Assignment 1

Hand-in Evaluation Deadline: 5:00 pm, 23th September
In-class Evaluation: L1: 2:40 pm - 2:50 pm, 27th September
L2: 9:40 am - 9:50 am, 27th September
L3: 2:40 pm - 2:50 pm, 26th September
L4: 4:40 pm - 4:50 pm, 27th September

1. For the following system of linear equation

$$\begin{cases} x_1 + x_2 + x_3 + 2x_4 + 3x_5 = -4 \\ 2x_1 + 3x_2 + x_3 + x_5 = -3 \\ 3x_1 + 4x_2 + 2x_3 + x_4 + x_5 = -1 \end{cases}.$$

   (a) Write down the coefficient matrix and the augmented matrix.
   (b) Find the solution set by applying elementary row operations on the augmented matrix.

2. Give a system of linear equations

$$\begin{cases} 2x_1 + 3x_2 - x_3 + 2x_4 = b_1 \\ -x_1 + 2x_2 + 3x_3 + 4x_4 = b_2 \\ 3x_1 + 8x_2 + x_3 + 8x_4 = b_3 \end{cases}.$$

   (a) Show that the system has an empty solution set if $2b_1 + b_2 \neq b_3$.
   (b) Show that the system has a non-empty solution set if $2b_1 + b_2 = b_3$ and find the solutions.

3. Let

$$\mathbf{A} = \begin{bmatrix} 0 & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & a_{m2} & \cdots & a_{mn} \end{bmatrix}.$$

   Show that the first column of A is still the zero column after each elementary row operation.

4. Describe the intersection of the three planes $u + v + w + z = 6$ and $u + w + z = 4$ and $u + w = 2$ (all in four-dimensional space). Is it a line or a point or an empty set? What is the intersection if the fourth plane $u = -1$ is included? Find a fourth equation that leaves us with no solution.

5. Sketch the three lines specified by the following equations and decide if the system of equations are solvable:

$$\begin{cases} x + 2y = 2 \\ x - y = 2 \\ y = 1 \end{cases}$$

   What happens to the solution set if all right-hand sides are zero? Is there any nonzero choice of right-hand sides that allows the three lines to intersect at the same point?

6. Determine if the following statements are true or not, if true, briefly explain the reason. If not, find a counter example.
(a) If A is a 5×5 matrix in reduced row-echelon form (r.r.e.f), delete the third row of A and obtain a new matrix B. Then B is also in r.r.e.f.
(b) If A is a 5×5 matrix in reduced row-echelon form (r.r.e.f), delete the last column of A and obtain a new matrix B. Then B is also a in r.r.e.f.
(c) If A is a 5×5 matrix in reduced row-echelon form (r.r.e.f), delete the third column of A and obtain a new matrix B. Then B is also a in r.r.e.f.

7. Give two examples of 4×5 augmented matrices satisfying three conditions: (i) all the entries either 0 or 1; (ii) there are only three pivot columns; and (iii) the corresponding system is consistent.

8. Find three assignments of a such that Gaussian elimination fails to give three pivots for the following matrix.
$$\begin{bmatrix} a & 2 & 3 \\ a & a & 4 \\ a & a & a \end{bmatrix}$$

9. Write the following problems as the matrix form $A\mathbf{x} = \mathbf{b}$ and solve them.
(a) Alice is twice as old as Bob now, but was three times as old as Bob 10 years ago. What are the ages of Alice and Bob now?
(b) The line $y = mx + c$ in an $x - y$ coordinate, contains points $(2, 5)$ and $(3, 7)$. Find m and c.

10. Find b such that the homogeneous system $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions.
$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 2 & 4 & b \end{bmatrix}$$