13.3) The Central Limit Theorem

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Reference

Tables, Graphics, and Figures from

Computational and Inferential Thinking: The Foundations of Data Science

Adhikari & DeNero (2019): Ch 14.4 The Central Limit Theorem

Ch 14.5 The Variability of the Sample Mean

https://www.inferentialthinking.com/

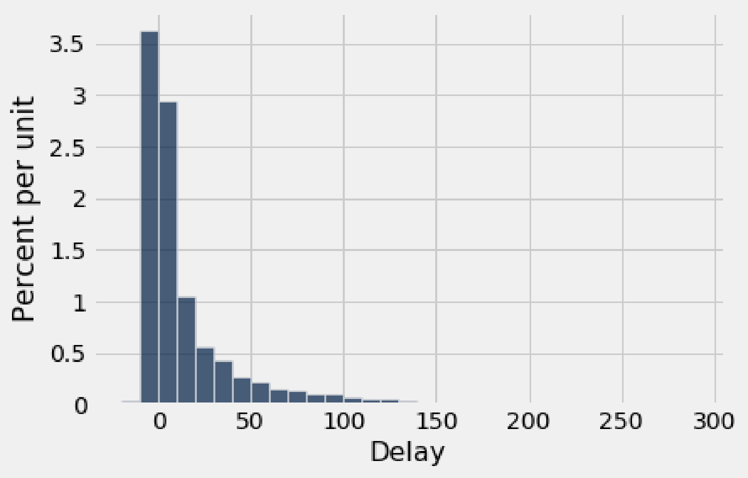
Average Flight Delay

```
from datascience import *
path_data = 'https://github.com/data-8/textbook/raw/gh-pages/data/'
united = Table.read_table(path_data + 'united_summer2015.csv')
```

```
import numpy as np
mean_delay = np.mean(united.column('Delay'))
sd_delay = np.std(united.column('Delay'))
mean_delay, sd_delay
```

```
(16.658155515370705, 39.480199851609314)
```

```
%matplotlib inline
import matplotlib.pyplot as plots
plots.style.use('fivethirtyeight')
united.select('Delay').hist(bins=np.arange(-20, 300, 10))
```



```
delay = united.select('Delay')
np.mean(delay.sample(400).column('Delay'))
```

20.705

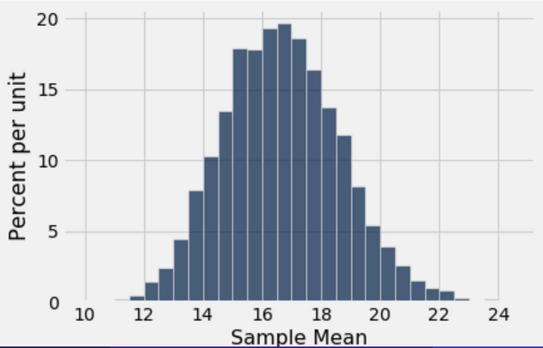
```
sample_size = 400
repetitions = 10000
means = make_array()

for i in np.arange(repetitions):
    sample = delay.sample(sample_size)
    new_mean = np.mean(sample.column('Delay'))
    means = np.append(means, new_mean)

results = Table().with_column(
    'Sample Mean', means)
```

```
results.hist(bins=np.arange(10, 25, 0.5))
```

The Central Limit Theorem: the probability distribution of the sum or average of a large random sample drawn with replacement will be roughly normal, regardless of the distribution of the population from which the sample is drawn



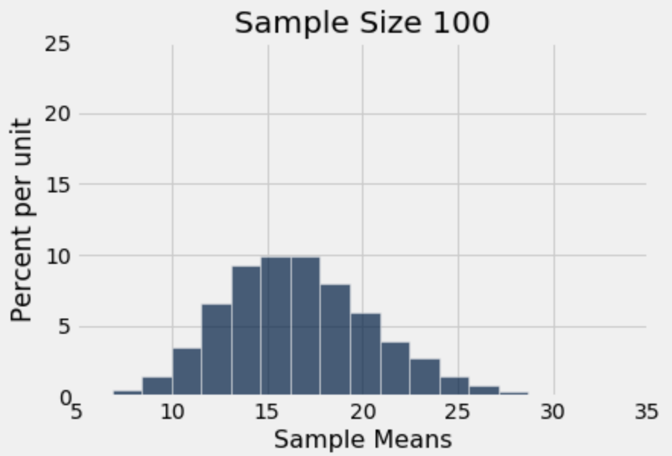
The Variability of the Sample Mean

```
def simulate_sample_mean(table, label, sample_size, repetitions):
    means = make_array()
    for i in range(repetitions):
        new_sample = table.sample(sample_size)
        new_sample_mean = np.mean(new_sample.column(label))
        means = np.append(means, new_sample_mean)

    sample_means = Table().with_column('Sample Means', means)
    sample_means.hist(bins=20)
    plots.xlabel('Sample Means')
    plots.title('Sample Size ' + str(sample_size))
    print("Sample size: ", sample_size)
    print("Population mean:", np.mean(table.column(label)))
    print("Average of sample means: ", np.mean(means))
    print("Population SD:", np.std(table.column(label)))
    print("SD of sample means:", np.std(means))
```

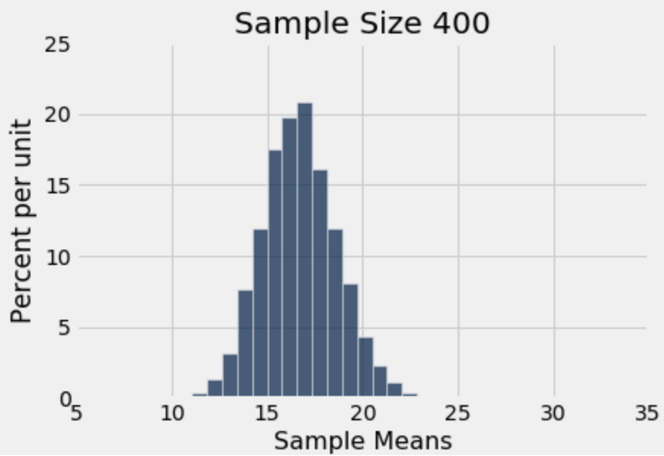
```
simulate_sample_mean(delay, 'Delay', 100, 10000)
plots.xlim(5, 35)
plots.ylim(0, 0.25);
```

```
Population SD: 39.480199851609314
SD of sample means: 3.950422448376376
```



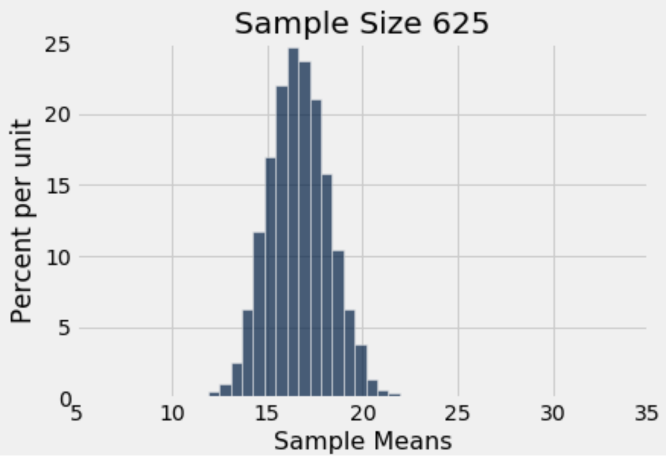
```
simulate_sample_mean(delay, 'Delay', 400, 10000)
plots.xlim(5, 35)
plots.ylim(0, 0.25);
```

```
Population SD: 39.480199851609314
SD of sample means: 1.9561426824492505
```



```
simulate_sample_mean(delay, 'Delay', 625, 10000)
plots.xlim(5, 35)
plots.ylim(0, 0.25);
```

```
Population SD: 39.480199851609314
SD of sample means: 1.5841431400443202
```



The SD of All the Sample Means

```
repetitions = 10000
sample_sizes = np.arange(25, 626, 25)
sd_means = make_array()
for n in sample_sizes:
    means = make_array()
    for i in np.arange(repetitions):
        means = np.append(means,
          np.mean(delay.sample(n).column('Delay')))
    sd_means = np.append(sd_means, np.std(means))

sd_comparison = Table().with_columns(
    'Sample Size n', sample_sizes,
    'SD of 10,000 Sample Means', sd_means,
    'pop_sd/sqrt(n)', pop_sd/np.sqrt(sample_sizes))
```

sd_comparison

Sample Size n	SD of 10,000 Sample Means	pop_sd/sqrt(n)
25	7.8629	7.89604
50	5.55068	5.58334
75	4.63241	4.55878
100	3.95522	3.94802
125	3.55819	3.53122

```
pop_sd = np.std(delay.column('Delay'))
```

39.480199851609314

```
sd_comparison.plot('Sample Size n')
```