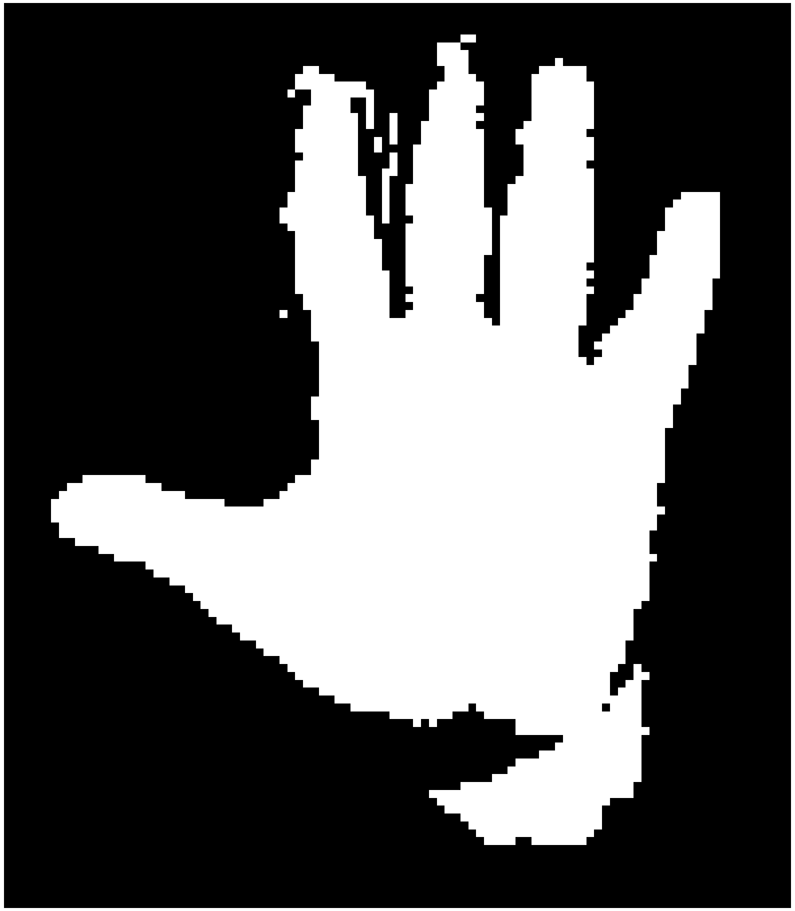
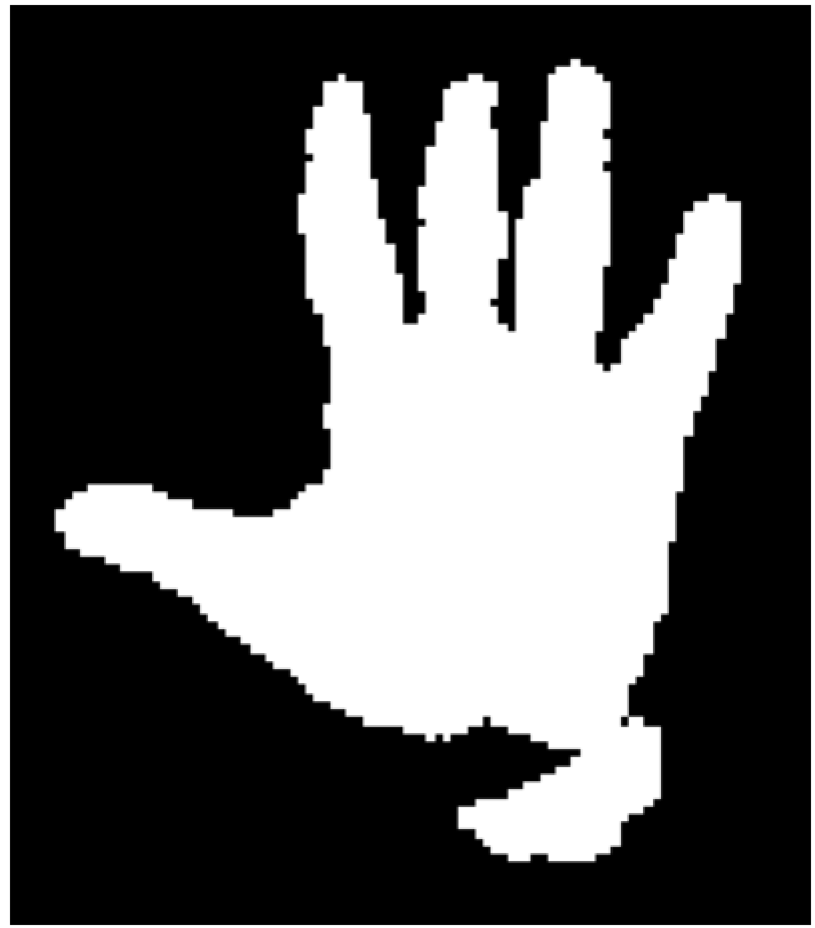
Mathematical morphology

http://www.imagemagick.org/Usage/morphology/





Introduction

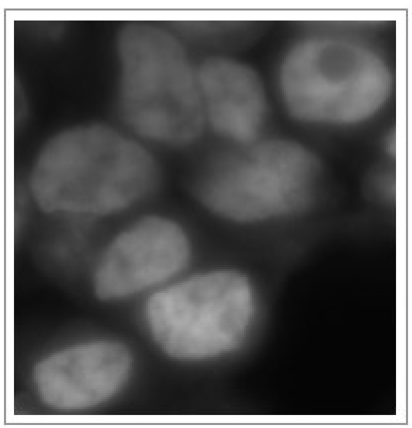
- Morphology: study of the shape (from Greek *μορφή* *e* *λογία*)
- Mathematical morphology is a **non-linear** theory of image analysis $(au(x) + v(x)) \otimes g(x) \overset{?}{=} a(u(x) \otimes g(x)) + (v(x) \otimes g(x))$
- It is not based on convolution (at least not on the type of convolution you are probably familiar with)
- It is based on the idea of **probing** an image with a smaller one to see how the image responds to it
  - By changing the shape and size of the probe, more information can be extracted

Introduction

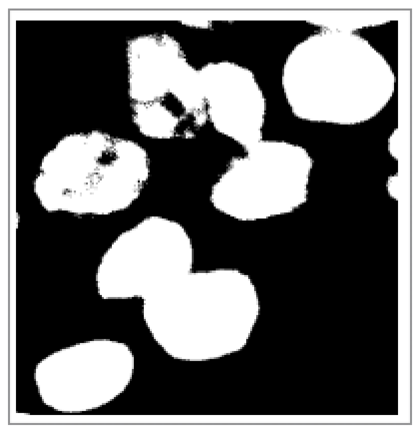
- The original theory, introduced in the late 1960, applies to **binary images**
  - Operators are applied after image thresholding
- The theory can be extended to **grayscale images**
  - In this case operators can be used before thresholding
- It is a very popular framework (effective and efficient) for image processing with several applications:
  - noise removal,
  - feature extraction,
  - shape enhancement (skeletonization, thinning, thickening)
  - image segmentation…

Preliminaries

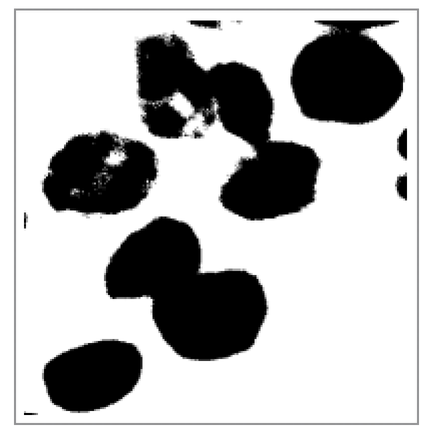
- The language of MM is set theory
  - Sets are used to represent objects in the image
- White pixels are considered the **foreground** and black pixels the **background**
  - Binary images are sets of foreground pixels



u



$I = \{x \in I | u(x) > t\}$

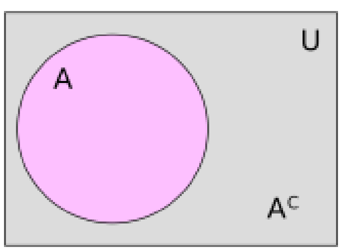


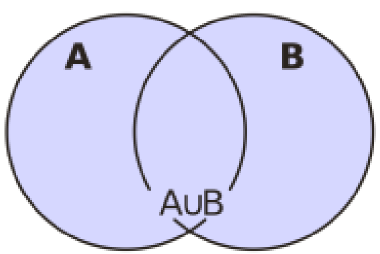
I^c

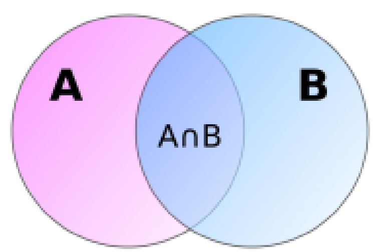
Preliminaries

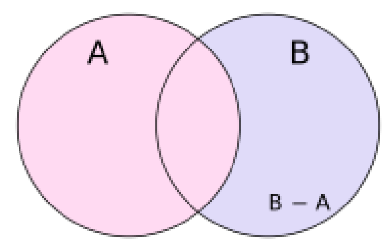
- Basic definitions. Let A be a set of points of a linear space X

$A^c = \{x \mid x \notin A\}$	Complement
$A \cup B = \{x \mid x \in A \ or \ x \in B\}$	Union
$A \cap B = \{x \mid x \in A \ and \ x \in B\}$	Intersection
$A - B = A \cap B^c$	Difference
$A_t = \{x \mid x = y + t \ for \ y \in A\}$	Translation
$\hat{A} = \{x \mid x = -y \ for \ y \in A\}$	Reflection



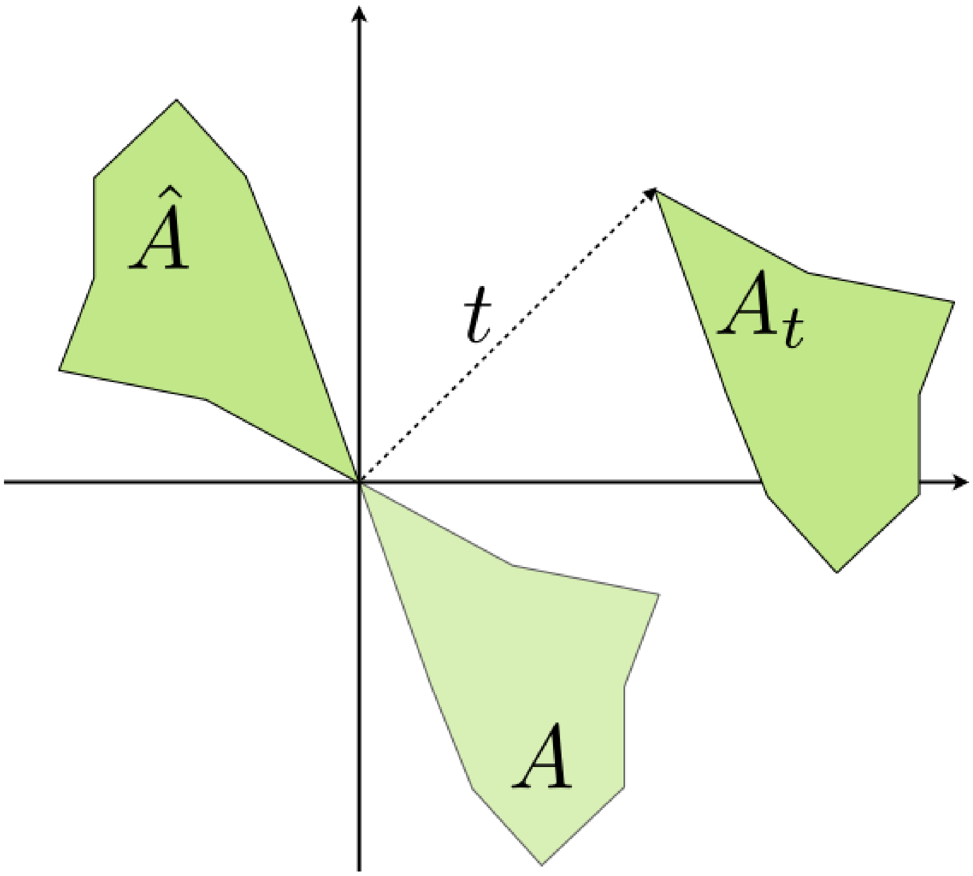






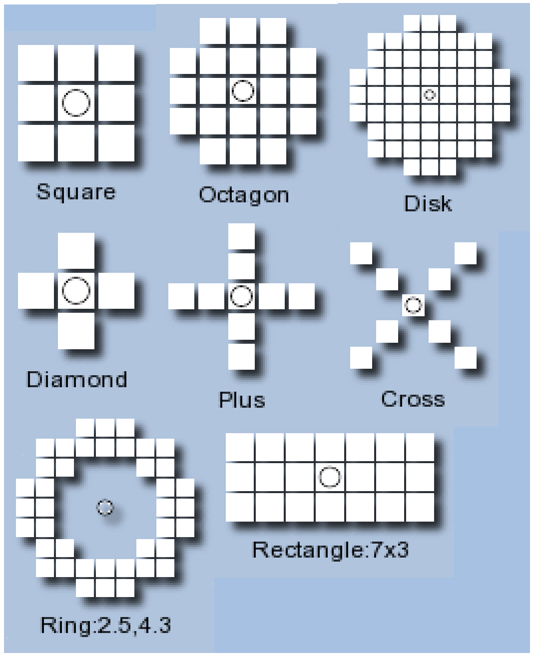
Preliminaries

- Translation and reflection are extensively used in MM



Preliminaries

- Translation and reflection are used to define operations based on **structuring elements**:
  - Small sets (objects) used to probe the image under analysis
- The SE consists of a set of points specified by their coordinates relative to an origin
  - If not specified, the origin is assumed to match the center of gravity of the SE
  - If a SE, say C, is symmetric about its origin, then $C = \hat{C}$



Basic operators

- Binary MM is based on two primitive operators: erosion and dilation

- **Erosion** of A by B:

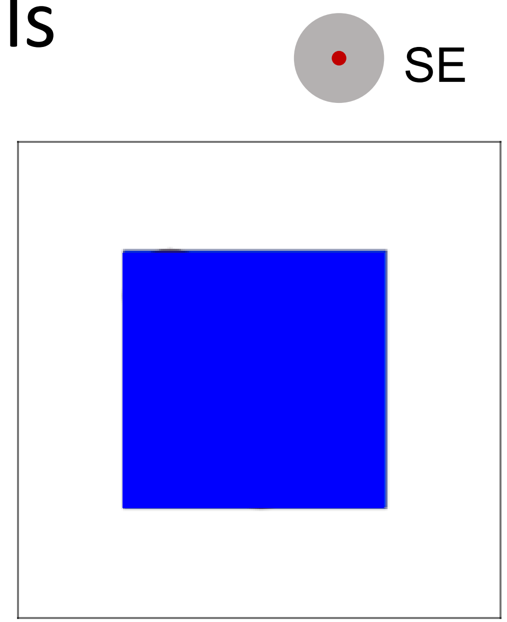
$$E(A,B)=\{z \mid B_z \subseteq A\}$$

- **Dilation** of A by B:

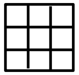
$$D(A,B)=\{z \mid \hat{B}_z \cap A \neq \varnothing\}$$

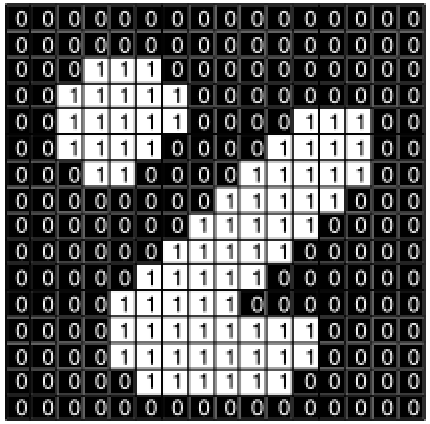
Erosion

- Erosion **thins** the image by removing FG pixels in the image where a translated version of the structuring element is not completely contained in the set of FG pixels
- The extent of thinning depends on the shape of the structuring element

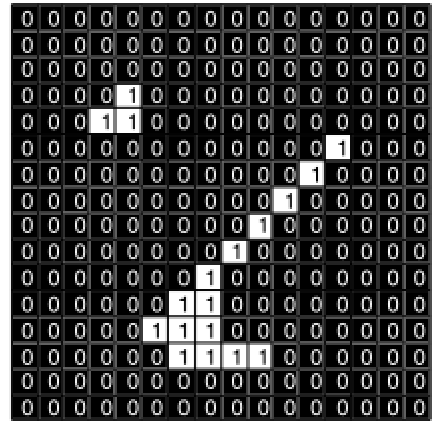


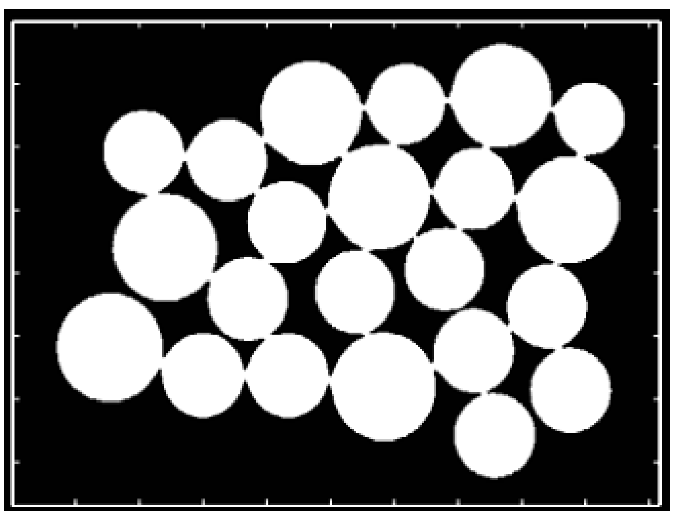
Erosion

SE 

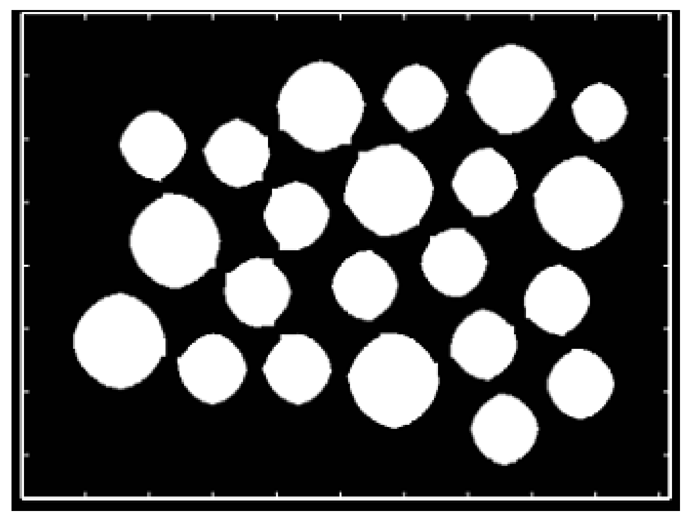


$\longrightarrow$



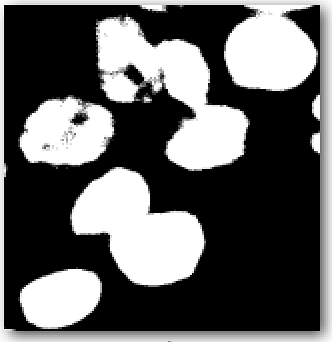


$\xrightarrow{?}$



Erosion

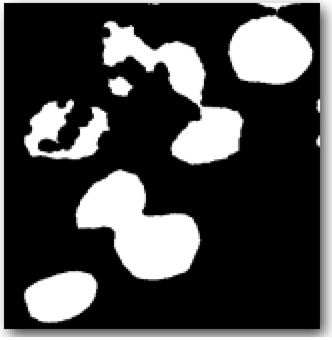
- Erosion is typically used to **remove protrusions and thin lines** that connect image regions
- The result depends on the size of the SE



A

$A \ominus D_3$

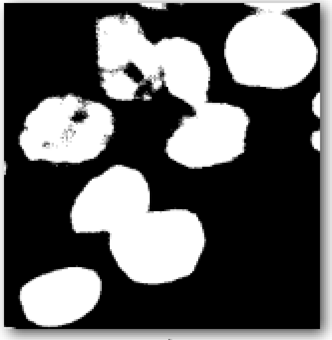
$A \ominus D_5$



$A \ominus D_{10}$

Erosion

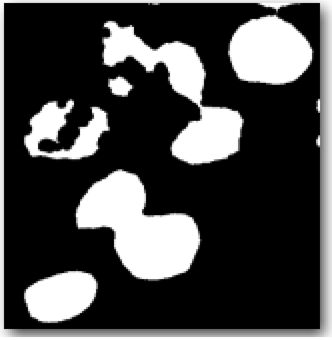
- Instead of eroding an image by a large SE, the image can be iteratively eroded by a small SE



A

$A \ominus D_3$

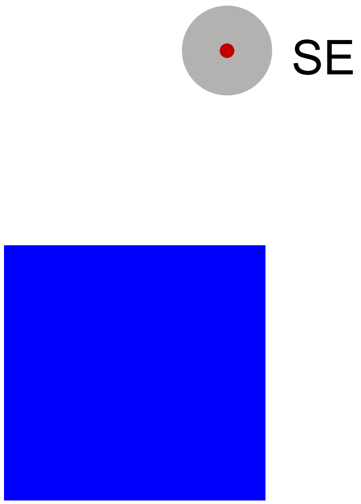
$(A \ominus D_3) \ominus D_3$



$((A \ominus D_3) \ominus D_3) \ldots \ominus D_3$

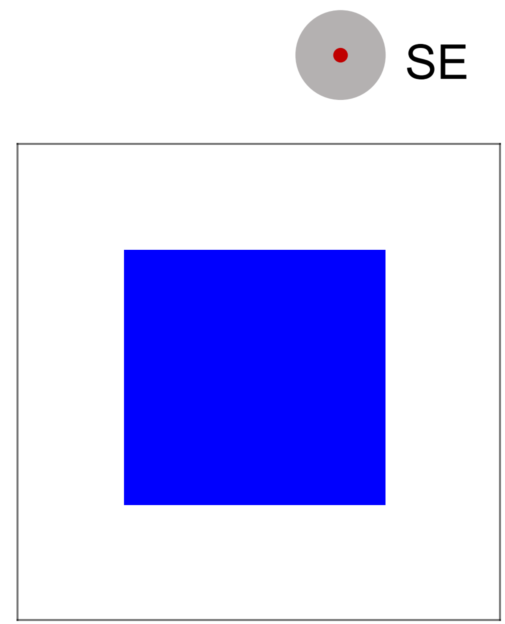
Dilation

- Dilation **thickens** the image by adding pixels where a reflected and translated version of the structuring element intersects the original set of FG pixels
- The extent of thickening depends on the shape of the structuring element
- For symmetric SE, a different geometric interpretation is usually adopted…

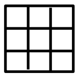


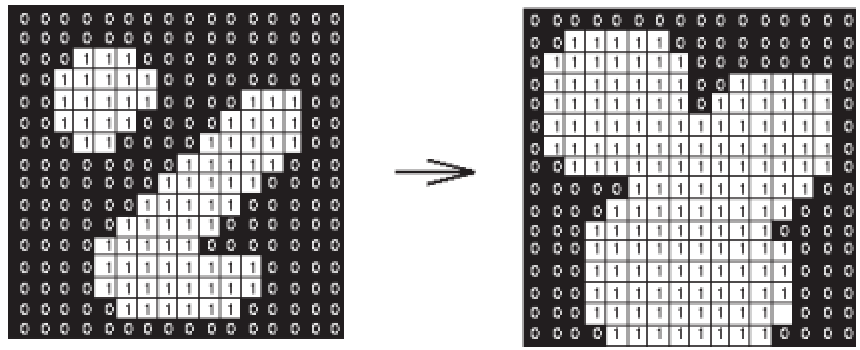
Dilation

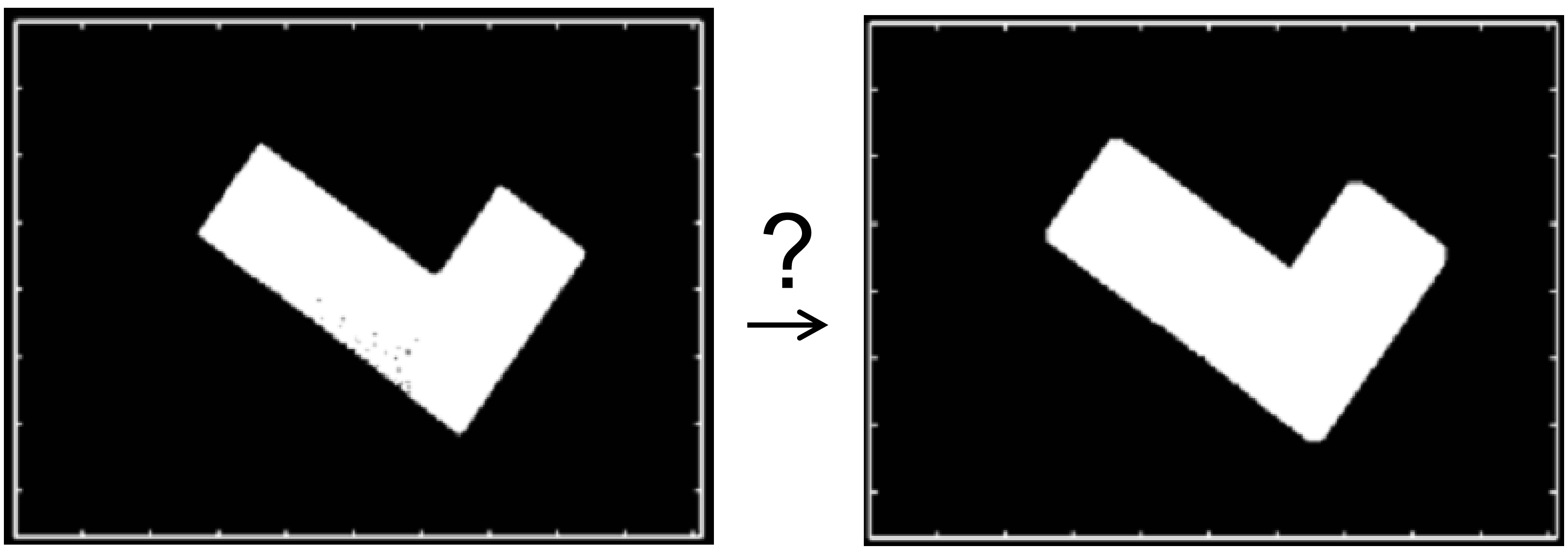
- Dilation **thickens** the image by reproducing the structuring element at every pixel present in the original image
- The extent of thickening depends on the shape of the structuring element



Dilation

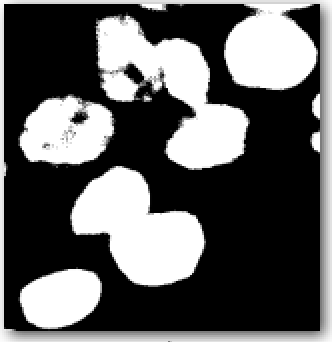
SE 





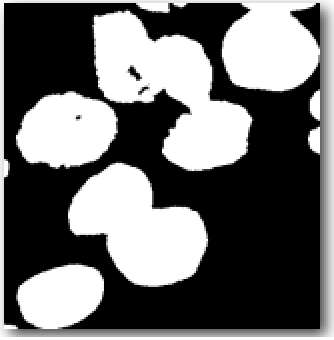
Dilation

- Dilation is typically used to **remove holes and thin gaps** so as to connect image regions
- The result depends on the size of the SE

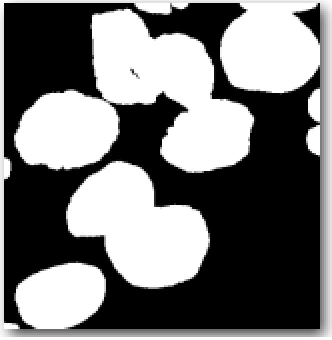


A

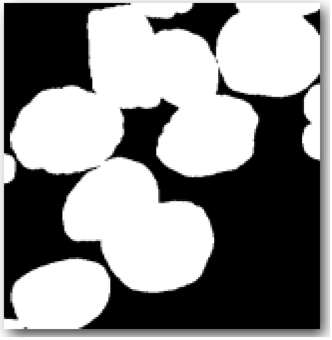
$A \oplus D_3$



$A \oplus D_5$

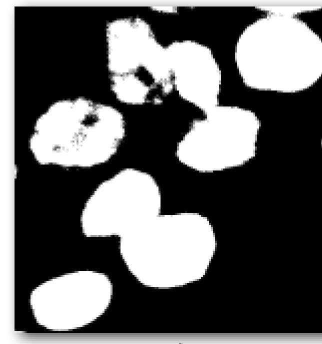


$A \oplus D_{10}$



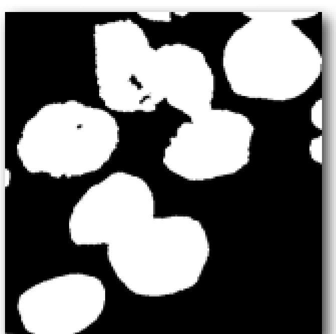
Dilation

- Instead of dilating an image by a large SE, the image can be iteratively dilated by a small SE

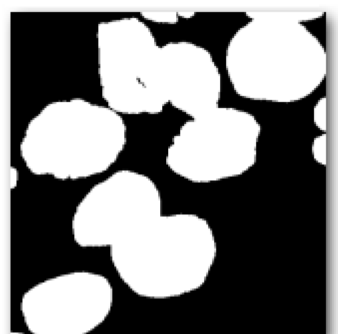


A

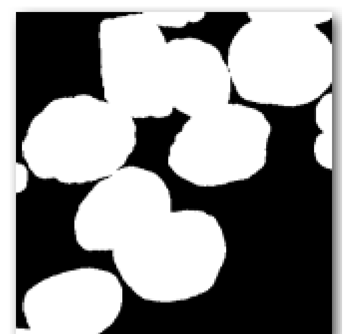
$A \oplus D_3$



$(A \oplus D_3) \oplus D_3$



$((A \oplus D_3) \oplus D_3) \ldots \oplus D_3$



Duality

- Erosion and dilation are duals of each other wrt complementation and reflection

$$E(A,B)^C = D\left(A^C, \hat{B}\right)$$

$$D(A,B)^C = E\left(A^C, \hat{B}\right)$$

Duality

- Proof

$$
\begin{aligned}
E(A,B)^C &= \{z \mid B_z \subseteq A\}^C \\
&= \{z \mid B_z \cap A^C = \varnothing\}^C \\
&= \{z \mid B_z \cap A^C \neq \varnothing\} \\
&= D(A^C, \hat{B})
\end{aligned}
$$

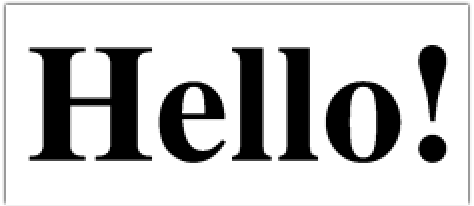
Duality

- Proof

$$
\begin{aligned}
D(A,B)^C &= \left\{ z \mid \hat{B}_z \cap A \neq \varnothing \right\}^C \\
&= \left\{ z \mid \hat{B}_z \cap A = \varnothing \right\} \\
&= \left\{ z \mid \hat{B}_z \subseteq A^C \right\} \\
&= E\left(A^C, \hat{B}\right)
\end{aligned}
$$

Duality

- In case of symmetric SE (which is usually the case), the duality principle states that we can perform any operation on A by performing the dual operation on A^C and taking the complement of the result

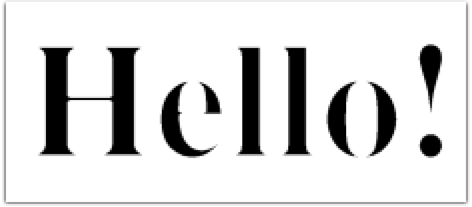


A

A^c

$A \ominus D_5$

$A^c \oplus D_5$

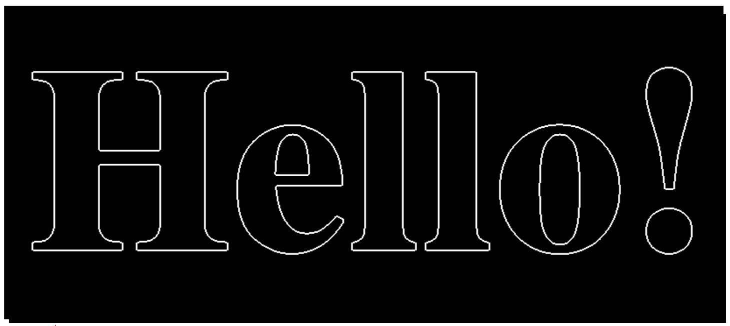


$A \oplus D_5$

$A^c \ominus D_5$

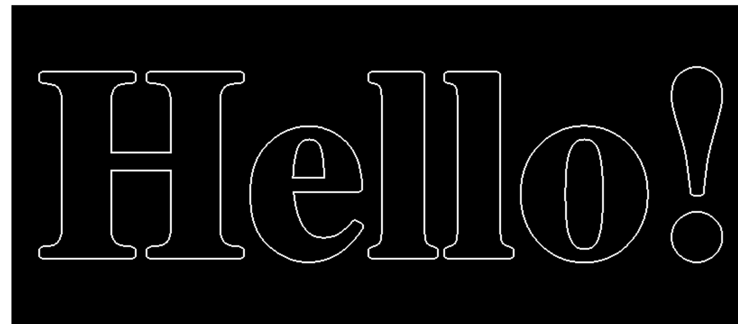
MM and spatial filtering

- Erosion and dilation can be used to define a **morphological** analogue to the image **gradient**:



$A - (A \ominus B_2)$

inner



$(A \oplus B_2) - A$

outer

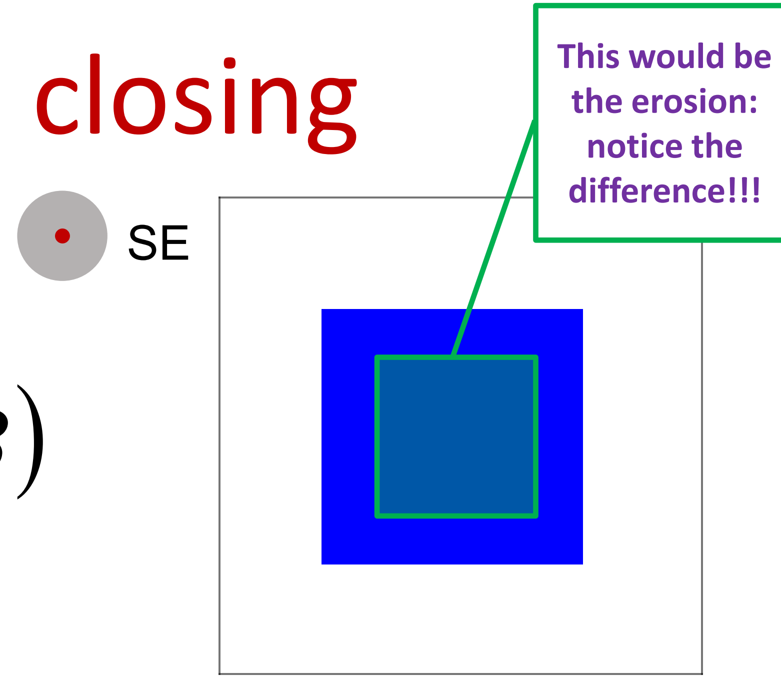
Beyond erosion and dilation

- Dilation expands the objects in the image whereas erosion shrinks them
- However, limiting ourselves to erosion and dilation wouldn't leave us with a very interesting theory
- The combination of erosion and dilation yields two important operators: Opening and Closing
  - **Opening** smoothes the object contour by removing thin protrusions and thin bridges
  - **Closing** smoothes the object contour by filling gaps and narrow breaks

Opening and closing

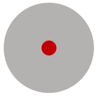
- Opening

$$A \circ B = D\left(E\left(A, B\right), B\right)$$



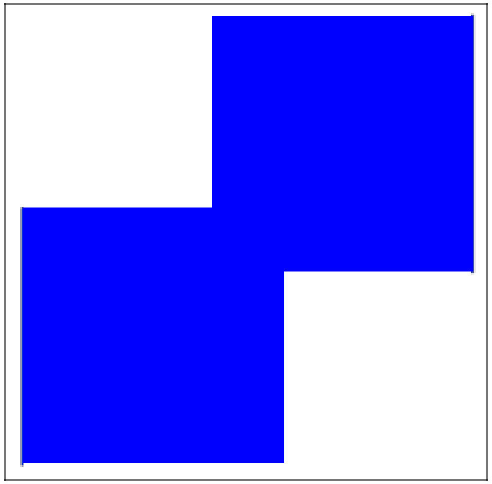
- Geometric interpretation:
  – A opened by B is the union of all translates of B that fit into A

Opening and closing



- Closing

$$A \bullet B = E\big(D(A, B), B\big)$$



- Geometric interpretation:
  - A closed by B is the complement of the union of all translates of B that do not intersect A

Duality of Open and Close

- Similarly to erode and dilate, also open and close are dual of each other with respect to set complementation and reflection

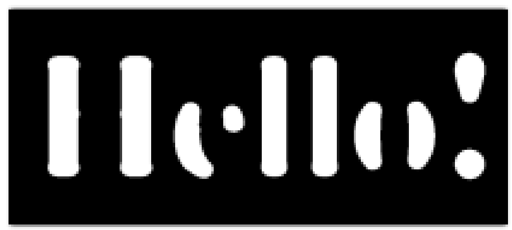
$$\left(A \circ B\right)^C = A^C \bullet \hat{B}$$

$$\left(A \bullet B\right)^C = A^C \circ \hat{B}$$

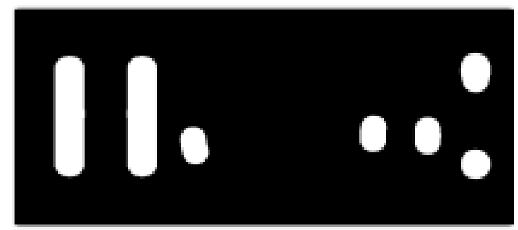
Opening and closing

A

$A \circ D_5$



$A \circ D_{15}$



$A \circ D_{21}$

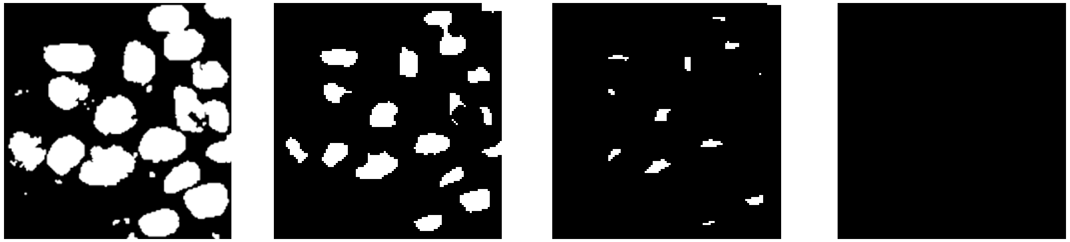
$A \bullet D_5$

$A \bullet D_{15}$

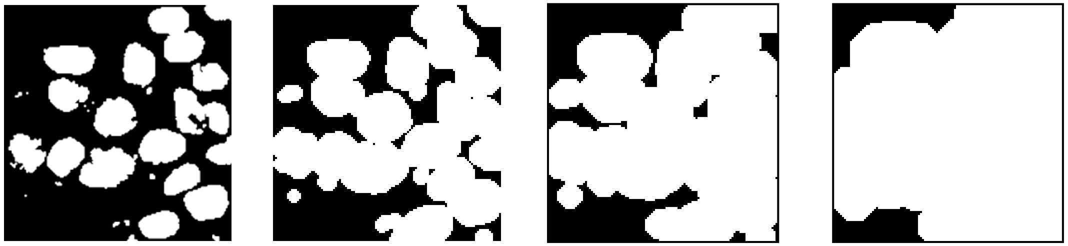
$A \bullet D_{21}$

Idempotence

- Iterated application of the erosion operator leads to the empty set



- Iterated application of the dilation operator leads to the entire image



- Does this hold also for opening and closing?

Idempotence

- A set A is said to be **B-open** if opening A with B has no effect:

$$\left(A \circ B\right) = A$$

- A set A is said to be **B-closed** if closing A with B has no effect:

$$\left(A \bullet B\right) = A$$

- **Idempotence** of opening and closing:
  - $\left(A \circ B\right)$ is B-open: $\left(A \circ B\right) \circ B = \left(A \circ B\right)$
  - $\left(A \bullet B\right)$ is B-closed: $\left(A \bullet B\right) \bullet B = \left(A \bullet B\right)$

Idempotence

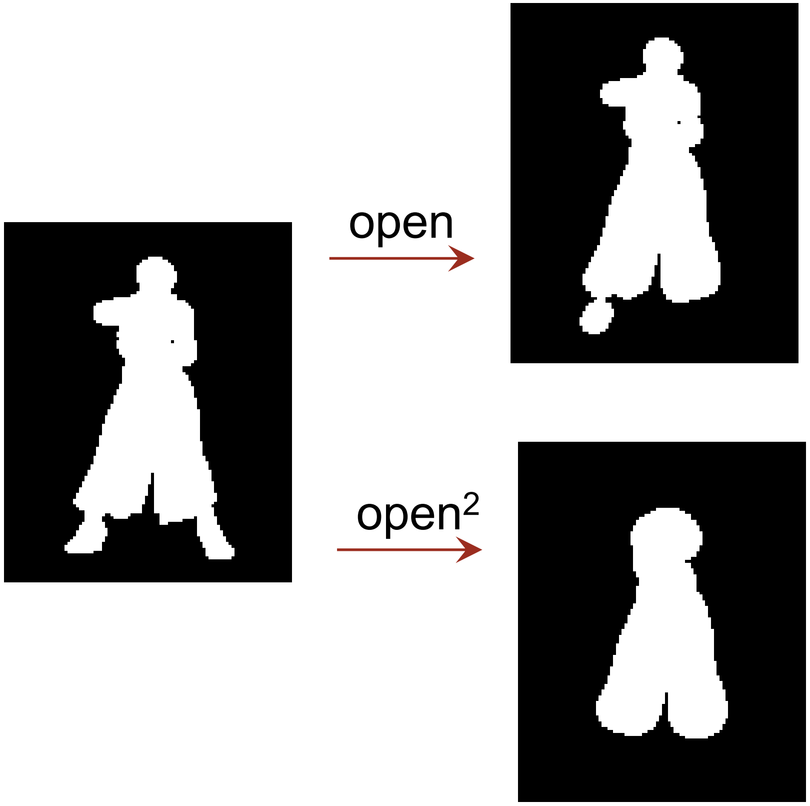
- Notice:
  - $(A \circ B)$ is B-open but it is not B-closed
  - $(A \bullet B)$ is B-closed but it is not B-open

- Since the iterated application of open (or close) is useless, to increase their effect the individual dilate and erode sub-methods are iterated:

$$A \overset{2}{\circ} B = D\left(D\left(E\left(E\left(A, B\right), B\right), B\right), B\right)$$

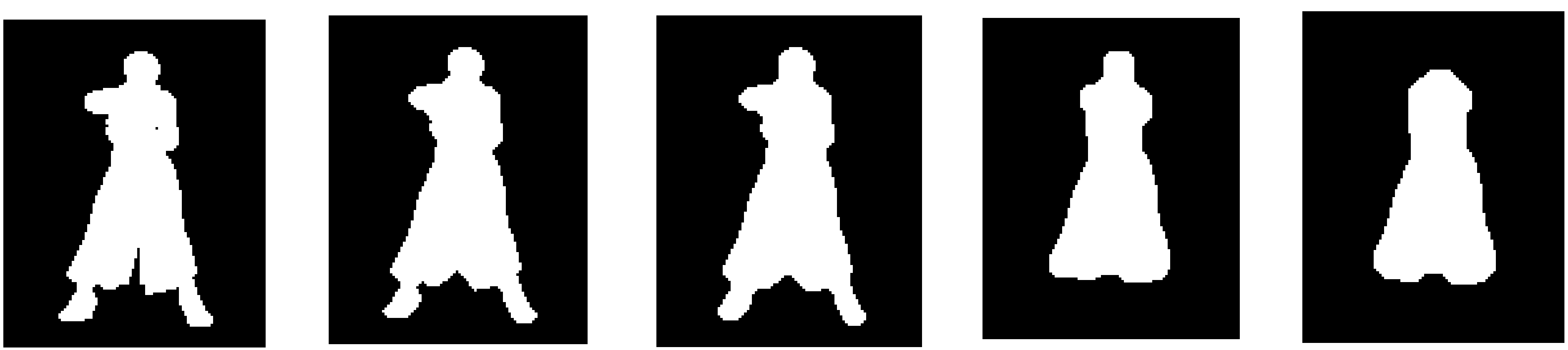
$$A \overset{2}{\bullet} B = E\left(E\left(D\left(D\left(A, B\right), B\right), B\right), B\right)$$

Morphological Open/Close



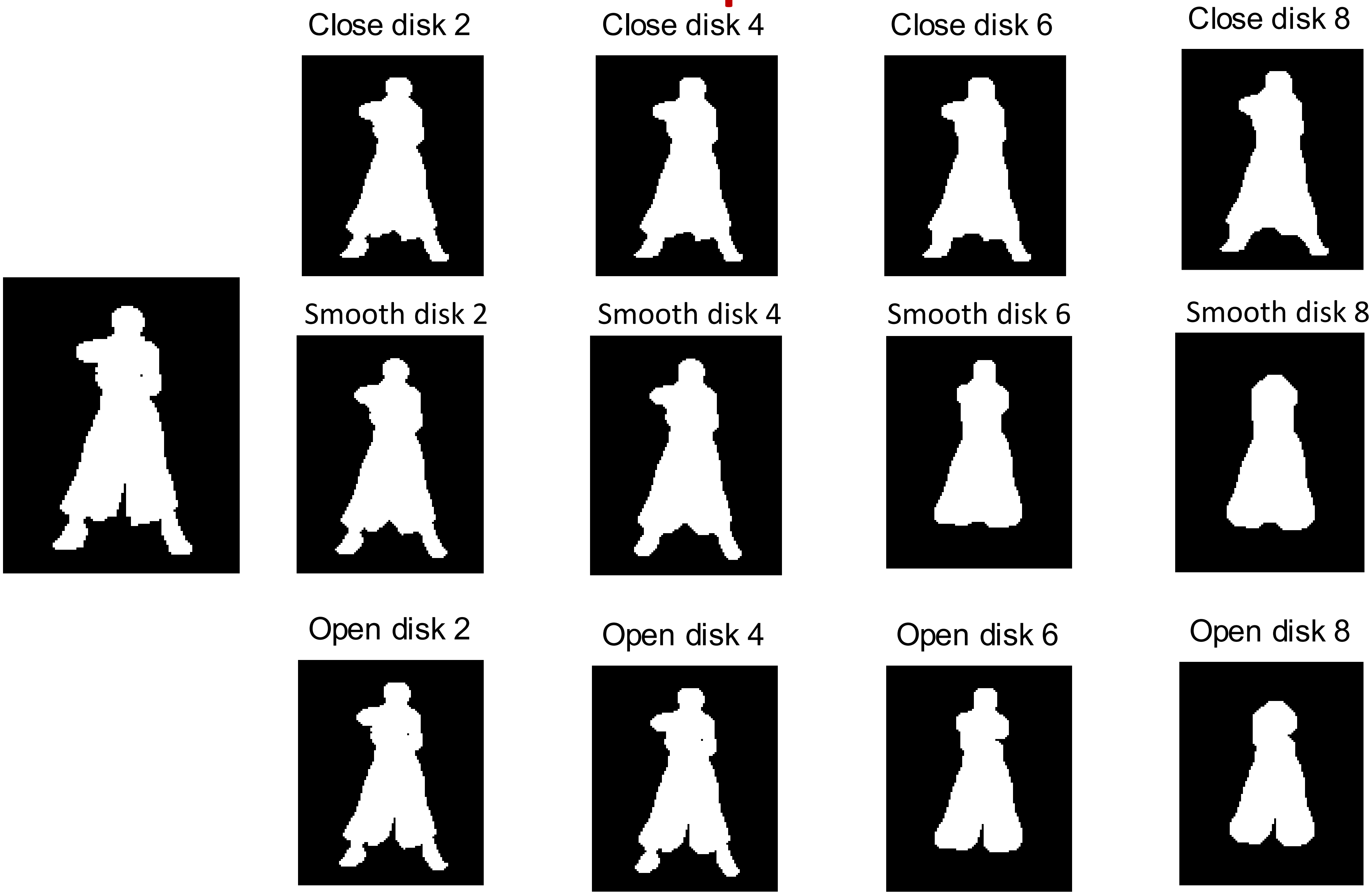
Morphological Smooth

- The **smooth** operator is obtained by applying a **open followed by a close**
  - This first removes any small objects then fills in and 'holes' or 'gaps' about the size of the SE
- The smooth operator is often repeated with slowly increasing sized SE, so as to slowly remove noise from images
- Iterated smoothing with the same SE is useless



smooth(disk(2)) **smooth(disk(4))** **smooth(disk(6))** **smooth(disk(8))**

Smooth vs Open vs Close



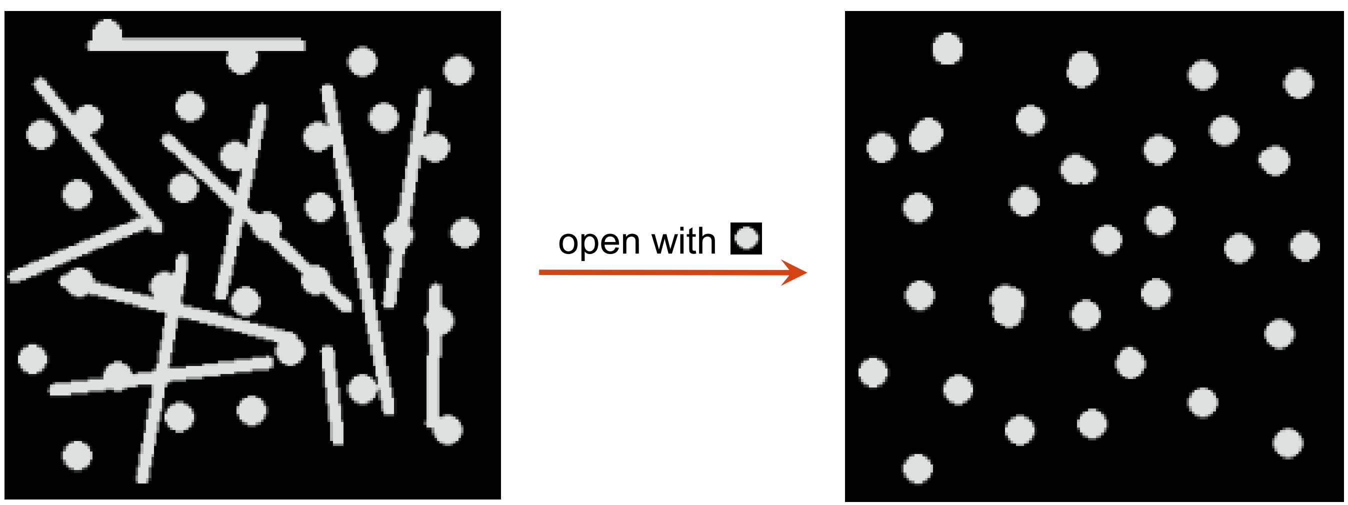
Other properties

- Opening:
  - $(A \circ B)$ is a subset of A
  - If C is a subset of D, then $(C \circ B)$ is a subset of $(D \circ B)$

- Closing:
  - A is a subset of $(A \bullet B)$
  - If C is a subset of D, then $(C \bullet B)$ is a subset of $(D \bullet B)$

More about opening

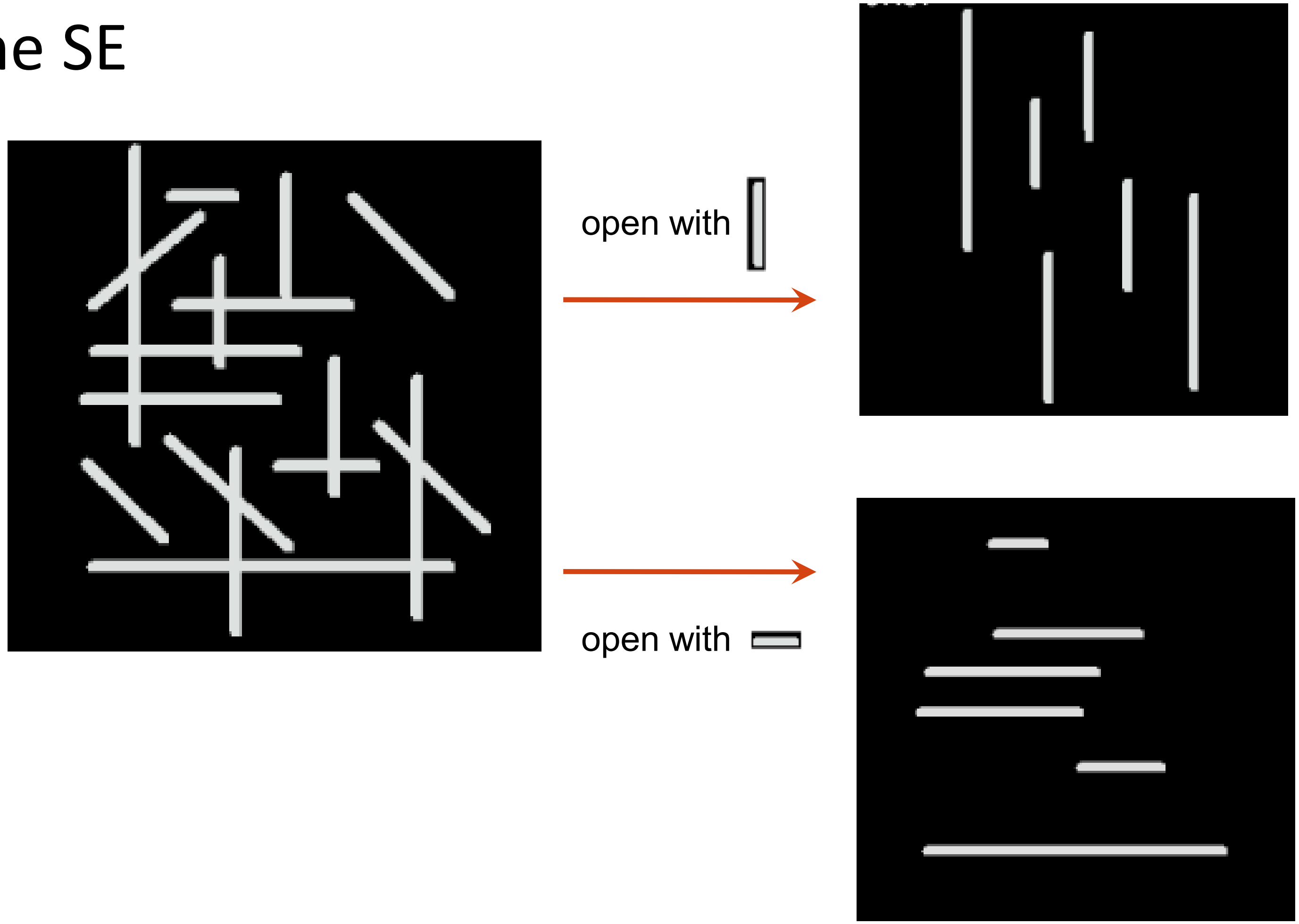
- The result of the opening operator is the union of all the parts of the image that match the SE
- This suggests a template matching analogy to highlight the parts of the image where a pattern corresponding to the template is represented



- However, there is a remarkable difference: In fact …

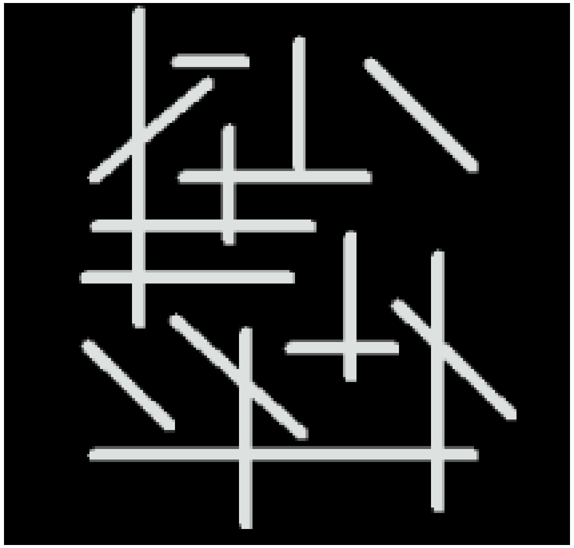
More about opening

- The result of the opening operator can be regarded as the union of all the parts of the image that match the SE



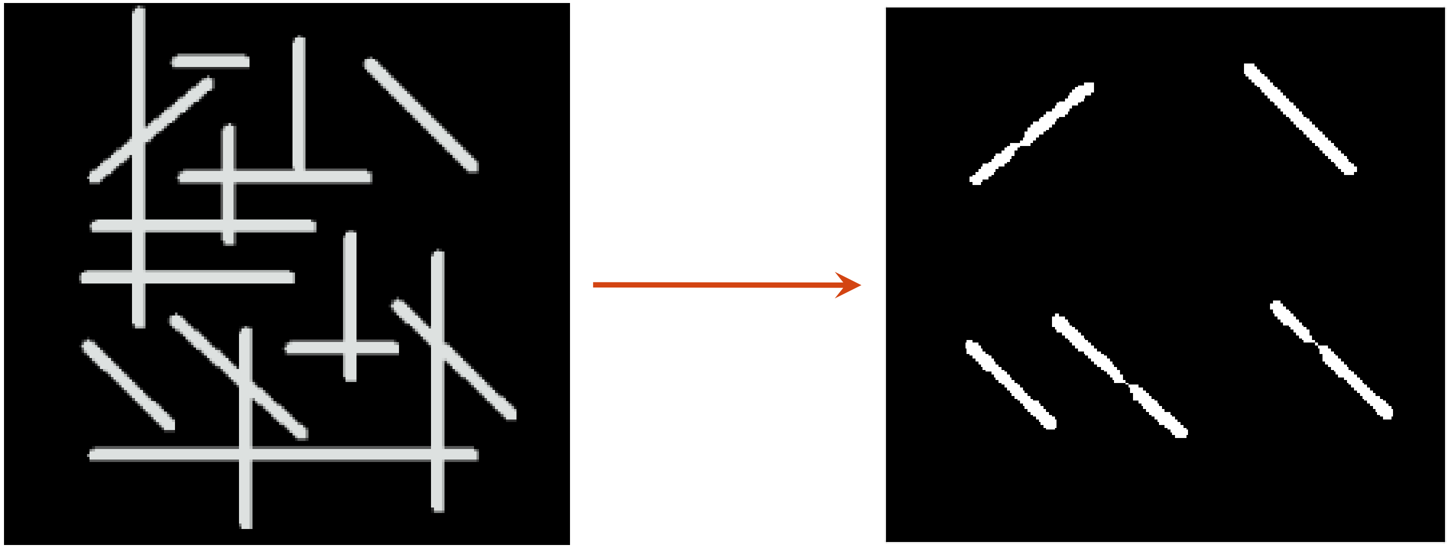
More about opening

- How to select the slanted lines?



More about opening

- How to select the slanted lines?
  - Select the horizontal lines, select the vertical ones and remove both from the original, then close with a disk



MM and spatial filtering

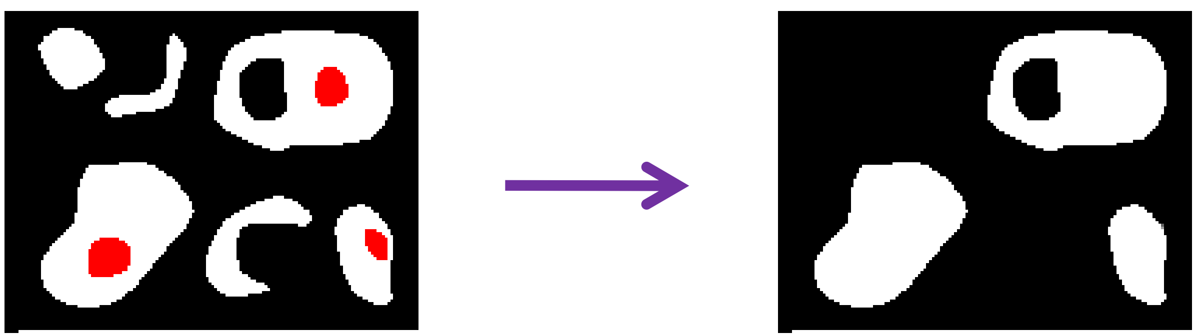
- Opening and closing are extremely good at removing shot (salt and pepper) noise

$(I \bullet D) \circ D$

$(I \circ D) \bullet D$

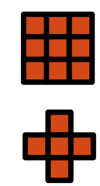
Morphological reconstruction

- In many cases it is necessary to process an image so as to **remove irrelevant regions:**
  - Retain only a subset of FG pixels, corresponding to some regions of interest
- The **morphological reconstruction** operator makes this possible under the assumption that a seed pixel (or patch) is available inside each region of interest



Morphological reconstruction

- The morphological reconstruction is used to **retain some connected components** of an image
- It is a process that involves two images and one SE
  - The first image is the **marker**: it contains the seeds for the reconstruction
  - The second image is the **mask**: it constrains the reconstruction
  - The SE implicitly defines the connectivity (4/8 neigh.)



- The process relies on the **geodesic dilation** and **geodesic erosion** operators

Geodesic dilation

- Let F and G denote the marker and mask images (both binary, $F \subseteq G$)
- The **geodesic dilation of size 1** of the marker F wrt the mask G is:

guarantees that the mask G will limit the growth of the marker F

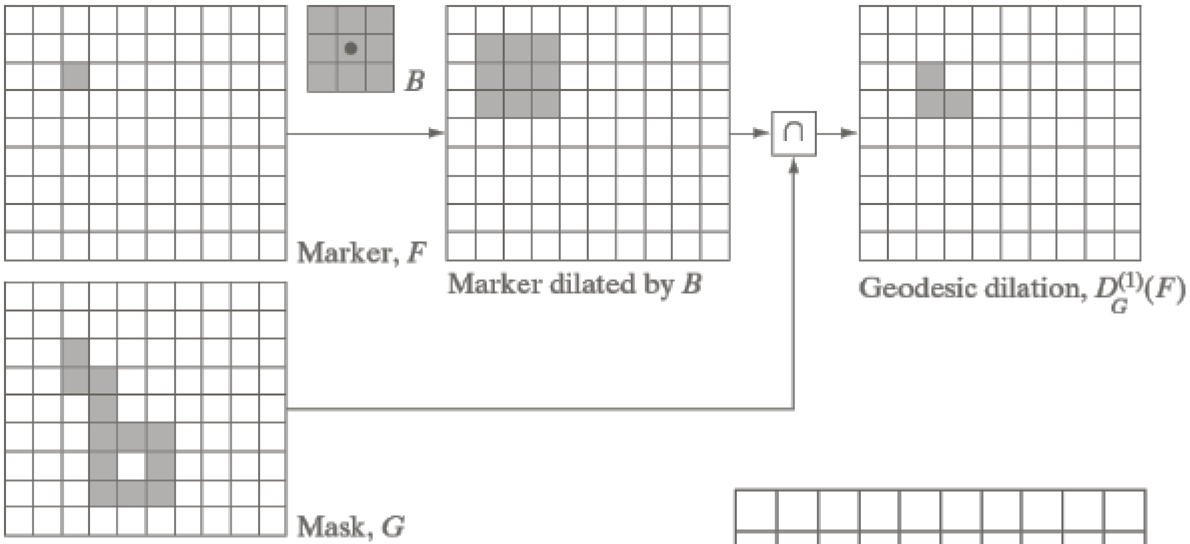
$$D_G^{(1)}(F) = (F \oplus B) \cap G$$

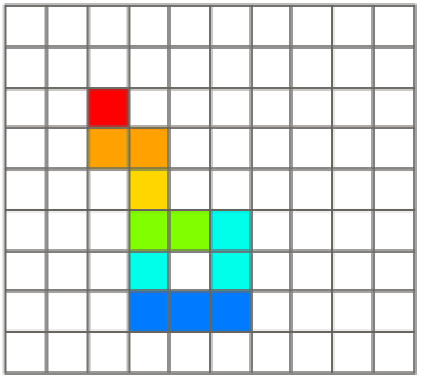
- The **geodesic dilation of size n** of the marker F wrt the mask G is:

$$D_G^{(n)}(F) = D_G^{(1)}\left(D_G^{(n-1)}(F)\right)$$

with $$D_G^{(0)}(F) = F$$

Geodesic dilation





Geodesic erosion

- Let F and G denote the marker and mask images (both binary)
- The **geodesic erosion of size 1** of the marker F wrt the mask G is:

guarantees that the mask G will limit the collapse of the marker F

$$E_G^{(1)}(F) = E(F, B) \cup G$$

- The **geodesic erosion of size n** of the marker F wrt the mask G is:

$$E_G^{(n)}(F) = E_G^{(1)}\left(E_G^{(n-1)}(F)\right)$$

with $$E_G^{(0)}(F) = F$$

Morphological reconstruction

- The **MREC by dilation** of a mask G by a marker F is the geodesic dilation of F wrt G iterated until stability

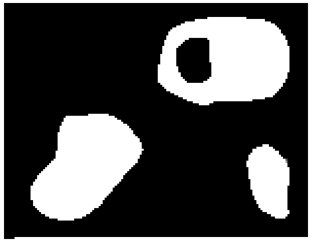
$$REC_G^D(F) = D_G^{(n)}(F) \quad \text{such that} \quad D_G^{(n)}(F) = D_G^{(n+1)}(F)$$

- The **MREC by erosion** of a mask G by a marker F is the geodesic erosion of F wrt G iterated until stability

$$REC_G^E(F) = E_G^{(n)}(F) \quad \text{such that} \quad E_G^{(n)}(F) = E_G^{(n+1)}(F)$$

Morphological reconstruction

- Sometimes it is necessary to retain only a subset of the connected components of the foreground data:
  - Retain only the regions of X that include the red marker Y



- This is obtained by the morphological reconstruction by dilation of the set X by the marker Y:

$$REC_X^D(Y)$$

Morphological reconstruction

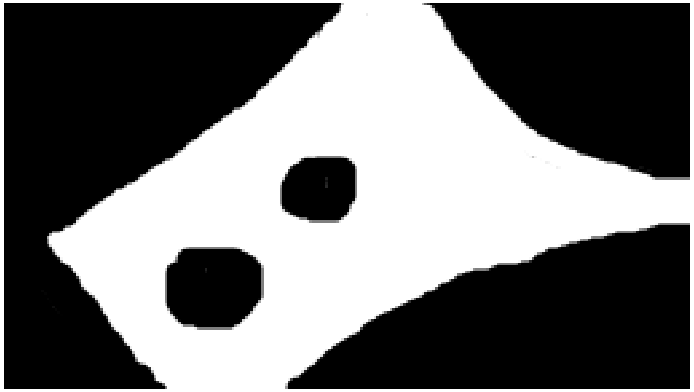
- The morphological reconstruction can be used for different purposes
- The result of its application depends on the marker, the mask and the SE (to encode the connectivity)
- If the marker is obtained by iteratively eroding the mask, the **open by reconstruction of size n** operator is defined:
  - This restores only those parts of the mask that remain after iterating the erosion by B

$$O_R^n(F) = REC_F^D(E(F, nB))$$

where $\boldsymbol{E(F, nB)}$ is the iterated (n times) erosion of F by B

Morphological reconstruction

- Can we define a fully automatic procedure for filling all the holes that are present in image regions?
- Let I be the image that we want to process for hole filling



Morphological reconstruction

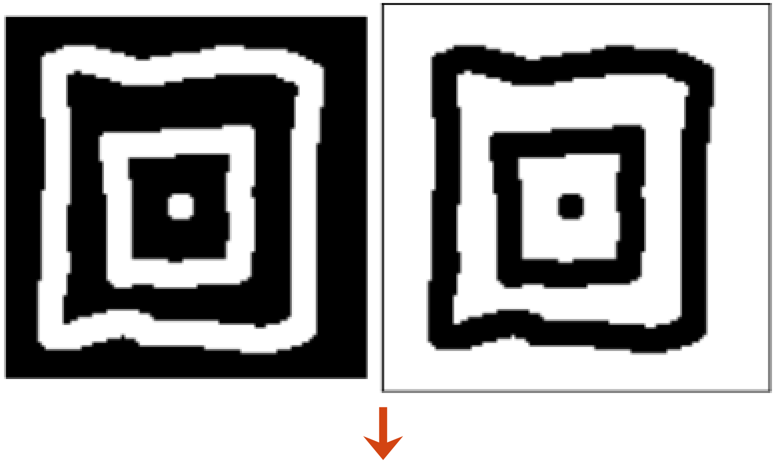
- Can we define a fully automatic procedure for filling all the holes that are present in image regions?
- Let I be the image that we want to process for hole filling
- Let the marker F be defined as 0 everywhere except at the image border where it is set to 1 – I

$$F(x,y)=\begin{cases}1-I(x,y) & \text{if } (x,y) \text{ is on the border} \\ 0 & \text{otherwise}\end{cases}$$

- The result of $\left[REC_{I^C}^{D}(F)\right]^C$ is a binary image equal to I with all holes filled

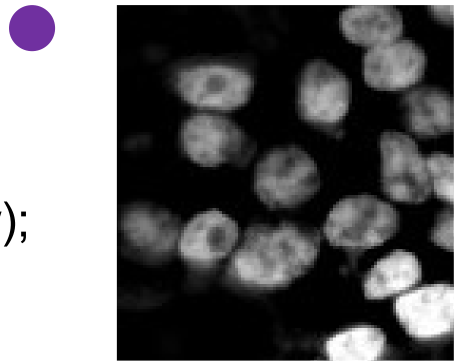
Morphological reconstruction

- How this procedure expected to operate on the following two images?



Morphological reconstruction

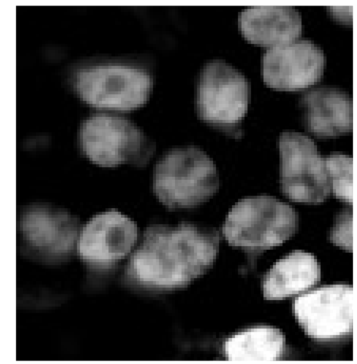
- How could we process the image so as to retain all the cells that are larger than a disk of radius 7 ?
  - img_bw = im2bw(img, graythresh(img));
  - se = strel('disk',7);
  - img_eroded = imerode(img_bw,se);
  - img_reco = imreconstruct(img_eroded, img_bw);



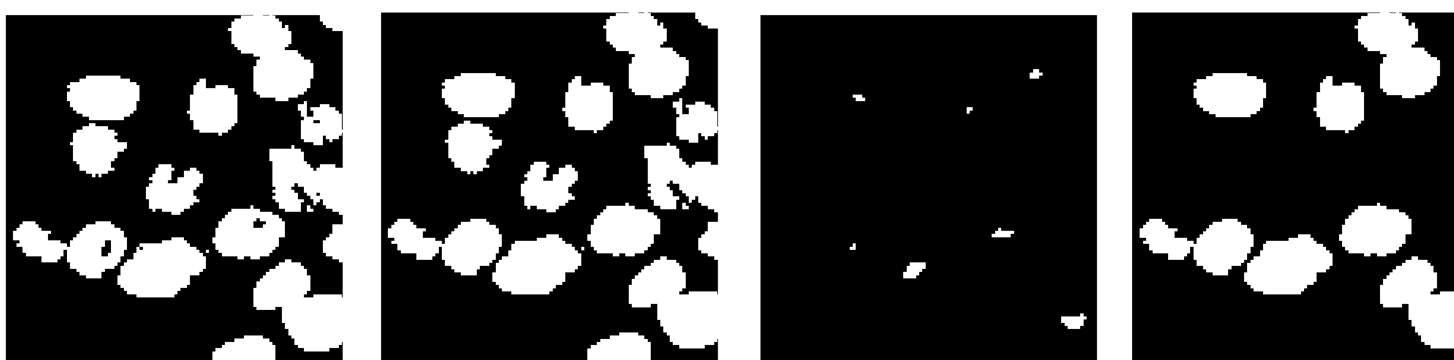
Notice that regions with one or more holes have been discarded: perform hole filling before open by reconstruction…

Morphological reconstruction

- How could we process the image so as to retain all the cells that are larger than a disk of radius 7 ?
  - img_bw = im2bw(img, graythresh(img));
  - se = strel('disk',7);
  - img_noholes = imfill(img_bw, 'holes');
  - img_eroded = imerode(img_noholes,se);
  - img_reco = imreconstruct(img_eroded, img_noholes);



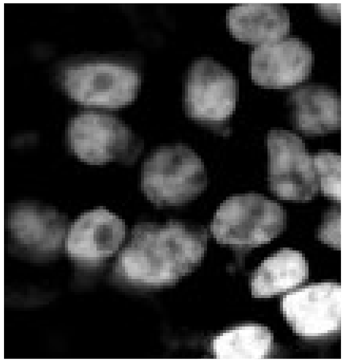
If img_bw is used as mask, regions are reconstructed with their holes inside

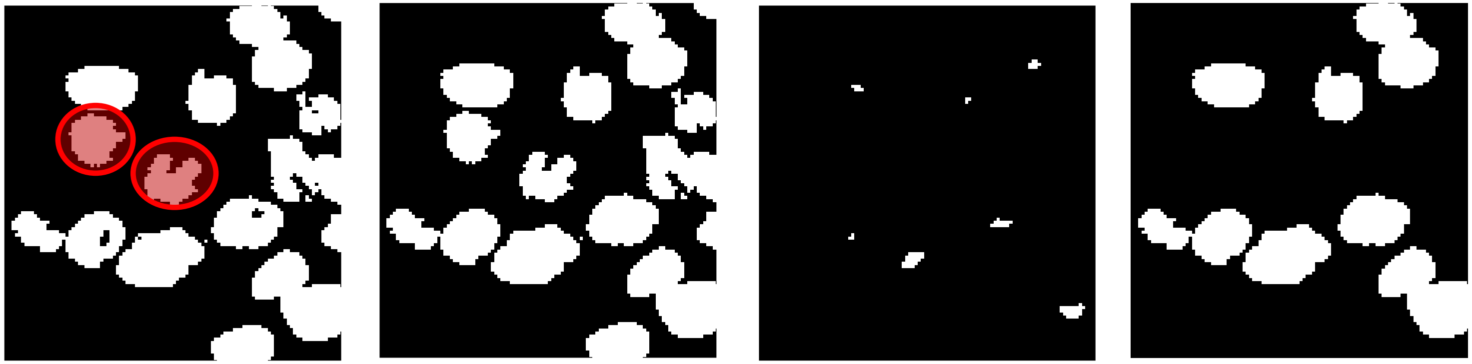


Morphological reconstruction

- How could we process the image so as to retain all the cells that are larger than a disk of radius 7 ?

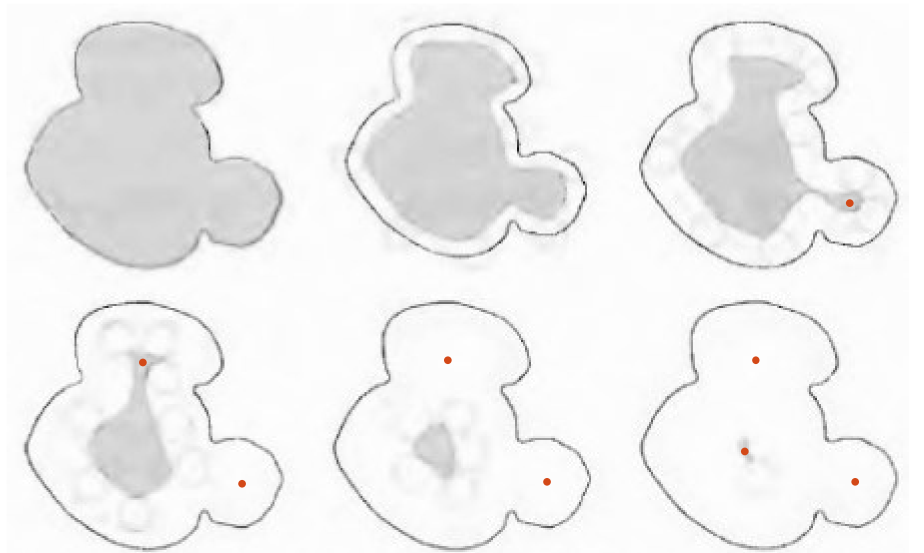
Notice that due to the results of thresholding (that removes some dark parts of the cells) some blobs are discarded although they are larger than a disk of radius 7





Ultimate erosion

- The **ultimate erosion** operator identifies a reduced set of seed points that support the reconstruction of region R (useful for robust marker selection)
- Consider the case of a region R that is iteratively eroded:
  - At some steps, one connected component may originate disconnected parts
  - The **union of residuals** of all connected components (just before they disappear) identifies the ultimate erosion of the original region R

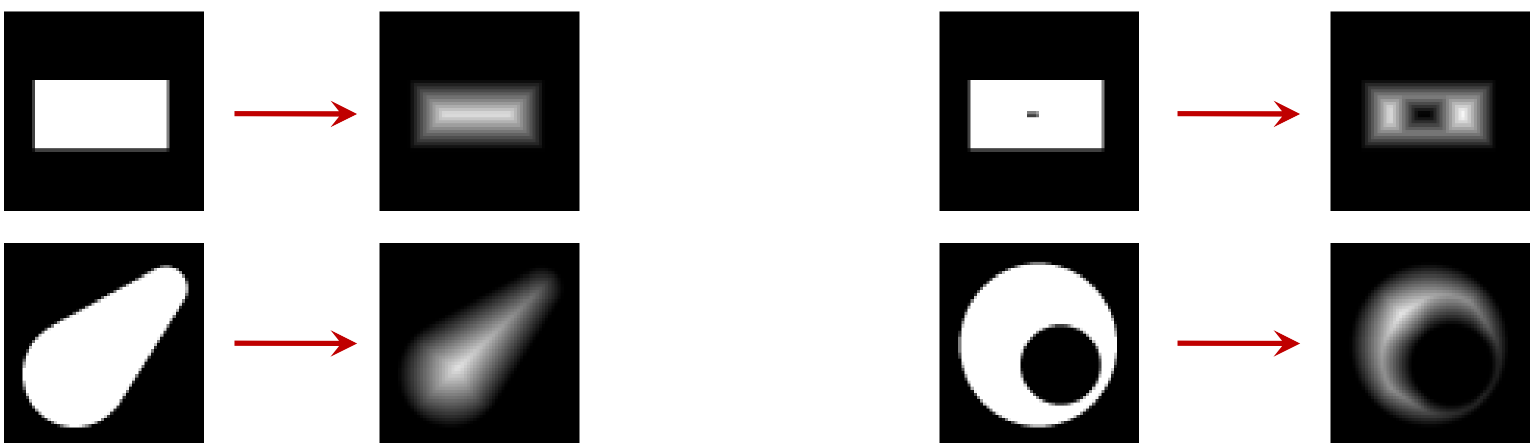


Distance transform

- In many cases it is necessary to apply the erosion operator iteratively (i.e. open by reconstruction of size n)
- A convenient way to perform this computation efficiently is through the **distance transform** operator

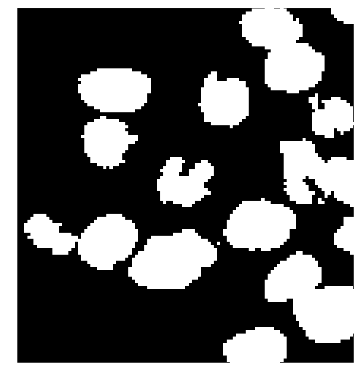
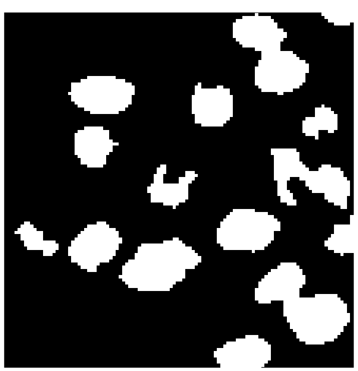
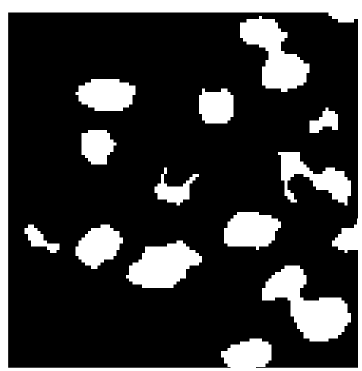
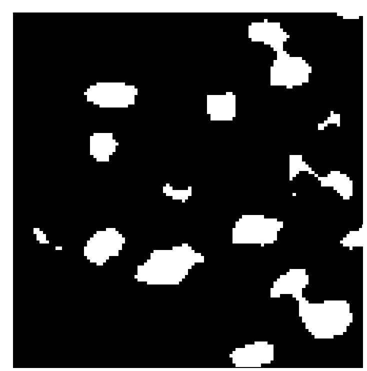
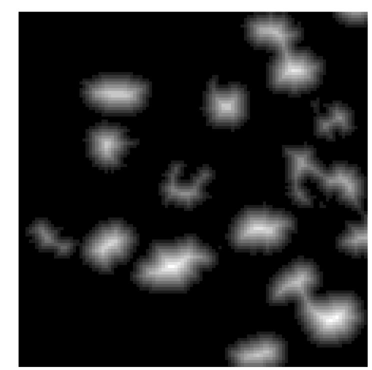
$$DT(A)[x] = \min_{y \in A^C} \|x - y\|$$

- The distance transform is a function that returns for each point $x \in A$ the distance to the nearest point in A^C



Distance transform

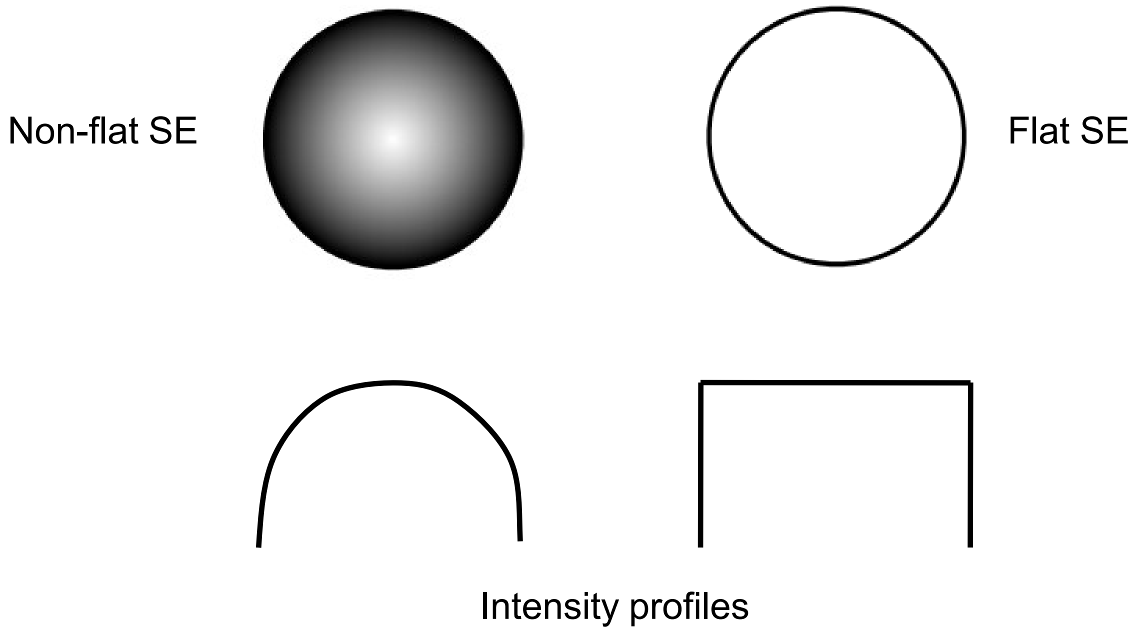
- Using the distance transform, the iterated application of erosion can be computed by simple thresholding
- Local maxima of the distance transform of a region R represent the ultimate erosion of R

LATTICE MORPHOLOGY

Introduction

- Morphological operators defined for binary images can be extended to operate on graylevel images
- Although in its most general form the framework allows also the structuring element to have graylevel values, we will focus on the case of **flat** (binary) SE



Erosion and dilation

- Let b a SE and S(b), its support, the set of points (u,v) such that b(u,v)=1
- **Erosion** of f(x,y) by b:

$$E(f,b)[x,y] = \min_{(u,v)\in S(b)} \{f(x+u, y+v)\}$$

- **Dilation** of f(x,y) by b:

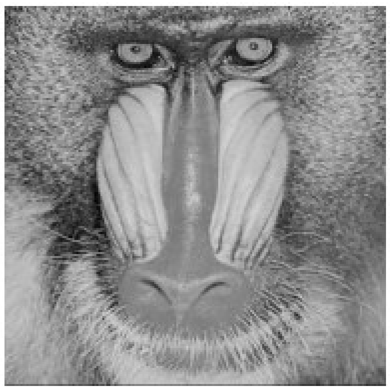
The minus sign accounts for the reflection of b = b(-x,-y)

$$D(f,b)[x,y] = \max_{(u,v)\in S(b)} \{f(x-u, y-v)\}$$

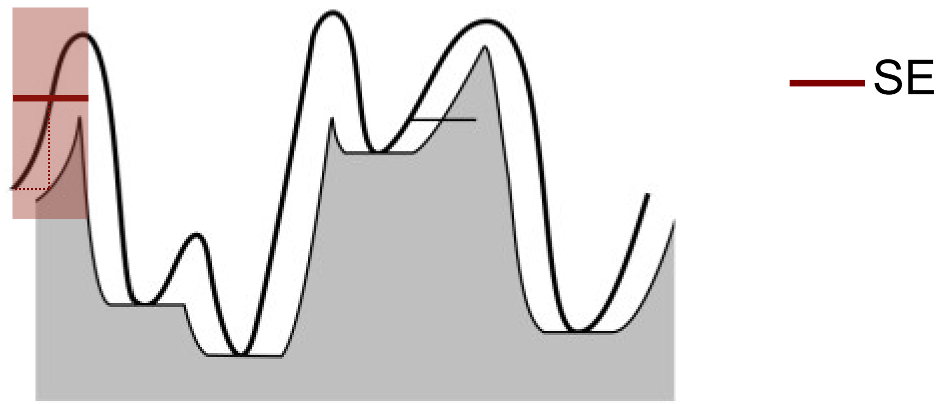
In case F is a binary image these definitions are consistent with binary operators

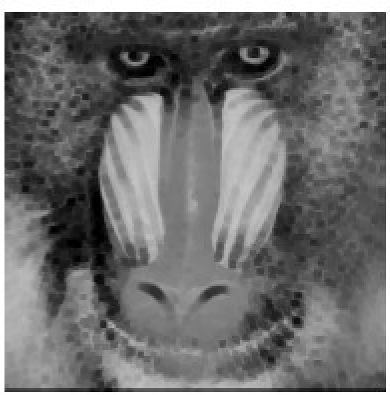
Erosion and dilation

- The eroded image is darker than the original



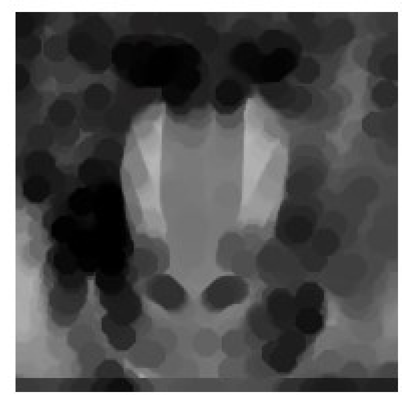
f





$f \ominus g_{D_3}$

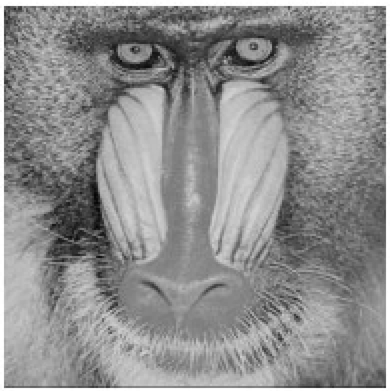
$f \ominus g_{D_5}$



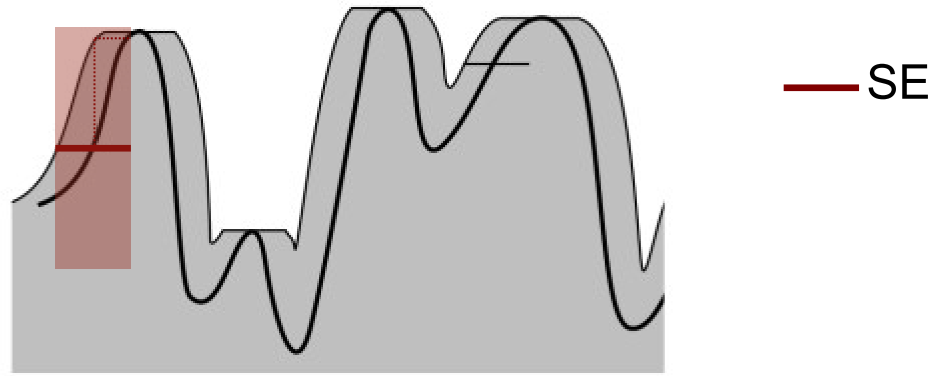
$f \ominus g_{D_{11}}$

Erosion and dilation

- The dilated image is lighter than the original



f



$f \oplus g_{D_3}$

$f \oplus g_{D_5}$

$f \oplus g_{D_{11}}$

Morphological gradient

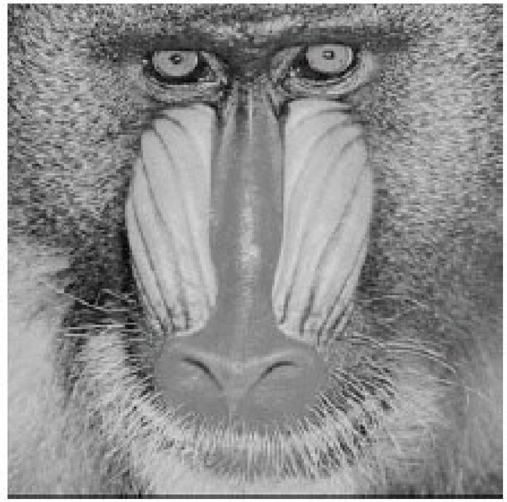
- Dilation and erosion can be combined to obtain the morphological gradient of an image

$$Gr(f,b) = D(f,b) - E(f,b)$$

- The dilation thickens image regions whereas the erosion shrinks them
  – Homogeneous regions are not affected, hence they are removed by the difference
  – As a result, edges are enhanced and homogeneous regions suppressed

Compared to the binary operator, the result is a grayscale image

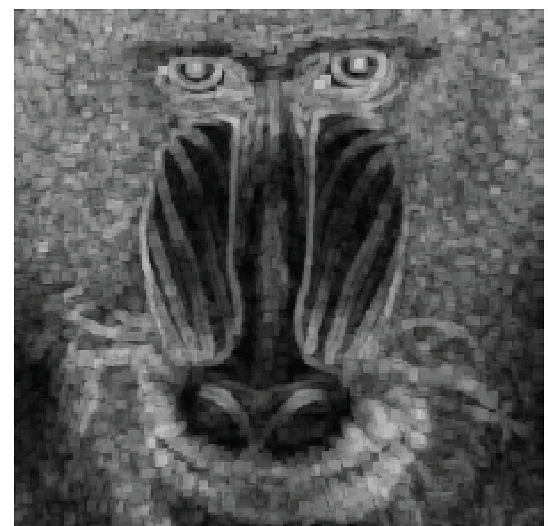
Morphological gradient



f

$(f \oplus g_{D_3}) - f$

$f - (f \ominus g_{D_3})$



$(f \oplus g_{D_3}) - (f \ominus g_{D_3})$

Beyond erosion and dilation

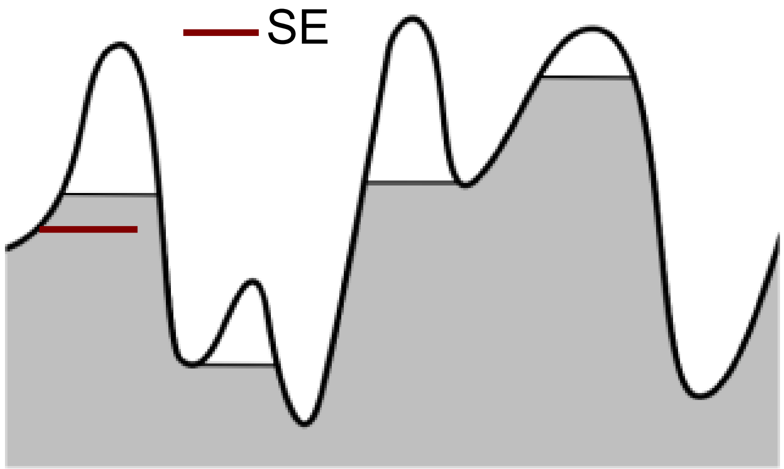
- Erosion and dilation can be used in combination to yield the opening and closing operators

- **Opening** of f by *b*: $\mathrm{f} \circ b = D\left(E\left(f, b\right), b\right)$

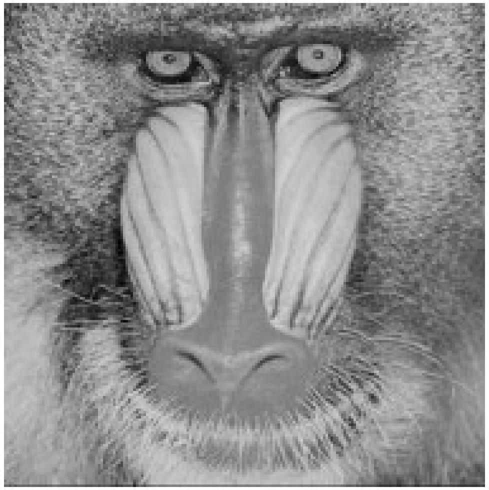
- **Closing** of f by *b*: $\mathrm{f} \bullet b = E\left(D\left(f, b\right), b\right)$

Opening

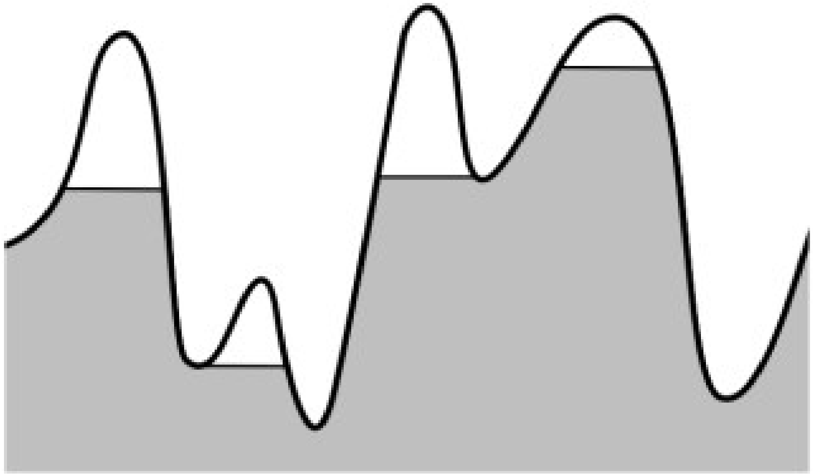
- Suppose that f(x,y) is viewed as a surface in 3D
- Opening of f by b can be viewed as pushing b up from below against the undersurface of f
- Opening removes small (smaller than b) bright details leaving the overall intensity and larger bright areas unaltered

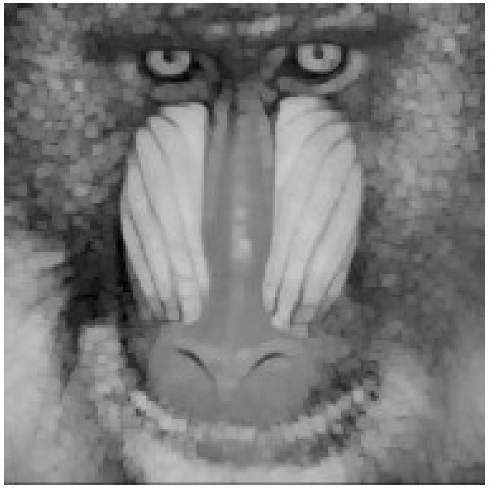


Opening



f



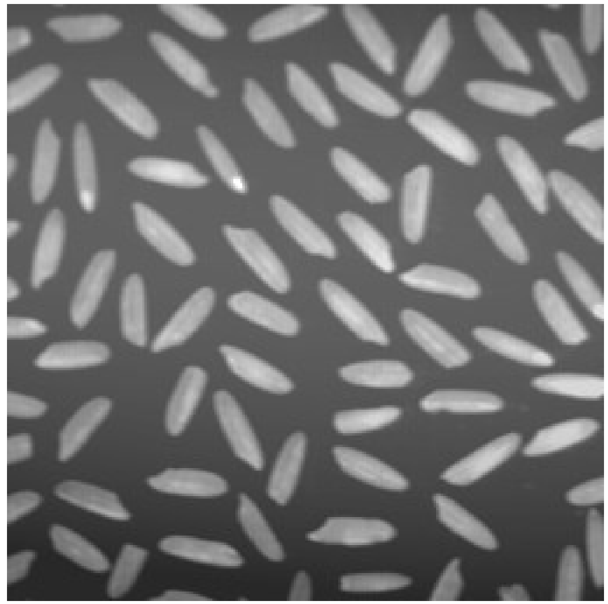


$f \circ g_{D_3}$

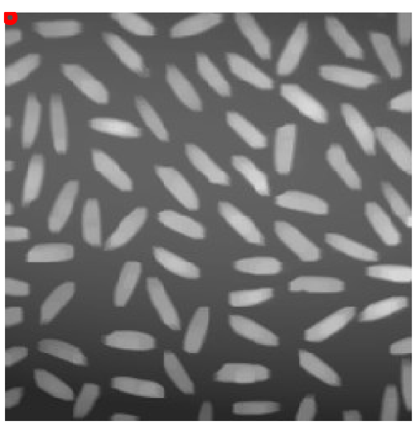
$f \circ g_{D_5}$

$f \circ g_{D_{11}}$

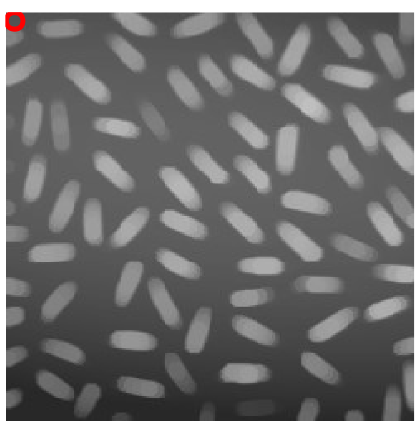
Opening



Radius 3



Radius 5

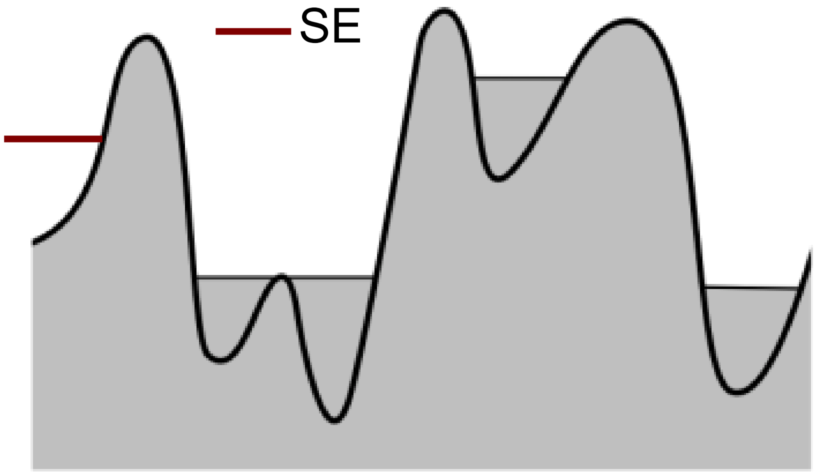


Radius 9

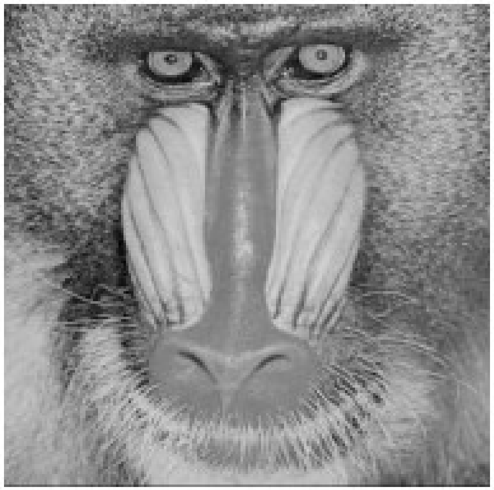
Radius 19

Closing

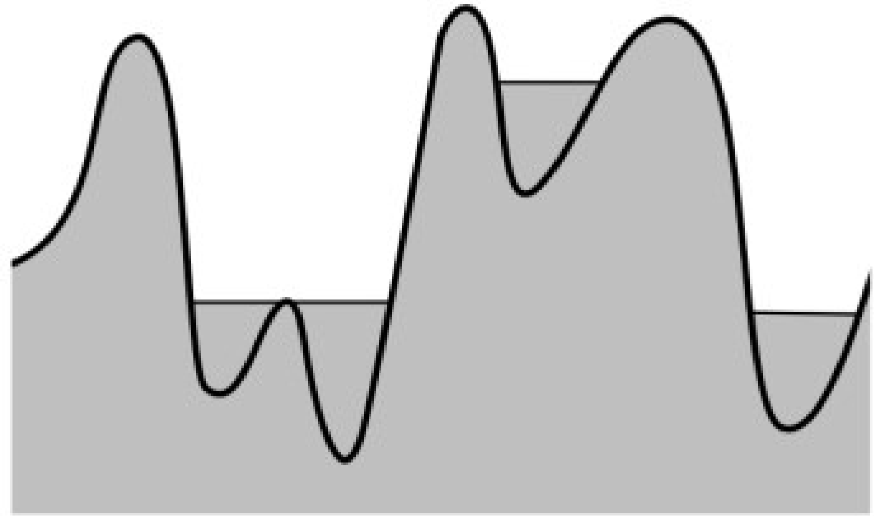
- Closing of f by b can be viewed as pushing b down over the surface of f
- Closing removes small (smaller than b) dark details leaving the overall intensity and larger dark areas unaltered



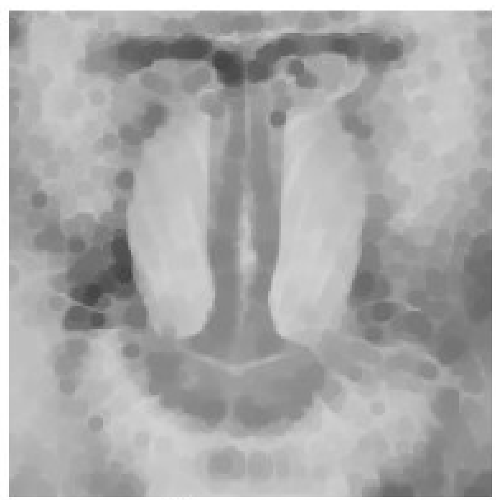
Closing



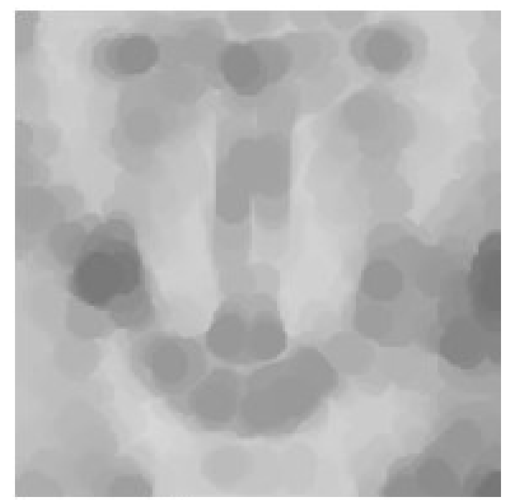
f



$f \bullet g_{D_3}$

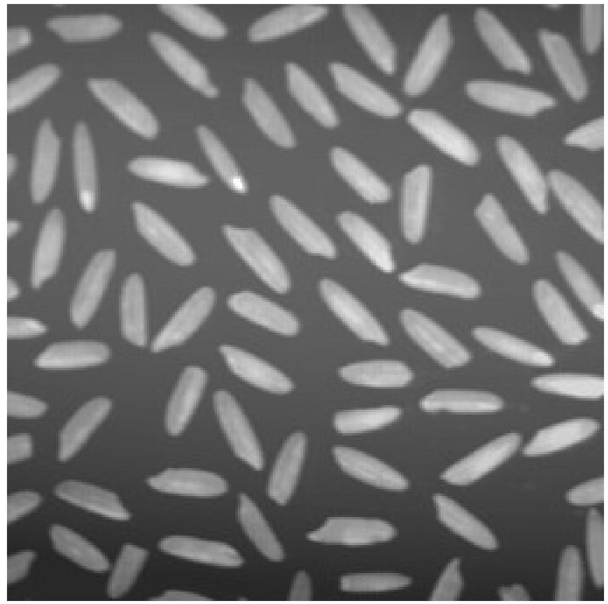


$f \bullet g_{D_5}$

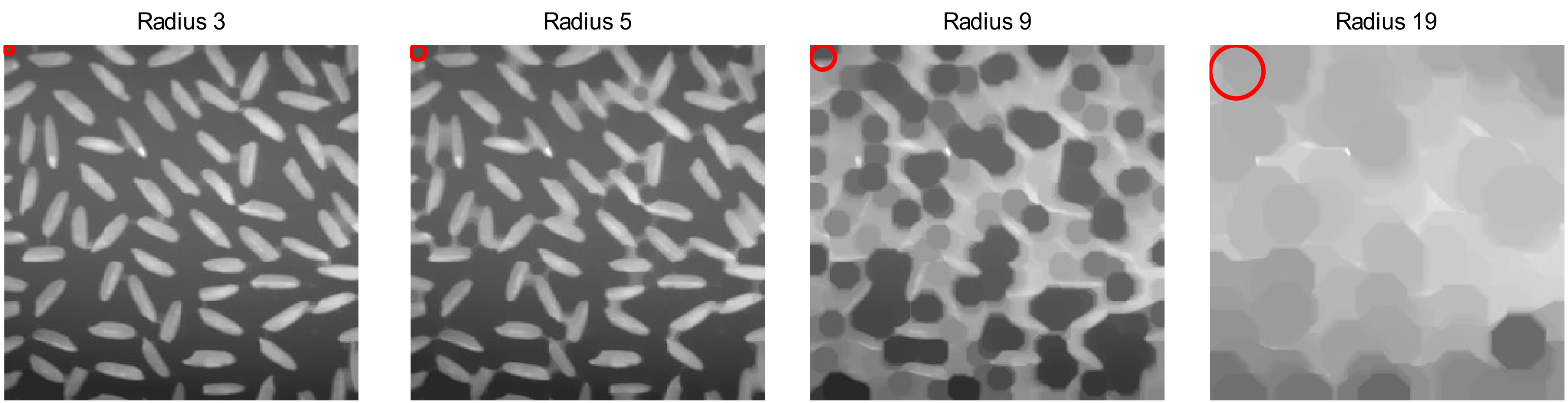


$f \bullet g_{D_{11}}$

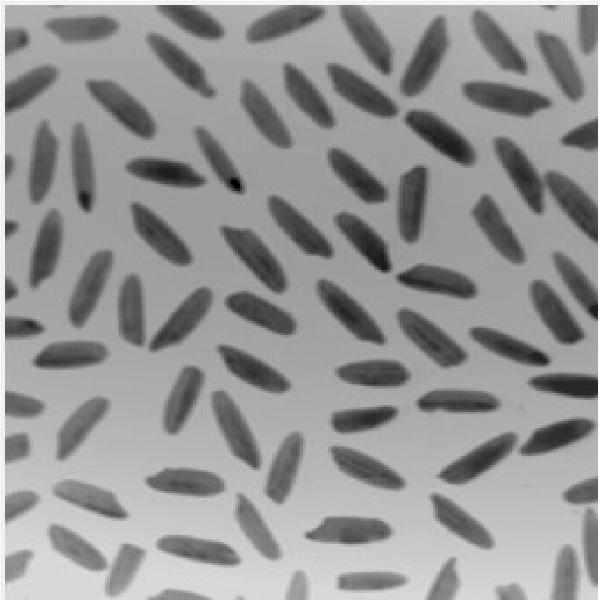
Closing



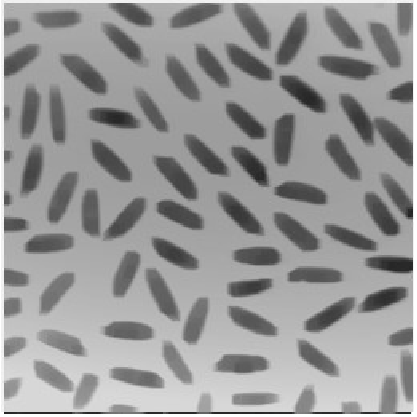
Does not seem to be really useful…
However, for images with dark FG over light BG…



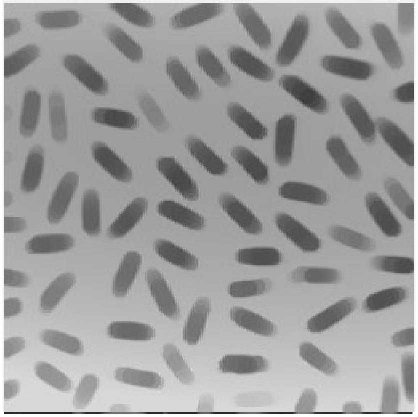
Closing



Radius 3

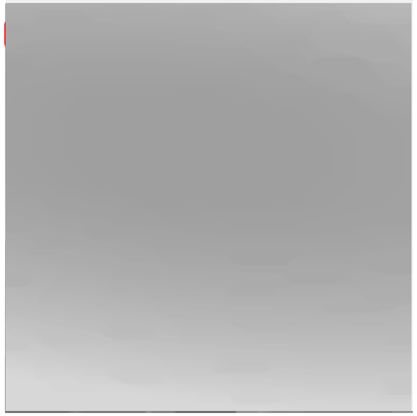


Radius 5



Radius 9

Radius 19



Duality

- Erosion and dilation for grayscale images are dual wrt function complementation and reflection of the SE

$$E(f,b)^C = D\left(f^C,\hat{b}\right)$$

$$D(f,b)^C = E\left(f^C,\hat{b}\right)$$

- The same holds for opening and closing

$$(f \circ b)^C = f^C \bullet \hat{b}$$

$$(f \bullet b)^C = f^C \circ \hat{b}$$

Top hat and bottom hat

- The residual of opening compared to the original image represents the **top hat** operator:

$$TH(f) = f - f \circ b$$

- Its dual, the **bottom hat** operator, is defined as the residual of the image after the closing is removed:

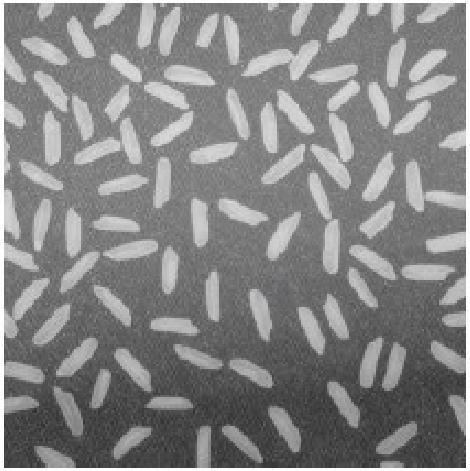
$$BH(f) = f \bullet b - f$$

- TopHat and BottomHat are sometimes referred to as **White TopHat** and **Black TopHat**, respectively

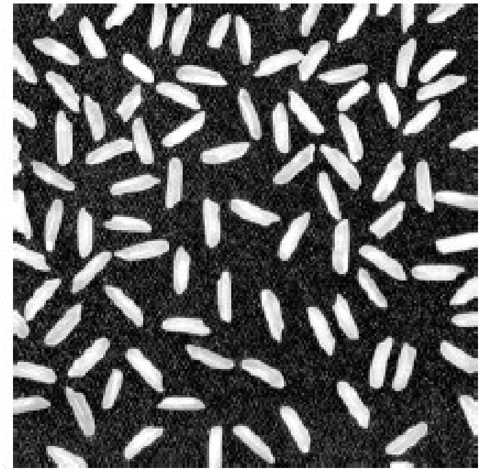
Top hat and bottom hat

- Top hat and bottom hat are used to **enhance the objects** in the image **that do not fit the SE**
  - Top hat enhances bright objects on dark background
  - Bottom hat enhances dark objects on bright background
- The size of the SE should be large enough so as **not to fit** the objects to be retained

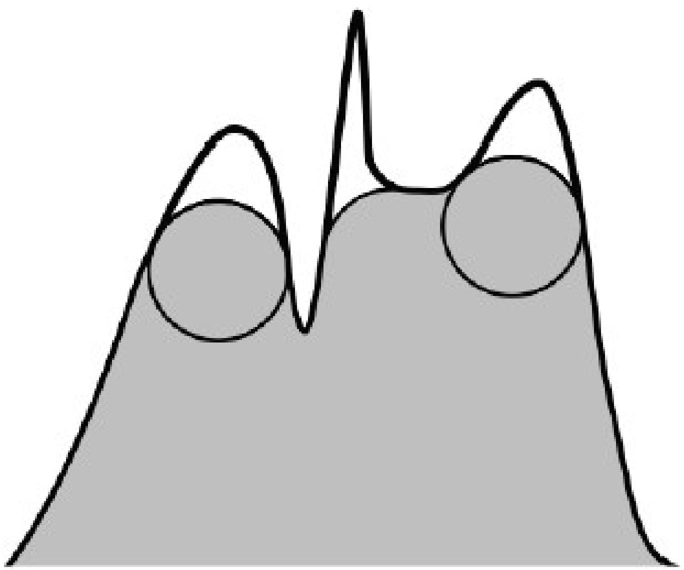
Top hat



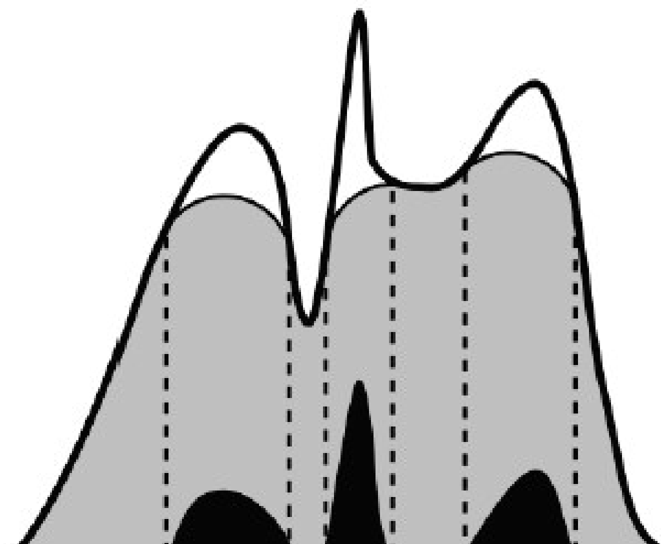
Original image



Top-hat $g_{D_{12}}$



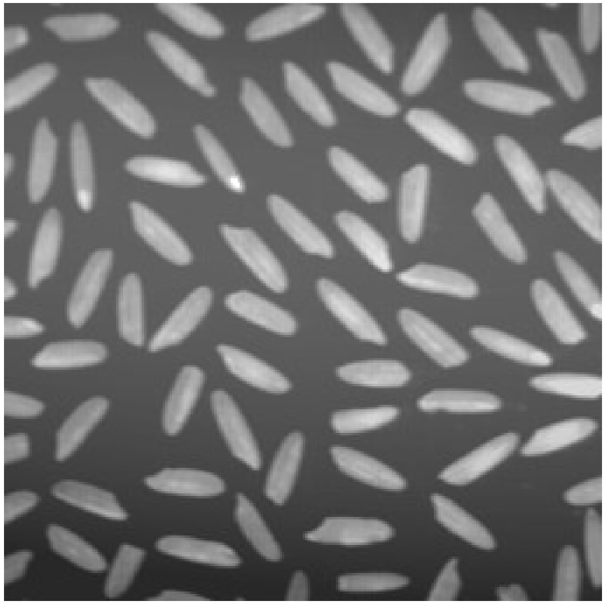
Opening

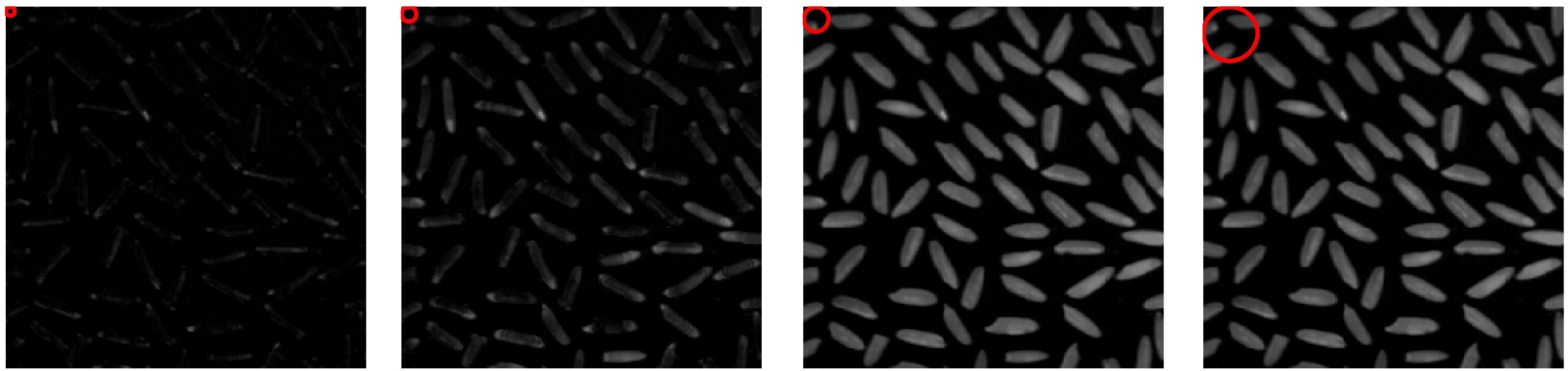


Opening and top-hat

Top hat

Original

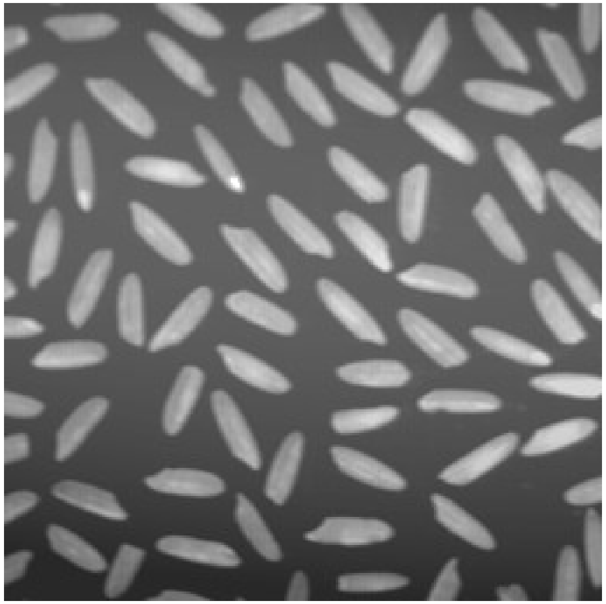


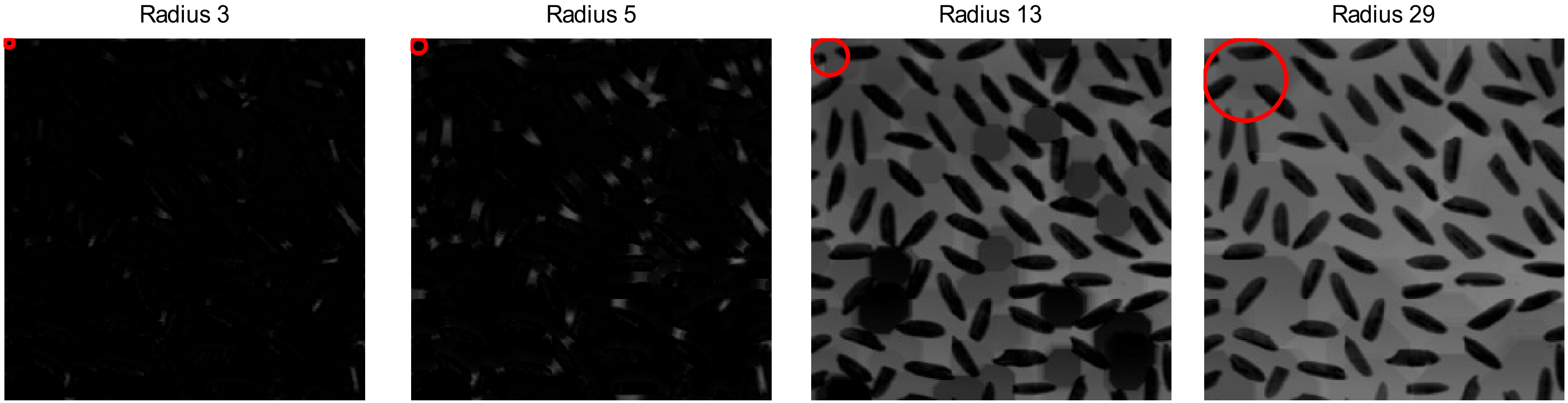


Radius 3 Radius 5 Radius 9 Radius 19

Bottom hat

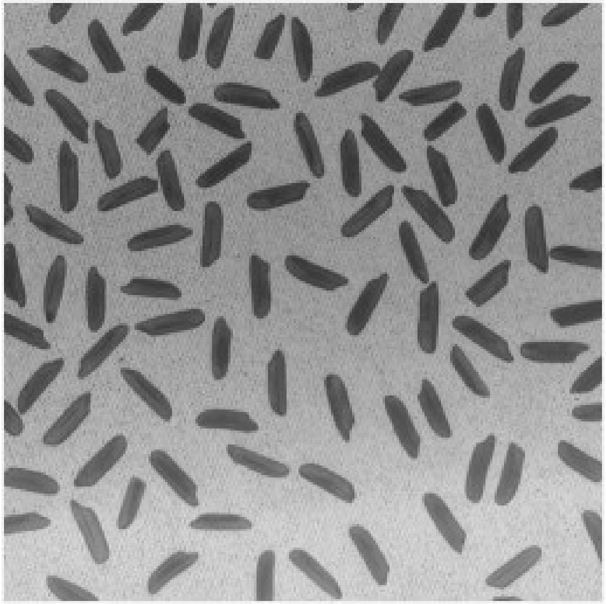
Original



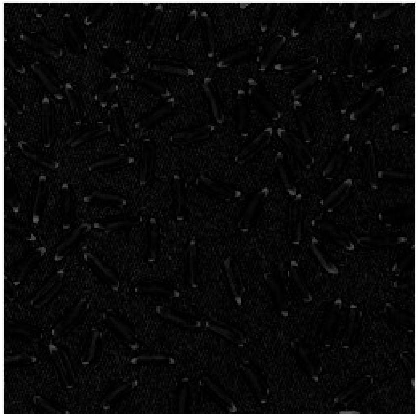


Bottom hat

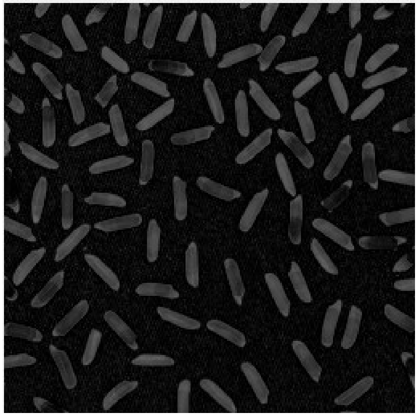
Original



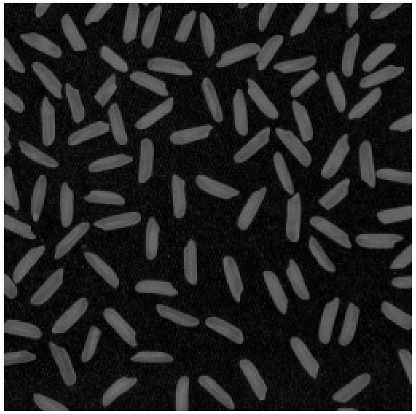
Radius 3



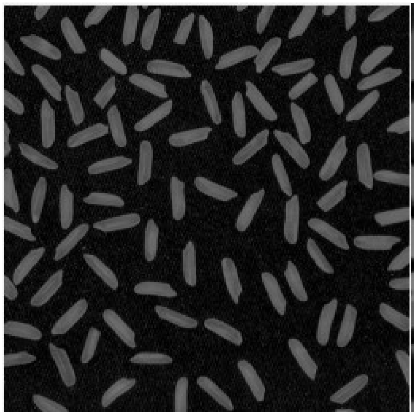
Radius 5



Radius 13



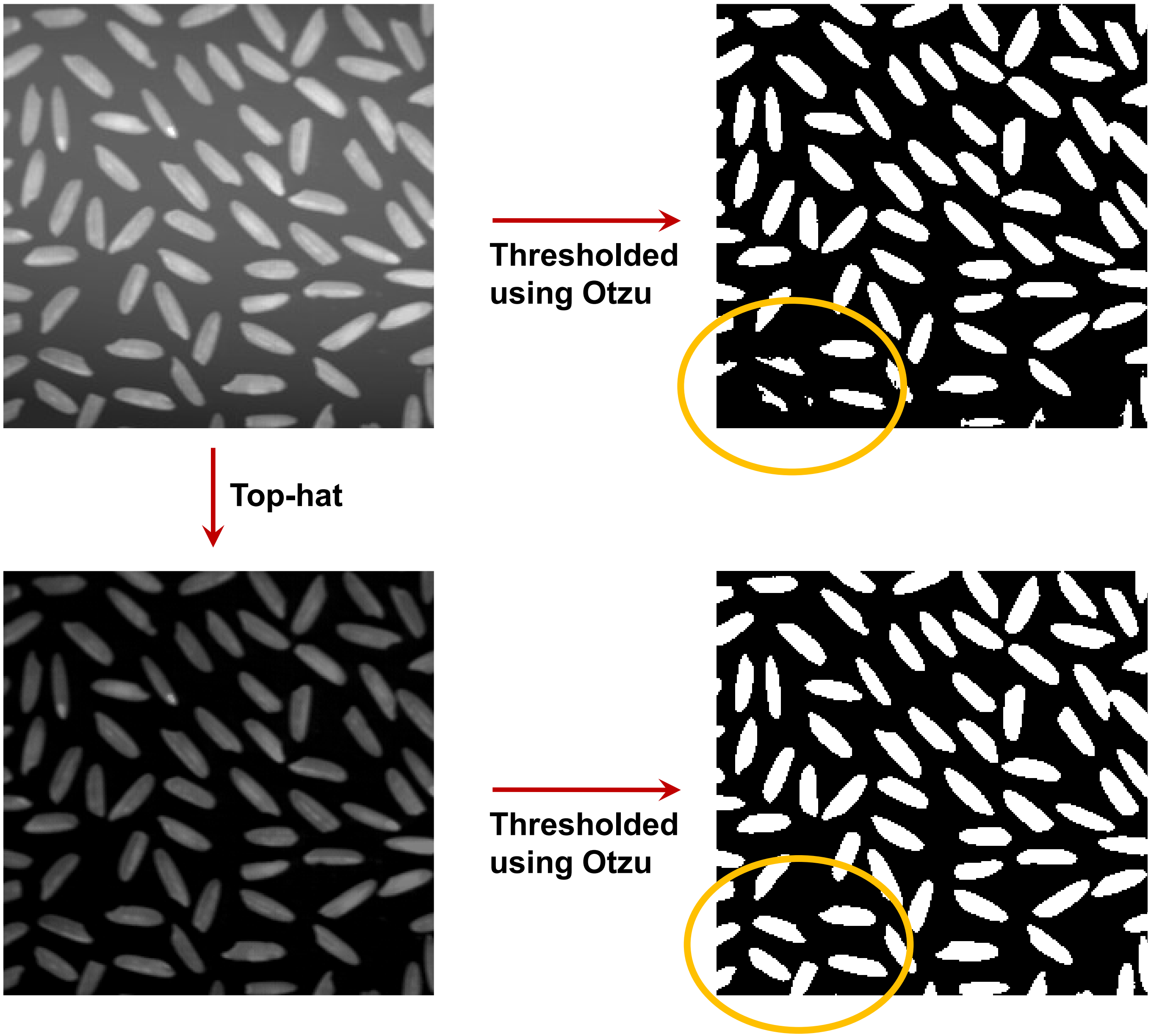
Radius 29



Top hat and bottom hat

- Top hat is frequently used to correct the effects of non uniform illumination
  - This is particularly convenient before applying a threshold

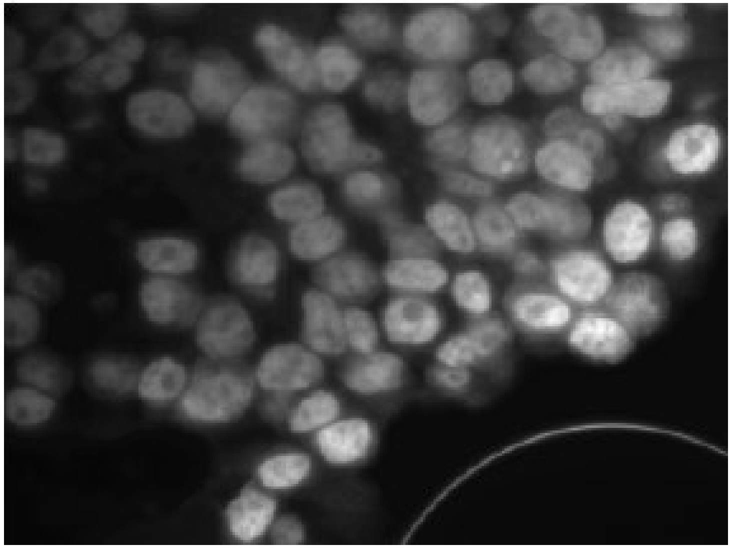
Top-hat for thresholding



Morphological contrast enhancement

- To enhance image contrast we can add the top hat filtered image to the image and then subtract the bottom hat filtered image

$$CE(f,b) = f + TH(f,b) - BH(f,b)$$

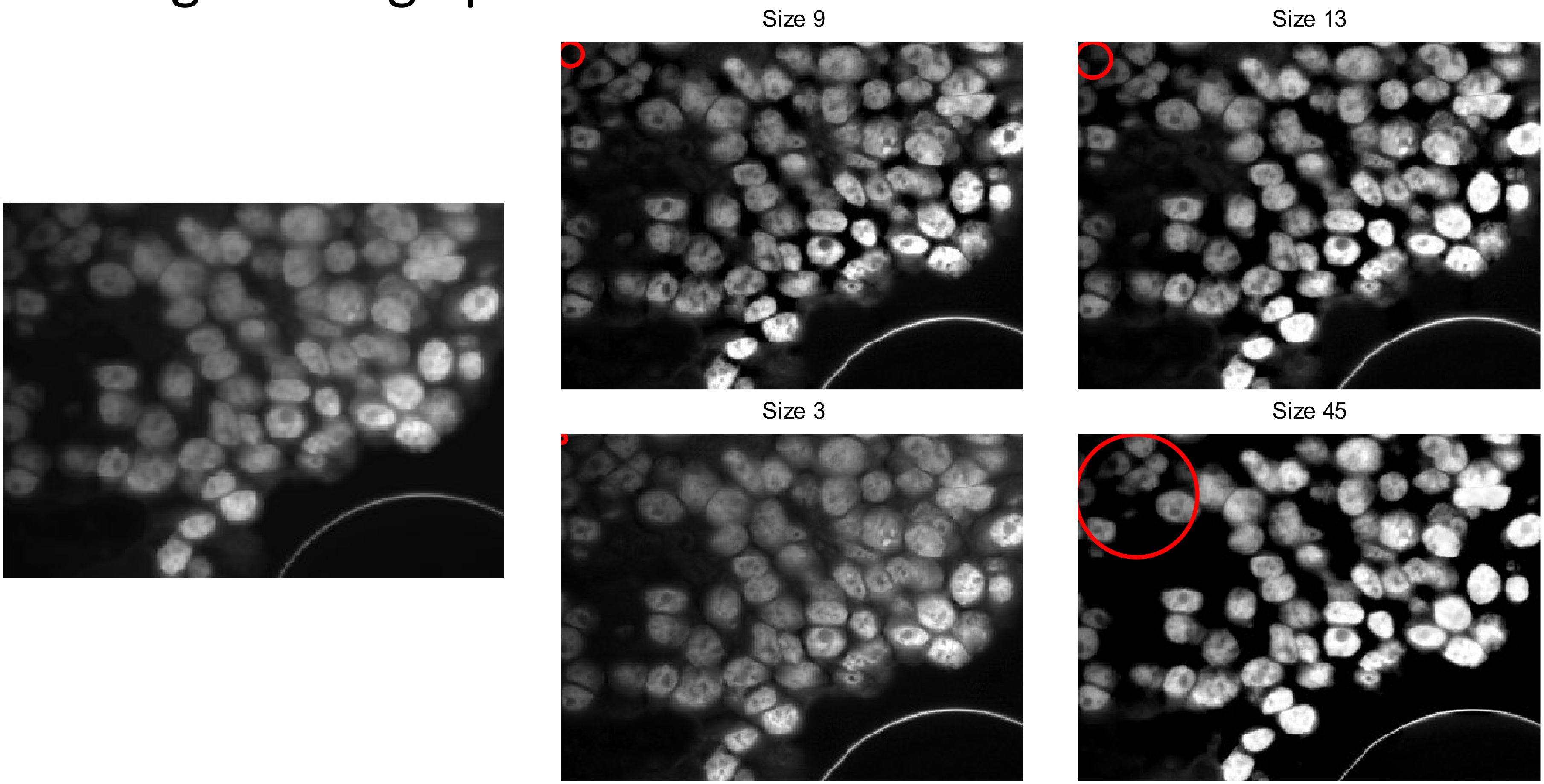


Original

Contrast enhanced

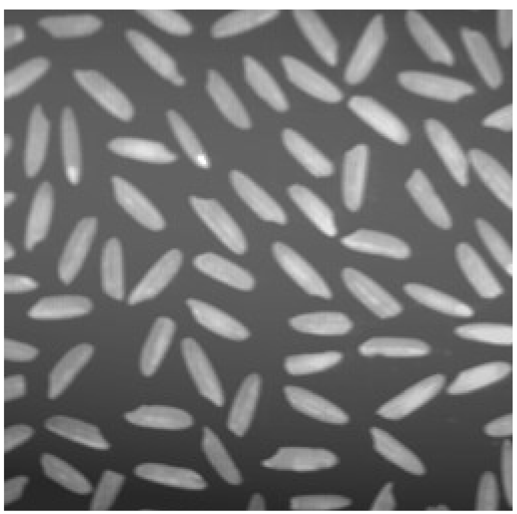
Morphological contrast enhancement

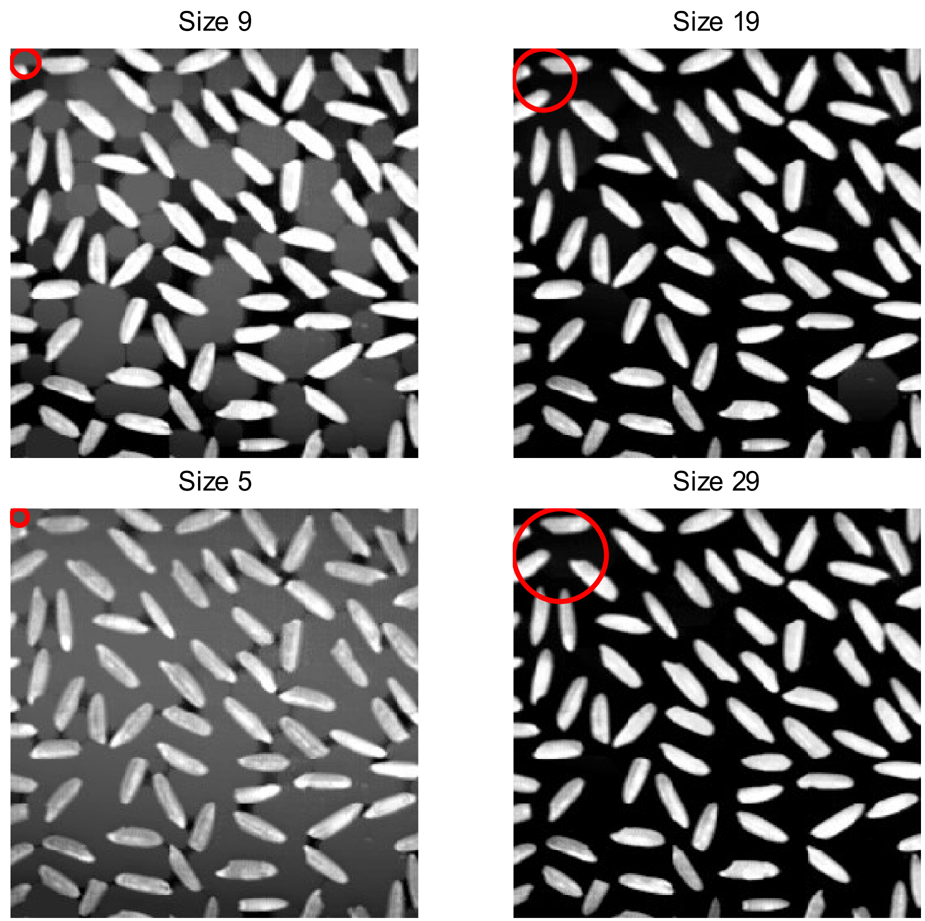
- b should be large enough not to be included in the largest image patch that we want to enhance



Morphological contrast enhancement

- b should be large enough not to be included in the largest image patch that we want to enhance





Morphological smoothing

- The combination of opening followed by closing is often effective for noise removal (particolarly isolated noise)

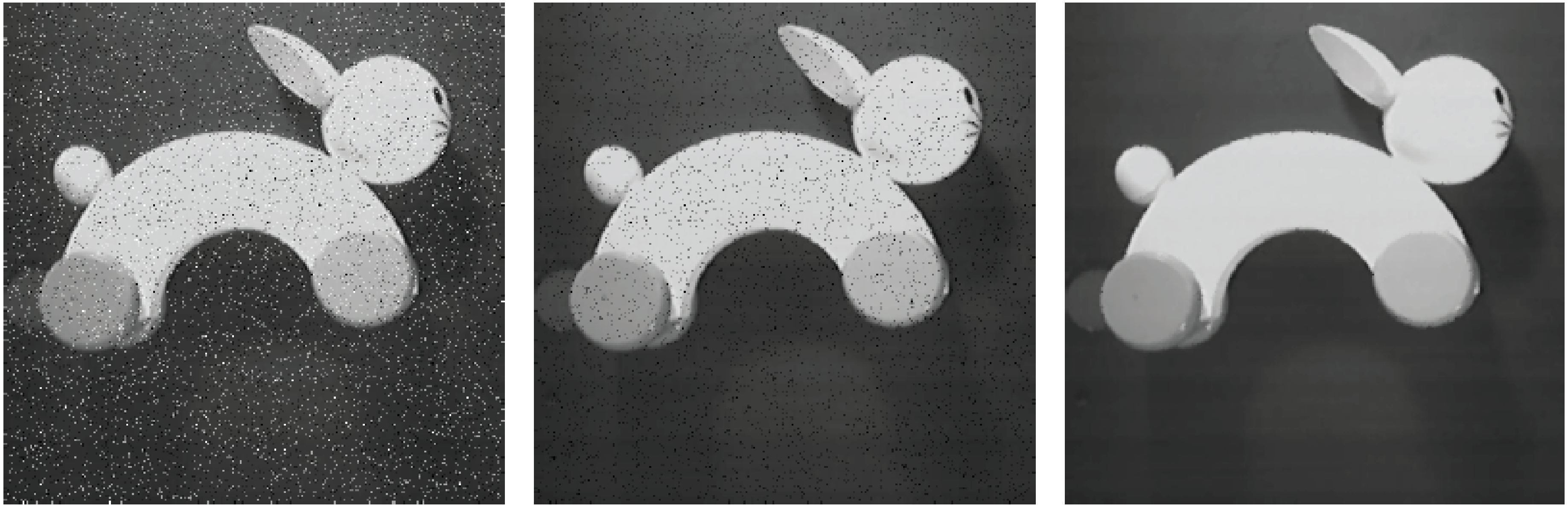


Image with salt-pepper noise | Opening | Closing

Geodesic dilation

- Grayscale morphological reconstruction extends the concepts defined for binary images
- Given a **marker** image f and a **mask** image g (same size) and such that $f \leq g$, the **geodesic dilation** of size 1 of f wrt g with SE b is

$$GD_g^1(f,b) = D(f,b) \wedge g$$

being $\wedge$ the point-wise minimum operator ($\vee$ the maximum)

Geodesic dilation

- Given a **marker** image f and a **mask** image g (same size) and such that f ≤ g, the **geodesic dilation** of size n of f wrt g with SE b is

$$GD_g^n(f,b) = GD_g^1\left(GD_g^{n-1}(f,b),b\right)$$

with $GD_g^0(f,b) = f$

Geodesic erosion

- The **geodesic erosion** is defined similarly

$$GE_g^1(f,b) = E(f,b) \vee g$$

$$GE_g^n(f,b) = GE_g^1\left(GE_g^{n-1}(f,b), b\right)$$

with $$GE_g^0(f,b) = f$$

Morphological reconstruction

- The **morphological reconstruction by dilation** of a grayscale mask image g by a grayscale marker image f is the geodesic dilation of f wrt g iterated until stability

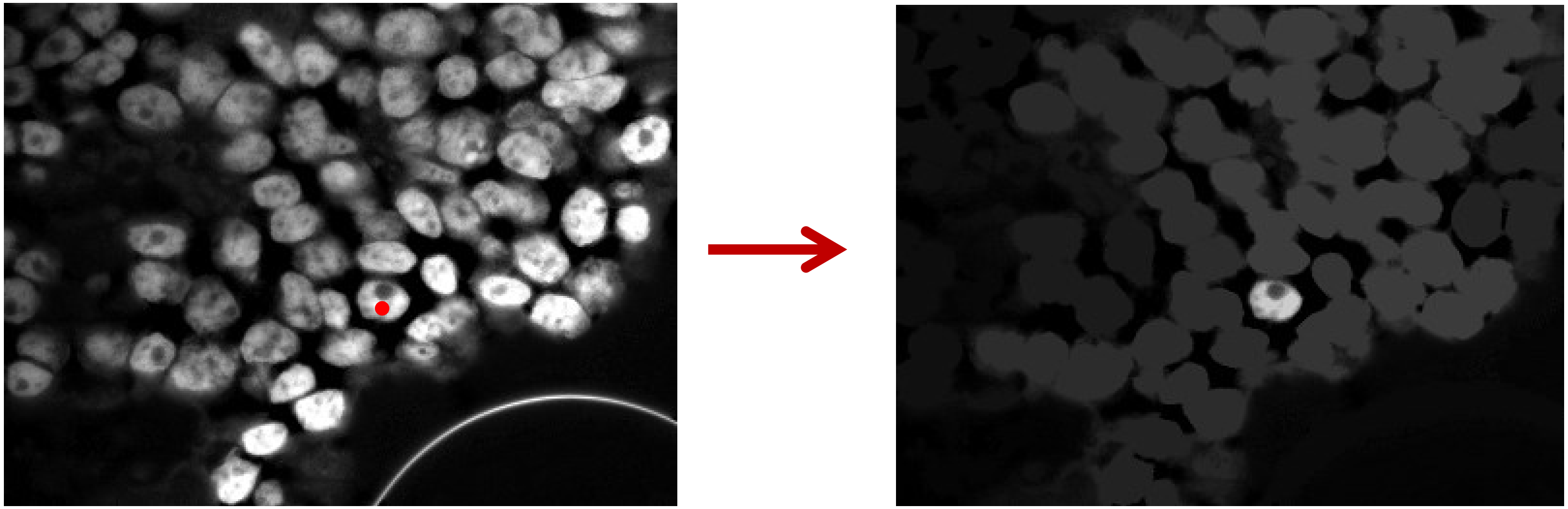
$$R_g^D(f,b) = GD_g^n(f,b)$$

such that $GD_g^n(f,b) = GD_g^{n+1}(f,b)$

- The **morphological reconstruction by erosion** is defined similarly

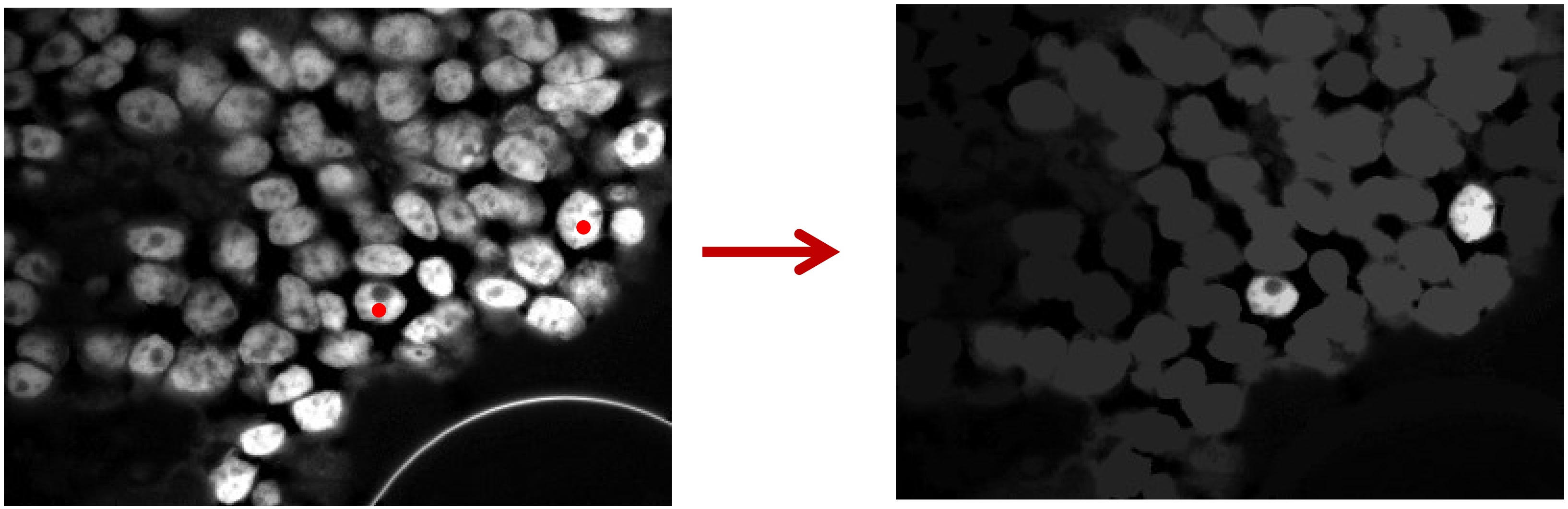
Morphological reconstruction

- Morphological reconstruction by dilation of the mask image shown below using as marker the red point



Morphological reconstruction

- Morphological reconstruction by dilation of the mask image shown below using as marker the two red points



Opening by reconstruction

- **Opening by reconstruction** of size n of a grayscale image f is the reconstruction by dilation of f from the erosion of size n of f

$$O_R^n(f,b) = R_f^D\left(E(f,nb),b\right)$$

being $E(f,nb)$ the result of n erosions of f by b

- As in the case of binary images, the goal of opening by reconstruction is to retain the image components that remain after erosion

Opening by reconstruction

- A useful technique is the **top-hat by reconstruction** consisting of subtracting from an image its opening by reconstruction

$$THbR^n(f,b) = f - O_R^n(f,b)$$

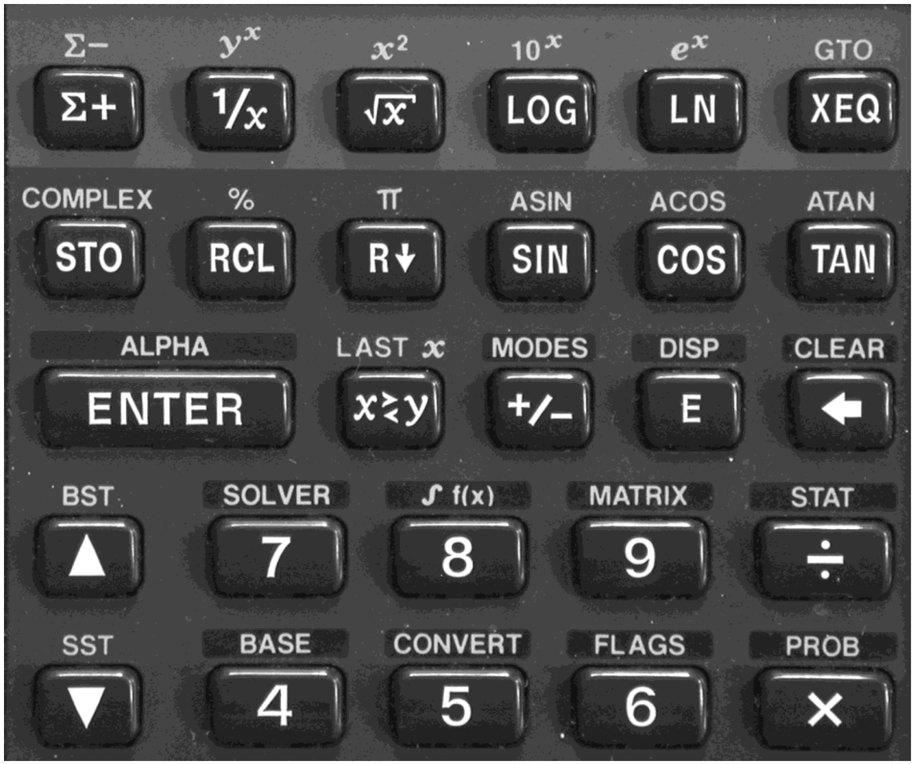
- This can be used to **remove noise of known form** from an image
  - Define a SE that fits the noise
  - Open by reconstruction using the SE so as to retain only the noise
  - Top-hat by subtracting the ObR to the original image

Opening by reconstruction

- **Open by reconstruction** and **top-hat by reconstruction** are adopted for two opposite goals:
  - There is a SE that fits the noise and does not fit the “interesting parts” of the image: apply THbR to the image with the SE
  - There is a SE that fits the “interesting parts” of the image and does not fit the noise: apply ObR to the image with the SE

An example

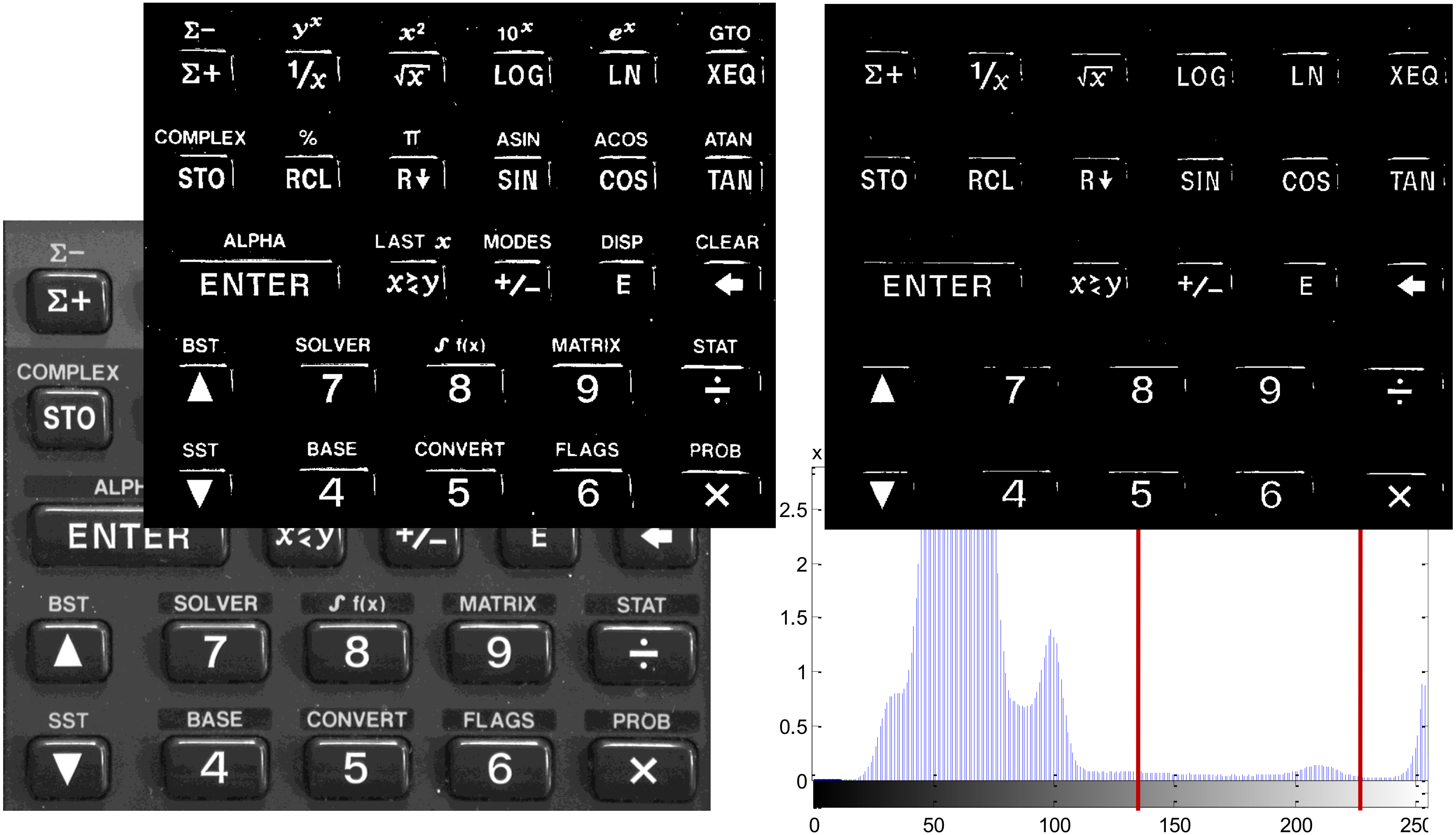
- We want to remove shading and reflection effects so as to retain only text/symbols useful to feed an OCR module



Noise: vertical and horizontal bright bars

An example

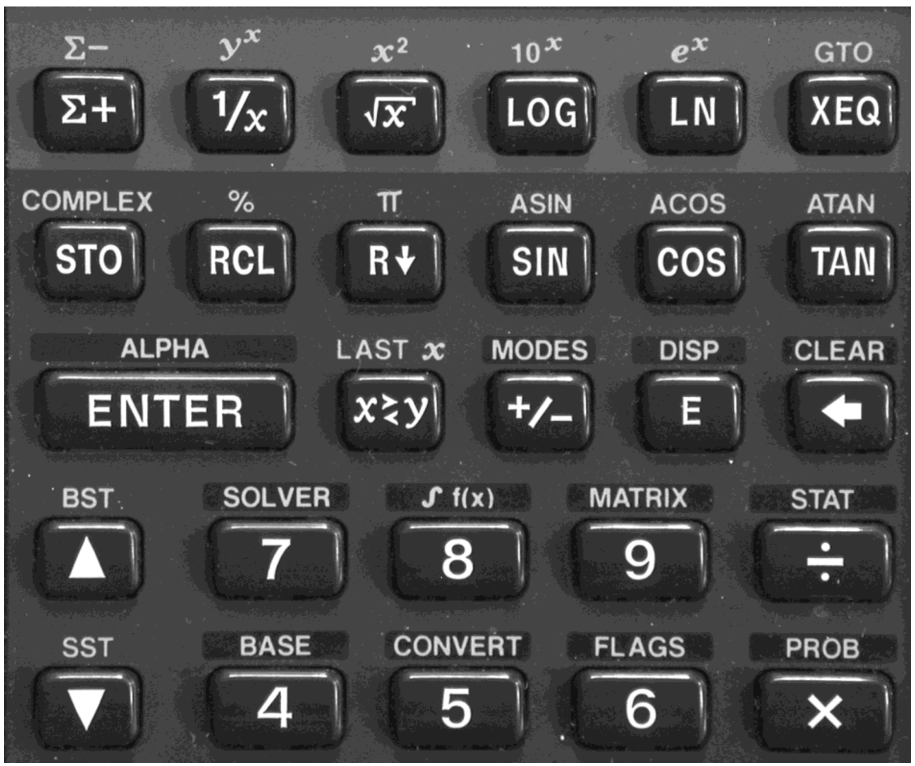
- Thresholding would not remove reflection effects …



An example

- The first step is to remove horizontal reflections along the top of each key. These can be isolated through the open by reconstruction with a horizontal bar structuring element:

imreconstruct(**imerode** (img, ones(1,71)), img));

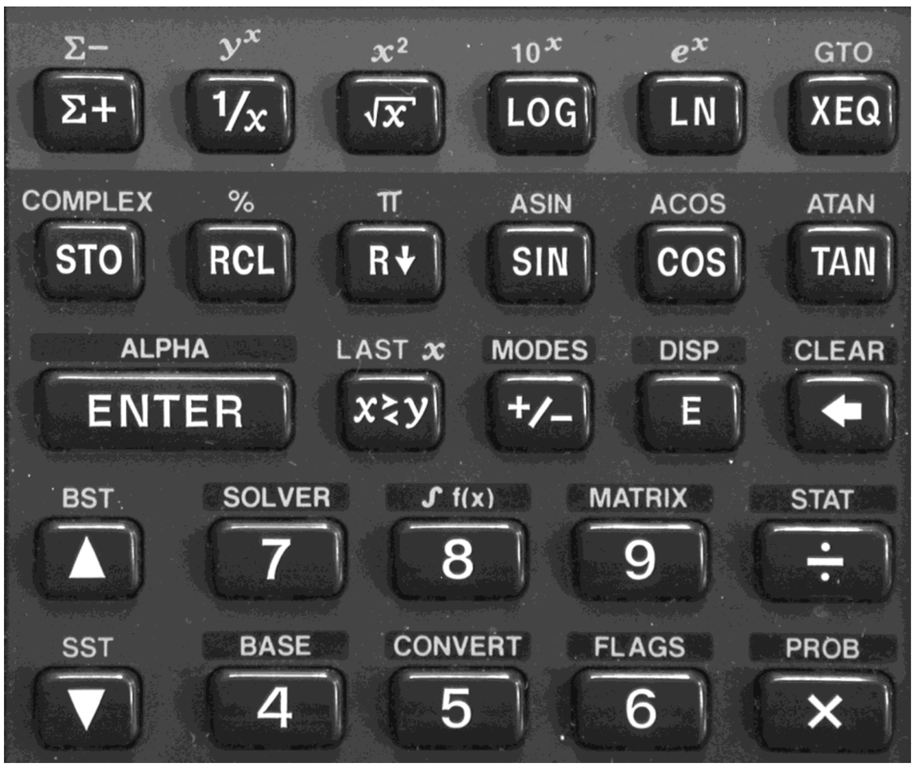


Slide one horizontal bar and replace the central pixel with the minimum

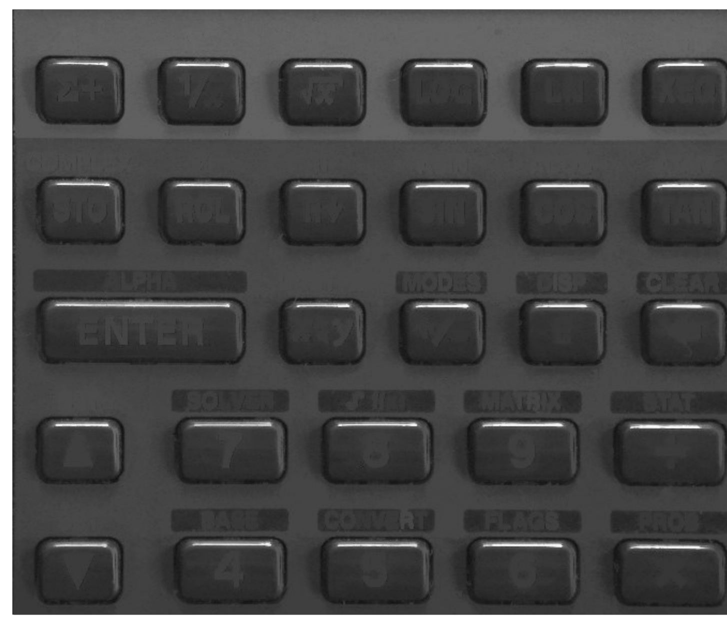
An example

- The first step is to remove horizontal reflections along the top of each key. These can be isolated through the open by reconstruction with a horizontal bar structuring element:

imreconstruct(**imerode** (img, ones(1,71)), img));

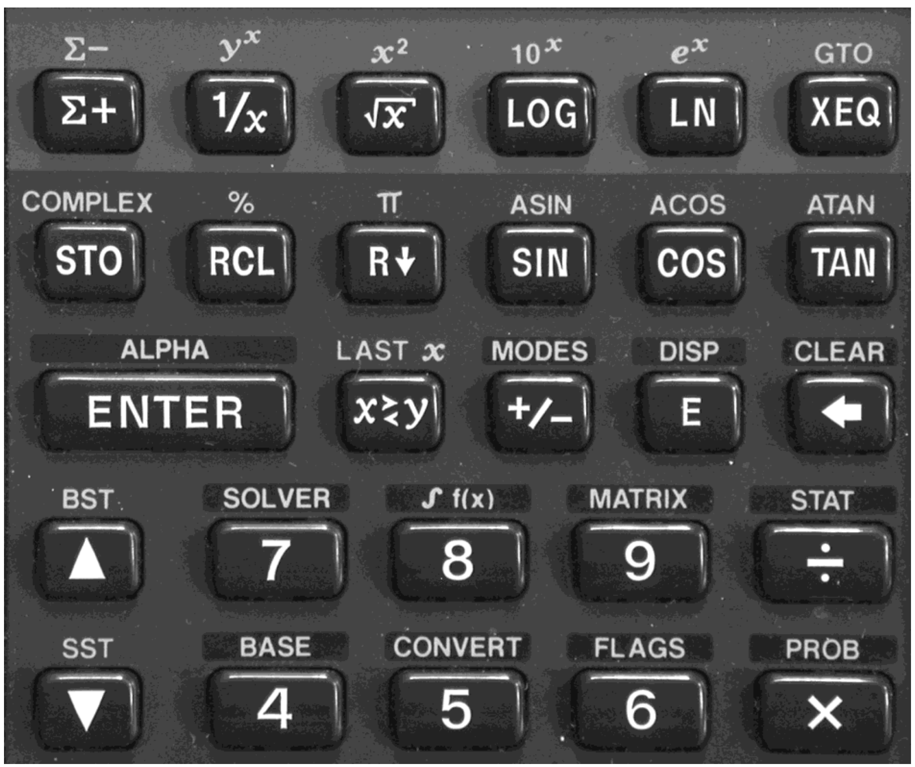


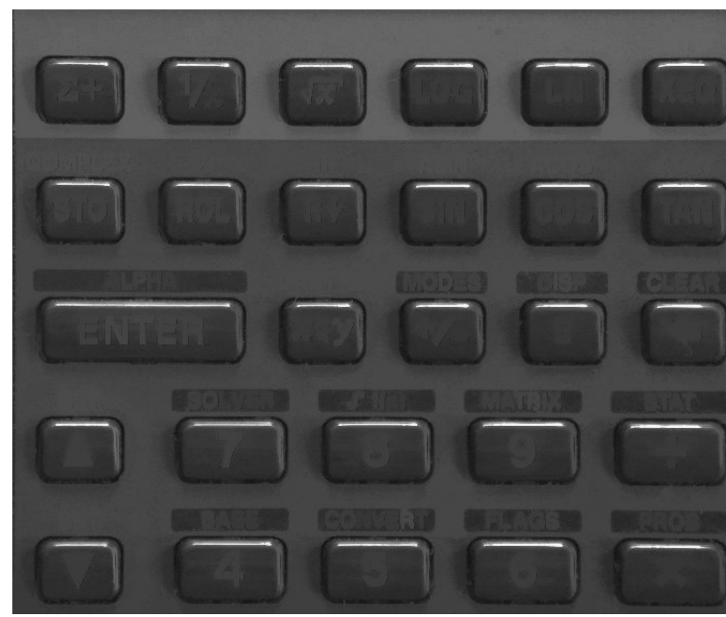
Reconstruction by dilation is performed using a 3x3 square SE



An example

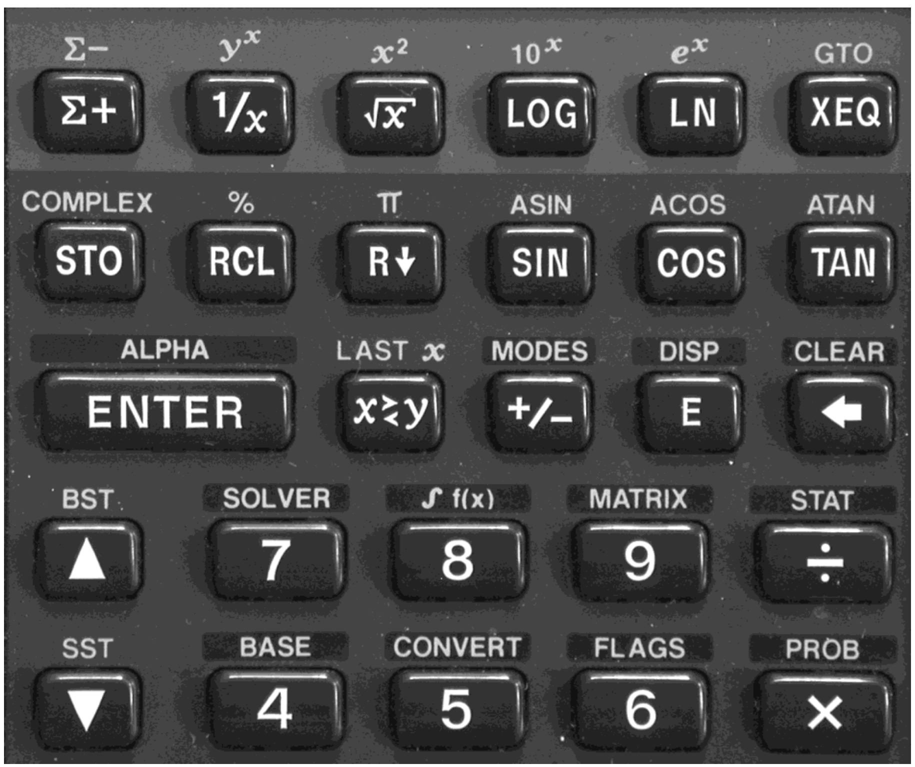
- Some bright regions are not reconstructed: all bright regions that do not fit a horizontal bar 71 pixel wide. These are exactly the characters that we want to extract from the original image …

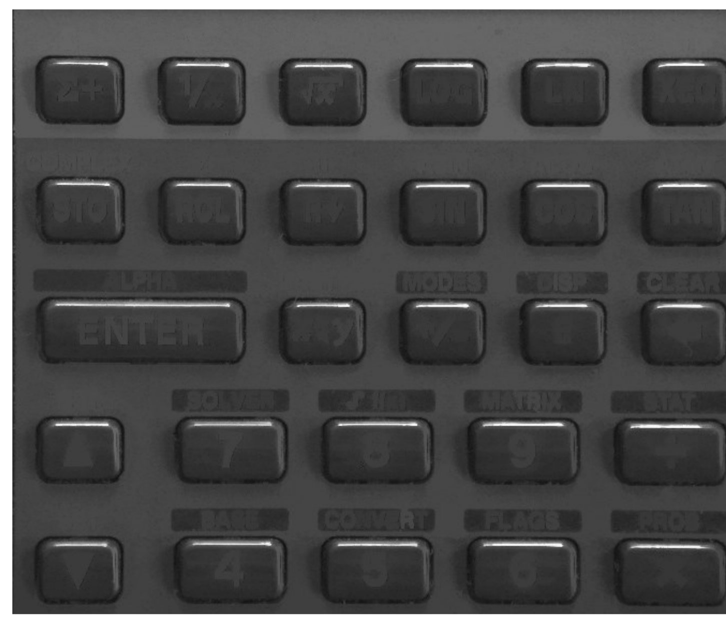




An example

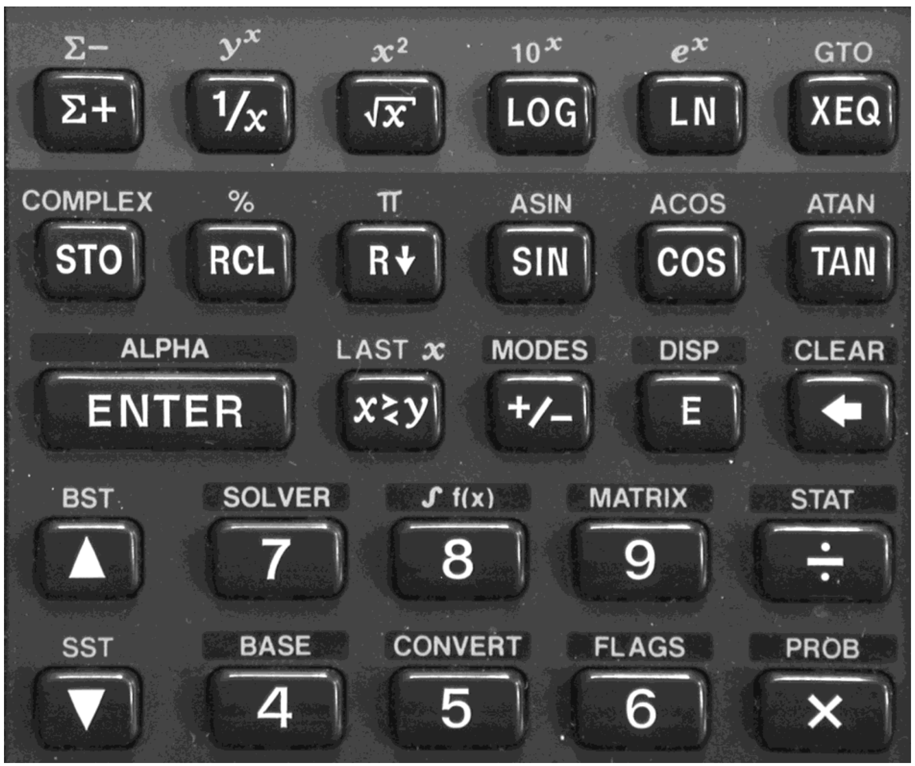
- The open by reconstruction should be subtracted to the original image yielding the top hat by reconstruction operator

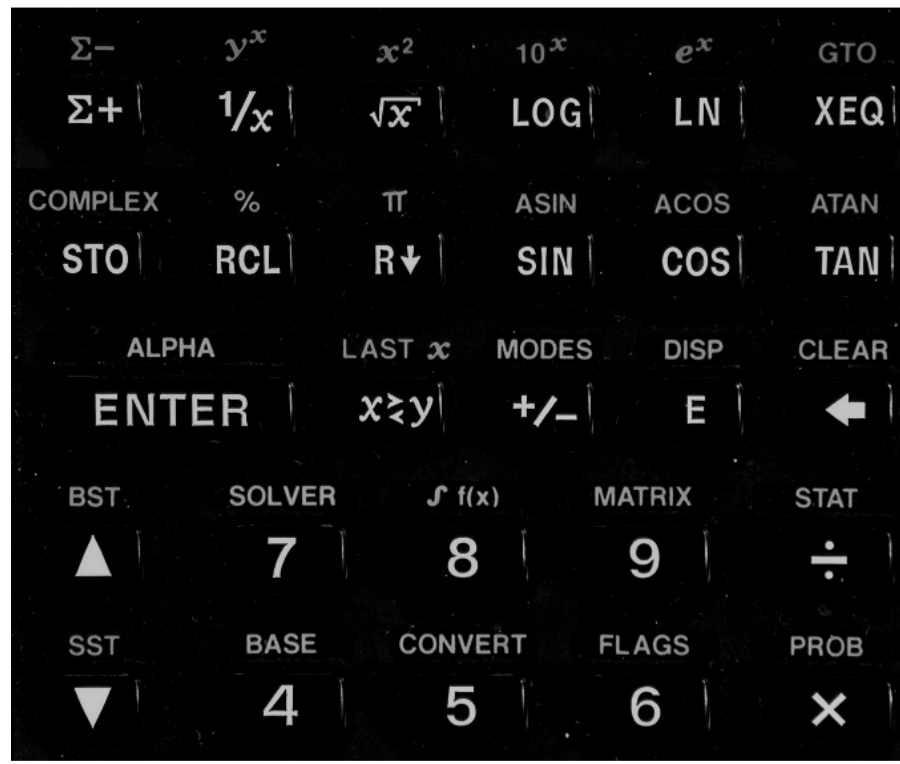




An example

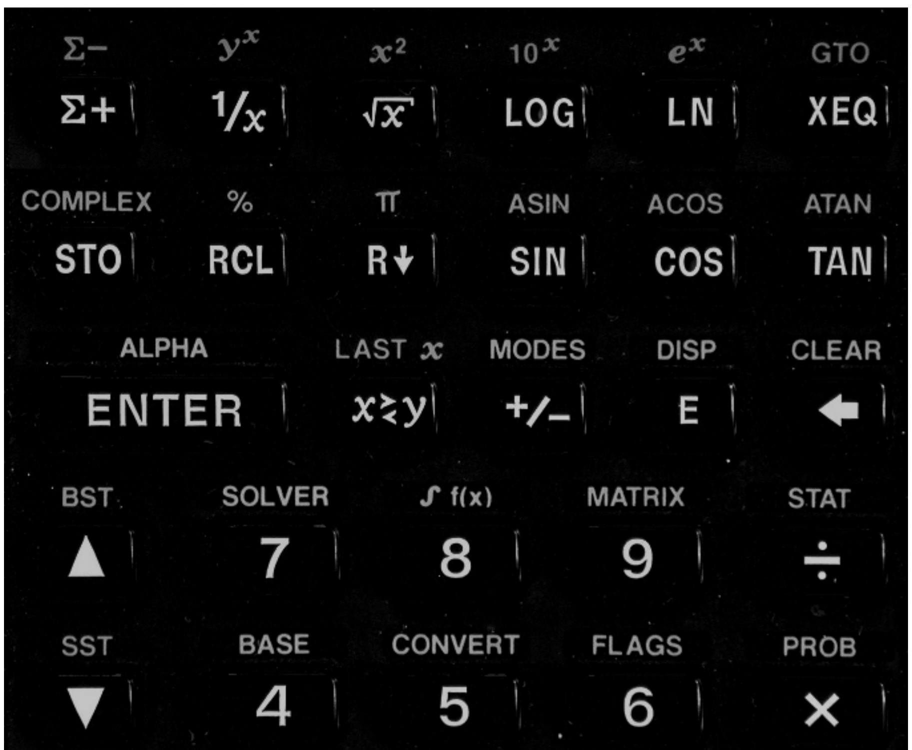
- The top hat by reconstruction has successfully removed horizontal reflections and rendered a more uniform background

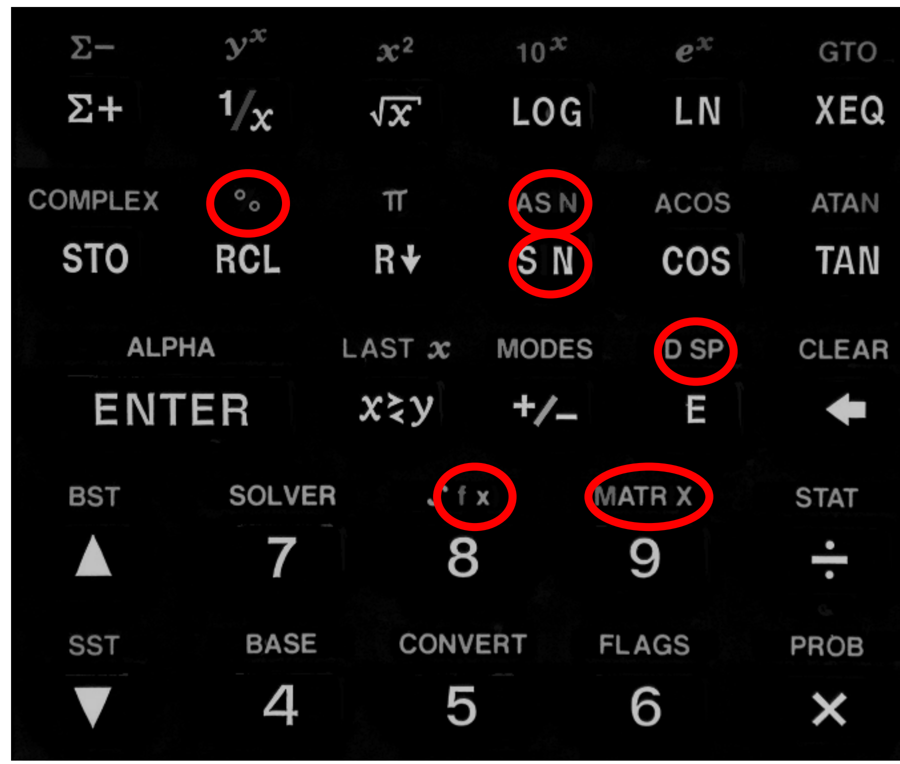




An example

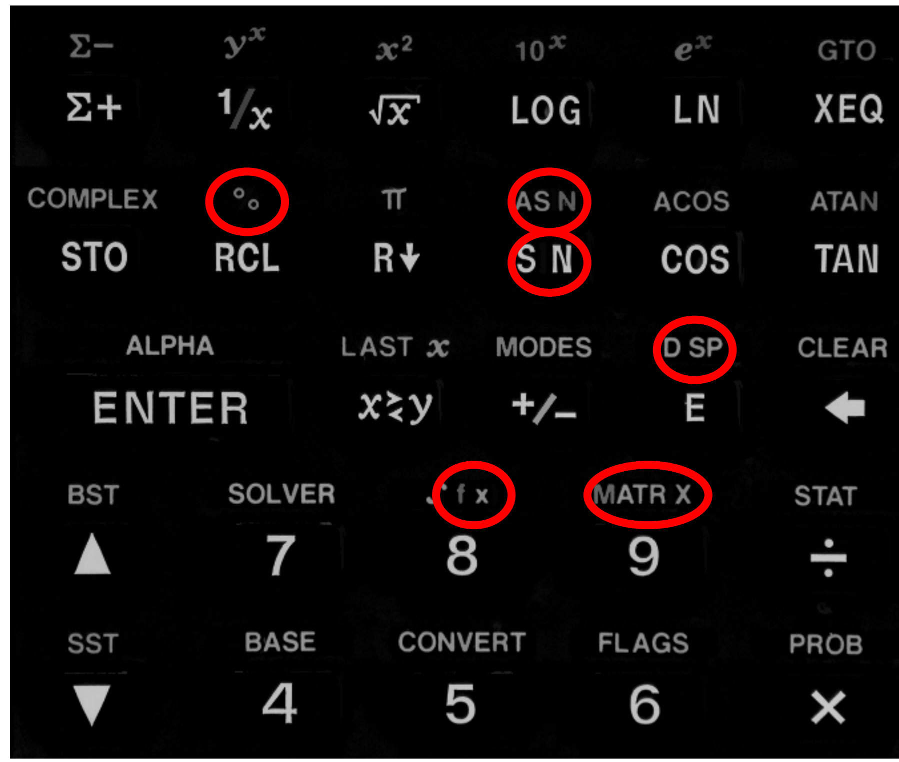
- Vertical reflections can be removed through open by reconstruction with a **horizontal bar** 11 pix wide (all isolated vertical elements are completely removed)





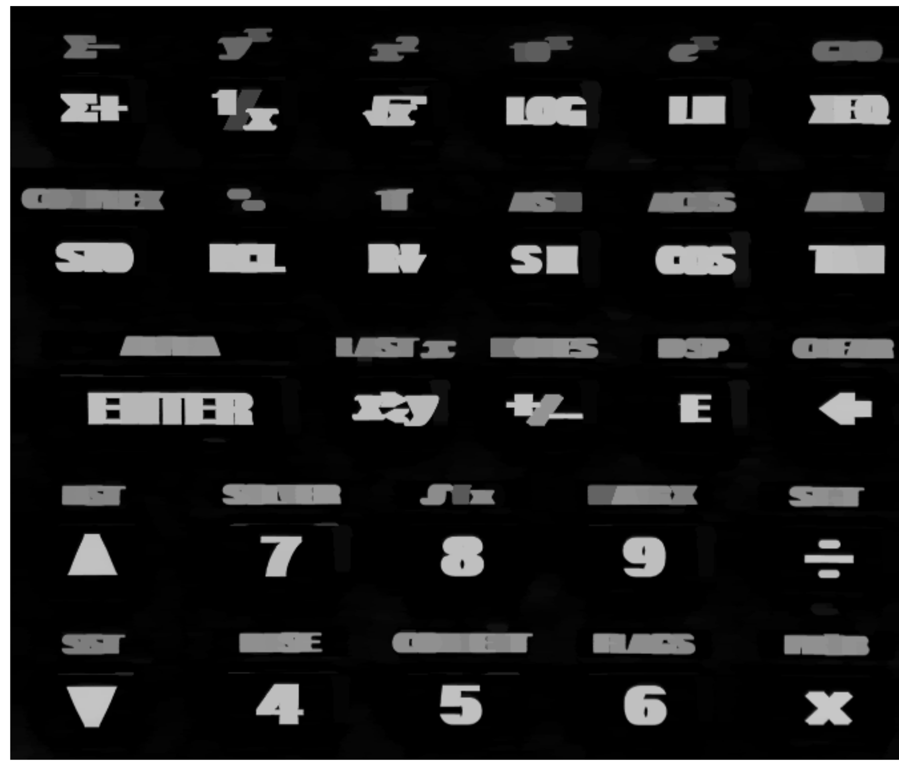
An example

- Vertical characters that have been suppressed by error are very close to other characters that have not been removed. They can be recovered by dilation with a horizontal bar 21 pix wide



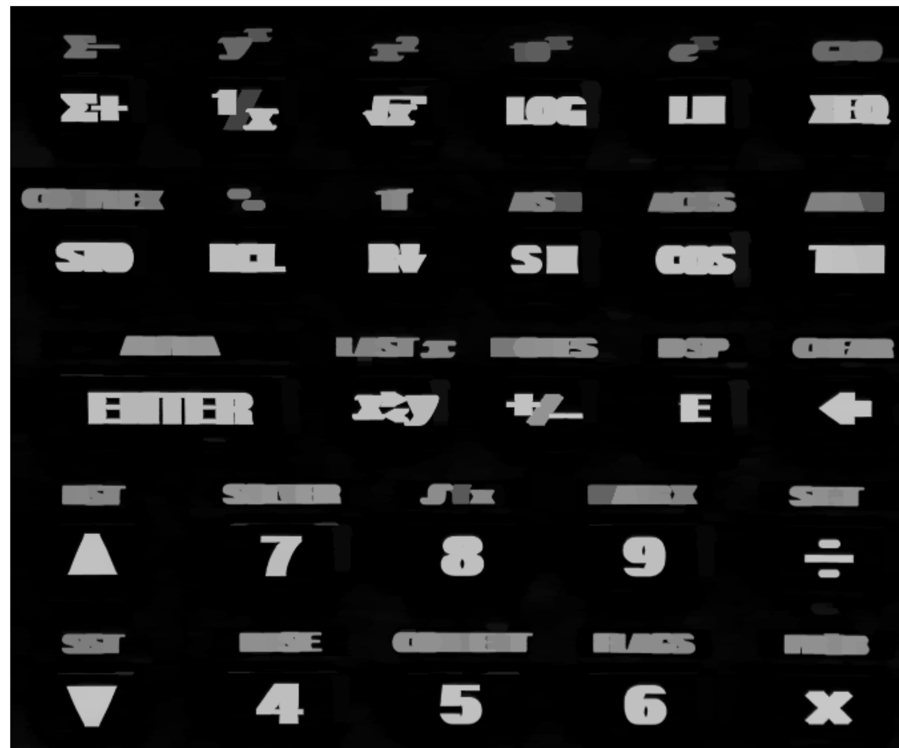
An example

- Vertical characters that have been suppressed by error are very close to other characters that have not been removed. They can be recovered by dilation with a horizontal bar 21 pix wide



An example

- The minimum between the two images below can be used to perform reconstruction by dilation using the first image as mask

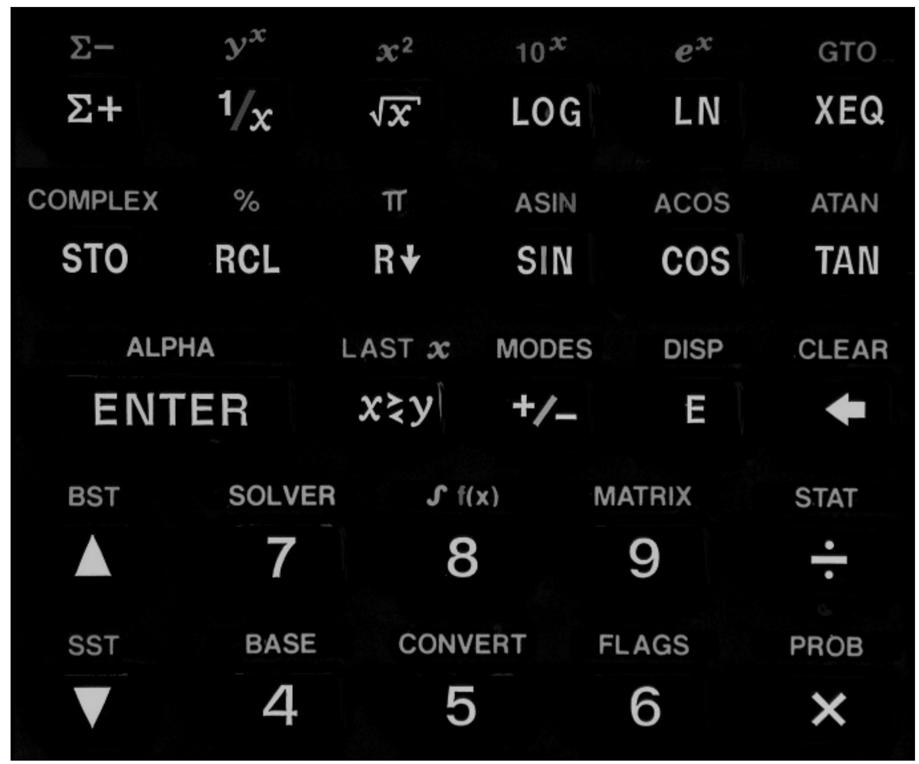


An example

- The minimum …

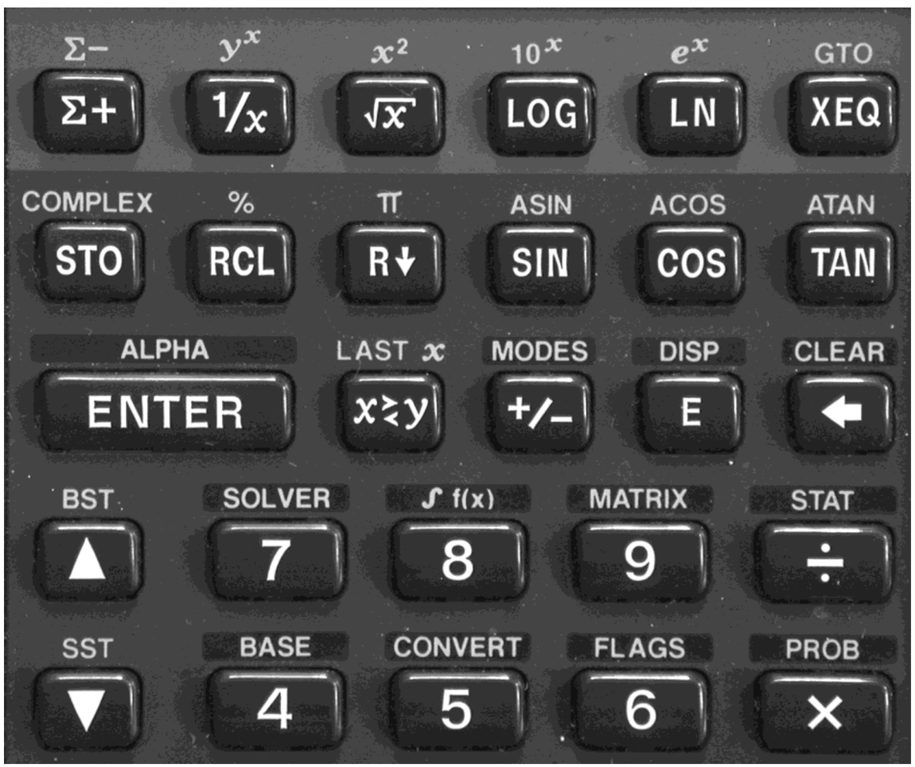
An example

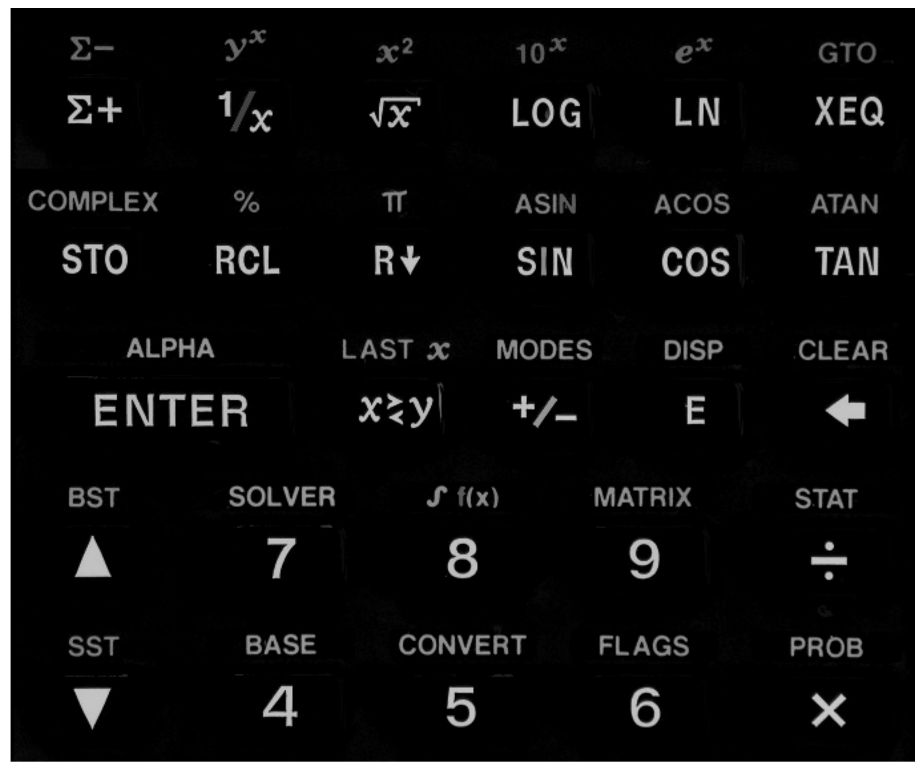
- The reconstructed image



An example

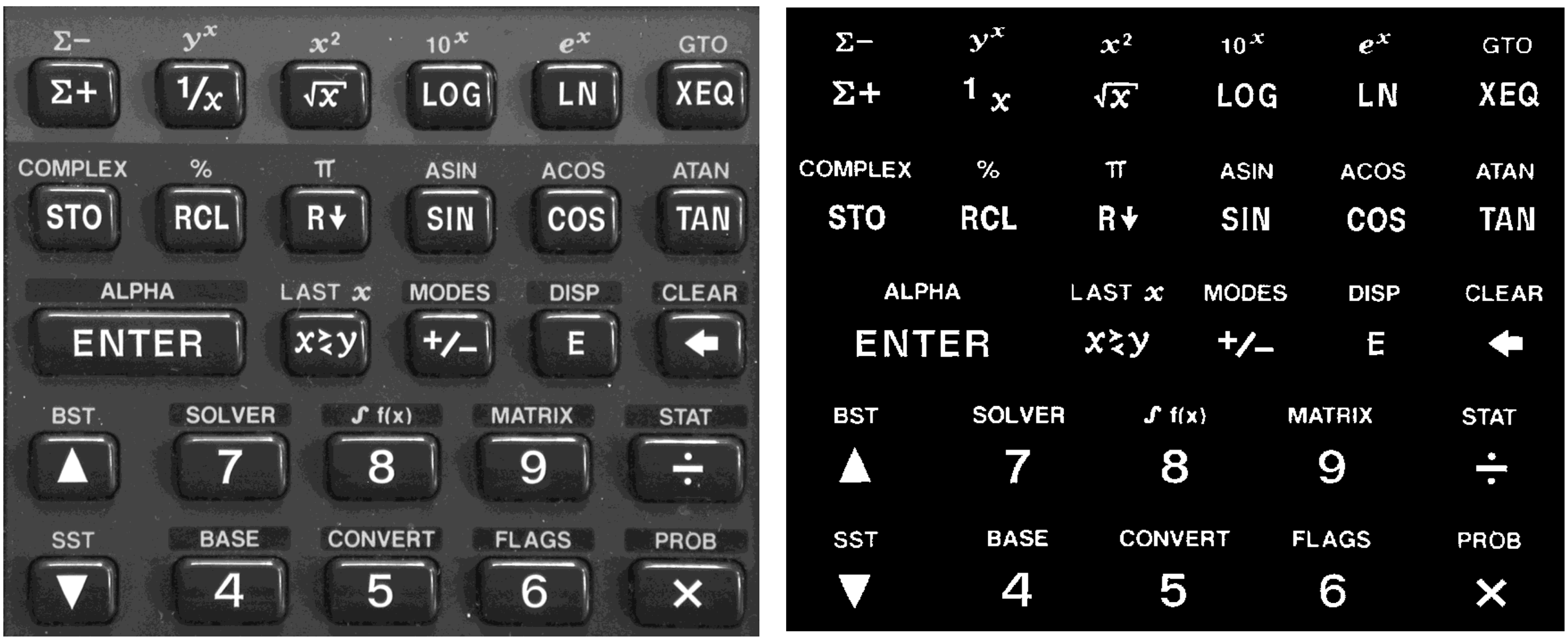
- The reconstructed image compared to the original





An example

- The thresholded (Otsu) reconstructed image compared to the original



Python code

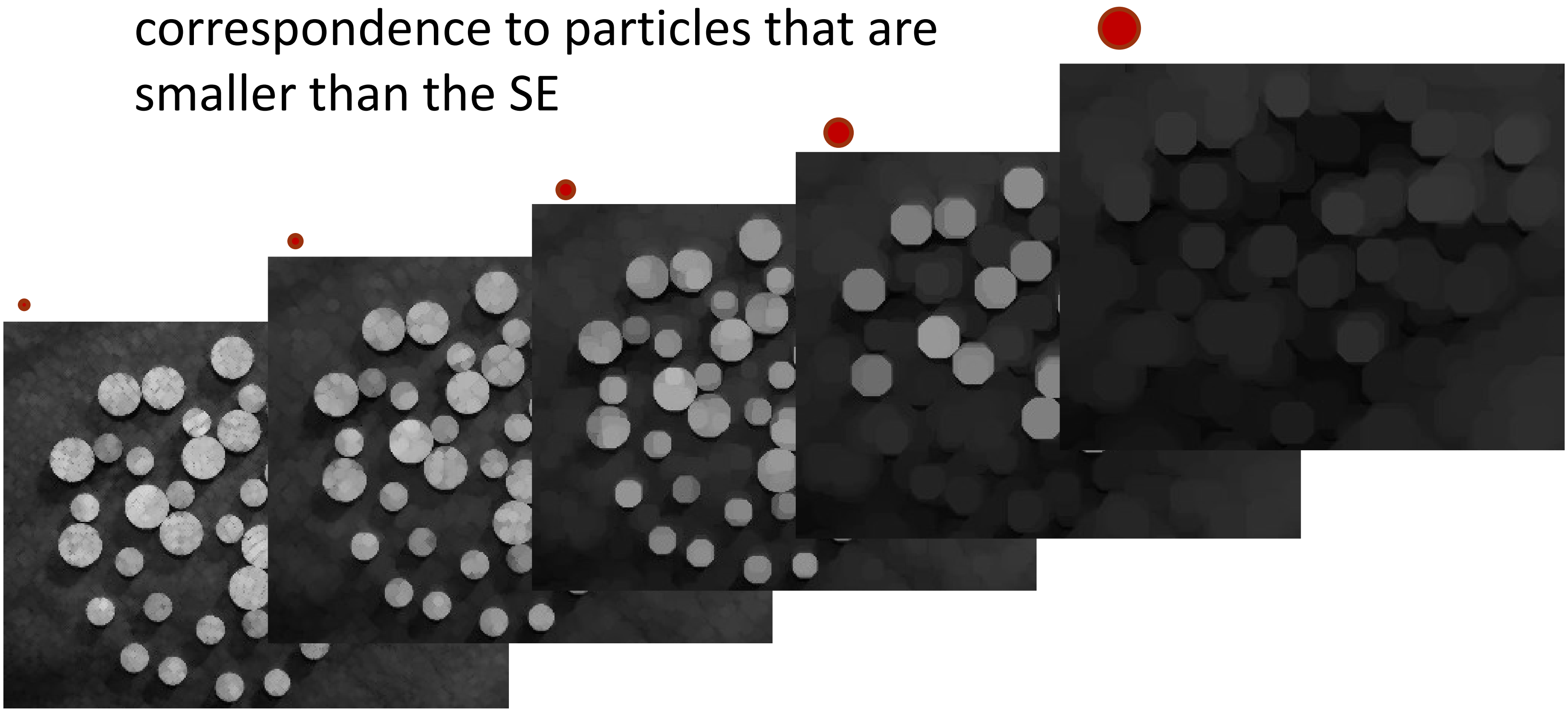
```
filename = 'calculator.tif'

img = cv2.imread(filename,0)

bar_hor = np.ones((1,71))
img_01 = cv2.erode(img, bar_hor)
img_02 = morpho_reconstruct(img, img_01)
img_03 = img - img_02

bar_hor = np.ones((1,11))
img_04 = cv2.erode(img_03, bar_hor)
img_05 = morpho_reconstruct(img_03, img_04)

img_06 = cv2.dilate(img_05, np.ones((1,21)))
img_07 = np.minimum(img_03, img_06)
img_08 = morpho_reconstruct(img_03, img_07)
```

Granulometry

- Granulometry deals with determining the **size distribution** of particles in an image
- Usually, particles are not well separated
  - This makes it difficult to isolate and count them
- MM can be used to estimate the distribution of particle size indirectly, without having to identify and measure each particle in the image
- **ASSUMPTION**: all particles have the same shape and this should match the SE

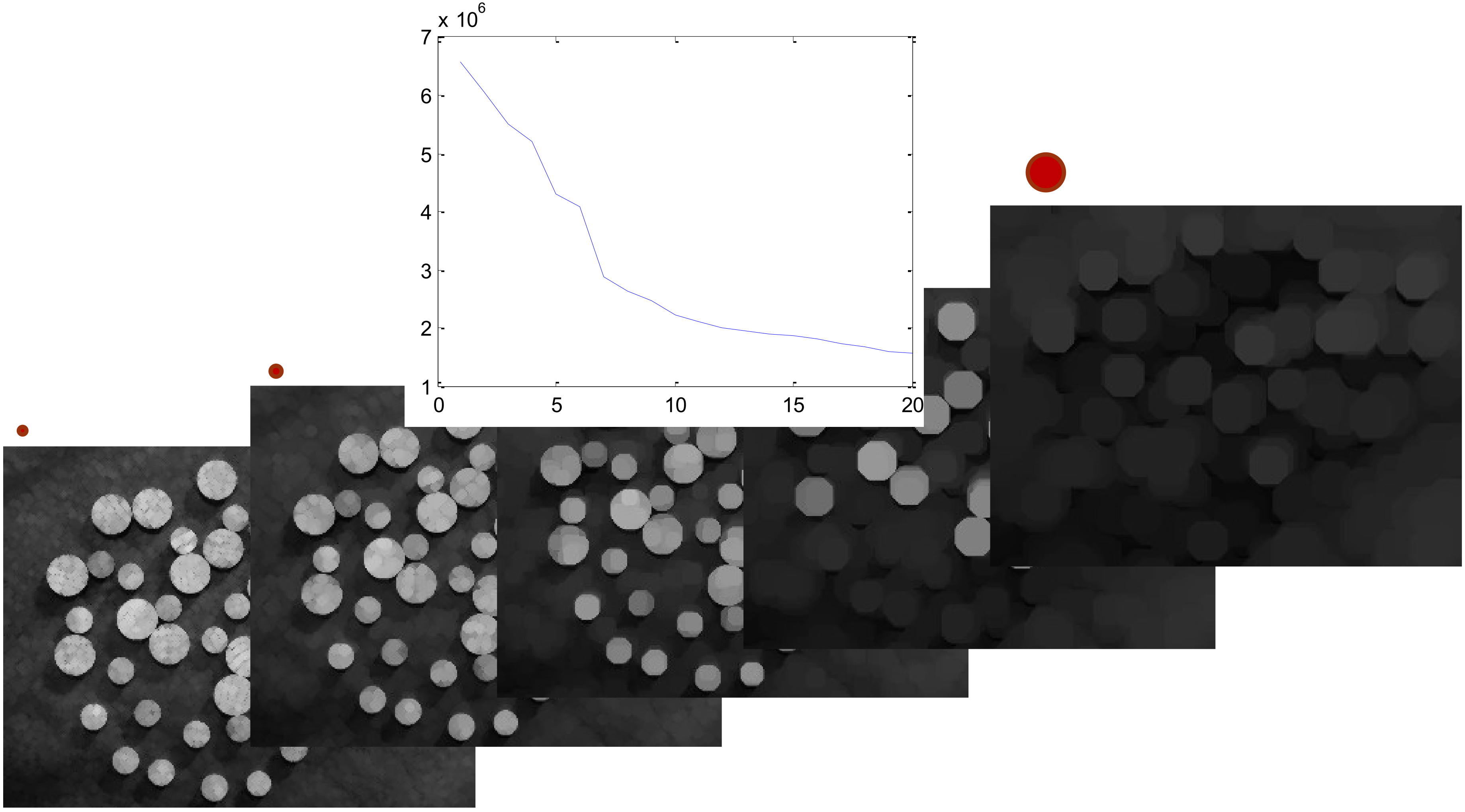
Granulometry

- Assuming that the particles are brighter than the background, the approach relies on application of the opening operator with SE of increasing size
- For a given SE size, opening supresses the output in correspondence to particles that are smaller than the SE



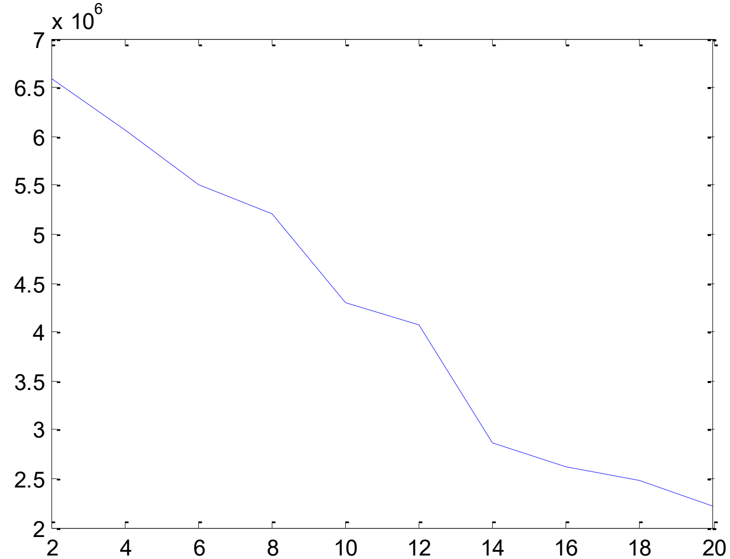
Granulometry

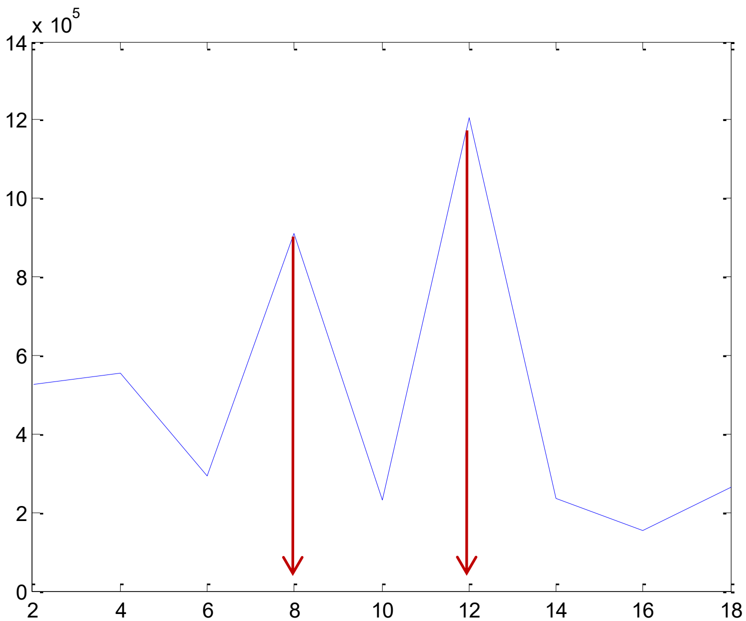
- At each opening step (SE size), the sum of output pixel values defines the **surface area**



Granulometry

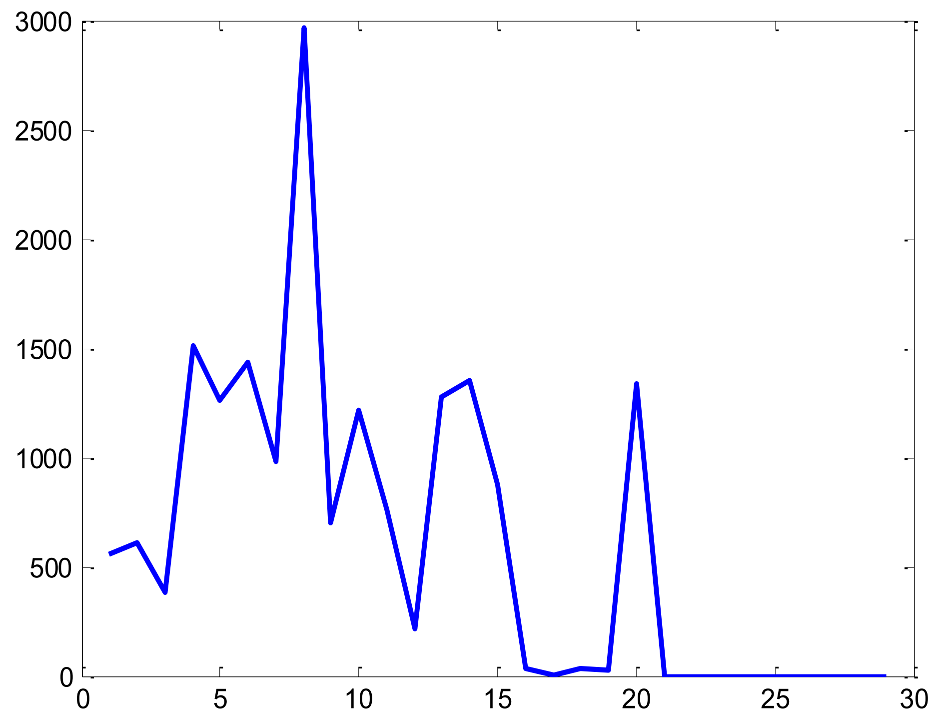
- Results are represented by the incremental differences of surface area values
- Peaks are expected when the size of the SE slightly exceeds the size of some particles



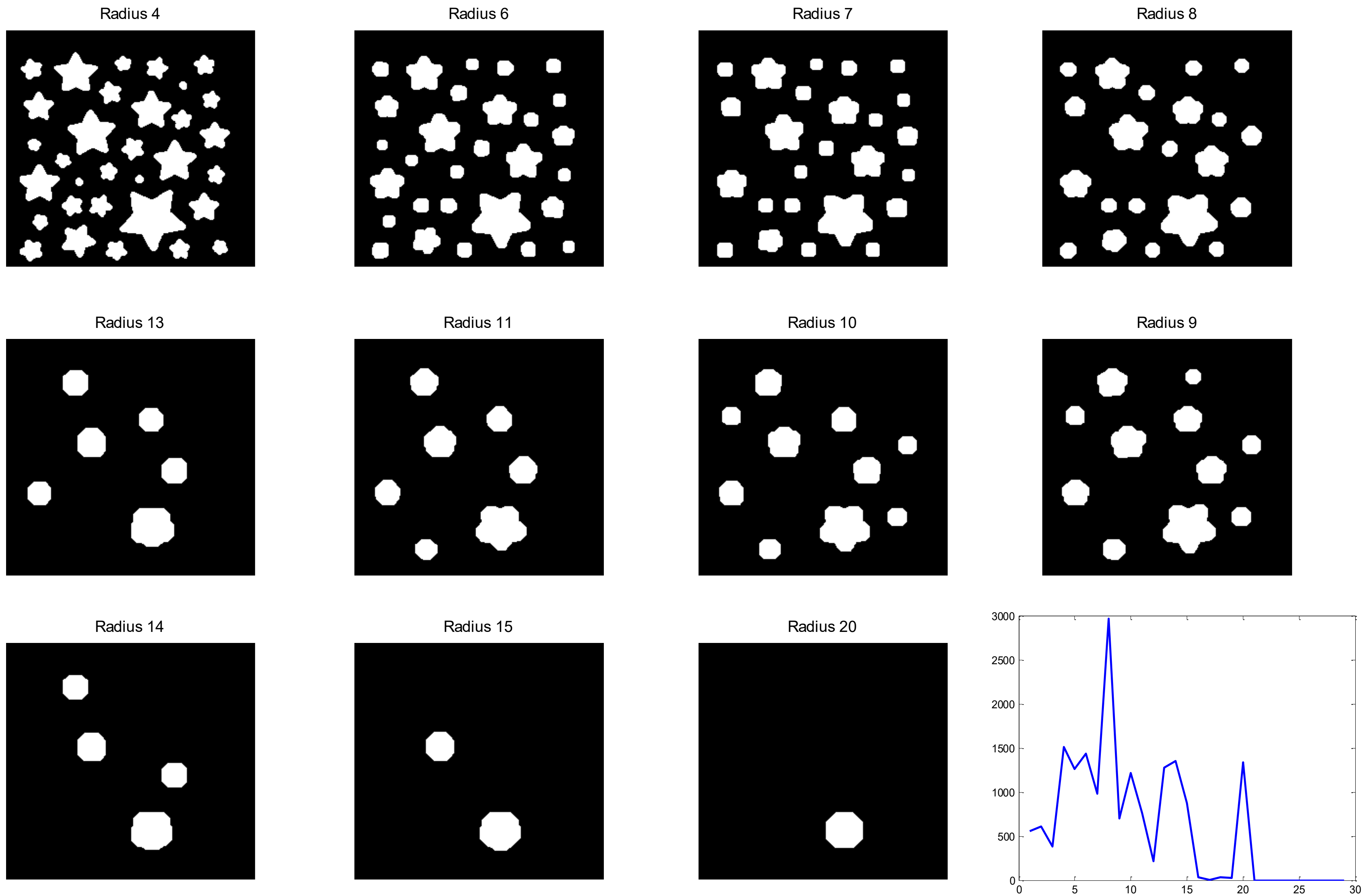


Granulometry

- Accurate results are obtained only if the shape of the objects matches the shape of the structuring element. In the case reported below the SE is a disk.

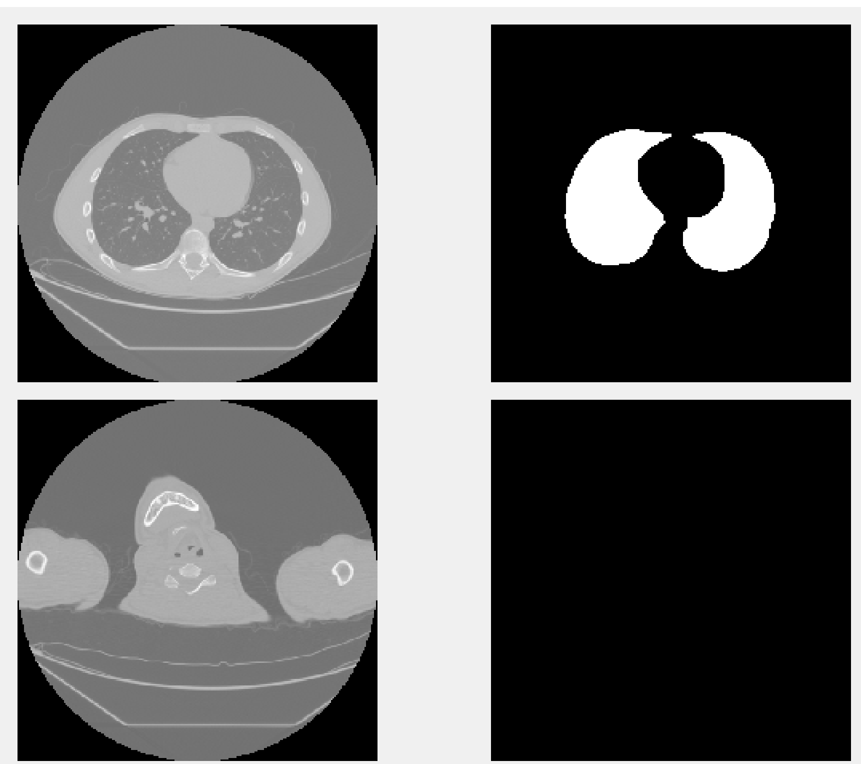


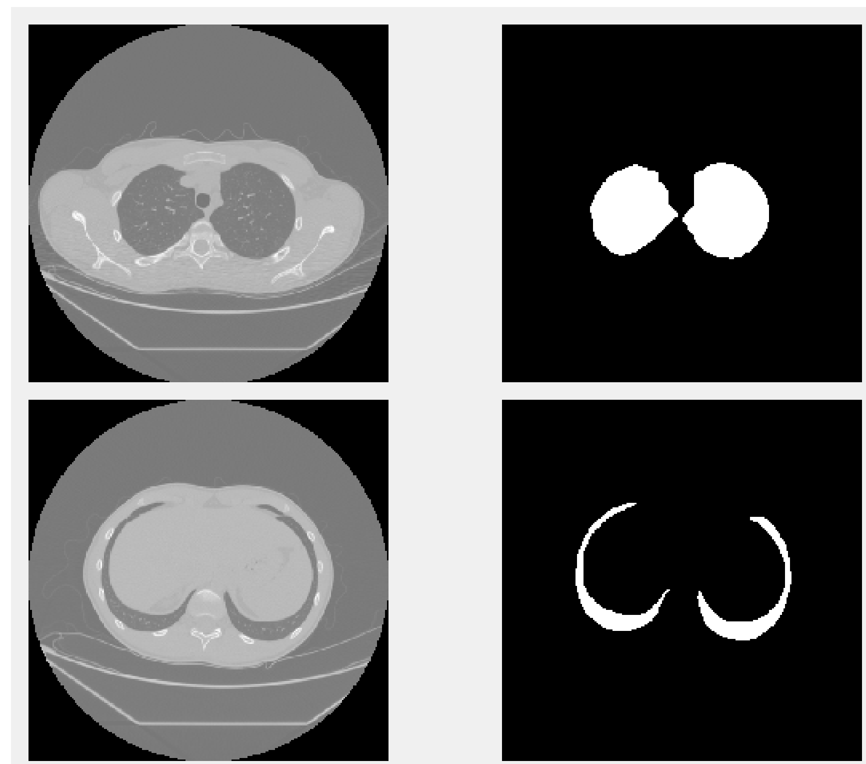
Granulometry



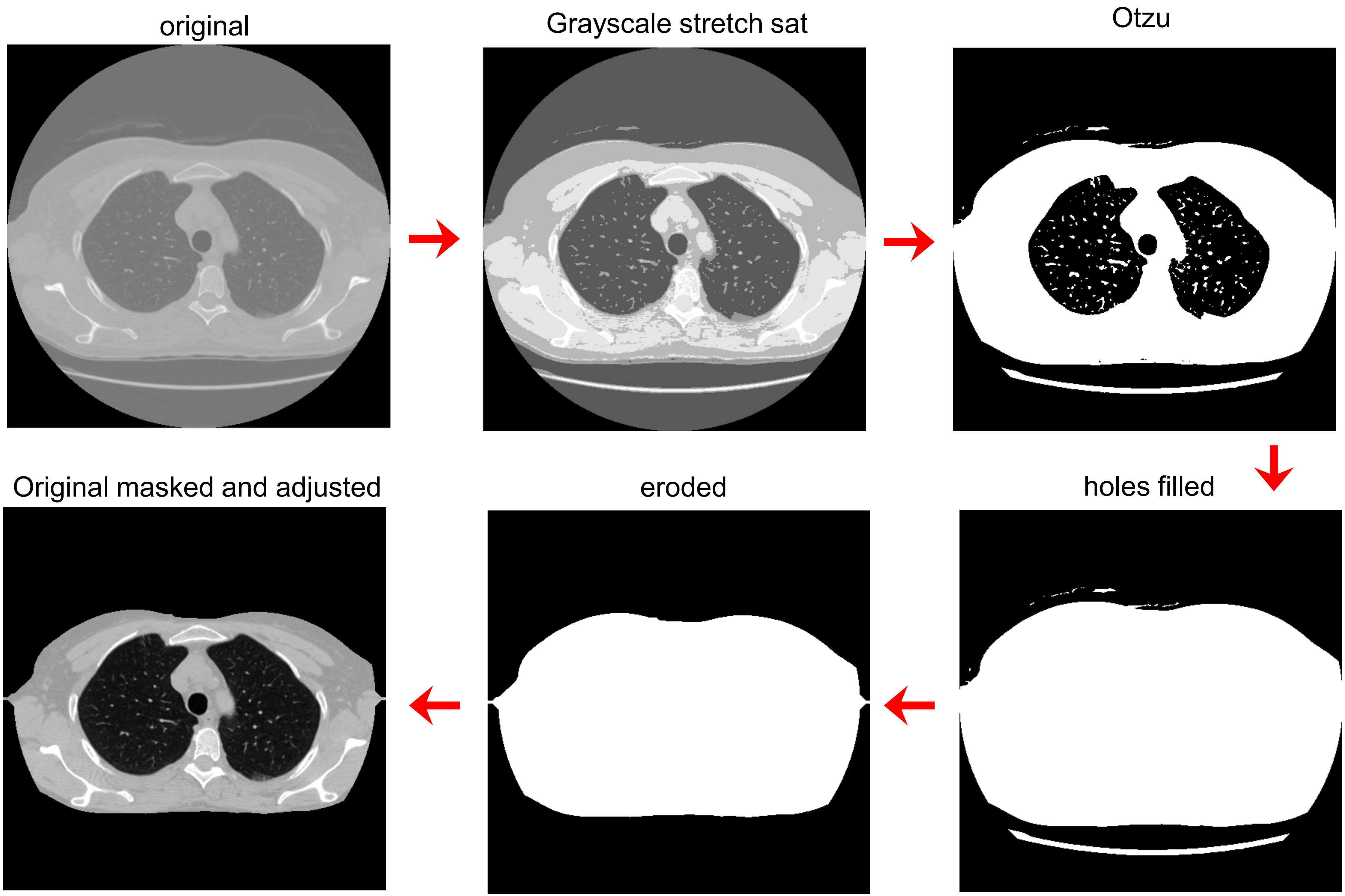
Morphological processing

- Define a processing pipeline that is able to extract a binary mask of lung regions in Computed Tomography slices of body chest





Discard regions outside the chest



Discard regions outside the chest