4) Analysis of Variance (ANOVA): Completely Randomized Designs

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Adaptation vs Mutation

Fact: Strains of bacteria die if exposed to certain virus, but some survives and reproduce fast

- In 1940s, both theories predict same average numbers of resistant bacteria
- But, Mutation Theory predicts a much higher variance
- 1969 Nobel Prize in Physiology/Medicine for Luria and Delbruck

Temperature (°C)									
175		194		213		231		250	
2.04	1.85	1.66	1.66	1.53	1.35	1.15	1.21	1.26	1.02
1.91	1.96	1.71	1.61	1.54	1.27	1.22	1.28	.83	1.09
2.00	1.88	1.42	1.55	1.38	1.26	1.17	1.17	1.08	1.06
1.92	1.90	1.76	1.66	1.31	1.38	1.16			

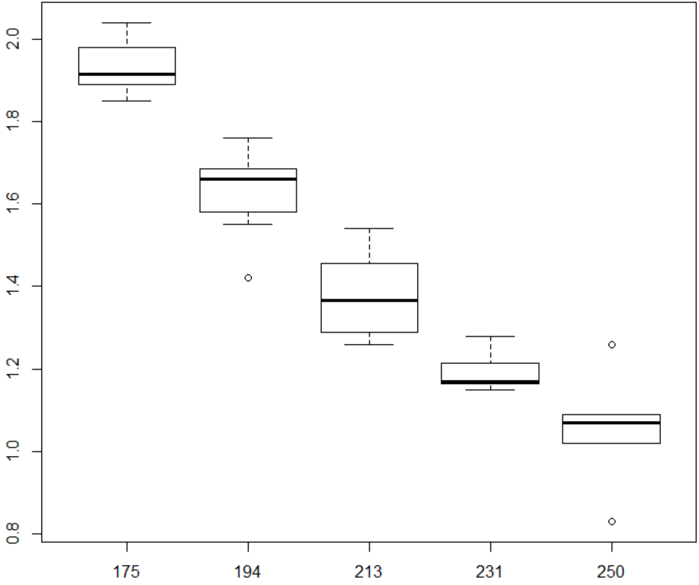
summary(resin)

attach(resin)

Statistic	N	Mean	St. Dev.	Min	Max
temp	37	210.081	26.144	175	250
y	37	1.465	0.326	0.830	2.040

Nelson (1990)

boxplot(y~temp)



Mechanics of ANOVA

$$y_{ij} - \mu = \alpha_i + \epsilon_{ij}$$

$$y_{ij} - \bar{y}_{\bullet\bullet} = (\bar{y}_{i\bullet} - \bar{y}_{\bullet\bullet}) + (y_{ij} - \bar{y}_{i\bullet})$$

$$y_{ij} - \bar{y}_{\bullet\bullet} = \hat{\alpha}_i + r_{ij}$$

$$(y_{ij} - \bar{y}_{\bullet\bullet})^2 = \hat{\alpha}_i^2 + r_{ij}^2 + 2\hat{\alpha}_i r_{ij}$$

$$SS_T = SS_{Trt} + SS_E + 2\sum_{i=1}^{g}\sum_{j=1}^{n_i} \hat{\alpha}_i r_{ij}$$

Generic ANOVA Table

Source	DF	SS	MS	F
Treatments	$g-1$	SS_{Trt}	$\frac{SS_{Trt}}{g-1}$	$\frac{MS_{Trt}}{MS_E}$
Error	$N-g$	SS_E	$\frac{SS_E}{N-g}$	

$$MS_{Trt} = \frac{1}{g-1} \sum_{i=1}^{g} \sum_{j=1}^{n_i} (\bar{y}_{i\bullet} - \bar{y}_{\bullet\bullet})^2 = \sum_{i=1}^{g} n_i \hat{\alpha}_i^2$$

$$MS_E = \frac{1}{N-g} \sum_{i=1}^{g} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{i\bullet})^2 = \hat{\sigma}^2$$

ANOVA Table

Dummy <- with(resin,as.factor(temp))

Result <- lm(y~Dummy)

anova(Result)

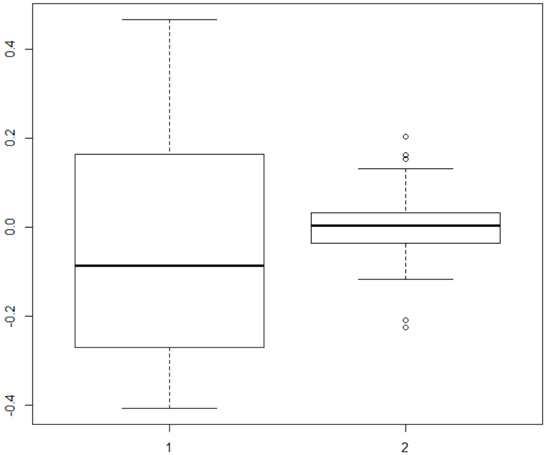
```
Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value    Pr(>F)
Dummy      4 3.5376 0.88441  96.363 < 2.2e-16 ***
Residuals 32 0.2937 0.00918
```

Side-by-Side Plots

yhat <- predict(Result); alpha <- yhat - 1.465

Residuals <- resid(Result); boxplot(alpha, Residuals)



summary(Result)

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.93250    0.03387  57.055  < 2e-16 ***
Dummy194    -0.30375    0.04790  -6.341 4.06e-07 ***
Dummy213    -0.55500    0.04790 -11.586 5.49e-13 ***
Dummy231    -0.73821    0.04958 -14.889 6.13e-16 ***
Dummy250    -0.87583    0.05174 -16.928  < 2e-16 ***
```

Dose-Response Modeling

$$\mu + \alpha_i = f(z_i; \theta)$$

$$\mu + \alpha_i = \theta_0 + \theta_1 z_i + \theta_2 z_i^2 + \ldots + \theta_{g-1} z_i^{g-1}$$

```
p1 <- lm(y~temp)
p2 <- lm(y~temp+I(temp^2))
p3 <- lm(y~temp+I(temp^2)+I(temp^3))
p4 <- lm(y~temp+I(temp^2)+I(temp^3)+I(temp^4))
```

```
stargazer(p1,p2,p3,p4, omit.stat=c("ser","f"),
type="text", out="Reg.txt")
```

Regression Results

	Dependent variable:			
	Lifetime (in hours)			
	(1)	(2)	(3)	(4)
temp	-0.012^{***}	-0.045^{***}	-0.037	0.076
	(0.001)	(0.011)	(0.187)	(3.750)
I(temp^2)		0.0001^{***}	0.00004	-0.001
		(0.00003)	(0.001)	(0.027)
I(temp^3)			0.00000	0.00000
			(0.00000)	(0.0001)
I(temp^4)				-0.000
				(0.00000)
Constant	3.956^{***}	7.418^{***}	6.827	0.970
	(0.139)	(1.156)	(12.987)	(195.724)
Observations	37	37	37	37
R^2	0.903	0.923	0.923	0.923
Adjusted R^2	0.900	0.919	0.916	0.914
Note:	$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$			

anova(p1,p2,p3,p4)

```
Analysis of Variance Table

Model 1: y ~ temp
Model 2: y ~ temp + I(temp^2)
Model 3: y ~ temp + I(temp^2) + I(temp^3)
Model 4: y ~ temp + I(temp^2) + I(temp^3) + I(temp^4)
  Res.Df     RSS Df Sum of Sq      F   Pr(>F)
1     35 0.37206
2     34 0.29372  1  0.078343 8.5361 0.006338 **
3     33 0.29370  1  0.000019 0.0020 0.964399
4     32 0.29369  1  0.000008 0.0009 0.976258
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’
```