# 1) Probability

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# Reference

Tables, Graphics, and Figures from

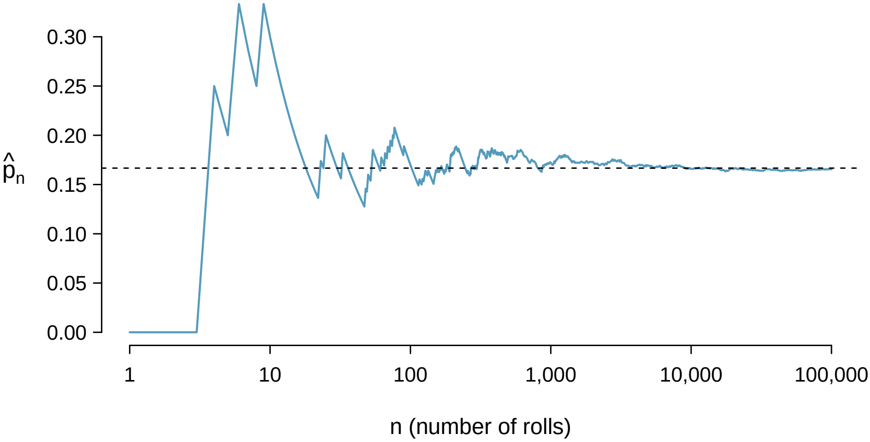
**Introductory Statistics with**

**Randomization and Simulation**

Diez et al. (2014): APPENDIX A - Probability

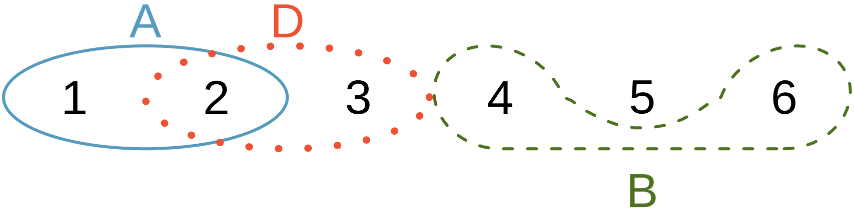
# Law of Large Numbers (LLN)

$$p = \frac{1}{6} \cong 0.167$$



# Disjoint or Mutually Exclusive Outcomes

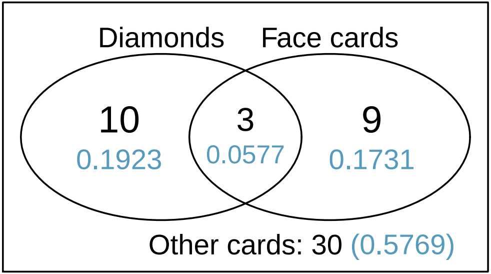
$$A = \{1, 2\};\ B = \{4, 6\};\ D = \{2, 3\}$$




$$P(A \cup B) = P(A) + P(B) = \frac{1}{3} + \frac{1}{3}$$

# Probabilities when Events are not Disjoint

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$




$$P(face.card \cup \diamondsuit)$$

$$P(face.card) + P(\diamondsuit) - P(face.card \cap \diamondsuit)$$

$$\frac{12}{52} + \frac{13}{52} - \frac{3}{52} = 0.423$$

# Independence

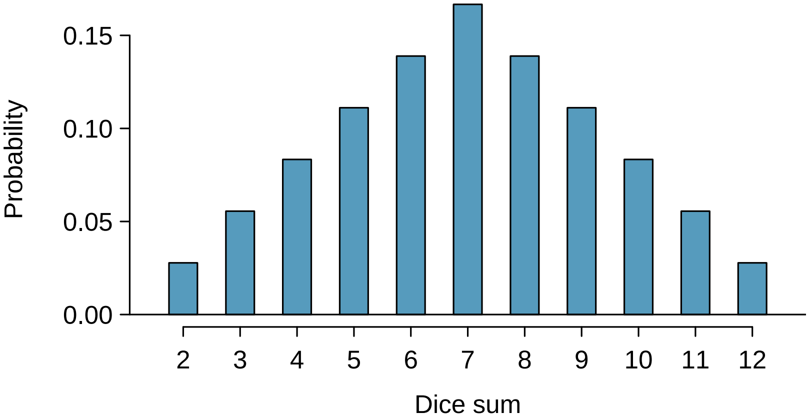
$$P(A \cap B) = P(A) \times P(B)$$

$$P(\heartsuit \cap ace) = P(A\heartsuit) = \frac{1}{52}$$

$$P(\heartsuit) \times P(ace)$$

$$\frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$$

# Probability Distribution of the Sum of Two Dice




| Dice sum | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Probability | $\frac{1}{36}$ | $\frac{2}{36}$ | $\frac{3}{36}$ | $\frac{4}{36}$ | $\frac{5}{36}$ | $\frac{6}{36}$ | $\frac{5}{36}$ | $\frac{4}{36}$ | $\frac{3}{36}$ | $\frac{2}{36}$ | $\frac{1}{36}$ |

# Independence and Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \times P(B)}{P(B)} = P(A)$$

Let X and Y represent the outcomes of rolling two dice

$$P(Y = 1|X = 1)$$

$$\frac{P(Y=1) \times (X=1)}{P(X=1)} = P(Y = 1)$$

## Smallpox in Boston, 1721 (Marginal and Joint Probabilities)

| | | inoculated | | |
|---|---|---|---|---|
| | | yes | no | Total |
| result | lived | 238 | 5136 | 5374 |
| | died | 6 | 844 | 850 |
| | Total | 244 | 5980 | 6224 |

Fenner (1988). Smallpox and Its Eradication. History of International Public Health, No 6. Geneva: World Health Organization.

$$P(result = died) = \frac{850}{6224} = 13.7\%$$

$$P(inoc. = yes) = \frac{244}{6224} = 3.9\%$$

$$P(result = died \cap inoc. = yes) = \frac{6}{6224} = 0.1\%$$

# Conditional Probability ($P(A|B) = \frac{P(A \cap B)}{P(B)}$)

| | | inoculated | | |
|---|---|---|---|---|
| | | yes | no | Total |
| result | lived | 0.0382 | 0.8252 | 0.8634 |
| | died | 0.0010 | 0.1356 | 0.1366 |
| | Total | 0.0392 | 0.9608 | 1.0000 |

$$P(result = died | inoculated = no)$$

$$\frac{P(result{=}died \cap inoculated{=}no)}{P(inoculated{=}no)} = \frac{0.1356}{0.9608} = 14.1\%$$

$$P(result = died | inoculated = yes)$$

$$\frac{P(result{=}died \cap inoculated{=}yes)}{P(inoculated{=}yes)} = \frac{0.0010}{0.0392} = 2.5\%$$

# Tree Diagrams


$$P(inoculated = yes) \times P(result = lived | inoculated = yes)$$

$$= 0.0392 \times 0.9754 = 0.0382$$

$$P(inoculated = yes \ \cap \ result = lived)$$