# 5) Sharp Regression Discontinuity Design

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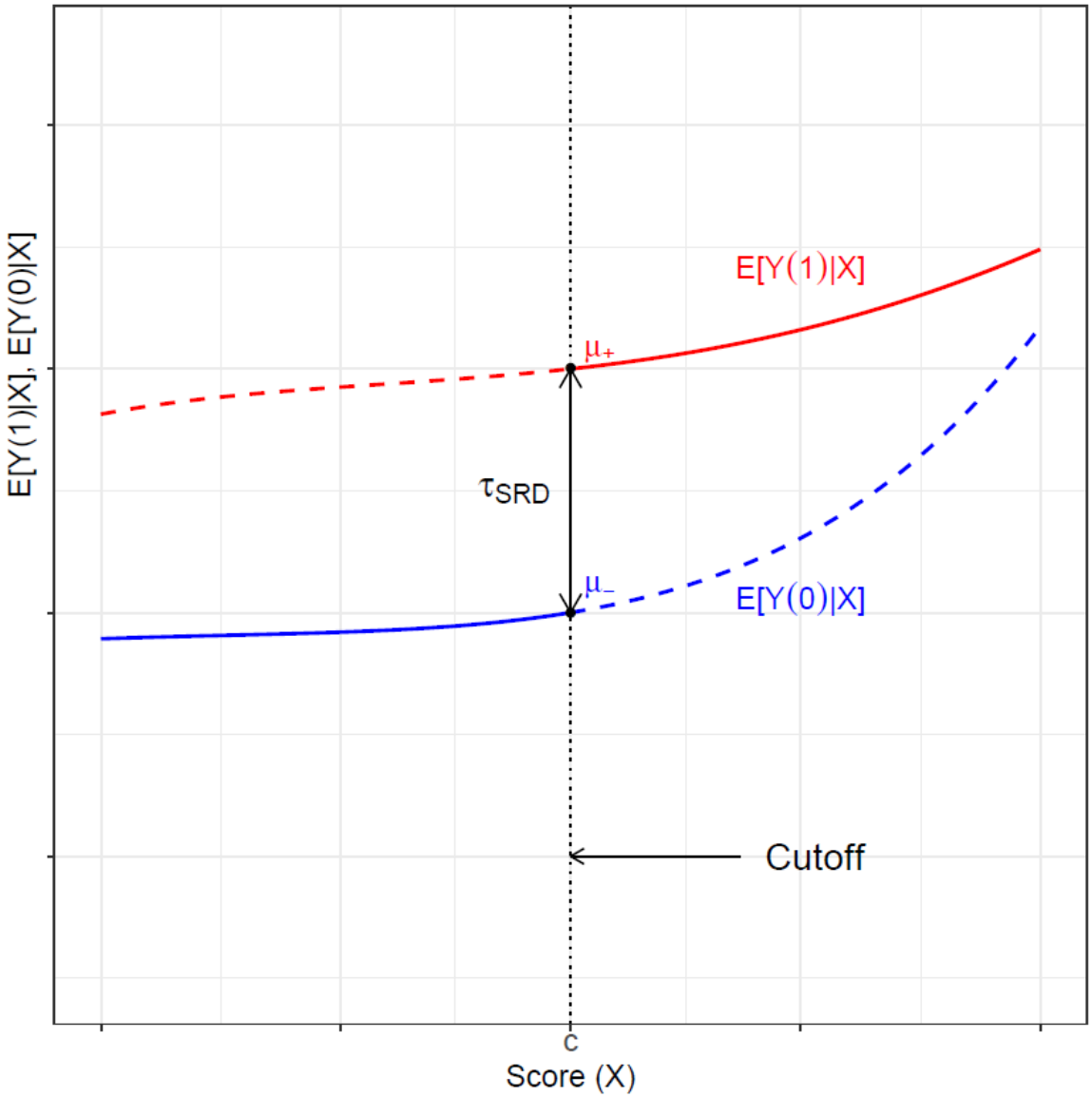
# Reference

Tables, Graphics, and Figures from

**A Practical Introduction to Regression Discontinuity Designs: Volume I**

Cattaneo et al. (2018)

# Treatment Effect in Sharp RD Design

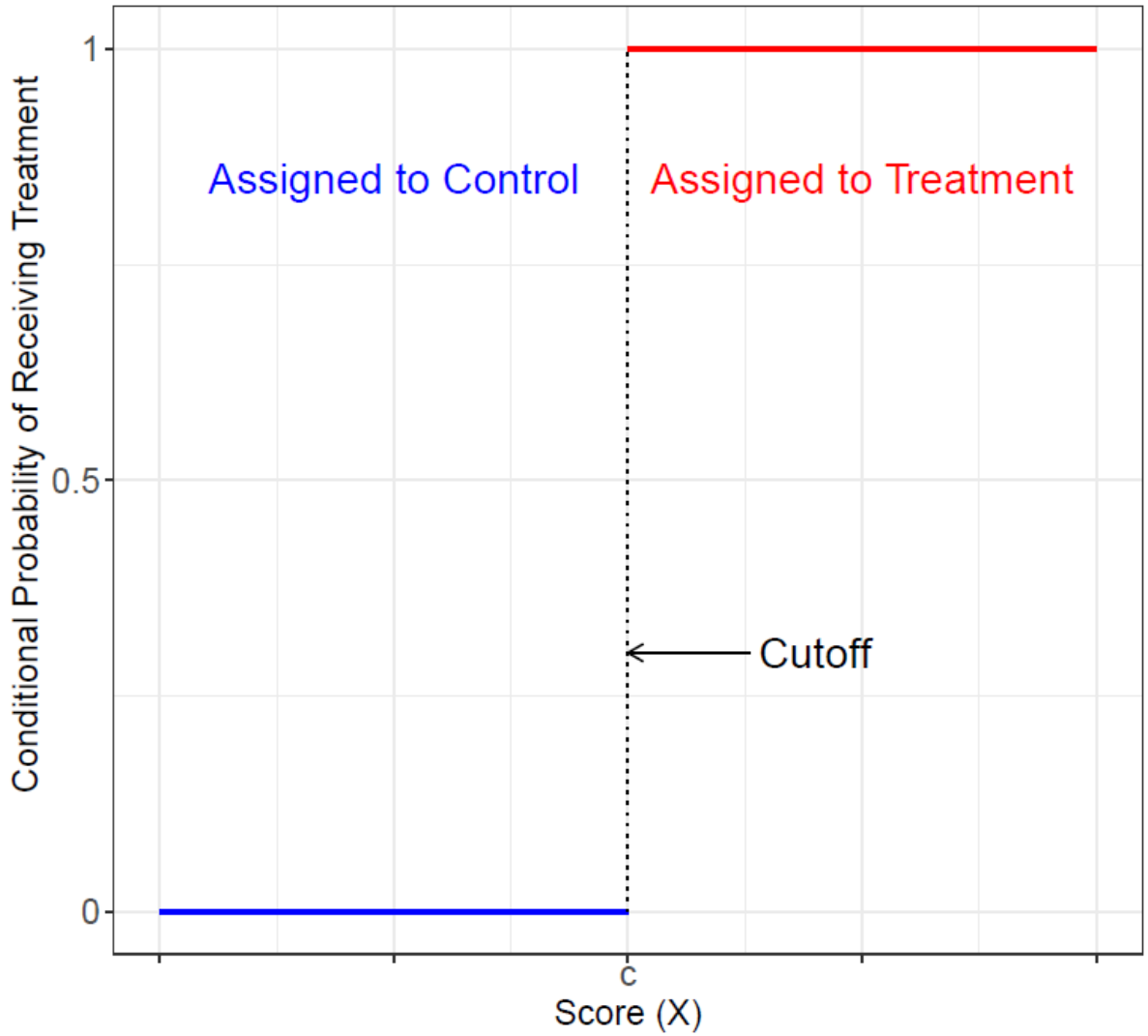


# Sharp RD

$$D_i = \begin{cases} 1 \ \ if \ \ x_i \geq x_0 \\ 0 \ \ if \ \ x_i < x_0 \end{cases}$$

$$y_i = \alpha + \rho D_i + \beta x_i + \delta D_i x_i + \eta_i$$

# Conditional Probability of Receiving Treatment



# Generalization of RD

$$y_i = f(x_i) + \rho D_i + \eta_i$$

$$y_i = \alpha + \beta_1 x_i + \beta_2 x_i^2 + ... + \beta_p x_i^p + \rho D_i + \eta_i$$

$$y_i = \alpha + \beta_{01}\tilde{x}_i + \beta_{02}\tilde{x}_i^2 + ... + \beta_{0p}\tilde{x}_i^p + \rho D_i + \beta_1^* D_i \tilde{x}_i + \beta_2^* D_i \tilde{x}_i^2 + ... + \beta_p^* D_i \tilde{x}_i^p + \eta_i$$

$$\tilde{x}_i = x_i - x_0$$

# Nonparametric Estimation Strategy

$$E[y_i|x_0 - \Delta \leq x_i < x_0] \cong E[y_{0i}|x_i = x_0]$$

$$E[y_i|x_0 \leq x_i < x_0 + \Delta] \cong E[y_{1i}|x_i = x_0]$$

$$\lim_{\Delta \to 0} E[y_i|x_0 \leq x_i < x_0+\Delta] - E[y_i|x_0-\Delta \leq x_i < x_0]$$

$$= E[y_{1i} - y_{0i}|x_i = x_0]$$

$$= \rho \textbf{ (Average Causal Effect)}$$

# Islamic Rule and the Empowerment of the Poor and Pious

**Y:** % of women aged 15 to 20 completed high school in 2000

**X:** vote margin of the Islamic candidate for mayor in the 1994 Turkish elections

**T:** electoral victory of the Islamic candidate in 1994

**Meyersson (2014) in Econometrica**

# Descriptive Statistics for Meyersson (2014)

| Variable | Mean | Median | Std. Dev. | Min. | Max. |
|---|---|---|---|---|---|
| Y | 16.306 | 15.523 | 9.584 | 0.000 | 68.038 |
| X | -28.141 | -31.426 | 22.115 | -100.000 | 99.051 |
| T | 0.120 | 0.000 | 0.325 | 0.000 | 1.000 |
| Percentage of men aged 15-20 with high school education | 19.238 | 18.724 | 7.737 | 0.000 | 68.307 |
| Islamic percentage of votes in 1994 | 13.872 | 7.029 | 15.385 | 0.000 | 99.526 |
| Number of parties receiving votes 1994 | 5.541 | 5.000 | 2.192 | 1.000 | 14.000 |
| Log population in 1994 | 7.840 | 7.479 | 1.188 | 5.493 | 15.338 |
| Percentage of population below 19 in 2000 | 40.511 | 39.721 | 8.297 | 6.544 | 68.764 |
| Percentage of population above 60 in 2000 | 9.222 | 8.461 | 3.960 | 1.665 | 27.225 |
| Gender ratio in 2000 | 107.325 | 103.209 | 25.293 | 74.987 | 1033.636 |
| Household size in 2000 | 5.835 | 5.274 | 2.360 | 2.823 | 33.634 |
| District center | 0.345 | 0.000 | 0.475 | 0.000 | 1.000 |
| Province center | 0.023 | 0.000 | 0.149 | 0.000 | 1.000 |
| Sub-metro center | 0.022 | 0.000 | 0.146 | 0.000 | 1.000 |

Note: the number of observations for all variables is 2,629

# RD Libraries

library(lpdensity)

library(rddensity)

library(rdrobust)

library(rdlocrand)

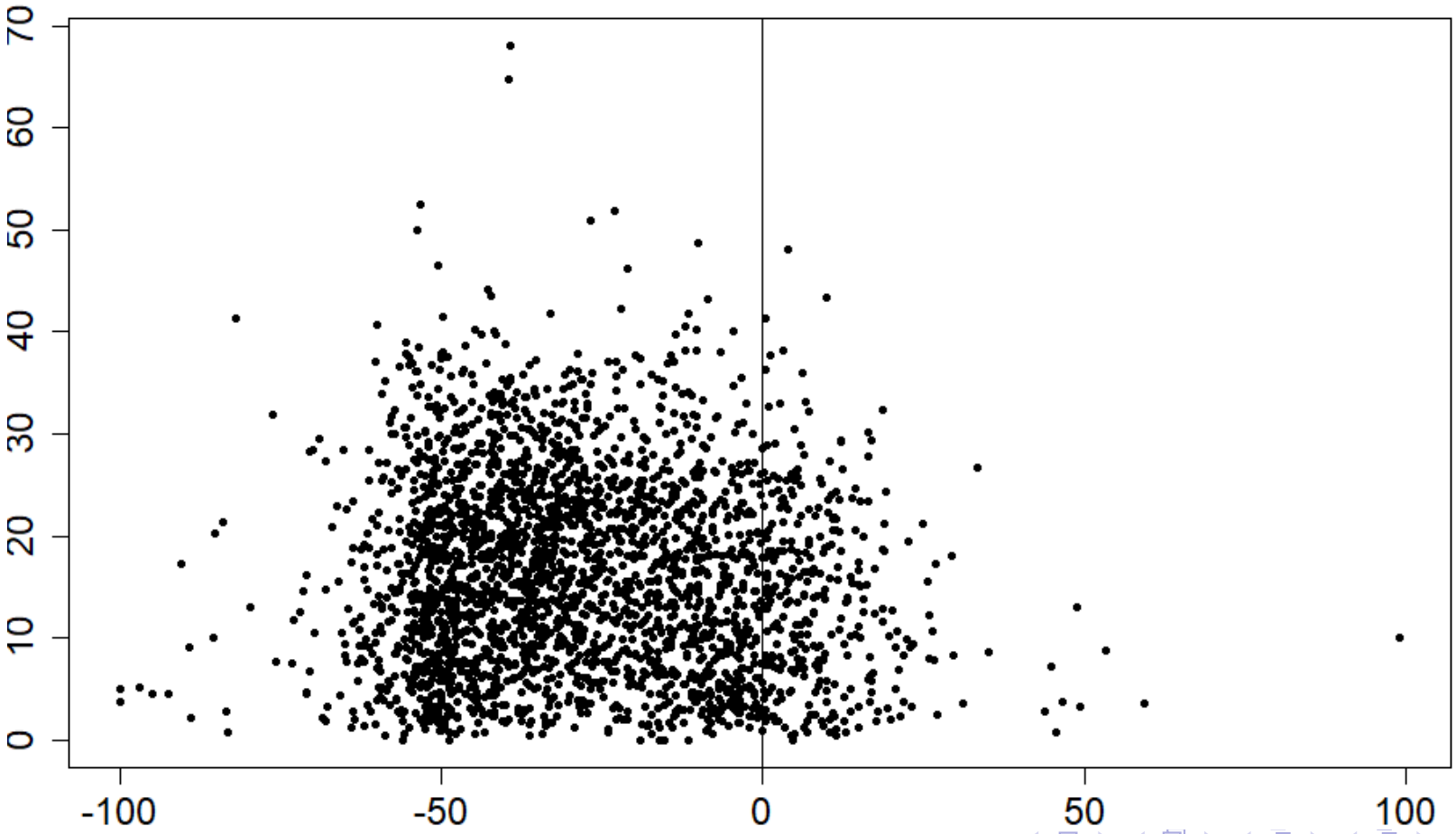
library(foreign); library(ggplot2)

library(TeachingDemos)

**https://sites.google.com/site/rdpackages/**

# plot(X, Y, xlab = "Score", ylab = "Outcome", col = 1, pch = 20, cex.axis = 1.5, cex.lab = 1.5)

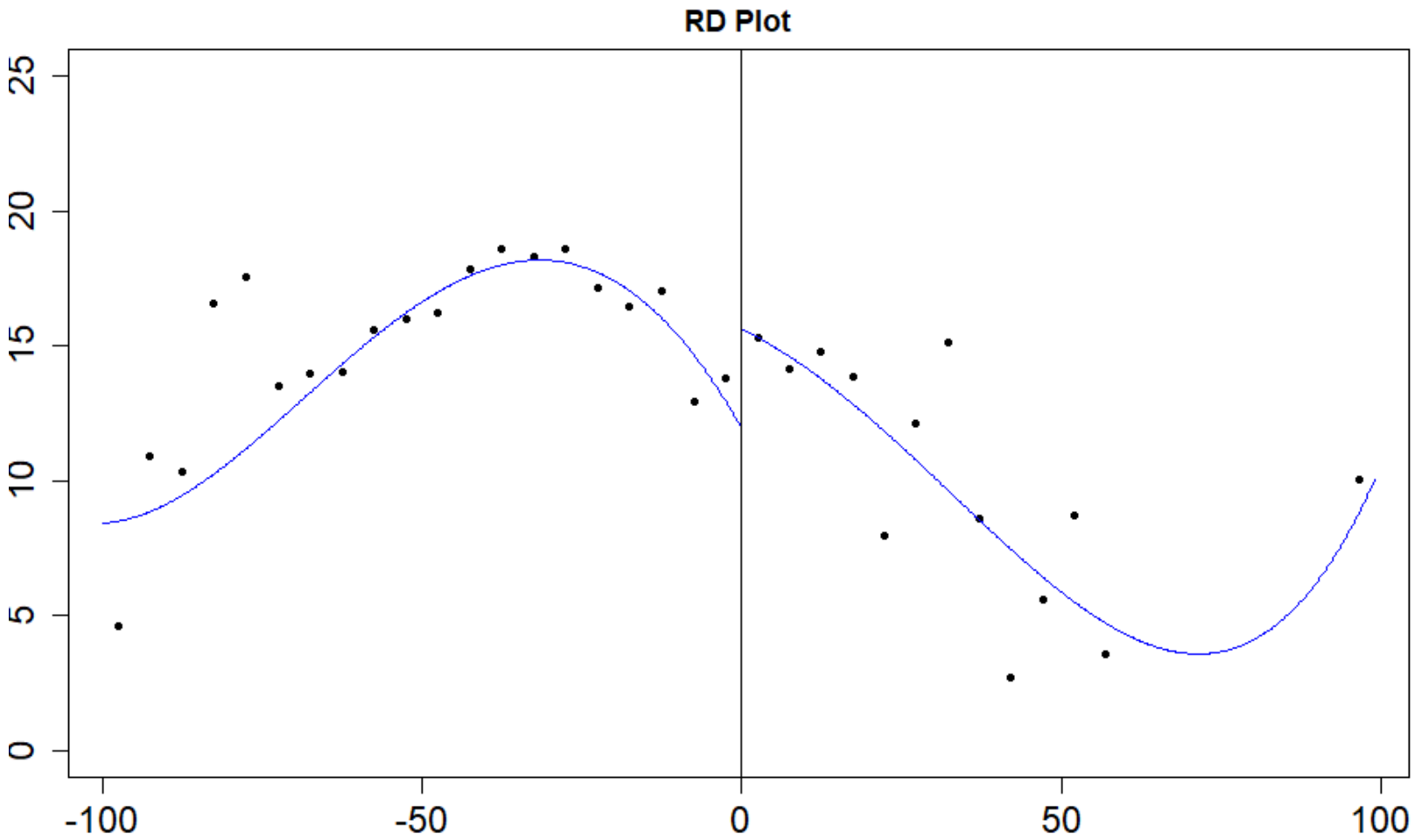
abline(v=0)



# Partition of Islamic Margin of Victory into 40 Bins of Equal Length

| Bin | Average Outcome in Bin | Number of Observations | Group Assignment |
|---|---|---|---|
| $\mathcal{B}_{-,1} = [-100, -95)$ | $\bar{Y}_{-,1} = 4.6366$ | 4 | Control |
| $\mathcal{B}_{-,2} = [-95, -90)$ | $\bar{Y}_{-,2} = 10.8942$ | 2 | Control |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $\mathcal{B}_{-,19} = [-10, -5)$ | $\bar{Y}_{-,19} = 12.9518$ | 149 | Control |
| $\mathcal{B}_{-,20} = [-5, 0)$ | $\bar{Y}_{-,20} = 13.8267$ | 148 | Control |
| $\mathcal{B}_{+,1} = [0, 5)$ | $\bar{Y}_{+,1} = 15.3678$ | 109 | Treatment |
| $\mathcal{B}_{+,2} = [5, 10)$ | $\bar{Y}_{+,2} = 13.9640$ | 83 | Treatment |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $\mathcal{B}_{+,19} = [90, 95)$ | $\bar{Y}_{+,19} = \text{NA}$ | 0 | Treatment |
| $\mathcal{B}_{+,20} = [95, 100]$ | $\bar{Y}_{+,20} = 10.0629$ | 1 | Treatment |

# out = rdplot(Y, X, nbins = c(20,20), binselect = 'esmv', y.lim = c(0,25), cex.axis = 1.5, cex.lab = 1.5)



# Sharp RD by OLS

```
out = lm(Y[X >= -20 & X <= 20] ~ X[X >= -20 & X <= 20] + T[X >= -20 & X <= 20] + T_X[X >= -20 & X <= 20])

summary(out)
```

```
Coefficients:
                          Estimate Std. Error t value Pr(>|t|)
(Intercept)               12.62254    0.77459  16.296  < 2e-16 ***
X[X >= -20 & X <= 20]     -0.24807    0.06723  -3.690 0.000238 ***
T[X >= -20 & X <= 20]      2.92708    1.23529   2.370 0.018024 *
T_X[X >= -20 & X <= 20]    0.12612    0.12459   1.012 0.311667
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
```

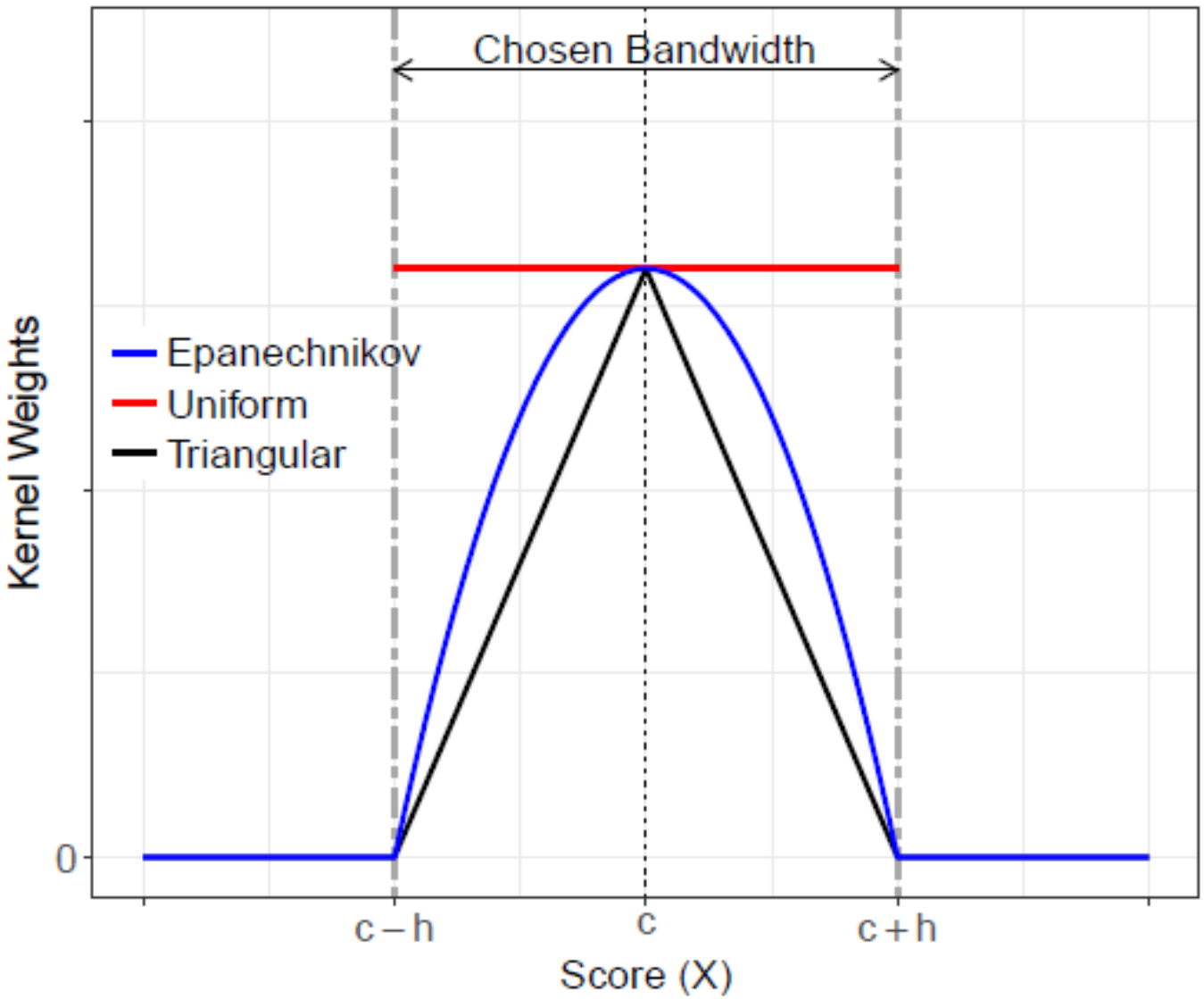
# Kernel: Local Linear Estimator

$$\min_{\alpha,\beta} \sum_{i=1}^{N} K\left(\frac{x_i - x}{h}\right)[y_i - \alpha - \beta(x_i - x)]^2$$

$$\begin{pmatrix} \hat{\alpha}(x) \\ \hat{\beta}(x) \end{pmatrix} = (Z'KZ)^{-1}Z'Ky$$

$$h \to \infty,\ \hat{m}(x) \to \hat{\alpha} + \hat{\beta}x$$

# Kernel Choice



# out = rdrobust(Y, X, kernel = "triangular", p = 1, bwselect = "mserd")

| | |
|---|---|
| Number of Obs. | 2629 |
| BW type | mserd |
| Kernel | Triangular |
| VCE method | NN |

| | | |
|---|---|---|
| Number of Obs. | 2314 | 315 |
| Eff. Number of Obs. | 529 | 266 |
| Order est. (p) | 1 | 1 |
| Order bias (p) | 2 | 2 |
| BW est. (h) | 17.239 | 17.239 |
| BW bias (b) | 28.575 | 28.575 |
| rho (h/b) | 0.603 | 0.603 |

| Method | Coef. | Std. Err. | z | P>\|z\| | [ 95% C.I. ] |
|---|---|---|---|---|---|
| Conventional | 3.020 | 1.427 | 2.116 | 0.034 | [0.223 , 5.817] |
| Robust | - | - | 1.776 | 0.076 | [-0.309 , 6.276] |

Z = cbind(data$vshr_islam1994, data$partycount, data$lpop1994, data$merkezi, data$merkezp, data$subbuyuk, data$buyuk)

out = rdrobust(Y, X, covs = Z, kernel = 'triangular', scaleregul = 1, p = 1, bwselect = 'mserd')

| | | |
|---|---|---|
| Number of Obs. | 2314 | 315 |
| Eff. Number of Obs. | 448 | 241 |
| Order est. (p) | 1 | 1 |
| Order bias (p) | 2 | 2 |
| BW est. (h) | 14.409 | 14.409 |
| BW bias (b) | 23.731 | 23.731 |
| rho (h/b) | 0.607 | 0.607 |

| Method | Coef. | Std. Err. | z | P>\|z\| | [ 95% C.I. ] |
|---|---|---|---|---|---|
| Conventional | 3.108 | 1.284 | 2.421 | 0.015 | [0.592 , 5.624] |
| Robust | - | - | 2.088 | 0.037 | [0.194 , 6.132] |