19) Multinomial, Conditional, and Mixed Logit

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Multinomial Models

$$p_j = Pr[y = j], \quad j = 1, ..., m$$

$$y_j = \begin{cases} 1 & if \quad y = j \\ 0 & if \quad y \neq j \end{cases}$$

$$f(y) = p_1^{y_1} \times ... \times p_m^{y_m} = \prod_{j=1}^{m} p_j^{y_j}$$

$$p_{ij} = Pr[y_i = j] = F_j(x_i, \beta)$$

Multinomial Logit (MNL)

$$p_{ij} = \frac{exp(x_i'\beta_j)}{\sum_{l=1}^{m} exp(x_i'\beta_l)}$$

$$j = 1, ..., m$$

$$0 < p_{ij} < 1 \text{ and } \sum_{j=1}^{m} p_{ij} = 1$$

Herriges and Kling (1999)

Variable	Obs	Mean	Std. Dev.	Min	Max
mode	1182	3.005076	.9936162	1	4
price	1182	52.08197	53.82997	1.29	666.11
crate	1182	.3893684	.5605964	.0002	2.3101
dbeach	1182	.1133672	.3171753	0	1
dpier	1182	.1505922	.3578023	0	1
dprivate	1182	.3536379	.4783008	0	1
dcharter	1182	.3824027	.4861799	0	1
pbeach	1182	103.422	103.641	1.29	843.186
ppier	1182	103.422	103.641	1.29	843.186
pprivate	1182	55.25657	62.71344	2.29	666.11
pcharter	1182	84.37924	63.54465	27.29	691.11
qbeach	1182	.2410113	.1907524	.0678	.5333
qpier	1182	.1622237	.1603898	.0014	.4522
qprivate	1182	.1712146	.2097885	.0002	.7369
qcharter	1182	.6293679	.7061142	.0021	2.3101
income	1182	4.099337	2.461964	.4166667	12.5

From wide form to long:

reshape long inc, i(id) j(year)

i		..... *x_ij*		
id	**sex**	**inc80**	**inc81**	**inc82**
1	0	5000	5500	6000
2	1	2000	2200	3300
3	0	3000	2000	1000

i	*j*		*x_ij*
id	**year**	**sex**	**inc**
1	80	0	5000
1	81	0	5500
1	82	0	6000
2	80	1	2000
2	81	1	2200
2	82	1	3300
3	80	0	3000
3	81	0	2000
3	82	0	1000

From long form to wide:

reshape wide inc, i(id) j(year)

Data: Wide Form

	mode	price	crate	pbeach	ppier	pprivate	pcharter
1	charter	182.93	.5391	157.93	157.93	157.93	182.93
2	charter	34.534	.4671	15.114	15.114	10.534	34.534
3	private	24.334	.2413	161.874	161.874	24.334	59.334
4	pier	15.134	.0789	15.134	15.134	55.93	84.93
5	private	41.514	.1082	106.93	106.93	41.514	71.014
6	charter	63.934	.3975	192.474	192.474	28.934	63.934
7	beach	51.934	.0678	51.934	51.934	191.93	220.93
8	charter	56.714	.0209	15.134	15.134	21.714	56.714
9	private	34.914	.0233	34.914	34.914	34.914	53.414
10	private	28.314	.0233	28.314	28.314	28.314	46.814

Dependent Variable and Case-Specific Regressors

Fishing mode	Freq.	Percent	Cum.
beach	134	11.34	11.34
pier	178	15.06	26.40
private	418	35.36	61.76
charter	452	38.24	100.00
Total	1,182	100.00	

Fishing mode	N(income)	mean(income)	sd(income)
beach	134	4.051617	2.50542
pier	178	3.387172	2.340324
private	418	4.654107	2.777898
charter	452	3.880899	2.050029

Alternative-Specific Regressors

Fishing mode	mean(pbeach)	mean(ppier)	mean(pprivate)	mean(pcharter)
beach	36	36	98	125
pier	31	31	82	110
private	138	138	42	71
charter	121	121	45	75

Fishing mode	mean(qbeach)	mean(qpier)	mean(qprivate)	mean(qcharter)
beach	0.28	0.22	0.16	0.52
pier	0.26	0.20	0.15	0.50
private	0.21	0.13	0.18	0.65
charter	0.25	0.16	0.18	0.69

mlogit mode income, baseoutcome(1) nolog

Multinomial logistic regression

Number of obs	= 1182
LR chi2(3)	= 41.14
Prob > chi2	= 0.0000
Log likelihood = -1477.1506	Pseudo R2 = 0.0137

mode	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
beach	(base outcome)					
pier						
income	-.1434029	.0532884	-2.69	0.007	-.2478463	-.0389595
_cons	.8141503	.228632	3.56	0.000	.3660399	1.262261
private						
income	.0919064	.0406637	2.26	0.024	.0122069	.1716058
_cons	.7389208	.1967309	3.76	0.000	.3533352	1.124506
charter						
income	-.0316399	.0418463	-0.76	0.450	-.1136571	.0503774
_cons	1.341291	.1945167	6.90	0.000	.9600457	1.722537

Wald Test of the Joint Significance of Income

test income

```
( 1)  [beach]income = 0
( 2)  [pier]income = 0
( 3)  [private]income = 0
( 4)  [charter]income = 0
      Constraint 1 dropped

           chi2(  3) =   37.70
         Prob > chi2 =    0.0000
```

Odds Ratios or Relative-Risk Ratios

$$\frac{Pr(y_i=j)}{Pr(y_i=1)} = exp(x_i'\beta_j)$$

The coefficient of income for pier was -0.1434

$$e^{\beta_{jr}} = e^{-0.1434} = 0.8664$$

One-unit ($1,000 monthly) increase in income leads to relative odds of choosing to fish from a pier rather than the beach that are 0.86 times what they were before the change

mlogit mode income, rr baseoutcome(1) nolog

Multinomial logistic regression

Log likelihood = -1477.1506

Number of obs	= 1182
LR chi2(3)	= 41.14
Prob > chi2	= 0.0000
Pseudo R2	= 0.0137

mode	RRR	Std. Err.	z	P>|z|	[95% Conf.	Interval]
beach	(base outcome)					
pier						
income	.8664049	.0461693	-2.69	0.007	.7804799	.9617896
private						
income	1.096262	.0445781	2.26	0.024	1.012282	1.18721
charter						
income	.9688554	.040543	-0.76	0.450	.8925639	1.051668

Predicted Probabilities

predict pmlogit1 pmlogit2 pmlogit3 pmlogit4, pr

summarize pmlogit* dbeach dpier dprivate dcharter, separator(4)

Variable	Obs	Mean	Std. Dev.	Min	Max
pmlogit1	1182	.1133672	.0036716	.0947395	.1153659
pmlogit2	1182	.1505922	.0444575	.0356142	.2342903
pmlogit3	1182	.3536379	.0797714	.2396973	.625706
pmlogit4	1182	.3824027	.0346281	.2439403	.4158273
dbeach	1182	.1133672	.3171753	0	1
dpier	1182	.1505922	.3578023	0	1
dprivate	1182	.3536379	.4783008	0	1
dcharter	1182	.3824027	.4861799	0	1

Sample Average Predicted Probability of the Third Outcome

margins, predict(outcome(3)) noatlegend

```
Predictive margins                                Number of obs   =       1182
Model VCE    : OIM

Expression   : Pr(mode==private), predict(outcome(3))
```

	Margin	Delta-method Std. Err.	z	P>|z|	[95% Conf.	Interval]
_cons	.3536379	.0137114	25.79	0.000	.326764	.3805118

Marginal Effect at Mean of Income Change for Outcome 3

$$\frac{\partial p_{ij}}{\partial x_i} = p_{ij}(\beta_j - \bar{\beta}_i), \qquad \bar{\beta}_i = \Sigma_l p_{il}\beta_l$$

The signs of the coefficients do not give the signs of the MEs

margins, dydx(*) predict(outcome(3)) atmean

```
Conditional marginal effects                    Number of obs   =       1182
Model VCE    : OIM
Expression   : Pr(mode==private), predict(outcome(3))
dy/dx w.r.t. : income
```

	Delta-method dy/dx	Std. Err.	z	P>|z|	[95% Conf.	Interval]
income	.0325985	.005692	5.73	0.000	.0214424	.0437547

Average Marginal Effect of Income Change for Outcome 3

margins, dydx(*) predict(outcome(3)) noatlegend

```
Average marginal effects                        Number of obs   =       1182
Model VCE    : OIM
Expression   : Pr(mode==private), predict(outcome(3))
dy/dx w.r.t. : income
```

	dy/dx	Delta-method Std. Err.	z	P>|z|	[95% Conf.	Interval]
income	.0317562	.0052589	6.04	0.000	.021449	.0420633

A one-unit change in income ($1,000) increase in monthly income, increases by 0.031 the probability of fishing from a private boat rather than from a beach, pier, or charter boat.

Conditional Logit (CL) or Mixed Logit Model

$$p_{ij} = \frac{exp(x'_{ij}\beta + z'_i\gamma_j)}{\sum_{l=1}^{m} exp(x'_{il}\beta + z'_i\gamma_l)}$$

$$j = 1, ..., m$$

One of the $\gamma_l = 0$ like MNL Model

Convert Data from Wide to Long Form

list mode price pbeach ppier pprivate pcharter in 1, clean

	mode	price	pbeach	ppier	pprivate	pcharter
1.	charter	182.93	157.93	157.93	157.93	182.93

generate id = _n

reshape long d p q, i(id) j(fishmode beach pier private charter) string

list in 1/4, clean noobs

id	fishmode	mode	price	crate	d	p	q	income
1	beach	charter	182.93	.5391	0	157.93	.0678	7.083332
1	charter	charter	182.93	.5391	1	182.93	.5391	7.083332
1	pier	charter	182.93	.5391	0	157.93	.0503	7.083332
1	private	charter	182.93	.5391	0	157.93	.2601	7.083332

asclogit d p q, case(id) alternatives(fishmode) casevars(income) basealternative(beach) nolog

```
Alternative-specific conditional logit          Number of obs      =      4728
Case variable: id                               Number of cases    =      1182
Alternative variable: fishmode                  Alts per case: min =         4
                                                               avg =       4.0
                                                               max =         4
                                                Wald chi2(5)       =    252.98
Log likelihood = -1215.1376                     Prob > chi2        =    0.0000
```

d	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
fishmode						
p	-.0251166	.0017317	-14.50	0.000	-.0285106	-.0217225
q	.357782	.1097733	3.26	0.001	.1426302	.5729337
beach	(base alternative)					
charter						
income	-.0332917	.0503409	-0.66	0.508	-.131958	.0653745
_cons	1.694366	.2240506	7.56	0.000	1.255235	2.133497
pier						
income	-.1275771	.0506395	-2.52	0.012	-.2268288	-.0283255
_cons	.7779593	.2204939	3.53	0.000	.3457992	1.210119
private						
income	.0894398	.0500671	1.79	0.074	-.0086898	.1875694
_cons	.5272788	.2227927	2.37	0.018	.0906132	.9639444

Predicted Probabilities

estimates store CL

predict pasclogit, pr

table fishmode, contents(mean d mean pasclogit sd pasclogit) cellwidth(15)

fishmode	mean(d)	mean(pasclogit)	sd(pasclogit)
beach	.1133672	.1133672	.1285042
charter	.3824027	.3824027	.1565869
pier	.1505922	.1505922	.1613722
private	.3536379	.3536379	.1664636

Marginal Effect at Mean (1)

estat mfx

Pr(choice = beach|1 selected) = .05248806

variable	dp/dx	Std. Err.	z	P>|z|	[	95% C.I.	]	X
p								
beach	-.001249	.000121	-10.29	0.000	-.001487	-.001011		103.42
charter	.000609	.000061	9.97	0.000	.000489	.000729		84.379
pier	.000087	.000016	5.42	0.000	.000055	.000118		103.42
private	.000553	.000056	9.88	0.000	.000443	.000663		55.257
q								
beach	.017794	.005971	2.98	0.003	.006091	.029496		.24101
charter	-.008677	.0029	-2.99	0.003	-.01436	-.002994		.62937
pier	-.001237	.000481	-2.57	0.010	-.002179	-.000294		.16222
private	-.00788	.002647	-2.98	0.003	-.013068	-.002691		.17121
casevars								
income	-.000721	.002319	-0.31	0.756	-.005266	.003823		4.0993

Marginal Effect at Mean (2)

Pr(choice = charter|1 selected) = .46206853

variable	dp/dx	Std. Err.	z	P>|z|	[95% C.I.]		X
p							
beach	.000609	.000061	9.97	0.000	.000489	.000729	103.42
charter	-.006243	.000441	-14.15	0.000	-.007108	-.005378	84.379
pier	.000764	.000071	10.69	0.000	.000624	.000904	103.42
private	.00487	.000452	10.77	0.000	.003983	.005756	55.257
q							
beach	-.008677	.0029	-2.99	0.003	-.01436	-.002994	.24101
charter	.088931	.027272	3.26	0.001	.035479	.142382	.62937
pier	-.010886	.003596	-3.03	0.002	-.017934	-.003839	.16222
private	-.069367	.021306	-3.26	0.001	-.111125	-.027609	.17121
casevars							
income	-.021734	.00666	-3.26	0.001	-.034787	-.00868	4.0993

Marginal Effect at Mean (3)

Pr(choice = pier|1 selected) = .06584968

variable	dp/dx	Std. Err.	z	P>|z|	[95% C.I.]		X
p							
beach	.000087	.000016	5.42	0.000	.000055	.000118	103.42
charter	.000764	.000071	10.69	0.000	.000624	.000904	84.379
pier	-.001545	.000138	-11.16	0.000	-.001816	-.001274	103.42
private	.000694	.000066	10.58	0.000	.000565	.000822	55.257
q							
beach	-.001237	.000481	-2.57	0.010	-.002179	-.000294	.24101
charter	-.010886	.003596	-3.03	0.002	-.017934	-.003839	.62937
pier	.022008	.007293	3.02	0.003	.007715	.036302	.16222
private	-.009886	.003283	-3.01	0.003	-.016321	-.00345	.17121
casevars							
income	-.009306	.002719	-3.42	0.001	-.014635	-.003977	4.0993

Marginal Effect at Mean (4)

Pr(choice = private|1 selected) = **.41959373**

variable	dp/dx	Std. Err.	z	P>|z|	[95% C.I.]		X
p							
beach	.000553	.000056	9.88	0.000	.000443	.000663	103.42
charter	.00487	.000452	10.77	0.000	.003983	.005756	84.379
pier	.000694	.000066	10.58	0.000	.000565	.000822	103.42
private	-.006117	.000444	-13.77	0.000	-.006987	-.005246	55.257
q							
beach	-.00788	.002647	-2.98	0.003	-.013068	-.002691	.24101
charter	-.069367	.021306	-3.26	0.001	-.111125	-.027609	.62937
pier	-.009886	.003283	-3.01	0.003	-.016321	-.00345	.16222
private	.087132	.026755	3.26	0.001	.034693	.139571	.17121
casevars							
income	.031761	.006554	4.85	0.000	.018915	.044608	4.0993