22) Generalized Additive Models (GAMs)

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Reference

Tables, Graphics, and Figures from

An Introduction to Statistical Learning

James et al. (2017): Chapters: 7.7, and 7.8.3

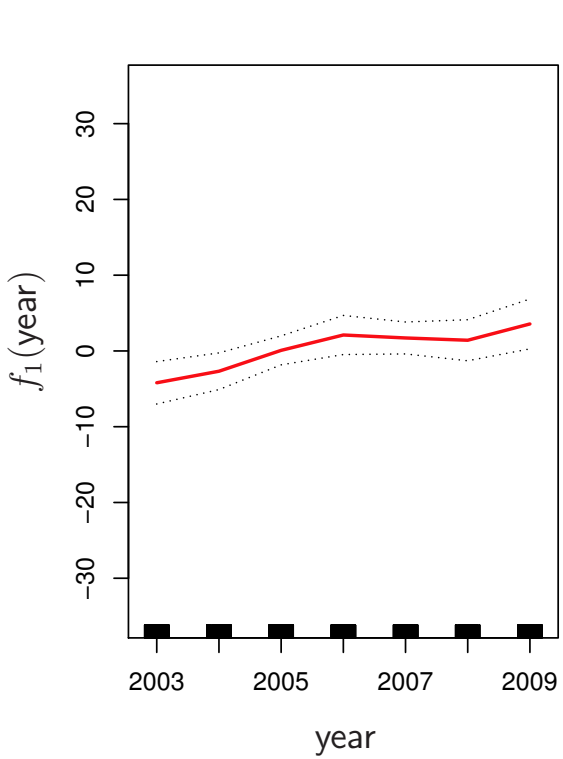
GAMs for Regression Problems

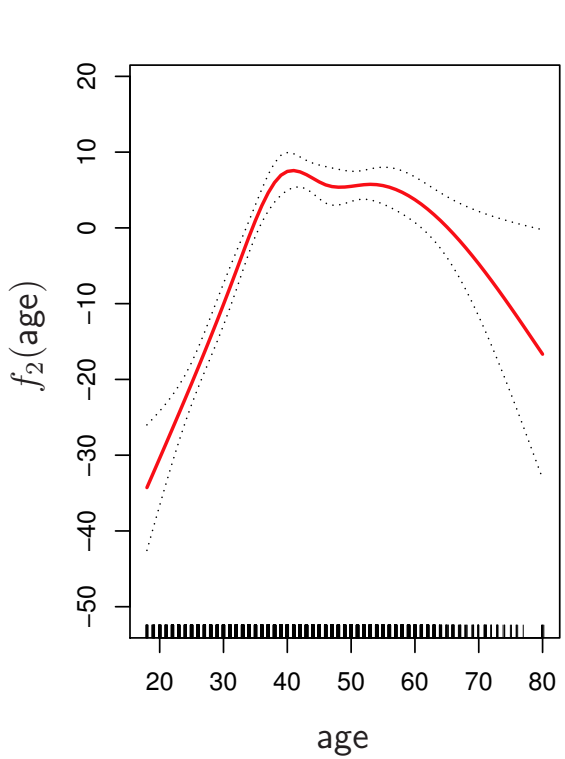
$$y_i = \beta_0 + \sum_{j=1}^{p} f_j(x_{ij}) + \epsilon_i$$

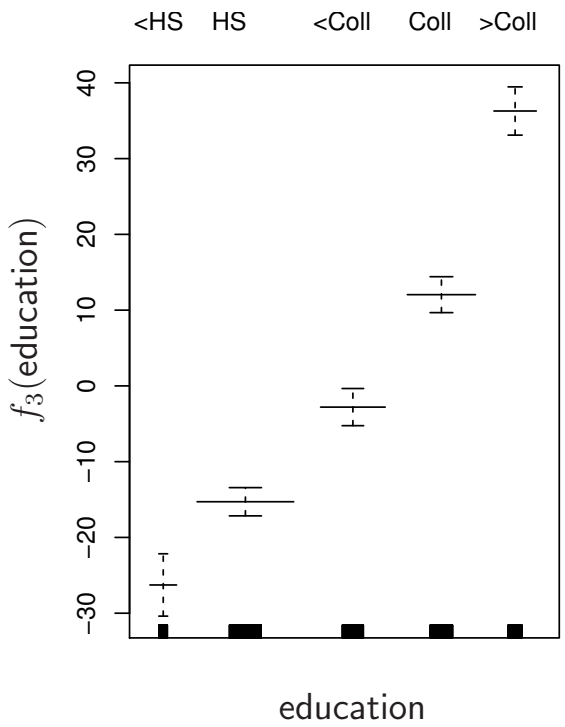
$$= \beta_0 + f_1(x_{i1}) + f_2(x_{i2}) + ... + f_p(x_{ip}) + \epsilon_i$$

$$wage =$$
$$\beta_0 + f_1(year) + f_2(age) + f_3(educ) + \epsilon$$

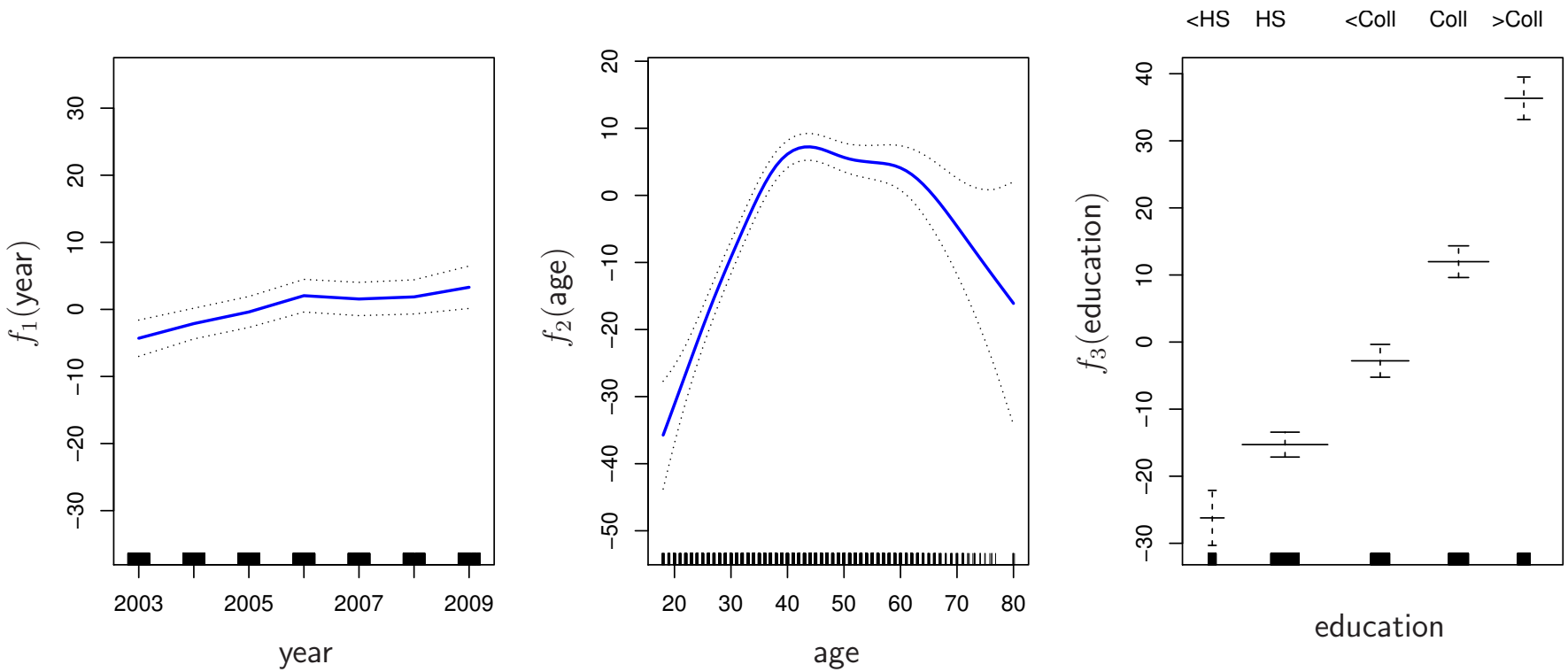
2 Natural Splines and 1 Step Function







2 Smoothing Splines and 1 Step Function



GAMs for Classification Problems

$$log\left[\frac{p(X)}{1-p(X)}\right] = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_p X_p$$

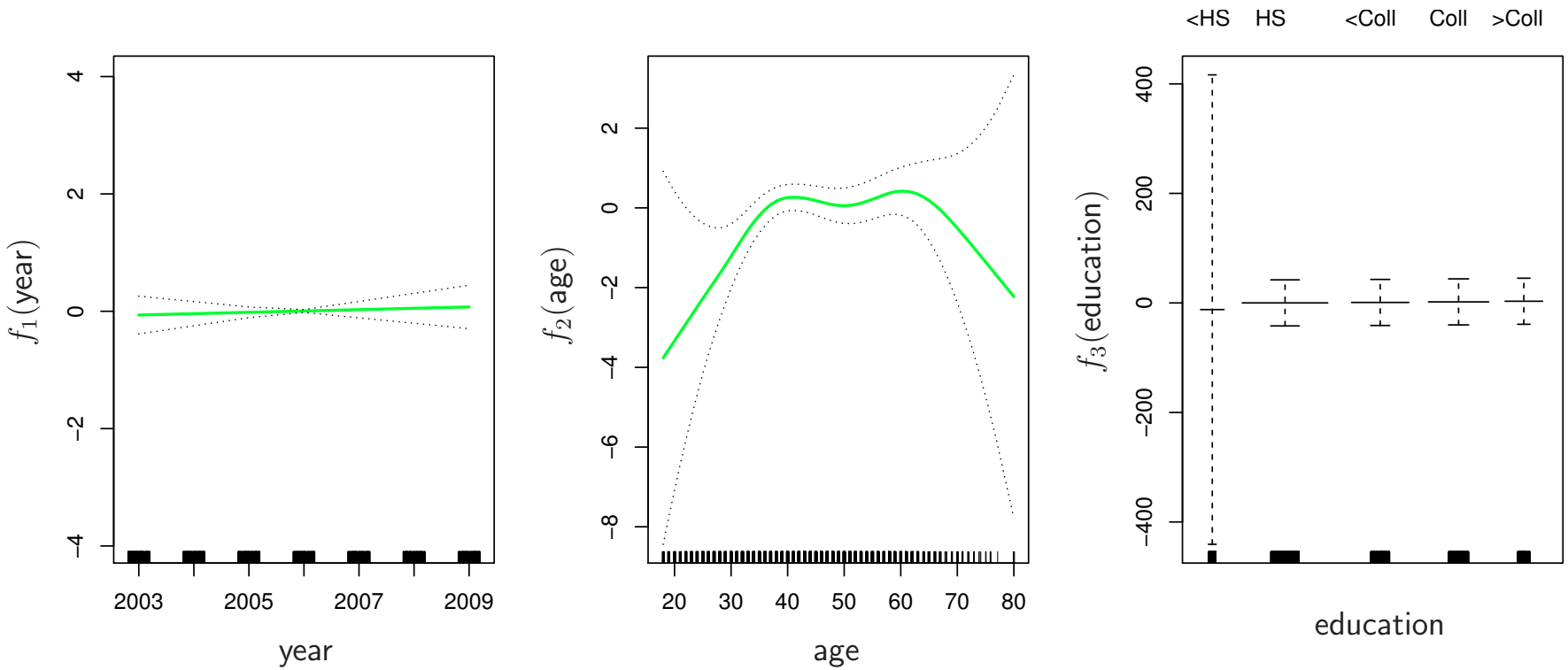
$$log\left[\frac{p(X)}{1-p(X)}\right] = \beta_0 + f_1(X_1) + f_2(X_2) + ... + f_p(X_p)$$

$$log\left[\frac{p(X)}{1-p(X)}\right] = \beta_0 + \beta_1 year + f_2(age) + f_3(educ)$$

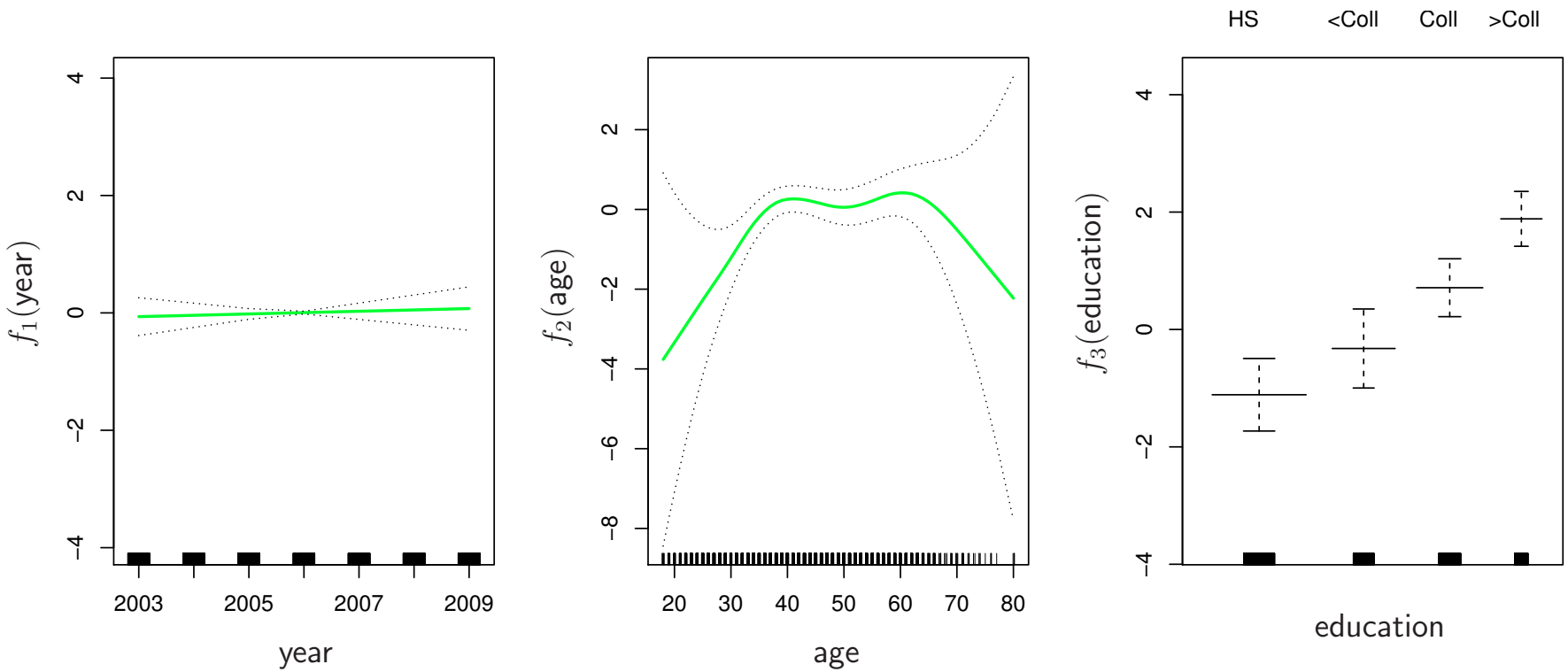
$$p(X) = Pr(wage > 250|year, age, educ)$$

Logistic Regression GAM (wage>250)

Linear, Smoothing Spline, and Step Function

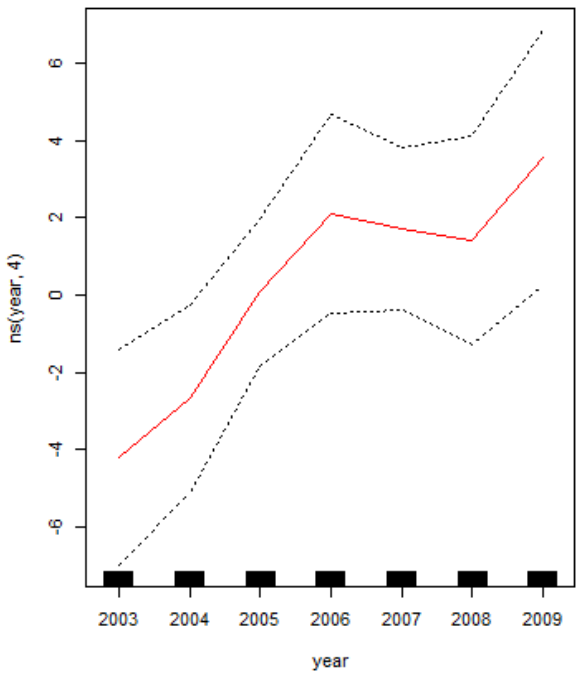


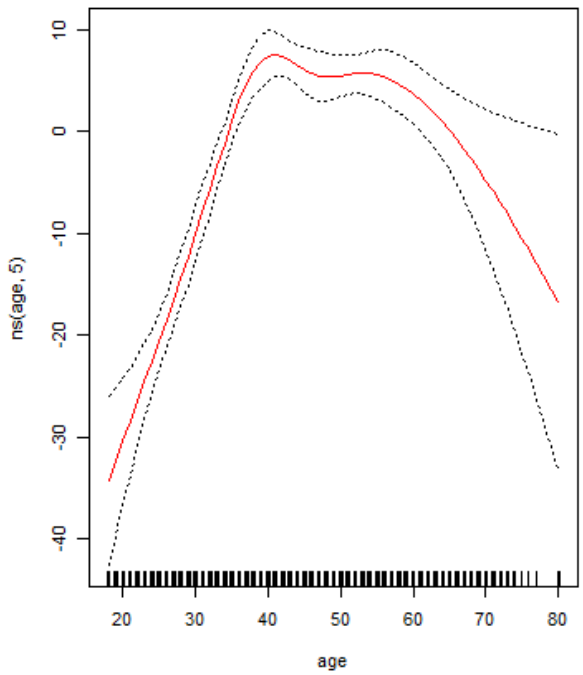
Excluding the Observations for which educ is $<$HS

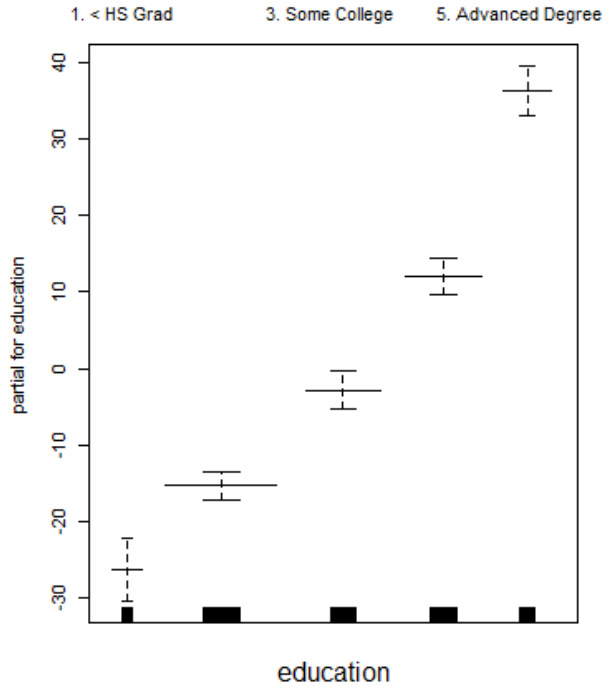


library(ISLR); attach(Wage); library(splines); library(gam)

```
gam1=gam(wage~ns(year,4)+ns(age,5)+education, data=Wage)
par(mfrow=c(1,3)); plot(gam1, se=TRUE,col="red")
```

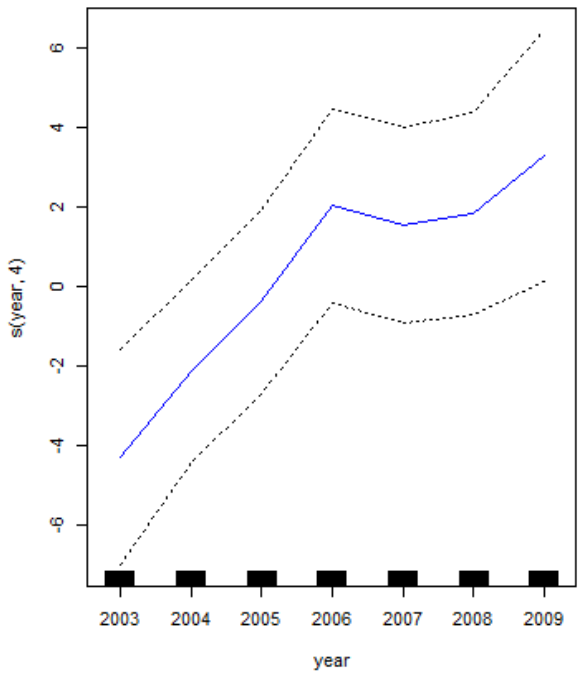


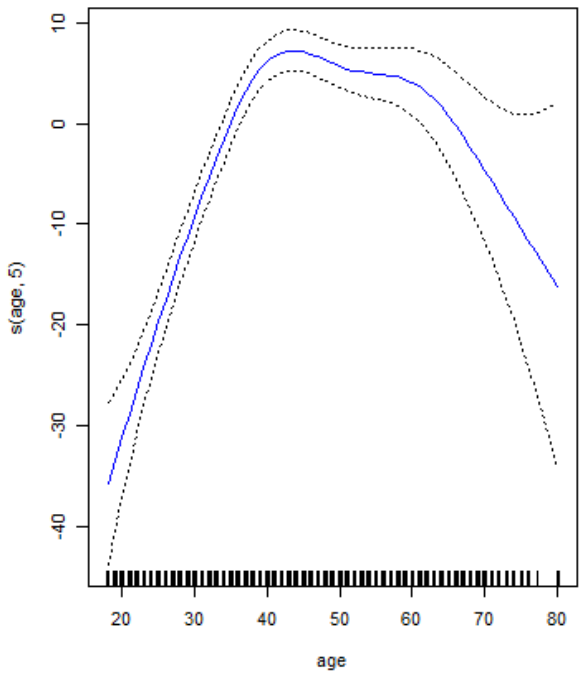


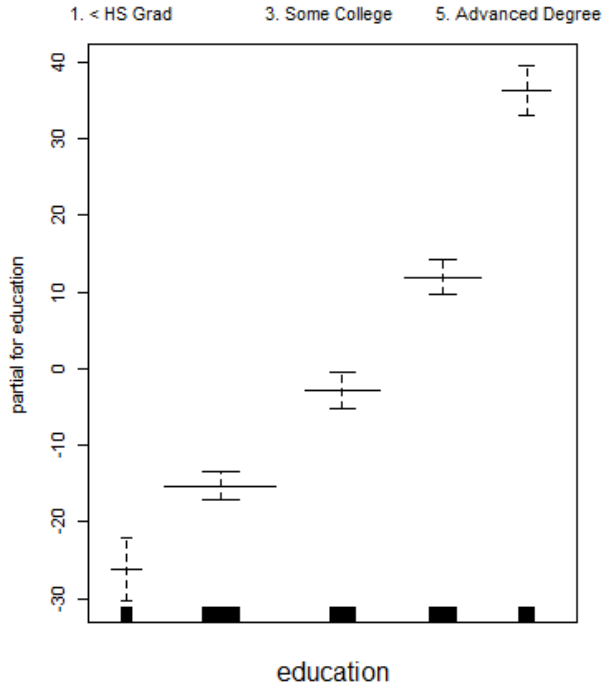


gam.m3=gam(wage~s(year,4)+s(age,5)+education, data=Wage)

```
par(mfrow=c(1,3)); plot(gam.m3, se=TRUE,col="blue")
```







gam.m1=gam(wage~s(age,5)+education, data=Wage)

gam.m2=gam(wage~year+s(age,5)+education, data=Wage)

anova(gam.m1,gam.m2,gam.m3,test="F")

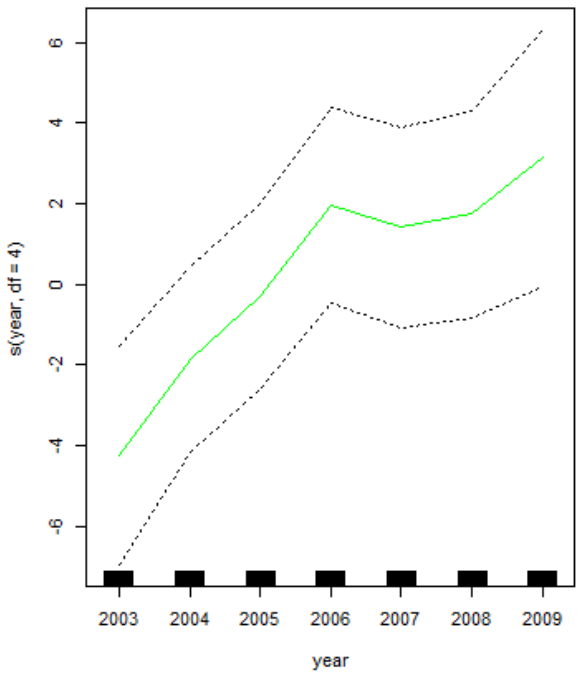
```
Model 1: wage ~ s(age, 5) + education
Model 2: wage ~ year + s(age, 5) + education
Model 3: wage ~ s(year, 4) + s(age, 5) + education
  Resid. Df Resid. Dev Df Deviance       F    Pr(>F)
1      2990    3711731
2      2989    3693842  1  17889.2 14.4771 0.0001447 ***
3      2986    3689770  3   4071.1  1.0982 0.3485661
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

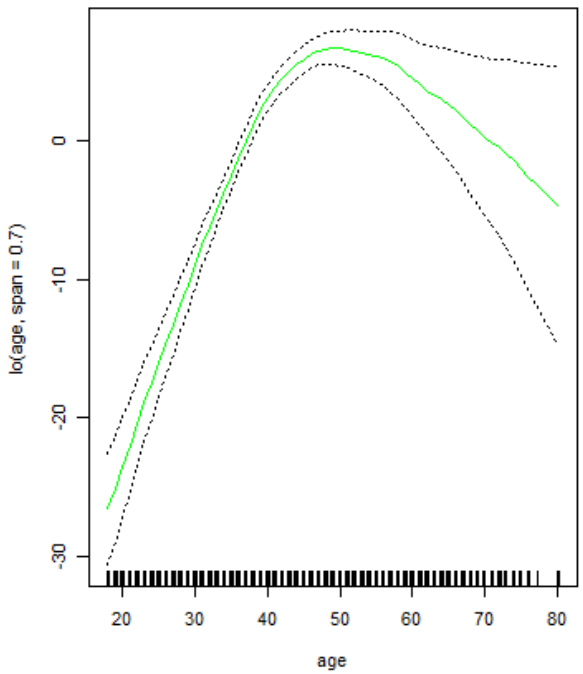
summary(gam.m3)

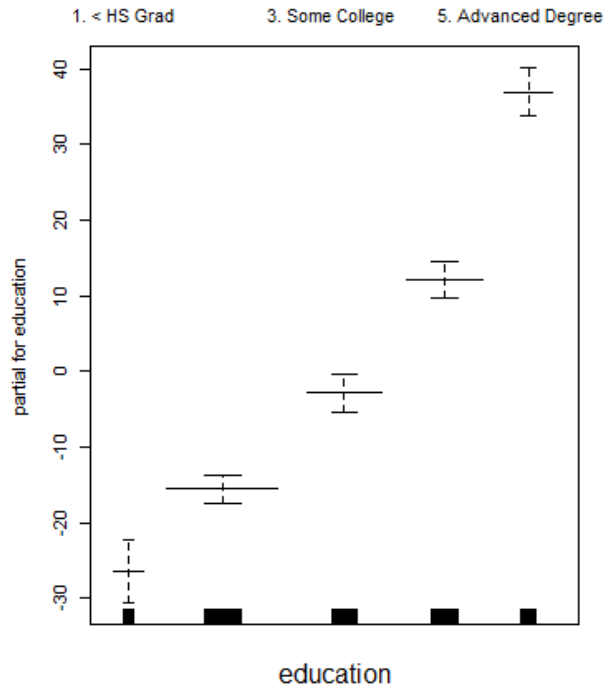
```
Anova for Parametric Effects
             Df  Sum Sq Mean Sq F value    Pr(>F)
s(year, 4)    1   27162   27162  21.981 2.877e-06 ***
s(age, 5)     1  195338  195338 158.081 < 2.2e-16 ***
education     4 1069726  267432 216.423 < 2.2e-16 ***
Residuals  2986 3689770    1236
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Anova for Nonparametric Effects
            Npar Df Npar F  Pr(F)
(Intercept)
s(year, 4)       3  1.086 0.3537
s(age, 5)        4 32.380 <2e-16 ***
education
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

gam.lo=gam(wage~s(year,df=4)+ lo(age,span=0.7)+education, data=Wage)

plot(gam.lo, se=TRUE,col="green")

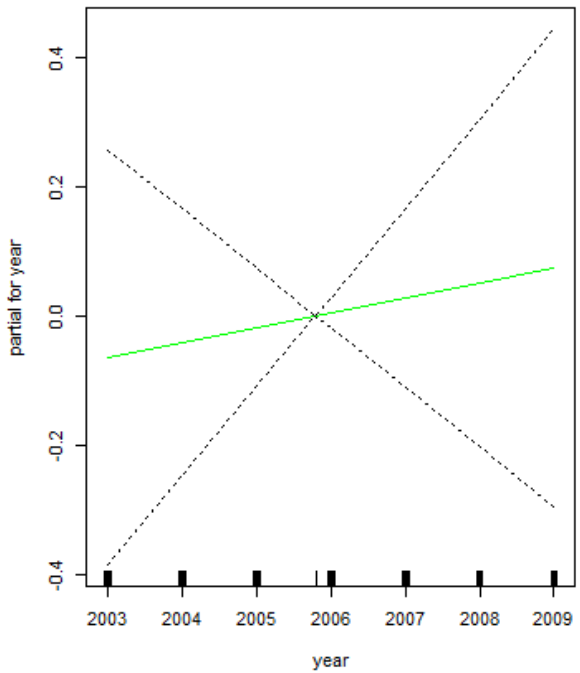


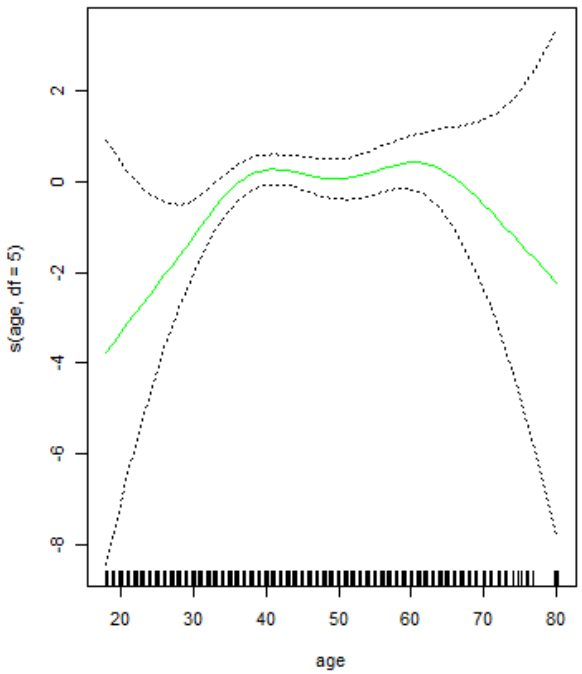


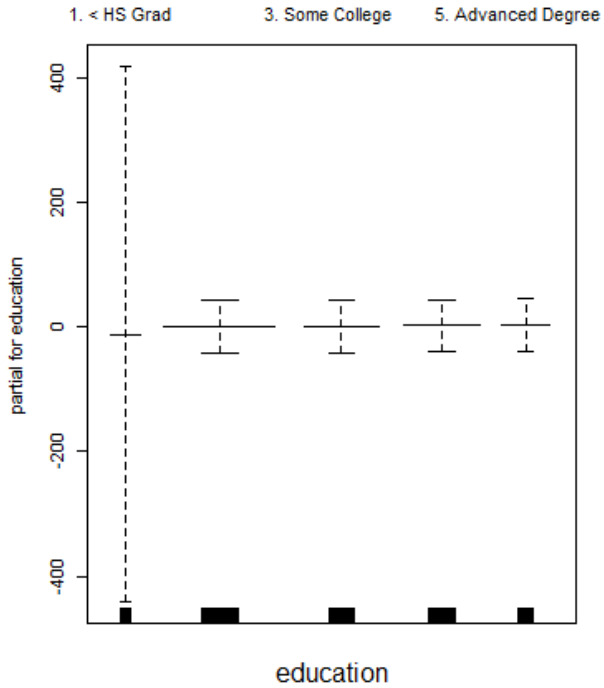


gam.lr=gam(I(wage>250)~year+s(age,df=5) +education, family=binomial,data=Wage)

par(mfrow=c(1,3)); plot(gam.lr,se=T,col="green")







table(education,I(wage>250))

education	FALSE	TRUE
1. < HS Grad	268	0
2. HS Grad	966	5
3. Some College	643	7
4. College Grad	663	22
5. Advanced Degree	381	45

gam.lr.s=gam(I(wage>250)~year+s(age,df=5)+education, family=binomial, data=Wage, subset=(education!="1. < HS Grad"))

plot(gam.lr.s,se=T,col="green")