8.2) Simultaneous Equations

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Reference

Wooldridge (2010). **Econometric Analysis of Cross Section and Panel Data.** Ch 9

https://ebookcentral.proquest.com/lib/wayne/detail.action?docID=3339196&

Housing Expenditures and Saving

$$housing = \alpha_1 saving + \beta_{11} inc + \beta_{12} educ + \beta_{13} age + u_1$$

$$saving = \alpha_2 housing + \beta_{21} inc + \beta_{22} educ + \beta_{23} age + u_2$$

Reduced Form for y_2

$$y_1 = \gamma_1 y_2 + z_{(1)}\delta_{(1)} + u_1$$

$$y_1 = \gamma_2 y_2 + z_{(2)}\delta_{(2)} + u_2$$

$$y_2 = z_{(1)}\pi_{(21)} + z_{(2)}\pi_{(22)} + v_2$$

$$\pi_{(21)} = \frac{\delta_{(1)}}{\gamma_2 - \gamma_1}, \; \pi_{(22)} = \frac{-\delta_{(2)}}{\gamma_2 - \gamma_1}, \; v_2 = \frac{u_1 - u_2}{\gamma_2 - \gamma_1}$$

Mroz (1987)

$$hours = \gamma_{12} log(wage) + \delta_{10} + \delta_{11} educ + \delta_{12} age + \delta_{13} kidslt6 + \delta_{14} kidsge6 + \delta_{14} nwifeinc + u_1$$

IVs: *exper* and $exper^2$

$$log(wage) = \gamma_{21} hours + \delta_{20} + \delta_{21} educ + \delta_{22} exper + \delta_{23} exper^2 + u_2$$

IVs: *age*, *kidslt6*, *kidsge6*, and *nwifeinc*

Estimation of Labor Supply Function

OLS

$$\hat{hours} = \underset{(340.1)}{2,114.7} - \underset{(54.22)}{17.41}log(wage) - \underset{(17.97)}{14.44}educ -$$
$$\underset{(5.53)}{7.73}age - \underset{(100.01)}{342.50}kidslt6 - \underset{(30.83)}{115.02}kidsge6 - \underset{(3.66)}{4.35}nwifeinc$$

2SLS

$$\hat{hours} = \underset{(594.2)}{2,432.2} + \underset{(480.74)}{1,544.82}log(wage) - \underset{(58.14)}{177.45}educ -$$
$$\underset{(9.58)}{10.78}age - \underset{(176.93)}{210.83}kidslt6 - \underset{(56.92)}{47.56}kidsge6 - \underset{(6.48)}{9.25}nwifeinc$$

Average Annual Hours $= 1{,}303$

Labor Supply Elasticity $= 1{,}544.82/\text{hours} \cong 1.2$

Using Cross Equation Restrictions to Achieve Identification

1) $y_1 = \gamma_{12} y_2 + \delta_{11} z_1 + \delta_{12} z_2 + \delta_{13} z_3 + u_1$

2) $y_2 = \gamma_{21} y_1 + \delta_{21} z_1 + \delta_{22} z_2 + u_2$

$$\delta_{12} = \delta_{22}$$

1) $y_1 - \hat{\delta}_{22} z_2 = \gamma_{12} y_2 + \delta_{11} z_1 + \delta_{13} z_3 + u_1$

IVs: (z_1, z_2, z_3)

Using Covariance Restrictions to Achieve Identification

$$\textbf{1)}\ y_1 = \gamma_{12} y_2 + \delta_{11} z_1 + \delta_{13} z_3 + u_1$$

$$\textbf{2)}\ y_2 = \gamma_{21} y_1 + \delta_{21} z_1 + \delta_{22} z_2 + \delta_{23} z_3 + u_2$$

$$Cov(u_1, u_2) = E(u_1 u_2) = 0$$

IVs for Eq1: (z_1, z_2, z_3)

IVs for Eq2: $(z_1, z_2, z_3, \hat{u}_1)$