8) Sharp Regression Discontinuity Design

Vitor Kamada

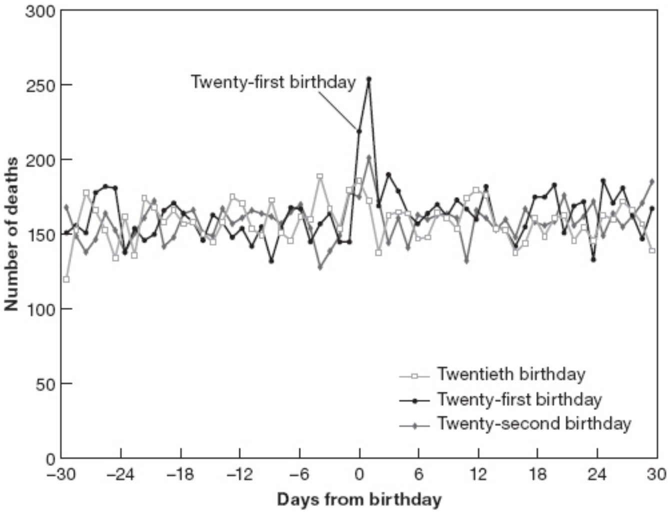
June 2017

Tables, Graphics, and Figures from

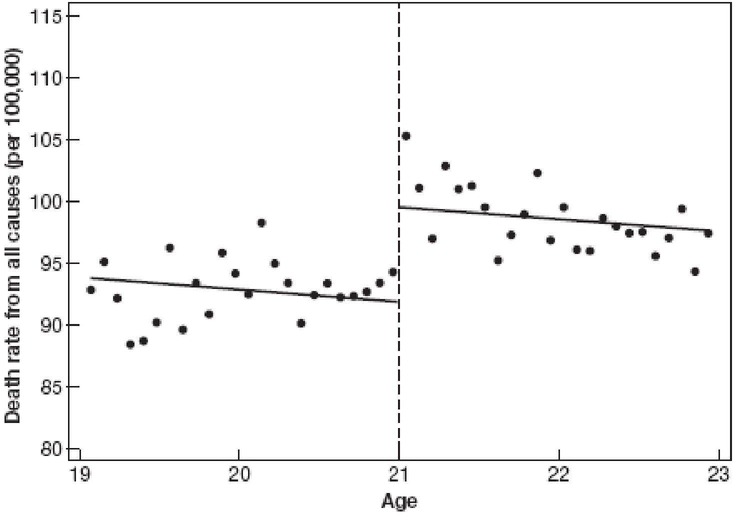
Mastering 'Metrics: The Path from Cause to Effect

Angrist & Pischke (2014): Chapter 4

Birthdays and Funerals from 1997 to 2003



A Sharp RD Estimate of MLDA Mortality Effects



Sharp RD

$$D_a = \begin{cases} 1 \;\; if \;\; a \geq 21 \\ 0 \;\; if \;\; a < 21 \end{cases}$$

$$\bar{M}_a = \alpha + \rho D_a + \gamma a + e_a$$

$\bar{M}_a$: death rate in month a

$$\hat{\rho} = 7.7$$

Carpenter and Dobkin (2011)

```
import os
os.chdir('C:\\Users\\Vitor\\Desktop\\
import pandas as pd
df = pd.read_stata('AEJfigs.dta')
df.head()
```

	agecell	all	allfitted	internal	internalfitted	external
0	19.068493	92.825401	91.706146	16.617590	16.738131	76.207817
1	19.150684	95.100739	91.883720	18.327684	16.920654	76.773056
2	19.232876	92.144295	92.049065	18.911053	17.098843	73.233238
3	19.315069	88.427757	92.202141	16.101770	17.272680	72.325981
4	19.397261	88.704941	92.342918	17.363520	17.442156	71.341415

	externalfitted	alcohol	alcoholfitted	homicide	homicidefitted	\
0	74.968010	0.639138	0.794344	16.316818	16.284573	
1	74.963066	0.677409	0.837575	16.859964	16.270697	
2	74.950226	0.866443	0.877835	15.219254	16.262882	
3	74.929466	0.867308	0.915115	16.742825	16.261148	
4	74.900757	1.019163	0.949407	14.947726	16.265511	

df.describe()

	agecell	all	allfitted	internal	internalfitted	\
count	50.000000	48.000000	50.000000	48.000000	50.000000	
mean	21.000000	95.672722	95.802841	20.285294	20.281301	
std	1.126957	3.831062	3.286415	2.253907	1.994682	
min	19.068493	88.427757	91.706146	15.977087	16.738131	
25%	20.075342	92.785929	93.040606	18.597654	18.674128	
50%	20.999995	95.686272	95.178303	20.288866	20.537065	
75%	21.924658	98.025751	97.786827	21.976349	21.658084	
max	22.931507	105.268349	102.891762	24.372910	24.043783	

	external	externalfitted	alcohol	alcoholfitted	homicide
count	48.000000	50.000000	48.000000	50.000000	48.000000
mean	75.387436	75.521538	1.257337	1.267447	16.912066
std	2.986008	2.269975	0.350312	0.259862	0.729982
min	71.341415	73.157860	0.639138	0.794344	14.947726
25%	73.042023	74.061251	0.996152	1.072381	16.611996
50%	74.813251	74.736385	1.211941	1.247127	16.985353
75%	77.242350	76.063623	1.470119	1.445450	17.288067
max	83.330986	81.783722	2.519309	1.817361	18.410973

Data Manipulation

```
import numpy as np
df['const'] = 1
df['age'] = df['agecell'] - 21
df['over21'] = np.where(df['agecell'] >=21, 1, 0)
df['over21_age'] = df['age']*df['over21']
df['age2'] = df['age']*df['age']
df['over21_age2'] = df['age2']*df['over21']
```

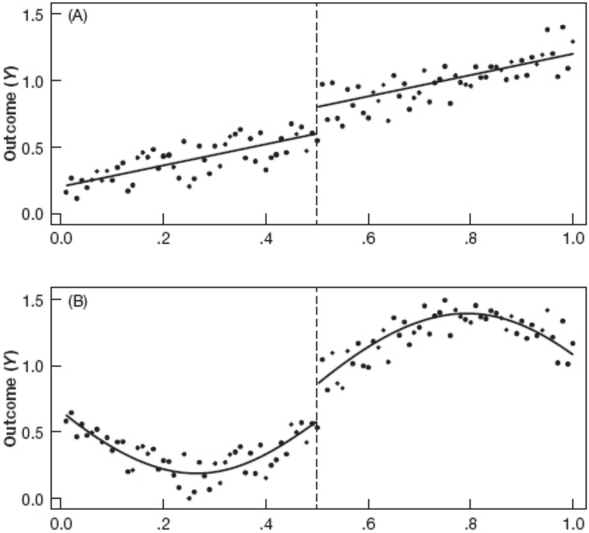
Index	const	age	over21	over21_age	age2	over21_age2
0	1	-1.9315071	0	-0	3.7307198	0
1	1	-1.8493156	0	-0	3.4199684	0
2	1	-1.7671242	0	-0	3.1227279	0
3	1	-1.6849308	0	-0	2.8389919	0

$\bar{M}_a = \alpha + \rho D_a + \gamma a + e_a$

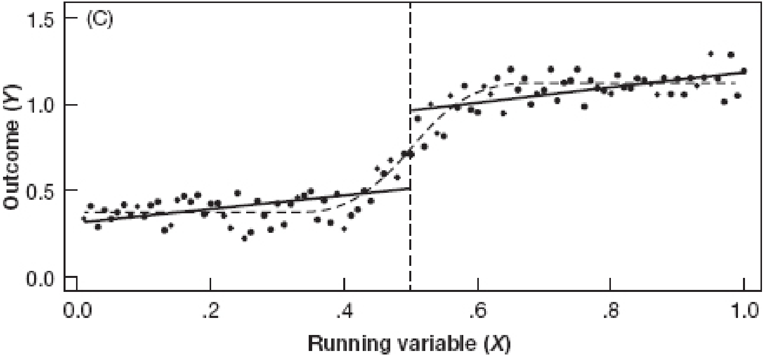
```
import statsmodels.api as sm
result1 = sm.OLS(df['all'], df[['const', 'age', 'over21']],
                    missing='drop').fit()
print(result1.summary())
```

```
Dep. Variable:                    all   R-squared:                       0.595
Model:                            OLS   Adj. R-squared:                  0.577
Method:                 Least Squares   F-statistic:                     32.99
Date:                Sat, 21 Jul 2018   Prob (F-statistic):           1.51e-09
Time:                        19:27:22   Log-Likelihood:                -110.41
No. Observations:                  48   AIC:                             226.8
Df Residuals:                      45   BIC:                             232.4
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         91.8414      0.805    114.083      0.000      90.220      93.463
age           -0.9747      0.632     -1.541      0.130      -2.249       0.299
over21         7.6627      1.440      5.320      0.000       4.762      10.564
```

Valid RD


(A)
Outcome (Y)
1.5
1.0
0.5
0.0
0.0
.2
.4
.6
.8
1.0
(B)
Outcome (Y)
1.5
1.0
0.5
0.0
0.0
.2
.4
.6
.8
1.0

Nonlinear Trend with no Discontinuity



Nonlinearities in an RD

$$\bar{M}_a = \alpha + \rho D_a + \gamma_1 a + \gamma_2 a^2 + e_a$$

$$\bar{M}_a = \alpha + \rho D_a + \gamma(a - a_0) + \delta[(a - a_0)D_a] + e_a$$

$$[\alpha + \rho + (\gamma + \delta)(a - a_0)] - [\alpha + \gamma(a - a_0)]$$

$$\text{TE} = \rho + \delta[(a - a_0)]$$

$\bar{M}_a = \alpha + \rho D_a + \gamma_1 a + \gamma_2 a^2 + e_a$

```
result3 = sm.OLS(df['all'],
                 df[['const', 'age', 'age2', 'over21']],
                 missing='drop').fit()
print(result3.summary())
```

```
Dep. Variable:                    all   R-squared:                       0.657
Model:                            OLS   Adj. R-squared:                  0.634
Method:                 Least Squares   F-statistic:                     28.12
Date:                Sat, 21 Jul 2018   Prob (F-statistic):           2.61e-10
Time:                        19:52:01   Log-Likelihood:                -106.38
No. Observations:                  48   AIC:                             220.8
Df Residuals:                      44   BIC:                             228.2
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         92.9027      0.837    110.994      0.000      91.216      94.590
age           -0.9747      0.588     -1.657      0.105      -2.160       0.211
age2          -0.8187      0.289     -2.835      0.007      -1.401      -0.237
over21         7.6627      1.339      5.721      0.000       4.963      10.362
```

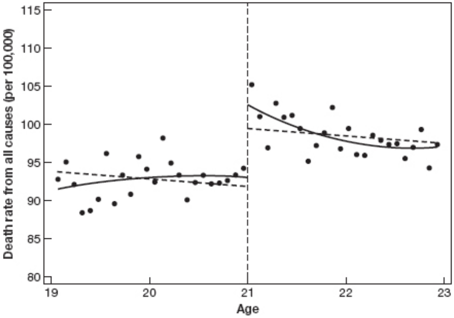
$\bar{M}_a = \alpha + \rho D_a + \gamma(a - a_0) + \delta[(a - a_0)D_a] + e_a$

```
result2 = sm.OLS(df['all'],
                 df[['const', 'age', 'over21', 'over21_age']],
                 missing='drop').fit()
print(result2.summary())
```

```
Dep. Variable:                    all   R-squared:                       0.668
Model:                            OLS   Adj. R-squared:                  0.645
Method:                 Least Squares   F-statistic:                     29.47
Date:                Sat, 21 Jul 2018   Prob (F-statistic):           1.33e-10
Time:                        19:54:34   Log-Likelihood:                -105.64
No. Observations:                  48   AIC:                             219.3
Df Residuals:                      44   BIC:                             226.8
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         93.6184      0.932    100.399      0.000      91.739      95.498
age            0.8270      0.819      1.010      0.318      -0.823       2.477
over21         7.6627      1.319      5.811      0.000       5.005      10.320
over21_age    -3.6034      1.158     -3.111      0.003      -5.937      -1.269
```

Concave to the Left and Convex Thereafter

$$\bar{M}_a = \alpha + \rho D_a + \gamma_1(a - a_0) + \gamma_2(a - a_0)^2 + (\delta_1[(a - a_0)D_a] + \delta_2[(a - a_0)^2 D_a] + e_a$$



TE: $\rho + \delta_1(a - a_0) + \delta_2(a - a_0)^2 = 9.5$

$$\bar{M}_a = \alpha + \rho D_a + \gamma_1(a - a_0) + \gamma_2(a - a_0)^2 + (\delta_1[(a - a_0)D_a] + \delta_2[(a - a_0)^2 D_a] + e_a$$

```python
result4 = sm.OLS(df['all'],
                 df[['const', 'age', 'age2',
                     'over21', 'over21_age', 'over21_age2']],
                 missing='drop').fit()
print(result4.summary())
```

	coef	std err	t	P>|t|	[0.025	0.975]
const	93.0729	1.404	66.301	0.000	90.240	95.906
age	-0.8306	3.290	-0.252	0.802	-7.470	5.809
age2	-0.8403	1.615	-0.520	0.606	-4.100	2.419
over21	9.5478	1.985	4.809	0.000	5.541	13.554
over21_age	-6.0170	4.653	-1.293	0.203	-15.407	3.373
over21_age2	2.9042	2.284	1.271	0.211	-1.706	7.514

Y = Mortality of Motor Vehicle Accidents

```
df1 = df[df['agecell'] >= 20]
df1 = df1[df1['agecell'] <= 22]
result5 = sm.OLS(df1['mva'],
                 df1[['const', 'age', 'age2',
                      'over21', 'over21_age', 'over21_age2']],
                 missing='drop').fit(cov_type='HC1')
print(result5.summary())
```

	coef	std err	z	P>|z|	[0.025	0.975]
const	30.1883	0.562	53.716	0.000	29.087	31.290
age	0.6801	3.816	0.178	0.859	-6.800	8.160
age2	4.4599	4.716	0.946	0.344	-4.783	13.702
over21	5.8925	1.329	4.433	0.000	3.287	8.498
over21_age	-15.1667	6.351	-2.388	0.017	-27.614	-2.720
over21_age2	6.9652	7.053	0.988	0.323	-6.858	20.789

Sharp RD Estimates of MLDA Effects on Mortality

Dependent variable	Ages 19–22 (1)	(2)	Ages 20–21 (3)	(4)
All deaths	7.66	9.55	9.75	9.61
	(1.51)	(1.83)	(2.06)	(2.29)
Motor vehicle accidents	4.53	4.66	4.76	5.89
	(.72)	(1.09)	(1.08)	(1.33)
Suicide	1.79	1.81	1.72	1.30
	(.50)	(.78)	(.73)	(1.14)
Homicide	.10	.20	.16	−.45
	(.45)	(.50)	(.59)	(.93)
Other external causes	.84	1.80	1.41	1.63
	(.42)	(.56)	(.59)	(.75)
All internal causes	.39	1.07	1.69	1.25
	(.54)	(.80)	(.74)	(1.01)
Alcohol-related causes	.44	.80	.74	1.03
	(.21)	(.32)	(.33)	(.41)
Controls	age	age, age^2, interacted with over-21	age	age, age^2, interacted with over-21
Sample size	48	48	24	24

RD Estimates of MLDA Effects on Mortality by Cause of Death