3) Seemingly Unrelated Regressions (SUR)

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System of Equations

$$housing = \beta_{10} + \beta_{11} houseprc + \beta_{12} foodprc + \beta_{13} clothprc + \beta_{14} income + \beta_{15} size + \beta_{16} age + u_1.$$

$$food = \beta_{20} + \beta_{21} houseprc + \beta_{22} foodprc + \beta_{23} clothprc + \beta_{24} income + \beta_{25} size + \beta_{26} age + u_2.$$

$$clothing = \beta_{30} + \beta_{31} houseprc + \beta_{32} foodprc + \beta_{33} clothprc + \beta_{34} income + \beta_{35} size + \beta_{36} age + u_3.$$

$$E(u_g | x_1, x_2, \ldots, x_G) = 0$$

$$g = 1, \ldots, G$$

Generalized Least Squares (GLS)

$$E(\epsilon\epsilon'|X) = \underset{(n\times n)}{\sigma^2 V(X)}$$

$$V^{-1} = C'C$$

$$Cy = CX\beta + C\epsilon$$

$$\hat{\beta}_{GLS} = (X'V^{-1}X)^{-1}X'V^{-1}y$$

GLS - Two Equations

$$y_g = X_g \beta_g + u_g, \qquad g = 1, 2$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\Sigma \otimes I_N = \begin{bmatrix} \sigma_{11} I_N & \sigma_{12} I_N \\ \sigma_{21} I_N & \sigma_{22} I_N \end{bmatrix}$$

$$\hat{\beta}_{GLS} = \begin{bmatrix} \sigma_{11} X_1' X_1 & \sigma_{12} X_1' X_2 \\ \sigma_{21} X_2' X_1 & \sigma_{22} X_2' X_2 \end{bmatrix}^{-1} \begin{bmatrix} \sigma_{11} X_1' y_1 + \sigma_{12} X_1' y_2 \\ \sigma_{21} X_2' y_1 + \sigma_{22} X_2' y_2 \end{bmatrix}$$

Seemingly Unrelated Regressions (SUR)

$$\begin{bmatrix} y_{i1} \\ \vdots \\ y_{iG} \end{bmatrix} = \begin{bmatrix} x_{i1}' & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & x_{iG}' \end{bmatrix} \begin{bmatrix} \beta_{i1} \\ \vdots \\ \beta_{iG} \end{bmatrix} + \begin{bmatrix} u_{i1} \\ \vdots \\ u_{iG} \end{bmatrix}$$

$$\hat{\beta}_{GLS} = \{X'(\Sigma^{-1} \otimes I_N)X\}^{-1}\{X'(\Sigma^{-1} \otimes I_N)y\}$$

$$Var(\hat{\beta}) = \{X'(\Sigma^{-1} \otimes I_N)X\}^{-1}$$

SUR Estimator = FGLS Estimator

1) Estimate each equation by OLS

2) Estimate Σ, using:

$\hat{u}_j = y_j - X_j\hat{\beta}_j$ and $\hat{\sigma}_{jj'} = \hat{u}_j'\hat{u}_{j'}/N$

3) Use $\hat{\Sigma}$ to obtain $\hat{\beta}_{FGLS}$

SUR Model for Hourly Wages and Hourly Benefits

$$r_{12} = .32$$

Explanatory Variables	*hrearn*	*hrbens*
educ	.459 (.069)	.077 (.008)
exper	−.076 (.057)	.023 (.007)
$exper^2$	.0040 (.0012)	−.0005 (.0001)
tenure	.110 (.084)	.054 (.010)
$tenure^2$	−.0051 (.0033)	−.0012 (.0004)
union	.808 (.408)	.366 (.049)
south	−.457 (.552)	−.023 (.066)
nrtheast	−1.151 (0.606)	−.057 (.072)
nrthcen	−.636 (.556)	−.038 (.066)
married	.642 (.418)	.058 (.050)
white	1.141 (0.612)	.090 (.073)
male	1.785 (0.398)	.268 (.048)
intercept	−2.632 (1.228)	−.890 (.147)

Wooldridge (2010)

Medical Expenditure Panel Survey (MEPS)

- Medicare-eligible population
- Aged $>$ 65 years
- Medicare does not cover all medical expenses
- People usually buy private insurance

ldrugexp: log of expenditure on prescribed drugs

ldrugexp: log of expenditure on all categories of medical services other than drugs

actlim: activity limitation

use mus05surdata.dta

summarize ldrugexp ltotothr age age2 educyr ///
actlim totchr medicaid private

Variable	Obs	Mean	Std. Dev.	Min	Max
ldrugexp	3,285	6.936533	1.300312	1.386294	10.33773
ltotothr	3,350	7.537196	1.61298	1.098612	11.71892
age	3,384	74.38475	6.388984	65	90
age2	3,384	5573.898	961.357	4225	8100
educyr	3,384	11.29108	3.7758	0	17
actlim	3,384	.3454492	.4755848	0	1
totchr	3,384	1.954492	1.326529	0	8
medicaid	3,384	.161643	.3681774	0	1
private	3,384	.5156619	.4998285	0	1

reg ldrugexp age age2 actlim totchr medicaid private

Source	SS	df	MS
Model	1260.35424	6	210.05904
Residual	4292.27045	3,278	1.30941746
Total	5552.62469	3,284	1.69081141

Number of obs	=	3,285
F(6, 3278)	=	160.42
Prob > F	=	0.0000
R-squared	=	0.2270
Adj R-squared	=	0.2256
Root MSE	=	1.1443

ldrugexp	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
age	.2764148	.0798228	3.46	0.001	.1199071	.4329224
age2	-.0018315	.0005306	-3.45	0.001	-.0028718	-.0007911
actlim	.357446	.0468032	7.64	0.000	.2656795	.4492125
totchr	.4035182	.0162256	24.87	0.000	.3717049	.4353316
medicaid	.0893386	.0600878	1.49	0.137	-.0284748	.2071521
private	.0775393	.0438092	1.77	0.077	-.0083569	.1634355
_cons	-4.402228	2.986652	-1.47	0.141	-10.25812	1.453664

reg ltotothr age age2 educyr actlim totchr private

Source	SS	df	MS		
Model	1342.02084	6	223.67014		
Residual	7371.09274	3,343	2.20493352		
Total	8713.11358	3,349	2.60170606		

Number of obs	=	3,350	
F(6, 3343)	=	101.44	
Prob > F	=	0.0000	
R-squared	=	0.1540	
Adj R-squared	=	0.1525	
Root MSE	=	1.4849	

ltotothr	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
age	.3173817	.1026155	3.09	0.002	.1161862	.5185772
age2	-.0020875	.0006823	-3.06	0.002	-.0034252	-.0007498
educyr	.0650207	.0072825	8.93	0.000	.0507421	.0792993
actlim	.7421208	.0601413	12.34	0.000	.6242032	.8600384
totchr	.295988	.0206706	14.32	0.000	.2554597	.3365164
private	.258998	.0546362	4.74	0.000	.1518741	.3661218
_cons	-6.1414	3.838185	-1.60	0.110	-13.66683	1.384028

$r_{12} = \hat{\sigma}_{12}/\sqrt{\hat{\sigma}_{11}\hat{\sigma}_{22}} = 0.17$

```
sureg (ldrugexp age age2 actlim totchr medicaid private) ///
    (ltotothr age age2 educyr actlim totchr private), corr
```

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
ldrugexp	**3,251**	**6**	**1.133657**	**0.2284**	**962.07**	**0.0000**
ltotothr	**3,251**	**6**	**1.491159**	**0.1491**	**567.91**	**0.0000**

```
Correlation matrix of residuals:

             ldrugexp  ltotothr
ldrugexp       1.0000
ltotothr       0.1741    1.0000

Breusch-Pagan test of independence: chi2(1) =     98.590, Pr = 0.0000
```

$$Nr_{12}^2 = 3251 \times 0.1741^2 = 98.54$$

SUR Results

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
ldrugexp						
age	.2630418	.0795316	3.31	0.001	.1071627	.4189209
age2	-.0017428	.0005287	-3.30	0.001	-.002779	-.0007066
actlim	.3546589	.046617	7.61	0.000	.2632912	.4460266
totchr	.4005159	.0161432	24.81	0.000	.3688757	.432156
medicaid	.1067772	.0592275	1.80	0.071	-.0093065	.2228608
private	.0810116	.0435596	1.86	0.063	-.0043636	.1663867
_cons	-3.891259	2.975898	-1.31	0.191	-9.723911	1.941394
ltotothr						
age	.2927827	.1046145	2.80	0.005	.087742	.4978234
age2	-.0019247	.0006955	-2.77	0.006	-.0032878	-.0005617
educyr	.0652702	.00732	8.92	0.000	.0509233	.0796172
actlim	.7386912	.0608764	12.13	0.000	.6193756	.8580068
totchr	.2873668	.0211713	13.57	0.000	.2458719	.3288618
private	.2689068	.055683	4.83	0.000	.1597701	.3780434
_cons	-5.198327	3.914053	-1.33	0.184	-12.86973	2.473077

bootstrap, reps(400) seed(10101) nodots: sureg ///

(ldrugexp age age2 actlim totchr medicaid private) ///

(ltotothr age age2 educyr actlim totchr private)

	Observed Coef.	Bootstrap Std. Err.	z	P>|z|	Normal-based [95% Conf.	Interval]
ldrugexp						
age	.2630418	.0743481	3.54	0.000	.1173222	.4087614
age2	-.0017428	.0004929	-3.54	0.000	-.0027089	-.0007766
actlim	.3546589	.0462869	7.66	0.000	.2639382	.4453795
totchr	.4005159	.0169809	23.59	0.000	.3672339	.4337979
medicaid	.1067772	.0642814	1.66	0.097	-.019212	.2327664
private	.0810116	.044791	1.81	0.071	-.0067771	.1688002
_cons	-3.891259	2.794579	-1.39	0.164	-9.368532	1.586015
ltotothr						
age	.2927827	.1062298	2.76	0.006	.0845762	.5009892
age2	-.0019247	.0007048	-2.73	0.006	-.0033061	-.0005434
educyr	.0652702	.0075052	8.70	0.000	.0505602	.0799802
actlim	.7386912	.0619353	11.93	0.000	.6173003	.8600821
totchr	.2873668	.0202824	14.17	0.000	.247614	.3271196
private	.2689068	.0548669	4.90	0.000	.1613696	.376444
_cons	-5.198327	3.991338	-1.30	0.193	-13.02121	2.624553

test age age2

```
 ( 1)  [ldrugexp]age = 0
 ( 2)  [ltotothr]age = 0
 ( 3)  [ldrugexp]age2 = 0
 ( 4)  [ltotothr]age2 = 0

           chi2(  4) =   16.55
         Prob > chi2 =    0.0024
```

test [ldrugexp]private = [ltotothr]private

```
 ( 1)  [ldrugexp]private - [ltotothr]private = 0

           chi2(  1) =    8.35
         Prob > chi2 =    0.0038
```

constraint 1 [ldrugexp]private = [ltotothr]private

```
sureg (ldrugexp age age2 actlim totchr medicaid private) ///
(ltotothr age age2 educyr actlim totchr private), constraints(1)
```

```
 ( 1)  [ldrugexp]private - [ltotothr]private = 0
```

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
ldrugexp						
age	.2707053	.0795434	3.40	0.001	.1148031	.4266076
age2	-.0017907	.0005288	-3.39	0.001	-.0028271	-.0007543
actlim	.3575386	.0466396	7.67	0.000	.2661268	.4489505
totchr	.3997819	.0161527	24.75	0.000	.3681233	.4314405
medicaid	.1473961	.0575962	2.56	0.010	.0345096	.2602827
private	.1482936	.0368364	4.03	0.000	.0760955	.2204917
_cons	-4.235088	2.975613	-1.42	0.155	-10.06718	1.597006
ltotothr						
age	.2780287	.1045298	2.66	0.008	.073154	.4829034
age2	-.0018298	.0006949	-2.63	0.008	-.0031919	-.0004677
educyr	.0703523	.0071112	9.89	0.000	.0564147	.0842899
actlim	.7276336	.0607791	11.97	0.000	.6085088	.8467584
totchr	.2874639	.0211794	13.57	0.000	.245953	.3289747
private	.1482936	.0368364	4.03	0.000	.0760955	.2204917
_cons	-4.62162	3.910453	-1.18	0.237	-12.28597	3.042727