Sample Final Part 2 – Solution

1)

(a) Let the random variable X denote the earned profits. Then the probability distribution of X is $p(0) = P(X = 0) = 0.05$, $p(20{,}000) = 0.75$, $p(50{,}000) = 0.20$.
(b) The expected value of X is $(0)(0.05) + (20{,}000)(0.75) + (50{,}000)(0.20) = \$25{,}000$.
(c) The variance of X is $(0–25{,}000)^2 (0.05) + (20{,}000–25{,}000)^2 (0.75) + (50{,}000–25{,}000)^2 (0.20) = 175{,}000$.
Hence $\sigma = \sqrt{175{,}000} \approx \$13{,}229$.

2)

(a) $E(X) = 1(0.50) + 2(0.50) = 1.5$ sandwiches
$Var(X) = (1 - 1.5)^2(0.50) + (2 - 1.5)^2(0.5) = 0.25$ sandwiches^2
(b) $E(Y) = 1(0.60) + 2(0.35) + 3(0.05) = 1.45$ drinks
$Var(Y) = (1 - 1.45)^2 (0.60) + (2 - 1.45)^2 (0.35) + (3 - 1.45)^2 (0.05) = 0.3475$ drinks^2
(c) $E(XY) = 1(0.4) + 2(0.2 + 0.1) + 4(0.25) + 6(0.05) = 2.3$
$Cov(X, Y) = 2.3 - 1.5(1.45) = 0.125$ sandwich-drinks
$Corr(X, Y) = Cov(X, Y)/(\sigma_x\sigma_y) = 0.125/(0.5 \times 0.589) \approx 0.424$
(d) Customers who buy more drinks also buy more sandwiches. The two are positively related, so those who buy more of one also purchase more of the other.
(e) $E(1.5X + 1Y) = 1.5(1.5) + 1(1.45) = \$3.70$
$Var(1.5X + 1Y) = 1.5^2 Var(X) + Var(Y) + 2(1.5)(1)Cov(X, Y) = 2.25(0.25) + 0.3475 + 3(0.125) = 1.285\ \$^2$
Hence, $SD(1.5X + 1Y) \approx \$1.13$.
(f) $E(Y/X) = 1(0.40 + 0.25) + 2(0.1) + 0.5(0.2) + 1.5(0.05) = 1.025$
The ratio of means, $\mu_y/\mu_x = 1.45/1.5 \approx 0.97$, is less than 1. These do not agree. In general, for positive random variables, $E(Y/X) \geq E(Y)/E(X)$.

3)

(a) Because the CPA specializes in similar businesses with comparable sales, we expect the adjustments to be of similar size. If the adjustment is the sum of many small corrections, then a normal model would be well-suited.
(b) Data for adjustments from prior years for this CPA and others who handle comparable types of businesses.

(c) About $\sigma = \$3{,}500$. This choice would mean that all but 2.5% (roughly) would save something since the mean of the normal model is 2 SDs less than zero.
(d) Probably not. These firms would likely be very different in size and nature of the adjustment. Some might be considerably larger than others. The distribution of adjustments for one CPA might be normal, but we should expect to see more outliers and perhaps skewness in the larger collection (as in the diamond example of the chapter).

4)

(a) 5%, and in this case it loses a lot!
(b) 5%. Either all of the bonds pay or they do not. These are not independent contracts. They all pay at the same time.
(c) The life insurance firm has independent customers, they don't all die at once. The hurricane bonds are not independent. The hurricane bonds are much more risky than life insurance.