# 2) Random Variables

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# Reference

Tables, Graphics, and Figures from

**Introductory Statistics with**

**Randomization and Simulation**

Diez et al. (2014): APPENDIX A3 - Probability

# Random Variable: Modelling

Two books are assigned for a statistics class: a textbook and its corresponding study guide. The bookstore determined 20% of enrolled students do not buy either book, 55% buy the textbook only, and 25% buy both books.

The textbook costs $137 and the study guide $33.

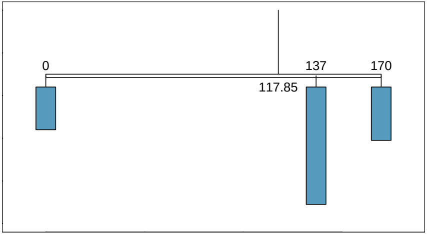
How much revenue should the bookstore expect from this class of 100 students?

# Expectation: $\mu = E(X) = \sum_{i=1}^{k} x_i P(X = x_i)$

| $i$ | 1 | 2 | 3 | Total |
|---|---|---|---|---|
| $x_i$ | \$0 | \$137 | \$170 | – |
| $P(X = x_i)$ | 0.20 | 0.55 | 0.25 | 1.00 |

$$x_1 P(X = x_1) + \ldots + x_k P(X = x_k)$$

$$= 0(.20) + 137(.55) + 170(.25) = \$117.85$$



# Variance and Standard Deviation

$$\sigma^2 = \sum_{j=1}^{k} (x_j - \mu)^2 P(X = x_j)$$

$$= (x_1 - \mu)^2 P(X = x_1) + \ldots + (x_k - \mu)^2 P(X = x_k)$$

| $i$ | 1 | 2 | 3 | Total |
|---|---|---|---|---|
| $x_i$ | \$0 | \$137 | \$170 | |
| $P(X = x_i)$ | 0.20 | 0.55 | 0.25 | |
| $x_i \times P(X = x_i)$ | 0 | 75.35 | 42.50 | 117.85 |
| $x_i - \mu$ | -117.85 | 19.15 | 52.15 | |
| $(x_i - \mu)^2$ | 13888.62 | 366.72 | 2719.62 | |
| $(x_i - \mu)^2 \times P(X = x_i)$ | 2777.7 | 201.7 | 679.9 | 3659.3 |

$$\sigma = \sqrt{3659.3} = \$60.49$$

# Linear Combinations of Random Variables

$$Z = aX + bY$$

$$E(Z) = aE(X) + bE(Y)$$

Leonard has invested \$6000 in Google and \$2000 in Exxon Mobil. Suppose Google and Exxon Mobil stocks have recently been rising 2.1% and 0.4% per month, respectively.

$$E(6000X + 2000Y)$$

$$6000(.021) + 2000(.004) = \$134$$

# Variability of Independent Random Variables

| | Mean ($\bar{x}$) | Standard deviation ($s$) | Variance ($s^2$) |
|---|---|---|---|
| GOOG | 0.0210 | 0.0846 | 0.0072 |
| XOM | 0.0038 | 0.0519 | 0.0027 |

$$Var(aX + bY) = a^2 Var(X) + b^2 Var(Y)$$

$$Var(6000X + 2000Y)$$

$$= 6000^2(.0072) + 2000^2(.0027) = 270,000$$

$$SD(6000X + 2000Y) = \sqrt{270,000} = \$520$$

# Covariance of Random Variables

$$\sigma_{xy} = Cov(X, Y) = E[(X - \mu_x)(Y - \mu_y)]$$

$$= E(XY - X\mu_y - \mu_x Y + \mu_x\mu_y)$$

$$= E(XY) - \mu_y E(X) - \mu_x E(Y) + \mu_x\mu_y$$

$$= E(XY) - \mu_x\mu_y$$

If $X$ and $Y$ are independent, then $Cov(X, Y) = 0$

# Variance for Dependent Random Variables

$$Var(X) = E[(X - \mu_x)^2]$$

$$Var(X + Y) = E[(X + Y - \mu_x - \mu_y)^2]$$

$$= E[(X - \mu_x + Y - \mu_y)^2]$$

$$= E[(X - \mu_x)^2 + (Y - \mu_y)^2 + 2(X - \mu_x)(Y - \mu_y)]$$

$$= Var(X) + Var(Y) + 2Cov(X, Y)$$

# Correlation for Random Variable

$$\rho = \frac{Cov(X,Y)}{\sqrt{Var(X) \times Var(Y)}} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

$$-1 \leq \rho \leq 1$$

Suppose $X$ can take only the three values$-1$, 0, and 1, and that each of these three values has the same probability. Let $Y = X^2$

$$E(XY) = E(X^3) = E(X) = 0$$

$\therefore$ $X$ and $Y$ are dependent, but uncorrelated

## Prove: $E(cX) = cE(X)$

$$E(X) = x_1 p_1 + x_2 p_2 + \ldots + x_k p_k$$

$$E(cX) = cx_1 p_1 + cx_2 p_2 + \ldots + cx_k p_k$$

$$= c(x_1 p_1 + x_2 p_2 + \ldots + x_k p_k)$$

$$= cE(X)$$

## **Prove:** $Var(cX) = c^2 Var(X)$

$$Var(X) = E[(X - \mu_x)^2]$$

$$Var(cX) = E[(cX - c\mu_x)^2]$$

$$= E[c^2(X - \mu_x)^2]$$

$$= c^2 E(X - \mu_x)^2$$

$$= c^2 Var(X)$$