A Source $\{a, a, a, a, a, a, a, a, a, a, b, b, b, b, b, b, c, c, c, c, c,$ $d, d, e, e, e, e, f, f, f, g\}$. Find the probabilities, and calculate

i) Standard Huffman code by placing composite symbol as high as possible.

ii) Average length and the Variance of the word lengths.

Solution:-

a	a	a	a	a	a.	a	a	a	a	— 10
b	b	b	b	b	b					— 06
c	c		c	c	c					— 05
										— 02
										— 04
d	d									— 03
e	e		e	e						— 01
f	f	f								
										31
g										

$P(a) = \dfrac{10}{31} = 0.322$

$P(b) = \dfrac{06}{31} = 0.193$

$P(c) = \dfrac{05}{31} = 0.161$

$P(d) = \dfrac{02}{31} = 0.064$

$P(e) = \dfrac{04}{31} = 0.129$

$P(f) = \dfrac{03}{31} = 0.096$

$P(g) = \dfrac{01}{31} = 0.032$

$a = 0.322 \Rightarrow 0.32 - ①$

$b = 0.193 \Rightarrow 0.19 - ②$

$c = 0.161 \Rightarrow 0.16 - ③$

$d = 0.065 \Rightarrow 0.07 - ④$

$e = 0.129 \Rightarrow 0.13 - ⑤$

$f = 0.096 \Rightarrow 0.01 - ⑥$

$g = 0.032 \Rightarrow 0.03 - ⑦$

a = 0.32 (11)   a = 0.32   a = 0.32 (11)   a = 0.32 (11) → α_4 = 0.39 (0) × = 0.61

b = 0.19 (00)   b = 0.19   → α_2 = 0.2 (01) → α_3 = 0.29 (10)   a = 0.32 ^{11} α_4 = 0.39

c = 0.16 (101)   c = 0.16   b = 0.19 (00) α_2 = 0.2 —01 α_3 = 0.29

e = 0.13 (100)   e = 0.13   c = 0.16 $\frac{101}{}$ b = 0.19 00   10

f = 0.1 (010)   e = 0.13 100 α_5 = 0.61 7
            0111 → α_1 = 0.1 —011 α_4 = 0.39 0
d = 0.07 ⌐   f = 0.1 ⌐
                                 010
g = 0.03 ⌐
         0100

a = 11

b = 00   Code length = $L = \sum P_i (\ell)$

c = 101
                  = 2 (0.32) + 2 (0.19) + 3 (0.16) + 4 (0.07) + 3 (0.13)
d = 0111

e = 100   + 3 (0.1) + 4 (0.03)

f = 010   L =   2.59 bits/pixel.

g = 0100

Arithmetic Encoding.

1) ~~Describe~~ Decode the five symbol sequence of message $a_1 a_2 a_3 a_4$
$a_1 a_2 a_3 ~~a_3~~ ~~a_4~~$ from a four symbol source is used.

Source Symbol	Probability	Initial Subinterval
a_1	0.2	[0.0, 0.2]
a_2	0.2 +0.2	[0.2, 0.4]
a_3	0.4 +0.4	[0.4, 0.8]
a_4	0.2 +0.8	[0.8, 1.0]
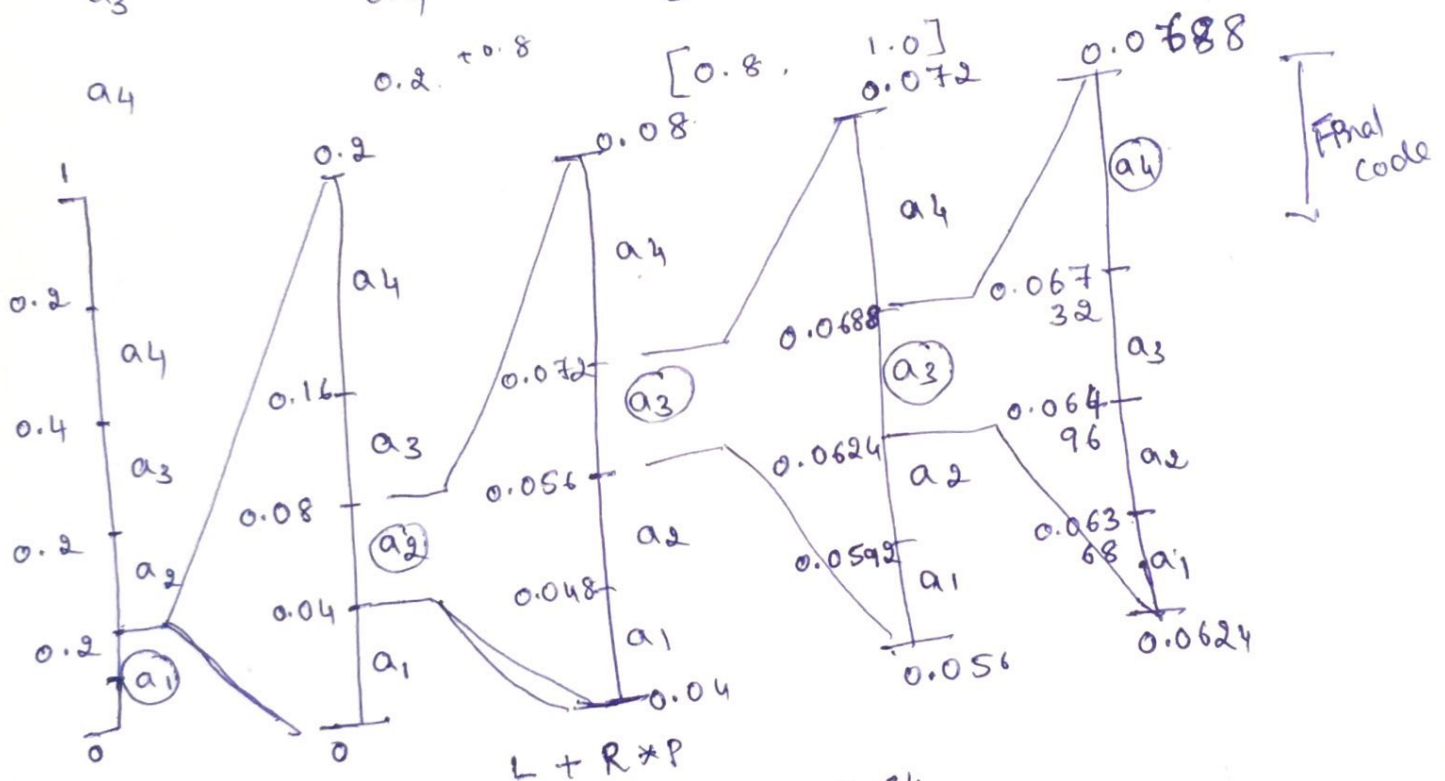
① st iteration
$R = H - L$

$R = 0.2 - 0$

$R = 0.2$

$$L + R * P$$

$a_1 = 0 + 0.2 * 0.2 = 0.04$

$a_2 = 0.04 + 0.2 \times 0.2 = 0.08$

$a_3 = 0.08 + 0.2 \times 0.4 = 0.16$

$a_4 = 0.16 + 0.2 \times 0.2 = 0.2$

②
$R = H - L$

$R = 0.08 - 0.04$

$R = 0.04$

$$L + R * P$$

$a_1 = 0.04 + 0.04 \times 0.2 = 0.048$

$a_2 = 0.048 + 0.04 \times 0.2 = 0.056$

$a_3 = 0.056 + 0.04 \times 0.4 = 0.072$

$a_4 = 0.072 + 0.04 \times 0.2 = 0.08$

③ $R = H - L = 0.072 - 0.056 = \underline{\underline{0.016}}$

$L + R * P$

$a_1 = 0.056 + 0.016 \times 0.2 = 0.0592$

$a_2 = 0.0592 + 0.016 \times 0.2 = 0.0624$

$a_3 = 0.0624 + 0.016 \times 0.4 = 0.0688$

$a_4 = 0.0688 + 0.016 \times 0.2 = 0.072$

④ $R = H - L = 0.0688 - 0.0624 = 0.0064$

$L + R * P$

$a_1 = 0.0624 + 0.0064 \times 0.2 = 0.06368$

$a_2 = 0.06368 + 0.0064 \times 0.2 = 0.06496$

$a_3 = 0.06496 + 0.0064 \times 0.4 = 0.06732.$

$a_4 = 0.06732 + 0.0064 \times 0.2 = 0.0688$