Interpreting the Contribution of Sensors in Blind Source Extraction by Means of Shapley Values

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***Abstract*—Several practical applications can be formulated as a problem of estimating a source of interest from a set of mixed data collected by different sensors. Although a lot of effort has been done to address the optimization task in signal extraction, there is a lack in the literature on how to evaluate the contribution of each sensor in the extraction process. In this letter, we propose a model-agnostic approach that can be used to interpret both the contribution of each sensor in the estimated source and the interaction effects between them. Our proposal is based on a solution concept from game theory, called Shapley value. Numerical experiments on synthetic and real data attest the use of our proposal in blind source extraction problems.**

***Index Terms*—signal extraction, sensors design, Shapley value**

I. Introduction

In Blind Source Extraction (BSE) [1] problems, the aim is to retrieve a signal of interest based on a set of observed mixed signals collected by a set of sensors. Several practical situations can be modeled as BSE problems. For instance, in fetal electrocardiogram (fetal ECG) [2], [3], [4], one extracts a signal that better represents the fetal heart beats based on the set of signals captured by electrodes on the maternal abdomen. In a recent work [5], the authors adopted a BSE formulation to extract acoustic emission signals and detect rail cracks. Another example consists in detecting bearing faults [6] from vibration signals and, therefore, prevent machine breakdowns.

BSE is generally tackled by an optimization task whose goal is to retrieve a signal that optimizes a predetermined property of interest, such as time-correlation [7], [8], [9], sparsity [10], smoothness [11], [12], quasi-periodicity [13] and cyclostationarity [14]. One may also have strategies that exploit more than one property simultaneously [2], [15]. As these properties vary for different applications (e.g., time-correlation for ECG data and cyclostationarity for communication signals), different algorithms have been proposed in the literature to deal with the optimization task.

Another research topic in BSE lies in evaluating the influence of sensors in sensor design. For example, in fetal ECG, there is an interest in strategically positioning the electrodes in order to improve the extraction of the fetal components [16]. Clearly, due to physical and financial constraints (e.g., cost reduction), only a subset of them (from all possible positions in the maternal abdomen) is selected to compose the array of sensors. Indeed, the use of an excessive number of electrodes during labor brings an inconvenience for the clinicians and ergonomic concerns for the mother [17]. Therefore, the use of a subset of sensors is of importance in real situations.

Although existing works deal with sensor selection [18] or sensor placement [19] problems, to the best of our knowledge, no effort was done to investigate the contribution of sensors based on a game theory paradigm widely used in machine learning explainability [20], [21], [22], [23], called Shapley value [24]. Therefore, in this paper, we propose an approach to interpret the importance of each sensor in BSE based on the Shapley value. When a set of players cooperate in a game, the Shapley value of a specific player indicates his/her marginal contribution on the achieved payoff. The cooperative game in this case refers to the BSE problem in the following way: the sensors (mixtures) can be seen as the players, and the retrieved signal can be seen as the game payoff when a subset of mixtures is available in the extraction procedure. Therefore, the Shapley value associated with each sensor will indicate its marginal contribution on the extracted signal. In the problem of designing an array of sensors (e.g., in fetal ECG), sensors with high marginal contributions should be considered to compose the array. Numerical experiments in both synthetic and real datasets attest our proposal.

The rest of this letter is organized as follows. In Sections II and III, we describe the theoretical aspects about the BSE problem and the Shapley value, respectively. Section IV describes the proposed Shapley value-based approach to evaluate and interpret sensors contribution in BSE. The numerical experiments are conducted in Section V. Finally, we present our conclusions and future perspectives in Section VI.

II. Blind source extraction problem

Assume a set of N signal sources $\mathbf{s}(k) = [s_1(k), s_2(k), \ldots, s_N(k)]^T$, where $k = 1, \ldots, K$ are the time samples. Consider that these sources are mixed by a linear and instantaneous process $\mathbf{A} \in \mathbb{R}^{M \times N}$ such that

$$\mathbf{x}(k) = \mathbf{A}\mathbf{s}(k), \tag{1}$$

is the set of M mixtures (i.e., $\mathbf{x}(k) = [x_1(k), x_2(k), \ldots, x_M(k)]^T$) collected by different sensors. Among $\mathbf{s}(k)$, suppose that we are interested in retrieving a single source $s_{n^*}(k)$, $n^* \in \{1, \ldots, N\}$. In BSE, one assumes that both the sources and the parameters of $\mathbf{A}$ are unknown.

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Therefore, in order to estimate $s_{n^*}(k)$, one needs to adjust an extraction vector $\mathbf{w} = [w_1, w_2, \ldots, w_M]^T$ such that

$$\hat{s}(k) = \mathbf{w}^T \mathbf{x}(k) \tag{2}$$

is as close as possible to $s_{n^*}(k)$. For this purpose, one formulates an optimization problem whose cost function represents a property expected from the extracted source $\hat{s}(k)$.

III. Shapley value in cooperative game theory

The Shapley value is a classical solution concept in cooperative game theory [24]. Assume that a set $\mathcal{M} = \{1, 2, \ldots, M\}$ of M players cooperate in order to achieve a common goal (e.g., increasing the profit from supply chain [25] or software development [26] alliances). Consider also a function $\upsilon : \mathcal{P}(\mathcal{M}) \to \mathbb{R}$, where $\mathcal{P}(\mathcal{M})$ is the power set of $\mathcal{M}$, that maps subsets of players to real numbers. $\upsilon(\mathcal{D})$ is referred to as the payoff (e.g., the profit) achieved in the game when players in $\mathcal{D}$ cooperate. When there is no player in the coalition, one assumes[1] $\upsilon(\emptyset) = 0$. The Shapley value of a player m indicates his/her marginal contribution on the payoff $\upsilon(\mathcal{M})$ achieved by the cooperation of all players. It is defined as follows:

$$\phi_m = \sum_{\mathcal{D} \subseteq \mathcal{M} \backslash \{m\}} \frac{(M - |\mathcal{D}| - 1)! \, |\mathcal{D}|!}{M!} \Delta_m \upsilon(\mathcal{D}), \tag{3}$$

where $\Delta_m \upsilon(\mathcal{D}) = \upsilon(\mathcal{D} \cup m) - \upsilon(\mathcal{D})$ and $|\mathcal{D}|$ represents the cardinality of subset $\mathcal{D}$. An important aspect of the Shapley value is that it satisfies several desired properties when sharing benefits from a cooperative game [27]. Among them, three are of interest for this work: symmetry, null player and efficiency. Symmetry means that if two players equally cooperate in the game, the associated Shapley values must also be equal. According to the null player property, if there is no gain when player m joins any coalition, his/her Shapley value must be zero (there is no profit to receive). Finally, the efficiency property ensures that $\sum_{m=1}^{M} \phi_m = \upsilon(\mathcal{M}) - \upsilon(\emptyset) = \upsilon(\mathcal{M})$, i.e., one may decompose the payoff of the grand coalition among players according to the Shapley values.

Besides the marginal contributions of players, one may also be interested in the interaction degree between them[2]. In other words, for two players m, m', one may evaluate if there is a positive (the contributions of both players is greater than the sum of their individual contributions) or a negative (the contributions of both players is lower than the sum of their individual contributions) effect between them. The Shapley interaction index is calculated as follows [29], [28]:

$$I_{m,m'} = \sum_{\mathcal{D} \subseteq \mathcal{M} \backslash \{m,m'\}} \frac{(M - |\mathcal{D}| - 2)! \, |\mathcal{D}|!}{(M-1)!} \Delta_{m,m'} \upsilon(\mathcal{D}), \tag{4}$$

where $\Delta_{m,m'} \upsilon(\mathcal{D}) = \upsilon(\mathcal{D} \cup \{m, m'\}) - \upsilon(\mathcal{D} \cup m) - \upsilon(\mathcal{D} \cup m') + \upsilon(\mathcal{D})$. In this case, $\gamma I_{m,m'} > 0$, $\gamma I_{m,m'} < 0$ or $I_{m,m'} = 0$, where $\gamma = \operatorname{sign}(\upsilon(\mathcal{M}) - \upsilon(\emptyset))$ is the sign function, indicate a positive (synergistic), a negative (redundant) or the absence of interaction effect between players m, m'.

[1] In cooperative game theory, one frequently adopts $\upsilon(\emptyset) = 0$. As will be discussed latter, in this work, we consider another value for $\upsilon(\emptyset)$.

[2] It is worth highlighting that the Shapley interaction index may be defined for any coalition of players. However, one only has clear interpretations for at most two players. See [28] for further details.

IV. The proposed Shapley value-based approach for BSE interpretability

The idea in this work is to use the Shapley value to interpret the importance of each sensor in BSE problems. We consider as players the set $\mathcal{M}$ of M sensors. Each sensor m provides a mixed signal which (possibly) contains information from the source of interest. Therefore, when we use more sensors, we have more available information in order to extract $s_{n^*}(k)$.

Let us define $\hat{s}_{\mathcal{D}}(k) = \mathbf{w}_{\mathcal{D}}^T \mathbf{x}_{\mathcal{D}}(k)$ as the estimated source when only the mixed signals $\mathbf{x}_{\mathcal{D}}(k)$ obtained by sensors in $\mathcal{D} \subseteq \mathcal{M}$ are used to adjust the extraction vector $\mathbf{w}_{\mathcal{D}}$. As the use of all sensors provides all possible information to retrieve $s_{n^*}(k)$, we assume that $\hat{s}_{\mathcal{M}}(k)$ is the best estimation that we could obtain. By modeling the BSE problem as a cooperative game, we here define the payoff $\upsilon_{BSE}(\mathcal{D})$ as a measure that quantifies the quality of $\hat{s}_{\mathcal{D}}(k)$ when estimating $\hat{s}_{\mathcal{M}}(k)$. By using the mean squared error (other measure could be adopted, such as the signal-to-interference ratio), the payoff can be defined by[3]

$$\upsilon_{BSE}(\mathcal{D}) = \frac{1}{K} \min \Bigg\{ \sum_{k=1}^{K} \left(\hat{s}_{\mathcal{M}}(k) - \hat{s}_{\mathcal{D}}(k) \right)^2, \sum_{k=1}^{K} \left(\hat{s}_{\mathcal{M}}(k) + \hat{s}_{\mathcal{D}}(k) \right)^2 \Bigg\}, \tag{5}$$

where the minimum operator $\min\{\cdot\}$ avoids miscalculations when $\hat{s}_{\mathcal{D}}(k) \approx -\hat{s}_{\mathcal{M}}(k)$. The Shapley value of sensor m (ϕ_m^{BSE}) and the Shapley interaction index between sensors m, m' ($I_{m,m'}^{BSE}$) are defined by replacing $\upsilon(\mathcal{D})$ by $\upsilon_{BSE}(\mathcal{D})$ in Equations (3) and (4), respectively. Note that, when $\mathcal{D} = \mathcal{M}$, $\upsilon_{BSE}(\mathcal{M}) = 0$. Therefore, in our proposal and based on the efficiency property, the Shapley values indicate the marginal contributions of sensors that cooperates to improve the prediction (or reducing the error) from $\hat{s}_{\emptyset}(k)$ to $\hat{s}_{\mathcal{M}}(k)$. It remains to define $\hat{s}_{\emptyset}(k)$. When no mixture is available, we simply assume that the retrieved signal is a vector sampled from an i.i.d. Gaussian distribution $\mathcal{N}(0, 1)$. The idea of sampling from a Gaussian distribution is to define a signal that carries no information from the source of interest. It consists in the "worst choice" when no information is available, so the most uncertain guess (in the sense of differential entropy) is considered. Surely, other assumptions can be adopted to generate $\hat{s}_{\emptyset}(k)$. Algorithm 1 presents the steps of our proposal.

It is worth highlighting that our proposal is model-agnostic, i.e., it can be used regardless how the sources were mixed or which BSE algorithm was adopted. It only requires that the adopted model is able to estimate the source of interest for any $\mathcal{D} \subseteq \mathcal{M}$. Moreover, another important aspect is that, in order to calculate the Shapley values, one must calculate $\upsilon_{BSE}(\mathcal{D})$ for all $\mathcal{D} \subseteq \mathcal{M}$. This means that one needs to address BSE problems for all coalitions of sensors. As this may pose a computational time constraint, existing techniques can be used to approximate the Shapley values with less effort [30], [20].

[3] After extracting both signals, we normalize them in order to avoid scaling ambiguity: $\hat{s}_{\mathcal{D}}(k) \leftarrow \left(\hat{s}_{\mathcal{D}}(k) - \mu_{\hat{s}_{\mathcal{D}}} \right) / \sigma_{\hat{s}_{\mathcal{D}}}$, where $\mu_{\hat{s}_{\mathcal{D}}}$ and $\sigma_{\hat{s}_{\mathcal{D}}}$ represent the mean and the standard deviation of vector $\hat{s}_{\mathcal{D}}(k)$, respectively.

Algorithm 1 (*Shapley values for BSE interpretability*)

Input: $\mathbf{x}(k)$
Output: ϕ_m^{BSE} and $I_{m,m'}^{BSE}$, for all $m, m' = 1, \ldots, M$
1: **Based on all mixtures, apply a BSE algorithm and conduct signal extraction:** $\hat{s}_{\mathcal{M}}(k) \leftarrow \mathbf{w}_{\mathcal{M}}^T \mathbf{x}_{\mathcal{M}}(k)$
2: **Normalize** $\hat{s}_{\mathcal{M}}(k)$: $\hat{s}_{\mathcal{M}}(k) \leftarrow (\hat{s}_{\mathcal{M}}(k) - \mu_{\hat{s}_{\mathcal{M}}}) / \sigma_{\hat{s}_{\mathcal{M}}}$
3: **Calculate the payoffs for all coalitions of sensors**:
for $\mathcal{D} \subseteq \mathcal{M}$ **do**
  if $\mathcal{D} = \emptyset$ **then**
    Generate: $\hat{s}_{\mathcal{D}}(k) \sim \mathcal{N}_k(0, 1)$
  else if $\mathcal{D} = m$ **then**
    Define: $\hat{s}_{\mathcal{D}}(k) \leftarrow x_m(k)$
  else
    Apply a BSE algorithm and extract the signal: $\hat{s}_{\mathcal{D}}(k) \leftarrow \mathbf{w}_{\mathcal{D}}^T \mathbf{x}_{\mathcal{D}}(k)$
  end if
  Normalize $\hat{s}_{\mathcal{D}}(k)$: $\hat{s}_{\mathcal{D}}(k) \leftarrow (\hat{s}_{\mathcal{D}}(k) - \mu_{\hat{s}_{\mathcal{D}}}) / \sigma_{\hat{s}_{\mathcal{D}}}$
  Calculate the payoff: $\upsilon_{BSE}(\mathcal{D})$ (Equation (5))
end for
5: **Calculate the Shapley values and interaction indices**: ϕ_m^{BSE} (Equation (3)) and $I_{m,m'}^{BSE}$ (Equation (4)), for all $m, m' = 1, \ldots, M$

V. Numerical experiments

In the sequel, we illustrate the application of our proposal in both synthetic and real mixed data.

A. Experiments on synthetic data

In the experiments with synthetic mixing data, we considered the signal sources presented in Figure 1, which were extracted from the ABio7 dataset available in ICALAB toolboxes [31]. We assumed as the source of interest the ECG ($s_3(k)$). As this signal is sparse in time, we formulate an optimization problem whose goal is to retrieve the sparsest signal from the set of mixtures; in that respect, we consider an approach based on the ℓ_1-norm. Therefore, for each coalition of sensors $\mathcal{D}$, we deal with the following problem:

$$\min_{\mathbf{w}} \left\| \mathbf{w}^T \mathbf{x}_{\mathcal{D}} \right\|_1 / \left\| \mathbf{w}^T \mathbf{x}_{\mathcal{D}} \right\|_2, \tag{6}$$

where $\|\cdot\|_1$ and $\|\cdot\|_2$ represent the ℓ_1 and ℓ_2-norm, respectively.

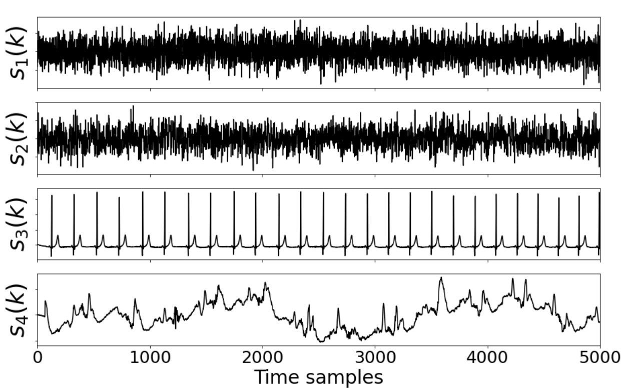


Fig. 1: Signal sources (synthetic data).

We evaluate our proposal by assuming the mixing matrix

$$\mathbf{A} = \begin{bmatrix} \alpha & \beta & \beta & \beta \\ \beta & \lambda & \eta & \beta \\ \beta & \beta & \alpha & \beta \\ \beta & \beta & \beta & \alpha \end{bmatrix}, \tag{7}$$

where the elements α, β, λ and η are defined according to the following scenarios ($\mathcal{U}(a, b)$ indicates a random value generated from a uniform distribution in the range $[a, b]$):

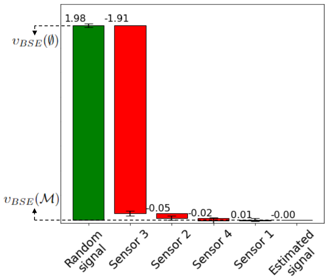
1) $\alpha, \lambda \sim \pm\mathcal{U}(0.95, 1.05)$ and $\beta, \eta \sim \pm\mathcal{U}(-0.05, 0.05)$;
2) $\alpha, \lambda \sim \pm\mathcal{U}(0.95, 1.05)$ and $\beta, \eta \sim \pm\mathcal{U}(0.45, 0.55)$;
3) $\alpha, \eta \sim \pm\mathcal{U}(0.95, 1.05)$ and $\beta, \lambda \sim \pm\mathcal{U}(-0.05, 0.05)$;
4) $\alpha, \beta, \eta, \lambda \sim \pm\mathcal{U}(0.45, 0.55)$.

The obtained results (average contribution and standard deviation over 500 simulations) are illustrated in Figure 2. The green bar represents the mean squared error $\upsilon_{BSE}(\emptyset)$ and the red bars indicate the contribution of sensors in decreasing the error until $\upsilon_{BSE}(\mathcal{M}) = 0$ (recall that, lower is the Shapley value, greater is the contribution when estimating $\hat{s}_{\mathcal{M}}(k)$). In all cases, we could recover the ECG signal when all sensors were used. In scenario (1), sensor 3 captured much more information from the source of interest in comparison to the other ones. Therefore, its contribution is practically maximal while the contribution of the other sensors are minimal (see Figure 2a). This illustrates the null player property, as there is no contribution from sensors 1, 2 and 4 when joining coalitions. For scenario (2), sensor 3 is still capturing more information from $s_3(k)$. However, the others sensors also captured some information from it. Therefore, although the contribution of sensor 3 is the highest one, it is not maximal, as the others sensors also contribute to retrieve the ECG (see Figure 2b). In scenario (3), both sensors 2 and 3 captured the same amount of information from $s_3(k)$. As a consequence, their contributions were the same (see Figure 2c). This reflects the symmetry property, as sensors 2 and 3 provide the same gain when joining coalitions. Finally, in scenario (4), all sensors captured similar information from $s_3(k)$. As also expected by the symmetry property, all of them have the same contribution towards the estimated source (see Figure 2d).

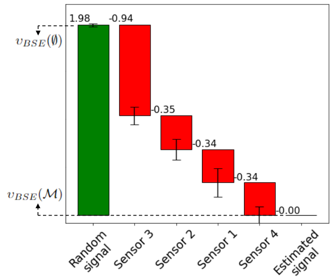
B. Experiments on real data

Aiming at validating our proposal in a real BSE scenario, we considered the ECG data provided by [32]. There are eight ECG signals (mixed data, as presented in Figure 3) obtained from a pregnant women and the goal is to extract the fetal ECG. In order to estimate such a signal, we adopted the strategy proposed by [7], whose idea is to extract a signal that maximizes its autocorrelation given a predetermined delay τ. We assumed the same delay as identified by the authors from $x_1(k)$ (an ECG where the fetal heart beats can be visualized), i.e., $\tau = 112$ (see [7] for more details on how to define τ as well as on the extraction algorithm). Figure 4 presents the estimated fetal ECG $s_{fECG}(k)$ based on all mixtures.

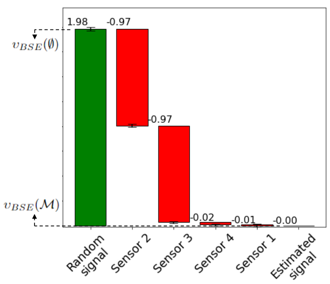
The application of our proposal leads to the sensors contributions described in Figure 5. Sensors 3 and 1 have the highest contribution on the estimated signal. This is visually



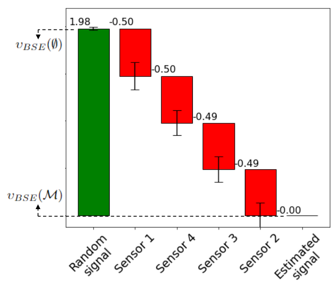
(a) Scenario (1).



(b) Scenario (2).



(c) Scenario (3).



(d) Scenario (4).

Fig. 2: Shapley values for each sensor (synthetic data).

Fig. 3: ECGs (mixed signals).

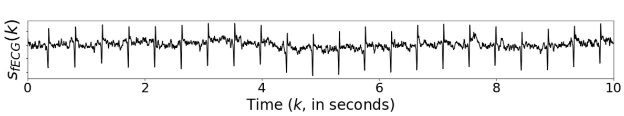


Fig. 4: Estimated fetal ECG.

in accordance with Figure 3, where the fetal heart beats could be noted on both $x_1(k)$ and $x_3(k)$. The better performance of sensor 3 over sensor 1 can be explained by positive interactions (see Figure 6) between the former and sensors 6 ($I_{3,6}^{BSE} = -0.37$), 7 ($I_{3,7}^{BSE} = -0.21$) and 8 ($I_{3,8}^{BSE} = -0.17$). Therefore, the strong contribution of sensor 3 is due to both the presence of fetal heart beats in the captured signal and the potential gain when used in combination with other sensors that essentially captured maternal signals.

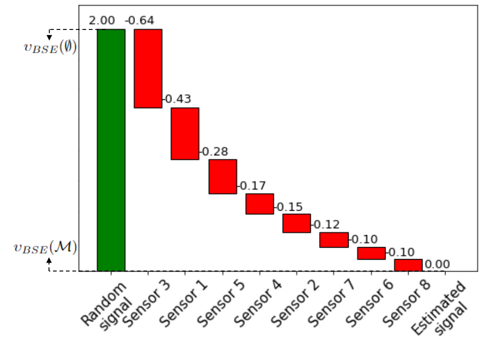


Fig. 5: Shapley values for each sensor (fetal ECG).

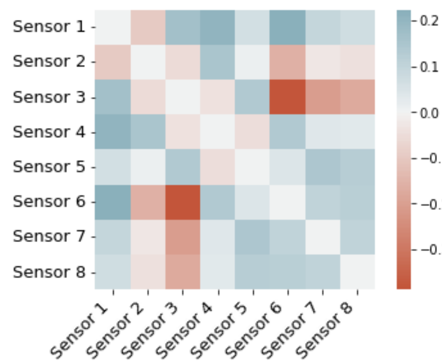


Fig. 6: Interaction effects between sensors (fetal ECG).

VI. Conclusions

This letter proposes an approach to evaluate the contribution of sensors in blind source extraction problems. Moreover, this approach also provides the interaction effects between sensors, i.e., if there are synergies or redundancies among them. An important aspect of our proposal is that it is model-agnostic, so it can be deployed in any signal extraction algorithm.

Numerical experiments illustrated the application of our proposal. Based on synthetic data, we demonstrated that the obtained sensors contributions are in accordance with the Shapley values properties. The greater the information from the source of interest captured by the sensor, the higher is its contribution when estimating such a signal. This conclusion is also attested by the results on real data. Moreover, we could note that synergies between sensors also increase their marginal contribution. It is worth mentioning that a mechanism to interpret sensors contributions can be useful, for instance, to identify the ones that have low impact on the extraction process and, therefore, could be removed from the set. As in the selection of array of electrodes for fetal ECG, this is useful to avoid unnecessary materials and save investments when (re)designing the set of sensors.

As we deal with recovering information from mixed signals, future works aim at extending our proposal to other signal processing tasks involving sensor arrays, such as the blind source separation problem. Moreover, another perspective consists in interpreting the importance of sensors in nonlinear mixture as well as in overdetermined models.

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