11) Simultaneous Equations and Three-Stage Least Squares (3SLS)

Vitor Kamada

August 2018

Reference

Stata Manual 15 (2017):

reg3 - Three-stage estimation for systems of simultaneous equations

gmm - Generalized method of moments estimation

Wooldridge (2010): Ch 9

Housing Expenditures and Saving

$$housing = \alpha_1 saving + \beta_{11} inc + \beta_{12} educ + \beta_{13} age + u_1$$

$$saving = \alpha_2 housing + \beta_{21} inc + \beta_{22} educ + \beta_{23} age + u_2$$

Reduced Form for y_2

$$y_1 = \gamma_1 y_2 + z_{(1)}\delta_{(1)} + u_1$$

$$y_1 = \gamma_2 y_2 + z_{(2)}\delta_{(2)} + u_2$$

$$y_2 = z_{(1)}\pi_{(21)} + z_{(2)}\pi_{(22)} + v_2$$

$$\pi_{(21)} = \frac{\delta_{(1)}}{\gamma_2 - \gamma_1}, \ \pi_{(22)} = \frac{-\delta_{(2)}}{\gamma_2 - \gamma_1}, \ v_2 = \frac{u_1 - u_2}{\gamma_2 - \gamma_1}$$

Mroz (1987)

$$hours = \gamma_{12} log(wage) + \delta_{10} + \delta_{11} educ + \delta_{12} age + \delta_{13} kidslt6 + \delta_{14} kidsge6 + \delta_{14} nwifeinc + u_1$$

IVs: *exper* and $exper^2$

$$log(wage) = \gamma_{21} hours + \delta_{20} + \delta_{21} educ + \delta_{22} exper + \delta_{23} exper^2 + u_2$$

IVs: *age*, *kidslt6*, *kidsge6*, and *nwifeinc*

Estimation of Labor Supply Function

OLS

$$\hat{hours} = \underset{(340.1)}{2,114.7} - \underset{(54.22)}{17.41}log(wage) - \underset{(17.97)}{14.44}educ -$$
$$\underset{(5.53)}{7.73}age - \underset{(100.01)}{342.50}kidslt6 - \underset{(30.83)}{115.02}kidsge6 - \underset{(3.66)}{4.35}nwifeinc$$

2SLS

$$\hat{hours} = \underset{(594.2)}{2,432.2} + \underset{(480.74)}{1,544.82}log(wage) - \underset{(58.14)}{177.45}educ -$$
$$\underset{(9.58)}{10.78}age - \underset{(176.93)}{210.83}kidslt6 - \underset{(56.92)}{47.56}kidsge6 - \underset{(6.48)}{9.25}nwifeinc$$

Average Annual Hours $= 1{,}303$

Labor Supply Elasticity $= 1{,}544.82/\text{hours} \cong 1.2$

Using Cross Equation Restrictions to Achieve Identification

1) $y_1 = \gamma_{12} y_2 + \delta_{11} z_1 + \delta_{12} z_2 + \delta_{13} z_3 + u_1$

2) $y_2 = \gamma_{21} y_1 + \delta_{21} z_1 + \delta_{22} z_2 + u_2$

$$\delta_{12} = \delta_{22}$$

1) $y_1 - \hat{\delta}_{22} z_2 = \gamma_{12} y_2 + \delta_{11} z_1 + \delta_{13} z_3 + u_1$

IVs: (z_1, z_2, z_3)

Using Covariance Restrictions to Achieve Identification

$$\textbf{1)}\ y_1 = \gamma_{12} y_2 + \delta_{11} z_1 + \delta_{13} z_3 + u_1$$

$$\textbf{2)}\ y_2 = \gamma_{21} y_1 + \delta_{21} z_1 + \delta_{22} z_2 + \delta_{23} z_3 + u_2$$

$$Cov(u_1, u_2) = E(u_1 u_2) = 0$$

IVs for Eq1: (z_1, z_2, z_3)

IVs for Eq2: $(z_1, z_2, z_3, \hat{u}_1)$

Klein's (1950): Model of the U.S. Economy

$$y = c + i + g$$

$$p = y - t - wp$$

$$k = L.k + i$$

$$w = wg + wp$$

c	Consumption
p	Private industry profits
wp	Private wage bill
wg	Government wage bill
w	Total wage bill
i	Investment

k	Capital stock
y	Total income/demand
g	Government spending
t	taxes + net exports
yr	year - 1931

System of Simultaneous Equations

$$c = \beta_0 + \beta_1 p + \beta_2 L.p + \beta_3 w + \epsilon_1$$

$$i = \beta_4 + \beta_5 p + \beta_6 L.p + \beta_7 L.k + \epsilon_2$$

$$wp = \beta_8 + \beta_9 y + \beta_{10} L.y + \beta_{11} yr + \epsilon_3$$

3SLS - Homoskedasticity

reg3 (c p L.p w) (i p L.p L.k) (wp y L.y yr), endog(w p y) exog(t wg g)

Eq 1: consump profits L.profits wagetot

	Coef.	Std. Err.	z	P>|z|
p				
--.	.1248904	.1081291	1.16	0.248
L1.	.1631439	.1004382	1.62	0.104
w	.790081	.0379379	20.83	0.000
_cons	16.44079	1.304549	12.60	0.000

Eq 2: invest profits L.profits L.capital

p				
--.	-.0130791	.1618962	-0.08	0.936
L1.	.7557238	.1529331	4.94	0.000
k				
L1.	-.1948482	.0325307	-5.99	0.000
_cons	28.17785	6.793768	4.15	0.000

Eq 3: wagepriv totinc L.totinc year

y				
--.	.4004919	.0318134	12.59	0.000
L1.	.181291	.0341588	5.31	0.000
yr	.149674	.0279352	5.36	0.000
_cons	1.797216	1.115854	1.61	0.107

GMM-3SLS Robust

```
gmm (eq1: c - {c: p L.p w _cons}) ///
(eq2: i - {i: p L.p L.k _cons}) ///
(eq3: wp - {wp: y L.y yr _cons}), ///
instruments(eq1: L.p L.k L.y yr t wg g) ///
instruments(eq2: L.p L.k L.y yr t wg g) ///
instruments(eq3: L.p L.k L.y yr t wg g) ///
winitial(unadjusted, independent) twostep
```

Eq 1: consump profits L.profits wagetot

p				
--.	.1107447	.0676899	1.64	0.102
L1.	.1591073	.0745102	2.14	0.033
w	.826938	.0312719	26.44	0.000
_cons	15.07861	.8425758	17.90	0.000

Eq 2: invest profits L.profits L.capital

p				
--.	.2175156	.0914721	2.38	0.017
L1.	.5527221	.1015973	5.44	0.000
k				
L1.	-.1699195	.0194554	-8.73	0.000
_cons	22.5153	3.781761	5.95	0.000

Eq 3: wagepriv totinc L.totinc year

y				
--.	.4373167	.0164508	26.58	0.000
L1.	.1314822	.0210174	6.26	0.000
yr	.1318394	.0183602	7.18	0.000
_cons	2.580174	.5096321	5.06	0.000