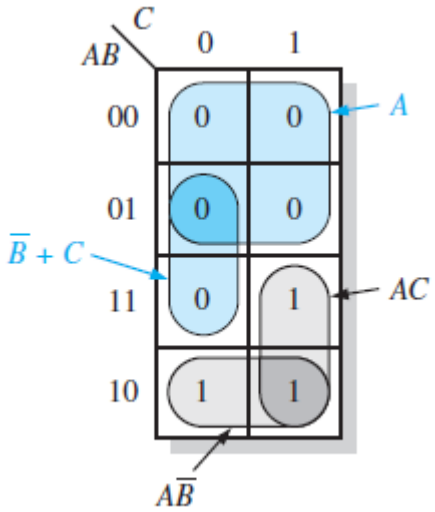


Boolean Algebra and Logic Simplification

- Karnaugh map

  - Simplification of POS expressions:

$$(A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + \overline{B} + C)$$



Final result: minimum POS expression: $A(\overline{B} + C)$

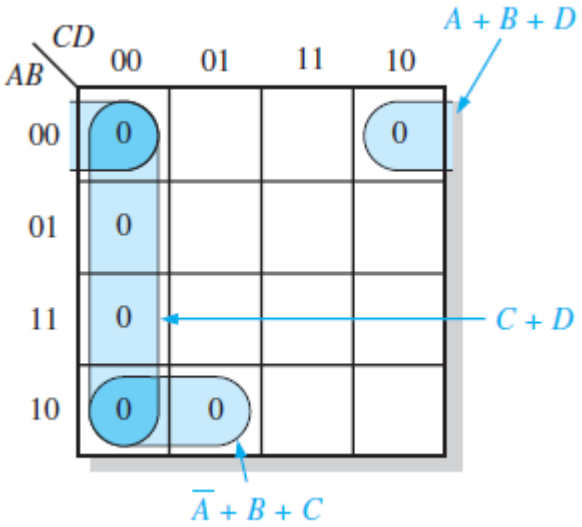
minimum SOP expression: $AC + A\overline{B}$

Boolean Algebra and Logic Simplification

- Karnaugh map

  - Simplification of POS expressions:

$$(B + C + D)(A + B + \overline{C} + D)(\overline{A} + B + C + \overline{D})(A + \overline{B} + C + D)(\overline{A} + \overline{B} + C + D)$$



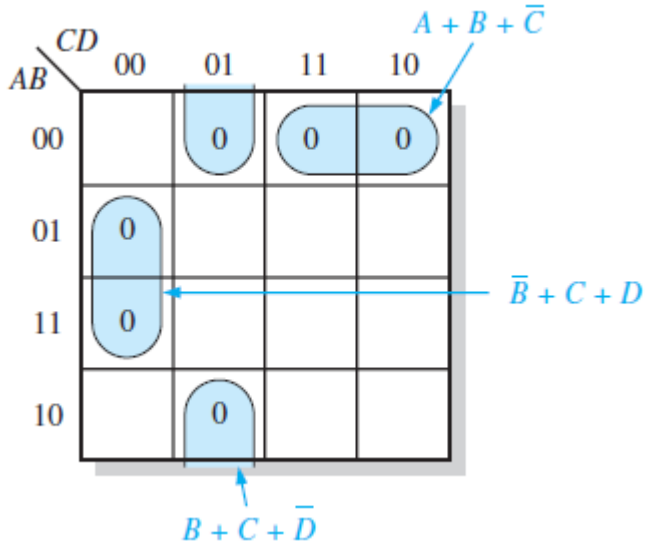
Final result: minimum POS expression: $(C + D)(A + B + D)(\overline{A} + B + C)$

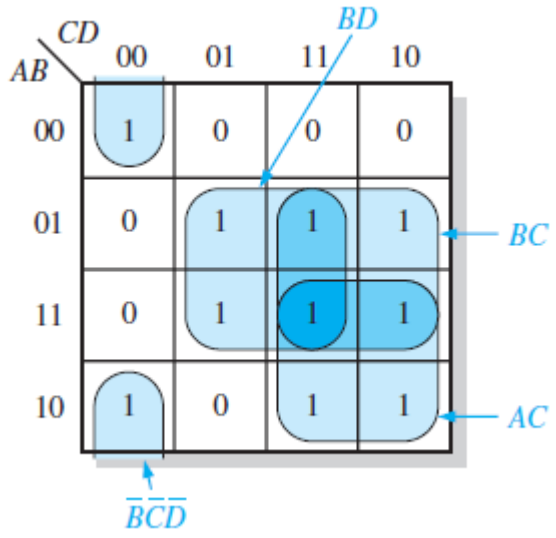
Boolean Algebra and Logic Simplification

- Using Karnaugh map to convert between SOP and POS

standard POS:

$$(\overline{A} + \overline{B} + C + D)(A + \overline{B} + C + D)(A + B + C + \overline{D})\bullet$$
$$(A + B + \overline{C} + \overline{D})(\overline{A} + B + C + \overline{D})(A + B + \overline{C} + D)$$





standard SOP:

$$\overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}B\overline{C}D + \overline{A}BCD + \overline{A}BC\overline{D} + ABC\overline{D} + A\overline{B}C\overline{D} +$$
$$A\overline{B}\overline{C}\overline{D} + AB\overline{C}D + A\overline{B}CD + ABCD$$

minimum POS:

$$(A + B + \overline{C})(\overline{B} + C + D)(B + C + \overline{D})$$

minimum SOP:

$$AC + BC + BD + \overline{B}\overline{C}\overline{D}$$