

# CIE 6020 Assignment 2

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1. Let  $X, Y, Z$  be three random variables with a joint probability mass function  $p(x, y, z)$ . The relative entropy between the joint distribution and the product of the marginal is

$$D(p(x, y, z) || p(x)p(y)p(z)) = E[\log \frac{p(x, y, z)}{p(x)p(y)p(z)}]$$

Expand this in terms of entropies. When is this quantity zero?

**Answer**

$$\begin{aligned} E[\log \frac{p(x, y, z)}{p(x)p(y)p(z)}] &= \sum_{z \in \mathcal{Z}} p(z) \sum_{y \in \mathcal{Y}} p(y | z) \sum_{x \in \mathcal{X}} p(x | y, z) \log \frac{p(x, y, z)}{p(x)p(y)p(z)} \\ &= \sum_{z \in \mathcal{Z}} p(z) \sum_{y \in \mathcal{Y}} p(y | z) \sum_{x \in \mathcal{X}} p(x | y, z) [\log p(x, y, z) - \log p(x)p(y)p(z)] \\ &= \end{aligned}$$