Assignment 4

Hand-in Evaluation Deadline: 5:00 pm, November 11th
In-class Evaluation: L1: 2:40 pm - 2:50 pm, November 15th
L2: 9:40 am - 9:50 am, November 15th

From this assignment on, the material in lectures may differ between {L1, L2} on the one hand and {L3, L4} on the other, and therefore the homework assignment will differ for {L1, L2} and {L3, L4}.

It is therefore **not** advisable to go to lecture L3 or L4 for the in-class homework evaluation if you attend L1 or L2!

1. Find a basis for $\operatorname{Null} A$.

(a) $A = \begin{bmatrix} 1 & 0 & -2 & 5 & 0 & 0 \\ 0 & 1 & 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 3 & 0 \\ 1 & 2 & 0 & 5 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 1 & 3 & -2 \end{bmatrix}$

(c) $A = \begin{bmatrix} 2 & -4 & 2 & 3 & 1 \\ 4 & -8 & 9 & 7 & 1 \\ -2 & 4 & -17 & -9 & 0 \end{bmatrix}$

2. Give a linear dependence relation for the columns of A, for the A given in 1(a), (b) and (c) above.

3. Find a basis for $\operatorname{Col} A$ and $\operatorname{Row} A$.

(a) $A = \begin{bmatrix} 0 & 2 & -1 & -5 \\ 0 & 4 & -2 & -3 \\ 0 & -2 & 1 & -16 \end{bmatrix}$

(b) $A = \begin{bmatrix} -6 & -7 & 3 & 12 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ -10 & -1 & 5 & 1 & 0 \end{bmatrix}$

4. Find $\operatorname{rank} A$ and $\dim \operatorname{Null} A$.

(a) $A = \begin{bmatrix} 1 & 0 & 3 & 5 & 13 & -1 & 0 \\ 0 & 1 & -3 & 4 & 3 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & -12 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 5 \end{bmatrix}$

(b) $A = \begin{bmatrix} 0 & 1 & 5 & -1 & 0 & 3 \\ 0 & 0 & 1 & -1 & 7 & -3 \\ 0 & 0 & 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

5. Let $A = \begin{bmatrix} -20 & 0 & -20 & -4 & 7 & 50 \\ 5 & 0 & 5 & 1 & 0 & 0 \\ -15 & 0 & -15 & -3 & 1 & 7 \\ 10 & 0 & 10 & 2 & 5 & 36 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 1 & \frac{1}{5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -\frac{1}{4} & \frac{3}{4} & -\frac{1}{2} \\ 0 & 1 & -\frac{17}{7} & \frac{34}{7} \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

   B is a row echelon form of matrix A, and C is a row echelon form of A^T.

   (a) Find a basis for Row A using B, and a basis for Col A^T using matrix C.

   (b) Choose any vector in your basis for Row A and show that it is in Col A^T.

   (c) Find a basis for Null A.

   (d) Choose any vector in your basis for Null A and show that it is not in Row A.

6. (a) If the null space of an 5×7 matrix A has dimension 3, what is rank A?

   (b) If the null space of an 10×8 matrix A has dimension 3, what is rank A?

   (c) If the row space of an 10×7 matrix A has dimension 6, what is the dimension of Col A?

   (d) If the A is a 4×5 matrix, what is the smallest possible dimension of Null A?

   (e) If the A is a 9×6 matrix, what is the smallest possible dimension of Null A?

7. Suppose A has rank 2. Show that you can write A as the sum of two rank-1 matrices.

   (If you want to get fancy: prove (by induction) that a matrix of rank k can be written as the sum of k rank-1 matrices. Don't hand this in.)

8. Determine $\det A$ by transforming A to upper triangular form.

   (a) $A = \begin{bmatrix} 2 & 1 & 5 \\ -6 & 0 & -14 \\ 4 & 17 & 17 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & 3.5 & 3.5 \\ -2 & -4 & -4 \\ 2 & 1 & 5 \end{bmatrix}$

   (c) Determine the volume of the parallelopiped defined by the following edges: from the origin to the point $(2, 1, 5)$, from the origin to the point $(-6, 0, -14)$, and from the origin to the point $(4, 17, 17)$.

9. Use only properties 1-8 of the determinant to find an expression for $\det B$ in terms of $\det A$, where $B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 5 & 2 & -6 & 3 \end{bmatrix} A$.

10. We will soon see in lecture that $\det A^T = \det A$. Recall that A is skew-symmetric if $A^T = -A$. Use these two facts (and the properties of the determinant) to conclude that skew-symmetric 3×3 matrices are not invertible.