23) Bayesian Structural Time-Series Model

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April 2019

Reference

Tables, Graphics, and Figures from:

Brodersen et al. (2015). **Inferring causal impact using Bayesian structural time-series models**. Annals of Applied Statistics: Vol. 9, No. 1, 247–274.

Bayesian Structural Time-Series Model

Observation Eq: $y_t = Z_t^T \alpha_t + \epsilon_t$

State Eq: $\alpha_{t+1} = T_t \alpha_t + R_t \eta_t$

$$\begin{matrix} \epsilon_t \sim N(0, \sigma_t^2) \\ \eta_t \sim N(0, Q_t) \end{matrix} \perp \text{all other unknowns}$$

Structural Parameters: Z_t, T_t, R_t

Local Level Model

$\alpha_t = \mu_t$, and $Z_t, T_t, R_t = 1$

$$y_t = \mu_t + \epsilon_t$$

$$\mu_{t+1} = \mu_t + \eta_t$$

$$\epsilon_t \sim N(0, \sigma_t^2) \text{ and } \eta_t \sim N(0, \tau_t^2)$$

	Best Estimator of y_{t+1}	**Model**
$\sigma_t^2 = 0$	y_t	Random Walk
$\tau_t^2 = 0$	$y_1, \ldots, y_t$	IID Gaussian Noise

Assembling the State-Space Model

$$y_t = \mu_t + \gamma_t + \sum_{j=1}^{J} x_{j,t}\beta_{j,t} + \epsilon_t$$

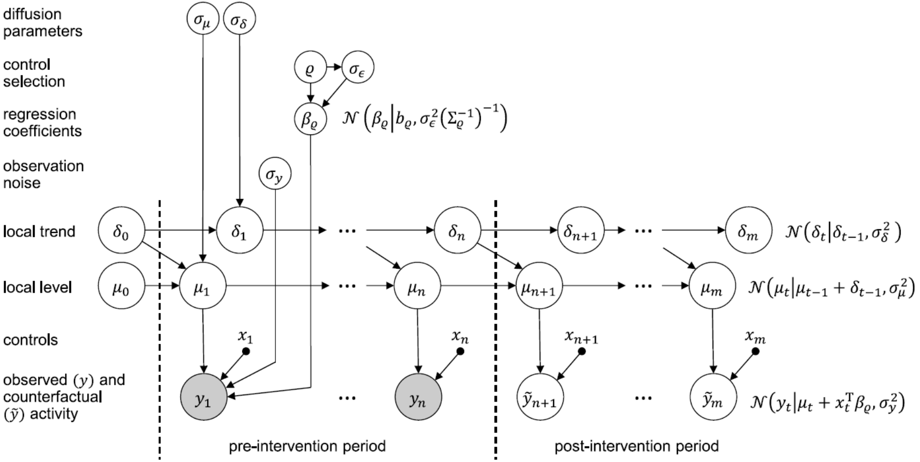
Trend: $\mu_{t+1} = \mu_t + \delta_t + \eta_{\mu,t}$

"Slope" of the trend: $\delta_{t+1} = \delta_t + \eta_{\delta,t}$

Seasonality: $\gamma_{t+1} = -\sum_{s=0}^{S-2} \gamma_{t-s} + \eta_{\gamma,t}$

Dynamic Coefficients: $\beta_{j,t+1} = \beta_{j,t} + \eta_{\beta,j,t}$

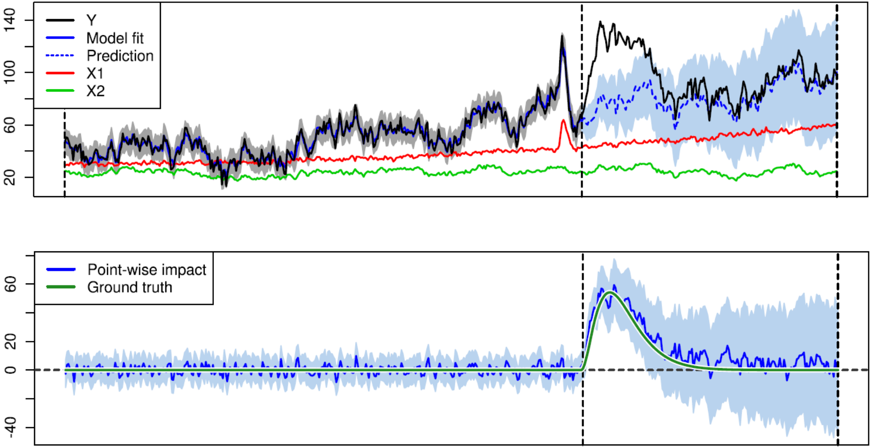
Static-Regression Variant of the State-Space Model



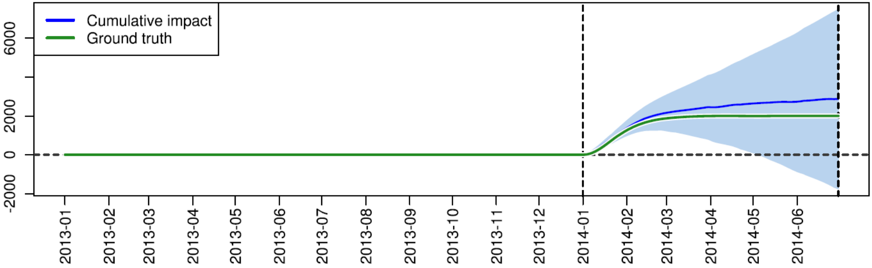
Bayesian Framework for Synthetic Control

- Time-series behaviour of the Y_t, prior to the intervention
- Behaviour of other time series $(X_1, X_2, ..)$ that were predictive of Y_t prior to the intervention
- Prior knowledge about the model parameters, ex: previous studies

Counterfactual Predictions



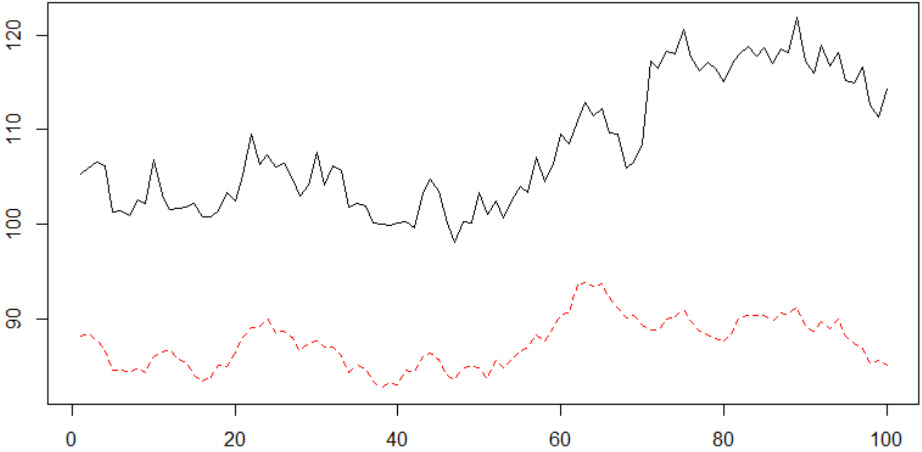
Cumulative Impact



Toy Example

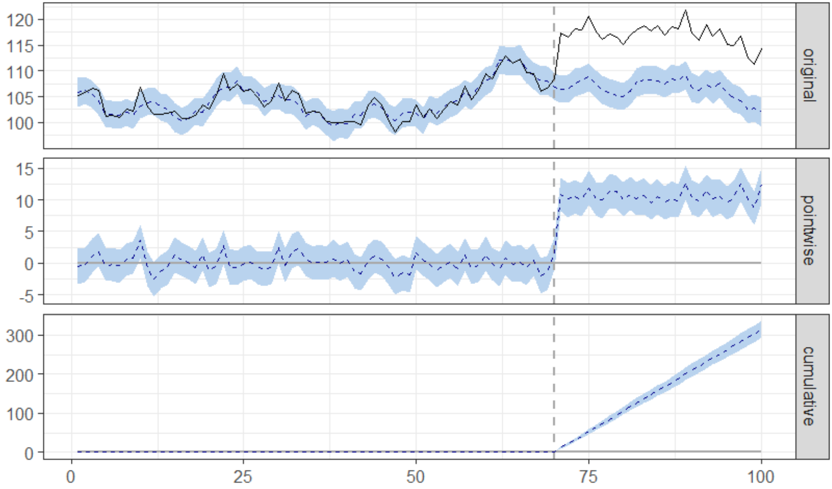
```
library(CausalImpact)
set.seed(1)
x1 <- 100 + arima.sim(model = list(ar = 0.999),
n = 100)
y <- 1.2 * x1 + rnorm(100)
y[71:100] <- y[71:100] + 10
data <- cbind(y, x1)
pre.period <- c(1, 70)
post.period <- c(71, 100)
```

matplot(data, type = "l")



impact<-CausalImpact(data, pre.period, post.period)

plot(impact)



summary(impact)

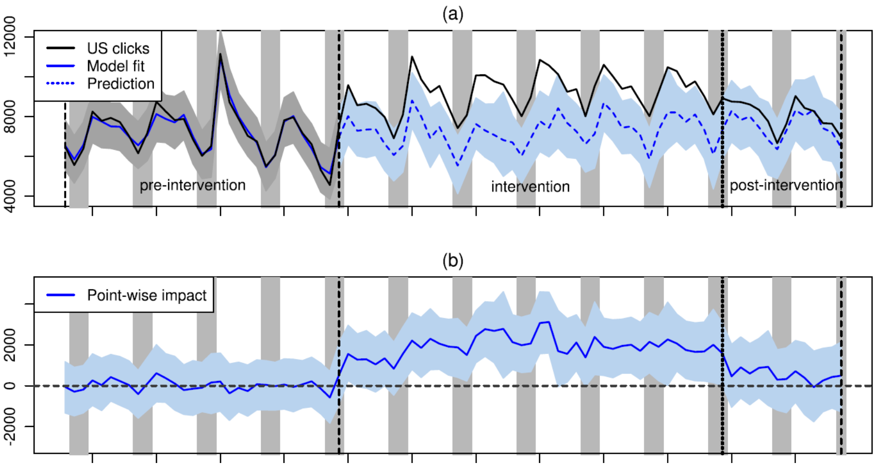
```
                          Average        Cumulative
Actual                    117            3511
Prediction (s.d.)         107 (0.38)     3196 (11.27)
95% CI                    [106, 107]     [3174, 3218]

Absolute effect (s.d.)    11 (0.38)      316 (11.27)
95% CI                    [9.8, 11]      [293.5, 337]

Relative effect (s.d.)    9.9% (0.35%)   9.9% (0.35%)
95% CI                    [9.2%, 11%]    [9.2%, 11%]

Posterior tail-area probability p:   0.001
Posterior prob. of a causal effect:  99.9%
```

Vaver and Koehler (2011)

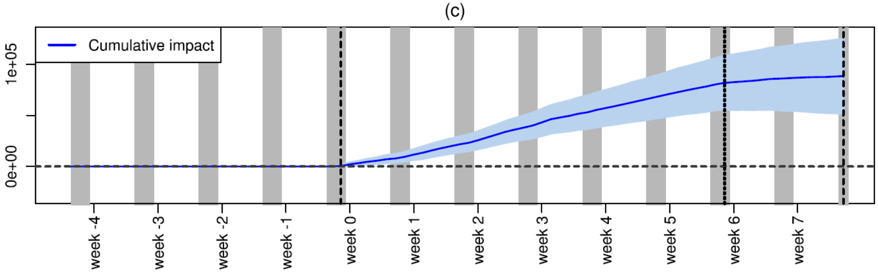
Campaign's causal effect on the # of times a user was directed to the advertiser's website from the Google search results page

- Ads went live for 6 consecutive weeks
- Ads were geo-targeted to a randomised set of 95 out of 190 designated market areas (DMAs)

Causal Effect of Online Advertising on Clicks in Treated Regions using Randomised Controls



Cumulative Impact of the Campaign on Clicks using Randomised Controls

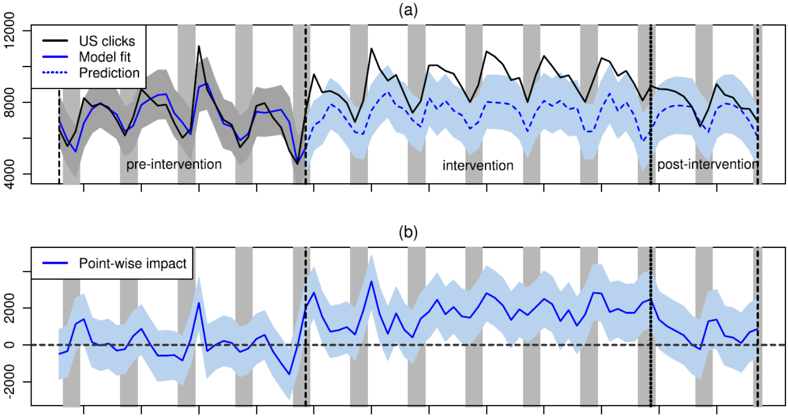


	Experiment	BSTS
ATET	84,700	88,400
Co.I/Cr.I	[19%,22%]	[13%,30%]
Relative Effect	21%	22%

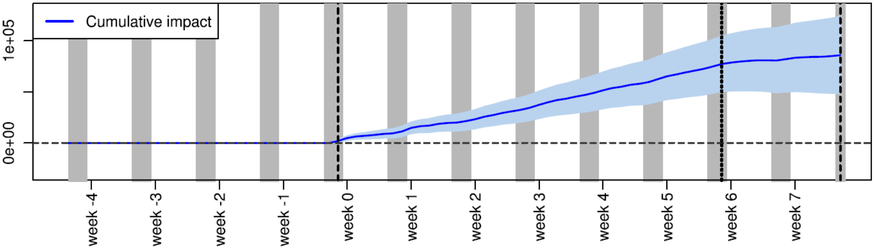
Causal Effect of Online Advertising on Clicks in Treated Regions using Observational Controls

Data from all control regions were discarded

Searches for keywords related to the advertiser's industry were used as controls (https://trends.google.com/trends)



Cumulative Impact of the Campaign on Clicks using Observational Controls

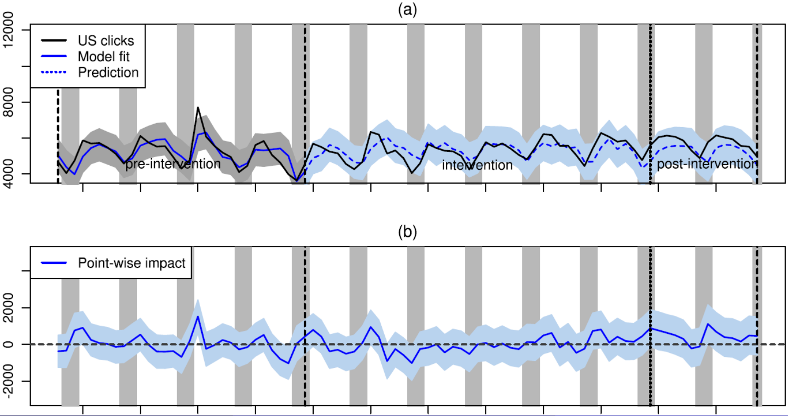


	Experiment	BSTS
ATET	84,700	85,900
Co.I/Cr.I	[19%,22%]	[12%,30%]
Relative Effect	21%	21%

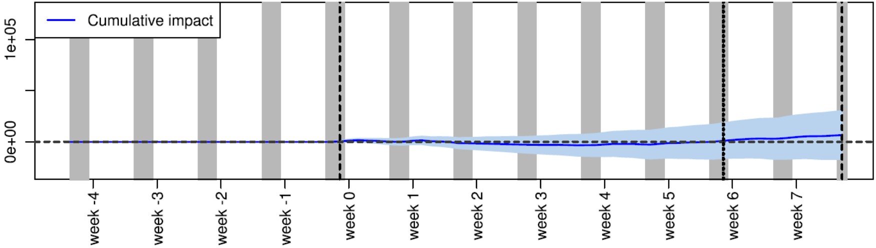
Causal Effect of Online Advertising on Clicks in Nontreated Regions

Data from all treated regions were discarded

Searches for keywords related to the advertiser's industry were used as controls (https://trends.google.com/trends)



Cumulative Impact of the Campaign on Clicks in Nontreated Regions



	Experiment	BSTS
ATET	84,700	-
Co.I/Cr.I	[19%,22%]	[-6%,10%]
Relative Effect	21%	2%