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Networks: Techonology, Economics and Society
EIE3280 Summer 2019
Assignment 2

Problem 1. Compute the pagerank vector π^* of the graph in Figure, for $\theta = 0.1, 0.3, 0.5, 0.85$. What do you observe?

Solution: Initially we characterize the graph in matrix H

$$H = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1/3 & 0 & 1/3 & 0 & 1/3 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Notice that there is a dangling node in row 5, thus we define the vector $\mathbf{w} = \big(0, 0, 0, 0, 1\big)^T$ represents the dangling node and obtain the second matrix $\hat{H}$ by

$$\hat{H} = H + \frac{1}{N}\mathbf{w}\mathbf{1}^T$$

$$\hat{H} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1/3 & 0 & 1/3 & 0 & 1/3 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \end{pmatrix}$$

To obtain the matrix G we substitute substitute corresponding entries by formula

$$G = \theta\hat{H} + (1-\theta)\frac{1}{N}\mathbf{1}\mathbf{1}^T$$

(1). When $\theta = 0.1$

$$G = \begin{pmatrix} 0.18 & 0.28 & 0.18 & 0.18 & 0.18 \\ 0.18 & 0.28 & 0.18 & 0.18 & 0.18 \\ 0.213 & 0.18 & 0.213 & 0.18 & 0.213 \\ 0.18 & 0.18 & 0.23 & 0.18 & 0.23 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{pmatrix}$$

Apply iterative approach to compute the ranking vector, we obtain

$$\pi^* = \begin{pmatrix} 0.1906 & 0.2256 & 0.1998 & 0.1840 & 0.1998 \end{pmatrix}^T$$

(2). When $\theta = 0.3$

$$G = \begin{pmatrix} 0.14 & 0.44 & 0.14 & 0.14 & 0.14 \\ 0.14 & 0.44 & 0.14 & 0.14 & 0.14 \\ 0.24 & 0.14 & 0.24 & 0.14 & 0.24 \\ 0.14 & 0.14 & 0.29 & 0.14 & 0.29 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{pmatrix}$$

Apply iterative approach to compute the ranking vector, we obtain

$$\pi^* = \begin{pmatrix} 0.1710 & 0.2899 & 0.1937 & 0.1516 & 0.1937 \end{pmatrix}^T$$

(3). When $\theta = 0.5$

$$G = \begin{pmatrix} 0.1 & 0.6 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.6 & 0.1 & 0.1 & 0.1 \\ 0.2667 & 0.1 & 0.2667 & 0.1 & 0.2667 \\ 0.1 & 0.1 & 0.35 & 0.1 & 0.35 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{pmatrix}$$

Apply iterative approach to compute the ranking vector, we obtain

$$\pi^* = \begin{pmatrix} 0.1471 & 0.3824 & 0.1765 & 0.1177 & 0.1765 \end{pmatrix}^T$$

(4). When $\theta = 0.85$

$$G = \begin{pmatrix} 0.03 & 0.88 & 0.03 & 0.03 & 0.03 \\ 0.03 & 0.88 & 0.03 & 0.03 & 0.03 \\ 0.3133 & 0.03 & 0.3133 & 0.03 & 0.3133 \\ 0.03 & 0.03 & 0.455 & 0.03 & 0.455 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{pmatrix}$$

Apply iterative approach to compute the ranking vector, we obtain

$$\pi^* = \begin{pmatrix} 0.0708 & 0.7030 & 0.0900 & 0.0453 & 0.0900 \end{pmatrix}^T$$

After calculating the result based on formula and corresponding θ, one can find that when θ is a small scalar, the distinction is not significant among webpages with more incoming links and webpages with less incoming links. And when θ increases, the numbers of incoming links have a much significant influence on the aggregate importance score, and therefore the distinction is more significant. □

Problem 2. (a) Solve for **b** in the following least squares problem, by hand or programming in any language:

$$\text{minimize}_b \qquad ||\mathbf{Ab} - \mathbf{c}||_2^2$$

where

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 1 & 1 & 0 \\ 0 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix} \qquad \text{and} \qquad \mathbf{c} = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 3 \end{pmatrix}$$

(b) Solve the above least squares problem again with regularization. Vary the regularization parameter λ for $\lambda = 0, 0.2, 0.4, \ldots, 5.0$, and plot both $||\mathbf{Ab} - \mathbf{c}||_2^2$ and $||\mathbf{b}||_2^2$ against λ.

Solution: (a) The objective function can be transformed by following

$$\begin{aligned}||\mathbf{Ab}-\mathbf{c}||_2^2 &= (\mathbf{Ab}-\mathbf{c})^T(\mathbf{Ab}-\mathbf{c}) \\ &= \mathbf{b}^T\mathbf{A}^T\mathbf{Ab} - 2\mathbf{b}^T\mathbf{A}^T\mathbf{c} + \mathbf{c}^T\mathbf{c}\end{aligned}$$

Take derivative with respective to $\mathbf{b}$ and set

$$\begin{aligned}2(\mathbf{A}^T\mathbf{A})\mathbf{b} - 2\mathbf{A}^T\mathbf{c} &= 0 \\ (\mathbf{A}^T\mathbf{A})\mathbf{b} &= \mathbf{A}^T\mathbf{c} \\ \begin{pmatrix}6 & 3 & 4\\ 3 & 6 & 3\\ 4 & 3 & 6\end{pmatrix}\mathbf{b} &= \begin{pmatrix}9\\6\\8\end{pmatrix}\end{aligned}$$

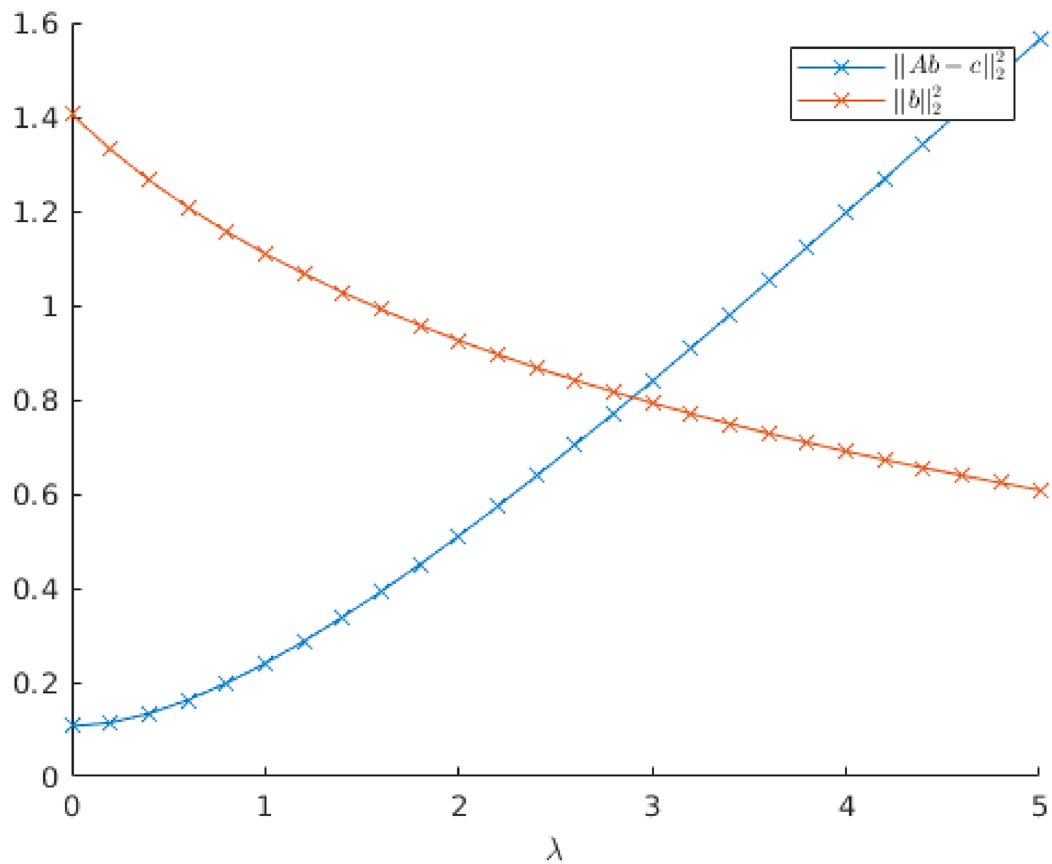
By solving the linear system we can obtain

$$\mathbf{b} = \begin{pmatrix}\frac{29}{28} & \frac{3}{14} & \frac{15}{28}\end{pmatrix}^T$$

(b). Take the derivative of $||\mathbf{Ab}-\mathbf{c}||_2^2 + \lambda||\mathbf{b}||_2^2$ respect to $\mathbf{b}$ and set it to zero

$$\begin{aligned}2\mathbf{A}^T(\mathbf{Ab}-\mathbf{c}) + 2\lambda\mathbf{b} &= 0 \\ (\mathbf{A}^T\mathbf{A}+\lambda)\mathbf{b} &= \mathbf{A}^T\mathbf{c}\end{aligned}$$

By solving the linear system and plot using computer software we obtain



Attachment: problem_2_b.m □

Problem 3. Consider 3 people making dependent estimates of a number, with the following expectations of errors and correlations of errors:

$$\begin{aligned}
E[\epsilon_1^2] &= 1773 \\
E[\epsilon_2^2] &= 645 \\
E[\epsilon_3^2] &= 1796 \\
E[\epsilon_1\epsilon_2] &= 1057 \\
E[\epsilon_1\epsilon_3] &= 970 \\
E[\epsilon_2\epsilon_3] &= 708
\end{aligned}$$

Compute the average of errors and the error of the average in this case.

Solution: The average of errors can be given by

$$\begin{aligned}
E_{AE} &= \frac{1}{N}\sum_{i=1}^{N} E_x[\epsilon_i^2(x)] \\
&= \frac{1}{3}(1773 + 645 + 1796) \\
&= \frac{4214}{3}
\end{aligned}$$

The error of the average can be given by

$$\begin{aligned}
E_{EA} &= \frac{1}{N^2}E_x[(\sum_{i=1}^{N} \epsilon_i(x))^2] \\
&= \frac{1}{9}(1773 + 645 + 1796 + 2 \times 1057 + 2 \times 970 + 2 \times 708) \\
&= 1076
\end{aligned}$$

$\square$