27) Spatial Econometrics with PySAL

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August 2018

Reference

Tables, Graphics, and Figures from:

Rey and Arribas-Bel (2018). **Geographic Data Science with PySAL**

http://darribas.org/gds_scipy16/

Texas Counties from the Census Bureau

```
import pysal as ps
import pandas as pd
import numpy as np
from pysal.contrib.viz import mapping as maps
shp_path = 'C:/Users/Vitor/Desktop/ECO 7110 E
data = ps.pdio.read_files(shp_path)
```

data.head()

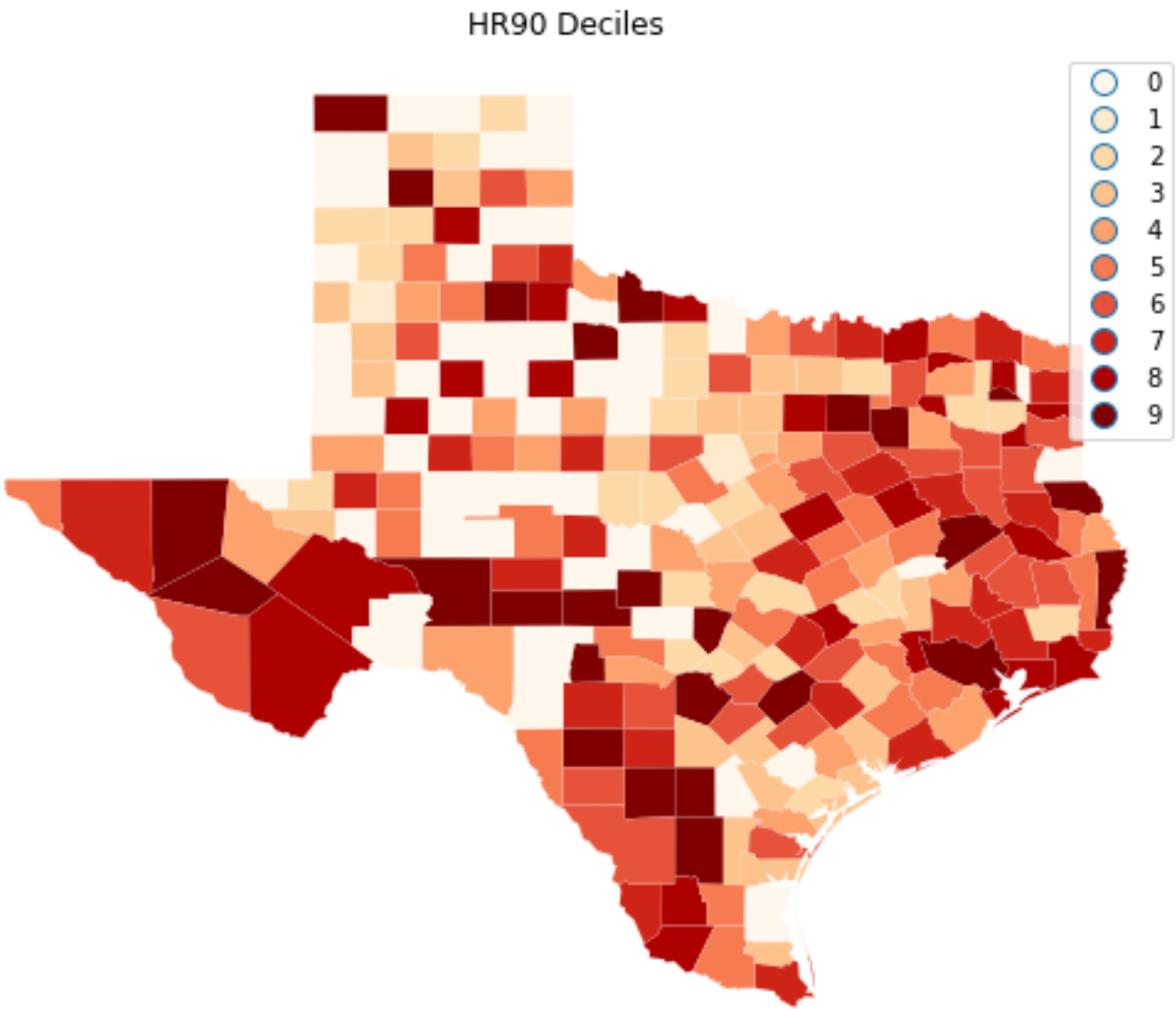
	NAME	STATE_NAME	STATE_FIPS	CNTY_FIPS	FIPS	STFIPS	COFIPS	FIPSNO
0	Lipscomb	Texas	48	295	48295	48	295	48295
1	Sherman	Texas	48	421	48421	48	421	48421
2	Dallam	Texas	48	111	48111	48	111	48111
3	Hansford	Texas	48	195	48195	48	195	48195
4	Ochiltree	Texas	48	357	48357	48	357	48357

	FH90	geometry
0	6.093580	<pysal.cg.shapes.Polygon object at 0x0000020C5...
1	3.869407	<pysal.cg.shapes.Polygon object at 0x0000020C5...
2	14.231738	<pysal.cg.shapes.Polygon object at 0x0000020C5...
3	7.125457	<pysal.cg.shapes.Polygon object at 0x0000020C5...
4	9.159159	<pysal.cg.shapes.Polygon object at 0x0000020C5...

Map Pattern

```
import matplotlib.pyplot as plt
import geopandas as gpd
tx = gpd.read_file(shp_path)
hr10 = ps.Quantiles(data.HR90, k=10)
f, ax = plt.subplots(1, figsize=(9, 9))
tx.assign(cl=hr10.yb).plot(column='cl',
          categorical=True, k=10, cmap='OrRd',
          linewidth=0.1, ax=ax,
          edgecolor='white', legend=True)
ax.set_axis_off()
plt.title("HR90 Deciles")
plt.show()
```

County Homicide Rates in 1990



Spatial Weights

Queen Contiguity: adjacency relationships as a binary indicator variable denoting whether or not a polygon shares an **edge or a vertex** with another polygon

KNN: distance to k nearest neighbors

Kernel: neighbors defined by bandwidth

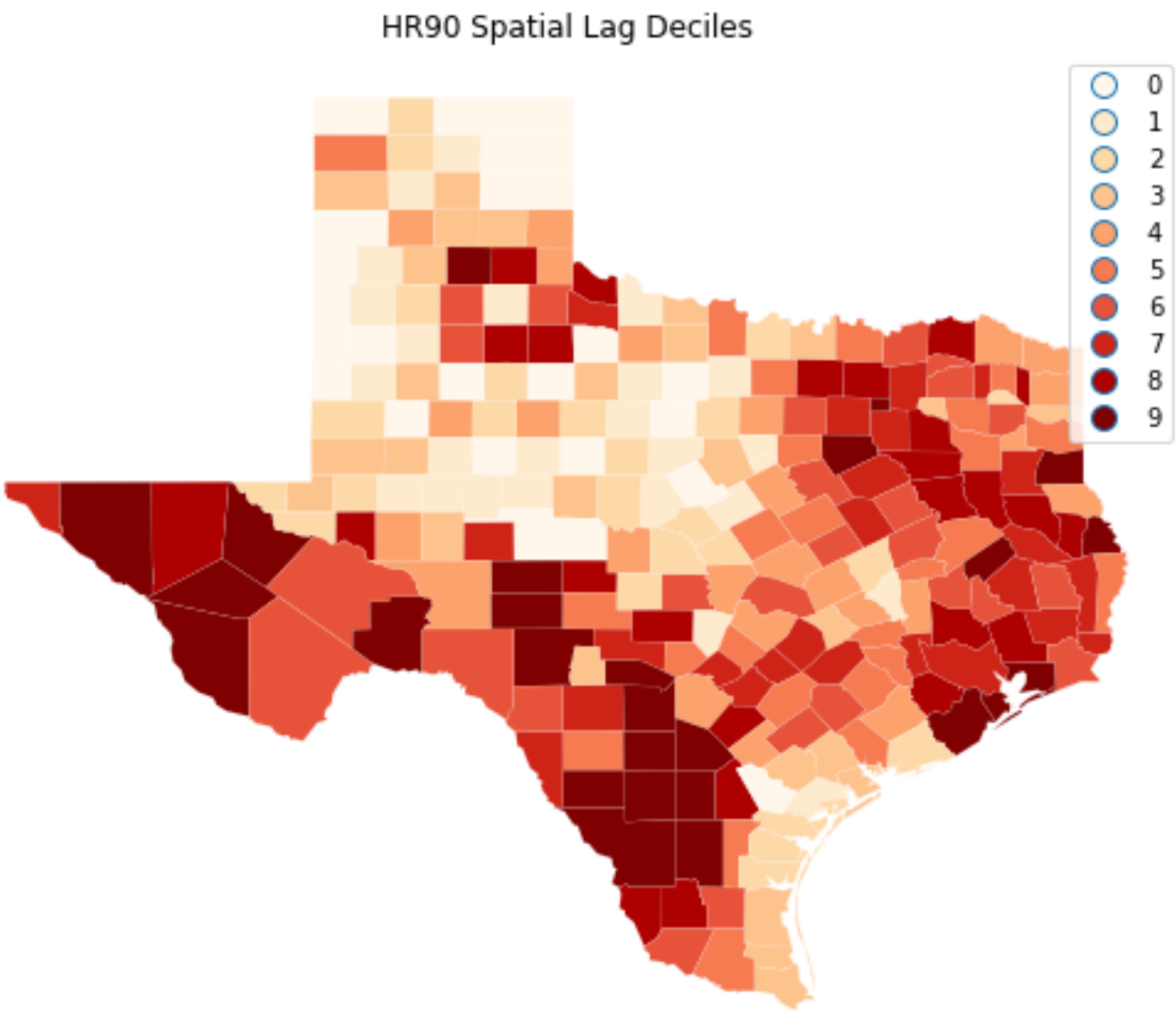
Spatial Lag: $\sum_j w_{i,j} HR90_j$

```
W = ps.queen_from_shapefile(shp_path)
W.transform = 'r'

HR90Lag = ps.lag_spatial(W, data.HR90)
HR90LagQ10 = ps.Quantiles(HR90Lag, k=10)

f, ax = plt.subplots(1, figsize=(9, 9))
tx.assign(cl=HR90LagQ10.yb).plot(column='cl',
                  categorical=True, k=10, cmap='OrRd',
                  linewidth=0.1, ax=ax,
                  edgecolor='white', legend=True)
ax.set_axis_off()
plt.title("HR90 Spatial Lag Deciles")
plt.show()
```

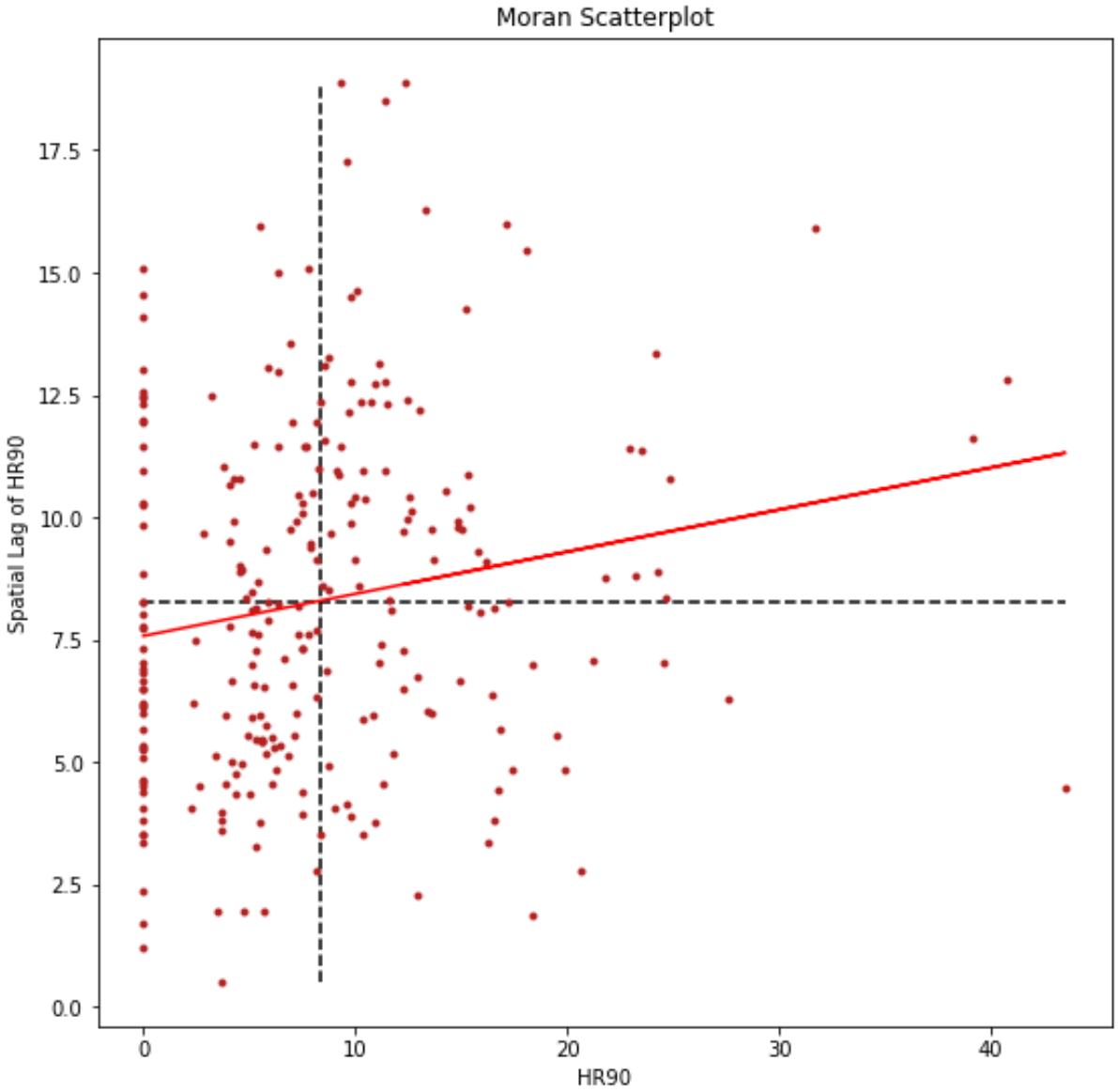
HR90 Spatial Lag Deciles



Moran Scatterplot

```
HR90 = data.HR90
b,a = np.polyfit(HR90, HR90Lag, 1)
f, ax = plt.subplots(1, figsize=(9, 9))
plt.plot(HR90, HR90Lag, '.', color='firebrick')
 # dashed vert at mean of the last year's PCI
plt.vlines(HR90.mean(), HR90Lag.min(), HR90Lag.max(),
           linestyle='--')
 # dashed horizontal at mean of lagged PCI
plt.hlines(HR90Lag.mean(), HR90.min(), HR90.max(),
           linestyle='--')
# red line of best fit using global I as slope
plt.plot(HR90, a + b*HR90, 'r')
plt.title('Moran Scatterplot')
plt.ylabel('Spatial Lag of HR90')
plt.xlabel('HR90')
plt.show()
```

$\sum_j w_{i,j} HR90_j$ vs HR90



Moran's Statistic (I)

```
I_HR90 = ps.Moran(data.HR90.values, W)
I_HR90.I, I_HR90.p_sim
```

(0.08597664031388977, 0.01)

```
b
```

0.085976640313889 5

Austin Properties Listed in AirBnb

http://insideairbnb.com/austin/index.html

```
import seaborn as sns
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import pysal as ps
import geopandas as gpd
sns.set(style="whitegrid")
abb_link = 'C:/Users/Vitor/Desktop/ECO 7110 E
lst = pd.read_csv(abb_link)
x = ['host_listings_count', 'bathrooms',
     'bedrooms', 'beds', 'guests_included']
```

Cleaning Data

```
def has_pool(a):
    if 'Pool' in a:
        return 1
    else:
        return 0

lst['pool'] = lst['amenities'].apply(has_pool)

yxs = lst.loc[:, x + ['pool', 'price']].dropna()
y = np.log(yxs['price'].apply(lambda x:
      float(x.strip('$').replace(',', '')))
      + 0.000001)
```

Fit the Regression

8 nearest neighbors

```
w = ps.knnW_from_array(lst.loc[yxs.index,
        ['longitude', 'latitude']].values)
w.transform = 'R'

m1 = ps.spreg.OLS(y.values[:, None],
         yxs.drop('price', axis=1).values,
         w=w, spat_diag=True,
                  name_x=yxs.drop('price',
                  axis=1).columns.tolist(),
                   name_y='ln(price)')
print(m1.summary)
```

$ln(P) = \alpha + \beta X + \epsilon$

Variable	Coefficient	Std.Error	t-Statistic	Probability
CONSTANT	4.0976886	0.0223530	183.3171506	0.0000000
host_listings_count	-0.0000130	0.0001790	-0.0726772	0.9420655
bathrooms	0.2947079	0.0194817	15.1273879	0.0000000
bedrooms	0.3274226	0.0159666	20.5067654	0.0000000
beds	0.0245741	0.0097379	2.5235601	0.0116440
guests_included	0.0075119	0.0060551	1.2406028	0.2148030
pool	0.0888039	0.0221903	4.0019209	0.0000636

DIAGNOSTICS FOR SPATIAL DEPENDENCE

TEST	MI/DF	VALUE	PROB
Lagrange Multiplier (lag)	1	255.796	0.0000
Robust LM (lag)	1	13.039	0.0003
Lagrange Multiplier (error)	1	278.752	0.0000
Robust LM (error)	1	35.995	0.0000
Lagrange Multiplier (SARMA)	2	291.791	0.0000

Spatially Lagged Exogenous Regressors

$$ln(P_i) = \alpha + \beta X_i + \delta \sum_j w_{ij} X'_i + \epsilon_i$$

```
w_pool = ps.knnW_from_array(lst.loc[yxs.index,
                    ['longitude', 'latitude']].values)
yxs_w = yxs.assign(w_pool=ps.lag_spatial(w_pool,
                                  yxs['pool'].values))

m2 = ps.spreg.OLS(y.values[:, None],
         yxs_w.drop('price', axis=1).values,
   w=w, spat_diag=True,
   name_x=yxs_w.drop('price', axis=1).columns.tolist(),
   name_y='ln(price)')
```

print(m2.summary)

Sum squared residual:	3070.363	F-statistic :	558.6139
Sigma-square :	0.533	Prob(F-statistic) :	0
S.E. of regression :	0.730	Log likelihood :	-6365.387
Sigma-square ML :	0.532	Akaike info criterion :	12746.773
S.E of regression ML:	0.7297	Schwarz criterion :	12800.053

Variable	Coefficient	Std.Error	t-Statistic	Probability
CONSTANT	4.0906444	0.0230571	177.4134022	0.0000000
host_listings_count	-0.0000108	0.0001790	-0.0603617	0.9518697
bathrooms	0.2948787	0.0194813	15.1365024	0.0000000
bedrooms	0.3277450	0.0159679	20.5252404	0.0000000
beds	0.0246650	0.0097377	2.5329419	0.0113373
guests_included	0.0076894	0.0060564	1.2696250	0.2042695
pool	0.0725756	0.0257356	2.8200486	0.0048181
w_pool	0.0188875	0.0151729	1.2448141	0.2132508

Spatially Lagged Endogenous Regressors

$$ln(P_i) = \alpha + \lambda \sum_j w_{ij} ln(P_i) + \beta X_i + \epsilon_i$$

```
m3 = ps.spreg.GM_Lag(y.values[:, None],
    yxs.drop('price', axis=1).values,
    w=w, spat_diag=True,
    name_x=yxs.drop('price',
                axis=1).columns.tolist(),
    name_y='ln(price)')

print(m3.summary)
```

Spatial 2SLS

Dependent Variable :	ln(price)	Number of Observations:	5767
Mean dependent var :	5.1952	Number of Variables :	8
S.D. dependent var :	0.9455	Degrees of Freedom :	5759
Pseudo R-squared :	0.4224		
Spatial Pseudo R-squared:	0.4056		

Variable	Coefficient	Std.Error	z-Statistic	Probability
CONSTANT	3.7085715	0.1075621	34.4784213	0.0000000
host_listings_count	-0.0000587	0.0001765	-0.3324585	0.7395430
bathrooms	0.2857932	0.0193237	14.7897969	0.0000000
bedrooms	0.3272598	0.0157132	20.8270544	0.0000000
beds	0.0239548	0.0095848	2.4992528	0.0124455
guests_included	0.0065147	0.0059651	1.0921407	0.2747713
pool	0.0891100	0.0218383	4.0804521	0.0000449
W_ln(price)	0.0785059	0.0212424	3.6957202	0.0002193

Instrumented: W_ln(price)
Instruments: W_bathrooms, W_bedrooms, W_beds, W_guests_included,
W_host_listings_count, W_pool

Spatial Durbin Model (SDM)

$$(I_n - \rho W)y = X\beta + WX\theta + \epsilon$$

$$y = \sum_{r=1}^{k} S_r(W)x_r + V(W)\epsilon$$

$$V(W) = (I_n - \rho W)^{-1}$$

$$S_r(W) = V(W)(I_n\beta_r + W\theta_r)$$

Summary Measures of Impacts

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \sum_{r=1}^{k} \begin{pmatrix} S_r(W)_{11} & S_r(W)_{12} & \cdots & S_r(W)_{1n} \\ S_r(W)_{21} & S_r(W)_{22} & & \\ \vdots & \vdots & \ddots & \\ S_r(W)_{n1} & S_r(W)_{n2} & & S_r(W)_{nn} \end{pmatrix} \begin{pmatrix} x_{1r} \\ x_{2r} \\ \vdots \\ x_{nr} \end{pmatrix} + V(W)\epsilon$$

$$\frac{\partial y_i}{\partial x_{jr}} = S_r(W)_{ij} \textbf{ and } \frac{\partial y_i}{\partial x_{ir}} = S_r(W)_{ii}$$

$$\bar{M}(r)_{direct} = n^{-1} tr(S_r(W))$$

$$\bar{M}(r)_{total} = n^{-1} \iota_n' (S_r(W)) \iota_n$$

$$\bar{M}(r)_{indirect} = \bar{M}(r)_{total} - \bar{M}(r)_{direct}$$

Spatial Autoregressive (SAR) Model

$$(I_n - \rho W)y = X\beta + \epsilon$$

$$y = \sum_{r=1}^{k} (I_n - \rho W)^{-1} I_n \beta_r x_r + (I_n - \rho W)^{-1} \epsilon$$

Total Impact

$$n^{-1} \iota_n' (I_n - \rho W)^{-1} \beta_r \iota_n = (1 - \rho)^{-1} \beta_r$$

Indirect Impact: $\frac{\beta_r}{(1-\rho)} - \beta_r$

Code: Direct & Indirect Impacts

```
b = m3.betas[:-1]
b

rho = m3.betas[-1]
rho

btot = b / (1.0 - rho) #total impact
bind = btot - b #indirect impact

x_names = ['NROOM','NBATH','PATIO','FIREPL','AC','GAR','AGE',
           'LOTSZ','SQFT']
varnames = ["CONSTANT"] + x + ["pool"]
print("             Variable       Direct       Indirect  \
    Total" )
for i in range(len(varnames)):
    print("%20s %12.3f %12.3f %12.3f" % (varnames[i],b[i][0],
                                         bind[i][0],btot[i][0]))
```

Direct & Indirect Impacts

$$ln(P_i) = \alpha + \lambda \sum_j w_{ij} ln(P_i) + \beta X_i + \epsilon_i$$

Variable	Direct	Indirect	Total
CONSTANT	3.709	0.316	4.025
host_listings_count	-0.000	-0.000	-0.000
bathrooms	0.286	0.024	0.310
bedrooms	0.327	0.028	0.355
beds	0.024	0.002	0.026
guests_included	0.007	0.001	0.007
pool	0.089	0.008	0.097