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A multiobjective integrated model for lot sizing and cutting stock problems

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ABSTRACT

In recent years, researchers have investigated a variety of approaches to integrating lot sizing and cutting stock problems due to their high importance in the manufacturing industry. Although the mono-objective integrated problem has been considered an excellent alternative for minimising global costs, it does not include all the multiple criteria involved in the manufacturing process. Thus, to address this issue, we use a multiobjective approach and explain its importance in providing various answers to the decision maker through the Pareto-optimal solution set. We analyse existing trade-offs and correlations between each cost of the integrated problem and the related decision variables. Several computational tests are performed, which validate the efficacy of our strategy.

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Manufacturing; lot sizing problems; cutting stock problems; integrated problems; multiobjective programming problems

1. Introduction

In the paper industry, a primary manufacturing process is to produce material objects with a specified length and paper type, using processing machines with a particular setup and capacity. The lot sizing problem (LSP) consists of planning the number of material objects that will be produced in a period to minimise the production, storage and setup costs, without exceeding the capacity of the machine while meeting consumer demands. Drexl and Kimms (1997), Jans and Degraeve (2008) and Glock, Grosse, and Ries (2014) have presented three surveys regarding relevant papers covering LSP models.

A secondary manufacturing process consists of cutting material objects into smaller pieces to meet the demand. In the one-dimensional case, a *cutting pattern* is a configuration in which the piece lengths are organised based on the object length to be cut. Since the cutting pattern may include a piece that is not part of the demand, it can be considered waste. These cut pieces can be stored to meet future demands in the long-term planning horizon. The cutting stock problem (CSP) consists of choosing the best cutting patterns and deciding the quantities to be used to minimise waste of material and storage costs and meet customer demands. The CSP has been widely studied in the literature; four surveys (Cheng, Feiring, & Cheng, 1994; Delorme, Iori, & Martello, 2016; Sweeney & Paternoster, 1992; Valério de Carvalho 2002) are dedicated to relevant papers about CSP models.

These two problems are interdependent, as in CSP, the waste of material depends on the length and number of produced objects in the LSP. The latter should consider the cutting process in the production planning of objects; otherwise, it can generate far more waste of material in the CSP (Gramani, França, & Arenales, 2009; Poltroniere, Poldi, Toledo, & Arenales, 2008). From this fact, some recent research has proposed integrating the LSP with the CSP in a mono-objective approach, with the purpose of improving the overall costs of production.

In a study about integrated process optimisation, Arbib and Marinelli (2005) integrated hierarchical decision levels and functional areas (short-term operations, mid-term planning, production, and purchase of materials), and by using leftovers of material, they achieved a further reduction of overall costs of about 43% on average. With some similarities to the model discussed here, in a furniture industry case (Gramani & França, 2006), the overall costs decreased 13% in comparison of taking decisions separately. Another example can be drawn from a paper industry process (Malik, Qiu, & Taplin, 2009) reduced the overall costs by about 25%. Also, Vanzela, Melega, Rangel, and de Araujo (2017) obtained better results for the integrated model when inventory costs and the demand for pieces are high. Similar mono-objective approaches and tailored solution algorithms are discussed by Alem and Morabito (2012), Ghidini, Alem, and Arenales (2007), Gramani, França, and Arenales (2011), Hendry, Fok, and Shek (1996), Nonas and

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Thorstenson (2000), Poltroniere et al. (2008), Silva, Alvelos, and Valério de Carvalho (2014), Suliman (2012), and Leao, Furlan, and Toledo (2017).

The mono-objective integrated approach provides just one optimal solution to be examined by a decision maker, which makes it difficult to assess its effectiveness because of the trade-off between the LSP and the CSP. Instead, the multiobjective integrated approach provides multiple solutions (Miettinen 1999), allowing a more extended study of the trade-off between these two problems by considering the features among different optimal solutions. Thus, we can infer that the multiobjective integrated approach is more convenient for manufacturing industries because it would allow them to organise the production according to their own criteria or apply a multicriteria decision analysis (Stewart, French, & Rios, 2013). This technique would help to choose the best alternative among the different optimal solutions obtained by multiobjective methods.

In the literature, we did not found multiobjective integrated approaches for these two problems but only found texts where the problems were considered isolated. In the case of the CSP, Wäscher (1990) minimises the storage costs, the extra cost of inputs and the waste of material during the cutting process from a multiobjective approach. Golfeto, Moretti, and Salles Neto (2009) propose a bi-objective model to minimise the waste of material and the machine setup, and they use a symbiotic genetic algorithm to solve their proposed model. Aliano Filho, Moretti, and Pato (2018) present a bi-objective optimisation problem to minimise the total number of different cutting patterns and solve it by applying new and classical methods. To address the LSP, Ustun and Demirtas (2008) connect the selection of raw material suppliers with LSP costs, and Rezaei and Davoodi (2011) introduce two multiobjective mixed integer non-linear models for a multi-period LSP involving multiple products and suppliers, where the cost, quality level and service level are considered in the objective vector, and after applying a genetic algorithm, the total costs are reduced significantly.

The central contribution of this research is to evaluate the trade-off between the LSP and CSP by analysing the cost variations of these two problems when we simultaneously minimise them. Also, we separate the objective function of the multiobjective problem into some other functions to compare, detect, analyse and study potential trade-offs and correlations at the heart of the problem. In this study, we show that there are several solution options for the integrated problem. We obtain these solutions by applying two multiobjective traditional methods of resolution. Last, we analyse the correlations among some decision variables and present the advantages of studying the LSP integrated with the CSP using a multiobjective approach.

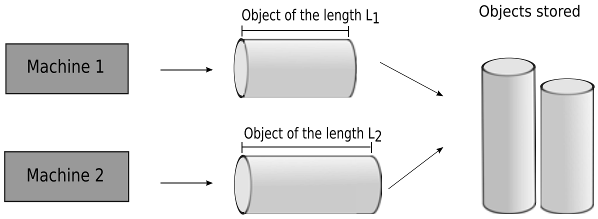

Figure 1. Papermaking processes: LSP production process.

The organisation of the article is as follows. Section 2 gives a formal description of the integrated problem as multiobjective optimisation programming. Section 3 briefly describes the two solution methods used. In Section 4, we present the main parameters used in the computational experiments and examine the computational results. Section 5 presents the conclusions.

2. Mathematical modelling

2.1. Description of the problem

The first part of the papermaking process consists of transforming cellulose pulp into a continuous and smooth sheet, which is rolled up as finished paper. The results of this process are several material objects (jumbos, master rolls, bars or simply "objects") that can be of different lengths. We assume that there are m available machines, $m = 1, \ldots, M$, producing objects of length L_m. We also consider multiple time periods, that is, the possibility of holding objects in stock and this can generate inventory costs. Setup costs are considered if the produced paper grammage changes because of the waste of material. In addition, each machine has a limited production capacity. Figure 1 illustrates the LSP manufacturing process, where two machines produce two objects of lengths L_1 and L_2, which are stored. Therefore, the LSP consists of deciding whether the machine must be prepared for production, the number of objects to be produced in a machine, and the number of objects to be stored to minimise production, storage and setup costs. This stage is labelled to Level 1 ("L1") of decisions and, because of the integration across time periods, it is classified as multi-period ("M") (Melega, de Araujo, & Jans, 2018).

The second stage of the papermaking process consists of cutting each object into smaller parts (smaller jumbos, rolls, items, formats, or pieces). Each required piece, i, $i = 1, \ldots, Nf$, is of length ℓ_i. The pieces can be stored to meet future demands on the planning horizon, but this generates storage

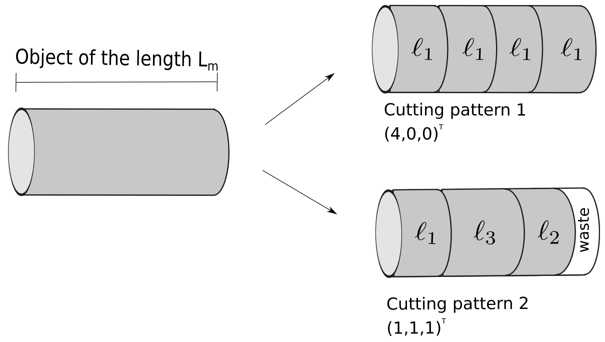

Figure 2. Papermaking processes: CSP production process.

costs. In some cases, storage of pieces is seen as a burden, for example, because they cannot be stored efficiently. Figure 2 illustrates this second stage of the production process, where different cutting patterns may imply different waste of material. Hence, the CSP consists of deciding the number of objects to be cut according to each cutting pattern and the number of pieces that must be stored to minimise the waste of material costs and storage costs. This stage is designated as Level 2 ("L2") of decisions and multi-period ("M") (Melega et al., 2018).

This article integrates these two papermaking processes and studies solutions employing a multi-objective optimisation approach, where the decision variables are linked and the manufacturing processes affect each other. The classification proposed by Melega et al. (2018) states that this integrated problem is L1/L2/-/M, where /-/ indicates the absence of Level 3 of decisions.

2.2. Multiobjective model

We introduce our mathematical formulation for modelling a production process of the paper industry as a bi-objective optimisation problem. We used a set of constraints similar to the one presented in Poltroniere et al. (2008). From the same study, we turned its mono-objective function into two new objective functions: one for the LSP and another for the CSP. We consider the following notation.

***Index*:**

LSP:

T: periods in the planning horizon; the period index is $t = 1, \dots, T$;
K: paper grammage types; the grammage index is $k = 1, \dots, K$;
M: machine types; the machine index is $m = 1, \dots, M$; each machine type m produces objects of length L_m.

CSP:

N_m: cutting pattern types of an object of length L_m; the cutting pattern index is $j = 1, \dots, N_m$;
Nf : demanded piece types; the piece index is $i = 1, \dots, Nf$;
$\{1, \dots, Nf\} \equiv S(1) \cup S(2) \cup \dots \cup S(K)$, where $S(k) = \{i|$ the piece type i is of grammage $k\}$.

***Parameters*:**

LSP:

c_{kmt}: cost to produce an object of grammage k in machine type m in period t;
h_{kt}: cost to store an object of grammage k at the end of period t;
s_{kmt}: setup cost to produce an object of grammage k in machine type m in period t;
C_{mt}: capacity of machine type m in period t;
ρ_k: specific weight of objects of grammage k;
D_{kt}: demand for paper of grammage k in period t;
b_{km}: object weight of grammage k produced in machine type m ($b_{km} = \rho_k L_m$);
f_{km}: amount of material waste of grammage k during the production of an object in machine type m.

In this article, the main LSP parameters are either proportional or related to the total weight of the object: C_{mt}, D_{kt}, and f_{km} are measured in tonnes; h_{kt} is the cost measured per tonne; s_{kmt} is a fraction of the c_{kmt}; c_{kmt} is a fraction of the object weight $b_{km} = \rho_k L_m$; and ρ_k is measured in kg/cm.

CSP:

σ_{it}: cost to store piece type i in period t;
cp_{kt}: cost of material waste of grammage k in the cutting process in period t;
$\mathbf{d}_{kt}$: vector of the demanded pieces; each component d_{ikt} represents the number of demanded piece type i, $i \in S(k)$, in period t;
η_{ik}: weight of piece i of grammage k ($\eta_{ik} = \rho_k \ell_i$);
$\mathbf{a}_{jm}$: vector of cutting patterns; each component a_{ijm} represents the number of demanded piece type i in the cutting pattern j of an object of length L_m;
p_{jm}: amount of material waste in the cutting pattern j of the object of length L_m;
Q: large number.

The main CSP parameters are related to either the total weight or length of the object: σ_{it} is the cost measured per tonne; cp_{kt} is the cost measured per centimetre; p_{jm} is measured in centimetre; and each d_{ikt} and a_{ijm} component is a nonnegative integer.

***Decision variables*:**

LSP:

x_{kmt}: the number of objects of grammage k produced in machine type m in period t;
w_{kmt}: the number of objects of grammage k produced in machine type m and stored at the end of period t;
z_{kmt}: binary variable; it takes a value of 1 if the object of grammage k is produced in machine type m in period t and 0 otherwise.

CSP:

y^j_{kmt}: the number of objects of grammage k produced in machine type m in period t that are cut according to cutting pattern j;

$\mathbf{e}_{kt}$: vector of the storage pieces; each component e_{ikt} represents the number of piece type i of grammage k stored at the end of period t.

The LSP total costs are modelled by function $F_1(x, w, z)$. It simultaneously measures three primary goals to be minimised: the costs of production, object storage and machine setup.

$$F_1(x, w, z) = \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{k=1}^{K}(c_{kmt}x_{kmt} + h_{kt}b_{km}w_{kmt} + s_{kmt}z_{kmt}) \tag{1}$$

The CSP total costs are modelled by function $F_2(y, e)$. This function simultaneously measures two primary goals to be minimised: the waste of material and storage of piece costs.

$$F_2(y, e) = \sum_{t=1}^{T}\sum_{k=1}^{K}cp_{kt}\sum_{m=1}^{M}\sum_{j=1}^{N_m}p_{jm}y_{kmt}^{j} + \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i\in S(k)}\sigma_{it}\eta_{ik}e_{ikt} \tag{2}$$

***Bi-objective mathematical model*:**

$$\text{Min } F = \big(F_1(x, w, z), F_2(y, e)\big) \tag{3}$$

s.t.

$$\sum_{m=1}^{M}(b_{km}x_{kmt} + b_{km}w_{k,m,t-1} - b_{km}w_{kmt}) \geq D_{kt}, \tag{4}$$
$$k = 1, \ldots, K, t = 1, \ldots, T;$$

$$\sum_{k=1}^{K}(b_{km}x_{kmt} + f_{km}z_{kmt}) \leq C_{mt}, \quad m = 1, \ldots, M, t = 1, \ldots, T; \tag{5}$$

$$x_{kmt} \leq Qz_{kmt}, \quad k = 1, \ldots, K, m = 1, \ldots, M, t = 1, \ldots, T; \tag{6}$$

$$\sum_{m=1}^{M}\sum_{j=1}^{N_m}\mathbf{a}_{jm}y_{kmt}^{j} + \mathbf{e}_{k,t-1} - \mathbf{e}_{kt} = \mathbf{d}_{kt}, \quad k = 1, \ldots, K, t = 1, \ldots, T; \tag{7}$$

$$\sum_{j=1}^{N_m}y_{kmt}^{j} = x_{kmt} + w_{k,m,t-1} - w_{kmt}, \tag{8}$$
$$k = 1, \ldots, K, m = 1, \ldots, M, t = 1, \ldots, T;$$

$$w_{km0} = 0, e_{ik0} = 0, \quad i = 1, \ldots, Nf, k = 1, \ldots, K, m = 1, \ldots, M; \tag{9}$$

$$x_{kmt}, w_{kmt} \in \mathbb{Z}^{+}, z_{kmt} \in \{0, 1\}, \tag{10}$$
$$k = 1, \ldots, K, m = 1, \ldots, M, t = 1, \ldots, T;$$

$$y_{kmt}^{j}, e_{ikt} \in \mathbb{Z}^{+}, i = 1, \ldots, Nf,$$
$$j = 1, \ldots, N_m, k = 1, \ldots, K, m = 1, \ldots, M, t = 1, \ldots, T. \tag{11}$$

The objective function (3) minimises the overall costs of the LSP, $F_1(x, w, z)$, and the CSP, $F_2(y, e)$. Constraints (4)–(6) are related to the LSP. Constraints (4) guarantees that the paper required, as measured in total weight, is met. Parameter D_{kt} represents the total paper of grammage k that must be available to be cut in period t. It is not known at first because it depends on the unknown amount of waste in the cutting process. As defined in Poltroniere et al. (2008): $D_{kt} = \sum_{i\in S(k)}\eta_{ik}d_{ikt} + waste$. Thus, we define the waste as equal to zero and allow the production to be greater than or equal to D_{kt}, in Constraints (4), to avoid infeasibility.

The constraints in (5) ensure that the machine capacity is not exceeded. If a machine is used, the constraints in (6) indicate that its setup costs must be computed. The constraints in (7) are related to the CSP and imply that the demand for pieces is met in each period, and for each k and t, we note that $\mathbf{a}_{jm}$, $\mathbf{e}_{kt}$, and $\mathbf{d}_{kt}$ have the same dimension, $|S(k)|$. The integration between the LSP and CSP is enforced by the constraints in (8); the number of produced objects plus the stock balance is equal to the number of cut objects. The constraints in (9) define the opening stock levels; and the last two sets of constraints, (10) and (11), simply specify the values allowed for the variables.

3. Multiobjective methods of resolution

In this section, we define some important concepts of multiobjective problems and briefly describe two well-known methods, taking into account Miettinen (1999), Aliano Filho et al. (2018), and Ehrgott and Gandibleux (2003), that we used to obtain Pareto-optimal solutions.

Definition 3.1. (*Multiobjective problem*) A multiobjective problem with n objective functions, $n > 1$, can be formulated as follows:

$$\begin{aligned} &\text{Min } \mathbf{z} = (F_1(x), F_2(x), \ldots, F_n(x)) \\ &\text{s.t. } \mathbf{x} \in \text{X}, \end{aligned} \tag{12}$$

where $z_k = F_k(x)$ is the value of the kth objective function to be minimised, $k = 1, \ldots, n$; $F = (F_1(x), F_2(x), \ldots, F_n(x))$ is the vector of the objective functions; $\mathbf{x} \in \mathbb{R}^n$ is the vector of the decision variables and $X \subseteq \mathbb{R}^n$ is the feasible space.

Definition 3.2. (*Pareto-optimal solution*) A solution $x^* \in X$ is Pareto-optimal if there is no other solution $x \in X$ such that $F_k(x) \leq F_k(x^*)$ for all $k = 1, \ldots, n$ and $F_i(x) < F_i(x^*)$ for at least one index i.

Definition 3.3. (*Ideal objective vector*) The Ideal objective vector $\mathbf{z}^*$ is obtained by minimizing each objective function that is individually subject to the constraints.

Definition 3.4. (*Nadir objective vector*) The Nadir objective vector $\mathbf{z}^{nad}$ represents the upper bounds of the Pareto-optimal set.

Definition 3.5. (*Supported solution*) Solution $x^* \in X$ is Pareto-optimal and supported if there is any

Table 1. Data of instances.

Dimension	Small (S)				Medium (M)				Large (L)				$K>1$			
Class	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Nf	3	3	4	4	5	5	6	6	7	7	8	8	4	4	4	4
T	3	4	3	4	3	4	3	4	3	4	3	4	3	4	3	4
K	1	1	1	1	1	1	1	1	1	1	1	1	2	2	3	3

vector $\mathbf{p} = (p_1, ..., p_n)^T$, where $p_k>0$ for all $k = 1, ..., n$, such that x^* is optimal solution of the problem:

$$\begin{aligned} \text{Min } & z = p_1F_1(x) + p_2F_2(x) + \cdots + p_nF_n(x) \\ \text{s.t. } & \mathbf{x} \in \text{X}. \end{aligned} \tag{13}$$

In the same sense, $x^* \in X$ is an unsupported Pareto-optimal solution if x^* is Pareto-optimal, but it is not an optimal solution of problem (13).

3.1. Weighting method

In the weighting method, we associate each objective function with a weighting coefficient and minimise the weighted sum of the objective functions, $F := p_1F_1 + p_2F_2$ (i.e., a weighted mono-objective problem). For each iteration of the computational tests, we change the weighting coefficients to obtain several Pareto-optimal solutions, in which each weight $p_i \in (0, 1)$ and $p_1 + p_2 = 1$.

In practice, the objective functions need to be normalised to prevent one of them from prevailing upon the other because of its greater magnitude. We use the normalisation

$$F_k^{norm}(x) = \frac{F_k(x) - z_k^*}{z_k^{nad} - z_k^*}, k = 1, 2, \tag{14}$$

where z_k^* is the kth component of the Ideal objective vector and z_k^{nad} is the kth component of the Nadir objective vector. Therefore, F_k^{norm} is the kth normalised objective function in the interval (0, 1).

The weighting method is widely used due to its simplicity. Nevertheless, this technique has a major disadvantage since it cannot determine unsupported Pareto-optimal solutions (Aliano Filho et al., 2018).

3.2. ε-constraint method

In the ε-constraint method, we select one of the objective functions to be minimised, and we convert all the other ones into constraints by setting an upper bound ε for each of them. If we set up the upper bounds in an appropriate manner, it is possible to reach all the Pareto-optimal solutions. Here, the LSP function $F_1(x, w, z)$ is minimised, and the CSP function $F_2(y, e)$ is converted into a constraint. Thus, (3) becomes

$$\text{Min } F = F_1(x, w, z), \tag{15}$$

and we include (16) in the set of constraints.

$$\sum_{t=1}^{T}\sum_{k=1}^{K} cp_{kt} \sum_{m=1}^{M}\sum_{j=1}^{N_m} p_{jm}y_{kmt}^{j} + \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i\in S(k)} \sigma_{it}\eta_{ik}e_{ikt} \le \varepsilon_2. \tag{16}$$

Considering the Ideal and Nadir vectors, we use upper bounds ε_2 by considering a number range from the Ideal to Nadir solution of the CSP component. As ε_2 increases from the Ideal solution, it becomes possible to find better solutions for the LSP objective function.

4. Numerical experiments

In this section, we measure the quality and performance of our integrated multiobjective approach. We have solved the proposed mathematical model using the solver IBM ILOG CPLEX Optimisation Studio. The two methods of resolution were coded in Python and executed on an Intel Core i5, with 24 GB of RAM and a Microsoft operating system (Windows). We used the *DOcplex* library as the linking code between CPLEX and Python.

This section is divided into three subsections. Section 4.1 presents the problem instances used in this study. Section 4.2 shows three strategies that we propose to solve Models (3)–(11). Finally, in Section 4.3, we present and discuss the results.

4.1. Problem instances

We used the 16 classes outlined in Table 1 to test our approach. The parameter values were generated randomly in the interval range described below, as found in Poltroniere et al. (2008): two types of machine are considered, that is, $M = 2$; the lengths of the objects are $L_1 = 540$ and $L_2 = 460$ cm; the specific weight ρ_k of all objects is 2 kg/cm; h_{kt} and σ_{it} are the costs measured per tonne; cp_{kt} is a cost measured per centimetre; C_{mt}, D_{kt} and f_{km} are measured in tonnes; p_{jm} is measured in centimetres; and c_{kmt} and s_{kmt} are proportional to the total weight of the object.

$c_{kmt} \in [0.015, 0.025]b_{km}$, where $b_{km} = \rho_k L_m$;

$s_{kmt} \in [0.03, 0.05]c_{kmt}$; $h_{kt} \in [0.0000075, 0.0000125]$;

$f_{km} \in [0.01, 0.05]b_{km}$;

$cp_{kt} = \frac{\sum_{m=1}^{M} c_{kmt}}{M} 10$; $\sigma_{it} = 0.5h_{kt}$;

$\ell_i \in [0.1, 0.3]\frac{\sum_{m=1}^{M} L_m}{M}$; $d_{ikt} \in [0, 300]$; $\eta_{ik} = \rho_k \ell_i$;

$$C_{mt} = \frac{b_{km}}{\sum_{m=1}^{M} b_{km}} \; Cap;$$
$$Cap = \phi \frac{\sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{k=1}^{K} \left((D_{kt}/M) + f_{km} \right)}{MT};$$
$$\phi = 1.24.$$

For each class, we solved 20 instances (the instances' data can be found in the following repository: https://github.com/BSCCampello/multiobjective). For each instance and each method of resolution, we have chosen to use 50 weighting coefficients, $p_i \in (0, 1)$, in the weighting method and 50 upper bounds ε_2 by considering an interval range from Ideal to Nadir solution of the CSP in the ε-constraint method. We used 50 points because our computational results showed that it is possible to obtain a Pareto front that is spreaded, which provides a good analysis of the trade-off between the LSP and CSP.

4.2. Design of experiment

We divided the computational experiments into three groups of tests. For each group, we tested the two methods of resolution described in Section 3 as follows.

Group 1. *Suboptimal solutions from a heuristic approach*: It is known that the two problems in the integrated mathematical model are NP-hard in general. Thus, we apply a heuristic approach to obtain a feasible solution for larger dimension problems in a reasonable time. Moreover, the availability of exact solutions for small instances allows evaluating the quality of the heuristic approach. Next, we describe the steps of the heuristic approach.

Step 1 - For each m, generate all possible cutting patterns.

Step 2 - For each m, sort the cutting patterns in ascending order according to their waste of material, and select the first n cutting patterns.

Step 3 - Solve the mathematical model described in Section 2 using the nM cutting patterns selected in Step 2.

We designed the heuristic based on the knowledge that as Nf increases, the number of cutting patterns grows exponentially (Suliman, 2001), as well as the number of variables of the mathematical model and, consequently, the computational time. We observed that for a value of n equal to 15, it was possible to solve the model in a reasonable amount of time. In this group of experiments, we used instances of small (S) and medium (M) dimensions (see Table 1).

Group 2. *Optimal Integer solutions from CPLEX solver*: It is possible to solve the problem using an exact approach for some small dimensional instances. Consequently, we can validate the trade-offs between the objective functions without the influence of suboptimal solutions from the heuristic approach since the exact solutions clarify the real trade-offs among the objective functions. In this group, we first generated all the possible cutting patterns; then, we solved the mathematical model presented in Section 2 directly through the CPLEX solver. To find the optimal solution using CPLEX, we determined 180 seconds as the time limit for each multiobjective method iteration. If it was not found within the time limit, we disregard the instance. In this group of experiments, we used the instances of dimension S shown in Table 1.

Group 3. *Relaxing the integrality of the variables*: Due to the difficulty of solving larger instances, the computational tests of the previous groups only covered small and medium-sized instances. In order to validate our proposed approach we developed a group of tests to solve large-sized instances. Here, we generate all the possible cutting patterns and solve the mathematical model using CPLEX. However, the integrality of the variables is relaxed, except for the binary variable z, regarding the setup costs. In this group of tests, we generate instances for all classes in Table 1.

In order to generate all the cutting patterns for the three groups of tests, we use the Python library *itertools*, which efficiently generates all the possible combinations of the pieces in an object. Then, we verify whether each combination is a feasible cutting pattern. The computational time to generate the cutting patterns and test whether it is feasible was low (under 30 seconds) for all instances.

In the next subsection, we close the numerical experiments with statistics summarising the test results and comments on the performances of our approach.

4.3. Results

The main purpose of this study is to evaluate the trade-off between the LSP and CSP by modelling them as a multiobjective integrated problem. In a suitable manner, we divided the objective function of the multiobjective problem into some other functions that can be compared. Thus, we can detect, analyse and study the potential trade-offs/correlations at the core of the problem using variables that have the greatest effect on each other. We examine the following issues: whether the increase in the total CSP costs involves a decrease in the total LSP costs or vice versa; whether there is just one optimal solution for an integrated problem with no trade-off; and what major advantages are achieved by using the multiobjective integrated approach.

4.3.1. Trade-off between the LSP and CSP

In Figure 3, we illustrate the Pareto front obtained from the ε-constraint method for one instance in Class 12 of Group 3. We chose this group because the instances with significant dimension provided a sufficient number of points for the Pareto front. Each point in the chart represents the total LSP costs in opposition to the total CSP costs by varying the bounds ε_2. Note that as the CSP costs increase, the LSP costs decrease, and vice versa, evidencing the trade-off between them. Hence, there are many solutions for the bi-objective integrated problem. This behaviour was observed in all the test groups and instance classes. Figure 4 illustrates some results obtained by the ε-constraint method and the weighting method for Classes 3, 5, and 10.

The trade-off noted in the results indicates a negative correlation between the objective functions. We think it is important to conduct an additional detailed analysis of each variable, reflected in various costs linked to the production process, to detect its interdependence. This analysis is described in Section 4.3.2.

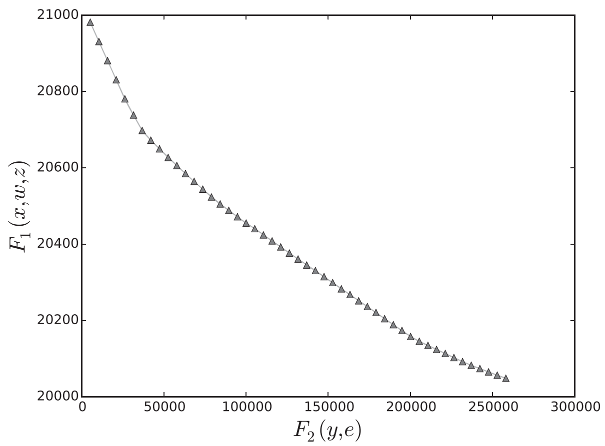

Figure 3. Pareto front from the ε-constraint method.

4.3.2. Correlation analysis

The test results are used to evaluate the statistical correlations among the objective functions. Considering that each part of the objective functions provides a set of observations (random variables), we check whether the increase or decrease of one variable is associated with the variation of another variable. The goal is to verify through descriptive statistical analysis whether there is a relationship between these variables. Therefore, we utilise dispersion graphs to exhibit whether there is a relationship between the parts of the objective functions and the Pearson product–moment correlation coefficient (Pearson's r), which is a measure of the linear correlation between them. We used the Python library *pearsonr* to calculate the Pearson's r.

Figure 5 shows the dispersion graphs obtained from the ε-constraint method for an instance in Class 11 of Group 3. It illustrates several tests conducted by splitting the bi-objective function into other suitable functions that can be compared with each other. Namely,

$g_1(x) \equiv \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{k=1}^{K}(c_{kmt}x_{kmt})$ is the LSP production costs;

$g_2(w) \equiv \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{k=1}^{K}(h_{kt}b_{km}w_{kmt})$ is the LSP storage costs;

$g_3(z) \equiv \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{k=1}^{K}(s_{kmt}z_{kmt})$ is the LSP setup costs;

$g_4(y) \equiv \sum_{t=1}^{T}\sum_{k=1}^{K}cp_{kt}\sum_{m=1}^{M}\sum_{j=1}^{N_m}(p_{jm}y^j_{kmt})$ is the CSP waste of material costs; and

$g_5(e) \equiv \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i\in S(k)}\sigma_{it}\eta_{ik}e_{ikt}$ is the CSP storage costs.

Each instance is solved using the ε-constraint method. We evaluated each one of these split

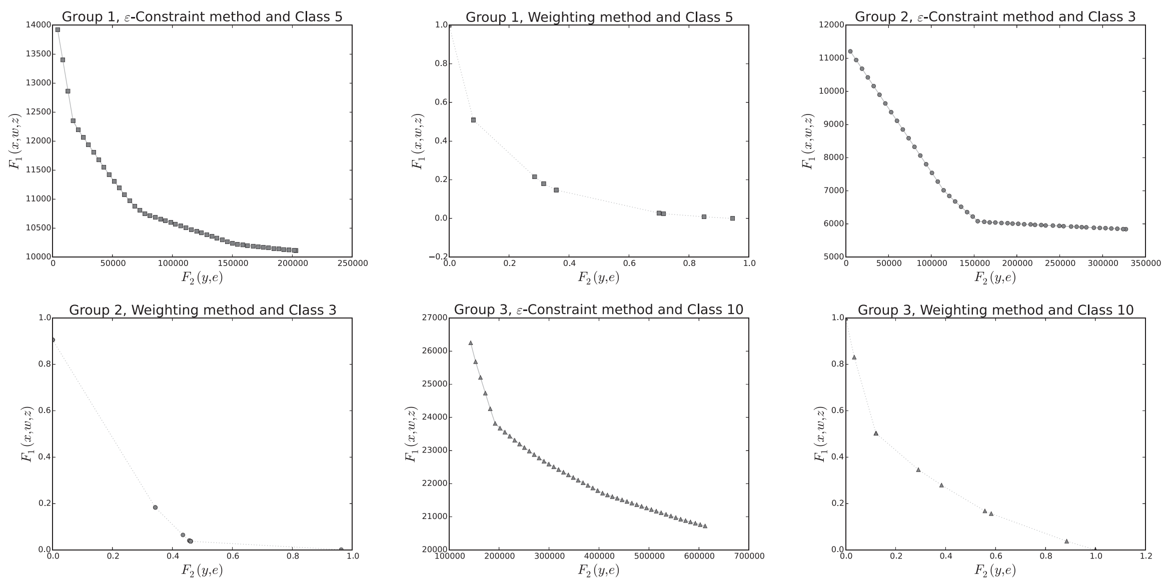

Figure 4. Pareto fronts from the weighting and ε-constraint method.

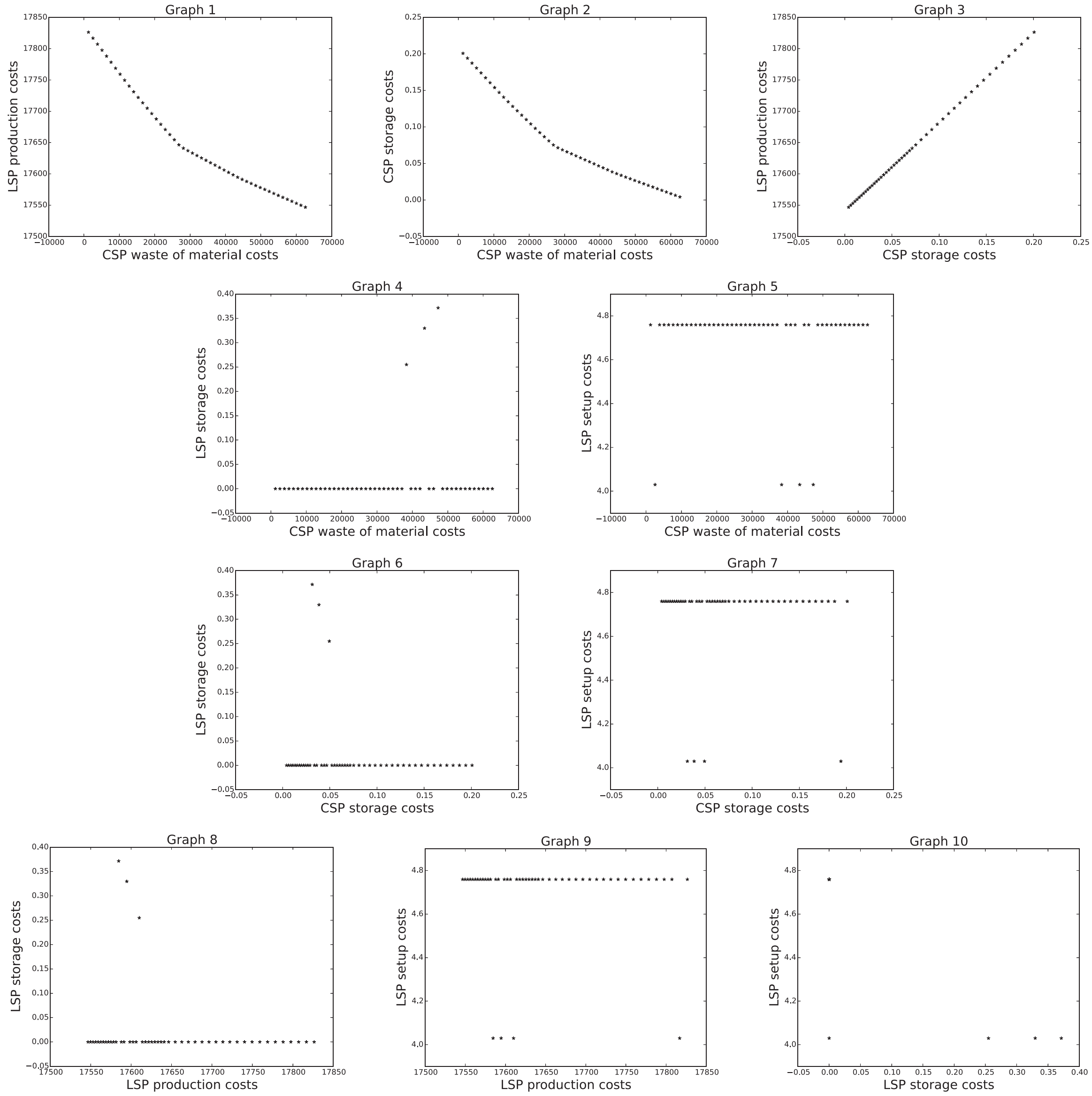

Figure 5. Comparisons from the dispersion graphs.

functions at the Pareto-optimal points found, and we compared the split functions with each other.

By varying the bound ε_2 in its interval range, we sought to illustrate whether there is interdependence or a correlation between two split functions (see Figure 5). In Graph 1 of Figure 5, note the negative correlation between LSP production and CSP waste of material costs. Similarly, in Graph 2, a negative correlation can be observed between CSP storage and CSP waste of material costs. However, as shown in Graph 3, the correlation between LSP production and CSP storage costs is positive. Given the above results, we can conclude that increasing the number of produced objects implies a reduction of the material wasted. There is also a negative correlation between the CSP costs; that is, the storage costs increase when the waste of material decreases. The other graphs in Figure 5 give no evidence of correlations between other split functions. The same behaviour can be observed in all test groups and instance classes.

Further details on the correlations among the split functions are presented in Table 2. For each instance and each method of resolution, we used the obtained by evaluating the split functions at the Pareto solution to calculate the Pearson's r. The entries of Table 2 (correlation matrix) are the average Pearson's r of the 20 instances per group, including the two methods of resolution. The observed values in Table 2 confirm the conclusions presented above. The table cells containing these entries with a Pearson's r from -1 to -0.8 are shaded to emphasise a negative and significant correlation. For example, in agreement with Pearson's r correlation analysis, we note in Group 2 of Class 1 that the correlation between $g_1(x)$ and $g_4(y)$, that is, between LSP production and CSP waste of material costs, is negative and significant. It is also equal to -0.93 on average. Yet, the correlation between $g_2(w)$ and $g_5(e)$, that is, between LSP storage and CSP storage costs, is not significant and is equal -0.45 on average.

Table 2. Pearson product–moment correlation coefficient among the split functions.

	$g_1(x)\times g_4(y)$	$g_4(y)\times g_5(e)$	$g_1(x)\times g_5(e)$	$g_2(w)\times g_4(y)$	$g_3(z)\times g_4(y)$	$g_2(w)\times g_5(e)$	$g_3(z)\times g_5(e)$	$g_1(x)\times g_2(w)$	$g_1(x)\times g_3(z)$	$g_2(w)\times g_3(z)$	$F_1\times F_2$
Class	Group 1										
1	−0.90	−0.85	0.92	0.44	−0.09	−0.41	0.09	−0.44	0.13	−0.43	−0.89
2	−0.91	−0.85	0.91	0.48	−0.05	−0.45	0.09	−0.46	0.10	−0.46	−0.91
3	−0.82	−0.83	0.99	0.33	0.08	−0.34	−0.12	−0.34	−0.12	−0.38	−0.82
4	−0.86	−0.84	0.92	0.24	0.13	−0.21	−0.14	−0.24	−0.13	−0.45	−0.86
5	−0.91	−0.88	0.95	0.33	0.08	−0.30	−0.05	−0.33	−0.05	−0.42	−0.91
6	−0.84	−0.75	0.81	0.26	0.15	−0.25	−0.09	−0.29	−0.14	−0.35	−0.71
7	−0.89	−0.82	0.93	0.22	0.04	−0.24	−0.06	−0.25	−0.03	−0.37	−0.89
8	−0.83	−0.81	0.87	0.04	0.02	−0.08	0.00	−0.09	0.01	−0.41	−0.82
Average	−0.87	−0.83	0.91	0.29	0.04	−0.29	−0.04	−0.30	−0.03	−0.41	−0.85
Class	Group 2										
1	−0.93	−0.93	0.99	0.44	−0.01	−0.45	0.01	−0.45	0.01	−0.36	−0.93
2	−0.90	−0.92	0.98	0.39	0.02	−0.40	0.02	−0.39	0.03	−0.55	−0.92
3	−0.85	−0.82	0.94	0.34	0.06	−0.32	−0.09	−0.35	−0.05	−0.42	−0.85
4	−0.87	−0.87	0.97	0.18	0.19	−0.19	−0.19	−0.22	−0.17	−0.40	−0.87
Average	−0.89	−0.89	0.97	0.39	0.02	−0.39	−0.02	−0.40	0.00	−0.44	−0.90
Class	Group 3										
1	−0.89	−0.83	0.93	0.24	−0.01	−0.23	0.05	−0.26	0.11	−0.41	−0.91
2	−0.87	−0.82	0.88	0.36	0.13	−0.39	−0.02	−0.35	0.00	−0.52	−0.89
3	−0.93	−0.92	0.99	0.23	0.14	−0.21	−0.10	−0.23	−0.07	−0.37	−0.93
4	−0.83	−0.84	0.98	0.22	0.23	−0.19	−0.21	−0.19	−0.18	−0.39	−0.83
5	−0.87	−0.88	1.00	0.14	0.20	−0.14	−0.21	−0.14	−0.20	−0.44	−0.87
6	−0.90	−0.90	0.99	0.35	0.25	−0.26	−0.28	−0.27	−0.25	−0.43	−0.90
7	−0.88	−0.89	0.99	0.05	0.27	−0.08	−0.26	−0.10	−0.24	−0.25	−0.88
8	−0.87	−0.88	0.99	0.12	0.19	−0.09	−0.24	−0.11	−0.22	−0.50	−0.87
9	−0.92	−0.93	0.99	0.00	0.19	0.00	−0.26	−0.01	−0.25	−0.53	−0.92
10	−0.91	−0.91	1.00	0.09	0.25	−0.10	−0.25	−0.10	−0.24	−0.59	−0.91
11	−0.81	−0.85	0.96	−0.11	0.07	0.10	−0.11	0.10	−0.14	−0.47	−0.89
12	−0.83	−0.83	0.97	0.07	0.18	−0.01	−0.22	−0.02	−0.21	−0.69	−0.84
13	−0.86	−0.88	0.99	0.16	0.25	−0.17	−0.22	−0.18	−0.22	−0.59	−0.86
14	−0.87	−0.88	0.99	0.29	0.15	−0.26	−0.18	−0.28	−0.16	−0.55	−0.87
15	−0.81	−0.82	0.99	0.21	0.26	−0.18	−0.27	−0.20	−0.25	−0.56	−0.81
16	−0.81	−0.81	0.99	0.08	0.22	−0.06	−0.25	0.07	−0.25	−0.72	−0.81
Average	−0.84	−0.85	0.99	0.18	0.21	−0.16	−0.22	−0.15	−0.21	−0.58	−0.85

From the results obtained using the ε-constraint method for an instance in Class 11 of Group 3, shown in the bar graphs in Figure 6, we have a more precise analysis of the variables' behaviour (measured by costs) when changing ε_2. Unlike in previous split functions, we include the CSP final remnant storage costs, measured by $\sum_{k=1}^{K}\sum_{i\in S(k)}\sigma_{iT}\eta_{ik}e_{ikT}$. That means we produce additional pieces above the required demands and store them at the end of the last period T. As can be seen in bar graphs 1, 5, and 6, as we gradually raise the bound, ε_2, the compared split function is reduced. In bar graph 4, we can note an increase in the CSP waste of material costs, but it is not possible to establish any conflicted relation in bar graphs 2 and 3.

By the correlation analysis, we can verify that reducing the production of objects not only results in a decrease in the storage costs of the pieces, including the final remnant storage costs, but it also results in an increase of the losses in cutting process. Thus, four variables influence each other, which should be considered in production planning. In practice, the decision maker has to decide between producing fewer objects, consequently having less storage for pieces and final remnants, but more losses in cutting process, or the opposite.

4.3.3. Pareto-optimal solutions and complexity analysis

In order to investigate whether the Pareto-optimal solutions are well spread, we use the following criteria. Let $u_1=(x_1,y_1,z_1,w_1,e_1)$ and $u_2=(x_2,y_2,z_2,w_2,e_2)$ be two Pareto-optimal solutions. If $|F_1(u_1)-F_1(u_2)|\geq 10^{-4}$ and $|F_2(u_1)-F_2(u_2)|\geq 10^{-4}$, we consider that u_1 and u_2 are different. Table 3 presents further details on the number of different Pareto solutions found by each method of resolution, the running time (in seconds), and the number of variables and constraints of the instances. For each instance, the methods of resolution are performed 50 times in accordance with the 50 upper bounds, ε_2, or 50 weighting coefficients $p_i\in(0,1)$. The computed time in Table 3 is the average time of the 20 instances per class. For each instance, we compute the number of variables and constraints. The values nv and nc are the average number of variables and constraints, respectively, of the 20 instances per class. These data provide some idea of the size and difficulty of solving each instance in each class. For the purposes of this paper, the running time obtained by our resolution approaches in each group can be considered satisfactory. Moreover, for each instance, each method of resolution can find up to 50 different Pareto solutions.

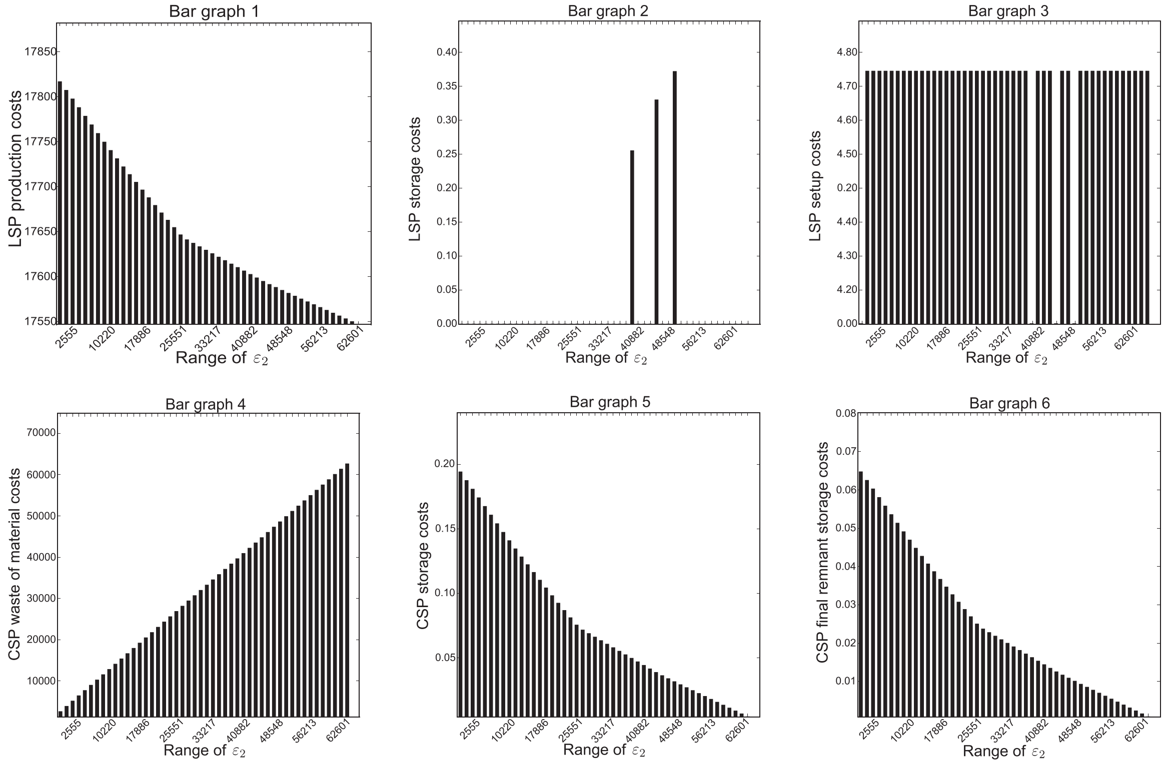

Figure 6. Comparisons from the bar graphs and variations of ε_2.

Table 3. Number of different Pareto solutions, running time, number of variables and number of constraints.

		Group 1				Group 2				Group 3			
Method	Class	nd	time (s)	nv	nc	nd	time (s)	nv	nc	nd	time (s)	nv	nc
ε-Constraint	1	39.9	24.6	110.6	31.0	41.9	136.2	120.2	31.0	34.2	5.4	121.5	31.0
	2	43.6	141.0	147.0	41.0	44.0	150.0	152.8	41.0	38.7	3.6	185.6	41.0
	3	42.1	117.0	120.0	34.0	47.0	36.0	259.5	34.0	48.0	4.8	307.2	34.0
	4	44.3	170.4	160.0	45.0	38.9	149.9	309.0	45.0	47.7	7.8	407.4	45.0
	5	43.8	136.2	123.0	37.0					45.6	6.0	539.4	37.0
	6	39.5	144.6	164.0	49.0					48.0	9.6	881.6	49.0
	7	41.7	79.2	126.0	40.0					45.6	10.2	1223.7	40.0
	8	46.0	90.0	168.0	53.0					48.0	14.4	1559.6	53.0
	9									46.6	16.2	1892.4	43.0
	10									45.6	25.2	3414.0	57.0
	11									43.4	27.0	3430.8	46.0
	12									45.2	39.0	4909.8	61.0
	13									46.8	20.8	540.0	61.0
	14									48.0	41.5	771.6	81.0
	15									47.9	48.6	929.1	88.0
	16									48.0	62.1	1112.8	117.0
Weighting	1	5.5	11.4	110.6	30.0	5.7	25.8	120.2	30.0	2.2	2.4	121.5	30.0
	2	4.7	47.4	147.0	40.0	4.4	43.8	152.8	40.0	3.2	1.8	185.6	40.0
	3	6.1	62.4	120.0	33.0	6.0	44.4	259.5	33.0	4.1	1.8	307.2	33.0
	4	6.8	76.2	160.0	44.0	4.9	48.7	309.0	44.0	6.2	3.0	407.4	44.0
	5	7.0	54.6	123.0	36.0					5.4	2.4	539.4	36.0
	6	6.2	19.2	164.0	48.0					4.8	3.0	881.6	48.0
	7	6.2	42.6	126.0	39.0					7.5	4.2	1223.7	39.0
	8	6.9	56.4	168.0	52.0					8.6	5.4	1559.6	52.0
	9									6.7	6.0	1892.4	42.0
	10									9.2	9.0	3414.0	56.0
	11									6.7	10.2	3430.8	45.0
	12									11.2	14.4	4909.8	60.0
	13									10.8	3.8	540.0	60.0
	14									14.1	7.2	771.6	80.0
	15									13.8	8.1	929.1	87.0
	16									16.5	10.9	1112.8	116.0

The value *nd* in Table 3 is the average number of different Pareto solutions found for the 20 instances per class. We observe that the ε-constraint method found from about 68–96% of the possible solutions, while the weighting method found from about 4–33% of the possible solutions. This finding could be explained by the fact that the weighting method cannot determine unsupported Pareto-optimal solutions. In this requirement, the ε-constraint method is more flexible than the weighting method, and it

Table 4. Heuristic performance based on the results of Groups 1 and 2.

	Small (S)							
Dimension	1		2		3		4	
Class	Gap (%)	CP (%)	Gap (%)	CP (%)	Gap (%)	CP (%)	Gap (%)	CP (%)
ε-Constraint	0	100	0.003	96.71	0.662	43.62	1.860	41.85
Weighting	0	100	0.107	96.71	0.000	43.62	4.900	41.85

can provide a broader range of solutions, which could be used by the decision maker in a more accurate way than the weighting method.

Solving the problem can be a difficult task once the model becomes relatively large. To illustrate the dimensions of the model, we calculate the total number of constraints, nc, and approximate number of variables, nv, of the model according to parameters K, M, T, Nf and N_m. Constraints (4)–(8) yield $nc = T(K+M) + KT(2M+Nf)$, so the highest power of nc is cubic; that is, nc has polynomial growth. Constraints (9)–(11) provide $nv = K(M+Nf) + KT(3M + Nf + \sum_{m=1}^{M} N_m)$. Each N_m, $m = 1, ..., M$, is the number of possible combinations in which the piece lengths are organised into an object of length L_m. In the simplest case (ignoring the issues of feasible combinations and Nf pieces for which there is a unit demand), enumerating all possible combinations requires at least $2^{Nf} - 1$ different combinations, where Nf is the number of required pieces. Therefore, nv has exponential growth (Arbel, 1993; Suliman, 2001). The result of the analytical calculation of nc and the data shown in Table 3, obtained by CPLEX, are the same, as expected. We should point out that in the ε-constraint method, there is an extra constraint because the objective function $F_2(y, e)$ becomes a constraint. Another important consideration is that the value of nv does not have exponential growth for Group 1 because we limit the N_m value, as explained in Section 4.2.

Table 4 shows the comparison between the results achieved from the optimal solution (Group 2) and the heuristic approach (Group 1). Given an instance, each successful execution of one of the ε-constraint and weighting methods for Group 2 and Group 1 provides the optimal objective function value FO* and the suboptimal objective function value FO, from which we calculate the gap equal to $\frac{FO - FO*}{FO} \times 100$. The complete running of an instance provides up to 50 gap values and thus the average gap value by instance is determined. In Table 4 the Gap column contains the average among all the average gap values for the 20 instances of a particular class. The CP value in Table 4 is the average portion of all cutting patterns for all the instances used by the heuristic approach. As mentioned in Section 4.1, the 50 upper bounds ε_2 used in the ε-constraint method are in the range from Ideal to Nadir solution. In this specific comparison, the ε_2 value must be the same for both groups because it is a parameter of the problem. Therefore, the range was calculated between Ideal and Nadir solutions obtained of each instance by Group 1, since the Ideal solution of the Group 2 might not be reached by Group 1, which would lead to infeasibility.

Table 4 presents the value Gap equal to zero for Class 1 because in this class the value of the CP is equal to all cutting patterns. Note that for Classes 2, 3, and 4, the value of the CP decreases to around 40% but the average gap increases to approximately 2 and 5% for the ε-constraint method and weighting method, respectively. The conclusion is that this heuristic approach is useful to small instances and it shows potential for medium instances.

4.3.4. Summary of the analysis

Based on the overall outcomes concerning the characteristics of the LSP integrated with the CSP, we assume that our multiobjective approach allows us to understand these two problems better. If it is necessary to generate low waste of material, it is also mandatory to increase the production of objects. This occurs because when many objects are available, it is possible to select better cutting patterns that incur little waste of material in the CSP, even if the selected cutting patterns exceed the demand over a particular period and the storage costs are paid, by cutting more pieces than required.

Conversely, when only a few objects need to be produced, the drawback is an increasing amount of wasted material. This point can be explained by Constraints (7) and (8) in the mathematical model. Concerning stock, in (8), the total number of produced objects plus the stock balance must be equal to the total number of cut objects, and in (7), the total number of cut objects following the cutting patterns must meet the pieces demand. Thus, if few objects are produced, generating low LSP costs, cutting patterns are selected to supply the demand, even if the waste of material increases. For clarity, we will present one small numerical example.

Suppose that the demand is two pieces that are 92 cm long and two pieces that are 115 cm long. If only one 460-cm long object is available, the cutting pattern to meet this demand must be $2 \times 92 + 2 \times 115 = 414$, which generates material waste equal to 46 cm. Now, suppose that two objects of 460-cm length are available; the cutting patterns that eliminate the waste of material would be $5 \times 92 = 460$ and $4 \times 115 = 460$. Therefore, in the second case, the total waste of material would be zero, and three 92-cm pieces and two 115-cm pieces would have to be stored. This surplus can be used over the planning horizon.

Each Pareto-optimal solution allows the decision maker to improve the production planning. Therefore, we believe that the multiobjective approach for integrated problems is mainly relevant for industries that use multiple-criteria ABC analysis (Flores & Whybark, 1987) in production planning. From a theoretical point of view, our findings suggest that it is worthwhile to further investigate the possibilities of our approach. For example, the five split functions can be used to create new approaches by considering novel multiobjective models. The correlations identified in this study may presume new 2-, 3-, or up to 4-objective optimisation models.

5. Conclusions

In this study, we introduce a multiobjective mathematical model to deal with the LSP integrated with the CSP in the paper industry, and we adequately transformed an earlier mono-objective model into a new bi-objective optimisation problem. The central aspect of this contribution is to evaluate the trade-off between LSP and CSP by analysing the cost variations of these two problems when we simultaneously minimise them. In a suitable manner, we separated the objective function of the multiobjective problem into some other functions to compare, detect, analyse, and study potential trade-offs/correlations at the heart of the problem. We illustrated our results through Pareto front, which highlighted several trade-offs/correlations among the separated functions in our approach. It was clear that increasing the total LSP costs implied decreasing the total CSP costs, and vice versa. The data confirmed many possible options of solutions for this integrated problem.

All correlations among the suitably separated functions have been validated and illustrated. The most important correlations are between the LSP production, CSP waste of material and CSP storage costs. Furthermore, we conclude that it is necessary to cut more than the demand to minimise the waste of material, even if remnant pieces must be stored in the final period. In practice, manufacturing industries that have a long planning horizon could use this strategy in their production process.

Our conclusions about the trade-off between the two problems can improve the technical decisions in the industrial sector, primarily because, in opposition to the mono-objective approach, the solutions obtained from multiobjective approaches provide more opportunities to the decision maker who plans the production. The results obtained from the multiobjective and heuristic approach presented in this article may also be useful for other industries, such as furniture and aluminum, which deal with two-dimensional cutting patterns applications.

In future research, we will consider a new multiobjective approach for this integrated problem by using a tri-objective optimisation model and by putting CSP waste of material costs in opposition to CSP storage costs. The setup costs of different cutting patterns could also be included in the CSP objective function.

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