6) Fixed Effects (FE) and Random Effects (RE)

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December 2018

Reference

Tables, Graphics, and Figures from:

Wooldridge (2010). **Econometric Analysis of Cross Section and Panel Data.** Ch 10.

Fixed Effects (Time-Demeaned)

$$y_{it} = x_{it}\beta + c_i + u_{it} \quad \textbf{(1)}$$

$$\bar{y}_i = \bar{x}_i\beta + c_i + \bar{u}_i \quad \textbf{(2)}$$

$$y_{it} - \bar{y}_i = (x_{it} - \bar{x}_i)\beta + u_{it} - \bar{u}_i$$

$$\ddot{y}_{it} = \ddot{x}_{it}\beta + \ddot{u}_{it}$$

Fixed Effects (Estimation)

$$\ddot{y}_{it} = \ddot{x}_{it}\beta + \ddot{u}_{it}$$

FE.1: $E(u_{it}|x_i, c_i) = 0, \quad t = 1, 2, ..., T$

FE.2: $rank[\sum_{t=1}^{T} E(\ddot{x}_i'\ddot{x}_i)] = K$

$$\hat{\beta}_{FE} = (\sum_{i=1}^{N}\sum_{t=1}^{T} \ddot{x}_{it}'\ddot{x}_{it})^{-1}(\sum_{i=1}^{N}\sum_{t=1}^{T} \ddot{x}_{it}'\ddot{y}_{it})$$

Fixed Effects (Inference)

FE.3: $E(u_i u_i' | x_i, c_i) = \sigma_u^2 I_T$

$$Avar(\hat{\beta}_{FE}) = \hat{\sigma}_u^2 (\sum_{i=1}^{N} \sum_{t=1}^{T} \ddot{x}_{it}' \ddot{x}_{it})^{-1}$$

$$Avar(\hat{\beta}_{FE}) = (\ddot{X}'\ddot{X})^{-1} [\sum_{i=1}^{N} \ddot{X}_i' \hat{\ddot{u}}_i \hat{\ddot{u}}_i' \ddot{X}_i] (\ddot{X}'\ddot{X})^{-1}$$

FE (Dummy Variable Regression)

$$y_{it} \text{ on } d1_i, d2_i, ... dN_i, x_{it},$$

$$t = 1, 2, ..., T; \; i = 1, 2, ..., N$$

Random Effects (RE.1)

$$y_{it} = x_{it}\beta + c_i + u_{it}$$

a) $E(u_{it}|x_i, c_i) = 0,\ t = 1, \ldots T$

b) $E(c_i|x_i) = E(c_i) = 0$, where $x_i = (x_{i1}, x_{i2}, \ldots x_{it})$

Random Effects (RE.2)

$$\text{rank } E(X_i'\Omega^{-1}X_i) = K$$

$$\hat{\beta}_{RE} = (\sum_{i=1}^{N} X_i'\hat{\Omega}^{-1}X_i)^{-1}(\sum_{i=1}^{N} X_i'\hat{\Omega}^{-1}y_i)$$

$$v_{it} = c_i + u_{it}$$

$$\Omega = E(v_i v_i') = \begin{pmatrix} \sigma_c^2 + \sigma_u^2 & \sigma_c^2 & \cdots & \sigma_c^2 \\ \sigma_c^2 & \sigma_c^2 + \sigma_u^2 & \cdots & \vdots \\ \vdots & & \ddots & \vdots \\ \sigma_c^2 & & & \sigma_c^2 + \sigma_u^2 \end{pmatrix}$$

Random Effects (RE.3)

$$\text{a) } E(u_i u_i' | x_i, c_i) = \sigma_u^2 I_T,$$

$$\text{b) } E(c_i^2 | x_i) = \sigma_c^2$$

$$v_{it} = c_i + u_{it}$$

$$Corr(v_{it}, v_{is}) = \frac{\sigma_c^2}{\sigma_c^2 + \sigma_u^2}, \; t \neq s$$

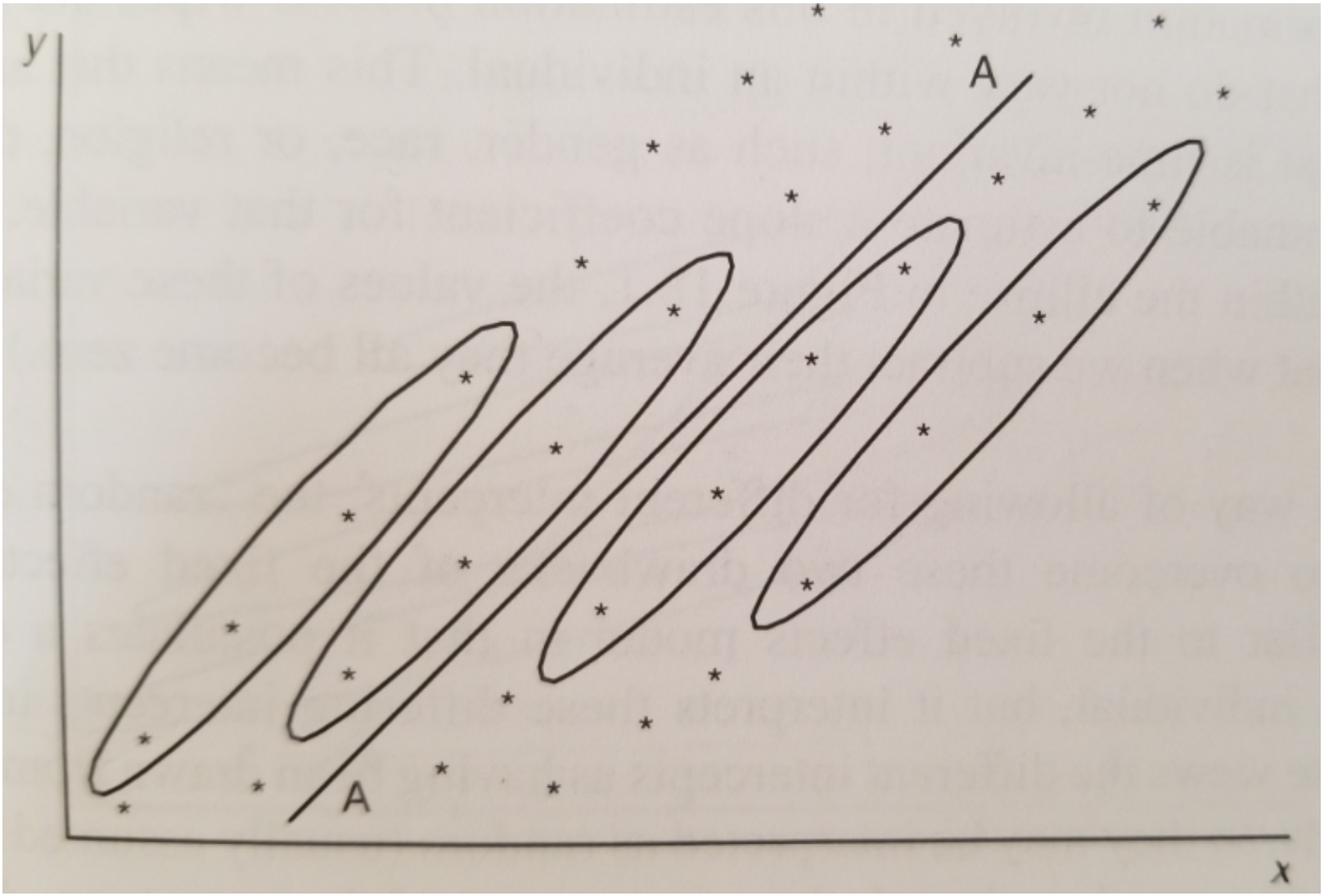
$$\frac{E(v_{it}, v_{is})}{E(v_{it}^2)} = \frac{E[(c_i + u_{it})(c_i + u_{is})]}{E(c_i^2) + 2E(c_i u_{it}) + E(u_{it}^2)}$$

Relationship between the RE and FE Estimators

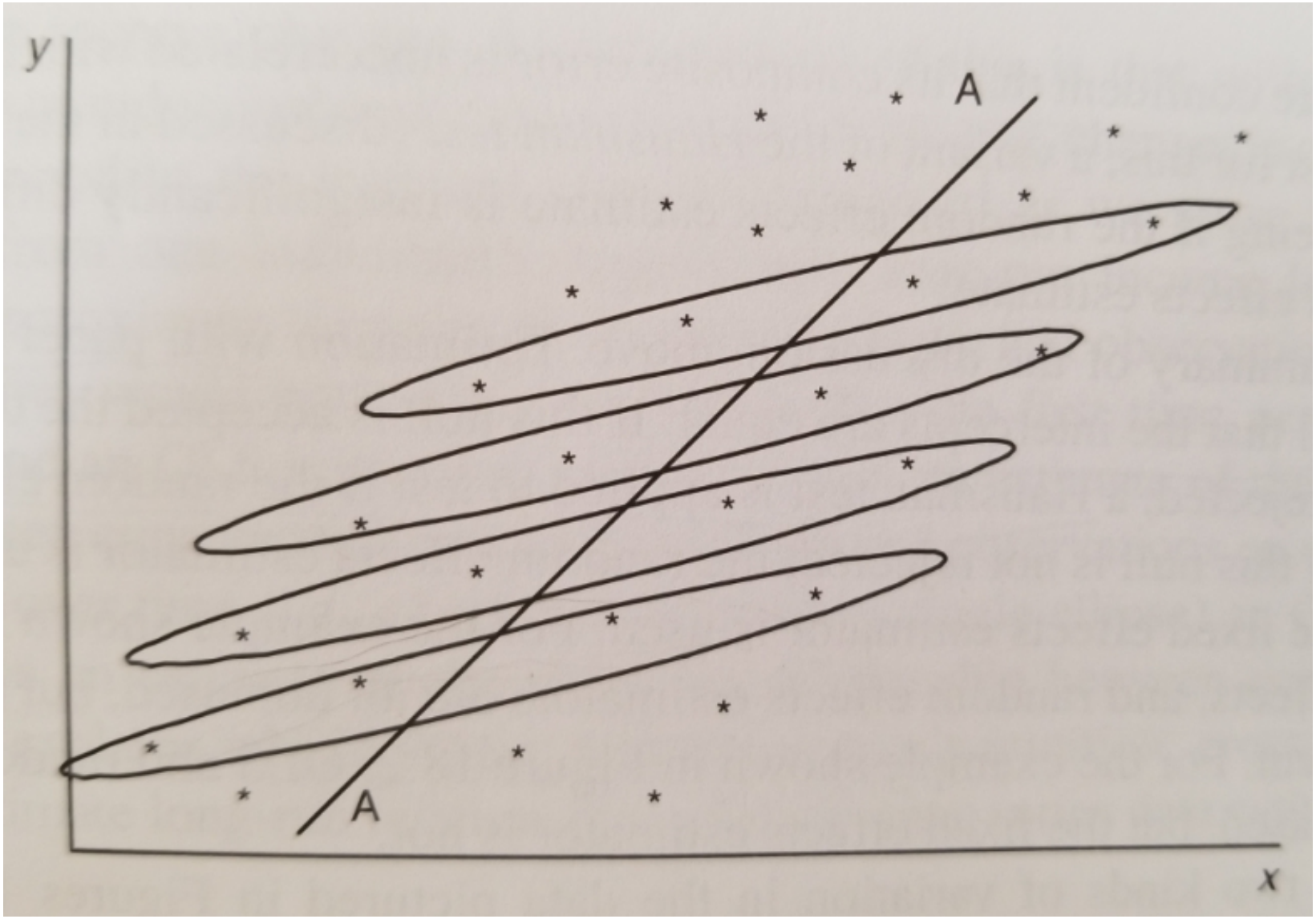
$$y_{it} - \lambda \bar{y}_i = (x_{it} - \lambda \bar{x}_i)\beta + v_{it} - \lambda \bar{v}_i$$

$$\lambda = 1 - \sqrt{\frac{\sigma_u^2}{\sigma_u^2 + T\sigma_c^2}}$$

Slope: OLS = FE = RE



Positive Correlation between x and the Intercept



Within and Between R^2

Within R^2: $\rho^2\{(y_{it} - \bar{y}_i), (x_{it}'\hat{\beta} - \bar{x}_i'\hat{\beta})\}$

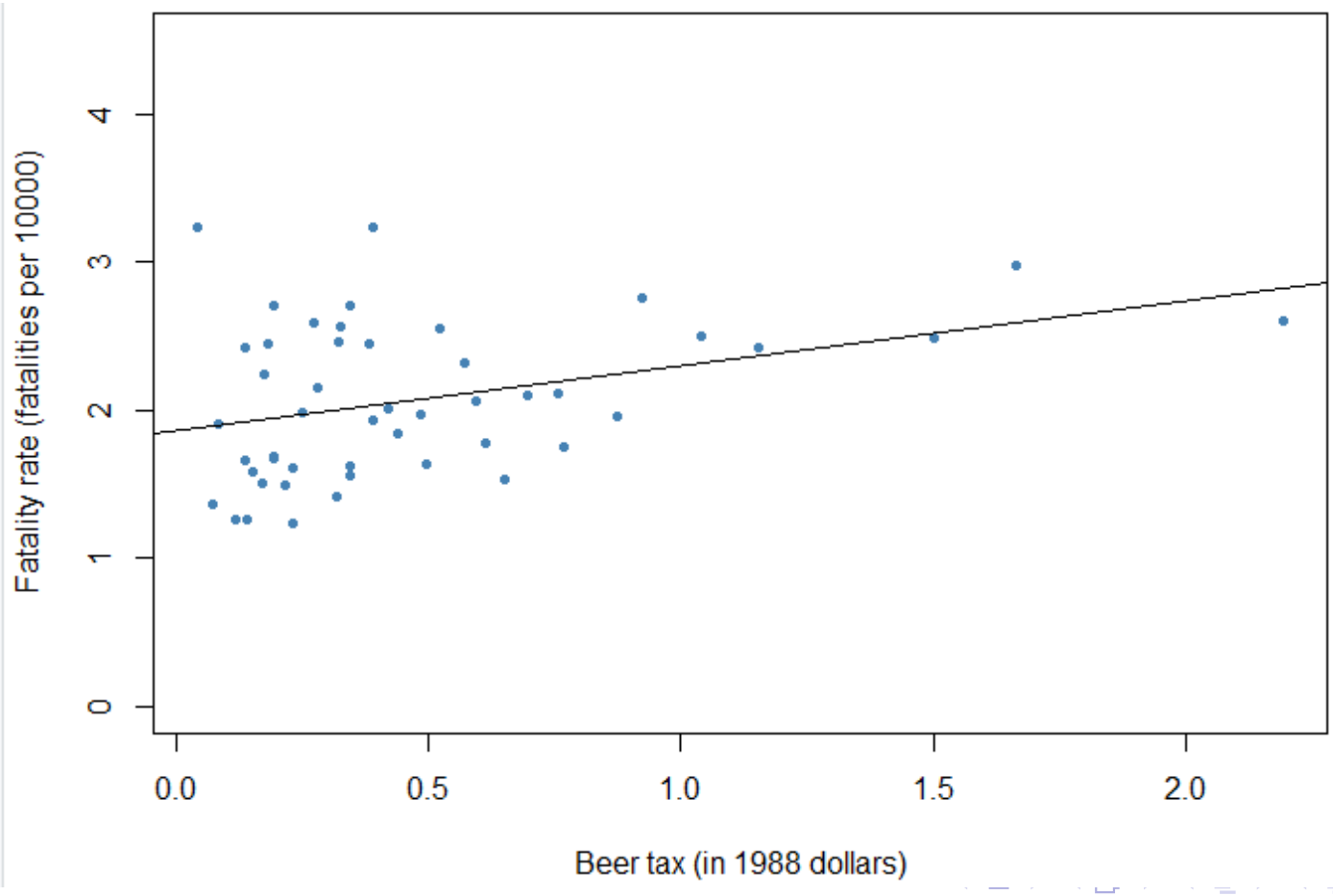
Between R^2: $\rho^2(\bar{y}_i, \bar{x}_i'\hat{\beta})$

Overall R^2: $\rho^2(y_{it}, x_{it}'\hat{\beta})$

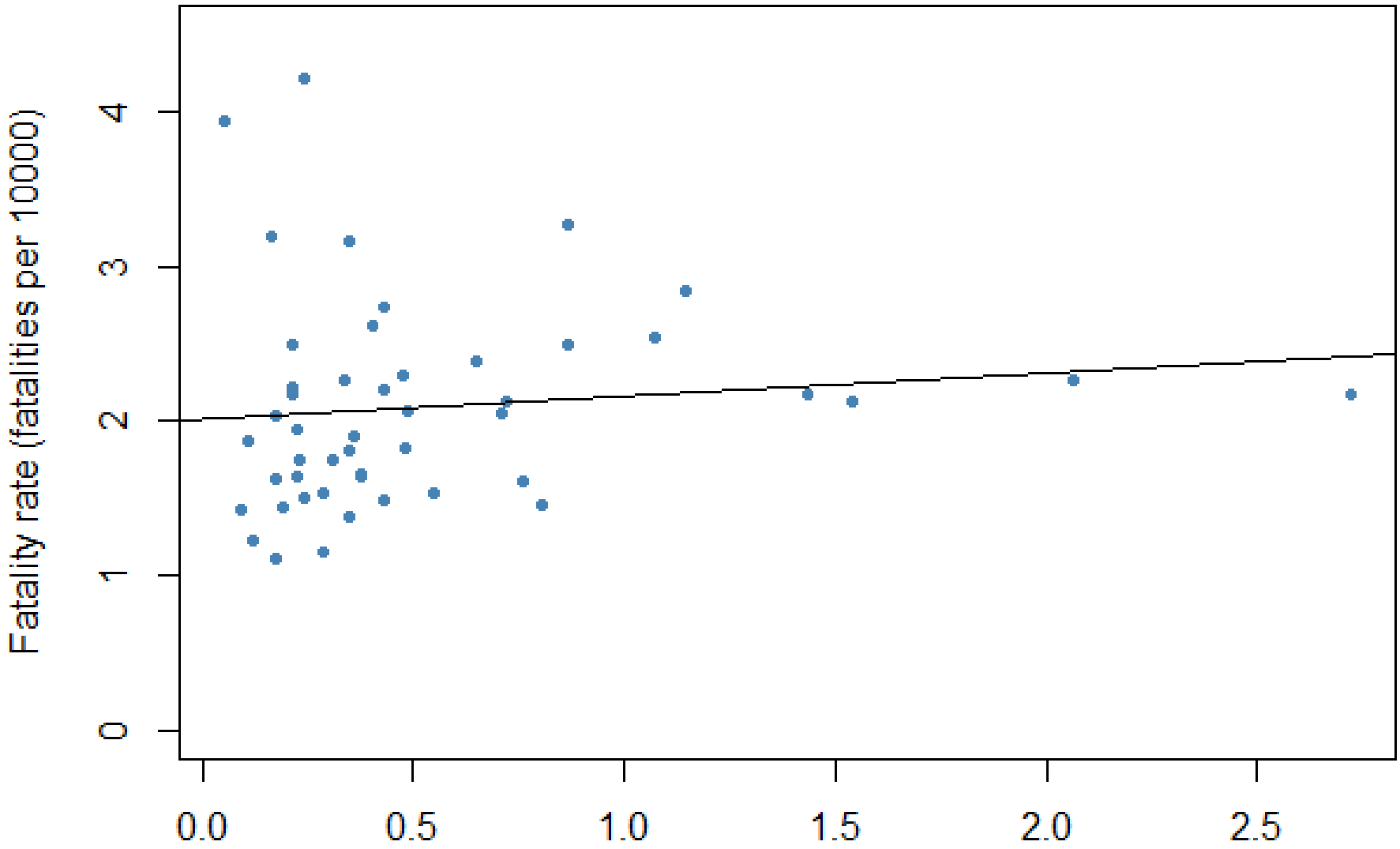
State Traffic Fatality Data from Christopher Ruhm

$$Deaths_{st} = \beta_0 + \beta_1 BeerTax_{st} + X_{st} + u_{st}$$

library(AER); library(plm); data(Fatalities)



Traffic Fatality Rates and Beer Taxes in 1982



RE <- plm(fatal_rate ~ beertax, data = Fatalities, index = c("state", "year"), model = "random")

summary(RE, vcov. = vcovHC, type = "HC1")

```
theta: 0.8622

Residuals:
      Min.    1st Qu.     Median    3rd Qu.       Max.
-0.471090 -0.120045 -0.021530  0.091011  0.964350

Coefficients:
             Estimate Std. Error t-value Pr(>|t|)
(Intercept)  2.067141   0.099971 20.6773   <2e-16
beertax     -0.052016   0.124176 -0.4189   0.6756
```

fe_Dummy <- lm(fatal_rate ~ beertax + state -1, data = Fatalities)

coeftest(fe_Dummy, vcov = vcovHC, type = "HC1")

```
          Estimate Std. Error t value  Pr(>|t|)
beertax -0.655874   0.203280 -3.2265  0.001398 **
stateal  3.477630   0.350784  9.9139 < 2.2e-16 ***
stateaz  2.909903   0.138146 21.0640 < 2.2e-16 ***
statear  2.822679   0.133998 21.0651 < 2.2e-16 ***
stateca  1.968161   0.029535 66.6385 < 2.2e-16 ***
stateco  1.993350   0.092783 21.4840 < 2.2e-16 ***
statect  1.615373   0.058131 27.7886 < 2.2e-16 ***
statede  2.170028   0.101284 21.4253 < 2.2e-16 ***
statefl  3.209500   0.228725 14.0321 < 2.2e-16 ***
```

Average traffic fatality rate is 2 per 10,000

Average real beer tax is $0.50 per case (in 1988 dollars)

FE_TD <- plm(fatal_rate ~ beertax, data = Fatalities, index = c("state", "year"), model = "within")

coeftest(FE_TD, vcov. = vcovHC, type = "HC1")

```
         Estimate Std. Error t value Pr(>|t|)
beertax -0.65587    0.28880  -2.271  0.02388 *
```

FETE<- plm(fatal_rate ~ beertax, data = Fatalities, index = c("state", "year"), model = "within", effect = "twoways")

coeftest(FETE, vcov = vcovHC, type = "HC1")

```
         Estimate Std. Error t value Pr(>|t|)
beertax -0.63998    0.35015 -1.8277  0.06865
```