

18) Capital Asset Pricing Model

Vitor Kamada

July 2018

Tables, Graphics, and Figures from
<https://www.quantopian.com/lectures>

Lecture 30 The Capital Asset Pricing Model

Capital Asset Pricing Model (CAPM)

- **Idiosyncratic Risk:** firm-specific, diversified away
- **Systematic Risk:** affects all market participants

$$E[\text{Return}] = \text{Risk-Free Rate of Return} + \text{Risk Premium}$$

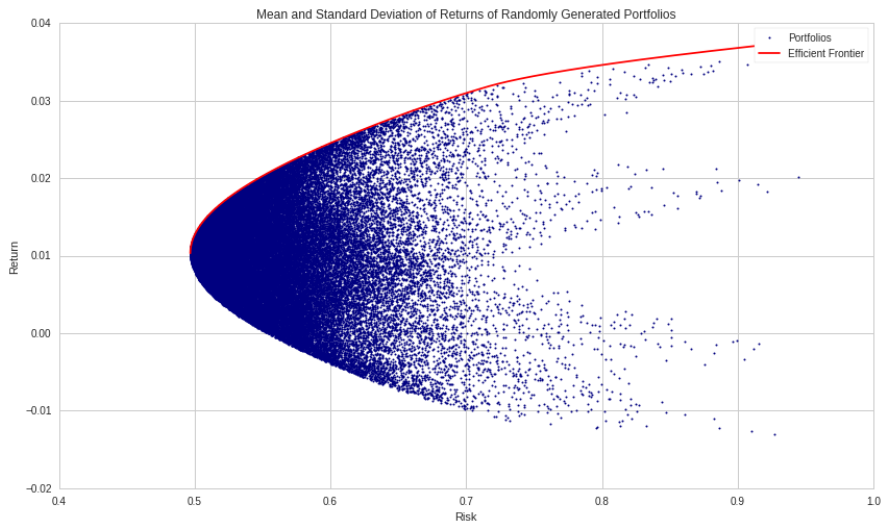
$$\text{Risk Premium of Asset} = \beta(\text{Market Risk Premium})$$

$$E[R_i] = R_F + \beta(E[R_M] - R_F)$$

Investors are:

- 1 able to trade without delay or cost and that everyone is able to borrow or lend money at the risk free rate
- 2 "mean-variance optimizers"

Mean and Standard Deviation of Returns



Sharpe Ratio derived in 1966 by William Sharpe, Nobel Prize in 1990

$$\text{Sharpe Ratio} = \frac{E(R_i) - R_F}{\sigma_i}$$

R_i : return of asset i

R_F : return of a risk-free security

σ_i : standard deviation of R_i

Capital Market Line (CAL)

$$E(R_p) = R_F + \left[\frac{E(R_M) - R_F}{\sigma_M} \right] \sigma_p$$

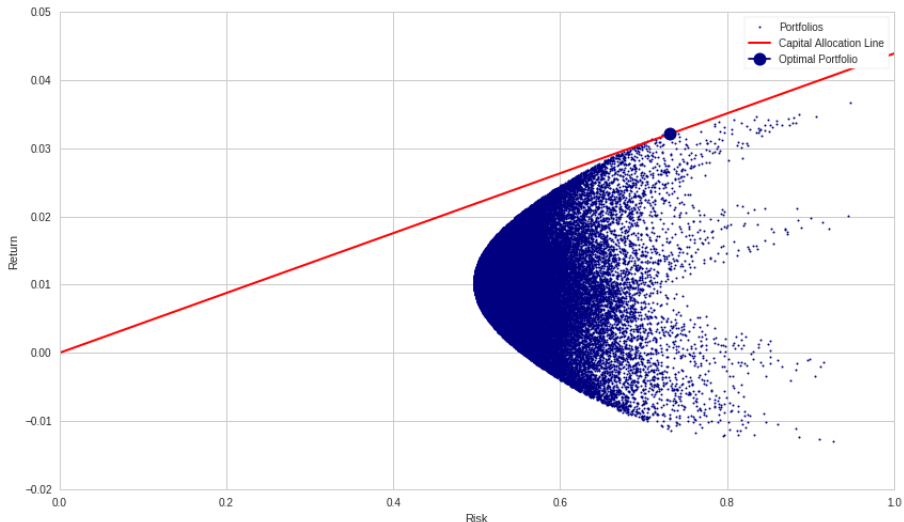
R_p : return of portfolio p

R_M : return of a market portfolio

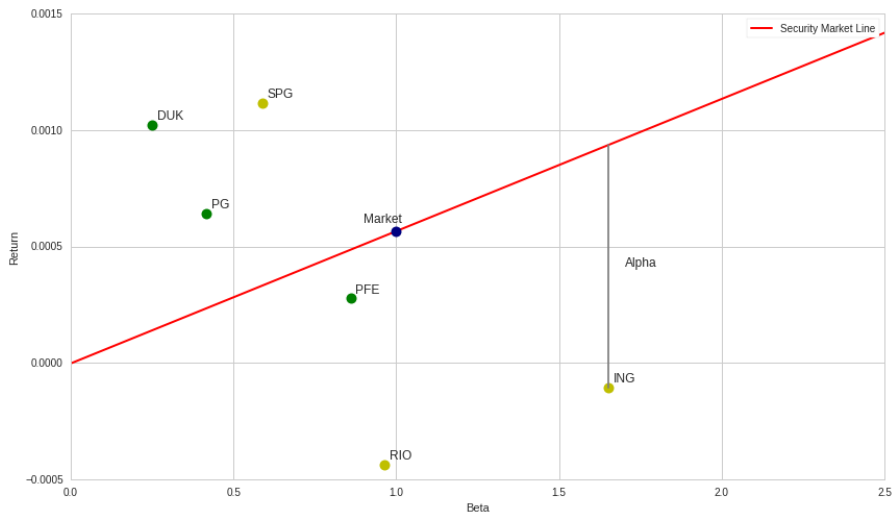
σ_M : standard deviation of R_M

σ_p : standard deviation of R_p

Slope of Capital Allocations Line = Sharpe Ratio



Security Markets Line: $E[R_i] = R_F + \beta_i(E[R_M] - R_F)$



$$E[R_i] = R_F + \beta(E[R_M] - R_F)$$

```
start_date = '2014-01-01'
end_date = '2014-12-31'
# choose stock
R = get_pricing('AAPL', fields='price',
                start_date=start_date, end_date=end_date).pct_change()[1:]

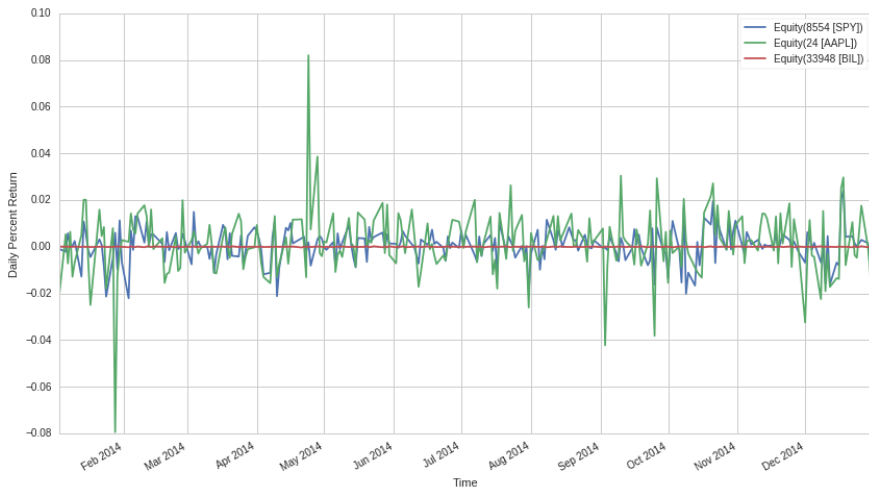
# risk-free proxy
R_F = get_pricing('BIL', fields='price',
                  start_date=start_date, end_date=end_date).pct_change()[1:]

# find it's beta against market
M = get_pricing('SPY', start_date=start_date,
                end_date=end_date, fields='price').pct_change()[1:]

AAPL_results = regression.linear_model.OLS(R-R_F, sm.add_constant(M)).fit()
AAPL_beta = AAPL_results.params[1]

M.plot()
R.plot()
R_F.plot()
plt.xlabel('Time')
plt.ylabel('Daily Percent Return')
plt.legend();
```

Daily Percent Return



AAPL_results.summary()

Dep. Variable:	y	R-squared:	0.192
Model:	OLS	Adj. R-squared:	0.189
Method:	Least Squares	F-statistic:	59.35
Date:	Mon, 28 Aug 2017	Prob (F-statistic):	3.12e-13
Time:	19:11:42	Log-Likelihood:	749.29
No. Observations:	251	AIC:	-1495.
Df Residuals:	249	BIC:	-1488.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P> t 	[95.0% Conf. Int.]
const	0.0010	0.001	1.331	0.185	-0.000 0.003
Equity(8554 [SPY])	0.8438	0.110	7.704	0.000	0.628 1.059

$$\text{predictions} = R_F + AAPL_beta * (M - R_F)$$

```
predictions.plot()  
R.plot(color='Y')  
plt.legend(['Prediction', 'Actual Return'])  
  
plt.xlabel('Time')  
plt.ylabel('Daily Percent Return');
```

