



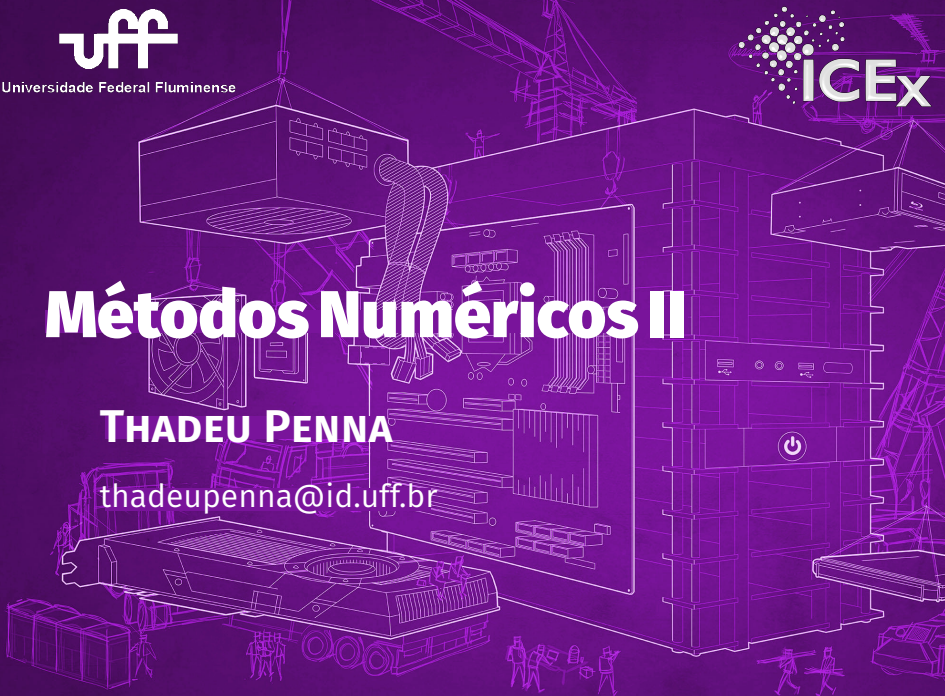
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Métodos Numéricos II

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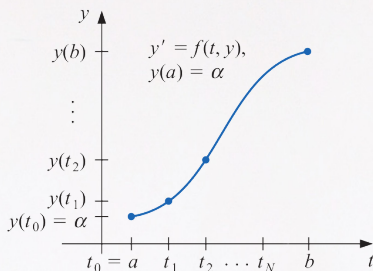
1. Problemas de Contorno

1.1 Diferenças finitas



PROBLEMAS DE CONTORNO

DIFERENÇAS FINITAS



Substituir as derivadas por diferenças.

$$y'(x_i) = \frac{1}{2h} [y(x_{i+1}) - y(x_{i-1}))]$$

$$y''(x_i) = \frac{1}{h^2} [y(x_{i+1}) - 2y(x_i) + y(x_{i-1}))]$$

Substituir em

$$y'' = p(x)y' + q(x)y + r(x)$$

Dados $y(a) = \alpha$ e $y(b) = \beta$ e reagrupando

$$\begin{aligned} & - \left(1 + \frac{h}{2} p(x_i) \right) y(x_{i-1}) \\ & + (2 + h^2 q(x_i)) y(x_i) \\ & - \left(1 - \frac{h}{2} p(x_i) \right) y(x_{i+1}) \\ & = -h^2 r(x_i) \end{aligned}$$

Que é um sistema de N equações tridiagonal.

$$\begin{bmatrix} 2 + h^2 q(x_1) & -1 + \frac{h}{2} p(x_1) & 0 & \dots & \dots & 0 \\ -1 - \frac{h}{2} p(x_2) & 2 + h^2 q(x_2) & -1 + \frac{h}{2} p(x_2) & \dots & \dots & 0 \\ 0 & -1 - \frac{h}{2} p(x_3) & 2 + h^2 q(x_3) & -1 + \frac{h}{2} p(x_3) & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ddots & \ddots & -1 + \frac{h}{2} p(x_{N-1}) & 2 + h^2 q(x_{N-1}) \\ 0 & 0 & \ddots & \ddots & -1 - \frac{h}{2} p(x_N) & 2 + h^2 q(x_N) \end{bmatrix}$$

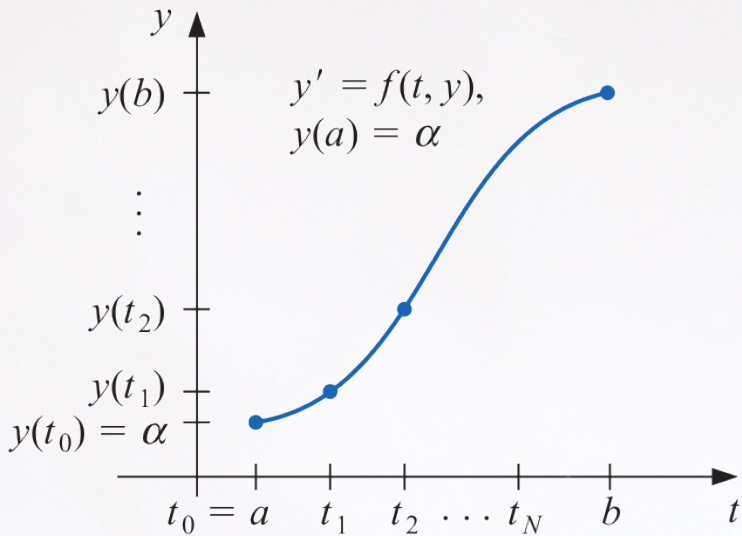
$$\times \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix} = \begin{bmatrix} -h^2 r(x_1) + (1 + \frac{h}{2} p(x_1)) y(x_0) \\ -h^2 r(x_2) \\ \vdots \\ -h^2 r(x_N) + (1 - \frac{h}{2} p(x_N)) y(x_{N+1}) \end{bmatrix}$$

RESULTADOS

2.05	-0.921	0	0	0	0	0	0	0	-0.348
-1.08	2.05	-0.921	0	0	0	0	0	0	-0.0235
0	-1.08	2.05	-0.921	0	0	0	0	0	-0.022
0	0	-1.08	2.05	-0.921	0	0	0	0	-0.02
0	0	0	-1.08	2.05	-0.921	0	0	0	-0.0174
0	0	0	0	-1.08	2.05	-0.921	0	0	-0.0145
0	0	0	0	0	-1.08	2.05	-0.921	0	-0.0112
0	0	0	0	0	0	-1.08	2.05	-0.921	-0.00762
0	0	0	0	0	0	0	-1.08	2.05	-0.096

x	w	exato	erro
0.00	-0.3		
0.16	-0.31198	-0.31195	2.9e-05
0.31	-0.31626	-0.31622	4.1e-05
0.47	-0.31274	-0.3127	4.1e-05
0.63	-0.30151	-0.30148	3e-05
0.79	-0.28286	-0.28284	1.3e-05
0.94	-0.25723	-0.25724	6.7e-06
1.10	-0.22527	-0.2253	2.4e-05
1.26	-0.18778	-0.18781	3.3e-05
1.41	-0.14567	-0.1457	2.8e-05
1.57	-0.1		





Resolva o problema de contorno

$$y'' = y' + 2y + \cos x, \quad 0 \leq x \leq \frac{\pi}{2}, \quad y(0) = -0.3, \quad y\left(\frac{\pi}{2}\right) = -0.1$$

Defina as funções $p(x)$, $q(x)$ e $r(x)$, mesmo que sejam simples. Isso vai facilitar modificar o programa para os próximos exercícios.

Use $N = 9$. Compare com o resultado exato e com o método de shooting.

$$y(x) = -\frac{1}{10} (\sin x + 3 \cos x)$$

