MAT3007 Assignment 5
Due at noon, Oct 31st, Wednesday

Problem 1 (50pts) A manager of an oil refinery has 8000 barrels of crude oil A and 5000 barrels of crude oil B allocated for production during the coming month. These resources can be used to make either gasoline, which sells for $38 per barrel, or home heating oil, which sells for $33 per barrel. There are three production processes with the characteristics specified in the following table (all quantities are in barrels):

	Process 1	Process 2	Process 3
Input Crude A	3	1	5
Input Crude B	5	1	3
Output Gasoline	4	1	3
Output Heating Oil	3	1	4
Cost	$51	$11	$40

1. First formulate a linear optimization problem that would help the manager maximize net revenue over the next month. Then use the simplex method to find an optimal solution. Please write down each iteration.

2. Suppose that the selling price of heating oil remains fixed over the next month, but the selling price of gasoline may rise. Using the simplex tableau, try to decide how high it can go without causing the optimal solution to change.

3. Now the manager has to consider waste disposal as well. Suppose per-unit Process 1 generates 4 tons of waste and per-unit Process 2 and 3 generate 3 tons and 5 tons of waste respectively. The refinery can dispose of 10000 tons of waste every month. What will be the optimal solution with this constraint?

Problem 2 (50pts). An insurance company is introducing three products: special risk insurance, mortgage insurance, and long-term care insurance. The expected profit is $500 per unit on special risk insurance, $250 per unit on mortgage insurance and $600 per unit on long term care insurance. The work requirements are as follows:

The management team wants to establish sales quotas for each product to maximize the total expected profit.

1. Formulate this problem as a linear optimization problem. Specify the decision variables, objective function, and constraints.

Department	Working hours per unit			Working hours available
	Special risk	Mortgage	Long-term care	
Underwriting	2	1	1	240
Administration	3	1	2	150
Claims	1	2	4	180

B	0	50	0	0	140	80 || 35400
4	0	0.5	0	1	-0.7	0.1 || 153
1	1	0	0	0	0.4	-0.2 || 24
3	0	0.5	1	0	-0.1	0.3 || 39

2. After solving the problem, the final simplex tableau (for the standard form) is given as below (the variables are in the natural order as in the description of the problem):

   Show the dual variables corresponding to the services of the three departments. Using complementarity conditions to explain why mortgage insurance is not sold.

3. Find the range of working hours available for claims to keep the current basis optimal.

4. Find the range of the expected profit on special risk insurance such that the current basis remains optimal.