16) Beta Hedging

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Reference

Tables, Graphics, and Figures from

https://www.quantopian.com/lectures

Lecture 31 Beta Hedging

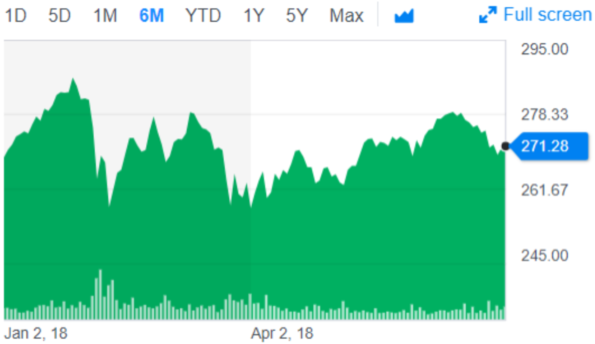
https://finance.yahoo.com/quote/TSLA/

$$Y_{TSLS} = \alpha + \beta X_{SPY} + \epsilon$$

Market Cap	**58.231B**
Beta	**0.91**
PE Ratio (TTM)	**N/A**
EPS (TTM)	**-13.97**
Earnings Date	**Jul 31, 2018 - Aug 6, 2018**
Forward Dividend & Yield	**N/A (N/A)**
Ex-Dividend Date	**N/A**
1y Target Est	**306.24**

SPDR S&P 500 ETF (SPY)

Net Assets	262.44B
NAV	270.74
PE Ratio (TTM)	N/A
Yield	1.80%
YTD Return	1.94%
Beta (3y)	1.00
Expense Ratio (net)	0.09%



Get Data

```
start = '2014-01-01'
end = '2015-01-01'
asset = get_pricing('TSLA', fields='price',
                    start_date=start, end_date=end)
benchmark = get_pricing('SPY', fields='price',
                    start_date=start, end_date=end)

# We have to take the percent changes to get to returns
# Get rid of the first (0th) element because it is NAN
r_a = asset.pct_change()[1:]
r_b = benchmark.pct_change()[1:]

# Let's plot them just for fun
r_a.plot()
r_b.plot()
plt.ylabel("Daily Return")
plt.legend();
```

TSLA and S&P 500 Return

$Y_{TSLS} = \alpha + \beta X_{SPY} + \epsilon$

```
X = r_b.values
Y = r_a.values

def linreg(x,y):
    # This just adds a column of 1s to our data
    x = sm.add_constant(x)
    model = regression.linear_model.OLS(y,x).fit()
    # Remove the constant now that we're done
    x = x[:, 1]
    return model.params[0], model.params[1]

alpha, beta = linreg(X,Y)
print 'alpha: ' + str(alpha)
print 'beta: ' + str(beta)
```

```
alpha: 0.00108062811902
beta: 1.92705010047
```

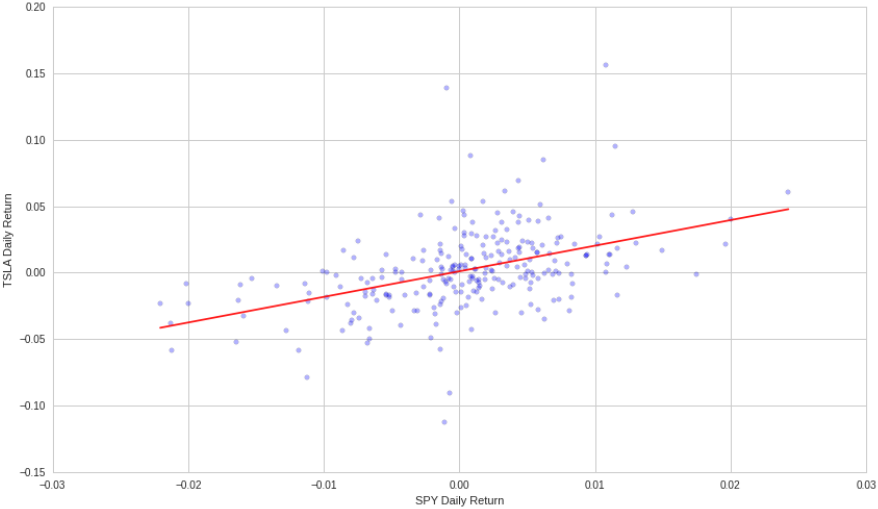
$\hat{Y}_{TSLS} = \alpha + \hat{\beta} X_{SPY} + \epsilon$

```
X2 = np.linspace(X.min(), X.max(), 100)
Y_hat = X2 * beta + alpha

# Plot the raw data
plt.scatter(X, Y, alpha=0.3)
plt.xlabel("SPY Daily Return")
plt.ylabel("TSLA Daily Return")

# Add the regression line, colored in red
plt.plot(X2, Y_hat, 'r', alpha=0.9);
```

$\alpha = 0.001$ and $\beta = 1.927$



Hedging

$$Y_{portfolio} = \alpha + \beta X_{SPY}$$

$$V = \text{Total Value of Portfolio}$$

$$\text{Short SPY: } -\beta V$$

$$\text{New Return:}$$

$$\alpha + \beta X_{SPY} - \beta X_{SPY} = \alpha$$

Implementing Hedging

```python
portfolio = -1*beta*r_b + r_a
portfolio.name = "TSLA + Hedge"

# Plot the returns of the portfolio
# as well as the asset by itself
portfolio.plot(alpha=0.9)
r_b.plot(alpha=0.5);
r_a.plot(alpha=0.5);
plt.ylabel("Daily Return")
plt.legend();
```

Portfolio Return Follows the Asset Fairly Closely

Mean Returns and the Volatilities

```
print "means: ", portfolio.mean(), r_a.mean()
print "volatilities: ", portfolio.std(), r_a.std()
```

```
means:  0.00108062811902 0.00202262496904
volatilities:  0.0272298767724 0.0304875405804
```

```
P = portfolio.values
alpha, beta = linreg(X,P)
print 'alpha: ' + str(alpha)
print 'beta: ' + str(beta)
```

```
alpha: 0.00108062811902
beta: 5.6898930012e-16
```

	Portfolio (TSLA + Hedge)	**TSLA**
Mean Daily Return	0.001	0.002
Volatilities	0.027	0.030
Beta (β)	0	1.927

Different Time Frame

```
start = '2015-01-01'
end = '2015-06-01'
asset = get_pricing('TSLA', fields='price',
                    start_date=start, end_date=end)
benchmark = get_pricing('SPY', fields='price',
                    start_date=start, end_date=end)

r_a = asset.pct_change()[1:]
r_b = benchmark.pct_change()[1:]
X = r_b.values
Y = r_a.values
alpha, beta = linreg(X,Y)
print 'Asset Out of Sample Estimate:'
print 'alpha: ' + str(alpha)
print 'beta: ' + str(beta)
```

```
alpha: 0.00114816439781
beta: 1.04339843544
```

Out of Sample Estimate (historical_beta=1.927)

```
portfolio = -1*historical_beta*r_b + r_a
P = portfolio.values
alpha, beta = linreg(X,P)
print 'Portfolio Out of Sample:'
print 'alpha: ' + str(alpha)
print 'beta: ' + str(beta)
```

```
alpha: 0.00114816439781
beta: -0.883651665033
```

```
portfolio.name = "TSLA + Hedge"
portfolio.plot(alpha=0.9)
r_a.plot(alpha=0.5);
r_b.plot(alpha=0.5)
plt.ylabel("Daily Return")
plt.legend();
```

Summary

$$Y_{TSLS} = \alpha + \beta X_{SPY} + \epsilon$$

	1/1/2014 - 1/1/2015	1/1/2015 - 6/1/2015
α	0.001	0.001
β	1.927	1.043

$$Portfolio = -\beta R_{SPY} + R_{TSLS}$$

$$= -\beta X_{SPY} + \alpha + \beta X_{SPY}$$

	Portfolio ($\beta = 1.927$)	
	1/1/2014 - 1/1/2015	1/1/2015 - 6/1/2015
α	0.001	0.001
β	0	-0.883

Returns of the Portfolio