Before we try to solve the problem, we first need to know what the problem is. Modelling mathematically the LP problem is the beginning of the process to find a solution (or not finding it).

Mathematically a LP problem is defined as:

$$Maximize\ f(\boldsymbol{x})$$

$$Subject\ to: \boldsymbol{x} \in \Omega$$

or

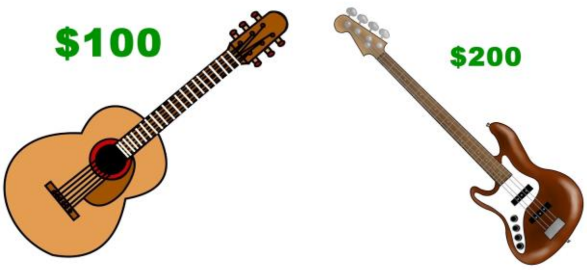
$$Minimize\ f(\boldsymbol{x})$$

$$Subject\ to: \boldsymbol{x} \in \Omega$$

Where **x** is the decision variable, Ω is the feasible set and f(**x**) is the objective function.

In simple words, we have a function that we want to find the optimum (min or max) but we have some constraints. Let us see one example and make it complex along the explanation to make it clear.

Example 1: Suppose you have a music store and you sell guitars and bass guitars, each guitar you sell for $100 and each bass for $200. Now you want to maximize the value of your sales.



In this first example, we will not give any constraints. Therefore, our problem will be:

$$Maximize\ f(x_1, x_2) = 100x_1 + 200x_2$$

Where x_1 are the number of guitars sold and x_2 are the number of bass guitars sold. Without any constraints, our problem became a problem without a definitive maximum, JUST SELL EVERYTHING!

Example 2: Now let us say you spend 10$ to sell a guitar and 15$ to sell a bass, and let us say we want to minimize your spending.

$$Minimize\ (10x_1 + 15x_2)$$

In this case, don't having any constraints can cause a logical problem, as one can see, we can sell "negative" products and make profit in this function, and obviously, this is wrong. So it is common to add the condition $x_n \geq 0$ to the problem (in the real world we do not produce negative products).

Example 3: After adding the non-negativity condition to the example 2, we have the following problem:

$$Minimize\ 10x_1 + 15x_2$$

$$x_1 \geq 0, x_2 \geq 0$$

Again, the solution for this problem is very simple, JUST DO NOT SELL! You will never find a better solution than this, but considering the "Example 1", that says to sell everything, this solution does not seems appropriate.

Example 4: If you sell a guitar for 100$ each and has a cost of 10$ each, then you are receiving 90$ for each and for the bass guitars you are receiving 185$, so we can reformulate our problem.

$$Maximize\ 90x_1 + 185x_2$$

$$x_1 \geq 0, x_2 \geq 0$$

Again, our problem seems to not to have a maximum solution, sell everything.

Example 5: Suppose now that the industry that produces the instruments to you to sell produce at the maximum 500 instruments (guitars or bass guitars) so now you have to make a choice of how much of each do you want. Moreover, to do that we add our first constraint.

$$Max\ 90x_1 + 185x_2$$

Subject to:

$$x_1 + x_2 \leq 500$$

$$x_1 \geq 0, x_2 \geq 0$$

Now our problem seems like a LP problem, but it is not too hard to tell that if we want to maximize our sells we just need to choose to buy 500 bass guitars from the industry and sell all of them. This is because we can sell the bass guitars for more money. So now we have a truly solution for our problem:

$x_1 = 0$ and $x_2 = 500$,

$f(x_1, x_2) = 90*0 + 185*500 = 92500.$

Example 6: Now suppose that a new music store opens and make you sell the bass guitars for the same price as the guitars (including the costs) so now guitars and bass guitars are sold for 90$ each. Modelling our problem gives:

$$Max\ 90x_1 + 90x_2$$

Subject to:

$$x_1 + x_2 \leq 500$$

$$x_1 \geq 0, x_2 \geq 0$$

Since we cannot sell 1.32 guitars or π bass guitars we have a great number of solutions (integer solutions) that solve our problem, and there is no difference between them. We can sell 500 guitars and none bass guitar, 250 of each, 100 of bass guitars and 400 guitars, etc… Our best solution is 90*500 = 45000. If we were working with real numbers in our variables we will had an infinity set of solutions, all of them giving the same result.

Example 7: The industry that produces the instruments just want to make one trip in one truck to you. They tell to you "Each bass guitar occupy 1.4 times the space of one guitar, and the truck can carry up to 400 guitars", and with that, for some reason you discover that you will spend only 5$ for each bass guitar, so now you sell the bass guitar for 100$ each, this creates a new problem.

$$Max\ 90x_1 + 100x_2$$

Subject to:

$$x_1 + x_2 \leq 500$$

$$x_1 + 1.4x_2 \leq 400$$
$$x_1 \geq 0, x_2 \geq 0$$

Now the problem became interesting, the first constraint tells that you can have up to 500 instruments, but the second constraint tells that you can have at the maximum 400 guitars or 400/1.4 bass guitars and the sum of the “volumes” of the instruments cannot pass 400. Since our bass guitar are sold for more money, we can imagine that if we just put on the truck the maximum capacity of bass guitars and the rest we put the guitars then we will have:

400/1.4 is 285 bass and we can put 1 guitar to fill up the truck (since 285*1,4 = 399).

Then we have 90*1 + 100*285 = 28590.

However, if we just sold 400 guitars we have 90*400 + 100*0 = 36000

This shows us that even with a better price there is a constraint that changes our perspective about our problem, so let’s add another one.

Example 8: You find out that if you sell less than 100000$ you go bankrupt, so:

$$Max\ 90x_1 + 100x_2$$

Subject to:

$$x_1 + x_2 \leq 500$$
$$x_1 + 1.4x_2 \leq 400$$
$$90x_1 + 100x_2 \geq 100000$$
$$x_1 \geq 0, x_2 \geq 0$$

Well since our maximum value in the last example was 36000$ it is impossible to obtain 100000$ so we had modelled our problem, but it does not have any feasible solution.

This example was to show that our example not always have a solution and constraints can be “less or equal to”, “more or equal to”, “equal to”.

Example 9: You pick a loan in the bank and now you just need to make more than 25000, but the loan was part of one agreement that you will sell at least 50 bass guitars on your store.

$$Max\ 90x_1 + 100x_2$$

Subject to:

$$x_1 + x_2 \leq 500$$

$$x_1 + 1.4x_2 \leq 400$$

$$90x_1 + 100x_2 \geq 25000$$

$$x_2 \geq 50$$

$$x_1 \geq 0, x_2 \geq 0$$

The constraint x_2 greater or equal to 50 cancel out the utility of the non-negative condition of x_2, but we let this condition to be part of the problem to see the scenario better.

Example 10: Finally, you can put any other constraints that appears in your problem, they do not need to be just sum, do not need to be all the variables, do not need to be all the same sign, etc.

$$Max\ 90x_1 + 100x_2$$

Subject to:

$$x_1 + x_2 \leq 500$$

$$x_1 + 1.4x_2 \leq 400$$

$$90x_1 + 100x_2 \geq 25000$$

$$x_2 \geq 50$$

$$8x_1 - 2.1x_2 \geq 67$$

$$x_1 \geq 0, x_2 \geq 0$$

When we put more and more constraints, add more and more variables to the problem it became harder to solve just by intuition, we need an algorithm to solve the problem and make the computer do all the work. There is many examples of modelling LP problems, in this work I will try to show some problems, models and implementation in R.