13) Logit and Probit I

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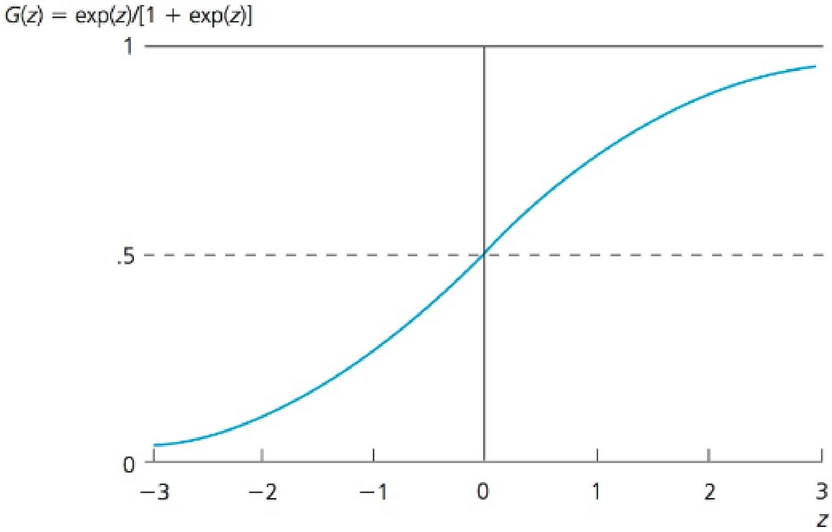
Reference

Tables, Graphics, and Figures from

Introductory Econometrics

Wooldridge, J. M. (2015): Chapter 17 - Limited Dependent Variable Models

Graph of the Logistic Function



Specifying Logit and Probit Models

$$P(y = 1|x) = G(\beta_0 + x\beta)$$

$$0 < G(z) < 1$$

$$G(z) = \frac{e^z}{1+e^z} = \Lambda(z)$$

$$G(z) = \Phi(z) = \int_{-\infty}^{z} \phi(v)dv$$

$$\phi(v) = (2\pi)^{-\frac{1}{2}} exp(-\frac{z^2}{2})$$

Latent Variable Model

$$y^* = \beta_0 + x\beta + e$$

$$y = 1[y^* > 0]$$

$$e \perp x$$

$$e \sim N(0,1) \text{ or } Logistic(0,1)$$

$$\therefore 1 - G(-z) = G(z)$$

Response Probability for y

$$P(y=1|x) = P(y^* > 0|x) = P[e > -(\beta_0 + x\beta)|x]$$

$$= 1 - G[-(\beta_0 + x\beta)] = G(\beta_0 + x\beta)$$

$$\frac{\partial p(x)}{\partial x_j} = g(\beta_0 + x\beta)\beta_j$$

$$g(z) = \frac{dG}{dz}(z) > 0$$

$G(\cdot)$ is a strictly increasing cdf

$$\phi(0) = \frac{1}{\sqrt{2\pi}} \approx .4, \qquad \lambda(0) = \frac{exp(0)}{[1+exp(0)]^2} \approx .25$$

Partial Effect of Continuous Variables on the Response Probability

$$\frac{\partial p(x)}{\partial x_j} = g(\beta_0 + x\beta)\beta_j$$

$$\frac{\partial p(x)}{\partial x_j} \Big/ \frac{\partial p(x)}{\partial x_h} = \frac{\beta_j}{\beta_h}$$

Binary: $\frac{\Delta p(x)}{\Delta x_1} = G(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k)$

$$-G(\beta_0 + \beta_2 x_2 + \ldots + \beta_k x_k)$$

Maximum Likelihood Estimation (MLE)

$$f(y|x_i;\beta) = [G(x_i\beta)]^y[1 - G(x_i\beta)]^{1-y}$$

$$\ell_i(\beta) = y_i log[G(x_i\beta)] + (1 - y_i) log[1 - G(x_i\beta)]$$

$$\mathcal{L}(\beta) = \sum_{i=1}^{n} \ell_i(\beta)$$

Pseudo R-squared

$$\text{Pseudo } R^2 = 1 - \frac{\mathcal{L}_{ur}}{\mathcal{L}_0}$$

$\mathcal{L}_0$ the model with only an intercept

$$|\mathcal{L}_{ur}| \leq |\mathcal{L}_0|$$

$$\tilde{y}_i = 1 \text{ if } G(\hat{\beta}_0 + x_i\hat{\beta}) \geq .5$$

$$\tilde{y}_i = 0 \text{ if } G(\hat{\beta}_0 + x_i\hat{\beta}) < .5$$

	$(y_i, \tilde{y}_i)$	
Correct Prediction	0,0	1,1
Incorrect Prediction	0,1	1,0

The % correctly predicted is the % of times that $y_i = \tilde{y}_i$

Table 17.1 Estimates of Labor Force Participation

Independent Variables	LPM (OLS)	Logit (MLE)	Probit (MLE)
nwifeinc	−.0034 (.0015)	−.021 (.008)	−.012 (.005)
educ	.038 (.007)	.221 (.043)	.131 (.025)
exper	.039 (.006)	.206 (.032)	.123 (.019)
$exper^2$	−.00060 (.00019)	−.0032 (.0010)	−.0019 (.0006)
age	−.016 (.002)	−.088 (.015)	−.053 (.008)
kidslt6	−.262 (.032)	−1.443 (.204)	−.868 (.119)
kidsge6	.013 (.014)	.060 (.075)	.036 (.043)
constant	.586 (.152)	.425 (.860)	.270 (.509)
Percentage correctly predicted	73.4	73.6	73.4
Log-likelihood value	—	−401.77	−401.30
Pseudo R-squared	.264	.220	.221

Partial Effect at the Average (PEA)

$$\Delta \hat{P}(y = 1|x) \approx [g(\hat{\beta}_0 + x\hat{\beta})\hat{\beta}_j]\Delta x_j$$

$$g(\hat{\beta}_0 + \bar{x}\hat{\beta}) = g(\hat{\beta}_0 + \hat{\beta}_1\bar{x}_1 + \hat{\beta}_2\bar{x}_2 + \ldots + \hat{\beta}_k\bar{x}_k)$$

Partial Effect of x_j for the "average" person in the sample.

Average Partial Effect (APE) or Average Marginal Effect (AME)

$$n^{-1} \sum_{i=1}^{n} [g(\hat{\beta}_0 + x_i\hat{\beta})\hat{\beta}_j]$$

AME is the average of the nonlinear function rather than the nonlinear function of the average

$$g[E(x\beta)] \neq E[g(x\beta)]$$

Table 17.2 Average Partial Effects for the Labor Force Participation Models

Independent Variables	LPM	Logit	Probit
nwifeinc	−.0034 (.0015)	−.0038 (.0015)	−.0036 (.0014)
educ	.038 (.007)	.039 (.007)	.039 (.007)
exper	.027 (.002)	.025 (.002)	.026 (.002)
age	−.016 (.002)	−.016 (.002)	−.016 (.002)
kidslt6	−.262 (.032)	−.258 (.032)	−.261 (.032)
kidsge6	.013 (.014)	.011 (.013)	.011 (.013)

Figure 17.2 Estimated Response Probabilities with Respect to Education for the Linear Probability and Probit Models