Sample Test 2 - Solution

1)

(a) $n = \dfrac{Z^2\pi(1-\pi)}{e^2} = \dfrac{2.5758^2(0.32)(1-0.32)}{0.03^2} = 1{,}604.17$ Use n = 1,605

(b) $n = \dfrac{Z^2\pi(1-\pi)}{e^2} = \dfrac{2.5758^2(0.32)(1-0.32)}{0.05^2} = 577.50$ Use n = 578

(c) The higher the precision you require of your confidence interval, the larger is the sample size required.

2)

(a) H_0: $\mu \geq \$100$ H_1: $\mu < \$100$

Decision rule: $d.f.$ = 74. If $t_{STAT} < -1.6657$, reject H_0.

Test statistic: $t_{STAT} = \dfrac{\bar{X}-\mu}{S/\sqrt{n}} = \dfrac{\$93.70-\$100}{\$34.55/\sqrt{75}} = -1.5791$

Decision: Since the test statistic of $t_{STAT} = -1.5791$ is greater than the critical bound of -1.6657, do not reject H_0. There is not enough evidence to conclude that the mean reimbursement for office visits to doctors paid by Medicare is less than \$100.

(b) H_0: $\pi \leq 0.10$. At most 10% of all reimbursements for office visits to doctors paid by Medicare are incorrect.

cont.

H_1: $\pi > 0.10$. More than 10% of all reimbursements for office visits to doctors paid by Medicare are incorrect.

Decision rule: If $Z_{STAT} > 1.645$, reject H_0.

Test statistic: $Z_{STAT} = \dfrac{p-\pi}{\sqrt{\dfrac{\pi(1-\pi)}{n}}} = \dfrac{0.16-0.10}{\sqrt{\dfrac{0.10(1-0.10)}{75}}} = 1.73$

Decision: Since $Z_{STAT} = 1.73$ is greater than the critical bound of 1.645, reject H_0. There is sufficient evidence to conclude that more than 10% of all reimbursements for office visits to doctors paid by Medicare are incorrect.

(c) To perform the t-test on the population mean, you must assume that the observed sequence in which the data were collected is random and that the data are approximately normally distributed.

(d) H_0: $\mu \geq \$100$. The mean reimbursement for office visits to doctors paid by Medicare is at least \$100.

H_1: $\mu < \$100$. The mean reimbursement for office visits to doctors paid by Medicare is less than \$100.

Decision rule: $d.f.$ = 74. If $t_{STAT} < -1.6657$, reject H_0.

Test statistic: $t_{STAT} = \dfrac{\bar{X}-\mu}{S/\sqrt{n}} = \dfrac{\$90-\$100}{\$34.55/\sqrt{75}} = -2.5066$

Decision: Since $t_{STAT} = -2.5066$ is less than the critical bound of -1.6657, reject H_0. There is enough evidence to conclude that the mean reimbursement for office visits to doctors paid by Medicare is less than \$100.

(e) H_0: $\pi \le 0.10$. At most 10% of all reimbursements for office visits to doctors paid by Medicare are incorrect.

H_1: $\pi > 0.10$. More than 10% of all reimbursements for office visits to doctors paid by Medicare are incorrect.

Decision rule: If $Z_{STAT} > 1.645$, reject H_0.

Test statistic: $Z_{STAT} = \dfrac{p-\pi}{\sqrt{\dfrac{\pi(1-\pi)}{n}}} = \dfrac{0.20-0.10}{\sqrt{\dfrac{0.10(1-0.10)}{75}}} = 2.89$

Decision: Since $Z_{STAT} = 2.89$ is greater than the critical bound of 1.645, reject H_0. There is sufficient evidence to conclude that more than 10% of all reimbursements for office visits to doctors paid by Medicare are incorrect.

3)

(a) H_0: $\mu_1 = \mu_2$ where Populations: 1 = Technology, 2 = Financial Institutions

H_1: $\mu_1 \neq \mu_2$

Decision rule: If $|t_{STAT}| > 2.0167$ or p-value < 0.05, reject H_0.

Test statistic:

$$t_{STAT} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = 2.6509 \qquad p\text{-value} = 0.0112$$

Decision: Since p-value < 0.05, reject H_0. There is enough evidence of a difference between the technology sector and the financial institutions sector with respect to mean brand value.

b) H_0: $\mu_1 = \mu_2$ where Populations: 1 = Technology, 2 = Financial Institutions

H_1: $\mu_1 \neq \mu_2$

Separate-Variances t Test for the Difference Between Two Means	
(assumes unequal population variances)	
Data	
Hypothesized Difference	**0**
Level of Significance	**0.05**
Population 1 Sample	
Sample Size	**20**
Sample Mean	**38308.1**
Sample Standard Deviation	**40463.5360**
Population 2 Sample	
Sample Size	**25**
Sample Mean	**16232.16**
Sample Standard Deviation	**9186.8383**
Intermediate Calculations	
Numerator of Degrees of Freedom	7265995251435590.0000
Denominator of Degrees of Freedom	353204331402340.0000
Total Degrees of Freedom	20.5716
Degrees of Freedom	20
Standard Error	9232.5948
Difference in Sample Means	22075.94
Separate-Variance t Test Statistic	**2.3911**
Two-Tail Test	
Lower Critical Value	**-2.0860**
Upper Critical Value	**2.0860**
***p*-Value**	**0.0267**
Reject the null hypothesis	

Decision rule: If $|t_{STAT}| > 2.0860$ or p-value < 0.05, reject H_0.

Since p-value < 0.05, reject H_0. There is enough evidence of a difference between the technology sector and the financial institutions sector with respect to mean brand value.

(c) The conclusions in (a) and (b) are the same. Since the sample variance of the technology sector is more than 19 times as big as that of the financial institutions sector, the test in (b) is the appropriate test to perform assuming that both samples are drawn from normally distributed populations.

4)

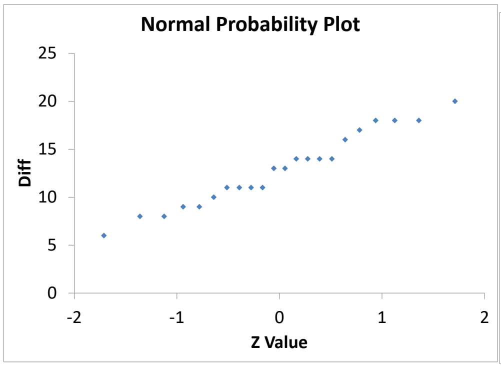
(a) $H_0 : \mu_D = 0$ vs. $H_1 : \mu_D \neq 0$

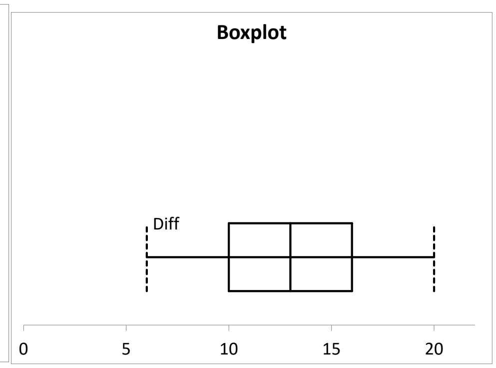
Test statistic: $t_{STAT} = \dfrac{\overline{D} - \mu_D}{\dfrac{S_D}{\sqrt{n}}}$ = 15.7396 p-value = 0.0000

Decision: Since p-value is virtually zero, reject H_0. There is evidence of a difference in the mean cellphone service rating between Verizon and AT&T.

(b) You must assume that the distribution of the differences between the mean measurements is approximately normal.

(c)





Both the boxplot and normal probability plot suggest that the distribution is normal.

(d)

$$\bar{D} \pm t \frac{S_D}{\sqrt{n}} = 12.8636 \pm 2.0796 \frac{3.8334}{\sqrt{22}} \qquad 11.16 \le \mu_D \le 14.56$$

You are 95% confident that difference in the mean cellphone service rating between Verizon and AT&T is somewhere between 11.16 and 14.56.