7) Generalized Method of Moments (GMM)

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Generalized Method of Moments (GMM)

$$Q_N(\beta) = \{\tfrac{1}{N}(y - X\beta)'Z\}W_N\{\tfrac{1}{N}Z'(y - X\beta)\}$$

$$\frac{\partial Q_n(\beta)}{\partial \beta} = -2[\tfrac{1}{N}X'Z]W_N[\tfrac{1}{N}Z'(y - X\beta)] = 0$$

$$\hat{\beta}_{GMM} = (X'ZW_NZ'X)^{-1}X'ZW_NZ'y$$

$$\hat{\beta}_{IV} = (Z'X)^{-1}Z'y$$

$$\hat{\beta}_{2SLS} = \{X'Z(Z'Z)^{-1}Z'X\}^{-1}X'Z(Z'Z)^{-1}Z'y$$

Optimal GMM

$$\hat{\beta}_{OGMM} = (X'Z\hat{S}^{-1}Z'X)^{-1}X'Z\hat{S}^{-1}Z'y$$

$$\hat{S} = \frac{1}{N}\sum_{i=1}^{N} \hat{u}^2 z_i z_i' = \frac{Z'DZ}{N}$$

If $E[u_i^2|z_i] = \sigma^2$, then $\hat{S} = \frac{s^2 Z'Z}{N}$

Medical Expenditure Panel Survey (MEPS): Individuals over the age of 65 years

ldrugexp: the log of total out-of-pocket expenditures on prescribed medications

hi_empunion: indicator for whether the individual holds either employer or union-sponsored health insurance

totchr: # of chronic conditions

sociodemographic variables: age, female, blhisp, and linc

ssiratio: ratio of an individual's social security income to the individual's income from all sources

multlc: if the firm is a large operator with multiple locations

Summary Statistics

global x2list totchr age female blhisp linc

summarize ldrugexp hi_empunion $x2list ssiratio multlc

Variable	Obs	Mean	Std. Dev.	Min	Max
ldrugexp	10,391	6.479668	1.363395	0	10.18017
hi_empunion	10,391	.3796555	.4853245	0	1
totchr	10,391	1.860745	1.290131	0	9
age	10,391	75.04639	6.69368	65	91
female	10,391	.5797325	.4936256	0	1
blhisp	10,391	.1703397	.3759491	0	1
linc	10,089	2.743275	.9131433	-6.907755	5.744476
ssiratio	10,391	.5206281	.3745878	-2.100647	9.25062
multlc	10,391	.0603407	.2381284	0	1

OLS Estimates

regress ldrugexp hi_empunion $x2list, vce(robust)

ldrugexp	Coef.	Robust Std. Err.	t	P>|t|
hi_empunion	.0738788	.0259848	2.84	0.004
totchr	.4403807	.0093633	47.03	0.000
age	-.0035295	.001937	-1.82	0.068
female	.0578055	.0253651	2.28	0.023
blhisp	-.1513068	.0341264	-4.43	0.000
linc	.0104815	.0137126	0.76	0.445
_cons	5.861131	.1571037	37.31	0.000

IV estimation of an exactly identified model: First Stage

ivregress 2sls ldrugexp (hi_empunion = ssiratio) ///
$x2list, vce(robust) first

hi_empunion	Coef.	Robust Std. Err.	t	P>|t|
totchr	.0127865	.0036655	3.49	0.000
age	-.0086323	.0007087	-12.18	0.000
female	-.07345	.0096392	-7.62	0.000
blhisp	-.06268	.0122742	-5.11	0.000
linc	.0483937	.0066075	7.32	0.000
ssiratio	-.1916432	.0236326	-8.11	0.000
_cons	1.028981	.0581387	17.70	0.000

ivregress 2sls ldrugexp (hi_empunion = ssiratio) ///
$x2list, vce(robust) first

ldrugexp	Coef.	Robust Std. Err.	z	P>|z|
hi_empunion	-.8975913	.2211268	-4.06	0.000
totchr	.4502655	.0101969	44.16	0.000
age	-.0132176	.0029977	-4.41	0.000
female	-.020406	.0326114	-0.63	0.531
blhisp	-.2174244	.0394944	-5.51	0.000
linc	.0870018	.0226356	3.84	0.000
_cons	6.78717	.2688453	25.25	0.000

IV, OGMM, and LIML

ivregress 2sls ldrugexp (hi_empunion = ssiratio) $x2list, vce(robust)
estimates store IV

ivregress gmm ldrugexp (hi_empunion = ssiratio) $x2list, wmatrix(robust)
estimates store OGMM

ivregress liml ldrugexp (hi_empunion = ssiratio) $x2list, vce(robust)
estimates store LIML

estimates table IV OGMM LIML, b(%9.4f) se

Variable	IV	OGMM	LIML
hi_empunion	-0.8976	-0.8976	-0.8976
	0.2211	0.2211	0.2211
totchr	0.4503	0.4503	0.4503
	0.0102	0.0102	0.0102
age	-0.0132	-0.0132	-0.0132
	0.0030	0.0030	0.0030
female	-0.0204	-0.0204	-0.0204
	0.0326	0.0326	0.0326
blhisp	-0.2174	-0.2174	-0.2174
	0.0395	0.0395	0.0395
linc	0.0870	0.0870	0.0870
	0.0226	0.0226	0.0226
cons	6.7872	6.7872	6.7872

Code: 2SLS, OGMM, and LIML

```
global ivmodel "ldrugexp (hi_empunion = ssiratio multlc) $x2list"
quietly ivregress 2sls $ivmodel
estimates store TwoSLS
quietly ivregress 2sls $ivmodel, vce(robust)
estimates store Rob_2SLS
quietly ivregress gmm $ivmodel, wmatrix(unadjusted)
estimates store GMM
quietly ivregress gmm $ivmodel, wmatrix(robust)
estimates store Rob_GMM
quietly ivregress liml $ivmodel, vce(robust)
estimates store Rob_LIML
estimates table TwoSLS Rob_2SLS GMM Rob_GMM Rob_LIML, b(%9.4f) se
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Estimation of an Overidentified Model

Variable	TwoSLS	Rob_2SLS	GMM	Rob_GMM	Rob_LIML
hi_empunion	-0.9899	-0.9899	-0.9899	-0.9933	-0.9957
	0.1922	0.2046	0.1922	0.2047	0.2059
totchr	0.4512	0.4512	0.4512	0.4510	0.4513
	0.0105	0.0103	0.0105	0.0103	0.0103
age	-0.0141	-0.0141	-0.0141	-0.0142	-0.0142
	0.0028	0.0029	0.0028	0.0029	0.0029
female	-0.0278	-0.0278	-0.0278	-0.0282	-0.0283
	0.0312	0.0322	0.0312	0.0322	0.0322
blhisp	-0.2237	-0.2237	-0.2237	-0.2231	-0.2241
	0.0387	0.0396	0.0387	0.0396	0.0396
linc	0.0943	0.0943	0.0943	0.0945	0.0947
	0.0212	0.0219	0.0212	0.0219	0.0220
_cons	6.8752	6.8752	6.8752	6.8778	6.8807
	0.2453	0.2579	0.2453	0.2580	0.2589

Overidentified Test (OID), Hansen's Test, and Sargan's Test

$$Q(\hat{\beta}) = \{\frac{1}{N}(y - X\hat{\beta})'Z\}\hat{S}^{-1}\{\frac{1}{N}Z'(y - X\hat{\beta})\}$$

$$Z'(y - X\hat{\beta}) \simeq 0, \text{ so } Q(\hat{\beta}) \simeq 0$$

$$Q(\hat{\beta}) \overset{a}{\sim} \chi^2_r,$$

r is the # of overidentifying restrictions

$$H_0 : E\{Z'(y - X\beta)\} = 0$$

Rejection means that at least one of the instruments is not valid

Test of Overidentifying Restrictions

ivregress gmm ldrugexp (hi_empunion = ssiratio multlc) $x2list, /// wmatrix(robust)

estat overid

$$H_0 : \text{Overidentifying Restriction is Valid}$$

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Test of overidentifying restriction:

Hansen's J chi2(1) = 1.04754 (p = 0.3061)
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Four Available Instruments

ivregress gmm ldrugexp (hi_empunion = ssiratio ///
lowincome multlc firmsz) $x2list, wmatrix(robust)

ldrugexp	Coef.	Robust Std. Err.	z	P>|z|
hi_empunion	-.8124043	.1846433	-4.40	0.000
totchr	.449488	.010047	44.74	0.000
age	-.0124598	.0027466	-4.54	0.000
female	-.0104528	.0306889	-0.34	0.733
blhisp	-.2061018	.0382891	-5.38	0.000
linc	.0796532	.0203397	3.92	0.000
_cons	6.7126	.2425973	27.67	0.000

estat overid

```
Test of overidentifying restriction:

Hansen's J chi2(3) = 11.5903 (p = 0.0089)
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