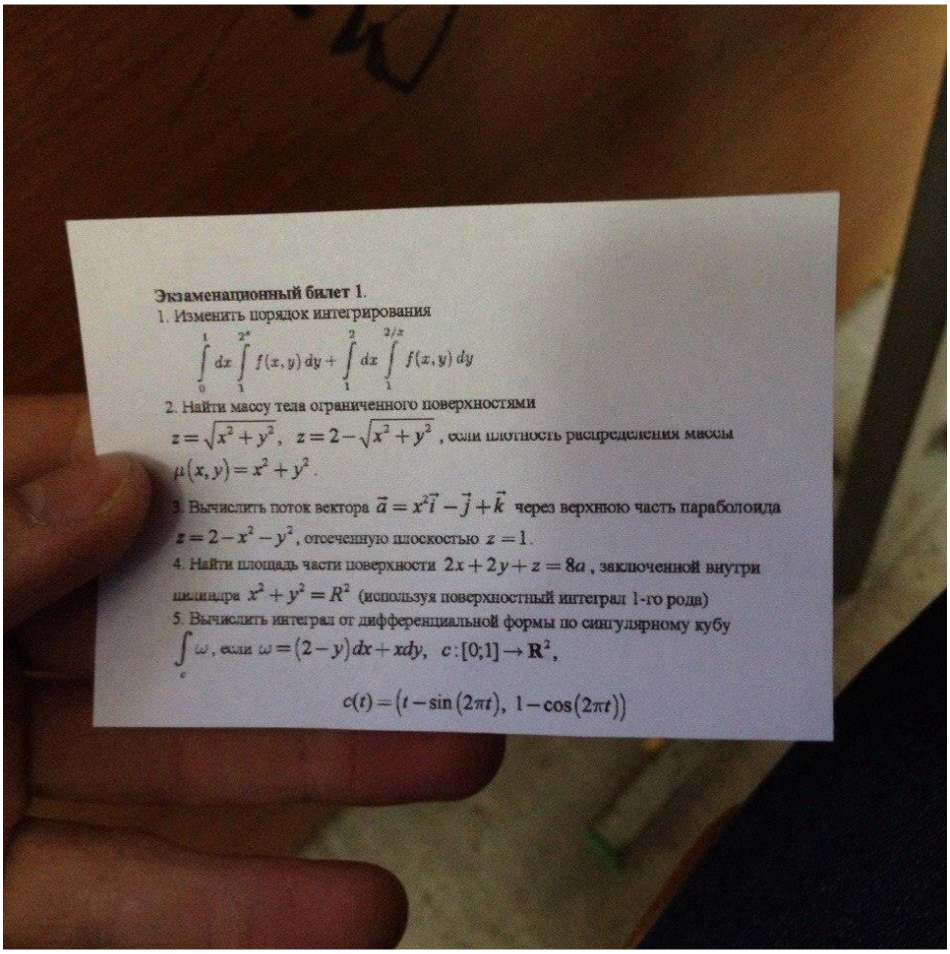


Экзаменационный билет 1.

1. Изменить порядок интегрирования

$$\int_0^1 dx \int_1^{2^x} f(x,y)\,dy + \int_1^2 dx \int_1^{2/x} f(x,y)\,dy$$

2. Найти массу тела ограниченного поверхностями $z=\sqrt{x^2+y^2}$, $z=2-\sqrt{x^2+y^2}$, если плотность распределения массы $\mu(x,y)=x^2+y^2$.

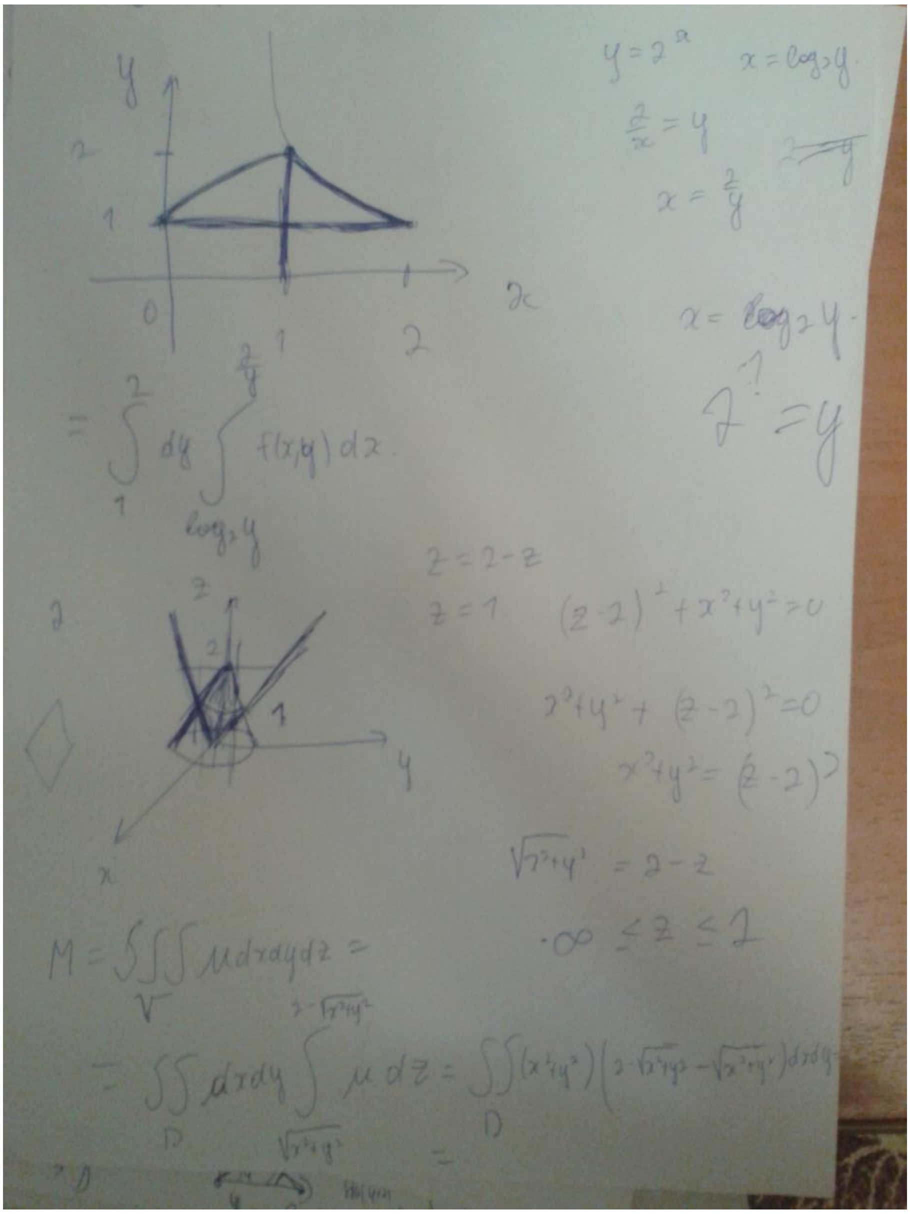
3. Вычислить поток вектора $\vec{a}=x^2\vec{i}-\vec{j}+\vec{k}$ через верхнюю часть параболоида $z=2-x^2-y^2$, отсеченную плоскостью $z=1$.

4. Найти площадь части поверхности $2x+2y+z=8a$, заключенной внутри цилиндра $x^2+y^2=R^2$ (используя поверхностный интеграл 1-го рода)

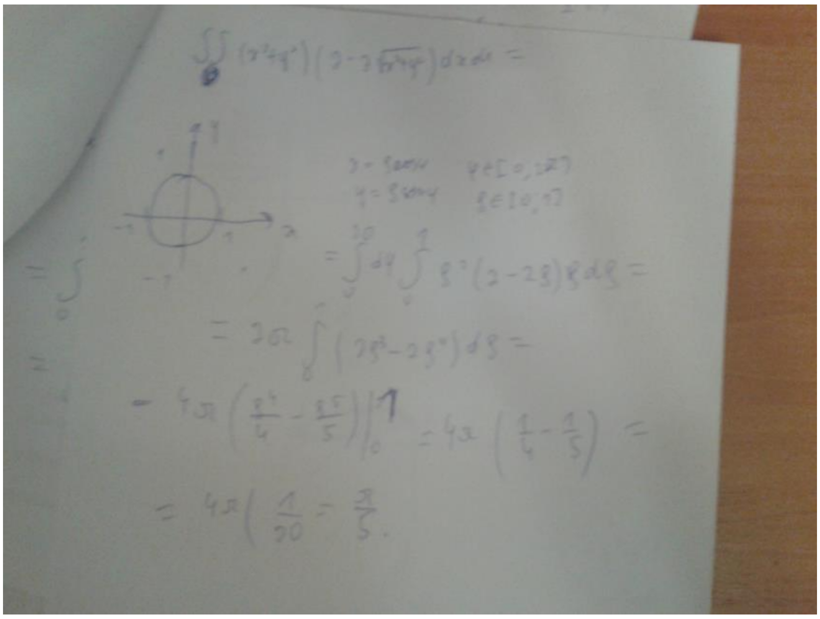
5. Вычислить интеграл от дифференциальной формы по сингулярному кубу $\int_c \omega$, если $\omega=(2-y)dx+xdy$, $c:[0;1]\to\mathbf{R}^2$,

$$c(t)=\big(t-\sin(2\pi t),\ 1-\cos(2\pi t)\big)$$

№1, №2

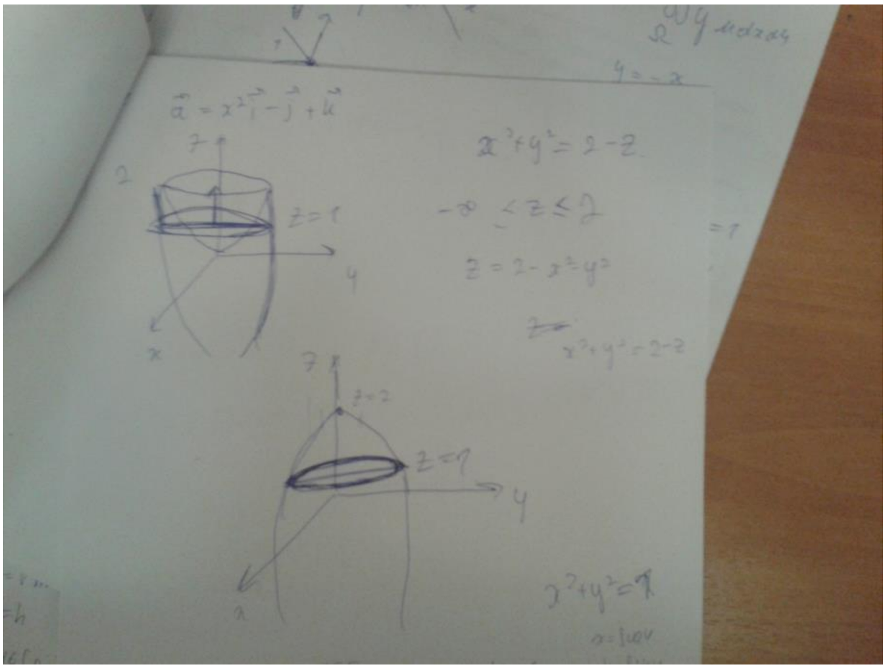


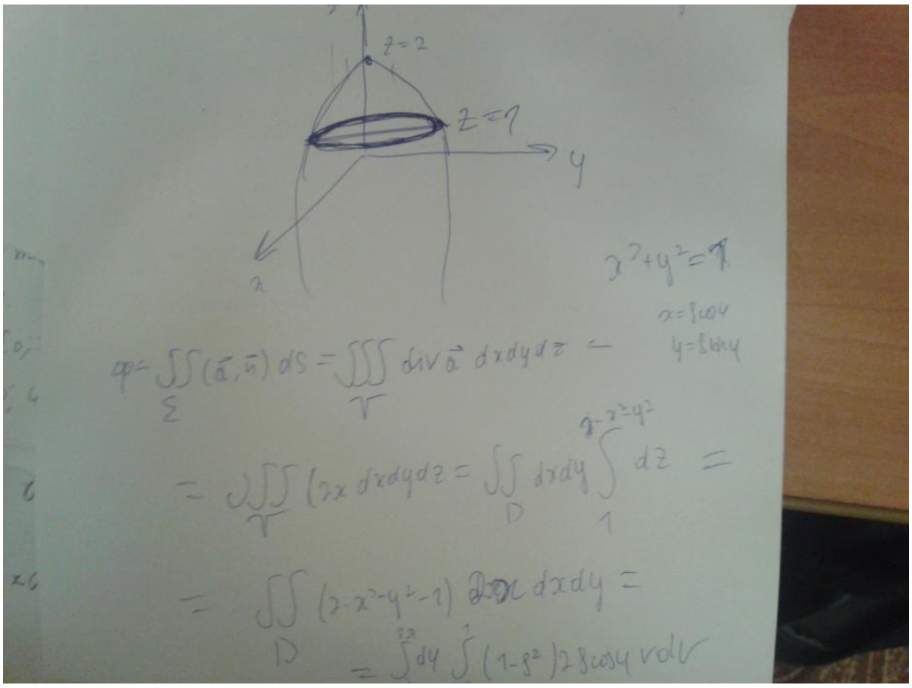
$y = 2^x$ $x = \log_2 y$
$\frac{2}{x} = y$
$x = \frac{2}{y}$
$x = \log_2 y$
$2^1 = y$
$= \int_1^2 dy \int_{\log_2 y}^{\frac{2}{y}} f(x,y)\, dx.$
2.
$z = 2 - z$
$z = 1$ $(z-2)^2 + x^2 + y^2 = 0$
$x^2 + y^2 + (z-2)^2 = 0$
$x^2 + y^2 = (z-2)^2$
$\sqrt{x^2+y^2} = 2 - z$
$-\infty \le z \le 2$
$M = \iiint_V \mu\, dx dy dz =$
$= \iint_D dx dy \int_{\sqrt{x^2+y^2}}^{2-\sqrt{x^2+y^2}} \mu\, dz = \iint_D (x^2+y^2)\left(2-\sqrt{x^2+y^2}-\sqrt{x^2+y^2}\right) dx dy =$



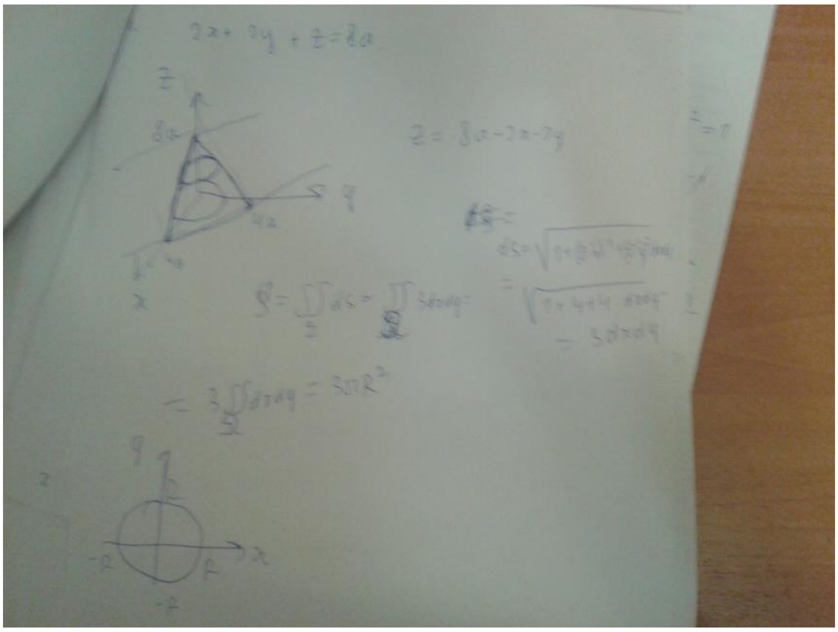
$\iint_D (x^2+y^2)(2-2\sqrt{x^2+y^2})\, dx dy =$
$x = \rho\cos\varphi$ $\varphi \in [0; 2\pi]$
$y = \rho\sin\varphi$ $\rho \in [0; 1]$
$= \int_0^{2\pi} d\varphi \int_0^1 \rho^2 (2 - 2\rho) \rho\, d\rho =$
$= 2\pi \int (2\rho^3 - 2\rho^4)\, d\rho =$
$= 4\pi \left(\frac{\rho^4}{4} - \frac{\rho^5}{5}\right)\Big|_0^1 = 4\pi\left(\frac{1}{4} - \frac{1}{5}\right) =$
$= 4\pi\left(\frac{1}{20}\right) = \frac{\pi}{5}.$

№ 3


a = x^2 i − j + k
x^2+y^2 = 2 − z
−∞ < z ≤ 2
z = 2 − x^2 − y^2
z=1
z=2
x^2+y^2=1


z=2
z=1
x^2+y^2=1
φ = ∬ (a, n) dS = ∭ div a dxdydz =
= ∭ 2x dxdydz = ∬ dxdy ∫ dz =
= ∬ (2−x^2−y^2−1) 2x dxdy =

№ 4,5


2x+2y+z=8a
z = 8a−2x−2y
= 3dxdy
= 3πR^2

5

$$\omega = (2-y)\,dx + x\,dy$$

$$x = t - \sin(2\pi t) \qquad dx$$

$$y = 1-\cos(2\pi t)$$

$$c^{*}\omega = \left(2 - 1 + \cos(2\pi t)\right)\left(1-\cos(2\pi t)\cdot 2\pi\right)dt + \qquad 1$$

$$+ \left(t - \sin(2\pi t)\right)\sin(2\pi t)\,2\pi\,dt \qquad (1+\cos(2\pi t))$$

$$\int_c \omega = \int_0^1 \quad t\Big|_0^1 - \frac{1}{2\pi}\sin(2\pi t)\Big|_0^1 (2\pi-1) -$$

$$-2\pi t\Big|_0^1 + \cancel{2\pi} - \cos(2\pi t)t\Big|_0^1 + \frac{1}{2\pi}\sin(2\pi t)\Big|_0^1 =$$

$$= 1 - 2\pi - 1 = -2\pi.$$

$$1 - 2\pi\cos(2\pi t) + \cos(2\pi t) - \cos^2(2\pi t)\,2\pi +$$

$$+ \left(t - \sin(2\pi t)\right)\sin(2\pi t)\,2\pi\,dt$$

$$\int_c \omega = \int_0^1 \quad t\Big|_0^1 - \frac{1}{2\pi}\sin(2\pi t)\Big|_0^1 (2\pi-1) -$$

$$-2\pi t\Big|_0^1 + \cancel{2\pi} - \cos(2\pi t)t\Big|_0^1 + \frac{1}{2\pi}\sin(2\pi t)\Big|_0^1 =$$

$$= 1 - 2\pi - 1 = -2\pi.$$

$$1 - 2\pi\cos(2\pi t) + \cos(2\pi t) - \cos^2(2\pi t)\,2\pi +$$

2

$y =$

$z = h$

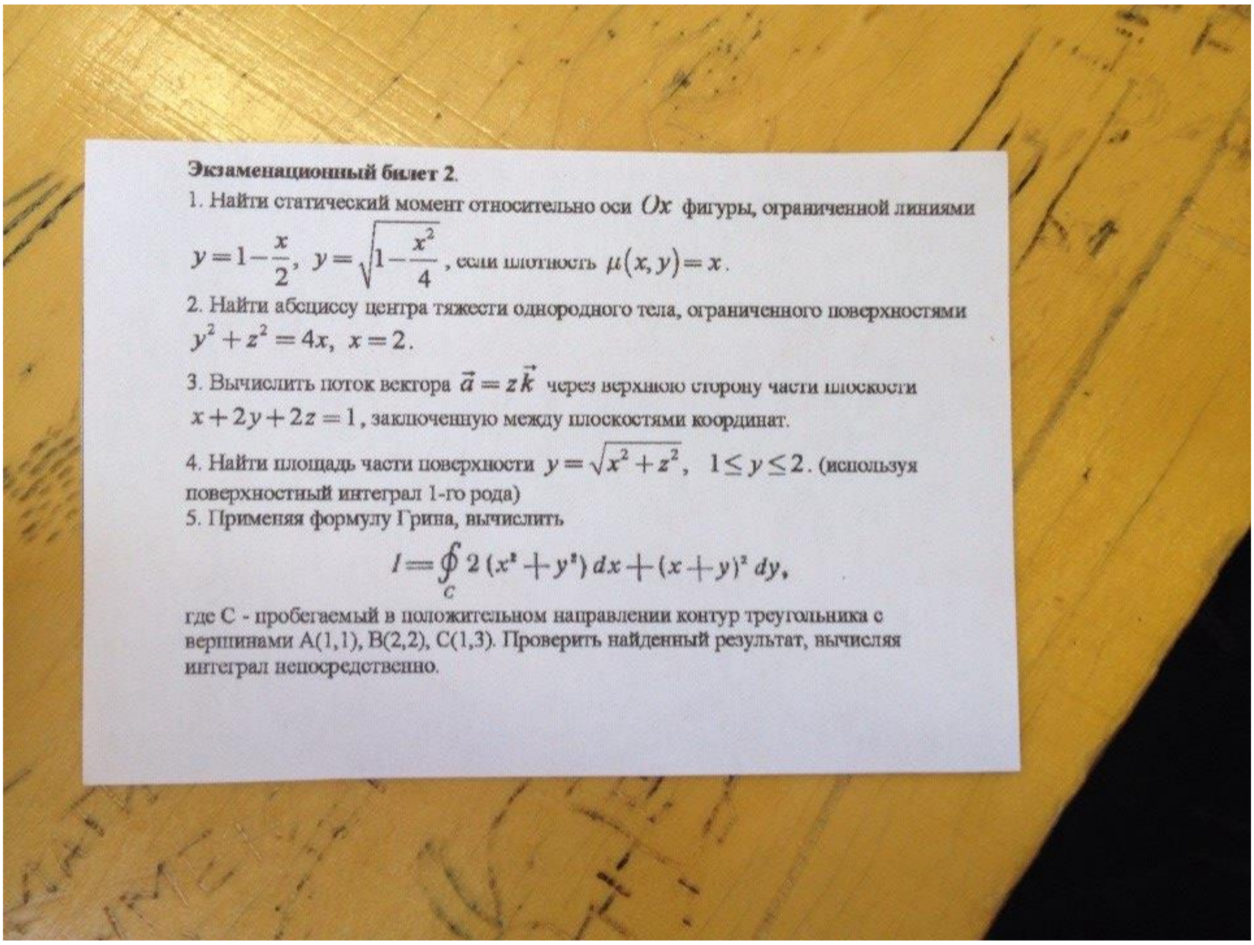
$h \in [$

$$+ t\sin(2\pi t)\,2\pi - \sin^2(2\pi t)\,2\pi =$$

$$= 1 - \cos(2\pi t)(2\pi - 1) - 2\pi + t\sin(2\pi t)\,2\pi$$

$$2\pi \int t\sin(2\pi t) = -\frac{1}{2\pi}\cos(2\pi t)\,t + \int \frac{1}{2\pi}\cos(2\pi t)\,dt -$$

$$= -\frac{1}{2\pi}\cos(2\pi t)\,t\Big|_0^1 + \frac{1}{4\pi^2}\sin(2\pi t)\Big|_0^1$$



Экзаменационный билет 2.

1. Найти статический момент относительно оси Ox фигуры, ограниченной линиями $y=1-\dfrac{x}{2},\ y=\sqrt{1-\dfrac{x^2}{4}}$, если плотность $\mu(x,y)=x$.

2. Найти абсциссу центра тяжести однородного тела, ограниченного поверхностями $y^2+z^2=4x,\ x=2.$

3. Вычислить поток вектора $\vec{a}=z\vec{k}$ через верхнюю сторону части плоскости $x+2y+2z=1$, заключенную между плоскостями координат.

4. Найти площадь части поверхности $y=\sqrt{x^2+z^2},\quad 1\le y\le 2$. (используя поверхностный интеграл 1-го рода)

5. Применяя формулу Грина, вычислить

$$I=\oint_C 2(x^2+y^2)\,dx+(x+y)^2\,dy,$$

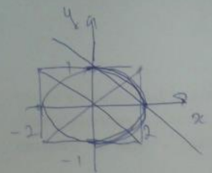
где C - пробегаемый в положительном направлении контур треугольника с вершинами A(1,1), B(2,2), C(1,3). Проверить найденный результат, вычисляя интеграл непосредственно.

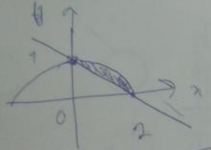
26.42, 26.46, 27.00, 27.05, 27.21, 27.26, 27.34, 27.45, 28.06, 28.11, 28.24, 28.32, 28.50

2 билет полностью по порядку

5 билет N 1

1.





$$S_x = \iint_\Omega y \mu \, dx\,dy$$

$$y = -\frac{x}{2} + 1$$

$$y^2 + \frac{x^2}{4} = 1.$$

$$\left(-\frac{x}{2}+1\right)^2 + \frac{x^2}{4} = 1$$

$$\frac{x^2}{4} + x + \cancel{1} + \frac{x^2}{4} = \cancel{1}$$

$$\frac{x^2}{2} + x = 0$$

$$x\left(\frac{1}{2}x - 1\right) = 0$$

$$x = 0 \quad x = 2$$

$$S_x = \iint \int_0^2 dx \int_{-\frac{x}{2}+1}^{\sqrt{1-\frac{x^2}{4}}} y\, x \, dy = \qquad \frac{y^2}{2}$$

$$= \int_0^2 x \left(\frac{1}{2} - \frac{x^2}{8} - \frac{\left(-\frac{x}{2}+1\right)^2}{2}\right) dx =$$

$$\left(-\frac{x}{2}+1\right)^2 + \frac{x^2}{4} = 1$$

$$\frac{x^2}{4} + x + \cancel{1} + \frac{x^2}{4} = \cancel{1}$$

$$S_x = \iint \int_0^2 dx \int_{-\frac{x}{2}+1}^{\sqrt{1-\frac{x^2}{4}}} y\, x \, dy = \qquad \frac{y^2}{2}$$

$$\frac{x^2}{2} + x = 0$$

$$x\left(\frac{1}{2}x - 1\right) = 0$$

$$x = 0 \quad x = 2$$

$$= \int_0^2 x \left(\frac{1}{2} - \frac{x^2}{8} - \frac{\left(-\frac{x}{2}+1\right)^2}{2}\right) dx =$$

$$= \int_0^2 x \left(\cancel{\frac{1}{2}} - \frac{x^2}{8} - \frac{x^2}{8} - \frac{x^2}{2} - \cancel{\frac{1}{2}}\right) dx =$$

$$= \int_0^2 x \left(-\frac{3x^2}{4}\right) dx = -\frac{3}{16} x^4 \Big|_0^2 =$$

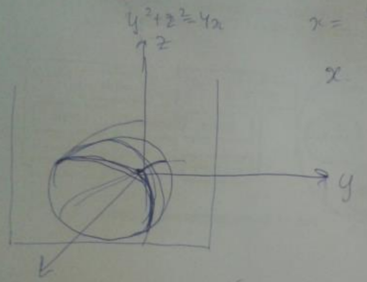
$$= -3$$

2

$$x = 2$$

$$y^2 + z^2 = 4x \qquad x = \frac{y^2+z^2}{4}$$

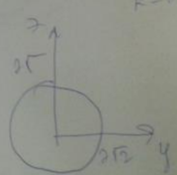
x



$$\mu = C.$$

8

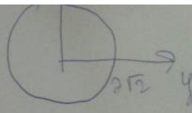
$R = 2\sqrt{2}$



$$x = \frac{\iiint_V C x \, dx\,dy\,dz}{\iiint_V C \, dx\,dy\,dz}$$

$$M = \iiint_V C \, dx\,dy\,dz = \iint_D dy\,dz \int_{\frac{y^2+z^2}{4}}^{2} dx =$$

$$\bar{x} = \frac{\iiint\limits_{V} x\,dx\,dy\,dz}{\iiint\limits_{V} c\,dx\,dy\,dz}$$



$$M = \iiint\limits_{V} c\,dx\,dy\,dz = \iint\limits_{D} dy\,dz \int\limits_{\frac{y^2+z^2}{4}}^{2} dx =$$

$$= \iint\limits_{D} \left(2 - \frac{y^2+z^2}{4}\right) dy\,dz = \int\limits_{0}^{2\pi} d\varphi \int\limits_{0}^{2\sqrt{2}} \left(2 - \frac{\rho^2}{4}\right)\rho\,d\rho =$$

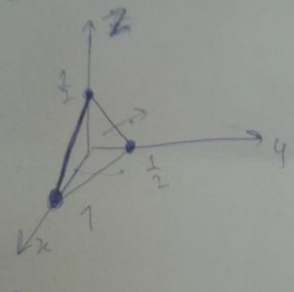
$x = \rho\cos\varphi$ $\quad \varphi \in [0; 2\pi]$

$y = \rho\sin\varphi$ $\quad \rho \in [0; 2\sqrt{2}]$

$$= 2\pi \left(\rho^2 - \frac{\rho^4}{16} \right) \Bigg|_{0}^{2\sqrt{2}} =$$

$$= 2\pi(8 - 4) = 8\pi$$

3. $\vec{a} = z\vec{k}$



$\vec{n}(1; 2; 2)$

$\vec{n}\left(\frac{1}{3}; \frac{2}{3}; \frac{2}{3}\right)$

$$z = \frac{1 - x - 2y}{2}$$

$$\Phi = \iint\limits_{\Sigma} (\vec{a}, \vec{n})\,dS = \iint\limits_{D} \frac{2}{3} z \cdot \frac{3}{2}\,dx\,dy =$$

$$dS = \sqrt{1 + (z'_x)^2 + (z'_y)^2}\,dx\,dy = \sqrt{1 + \frac{1}{4} + 1}\,dx\,dy = \frac{3}{2}\,dx\,dy$$

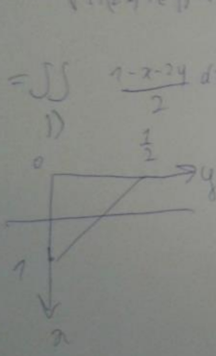
$$= \iint\limits_{D} \frac{1 - x - 2y}{2}\,dx\,dy = \int\limits_{0}^{1} dx \int\limits_{0}^{-\frac{x}{2}+\frac{1}{2}} \left(\frac{1}{2} - x - y\right) dy =$$

$$z = \frac{1 - x - 2y}{2}$$

$$\Phi = \iint\limits_{\Sigma} (\vec{a}, \vec{n})\,dS = \iint\limits_{D} \frac{2}{3} z \cdot \frac{3}{2}\,dx\,dy =$$

$$dS = \sqrt{1 + (z'_x)^2 + (z'_y)^2}\,dx\,dy = \sqrt{1 + \frac{1}{4} + 1}\,dx\,dy = \frac{3}{2}\,dx\,dy$$

$$= \iint\limits_{D} \frac{1 - x - 2y}{2}\,dx\,dy = \int\limits_{0}^{1} dx \int\limits_{0}^{-\frac{x}{2}+\frac{1}{2}} \left(\frac{1}{2} - x - y\right) dy =$$



$$= \int\limits_{0}^{1} \left(\frac{1}{2}y - xy - \frac{y^2}{2} \right) \Bigg|_{0}^{-\frac{x}{2}+\frac{1}{2}} dx =$$

$2y = 1 - x$

$y = -\frac{x}{2} + \frac{1}{2}$

$$= \int\limits_{0}^{1} \left(\frac{1}{2}\left(-\frac{x}{2}+\frac{1}{2}\right) - x\left(-\frac{x}{2}+\frac{1}{2}\right) - \frac{\left(-\frac{x}{2}+\frac{1}{2}\right)^2}{2} \right) dx =$$

$$= \int\limits_{0}^{1} \left(-\frac{x}{4} + \frac{1}{4} + \cancel{\frac{x^2}{2}} - \cancel{\frac{x^2}{2}} - \frac{1}{32} - \frac{1}{8}x + \frac{x^2}{8} \right) dx =$$

5 $\boxed{\text{5 билет}}$

$$\iint\limits_D f(x,y)\,dx\,dy \qquad y \geq x^2 \qquad y \leq \frac{1}{2}x^2 + \frac{1}{2}$$

$$3 = \int_0^1 \left(-\frac{5x}{8} + \frac{1}{32} + \frac{x^2}{8}\right)dx = -\frac{5x^2}{16}\Big|_0^1 + \frac{1}{32}x\Big|_0^1 + \frac{x^3}{24}\Big|_0^1 =$$

$$= -\frac{5}{16} + \frac{1}{32} + \frac{1}{24} = -\cancel{\frac{9}{32} + \frac{1}{24}} = \frac{1}{24}$$

4

$$y = \sqrt{x^2+z^2}$$

$$1 \leq y \leq 2$$

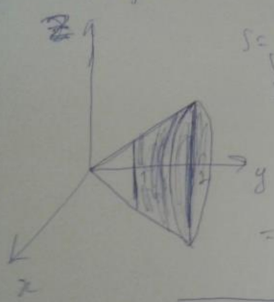
$$S = \iint\limits_\Sigma dS =$$

$$dS = \sqrt{1 + (y'_x)^2 + (y'_z)^2}\ dx\,dz$$

4

$$y = \sqrt{x^2+z^2}$$

$$1 \leq y \leq 2$$



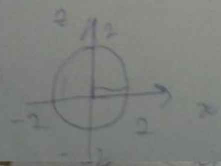
$$S = \iint\limits_\Sigma dS =$$

$$dS = \sqrt{1 + (y'_x)^2 + (y'_z)^2}\ dx\,dz$$

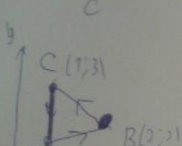
$$= \sqrt{1 + \frac{x^2}{x^2+z^2} + \frac{z^2}{x^2+z^2}}\ dx\,dz =$$

$$= \sqrt{\cancel{x^2+z^2}}\ \sqrt{2}\,dx\,dz$$

$$= \iint\limits_\Omega \sqrt{2}\,dx\,dz = \sqrt{2}\,\pi R^2\Big|_{R=2} = \pi\sqrt{2}\,4 = 4\pi\sqrt{2}$$



$$I = \oint\limits_C 2(x^2+y^2)\,dx + (x+y)^2\,dy$$



$y = x$
$y = 4 - x$

$$I \iint\limits_\Omega 2(x+y) - 4y\,dx\,dy = \int_1^2 dx \int_x^{4-x} (2x - 2y)\,dy =$$

$$= 2\int_1^2 dx \int_x^{4-x} (x-y)\,dy = 2\int_1^2 \left(xy - \frac{y^2}{2}\right)\Big|_x^{4-x} dx =$$

$$= 2\int_1^2 \left(x(4-x) - xx\right) - \left(\frac{(4-x)^2}{2} - \frac{x^2}{2}\right)dx =$$

$$= 2\int_1^2 \left(4x - x^2 - x^2\right) - \left(8 - 4x + \cancel{\frac{x^2}{2}} - \cancel{\frac{x^2}{2}}\right)dx =$$

$$I = \iint\limits_{\Omega} 2(x+y) - 4y \, dxdy = \int_1^2 dx \int_x^{4-x} (2x-2y)\,dy =$$

$$= 2\int_1^2 dx \int_x^{4-x} (x-y)\,dy = 2\int_1^2 \left(xy - \frac{y^2}{2}\right)\Big|_x^{4-x} dx =$$

$$= 2\int_1^2 \left(x(4-x) - x\cdot x\right) - \left(\frac{(4-x)^2}{2} - \frac{x^2}{2}\right) dx =$$

$$= 2\int_1^2 (4x - x^2 - x^2) - \left(8 - 4x + \cancel{\frac{x^2}{2}} - \cancel{\frac{x^2}{2}}\right) dx =$$

$$= 2\int_1^2 (4x - 2x^2 - 8 + 4x)\,dx = 2\int_1^2 (-2x^2+8x-8)\,dx =$$

$$= 2\int_1^2 (-2x^2+8x-8)\,dx = 2\left(-\frac{2x^3}{3} + \frac{8x^2}{2} - 8x\right)\Big|_1^2 =$$

$$= 2\left(\left(-\frac{2\cdot 8}{3} + 4\cdot 4 - 16\right) - \left(-\frac{2}{3} + 4 - 8\right)\right) =$$

$$= 2\left(-\frac{16}{3} + \cancel{16} - \cancel{16} + \frac{2}{3} + 4\right) = 2\left(-\frac{14}{3} + 4\right) =$$

$$= 8 - \frac{28}{3} = \frac{24-28}{3} = -\frac{4}{3}$$

$$I = \int_{C1} + \int_{C2} + \int_{C3}$$

$C1$: $y = x$

$x \in [1;2]$

$$I_1 = \int_1^2 2(x^2+x^2)\,dx + (2x)^2\,dx =$$

$$= \int_1^2 4x^2 + 4x^2 = \int_1^2 8x^2\,dx = \frac{8x^3}{3}\Big|_1^2 = \frac{8\cdot 8}{3} - \frac{8}{3} =$$

$$= \frac{8\cdot 7}{3}$$

I_2 $x \in [2;1]$ $\quad I_2 = \int^1 2(x^2+(4-x)^2) - (x+4-x)^2\,dx =$

$= \frac{8\cdot 7}{3}$

$I_2 \quad x \in [2; 1]$

$y = 4 - x$

$dy = -dx$

$I_2 = \int_2^1 2(x^2 + (4-x)^2) - (x + 4 - x)^2 dx =$

$= \int_2^1 2(x^2 + 16 - 8x + x^2) - 16 dx =$

$= \int_2^1 4x^2 - 16x + 32 - 16 dx = \int_2^1 4x^2 - 16x + 16 dx = \left(\frac{4x^3}{3} - \frac{16x^2}{2} + 16x\right)\Big|_2^1 = \left(\frac{4x^3}{3} - 8x^2 + 16x\right)\Big|_2^1 = \left(\frac{4}{3} - 8 + 16\right) - \left(\frac{8\cdot 4}{3} - 32 + 32\right) = \frac{4}{3} \cancel{- 2 + 16 - \frac{8\cdot 4}{3} + 8 - 32} =$

$\cancel{= -2 + - \frac{4\cdot 7}{3}}$

$= \frac{4}{3} + 8 \cancel{\frac{8\cdot 4}{3}} =$

$= 8 - \frac{24}{3} = \frac{4}{3} + 8 - \frac{24}{3} = 8 - \frac{28}{3}$

$I_3 =$

$dx = 0$

$y = y \quad y \in [3; 1] \quad x = 1$

$I_3 = \int_3^1 (1+y)^2 dy = \int_3^1 (1 + 2y + y^2) dy = \left(y + y^2 + \frac{y^3}{3}\right)\Big|_3^1$

$= \left(1 + 1 + \frac{1}{3}\right) - (3 + 9 + 9) =$

$= 2 + \frac{1}{3} - 21 = -19 + \frac{1}{3}$

$\frac{8\cdot 7}{3} + \frac{1}{3} - 19 + 2 - \frac{4\cdot 7}{3} = -21 + \frac{56 + 1 - 28}{3} =$

$= -21 + \frac{29}{3}$

$\cancel{\frac{8\cdot 7}{3} - 19 + \frac{1}{3} + 8 - \frac{7\cdot 4}{3} = -11}$

$\cancel{-19 + \frac{1}{3} + \frac{8\cdot 7}{3} + 8 - \frac{28}{3}}$

$-19 + \frac{1}{3} + 8 - \frac{28}{3} + \frac{8\cdot 7}{3} =$

$= -11 + \frac{57}{3} - \frac{28}{3} = -11 + \frac{29}{3} =$

$= -\frac{4}{3}$