MAT2040 Assignment 1 117010285 . Wu Chenhao

1. (a). coefficient matrix: $\begin{bmatrix} 1 & 1 & 1 & 2 & 3 \\ 2 & 3 & 1 & 0 & 1 \\ 3 & 4 & 2 & 1 & 1 \end{bmatrix}$ Augmented Matrix: $\left[\begin{array}{ccccc|c} 1 & 1 & 1 & 2 & 3 & -4 \\ 2 & 3 & 1 & 0 & 1 & -3 \\ 3 & 4 & 2 & 1 & 1 & -1 \end{array}\right]$

(b) $\left[\begin{array}{ccccc|c} 1 & 1 & 1 & 2 & 3 & -4 \\ 2 & 3 & 1 & 0 & 1 & -3 \\ 3 & 4 & 2 & 1 & 1 & -1 \end{array}\right] \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 2 & 3 & -4 \\ 0 & 1 & -1 & -4 & -5 & 5 \\ 0 & 1 & -1 & -5 & -8 & 11 \end{array}\right] \xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 2 & 3 & -4 \\ 0 & 1 & -1 & -4 & -5 & 5 \\ 0 & 0 & 0 & 1 & 3 & -6 \end{array}\right]$

Solution Set: $\left\{ \begin{bmatrix} -2x_3 + 10x_5 + 2 \\ x_3 - 7x_5 - 19 \\ x_3 \\ -3x_5 - 6 \\ x_5 \end{bmatrix} \middle| \ x_3, x_5 \in \mathbb{R} \right\}$

2. (a) The linear system can be represented as an augmented matrix as following and then we can apply row operations to simplify the matrix

$\begin{bmatrix} 2 & 3 & -1 & 2 & b_1 \\ -1 & 2 & 3 & 4 & b_2 \\ 3 & 8 & 1 & 8 & b_3 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} -1 & 2 & 3 & 4 & b_2 \\ 2 & 3 & -1 & 2 & b_1 \\ 3 & 8 & 1 & 8 & b_3 \end{bmatrix} \xrightarrow{R_1 \to -1R_1} \begin{bmatrix} 1 & -2 & -3 & -4 & -b_2 \\ 2 & 3 & -1 & 2 & b_1 \\ 3 & 8 & 1 & 8 & b_3 \end{bmatrix}$

$\xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & -2 & -3 & -4 & -b_2 \\ 0 & 7 & 5 & 10 & b_1 + 2b_2 \\ 0 & 14 & 10 & 20 & b_3 + 3b_2 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 2R_2} \begin{bmatrix} 1 & -2 & -3 & -4 & b_2 \\ 0 & 7 & 5 & 10 & b_1 + 2b_2 \\ 0 & 0 & 0 & 0 & b_3 - 2b_1 - b_2 \end{bmatrix}$

Intuitively, the solution set should satisfy Row 3 which is $0 = b_3 - 2b_1 - b_2$

Hence, if $b_3 - 2b_1 - b_2 \neq 0$, then the linear system has no solution.

(b) If $2b_1 + b_2 = b_3$, then the solution set can be obtained and formulated as following:

Solution Set: $\left\{ \begin{bmatrix} -\frac{2}{7}b_1 - \frac{3}{7}b_2 + \frac{11}{7}x_3 + \frac{8}{7}x_4 \\ \frac{1}{7}b_1 + \frac{2}{7}b_2 - \frac{5}{7}x_3 - \frac{10}{7}x_4 \\ x_3 \\ x_4 \end{bmatrix} \middle| \ x_3, x_4 \in \mathbb{R} \right\}$

3. In practical process, row operations are made under a minimum unit, entries. Hence, if row operations do not affect the value of first column entries, then after several row operations, the first column will still be a zero column.

Proof: For $R_i \leftrightarrow R_j \Rightarrow a_{i1} \leftrightarrow a_{j1}. \Rightarrow 0 \leftrightarrow 0$

For $R_i \rightarrow \alpha R_i \Rightarrow a_{i1} \longrightarrow \alpha a_{i1} \Rightarrow 0 \longrightarrow \alpha 0 \longrightarrow 0$

For $R_i \rightarrow R_i + \beta R_j \Rightarrow a_{i1} \longrightarrow a_{i1} + \beta a_{j1} \Rightarrow 0 \longrightarrow 0 + \beta 0 \longrightarrow 0$

4. As a trivial thinking, the intersection of planets should satisfy both planet equations, which can contribute to a linear system:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 6 \\ 1 & 0 & 1 & 1 & 4 \\ 1 & 0 & 1 & 0 & 2 \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \begin{bmatrix} 1 & 1 & 1 & 1 & 6 \\ 0 & -1 & 0 & 0 & -2 \\ 0 & -1 & 0 & -1 & -4 \end{bmatrix} \xrightarrow{R_3 \to R_3 - R_2} \begin{bmatrix} 1 & 1 & 1 & 1 & 6 \\ 0 & -1 & 0 & 0 & -2 \\ 0 & 0 & 0 & -1 & -2 \end{bmatrix}$$

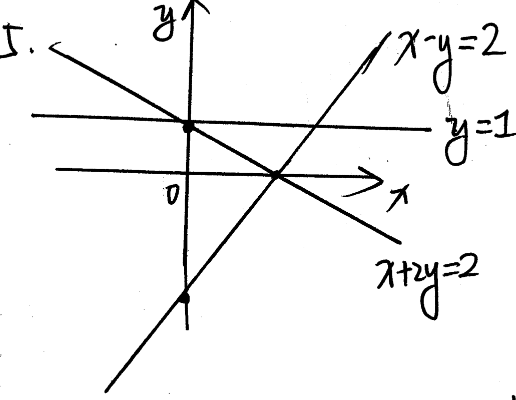
$$\xrightarrow{R_1 \to R_1 + R_2} \begin{bmatrix} 1 & 0 & 1 & 1 & 4 \\ 0 & -1 & 0 & 0 & -2 \\ 0 & 0 & 0 & -1 & -2 \end{bmatrix} \xrightarrow{R_1 \to R_1 + R_3} \begin{bmatrix} 1 & 0 & 1 & 0 & 2 \\ 0 & -1 & 0 & 0 & -2 \\ 0 & 0 & 0 & -1 & -2 \end{bmatrix} \xrightarrow[R_3 \to -R_3]{R_2 \to -R_2} \begin{bmatrix} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Hence: we can obtain a solution set that: $\left\{ \begin{bmatrix} -v+2 \\ v \\ 2 \\ 2 \end{bmatrix} \middle| \ v \in \mathbb{R} \right\}$ which depends on independent variable v, and therefore, the linear system has infinite many solutions, giving the intersections a line.

In case that $u=-1$ included, v can be determined as 3, the intersection is a point.

As what we've done before, z can be determined as $z=2$, if the fourth equation is given that $z=1$, then they'll contradict to each other, which gives no solution to the linear system.

5.



Intuitively, the linear system is not solvable.

If right-hand sides are zero, then (0,0) will be passed by 3 lines, which is the solution of the linear system.

~~That~~ Let say the linear system satisfy eq. R_1 & eq. R_2, which will give a solution set $\left\{\begin{bmatrix}2\\1\end{bmatrix}\right\}$, ~~若~~ then if R_3 gives $y=1$, the right-hand side will definitely not be zero. In this case, equation set is:

$$\begin{cases} x+2y=4 \\ x-y=1 \\ y=1 \end{cases}$$

6. (a). True. Reason: Deleting a Row will ~~have 2 possible~~ reduce the number of zeros under the leading "1"s, but won't add an obitary number under them.

(b). True. Reason: Deleting the last Column can possibly delete the rightmost pivot Column. But due to the reason that it is ~~the~~ RIGHTMOST column, the obtained new Matrix will ~~not~~ still be the rref form.

(b). False: Counter:

$$\begin{bmatrix} 1&1&1&1&1 \\ 0&1&1&1&1 \\ 0&0&1&2&1 \\ 0&0&0&1&1 \\ 0&0&0&0&1 \end{bmatrix} \xrightarrow{\text{Delete } C_3} \begin{bmatrix} 1&1&1&1 \\ 0&1&1&1 \\ 0&0&2&1 \\ 0&0&1&1 \\ 0&0&0&1 \end{bmatrix}$$

7.

$$\begin{bmatrix} 1&0&0&0&\cancel{0}&1 \\ 0&1&0&0&\cancel{0}&1 \\ 0&0&1&0&&1 \\ 0&0&0&0&&0 \end{bmatrix}$$

8.

$$\begin{bmatrix} a&2&3 \\ a&a&4 \\ a&a&a \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \begin{bmatrix} a&2&3 \\ 0&a-2&1 \\ 0&a-2&a-3 \end{bmatrix} \xrightarrow{R_3 \to R_3 - R_2} \begin{bmatrix} a&2&3 \\ 0&a-2&1 \\ 0&0&a-4 \end{bmatrix}$$

Gaussion-Jordan Elimination fails if $a=0, 2,$ or 4

9.(a). $A=\begin{bmatrix}1 & -2\\ 1 & -3\end{bmatrix}$ $x=\begin{bmatrix}x_1\\ x_2\end{bmatrix}$ $B=\begin{bmatrix}0\\ -20\end{bmatrix}$ $\Rightarrow \begin{bmatrix}1 & -2\\ 1 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ -20\end{bmatrix}$

Solved Solution Set: $\cancel{\begin{bmatrix}40\\ 20\end{bmatrix}}$ $\begin{bmatrix}40\\ 20\end{bmatrix}$

(b) $A=\begin{bmatrix}2 & 1\\ 3 & 1\end{bmatrix}$ $x=\begin{bmatrix}m\\ c\end{bmatrix}$ $B=\begin{bmatrix}5\\ 7\end{bmatrix}$ $\Rightarrow \begin{bmatrix}2 & 1\\ 3 & 1\end{bmatrix}\begin{bmatrix}m\\ c\end{bmatrix}=\begin{bmatrix}5\\ 7\end{bmatrix}$

Solved Solution Set: $\begin{bmatrix}2\\ 1\end{bmatrix}$

10. $\begin{bmatrix}1 & 0 & -1 & 1\\ 0 & 1 & 2 & 1\\ 0 & 0 & 1 & -1\\ 1 & 2 & 4 & b\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\\ 0\end{bmatrix} \Rightarrow \begin{bmatrix}1 & 0 & -1 & 1 & 0\\ 0 & 1 & 2 & 1 & 0\\ 0 & 0 & 1 & -1 & 0\\ 1 & 2 & 4 & b & 0\end{bmatrix}$

To give a linear system ~~have~~ infinite many solutions, R_4 must be a linear combination of R_1, R_2 & R_3, which may be $R_4 = R_1 + 2R_2 + R_3 \Rightarrow b = 1 + 2\times 1 - 1 = 2$