# 6) Normal Distribution

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# Reference

Tables, Graphics, and Figures from

**Introductory Statistics with**

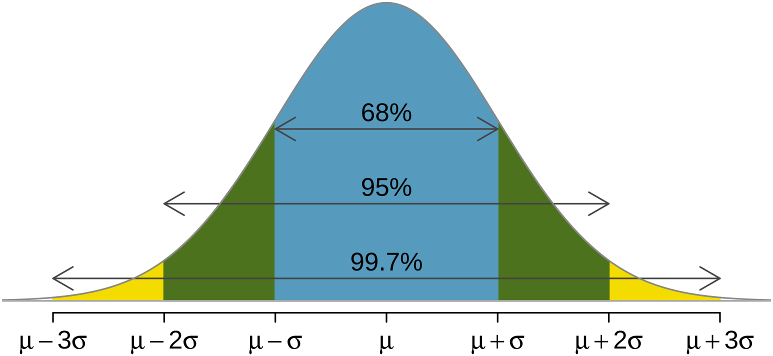
**Randomization and Simulation**

Diez et al. (2014): Ch 2 - Foundation for inference

# Central Limit Theorem (CLT)

$$Y = X_1 + X_2 + X_3 + \ldots + X_n$$

If $Xs$ are independent and $n$ large enough, then $Y \sim N(\mu, \sigma^2)$



# Standardizing with Z scores

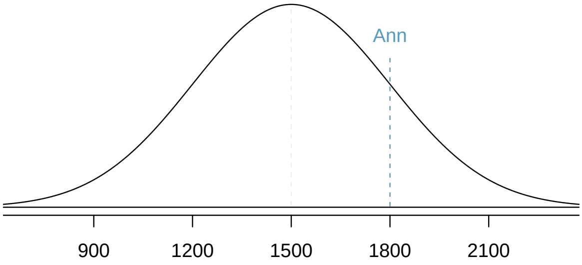
Ann scored 1800 on SAT

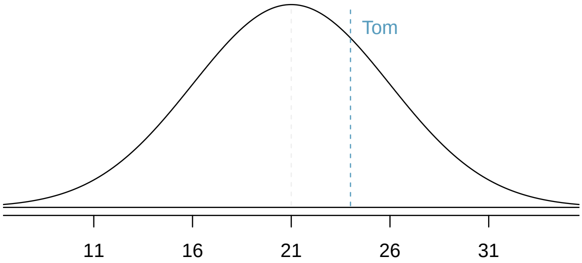
Tom scored 24 on his ACT

| | SAT | ACT |
|---|---|---|
| Mean | 1500 | 21 |
| SD | 300 | 5 |

$$Z = \frac{X-\mu}{\sigma} = \frac{24-21}{5} = 0.6$$

# SAT and ACT



# Normal Probability Table

| $Z$ | Second decimal place of $Z$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0.00 | 0.01 | 0.02 | *0.03* | **0.04** | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| *0.4* | 0.6554 | 0.6591 | 0.6628 | *0.6664* | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| **0.8** | 0.7881 | 0.7910 | 0.7939 | 0.7967 | **0.7995** | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |

$$P(z < 0.43) \cong 66.6\%$$

$$P(z < 0.84) \cong 80\%$$

## Normal Probability in R

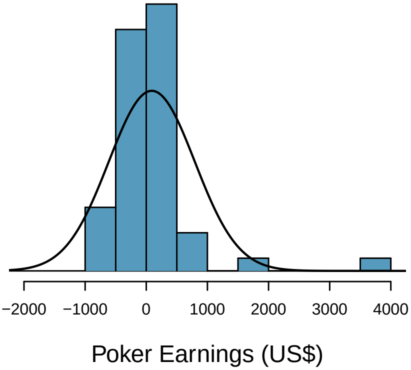
pnorm(0.43, 0, 1)

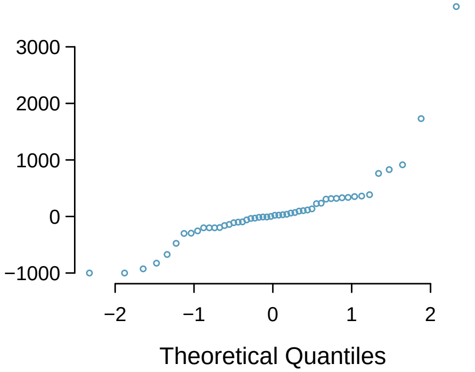
0.6664022

qnorm(0.6664022,0,1)

0.43

# Normal Probability Plot



# Opportunity Cost

$$H_0 : p_t = p_c \text{ vs } H_A : p_t > p_c$$

| | decision | | |
|---|---|---|---|
| | buy DVD | not buy DVD | Total |
| control group | 56 | 19 | 75 |
| treatment group | 41 | 34 | 75 |
| Total | 97 | 53 | 150 |

| | decision | | |
|---|---|---|---|
| | buy DVD | not buy DVD | Total |
| control group | 0.747 | 0.253 | 1.000 |
| treatment group | 0.547 | 0.453 | 1.000 |
| Total | 0.647 | 0.353 | 1.000 |

# Standard Error

$$\hat{p}_t = 0.453 \text{ and } \hat{p}_c = 0.253$$

$$n_t = 75 \text{ and } n_c = 75$$

$$SE_{p_t - p_c} = \sqrt{\frac{p_t(1-p_t)}{n_t} + \frac{p_c(1-p_c)}{n_c}} = 0.078$$

$$Z = \frac{p_t - p_c - 0}{SE_{p_t - p_c}} = \frac{0.2 - 0}{0.078} = 2.56$$

1-pnorm(2.56, 0, 1)

$$P(z > 2.56) = 0.0052$$

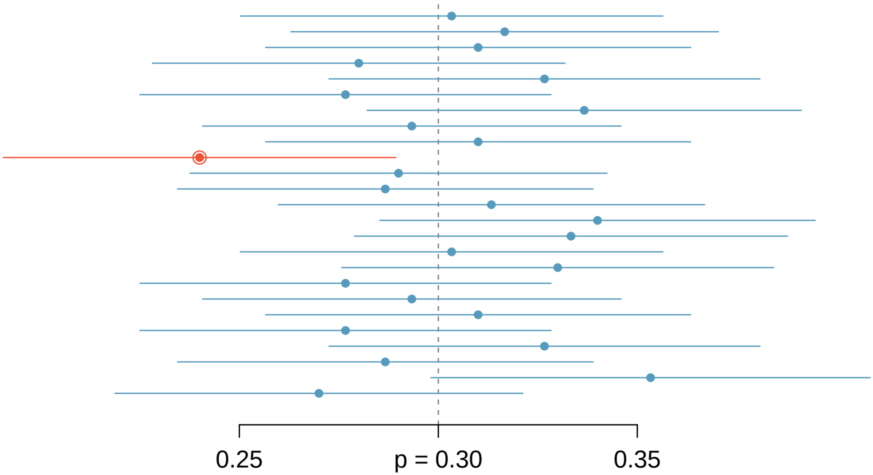
# 95% Confidence Interval

$$p \pm z_{2.5\%} \times SE$$

$$0.2 \pm 1.96 \times 0.078 = [0.047, 0.353]$$

**We are 95% confident that the DVD purchase rate resulting from the treatment is between 4.7% and 35.3% lower than in the control group.**

# 95% Confidence Intervals for $p = 0.3$



# Margin of Error (ME)

$$p \pm z_{2.5\%} \times SE$$

$$0.2 \pm 1.96 \times 0.078 = [0.047, 0.353]$$

$$ME = z_{2.5\%} \times SE$$

$$= 1.96 \times 0.078 \cong 0.1528 = 15.28\%$$

# Margin of Error and Sample Sizes

| n | Margin of Error |
|---|---|
| 100 | 10% |
| 400 | 5% |
| 625 | 4% |
| 1,112 | 3% |
| 2,500 | 2% |
| 10,000 | 1% |

# Presidential Election in 1936

| Companies | Sample Size | Landon | Roosevelt |
|---|---|---|---|
| Literary Digest | 10M | 57% | 43% |
| Gallup | 50K | 44% | 56% |
| Actual Result | | 37% | 62% |