15) Capital Asset Pricing Model

Vitor Kamada

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Reference

Tables, Graphics, and Figures from

https://www.quantopian.com/lectures

Lecture 30 The Capital Asset Pricing Model

Capital Asset Pricing Model (CAPM)

- **Idiosyncratic Risk**: firm-specific, diversified away
- **Systematic Risk**: affects all market participants

$$\mathrm{E}[\mathrm{Return}] = \mathrm{Risk\text{-}Free}\ \text{Rate of Return} + \text{Risk Premium}$$

$$\text{Risk Premium of Asset} = \beta(\text{Market Risk Premium})$$

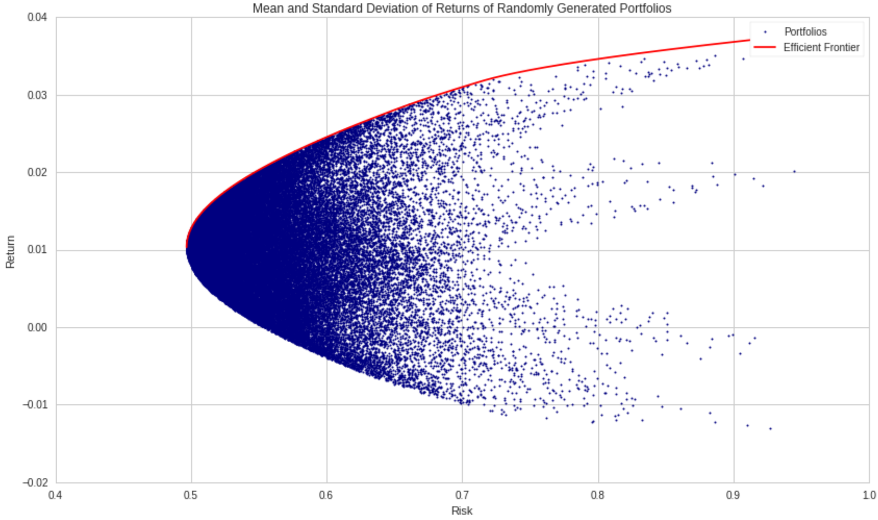
$$E[R_i] = R_F + \beta(E[R_M] - R_F)$$

CAPM Assumptions

Investors are:

1. able to trade without delay or cost and that everyone is able to borrow or lend money at the risk free rate
2. "mean-variance optimizers"

Mean and Standard Deviation of Returns



Sharpe Ratio derived in 1966 by William Sharpe, Nobel Prize in 1990

$$\text{Sharpe Ratio} = \frac{E(R_i) - R_F}{\sigma_i}$$

R_i: return of asset i

R_F: return of a risk-free security

σ_i: standard deviation of R_i

Capital Market Line (CAL)

$$E(R_p) = R_F + [\frac{E(R_M)-R_F}{\sigma_M}]\sigma_p$$

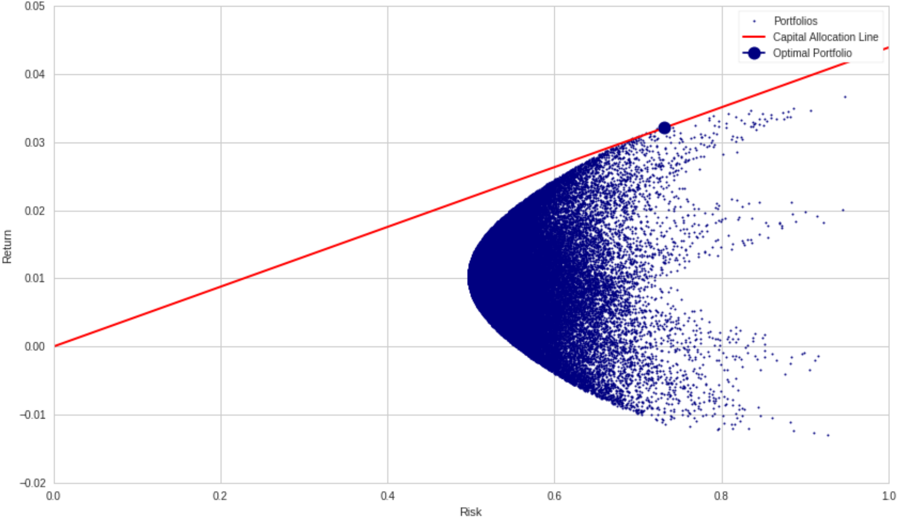
R_p: return of portfolio p

R_M: return of a market portfolio

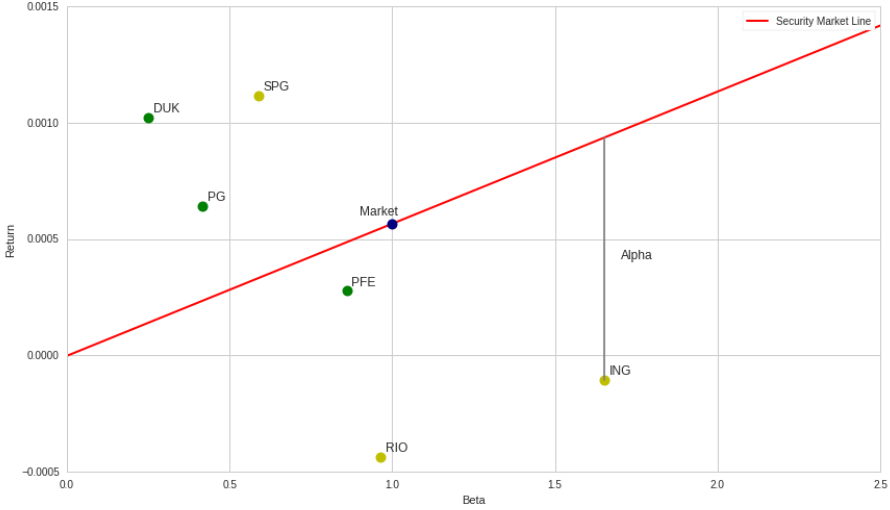
σ_M: standard deviation of R_M

σ_p: standard deviation of R_p

Slope of Capital Allocations Line = Sharpe Ratio



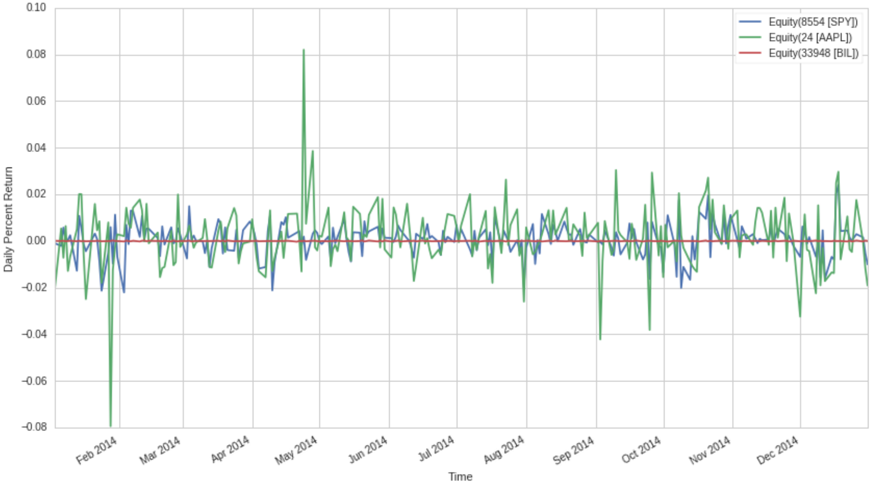
Security Markets Line: $E[R_i] = R_F + \beta_i(E[R_M] - R_F)$



$E[R_i] = R_F + \beta(E[R_M] - R_F)$

```
start_date = '2014-01-01'
end_date = '2014-12-31'
# choose stock
R = get_pricing('AAPL', fields='price',
                start_date=start_date, end_date=end_date).pct_change()[1:]

# risk-free proxy
R_F = get_pricing('BIL', fields='price',
                start_date=start_date, end_date=end_date).pct_change()[1:]

# find it's beta against market
M = get_pricing('SPY', start_date=start_date,
                end_date=end_date, fields='price').pct_change()[1:]

AAPL_results = regression.linear_model.OLS(R-R_F, sm.add_constant(M)).fit()
AAPL_beta = AAPL_results.params[1]

M.plot()
R.plot()
R_F.plot()
plt.xlabel('Time')
plt.ylabel('Daily Percent Return')
plt.legend();
```

Daily Percent Return



AAPL_results.summary()

Dep. Variable:	y	**R-squared:**	0.192
Model:	OLS	**Adj. R-squared:**	0.189
Method:	Least Squares	**F-statistic:**	59.35
Date:	Mon, 28 Aug 2017	**Prob (F-statistic):**	3.12e-13
Time:	19:11:42	**Log-Likelihood:**	749.29
No. Observations:	251	**AIC:**	-1495.
Df Residuals:	249	**BIC:**	-1488.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[95.0% Conf. Int.]**
const	0.0010	0.001	1.331	0.185	-0.000 0.003
Equity(8554 [SPY])	0.8438	0.110	7.704	0.000	0.628 1.059

predictions = R_F + AAPL_beta*(M - R_F)

```
predictions.plot()
R.plot(color='Y')
plt.legend(['Prediction', 'Actual Return'])

plt.xlabel('Time')
plt.ylabel('Daily Percent Return');
```