1) Probability

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Reference

Tables, Graphics, and Figures from

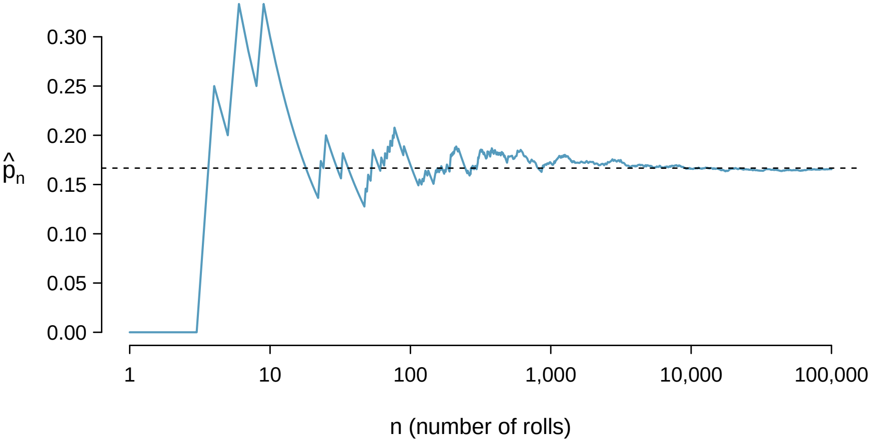
Introductory Statistics with

Randomization and Simulation

Diez et al. (2014): APPENDIX A - Probability

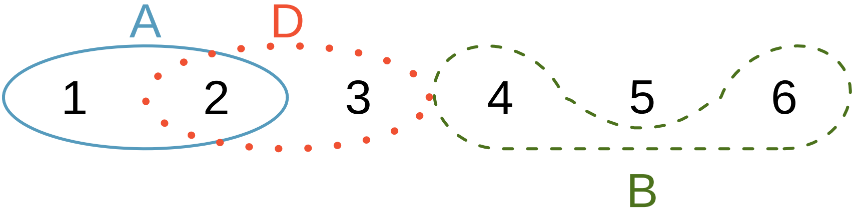
Law of Large Numbers (LLN)

$$p = \frac{1}{6} \cong 0.167$$



Disjoint or Mutually Exclusive Outcomes

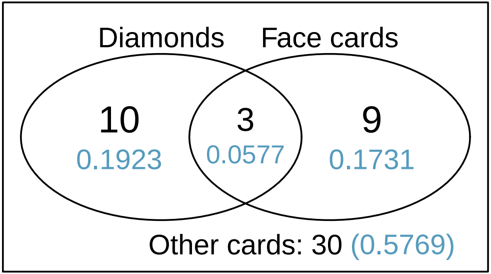
$$A = \{1, 2\};\ B = \{4, 6\};\ D = \{2, 3\}$$



$$P(A \cup B) = P(A) + P(B) = \frac{1}{3} + \frac{1}{3}$$

Probabilities when Events are not Disjoint

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

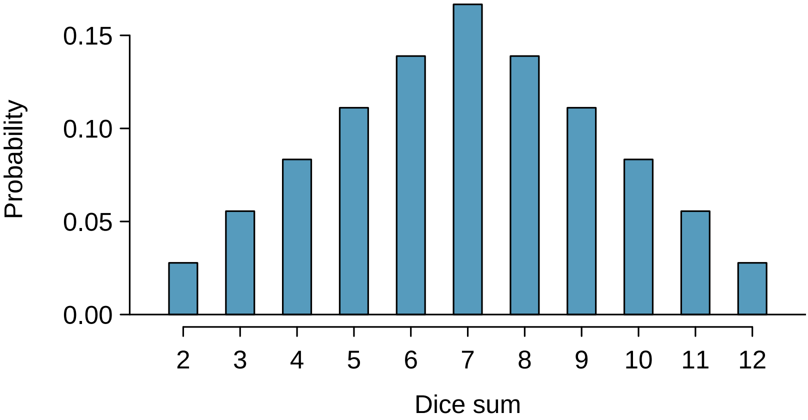


$$P(face.card \cup \diamondsuit)$$

$$P(face.card) + P(\diamondsuit) - P(face.card \cap \diamondsuit)$$

$$\frac{12}{52} + \frac{13}{52} + \frac{3}{52} = 0.423$$

Probability Distribution



Dice sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Independence

$$P(A \cap B) = P(A) \times P(B)$$

$$P(\heartsuit \cap ace) = P(A\heartsuit) = \frac{1}{52}$$

$$P(\heartsuit) \times P(ace)$$

$$\frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$$

Marginal and Joint Probabilities

		inoculated		
		yes	no	Total
result	lived	238	5136	5374
	died	6	844	850
	Total	244	5980	6224

$$P(result = died) = \frac{850}{6224} = 13.7\%$$

$$P(result = died \cap inoculated = yes)$$

$$= \frac{6}{6224} = 0.1\%$$

Conditional Probability

		inoculated		
		yes	no	Total
result	lived	0.0382	0.8252	0.8634
	died	0.0010	0.1356	0.1366
	Total	0.0392	0.9608	1.0000

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(result = died | inoculated = no)$$

$$\frac{P(result = died \cap inoculated = no)}{P(inoculated = no)}$$

$$\frac{0.1356}{0.9608} = 14.1\%$$

Tree Diagrams

$$P(inoculated = yes \ \cap \ result = lived)$$

$$P(inoculated = yes) \times P(result = lived|inoculated = yes)$$

$$= 0.0392 \times 0.9754 = 0.0382$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \times P(B)}{P(B)} = P(A)$$

Let X and Y represent the outcomes of rolling two dice

$$P(Y = 1|X = 1)$$

$$\frac{P(Y=1) \times (X=1)}{P(X=1)} = P(Y = 1)$$