CSC3001: Discrete Mathematics
Assignment 3

Instructions:

1. Print out this question paper (**two-sided**) and write down your full working on the blank area.
2. You can have discussions with your classmates. However, make sure all the solutions you submit are your own work. Any plagirism will be given **ZERO** mark.
3. Submission of this assignment should **NOT** be later than **11:59am on 30th of November**.
4. Before your submission, please **make a softcopy** of your work for further discussion in a tutorial.
5. After making your softcopy, submit your assignment to the dropbox located on the 4th floor in Chengdao Building.

Student Number: ____________________ **Name:** _______________

1. (*20 points*) For the stable matching problem with equal number of boys and girls, prove that there is always a girl who does not receive any proposals until the last day of the marrying procedure.

2. (*20 points*) Let G be a bipartite graph with a bipartition (A, B) where $|A| = |B| = n$. Suppose that all the vertices in A have distinct positive degrees. Prove that G has a perfect matching.

3. (*20 points*) Let $G = (V, E)$ be a graph. The *complement* $\overline{G}$ of G is the graph on the same vertex set V such that two distinct vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G. Suppose that G is isomorphic to $\overline{G}$. Prove that G is connected.

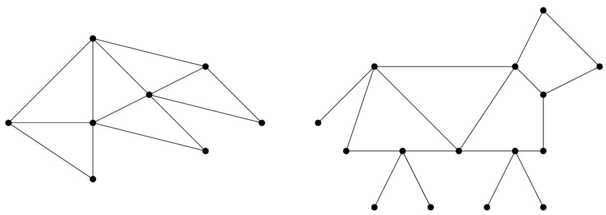
4. (*20 points*) There are a few lockers in a building for the delivery men to deposite parcels. Each locker stores only one parcel at a time. There were 7 residents who picked up the parcels on a day. The system records the deposite time and pick-up time for each parcel as follows. (**Note:** the letters in the table entries indicate the different parcels.)

Delivery man / Deposite time	Rabit	Turtle	Dinosaur	Wolf
7:05	A			
8:13		B		
8:20		C		
8:57			D	
10:04				E
11:51	F			
11:53	G			
14:11				H

Residents / Pick-up time	Kiwi	Moa	Morepork	Tuatara	Kakapo	Penguin	Emu
9:31					C		
10:50							A
11:58			E				
12:01					F		
12:42	G						
15:23				H			
16:15						B	
17:35		D					

Model this problem as a graph problem and determine the least number of lockers in the building.

5. (*20 points*) Let G be a simple graph such that it can be drawn on a plane and its edges intersect only at their endpoints. If all vertices of G lie on the unbounded face in this case, then G is *outerplanar*. The following are two examples of outerplanar graphs.



(a) Suppose that G has $n \geq 2$ vertices and m edges. Determine the maximum value of m and prove your claim.

(b) Does there exist a planar graph that is not outerplanar? If yes, give an example and explain why; otherwise, prove your claim.

6. (*10 points*) [**Bonus qustion**] An automorphism of a graph G is an isomorphism from G to G. Determine the total number of automorphisms of the following graph.