5.1) Generalized Least Squares (GLS)

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Reference

Tables, Graphics, and Figures from:

Hansen (2018). **Econometrics.** Ch 4.8 to 4.19.

Known and Nonsingular $V(X)$

$$E(\epsilon\epsilon'|X) = \underset{(n\times n)}{\sigma^2 V(X)}$$

$$V^{-1} = C'C$$

$$\tilde{y} = Cy,\ \tilde{X} = CX,\ \tilde{\epsilon} = C\epsilon$$

$$\tilde{y} = \tilde{X}\beta + \tilde{\epsilon}$$

Strict Exogeneity

$$E(\tilde{\epsilon}|\tilde{X})$$

$$E(\tilde{\epsilon}|X)$$

$$E(C\epsilon|X)$$

$$CE(\epsilon|X)$$

$$0$$

Variance of the Transformed Error $\tilde{\epsilon}$ is Spherical

$$E(\tilde{\epsilon}\tilde{\epsilon}'|\tilde{X})$$

$$E(\tilde{\epsilon}\tilde{\epsilon}'|X)$$

$$C\sigma^2 VC'$$

$$\sigma^2 CVC'$$

$$\sigma^2 I_n$$

Generalized Least Squares (GLS) Estimator

$$\hat{\beta}_{GLS} = (\tilde{X}'\tilde{X})^{-1}\tilde{X}'\tilde{y}$$

$$[(CX)'(CX)]^{-1}(CX)'Cy$$

$$(X'C'CX)^{-1}(X'C'Cy)$$

$$(X'V^{-1}X)^{-1}X'V^{-1}y$$

GLS: Conditional Variance

$$Var(\hat{\beta}_{GLS}|X)$$

$$(X'V^{-1}X)^{-1}X'V^{-1}Var(y|X)V^{-1}X(X'V^{-1}X)^{-1}$$

$$(X'V^{-1}X)^{-1}X'V^{-1}(\sigma^2 V)V^{-1}X(X'V^{-1}X)^{-1}$$

$$\sigma^2(X'V^{-1}X)^{-1}$$

Weighted Least Squares (WLS)

$$Var(\epsilon_i|X) = E(\epsilon_i^2|X) = \sigma^2 v_i(X)$$

$$\tilde{y}_i = \frac{y_i}{\sqrt{v_i(X)}}, \tilde{X} = \frac{X_i}{\sqrt{v_i(X)}},$$
$$\tilde{\epsilon} = \frac{\epsilon_i}{\sqrt{v_i(X)}}$$

$$\tilde{y} = \tilde{X}\beta + \tilde{\epsilon}$$

Papke (1995)

library(foreign); library(lmtest); library(car); library(stargazer)

d401k<-
read.dta("https://github.com/VitorKamada/ECO7100/raw/master/Data/401ksubs.dta")

summary(d401k); stargazer(d401k)

Statistic	N	Mean	St. Dev.	Min	Max
e401k	9,275	0.392	0.488	0	1
inc	9,275	39.255	24.090	10.008	199.041
marr	9,275	0.629	0.483	0	1
male	9,275	0.204	0.403	0	1
age	9,275	41.080	10.300	25	64
fsize	9,275	2.885	1.526	1	13
nettfa	9,275	19.072	63.964	−502.302	1,536.798
p401k	9,275	0.276	0.447	0	1
pira	9,275	0.254	0.436	0	1
incsq	9,275	2,121.192	3,001.469	100.160	39,617.320
agesq	9,275	1,793.653	895.649	625	4,096

R Code for WLS and Heteroskedasticity

```
OLS <- lm(nettfa ~ inc + I((age-25)^2) + male + e401k,
  data=d401k, subset=(fsize==1))
OLSRobRef <- coeftest(OLS,hccm)
OLSRob <- coeftest(OLS, vcov=hccm(OLS, type="hc0"))

wlsreg <- lm(nettfa ~ inc + I((age-25)^2) + male + e401k,
  weight=1/inc, data=d401k, subset=(fsize==1))
WLS <- coeftest(wlsreg)
WLSRob <- coeftest(wlsreg,hccm)
```

R Code for Table of Results

```
library(stargazer)
stargazer(OLSRob,OLSRobRef,WLS,WLSRob,
  title="Regression Results",
  dep.var.labels="Net Financial Wealth",
  column.labels=c("OLSRob","OLSRobRef",
     "WLS","WLSRob"),
  no.space=TRUE,
  column.sep.width = "1pt",
  omit.stat=c("ser","f"))
```

Regression Results

	Dependent variable:			
	Net Financial Wealth			
	OLSRob	OLSRobRef	WLS	WLSRob
	(1)	(2)	(3)	(4)
inc	0.771***	0.771***	0.740***	0.740***
	(0.099)	(0.100)	(0.064)	(0.075)
I((age - 25)^2)	0.025***	0.025***	0.018***	0.018***
	(0.004)	(0.004)	(0.002)	(0.003)
male	2.478	2.478	1.841	1.841
	(2.056)	(2.065)	(1.564)	(1.313)
e401k	6.886***	6.886***	5.188***	5.188***
	(2.284)	(2.292)	(1.703)	(1.574)
Constant	−20.985***	−20.985***	−16.703***	−16.703***
	(3.491)	(3.520)	(1.958)	(2.248)
Note:				*p<0.1; **p<0.05; ***p<0.01