

H2 Logic Diaries

Foundational Development Record

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Diary 1 Interpretation of H1.5 Failures

1.1 Purpose

This diary establishes the empirical and logical motivation for the H2 framework. It does not introduce new physics, mechanisms, or hypotheses. Its role is strictly diagnostic: to identify what failed in H1.5 and why that failure cannot be ignored or repaired retroactively.

1.2 Immutable Inputs

The following results are taken as fixed and non-negotiable:

- H1 is a frozen phenomenological nonlocal baryon-convolved model.
- H1.5 tested H1 against BTFR and RAR with no refitting, tuning, or adaptation.
- All numerical procedures were verified to machine precision.
- The SPARC galaxy sample is treated as fixed.

1.3 Observed Diagnostic Outcomes

- Global BTFR ordering is preserved across the galaxy sample.
- The RAR exhibits large scatter when considered globally.
- Radial decomposition of the RAR scatter reveals a structured pattern:
 - Inner region ($r < 0.3$): large scatter (failure regime).
 - Intermediate region ($0.3 \leq r < 0.7$): transitional behaviour.
 - Outer region ($r \geq 0.7$): low scatter (success regime).

1.4 Negative Conclusions

The following explanations are explicitly ruled out:

- Numerical instability or resolution artefacts.
- Global mis-normalisation of the kernel.
- Data selection bias.
- Stochastic noise.
- Radius as a causal control variable.

1.5 Core Interpretation

The failure of H1 is identified as a consequence of applying a single, radius-blind nonlocal smoothing scale to regions with fundamentally different dynamical stiffness.

H1 succeeds where baryonic acceleration fields are spatially smooth and fails where these fields exhibit strong local gradients.

This failure mode is therefore attributed to **over-smoothing of dynamically stiff inner regions**, not to the absence of a global dynamical mechanism.

1.6 Implication for H2

H2 is motivated as a minimal corrective framework whose sole purpose is to regularise the nonlocal response in stiff regimes while preserving H1 behaviour in smooth regimes.

No claims beyond this diagnostic necessity are made at this stage.

Diary 2 State Variable Definition

2.1 Purpose

This diary defines the minimal state variable required to control the H2 adaptive response. The role of this variable is purely operational: it determines *where* the frozen H1 kernel is allowed to deform and *where* it must remain unchanged.

No physical interpretation beyond this control function is assumed.

2.2 Design Constraints

The state variable must satisfy the following non-negotiable conditions:

- It must be locally computable from quantities already present in H1.5.
- It must be scalar, continuous, and dimensionless.
- It must not explicitly depend on galactocentric radius.
- It must be invariant under global rescaling of mass and length.
- It must be applicable uniformly across all galaxies.

Any candidate failing one or more of these constraints is rejected.

2.3 Rejected Candidates

Several natural choices are ruled out:

- **Radius:** purely geometric and not causally diagnostic.
- **Baryonic acceleration magnitude** (g_{bar}) alone: insufficient to distinguish smooth from stiff regions.
- **Surface density proxies:** implicitly introduce scale dependence.

These quantities correlate with inner regions but do not encode structural complexity.

2.4 Adopted State Variable

The selected state variable is a dimensionless stiffness parameter defined as:

$$\chi(x) = \frac{|\nabla g_{\text{bar}}(x)|}{g_{\text{bar}}(x)/R_{\text{ref}}}$$

where:

- $g_{\text{bar}}(x)$ is the baryonic acceleration field,

- $\nabla g_{\text{bar}}(x)$ is its spatial gradient,
- R_{ref} is a fixed reference length associated with numerical resolution or disk scale.

2.5 Dimensional Consistency

The numerator and denominator both have dimensions of acceleration per length, ensuring that χ is dimensionless by construction.

As a result, χ is invariant under uniform rescaling of galaxy size or mass.

2.6 Operational Interpretation

The stiffness parameter partitions dynamical regimes as follows:

- $\chi \ll 1$: smooth, slowly varying acceleration fields.
- $\chi \sim 1$: transitional regimes.
- $\chi \gg 1$: stiff regions with strong local gradients.

Importantly, this partition emerges from local field structure rather than imposed geometric tagging.

2.7 Numerical Definition of the Gradient Operator

For diagnostic and implementation clarity, the gradient of the baryonic acceleration field is defined numerically on the same spatial grid used in the H1.5 pipeline.

The gradient ∇g_{bar} is evaluated using second-order central finite differences on a Cartesian grid, with the magnitude computed as:

$$|\nabla g_{\text{bar}}| = \sqrt{(\partial_x g_{\text{bar}})^2 + (\partial_y g_{\text{bar}})^2 + (\partial_z g_{\text{bar}})^2}$$

At domain boundaries, one-sided finite differences are used to avoid wraparound artefacts.

This definition introduces no additional physical assumptions and ensures that χ is computed consistently across all galaxies.

2.8 Low-Acceleration Regularisation

In regions where the baryonic acceleration approaches zero, numerical evaluation of χ may become unstable due to division by small values of g_{bar} .

To prevent spurious divergence in dynamically irrelevant regions, a small regularisation floor ε is introduced:

$$\chi = \frac{|\nabla g_{\text{bar}}|}{\max(g_{\text{bar}}, \varepsilon) / R_{\text{ref}}}$$

The parameter ε is chosen to be several orders of magnitude below typical inner-galaxy accelerations and affects only a negligible fraction of grid cells at the outskirts of the computational domain.

This regularisation has no impact on the inner-region diagnostics targeted by H2 and serves solely to ensure numerical robustness.

2.9 Role Within H2

Within the H2 framework, χ acts exclusively as a *control knob* for kernel deformation. It does not modify baryonic inputs, introduce new forces, or encode temporal or historical information.

If χ fails to correlate with the failure regions identified in Diary 1, the H2 approach is invalidated at the state-variable level and must be abandoned or redefined.

Diary 3 Kernel Deformation Rule

3.1 Purpose

This diary defines the minimal modification introduced by H2. No new fields, forces, or response channels are added. Only the effective nonlocal smoothing scale of the H1 kernel is allowed to vary, under strict control of the state variable defined in Diary 2.

The kernel shape itself remains unchanged.

3.2 Frozen H1 Operator

In H1, the total acceleration field is computed as a nonlocal convolution:

$$g_{\text{tot}}(x) = \int K_{L_0}(|x - x'|) g_{\text{bar}}(x') d^3 x'$$

where:

- K_{L_0} is a fixed-shape kernel,
- L_0 is a global, galaxy-independent length scale.

This operator is left intact in the outer, low-stiffness regime.

3.3 H2 Modification Principle

H2 modifies the operator only through a state-dependent replacement:

$$L_0 \longrightarrow L_{\text{eff}}(\chi)$$

All other aspects of the convolution remain unchanged.

This ensures maximal continuity with H1.

3.4 Structural Constraints

Any admissible deformation rule must satisfy the following constraints:

- **Monotonicity:** increasing χ must reduce L_{eff} .

- **Outer-regime recovery:**

$$\lim_{\chi \rightarrow 0} L_{\text{eff}} = L_0$$

- **Inner-regime contraction:** L_{eff} must remain finite and bounded from below by numerical resolution.
- **Smoothness:** $L_{\text{eff}}(\chi)$ must be at least C^1 .
- **Globality:** no per-galaxy tuning or local free parameters.

Any deformation violating these conditions is rejected.

3.5 Canonical Minimal Deformation

The simplest admissible deformation rule is defined as:

$$L_{\text{eff}}(\chi) = \frac{L_0}{1 + \alpha \chi}$$

where α is a fixed, dimensionless constant applied uniformly across the entire galaxy sample.

3.6 Interpretational Discipline

The parameter α functions as a global regularisation strength rather than a fitted quantity. Its role is to set the sensitivity of the kernel length to local stiffness, not to optimise empirical agreement.

No interpretation beyond this operational role is assigned.

3.7 Provisional Status

This deformation rule is provisional. It exists to be tested, constrained, or rejected in Diary 4. Failure of this rule does not invalidate the H2 programme, only this specific functional form.

3.8 Kernel Normalisation Under Deformation

The deformation $L_0 \rightarrow L_{\text{eff}}(\chi)$ modifies the spatial extent of the kernel and may, in principle, alter its integrated strength.

Two distinct implementation choices are therefore identified:

- **Scale-preserving mode:** the kernel is applied without renormalisation, allowing the integrated response to vary with L_{eff} .
- **Force-preserving mode:** the kernel is explicitly renormalised to preserve its total integrated weight under deformation.

Unless otherwise stated, H2 adopts the scale-preserving mode by default. This choice is diagnostic rather than physical and is intended to minimise implicit assumptions.

If Test 1 of Diary 4 (structural integrity) fails, the force-preserving mode may be evaluated as a secondary diagnostic check.

3.9 Regularisation Strength and Robustness

The parameter α controls the sensitivity of the kernel scale to local stiffness. It is treated as a global regularisation strength rather than a fitted parameter.

Unless explicitly stated otherwise, all H2 diagnostics adopt a fixed value $\alpha = 1$.

To assess robustness, a limited sensitivity scan over the interval $\alpha \in [0.5, 2.0]$ may be performed. This scan is not used for optimisation and does not affect parameter selection.

Any dependence of diagnostic outcomes on fine adjustment of α is considered grounds for rejecting the deformation rule.

Diary 4 Diagnostic Simulation Plan

4.1 Purpose

This diary defines the diagnostic protocol for evaluating the H2 framework. Its goal is not to demonstrate success, but to establish clear, pre-registered tests whose outcomes determine whether H2 is viable, needs revision, or must be abandoned.

No interpretation or mechanism is introduced at this stage.

4.2 Inputs

The following inputs are treated as fixed:

- SPARC baryonic mass models used in H1 and H1.5.
- The frozen H1 numerical pipeline.
- Derived baryonic acceleration field $g_{\text{bar}}(x)$.

The following quantities are computed without additional fitting:

- Spatial gradient $\nabla g_{\text{bar}}(x)$.
- Stiffness parameter $\chi(x)$.
- Effective kernel scale $L_{\text{eff}}(\chi)$.

4.3 Recomputed Outputs

Using the H2-modified operator, the following quantities are recomputed:

- Total acceleration field $g_{\text{tot}}^{\text{H2}}(x)$.
- Rotation curves.
- Radial Acceleration Relation (RAR) diagnostic points.

No additional observables are introduced.

4.4 Pre-Registered Diagnostic Tests

Test 1: Structural Integrity The outer-region behaviour ($r \geq 0.7$) must reproduce H1 results within numerical tolerance. The global BTFR ordering must remain unchanged.

Failure of this test invalidates the deformation approach.

Test 2: Correlation Gate Regions of high stiffness ($\chi \gg 1$) must spatially correlate with the regions exhibiting large H1.5 RAR residuals.

Failure of this test invalidates the choice of state variable.

Test 3: Inner-Region Response In the inner region ($r < 0.3$), the RAR scatter must decrease or exhibit systematic directional improvement.

Absence of improvement invalidates the deformation rule defined in Diary 3.

4.5 Forbidden Practices

The following practices are explicitly prohibited:

- Parameter tuning to RAR or rotation curves.
- Per-galaxy optimisation of α or L_{eff} .
- Post-hoc metric selection.
- Selective reporting of successful subsamples.

Any result obtained using these practices is considered invalid.

4.6 Rewind Map

The H2 development process includes explicit rewind conditions:

- Failure of Test 2 → return to Diary 2.
- Failure of Test 3 → return to Diary 3.
- Failure of Test 1 → abandon the adaptive-kernel approach.

Rewind is considered a valid and expected outcome.

4.7 Status

At this stage, H2 is defined as a diagnostic framework. No claims of explanatory power, universality, or physical mechanism are made.

Diary 5 Mathematical Consistency and Limits

5.1 Purpose

This diary establishes the mathematical consistency requirements of the H2 framework. Its role is preventative: to ensure that the adaptive kernel deformation introduced in

Diaries 2 and 3 does not introduce dimensional inconsistencies, hidden degrees of freedom, pathological limits, or numerical instabilities.

No empirical claims are made here.

5.2 Dimensional Analysis

The H2 deformation rule is defined as:

$$L_{\text{eff}}(\chi) = \frac{L_0}{1 + \alpha \chi}$$

where:

- L_0 has dimensions of length,
- χ is dimensionless by construction,
- α is dimensionless.

Therefore, L_{eff} preserves correct dimensionality and introduces no hidden physical scale.

5.3 Consistency of the State Variable

The stiffness parameter is defined as:

$$\chi(x) = \frac{|\nabla g_{\text{bar}}(x)|}{g_{\text{bar}}(x)/R_{\text{ref}}}$$

Dimensional inspection yields:

- g_{bar} : acceleration,
- $|\nabla g_{\text{bar}}|$: acceleration per unit length,
- $g_{\text{bar}}/R_{\text{ref}}$: acceleration per unit length.

Thus, χ is dimensionless and invariant under global rescaling of mass and length.

5.4 Limiting Behaviour

Outer-Regime Limit In the limit of vanishing stiffness:

$$\lim_{\chi \rightarrow 0} L_{\text{eff}} = L_0$$

This guarantees exact recovery of H1 behaviour in smooth regions.

Inner-Regime Limit In the formal limit of large stiffness:

$$\lim_{\chi \rightarrow \infty} L_{\text{eff}} = 0$$

In practice, numerical resolution enforces a finite lower bound L_{\min} , preventing singular behaviour.

5.5 Smoothness and Regularity

The deformation rule is smooth for all $\chi \geq 0$. All derivatives of $L_{\text{eff}}(\chi)$ are bounded for finite χ .

This avoids:

- discontinuous force responses,
- regime-switch artefacts,
- stiffness-induced numerical instabilities.

5.6 Absence of Hidden Degrees of Freedom

The H2 framework introduces:

- no new fitted parameters,
- no per-galaxy freedoms,
- no empirical interpolation functions.

The parameter α functions as a global regularisation strength rather than a tuning parameter.

5.7 Symmetry and Invariance

The formulation preserves:

- translational invariance,
- rotational invariance,
- scale covariance under global rescaling.

No preferred radius, mass scale, or acceleration scale is introduced.

5.8 Pathologies Explicitly Excluded

By construction, H2 avoids:

- singular kernels,
- nonlocal feedback loops,
- history-dependent operators,
- violations of causality.

Any extension violating these conditions must be treated as a separate framework.

5.9 Status

At this stage, the H2 framework is mathematically well-posed, dimensionally consistent, numerically regular, and explicitly falsifiable at the diagnostic level.

Diary 6 Alternative Deformation Families

6.1 Purpose

This diary enumerates admissible alternative kernel deformation families that satisfy all constraints established in Diaries 1 through 5. Its purpose is not to propose additional freedom, but to document the limited design space within which H2 is allowed to operate.

All alternatives listed here are mathematically equivalent in intent and must be evaluated solely on diagnostic performance.

6.2 Design Constraints Recap

Any admissible deformation family must satisfy:

- Dependence on the state variable χ only.
- Monotonic decrease of L_{eff} with increasing χ .
- Exact recovery of L_0 as $\chi \rightarrow 0$.
- Finite lower bound enforced by numerical resolution.
- Smoothness (at least C^1 continuity).
- No additional free parameters beyond a single global scale factor.

Any functional form violating these constraints is excluded.

6.3 Alternative Family A: Saturating Rational Form

$$L_{\text{eff}}(\chi) = \frac{L_0}{1 + \alpha \chi / (1 + \chi)}$$

This form limits the rate of contraction at large χ while preserving monotonicity and outer-regime recovery.

6.4 Alternative Family B: Exponential Suppression

$$L_{\text{eff}}(\chi) = L_0 \exp(-\alpha \chi)$$

This form provides stronger suppression in high-stiffness regions but risks over-contraction if not carefully bounded.

6.5 Alternative Family C: Logarithmic Softening

$$L_{\text{eff}}(\chi) = \frac{L_0}{1 + \alpha \ln(1 + \chi)}$$

This form introduces a slower response to stiffness gradients and may preserve more inner-region coherence.

6.6 Equivalence Principle

All deformation families listed above are considered equivalent *a priori*. No preference is assigned without diagnostic justification.

Selection among them, if any, must be based exclusively on the pre-registered tests defined in Diary 4.

6.7 Non-Admissible Extensions

The following are explicitly excluded from H2:

- Multi-parameter deformation rules.
- Non-monotonic response functions.
- Galaxy-specific functional forms.
- Deformations dependent on history or cumulative effects.

Any such extension constitutes a separate framework and must not be retrofitted into H2.

Diary 7 Failure Modes and Null Tests

7.1 Purpose

This diary defines the explicit failure modes of the H2 framework. Its goal is to pre-register conditions under which H2 must be considered unsuccessful, inconclusive, or invalid.

Failure is treated as an informative outcome, not an error.

7.2 Null Hypothesis

The null hypothesis for H2 is that the inner-region RAR scatter observed in H1.5 is not reducible by adaptive kernel deformation constrained solely by local baryonic field structure.

H2 must be rejected if this hypothesis cannot be falsified.

7.3 Primary Failure Modes

H2 is considered invalid if any of the following occur:

- No correlation is found between χ and H1.5 residual structure.
- Inner-region RAR scatter remains unchanged within uncertainties.
- Outer-region behaviour is degraded relative to H1.
- Global BTFR ordering is disrupted.

7.4 Secondary Failure Modes

H2 is considered inconclusive if:

- Modest improvements occur only for narrow subsamples.

- Improvements depend sensitively on the chosen deformation family.
- Diagnostic gains are comparable to numerical noise levels.

Such outcomes require either reformulation or abandonment.

7.5 Control Tests

The following null tests must be performed:

- Randomised χ field with identical statistical distribution.
- Fixed-radius deformation replacing χ .
- Uniform contraction of L_0 without state dependence.

If these controls perform comparably to H2, the framework is invalidated.

7.6 Interpretational Boundaries

Even if H2 succeeds diagnostically, the following conclusions are explicitly disallowed:

- Claims of new fundamental interactions.
- Claims of modified gravity.
- Claims of emergent dynamics beyond phenomenology.

Any such interpretation must be deferred to future, independent work.

7.7 Terminal Status

H2 is defined as a finite, testable, and falsifiable framework. Upon completion of its diagnostic evaluation, it must be either:

- Accepted as a valid phenomenological correction, or
- Rejected and archived without revisionist reinterpretation.

No open-ended extension is permitted within the H2 scope.

7.8 No-Free-Lunch Constraint

Any apparent improvement in inner-region diagnostics must not be accompanied by a compensating degradation elsewhere.

In particular, H2 is considered invalid if reductions in inner-region RAR scatter are achieved at the cost of:

- increased scatter in intermediate or outer regions,
- degradation of BTFR ordering,
- systematic bias in total acceleration normalisation.

H2 is required to improve diagnostics without redistributing error across scales.

7.9 Dimensional Blindness Control Test

To test whether the stiffness parameter χ carries genuine structural information, the following control experiment must be performed:

- Replace $\chi(x)$ with a synthetic scalar field that preserves its global distribution but is spatially uncorrelated with ∇g_{bar} .

If comparable diagnostic improvement is obtained under this substitution, the H2 framework is invalidated.

This test ensures that H2 responds to meaningful local structure rather than to generic scalar modulation.

7.10 Scope Firewall

Regardless of diagnostic outcome, the H2 framework does not license claims regarding:

- modified gravity,
- new fundamental interactions,
- non-baryonic degrees of freedom,
- cosmological implications.

H2 is defined strictly as a phenomenological, rotation-curve–level diagnostic framework.

Any physical interpretation must be pursued in future, independent work and is explicitly outside the scope of H2.