In [176…

```
import pyreadr
import numpy as np
result = pyreadr.read_r("SMRi ordered - Brazil")   # também lê .RData
obj = next(iter(result.values()))        # pega o único objeto do .rds

# se for um data frame R, vira um pandas.DataFrame
import pandas as pd
df = obj if isinstance(obj, pd.DataFrame) else None
df
```

Out[176…

	C349_F	C509_F	C349_M	C509_M	C169_F	C61_M	C539_F	C159_F	C169_
0	1.123807	0.661231	0.986634	0.707814	1.221911	0.742698	1.529891	0.683831	1.3146
1	0.443441	0.610168	0.448988	1.158253	0.574836	0.669648	1.016424	0.642775	0.8570
2	0.879448	0.449045	0.890769	0.604989	1.166260	0.811681	0.525437	0.498478	0.8600
3	1.020218	0.422304	1.001927	0.662585	0.601234	1.225795	0.519147	0.668390	0.7399
4	0.443073	0.354335	0.906988	0.692725	0.575117	0.642738	0.642628	0.642295	0.6613
...	...	...	...	...	...	...	...	...	
552	0.795859	0.878455	0.941837	0.501845	0.541944	0.668475	0.851317	0.613098	0.5979
553	0.736944	0.657660	0.301150	0.686377	0.532942	0.702059	0.835282	0.613446	0.6168
554	0.574846	0.596009	0.625308	0.505195	0.645011	0.514871	0.851198	0.707966	0.4493
555	1.210127	0.824201	0.832557	1.017706	0.747300	1.087854	1.233877	1.062437	0.5349
556	0.342973	0.419991	0.913603	1.234261	0.444595	0.794759	0.454757	0.277496	1.0593

557 rows × 30 columns

In [177…

```
vars_cols = df.var()
vars_cols.head()
```

Out[177…

```
C349_F    0.674739
C509_F    0.179864
C349_M    0.568566
C509_M    0.175342
C169_F    0.355013
dtype: float64
```

In [178…

```
df_std = df.copy()

#vamos padronizar as colunas
mu = df_std.mean(axis=0)
sd = df_std.std(axis=0, ddof=0)  # desvio padrão populacional (ddof=0)

# evita divisão por zero (coluna constante). Aqui descartamos constantes.
keep = sd > 0
```

```python
# Y é o DataFrame padronizado
Y = ((df_std.loc[:, keep] - mu[keep]) / sd[keep]).fillna(0.0)
print(Y.values, Y.shape)
Y.var().head()
```

```
[[ 0.36497084 -0.18883623  0.27358684 ...  0.17555823  1.919834
  -0.54308607]
 [-0.46404909 -0.30934677 -0.44008088 ... -0.18922218 -0.1473854
  -0.44313775]
 [ 0.06722115 -0.68960224  0.14633648 ...  0.05572086  0.17285073
  -0.75892984]
 ...
 [-0.30393368 -0.34276097 -0.20603454 ... -1.02813628 -0.87380289
  -0.46704425]
 [ 0.47015061  0.19577766  0.06906562 ... -0.29516127  0.58097297
  -0.02770818]
 [-0.58646782 -0.75816981  0.17664611 ... -0.04401604  1.72514341
  -0.75674525]] (557, 30)
```

Out[178…

```
C349_F    1.001799
C509_F    1.001799
C349_M    1.001799
C509_M    1.001799
C169_F    1.001799
dtype: float64
```

In [179…

```python
# Agora sim, podemos fazer a SVD
from numpy.linalg import svd

# X = M S Vt
M, S, Vt = svd(Y.values, full_matrices=False)
print(M.shape, S.shape, Vt.shape)  # M:(n×q), S:(q,), Vt:(q×q)
```

```
(557, 30) (30,) (30, 30)
```

In [180…

```python
for i in S:
    print(f"{i:.3f}", end=" ")
```

```
94.025 35.695 28.201 25.718 23.597 22.430 20.122 19.006 17.952 17.604 17.224 15.466
15.254 14.915 13.772 12.911 12.427 11.966 11.567 10.799 10.383 10.152 9.859 9.654 9.
094 8.811 8.363 7.878 7.549 5.839
```

In [181…

```python
# Gavish-Donoho para escolher K
n, q = Y.shape

beta = min(q / n, n / q)
sigma_median = np.median(S)
omega = 0.56*beta**3 - 0.95*beta**2 + 1.82*beta + 1.43
tau = omega * sigma_median
k = np.sum(S >= tau)
print(f"n={n}, q={q}, beta={beta:.3f}, omega={omega:.3f}, tau={tau:.3f}, K={K}")
```

```
n=557, q=30, beta=0.054, omega=1.525, tau=20.351, K=6
```

The SVD approximation in (1) implies that the jth column $\mathbf{x}_j$ of the matrix $\mathbf{X}$, associated with the jth cancer, can be approximately written as

$$\begin{aligned} \mathbf{x}_j &\approx \sigma_1\, \mathbf{v}_1[j] \times \mathbf{m}_1 + \cdots + \sigma_k\, \mathbf{v}_k[j] \times \mathbf{m}_k \\ &= \mathbf{W}_{1,j} \times \mathbf{m}_1 + \cdots + \mathbf{W}_{k,j} \times \mathbf{m}_k \end{aligned} \tag{3}$$

When the matrix $\mathbf{S}$ is dominated by a few large elements while the rest are relatively smaller, then it is possible to provide a good approximation to the data matrix $\mathbf{X}$ considering just part of the matrices $\mathbf{M}$ and $\mathbf{W}$. Defining k as the amount of significant values in the $\mathbf{S}$ matrix, $\mathbf{M_k}$ can be written as the $n \times k$ matrix containing the first k latent maps and $\mathbf{W_k}$ of dimension $k \times q$

being obtained selecting the first k rows of the matrix $\mathbf{W}$. Thus, $\mathbf{X}$ is approximated by:

$$\mathbf{X} \approx \mathbf{M_k W_k} \tag{4}$$

With this, the n-dimensional column-vector of the jth cancer is given approximately by:

$$\mathbf{x}_j \approx \mathbf{M_k W_k}[\cdot, j]$$

where $\mathbf{W_k}[\cdot, j]$ is the jth column of $\mathbf{W_k}$.

In [182…

```
import numpy as np

def xj_via_sum(M, S, Vt, j, k):
    """
    Implementa x_j ≈ sum_{l=1..k} sigma_l * v_l[j] * m_l
    M: (n x q)   -> colunas m_l
    S: (q,)      -> valores singulares
    Vt: (q x q)  -> V^T
    j: índice 1-based da coluna
    k: posto da aproximação
    """
    x = np.zeros(n, dtype=M.dtype)

    for l in range(k):
        coeff = S[l] * Vt[l, j-1] #sigma_l * v_l[j]
        x += coeff * M[:, l]
    return x



def xj_via_mat(M, S, Vt, j, k):
    """
    Forma matricial: x_j ≈ M_k @ W_k[:, j], com W = S V^T.
```

```
    """
    Mk = M[:, :k]                        # (n x k)
    Wk_col_j = S[:k] * Vt[:k, j-1]   # (k,) == [sigma_l * v_l[j]]_{l=1..k}
    return Mk @ Wk_col_j

k=6
cols = [1, 10, 20, 30]  # colunas pedidas

for j in cols:
    x_sum = xj_via_sum(M, S, Vt, j, k)
    x_mat = xj_via_mat(M, S, Vt, j, k)
    assert np.allclose(x_sum, x_mat)
    print(f"x_{j} via sum: {x_sum[:5]} ...")
    print(f"x_{j} via mat: {x_mat[:5]} ...")
```

```
x_1 via sum: [ 0.22639466 -0.12476106 -0.2072851   0.12235484 -0.10337673] ...
x_1 via mat: [ 0.22639466 -0.12476106 -0.2072851   0.12235484 -0.10337673] ...
x_10 via sum: [ 0.04123333 -0.23482643 -0.3269082  -0.21295424 -0.237691  ] ...
x_10 via mat: [ 0.04123333 -0.23482643 -0.3269082  -0.21295424 -0.237691  ] ...
x_20 via sum: [ 0.48453218 -0.35525627 -0.41890322 -0.99923011  0.01963938] ...
x_20 via mat: [ 0.48453218 -0.35525627 -0.41890322 -0.99923011  0.01963938] ...
x_30 via sum: [-0.61149713 -0.48139699  0.06270461 -0.23945405 -0.06693494] ...
x_30 via mat: [-0.61149713 -0.48139699  0.06270461 -0.23945405 -0.06693494] ...
```

In [183…

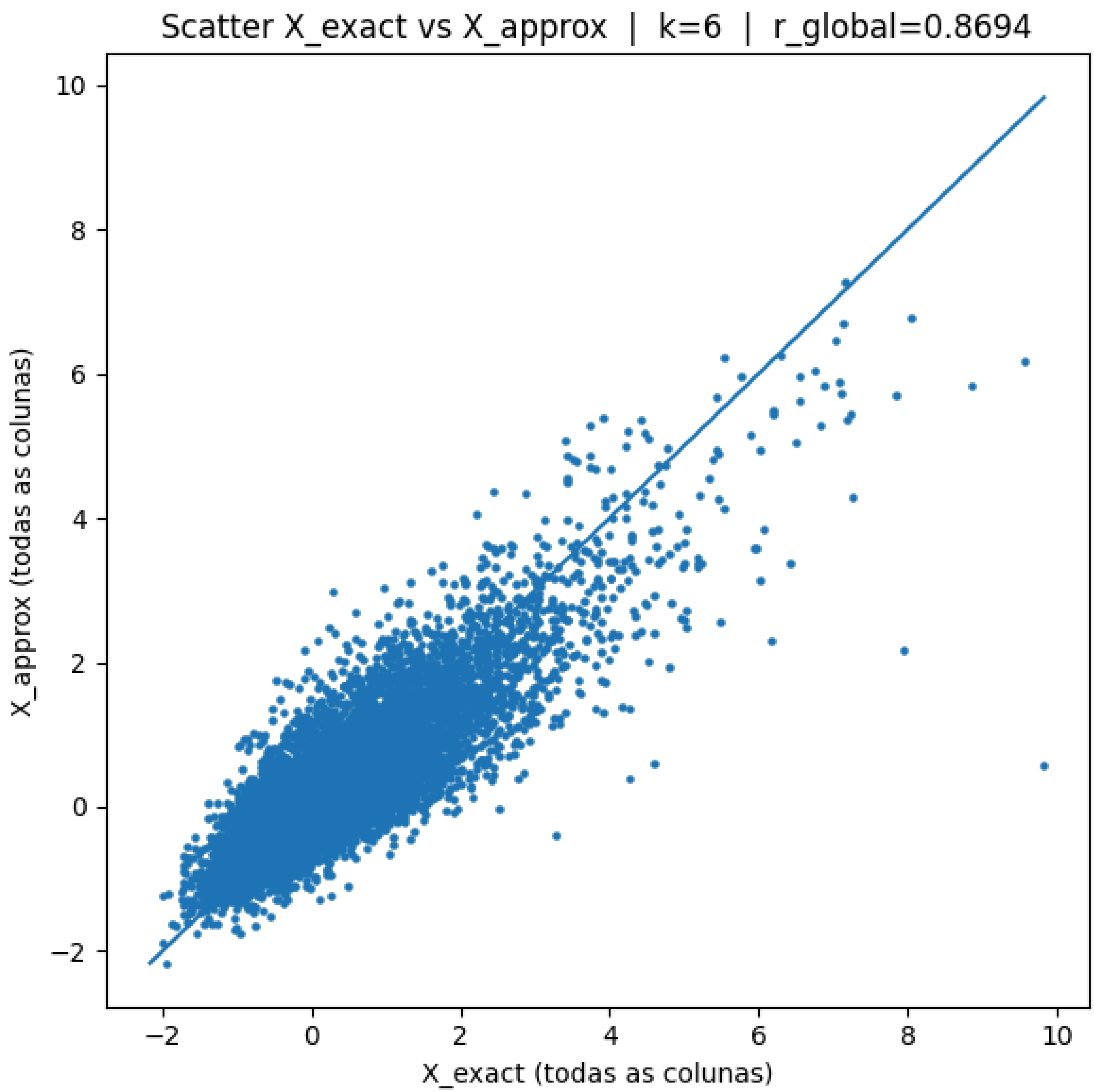
```
#os X_j dos dados são basicamente quando k vale o total de valores singulares, ou s
#vamos calcular os X_j para k = 30 e comparar com os X^_j aproximados.

k_full   = len(S)  # posto efetivo retornado pela SVD (<= min(n,q)) #30
k_approx = 6
cols = [1, 10, 20, 30]  # 1-based como no artigo
for j in cols:
    x_approx = xj_via_mat(M,S,Vt,j,k_approx)
    x_exact = xj_via_mat(M,S,Vt,j,k_full)
    print(f"x_{j} approx (k=6): {x_approx[:5]} ...")
    print(f"x_{j} exact  (k=30): {x_exact[:5]} ...")

    data_col = Y.iloc[:, j-1].to_numpy()
    # verificação do "x_exato" com a coluna dos dados reais #tem que retornar True.
    data_match = np.allclose(x_exact, data_col)
    print(f"exact equals data column? {data_match}\n")
```

```
x_1 approx (k=6): [ 0.22639466 -0.12476106 -0.2072851   0.12235484 -0.10337673] ...
x_1 exact  (k=30): [ 0.36497084 -0.46404909  0.06722115  0.23874855 -0.46449772] ...
exact equals data column? True

x_10 approx (k=6): [ 0.04123333 -0.23482643 -0.3269082  -0.21295424 -0.237691  ] ...
x_10 exact  (k=30): [-0.49941167  0.3662299  -0.53299605 -0.62912132 -0.19513286]
...
exact equals data column? True

x_20 approx (k=6): [ 0.48453218 -0.35525627 -0.41890322 -0.99923011  0.01963938] ...
x_20 exact  (k=30): [ 0.19897742  0.01676031  0.00443683 -0.64532072  0.9914579 ]
...
exact equals data column? True

x_30 approx (k=6): [-0.61149713 -0.48139699  0.06270461 -0.23945405 -0.06693494] ...
x_30 exact  (k=30): [-0.54308607 -0.44313775 -0.75892984 -0.72838098 -0.46689166]
...
exact equals data column? True
```

In [184…

```python
import matplotlib.pyplot as plt

k_full   = len(S)  # posto efetivo retornado pela SVD (<= min(n,q)) #30
k_approx = 6
per_col_r = {}

cols = np.arange(1,len(S)+1).tolist()

xs_all = []   # exato (eixo X)
ys_all = []   # aprox (eixo Y)

for j in cols:

    x_approx = xj_via_mat(M,S,Vt,j,k_approx)
    x_exact = xj_via_mat(M,S,Vt,j,k_full)

    xs_all.append(x_exact)
    ys_all.append(x_approx)

    # correlação por coluna
    rj = np.corrcoef(x_exact, x_approx)[0,1] #np.corrcoef retorna a matriz de corre
    per_col_r[j] = rj

# concatena todos os pontos (todas as colunas)
xs_all = np.concatenate(xs_all)
ys_all = np.concatenate(ys_all)

#correlação global (todas os pontos)
r_all = np.corrcoef(xs_all, ys_all)[0,1]

#plot
plt.figure(figsize=(6,6))
plt.scatter(xs_all, ys_all, s=5)

# linha y = x para referência
vmin = float(min(xs_all.min(), ys_all.min()))
```

```
vmax = float(max(xs_all.max(), ys_all.max()))

plt.plot([vmin, vmax], [vmin, vmax])        # Linha 45°
plt.xlabel("X_exact (todas as colunas)")
plt.ylabel("X_approx (todas as colunas)")
plt.title(f"Scatter X_exact vs X_approx  |  k={k_approx}  |  r_global={r_all:.4f}")
plt.tight_layout()
plt.show()
```



In [185…

```
print("Correlação por coluna (Pearson):")
for j in cols:
    print(f"  j={j:2d}: r = {per_col_r[j]:.4f}")
```

```
Correlação por coluna (Pearson):
  j= 1: r = 0.9088
  j= 2: r = 0.8987
  j= 3: r = 0.9312
  j= 4: r = 0.8899
  j= 5: r = 0.9186
  j= 6: r = 0.8664
  j= 7: r = 0.8837
  j= 8: r = 0.8895
  j= 9: r = 0.9068
  j=10: r = 0.8913
  j=11: r = 0.8631
  j=12: r = 0.9179
  j=13: r = 0.8866
  j=14: r = 0.8826
  j=15: r = 0.8257
  j=16: r = 0.8298
  j=17: r = 0.8567
  j=18: r = 0.8805
  j=19: r = 0.8739
  j=20: r = 0.7992
  j=21: r = 0.9051
  j=22: r = 0.8881
  j=23: r = 0.8873
  j=24: r = 0.7682
  j=25: r = 0.8385
  j=26: r = 0.8863
  j=27: r = 0.7070
  j=28: r = 0.8218
  j=29: r = 0.8443
  j=30: r = 0.8977
```

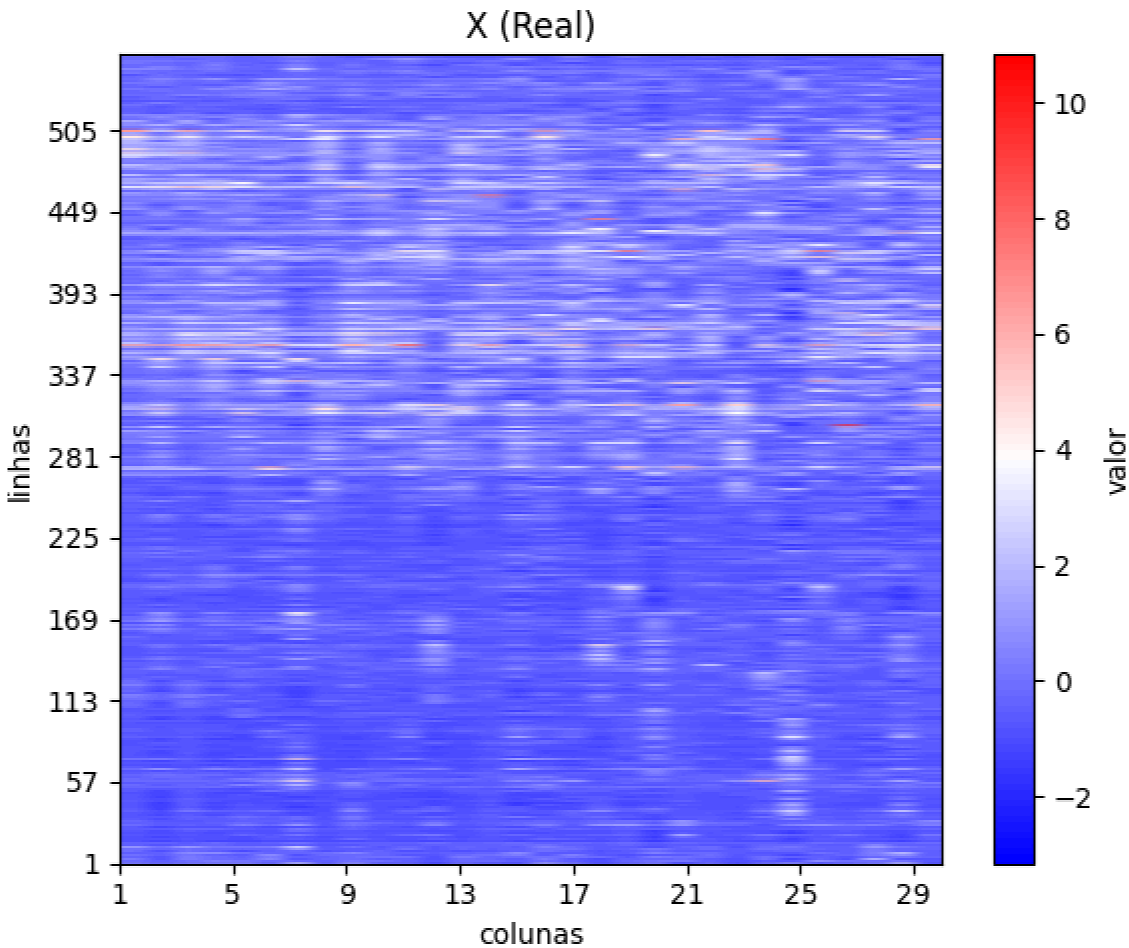
In [186…

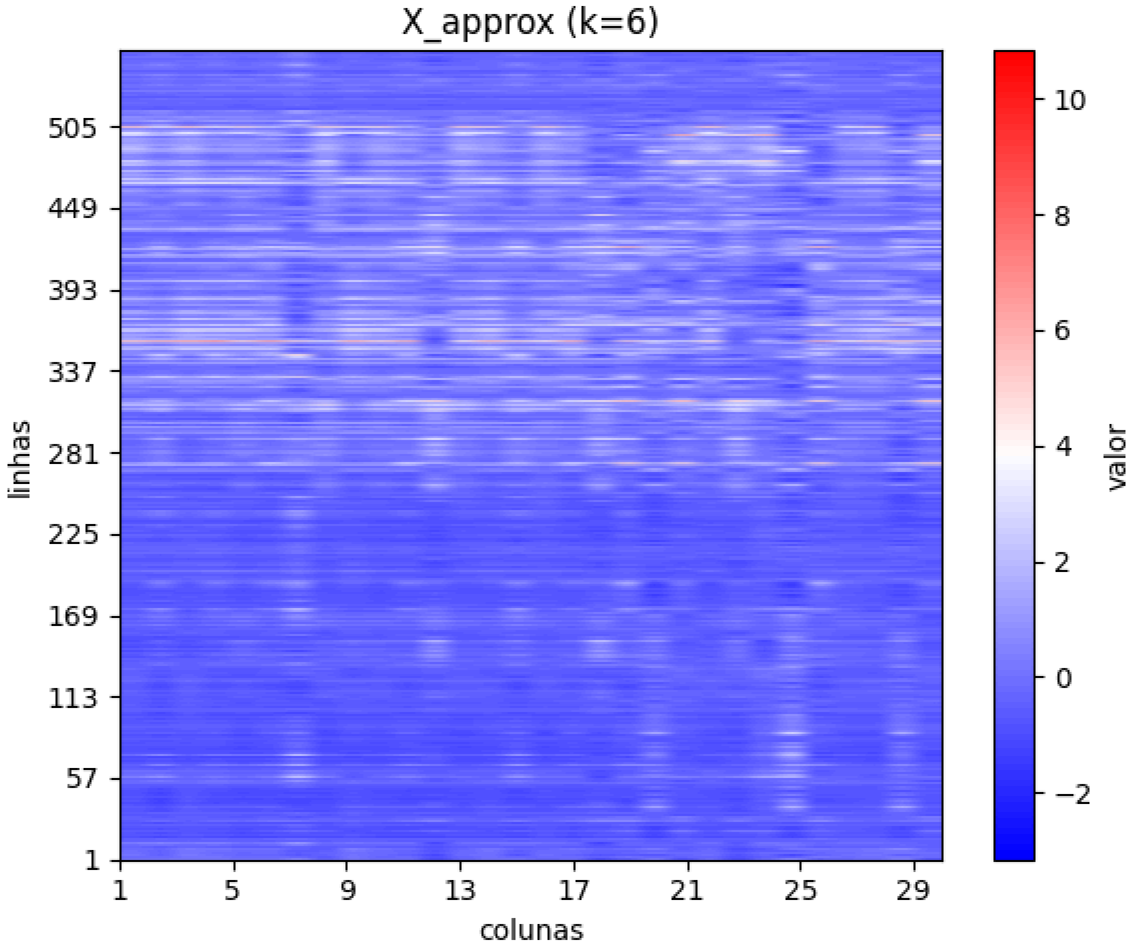
```python
#Funcao adaptada do R para grafico de imagem

def imagem_py(y, mini=-1, maxi=1, cores=0, title=""):
    """
    y     : matriz (np.array ou pandas DataFrame) a exibir
    mini  : valor mínimo do mapeamento de cores
    maxi  : valor máximo do mapeamento de cores
    cores : 0 = escala de cinza; 1 = azul-branco-vermelho
    title : título opcional
    """
    # aceita DataFrame ou array
    if hasattr(y, "values"):
        y = y.values
    y = np.asarray(y)
    nr, nc = y.shape

    # espaçamento dos ticks "inteligente"
    spr = 1 if nr <= 5 else round(nr / 10)
    spc = 1 if nc <= 5 else round(nc / 7)

    # paleta (equivalente ao cores do R)
    # 0: branco-cinza-preto  |  1: azul-branco-vermelho
    cmap = "gray" if cores == 0 else "bwr"
```

```
    # limites de cor
    vmin = mini
    vmax = maxi

    # plot
    fig, ax = plt.subplots(figsize=(6, 5))
    im = ax.imshow(
        y,
        origin="lower",          # 1ª linha na base (equivalente ao t(y[nr:1, ]) do
        vmin=vmin, vmax=vmax,
        cmap=cmap,
        aspect="auto",
        extent=[1, nc, 1, nr],   # eixos em 1..nc (x) e 1..nr (y)
    )

    # ticks e rótulos
    ax.set_xticks(np.arange(1, nc+1, spc))
    ax.set_yticks(np.arange(1, nr+1, spr))
    ax.set_xlabel("colunas")
    ax.set_ylabel("linhas")
    if title:
        ax.set_title(title)

    # colorbar
    cbar = plt.colorbar(im, ax=ax)
    cbar.set_label("valor")

    plt.tight_layout()
    plt.show()


#  construir X_approx a partir da SVD e plotar X vs X_approx
#X_approx está represnetado em (4)
def reconstruir_X_approx(M, S, Vt, k):
    """
    Retorna a aproximação de posto k: X_k = M_k @ diag(S_k) @ Vt_k
    """
    Mk = M[:, :k]                       # (n x k)
    Wk= S[:k, None] * Vt[:k, :]
    print(Wk.shape, Mk.shape)
    return Mk @ Wk

# Escolha o k para a aproximação
k_approx = 6

# A matriz original está num DataFrame Y, devo usar Y.values como X
X = Y.values

X_approx = reconstruir_X_approx(M, S, Vt, k_approx)

# Escalas de cor iguais para comparar (usa min/max globais)
gmin = float(min(X.min(), X_approx.min()))
gmax = float(max(X.max(), X_approx.max()))

# Plot das duas imagens
```

```
imagem_py(X,         mini=gmin-1, maxi=gmax+1, cores=1, title="X (Real)")
imagem_py(X_approx, mini=gmin-1, maxi=gmax+1, cores=1, title=f"X_approx (k={k_appro
```

```
(6, 30) (557, 6)
```




X_approx (k=6)
linhas
colunas
valor
505
449
393
337
281
225
169
113
57
1
1
5
9
13
17
21
25
29
10
8
6
4
2
0
−2