22) Principal Components Analysis

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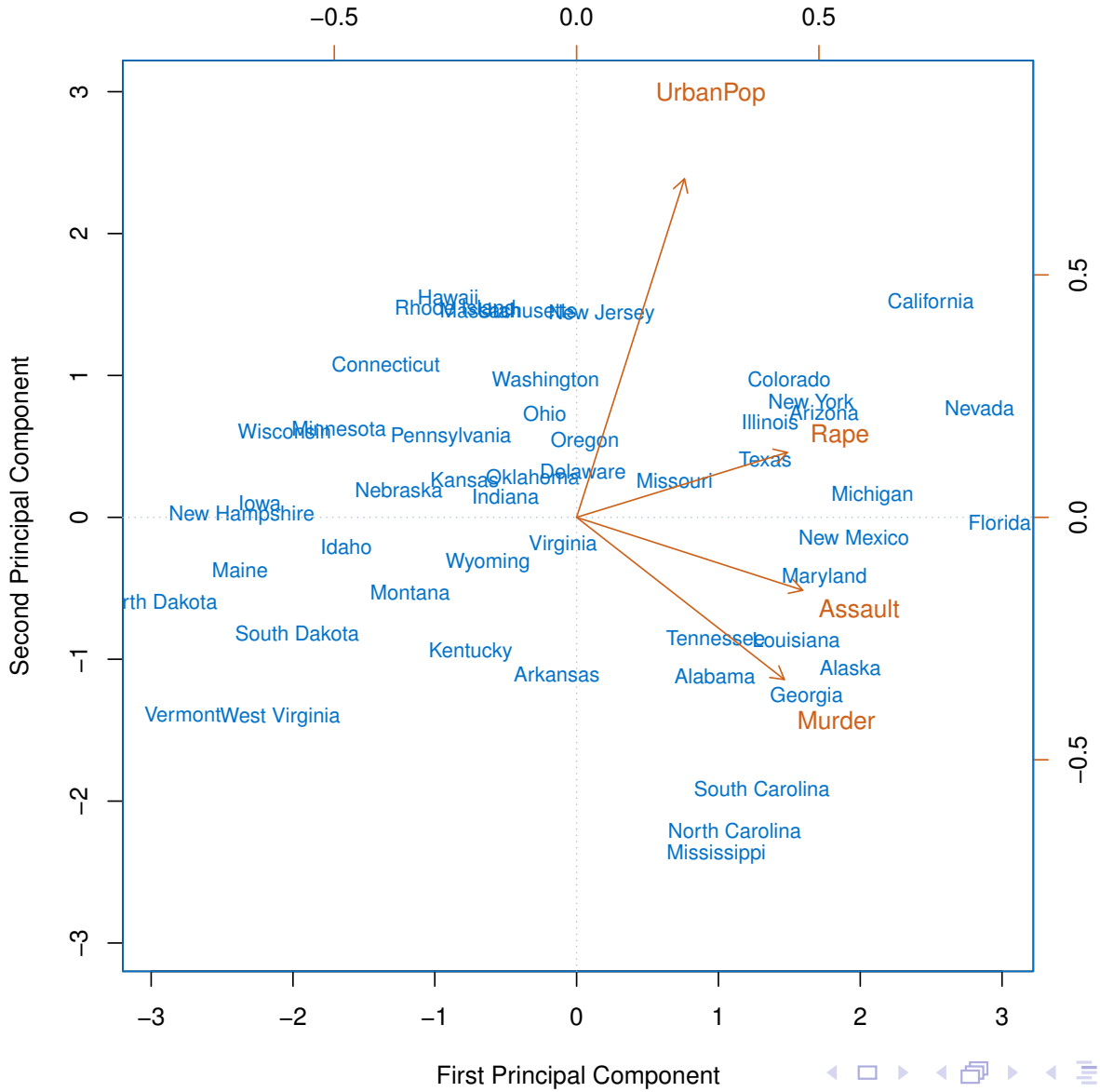
Reference

Tables, Graphics, and Figures from

James et al. (2017): Ch 10.2

USArrests Data



Principal Component Analysis (PCA)

$$Z_1 = \phi_{11} X_1 + \phi_{21} X_2 + \ldots + \phi_{p1} X_p$$

$$z_{i1} = \phi_{11} x_{i1} + \phi_{21} x_{i2} + \ldots + \phi_{p1} x_{ip}$$

$$\underset{\phi_{11},\ldots,\phi_{p1}}{max} \left\{ \frac{1}{n} \sum_{i=1}^{n} \left(\sum_{j=1}^{p} \phi_{j1} x_{ij} \right)^2 \right\}$$

$$\text{subject to } \sum_{j=1}^{p} \phi_{j1}^2 = 1$$

$$z_{i2} = \phi_{12} x_{i1} + \phi_{22} x_{i2} + \ldots + \phi_{p2} x_{ip}$$

Maximum Variance

$$z_1 = Xv_1 = u_1 d_1$$

$$Var(z_1) = \frac{d_1^2}{n}$$

Subsequent Principal Components z_j have maximum variance $\frac{d_j^2}{n}$, subject to being orthogonal to the earlier ones

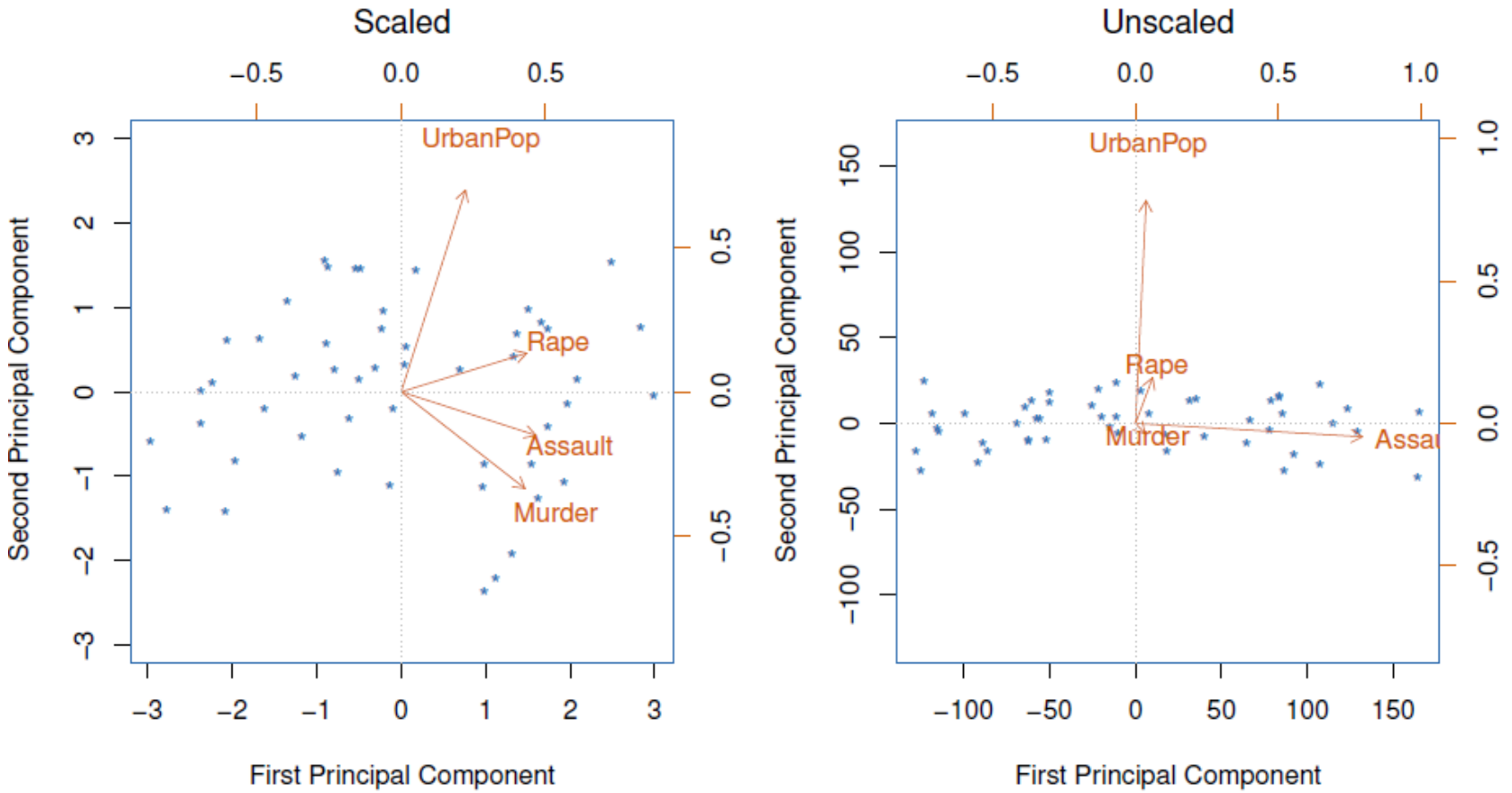
First and Second Principal Component

	PC1	PC2
Murder	0.5358995	−0.4181809
Assault	0.5831836	−0.1879856
UrbanPop	0.2781909	0.8728062
Rape	0.5434321	0.1673186

Scaling the Variables

Assault per 100 people rather per 100,00 people

Variance for Murder, Rape, Assault, and UrbanPop: 18.97, 87.73, 6945.16, and 209.5



Proportion of Variance Explained (PVE)

$$PVE = \frac{\frac{1}{n}\sum_{i=1}^{n} z_{im}^2}{\sum_{j=1}^{p} Var(X_j)}$$

$$\sum_{j=1}^{p} Var(X_j) = \sum_{j=1}^{p} \frac{1}{n} \sum_{i=1}^{n} x_{ij}^2$$

$$\frac{1}{n} \sum_{i=1}^{n} z_{im}^2 = \frac{1}{n} \sum_{i=1}^{n} \left(\sum_{j=1}^{p} \phi_{jm} x_{ij} \right)^2$$

Cumulative Proportion of Variance Explained