10) Multiple Linear Regression

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January 2018

Reference

Tables, Graphics, and Figures from

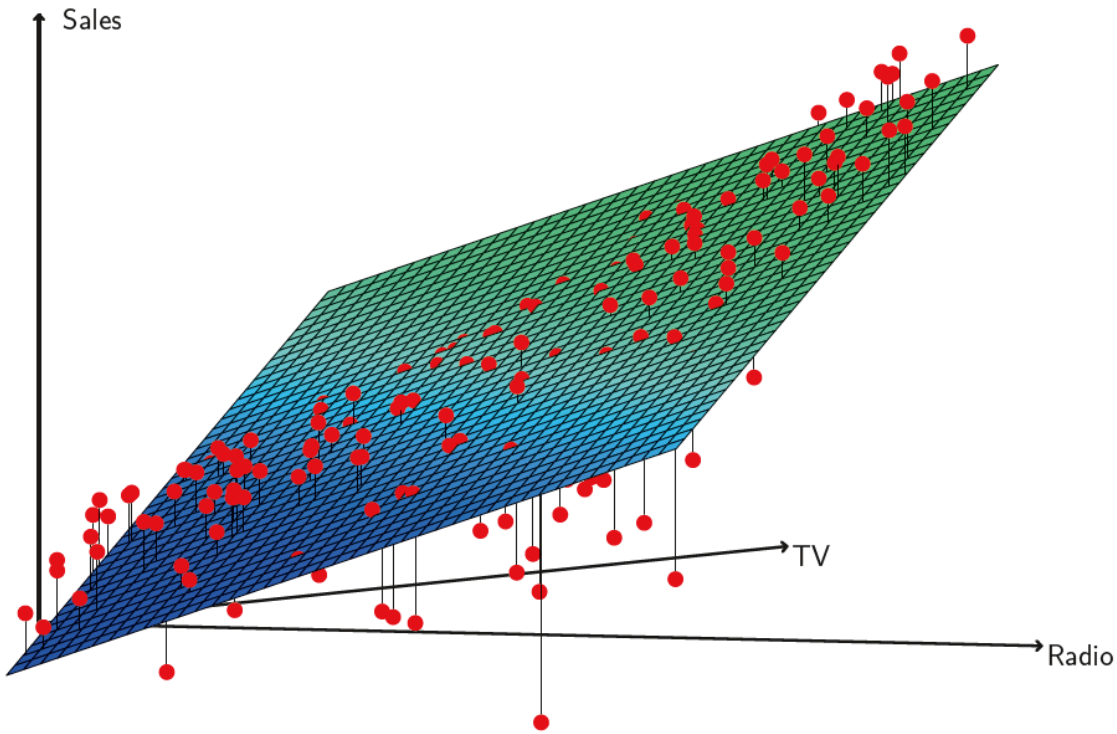
An Introduction to Statistical Learning

James et al. (2017): Chapters: 3.2 and 3.6.3

Multiple Linear Regression

$$Y = \beta_0 + \beta_1 X_1 + ... + \beta_p X_p + \epsilon$$

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2$$



Simple vs Multiple Regression

	Coefficient	Std. error	t-statistic	p-value
Intercept	9.312	0.563	16.54	< 0.0001
radio	0.203	0.020	9.92	< 0.0001

	Coefficient	Std. error	t-statistic	p-value
Intercept	12.351	0.621	19.88	< 0.0001
newspaper	0.055	0.017	3.30	0.00115

	Coefficient	Std. error	t-statistic	p-value
Intercept	2.939	0.3119	9.42	< 0.0001
TV	0.046	0.0014	32.81	< 0.0001
radio	0.189	0.0086	21.89	< 0.0001
newspaper	-0.001	0.0059	-0.18	0.8599

Correlation Matrix

$$r = \frac{cov(x,y)}{s_x s_y}$$

	TV	radio	newspaper	sales
TV	1.0000	0.0548	0.0567	0.7822
radio		1.0000	0.3541	0.5762
newspaper			1.0000	0.2283
sales				1.0000

F-statistic

$$H_0 : \beta_1 = \beta_2 = ... = \beta_p = 0$$

$$H_a : \text{at least one } \beta_j \neq 0$$

$$F = \frac{\frac{TSS - RSS}{p}}{\frac{RSS}{n-p-1}}$$

Quantity	Value
Residual standard error	1.69
R^2	0.897
F-statistic	570

Test if q Coefficients are Zero

$$H_0 : \beta_3 = \beta_4 = \beta_5 = 0$$

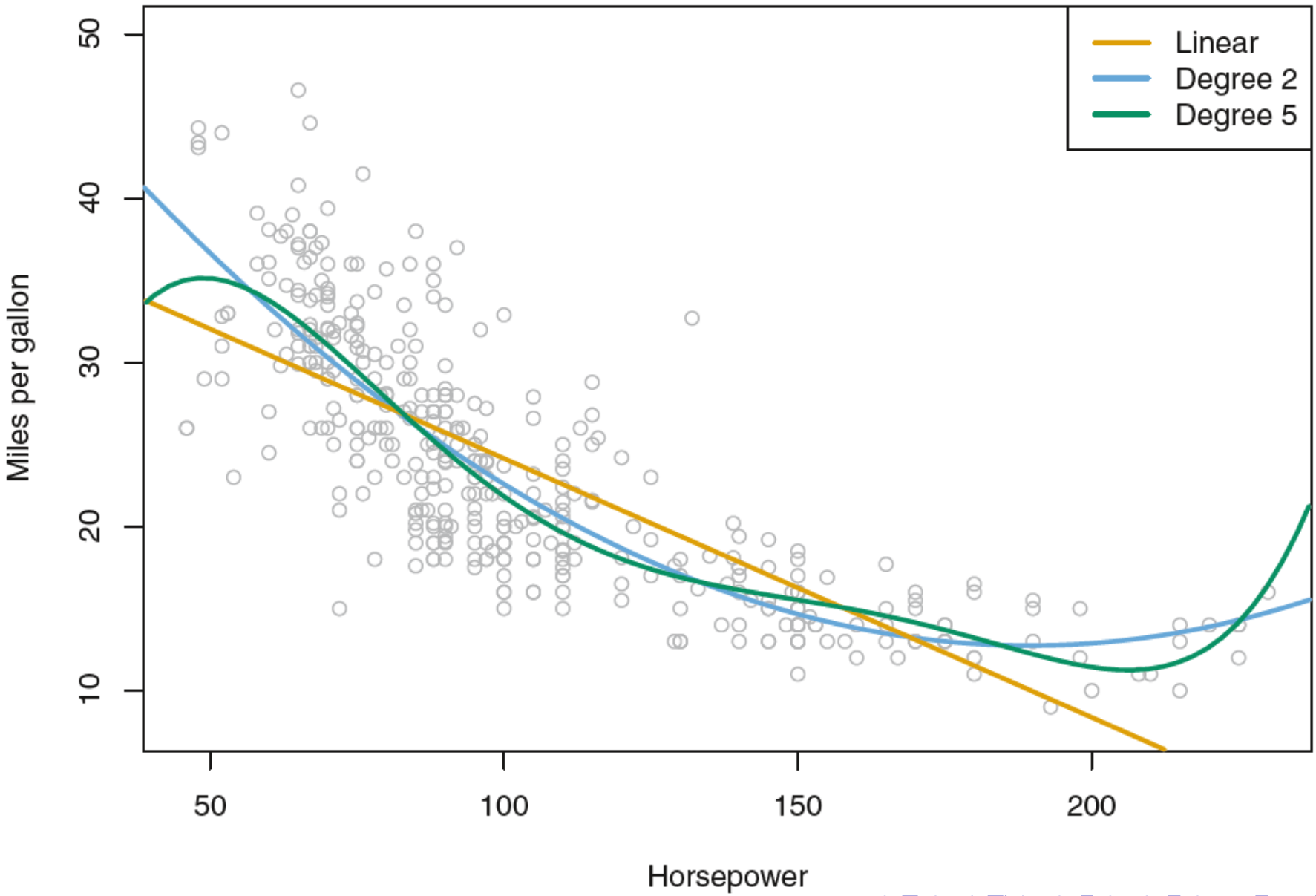
$$F = \frac{\frac{RSS_r - RSS_{ur}}{q}}{\frac{RSS_{ur}}{n-p-1}}$$

Interaction Effect

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \epsilon$$

	Coefficient	Std. error	t-statistic	p-value
Intercept	6.7502	0.248	27.23	< 0.0001
TV	0.0191	0.002	12.70	< 0.0001
radio	0.0289	0.009	3.24	0.0014
TV×radio	0.0011	0.000	20.73	< 0.0001

Auto Data Set



Non-linear Relationships

$$mpg = \beta_0 + \beta_1 hp + \beta_2 hp^2 + \epsilon$$

	Coefficient	Std. error	t-statistic	p-value
Intercept	56.9001	1.8004	31.6	< 0.0001
horsepower	-0.4662	0.0311	-15.0	< 0.0001
horsepower^2	0.0012	0.0001	10.1	< 0.0001

lm.fit=lm(medv~lstat +age,data=Boston)

```
Residuals:
    Min      1Q  Median      3Q     Max
-15.981  -3.978  -1.283   1.968  23.158

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 33.22276    0.73085  45.458  < 2e-16 ***
lstat       -1.03207    0.04819 -21.416  < 2e-16 ***
age          0.03454    0.01223   2.826  0.00491 **
---
Signif. codes:
0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.173 on 503 degrees of freedom
Multiple R-squared:  0.5513,    Adjusted R-squared:  0.5495
F-statistic:   309 on 2 and 503 DF,  p-value: < 2.2e-16
```

lm.fit=lm(medv~.,data=Boston)

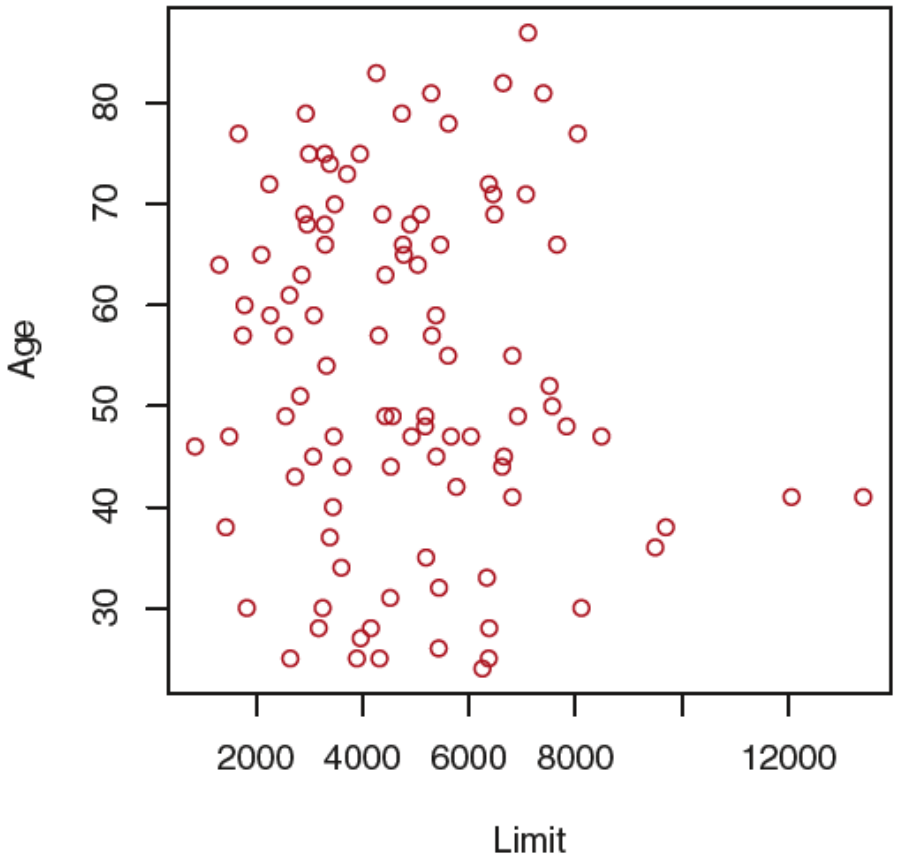
	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	3.646e+01	5.103e+00	7.144	3.28e-12	***
crim	-1.080e-01	3.286e-02	-3.287	0.001087	**
zn	4.642e-02	1.373e-02	3.382	0.000778	***
indus	2.056e-02	6.150e-02	0.334	0.738288	
chas	2.687e+00	8.616e-01	3.118	0.001925	**
nox	-1.777e+01	3.820e+00	-4.651	4.25e-06	***
rm	3.810e+00	4.179e-01	9.116	< 2e-16	***
age	6.922e-04	1.321e-02	0.052	0.958229	
dis	-1.476e+00	1.995e-01	-7.398	6.01e-13	***
rad	3.060e-01	6.635e-02	4.613	5.07e-06	***
tax	-1.233e-02	3.760e-03	-3.280	0.001112	**
ptratio	-9.527e-01	1.308e-01	-7.283	1.31e-12	***
black	9.312e-03	2.686e-03	3.467	0.000573	***
lstat	-5.248e-01	5.072e-02	-10.347	< 2e-16	***

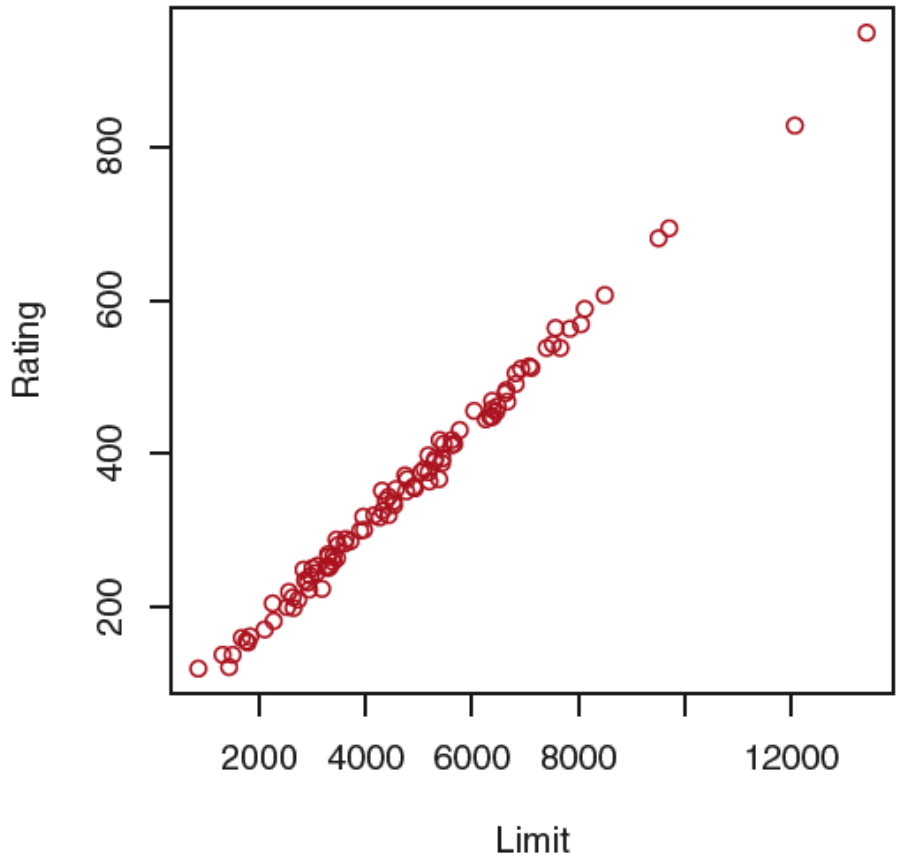
Multicollinearity

Y = balance

		Coefficient	Std. error	t-statistic	p-value
Model 1	Intercept	−173.411	43.828	−3.957	< 0.0001
	age	−2.292	0.672	−3.407	0.0007
	limit	0.173	0.005	34.496	< 0.0001
Model 2	Intercept	−377.537	45.254	−8.343	< 0.0001
	rating	2.202	0.952	2.312	0.0213
	limit	0.025	0.064	0.384	0.7012

Scatterplots: Credit Data Set





Variance Inflation Factor (VIF)

$$VIF(\hat{\beta}_j) = \frac{1}{1 - R^2_{X_j|X_{-j}}}$$

library(car); vif(lm.fit)

```
    crim       zn    indus     chas      nox       rm
1.792192 2.298758 3.991596 1.073995 4.393720 1.933744
     age      dis      rad      tax  ptratio    black
3.100826 3.955945 7.484496 9.008554 1.799084 1.348521
   lstat
2.941491
```

$>$ 5 or 10 indicates a problematic collinearity

summary(lm(medv~lstat*age,data=Boston))

```
Residuals:
    Min      1Q  Median      3Q     Max
-15.806  -4.045  -1.333   2.085  27.552

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 36.0885359  1.4698355  24.553  < 2e-16 ***
lstat       -1.3921168  0.1674555  -8.313 8.78e-16 ***
age         -0.0007209  0.0198792  -0.036   0.9711
lstat:age    0.0041560  0.0018518   2.244   0.0252 *
---
Signif. codes:
0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.149 on 502 degrees of freedom
Multiple R-squared:  0.5557,    Adjusted R-squared:  0.5531
F-statistic: 209.3 on 3 and 502 DF,  p-value: < 2.2e-16
```

lm.fit2=lm(medv~lstat+I(lstat^2))

```
Residuals:
     Min       1Q   Median       3Q      Max
-15.2834  -3.8313  -0.5295   2.3095  25.4148

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 42.862007   0.872084   49.15   <2e-16 ***
lstat       -2.332821   0.123803  -18.84   <2e-16 ***
I(lstat^2)   0.043547   0.003745   11.63   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 5.524 on 503 degrees of freedom
Multiple R-squared:  0.6407,    Adjusted R-squared:  0.6393
F-statistic: 448.5 on 2 and 503 DF,  p-value: < 2.2e-16
```

anova(lm.fit,lm.fit2)

```
Model 1: medv ~ lstat
Model 2: medv ~ lstat + I(lstat^2)
  Res.Df   RSS Df Sum of Sq     F    Pr(>F)
1    504 19472
2    503 15347  1    4125.1 135.2 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```