FEEDBACK LINEARIZATION BASED CONTROL OF NON-LINEAR SYSTEMS WITH UNKNOWN DISTURBANCES

Major Project- I Report

Submitted in the partial fulfillment for the award of

Degree of

BACHELOR OF TECHNOLOGY
IN ELECTRICAL ENGINEERING

Under the guidance

of

Dr. Bharat Bhushan Sharma

DEPARTMENT OF ELECTRICAL ENGINEERING
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CANDIDATE'S DECLARATION

We hereby certify that the work which is being presented in the project titled "Feedback linearization based control of non-linear systems with unknown disturbances" in partial fulfillment of the requirements for the award of the degree of **Bachelor of Technology** and submitted in the Department of Electrical Engineering of the National Institute of Technology Hamirpur, is an authentic record of our own work carried out during a period from July 2022 to December 2022 under the guidance of Dr. Bharat Bhushan Sharma, Assistant Professor, Department of Electrical Engineering, National Institute of Technology, Hamirpur.

The matter presented in this report has not been submitted by us for the award of any other degree of this or any other Institute/University.

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The project Viva-Voce Examination of Vivek Kumar, Priyanshu Kaushal, Dipesh Kumar, Siddharth Saini and Paras Verma has been held on ……………

Signature of External Examiner

Acknowledgement

We owe a debt of thanks to a number of eminent persons for their assistance with this project, the first and foremost of which is Dr. Bharat Bhushan Sharma, our supervisor and guide, for his professional insight and unwavering support throughout. This work has been heavily influenced by his perceptive inspiration, encouragement, and understanding.

We would like to express our profound gratitude to the entire faculty of the Electrical Engineering Department at NIT Hamirpur for their direction, inspiration, cooperation, and consistent encouragement throughout the project; without their unwavering support, this project would not have been successful.

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Finally, we owe everything to almighty to shower his blessings so that our efforts reach the destination.

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ABSTRACT

Non-linearity is an inevitable part of the majority of the systems we come across. In general, a non-linear system controller is a type of control system that is designed to regulate the behaviour of a non-linear system. Non-linear systems are complex and often exhibit unpredictable behaviour, making them difficult to control using traditional linear control methods.

To address this challenge, non-linear system controllers use advanced algorithms and techniques to model and analyse the system, and to generate appropriate control signals that can effectively manage its behaviour.

One of the key challenges in designing a non-linear system controller is the need to accurately model the system's behaviour, including its various non-linearities and dynamics. This requires the use of advanced mathematical techniques, such as non-linear system identification and non-linear control theory, to accurately capture the system's behaviour and to develop appropriate control strategies.

Non-linear system controllers may incorporate advanced techniques such as adaptive control, robust control, and optimal control, to account for uncertainty and to generate control signals that are robust to disturbances.

Overall, the design and implementation of a non-linear system controller require. By effectively addressing the challenges of non-linearity, uncertainty, and disturbance, a non-linear system controller can help to improve the performance, stability, and reliability of complex non-linear systems.

A feedback linearization controller and a disturbance observer are combined in this study to form a robust controller. Despite being subjected to an unknowable external force, this controller is able to tame the chaotic behaviour of a nonlinear system. To prove that the suggested strategy works, numerical simulations are run.

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Chapter 1
Introduction

Control problems are significant because they are a fundamental part of many different fields, including engineering, economics, and computer science. In general, a control problem involves designing a system that can achieve a desired behavior or objective by taking certain actions based on sensory information[1-8].

This type of problem is important because it allows us to design systems that can respond to changing conditions and adapt to different situations.

For example, in engineering, control problems are used to design systems for controlling the movement of robots or vehicles, regulating temperature in buildings, or managing power systems. In economics, control problems are used to design policies that can help manage the economy and achieve specific goals, such as reducing inflation or increasing employment. In computer science, control problems are used to design algorithms that can solve complex problems and make decisions based on data. The ability to design systems that can effectively control their behavior is crucial for various fields and applications.

All control systems can be categorized as linear and non-linear systems:

a) Linear control systems
b) Non-Linear control systems

1.1 Linear control systems.

Linear control systems are systems that can be modeled using linear mathematical equations. In other words, the system's behavior is determined by a set of linear equations that describe how the system's variables change over time. This type of system is important because it is relatively simple to analyze and design, and many real-world systems can be approximated as linear systems.

As linear control systems are those that work on the basis of the homogeneity and additivity principles.

1.2 Non-linear Control Systems

Non-linear control systems are systems that cannot be accurately modeled using linear equations. In other words, the system's behavior is determined by non-linear equations, which are equations that are not proportional to the variables they describe [9-13]. Non-linear systems are often more complex and difficult to analyze than linear systems, but they can also exhibit more interesting and diverse behavior. Many real-world systems, such as biological systems and mechanical systems with friction, are non-linear. Because of their complexity, the design and analysis of non-linear control systems often require more advanced mathematical techniques than those used for linear systems.

We can say that a control system is nonlinear if it doesn't follow the principle of homogeneity. In real life, none of the systems that control things are linear (linear control systems only exist in theory). The describing function is a way to analyze some nonlinear control problems in a rough way.

1.3 Challenges of non-linear domain

Amongst the spectrum, one of the main challenges in the field of non-linear control is the difficulty of analyzing and designing non-linear systems. Because non-linear equations are not proportional to the variables they describe, they are often more complex and difficult to solve than linear equations. This can make it challenging to understand the behavior of a non-linear system and to predict how it will respond to different inputs and stimuli.

Another challenge in the field of non-linear control is the fact that many real-world systems are non-linear, and therefore cannot be accurately modeled or controlled using linear techniques. This can make it difficult to design control systems for these real-world systems, and it may require the use of more complex and advanced methods like Feedback linearization, disturbance observer, the Lyapunov method, and many more.

Some non-linear systems may exhibit chaotic behavior, where small changes in the system's initial conditions can lead to large and unpredictable changes in its behavior over time. Other non-linear systems may exhibit periodic behavior, where the system's variables follow a regular and repeating pattern. Still other non-linear systems may exhibit more complex behavior, such as bifurcations or attractors. Overall, there is a wide variety of different types of non-linear systems, and each type has its own unique characteristics and properties.

1.4 Chaotic behavior of non-linear systems

Chaotic behavior is a type of behavior exhibited by some non-linear systems. In a chaotic system, small changes in the system's initial conditions can lead to large and unpredictable changes in its behavior over time. Indicating that it is difficult or impossible to predict the long-term behavior of a chaotic system, even if we have a detailed understanding of the system's equations and parameters[14].

This behavior is often characterized by the presence of sensitive dependence on initial conditions, where small differences in the initial conditions of the system can lead to vastly unpredictable changes in its behavior over time. This type of behavior is commonly observed in many real-world systems, such as weather patterns, population dynamics, and the motion of celestial bodies. The behavior is often characterized by a seemingly random and irregular pattern of behavior, and it can make it difficult to predict the long-term behavior of a chaotic system. The study of chaotic behavior in non-linear systems is an active area of research in many different fields, including physics, mathematics, and engineering.

Chapter 2
Literature Review

2.1 Robust Control

Robust control is a type of control strategy that is designed to ensure the stability and performance of a control system in the presence of uncertainty. In other words, robust control is concerned with designing control systems that can operate effectively in real-world environments, where there may be variations in the system's parameters or external disturbances. The goal of robust control is to ensure that the system is able to maintain its desired behavior, even in the face of these uncertainties.

One of the main challenges in control theory is dealing with uncertainty, because many real-world systems are subject to variations in their parameters or external disturbances. For example, a robot control system may need to handle variations in the robot's actuators or changes in the environment, such as obstacles or changes in lighting. A control system for a power grid may need to handle variations in the demand for electricity or changes in the availability of power sources. In these cases, traditional control techniques, which are based on the assumption of perfect knowledge of the system's parameters and conditions, may not be effective.

Robust control addresses this challenge by using advanced mathematical techniques to design control systems that are resilient to uncertainty. These techniques allow the control system to adapt to changes in the system's parameters or external disturbances, and to maintain its desired behavior even in the face of these variations. For example, a robust control system may use feedback to monitor the system's state and adjust its control inputs accordingly. This can help the system to respond to changes in its environment and to maintain its desired behavior.

There are many different techniques and methods that can be used for robust control, and they vary depending on the specific application and the type of system being controlled. Some common approaches to robust control include the use of Lyapunov stability theory, robust optimization, and robust adaptive control. These methods allow the control system to be designed in a way that is robust to uncertainty, while still maintaining good performance and stability.

In general Robust control serves as reliable aspect in control theory for controlling systems, because it allows us to design control systems that can operate effectively in real-world environments. By using robust control techniques, it is possible to design control systems that are more resilient and reliable, and that can maintain their desired behavior even in the face of uncertainty. This is crucial for many different applications, including robotics, power systems, and many others, where control systems must operate in uncertain and changing environments.

2.2 Feedback Linearization

Feedback linearization is a technique in control theory that is used to linearize the dynamics of a non-linear system. In other words, feedback linearization is a method for converting the equations that describe a non-linear system into a set of linear equations, which are much easier to analyze and design. This technique is important because it allows us to use the powerful tools and techniques of linear control theory to design and analyze non-linear systems.

The need for feedback linearization arises from the fact that many real-world systems are non-linear, and therefore cannot be accurately described or controlled using linear techniques. For example, many mechanical systems, such as robots or vehicles, have non-linear dynamics due to factors such as friction or elasticity. Similarly, many biological systems, such as the human body or ecological systems, have non-linear dynamics due to the complex interactions between their components. In these cases, non-linear control techniques must be used to design and analyze the systems.

Feedback linearization is a method for overcoming this challenge by using feedback control to linearize the dynamics of a non-linear system. This is done by using sensors to measure the system's state, and then using feedback control to adjust the system's inputs in such a way that the resulting equations of motion are linear. This allows the system's behavior to be described using linear equations, which can then be analyzed and designed using the powerful tools and techniques of linear control theory.

There are many different techniques and methods that can be used for feedback linearization, and they vary depending on the specific application and the type of system being controlled. Some common approaches to feedback linearization include the use of coordinate transformations, state feedback, and dynamic feedback linearization.

These methods allow the control system to be designed in a way that ensures its stability and performance, by using feedback to linearize the system's dynamics and make it amenable to linear control techniques.

2.3 Disturbance Observer

A disturbance observer is a type of control system that is designed to estimate and compensate for external disturbances that affect the system's behavior. In other words, a disturbance observer is a device or algorithm that is used to identify and counteract the effects of external disturbances on a control system. This type of system is important because it allows the control system to maintain its desired behavior, even in the presence of external disturbances.

The use of disturbance observers is motivated by the fact that many real-world control systems are subject to external disturbances that can affect their behavior. For example, a robot control system may be affected by external forces, such as wind or friction, that can push or pull the robot off course. A control system for a power grid may be affected by changes in the demand for electricity or the availability of power sources. In these cases, the external disturbances can cause the control system to deviate from its desired behavior, and this can reduce the system's performance and reliability.

Disturbance observers are designed to address this problem by estimating the effects of external disturbances on the system's behavior, and by using this information to compensate for the disturbances. This is typically done using feedback control, where the disturbance observer uses sensors to measure the system's state, and then adjusts the control inputs in order to compensate for the effects of the disturbance. By using this approach, the disturbance observer can help the control system to maintain its desired behavior, even in the presence of external disturbances.

There are many different techniques and methods that can be used for disturbance observer design, and they vary depending on the specific application and the type of system being controlled. Some common approaches to disturbance observer design include the use of robust control techniques, adaptive control, and state observer design. These methods allow the disturbance observer to be designed in a way that is effective at estimating and compensating for external disturbances, while still maintaining good performance and stability.

2.4 Lyapunov stability

Lyapunov stability is a concept in control theory that is used to analyze the stability of a control system. In general, a control system is said to be Lyapunov stable if it is able to maintain its desired behavior, even in the presence of disturbances or variations in the system's parameters. This concept is named after the Russian mathematician Aleksandr Lyapunov, who first developed the theory of stability in the late 1800s.

The concept of Lyapunov stability is important because it provides a mathematical framework for understanding the stability of a control system. In particular, Lyapunov stability is based on the idea that a control system is stable if it has a Lyapunov function, which is a mathematical function that can be used to measure the system's deviation from its desired behavior. A Lyapunov function is typically chosen so that it has the property that it decreases over time if the system is following its desired behavior, and increases if the system is deviating from its desired behavior. This allows the Lyapunov function to be used as a measure of the system's stability, and to provide a basis for analyzing and designing stable control systems.

There are many different techniques and methods that can be used for analyzing and designing control systems using Lyapunov stability. These methods vary depending on the specific application and the type of system being controlled, but some common approaches include Lyapunov stability analysis, Lyapunov synthesis, and Lyapunov-based control design. These methods allow the control system to be designed in a way that ensures its stability, by using the Lyapunov function to measure and control the system's deviation from its desired behavior.

In conclusion, Lyapunov stability is an important concept in control theory, because it provides a mathematical framework for understanding the stability of a control system. By using Lyapunov stability analysis and design methods, it is possible to design control systems that are able to maintain their desired behavior, even in the presence of disturbances or variations in the system's parameters. This is crucial for many different applications, including robotics, power systems, and many others, where control systems must operate in uncertain and changing environments.

A Lyapunov function is a scalar function established on phase space that can be used to show an equilibrium point's stability.
Suppose V(X) be a continuously differentiable function in the origin's neighbourhood U. If the following requirements are satisfied, the function V(X) is known as the Lyapunov function for an autonomous system X' = f(x).

1. $V(X) > 0$ for all $X \in U \setminus \{0\}$
2. $(dV/dt) \leq 0$ for all $X \in U$
3. $V(0) = 0$

2.4.1 Stability Theorem in the Lyapunov Sense

If a Lyapunov function V(X) exists in the neighbourhood U of an autonomous system's zero solution X = 0, the system's equilibrium point X = 0 is Lyapunov stable.

2.4.2 Asymptotic Stability Theorem

If a Lyapunov function $V(X)$ with a negative definite derivative $(dV/dt) < 0$ for all $X \in U \setminus \{0\}$ exists in the neighbourhood U of an autonomous system's zero solution $X = 0$, then the system's equilibrium point $X = 0$ is asymptotically stable.

2.4.3 Condition for stability

1. $V(X) > 0$ for all $X \in U \setminus \{0\}$
2. $(dV/dt) < 0$ for all $X \in U$
3. $V(0) = 0$

As can be seen, the total derivative dV/dt in the vicinity of the origin must be strictly negative in order to be asymptotically stable.

2.5 Duffing's Oscillator

Duffing's oscillator is a type of non-linear oscillator that exhibits chaotic behavior. It is named after the German mathematician Georg Duffing[15], who first studied this system in the early 1900s. The Duffing oscillator is described by a non-linear differential equation, which describes how the oscillator's position and velocity change over time. This equation contains a cubic non-linear term, which is what gives the system its chaotic behavior.

The Duffing oscillator is interesting because it provides a simple but representative example of a chaotic system. In particular, the Duffing oscillator exhibits sensitive dependence on initial conditions, which is a defining characteristic of chaotic systems. This means that small changes in the oscillator's initial position and velocity can lead to large and unpredictable changes in its behavior over time. This makes it difficult or impossible to predict the long-term behavior of the Duffing oscillator, even if we have a detailed understanding of its equations and parameters.

The Duffing oscillator is commonly used as a model system for studying the dynamics of non-linear systems and the phenomenon of chaos. It is also used in many different fields, including engineering, physics, and biology, as a simple but representative example of a chaotic system. By studying the behavior of the Duffing oscillator, researchers are able to gain insight into the complex and unpredictable behavior of non-linear systems, and to develop methods for analyzing and controlling these systems.

2.6 Genesio System

The Genesio system is a type of non-linear system that is commonly used to study the behavior of unstable systems. It is named after the Italian mathematician Tullio Genesio[16], who first studied this system in the late 1960s. The Genesio system is described by a set of non-linear differential equations, which describe how the system's variables change over time. The Genesio system is interesting because it can exhibit a wide range of behaviors, including stable, periodic, and chaotic behavior.

The Genesio system is useful as a model for studying the dynamics of non-linear systems, because it can exhibit a wide range of behaviors depending on the values of its parameters. In particular, the Genesio system can exhibit stable behavior, where the system's variables follow a fixed and predictable pattern, or chaotic behavior, where the system's variables follow a complex and unpredictable pattern.

This makes it a useful tool for studying the transition from stable to chaotic behavior in non-linear systems, and for understanding how different system parameters can affect the system's behavior. The Genesio system is commonly used in many different fields, including engineering, physics, and biology, as a simple but representative example of a non-linear system.

Chapter 3

Problem Formulation and Main Result

Non-linear systems exhibiting chaotic behavior are described in this project. The phenomenon of chaos in nonlinear systems is fascinating. There are many unique features of a chaotic system. These include sensitivity to initial conditions, a wide range of possible states, and fractal motion in phase space.

However, in specific engineering contexts, chaotic behavior is seen as undesirable due to the damage it can do to a system's performance and the bounds it can place on how far the system can operate. As a result, it is crucial to identify the characteristics of chaotic motion and use them as a basis for the development of strategies for bringing order out of chaos.

As the controlled trajectory is close to the target one, the method uses time-dependent perturbations as feedback to an approachable system parameter to propel the system into a periodic orbit within a chaotic system. This work proposes a reliable controller that integrates a linearization controller with feedback and a disturbance observer.

This controller can stabilize a nonlinear system whose external excitation is also unknown. Finally, Duffing's oscillator and the Genesio system are used to show that the proposed chaos control scheme works as intended.

Considering an n^{th} order nonlinear dynamical system with the following structure

$$\dot{x}_i = \dot{x}_{i+1},\ i=1,\ldots,\ n\text{-}1, \tag{1}$$

$$\dot{x}_n = f(x) + d(t) + u(t) \equiv f(x) + z(t) \tag{2}$$

Here

$x = [x_1, x_2,\ \ldots x_n]^{\mathrm{T}}$ is the state vector

f(x) = unknown function (maybe nonlinear)
d(t) = unknown external excitation
u(t) = control input
T = sampling time

Assume that our goal is to manipulate the system so that the state converges as closely as possible to the area surrounding the origin, even in the existence of an unidentified disturbance. Suggesting a robust controller that combines a feedback linearization controller with a disturbance observer in order to accomplish the desired result. Here we define:

$$\hat{z} = \hat{\dot{x}}_n - f(x) \tag{3}$$

where $\hat{\dot{x}}_n$ is the estimations of $\dot{x}_n$. If the sampling time is small enough, we can get the following approximation

$$\hat{z}(t) \approx z(t-T) \tag{4}$$

The disturbance observer is defined as follows

$$w = \hat{z} - u(t-T) \approx d(t-T) \tag{5}$$

Next, the feedback linearization controller is chosen as the form

$$v = -f(x) - C^T x \tag{6}$$

Where

$$C = [\, c_1, c_2, \ldots, c_n]^T$$

The selection of c is concerned with the robust stability of the closed-loop system.

The whole control force is defined as follows:

$$u(t) = v(t) - w(t) \tag{7}$$

Now

$$\dot{x}_n = - C^T x + \varepsilon(t) \tag{8}$$

$$\varepsilon(t) \approx d(t) - d(t\text{-}T) \tag{9}$$

Equation (8) can be rewritten as

$$\dot{x} = Ax + \varepsilon(\mathrm{t}) \tag{10}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & & 0 \\ & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -c_1 & -c_2 & -c_3 & & -c_n \end{bmatrix} \tag{11}$$

C is the gain parameter and we choose C such that system matrix A has distinct negative eigen values.

The derivative of the Lyapunov function along the system trajectory can show that a smaller sampling time and a large enough λ_{min} can suppress the influence of the external disturbance effectively.

The control law can be simplified as:

$$\begin{aligned} u(t) &= v(t) - w(t) \\ &= -f(x) - C^T x - \hat{z} + u(t-T) \\ &= -f(x) - C^T x - \hat{\dot{x}}_n + f(x) + u(t-T) \end{aligned}$$

$$\therefore \; u(t) = -C^T x - \hat{\dot{x}}_n + u(t-T) \tag{12}$$

This is a linear controller due to the absence of the function f. Therefore, the control law can be applied to any system satisfying the form of Eq. (1) without estimating the function f. If we adopt the following approximated differential:

$$\hat{\dot{x}}_n(k) = \frac{1}{T}\big(x_n(k) - x_n(k-1)\big) \tag{13}$$

$$\hat{\dot{x}}_n(k) = \frac{1}{T}\big(x_n(k) - x_{n-1}(k)\big) \tag{14}$$

where k denotes the k^{th} sampling.

The discrete version of the control law can be represented as:

$$u(k) = -c^T x(k) - \frac{1}{T}(x_n(k) - x_{n-1}(k)) + u(k-1) \tag{15}$$

Chapter 4
Numerical Simulation

Here, the suggested chaos control scheme is shown to work by being applied to a prototypical chaotic system, the Duffing's oscillator.

4.1.1 Duffing's Oscillator

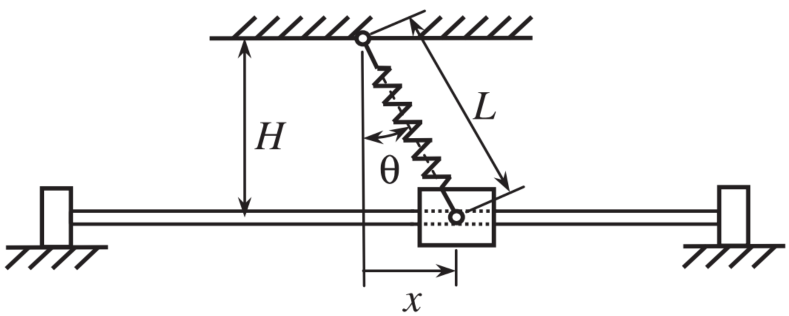

Figure 4.1 A mass-spring mechanical system

Consider a mass-spring mechanical system shown in above figure, in which the mass slides on a horizontal linear guide and is attached to a pivot point through an ideal spring.

The mass is being acted upon by a controlling force u and disturbed by some other external force d.

Taking the length of the spring to be L_0, where $L_0 > H$, and assume that it maintains its straightness at all times. Let's call the inclination of the spring with respect to the vertical axis at the pivot point h. Our working definition of x is the distance moved away from the starting point.

4.1.2 Free body diagram of the mass

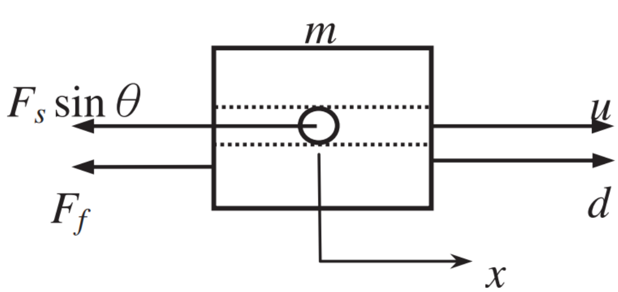

Figure 4.2 Free body diagram of the mass

The dynamic equation of the system in the horizontal direction can be written as:

$$M\ddot{x} = -F_s sin\theta - F_f + d + u \tag{16}$$

Here,

F_s = Restoring Force
F_f = Resistive Force due to friction
u = Control Force
d = disturbed by an unknown external force

The horizontal component of the spring force can be represented as:

$$F_s sin\theta = K\left(1 - \frac{Lo}{L}\right)(\mathrm{L}\sin\theta) = \mathrm{K}\left(1 - \frac{Lo}{\sqrt{H^2+x^2}}\right)x \tag{17}$$

In this example, the parameters are considered as: M = 1, C = 0.25, K = 11.5, L_0 = 2, and H=$\sqrt{3}$. The dynamic system can be represented as:

$$\dot{x}_1 = x_2, \tag{18}$$

$$\dot{x}_2 = -11.5\left(1 - \frac{2}{\sqrt{3+x_1^2}}\right)x_1 - 0.25x_2 + \sin(0.5t) + u, \tag{19}$$

$$\equiv f(x) + d + u,$$

Where,

$$x = [x_1, x_2]^T$$

$$\equiv [x, \dot{x}]^T$$

Using Lyapunov Stability Theorem, deriving the equation for control input:

$$v = \frac{1}{2}[x^T x] \tag{20}$$

Here, v is Lyapunov function

$$\dot{v} = x_1\dot{x}_1 + x_2\dot{x}_2 \qquad \text{(For Duffing Oscillator)}$$

$$\dot{v} = x_1 x_2 + x_2\dot{x}_2 \tag{21}$$

$$\dot{v} = x_1 x_2 + x_2\left(-11.5\left(1 - \frac{2}{\sqrt{3+x_1^2}}\right)x_1 - 0.25x_2 + \sin(0.5t) + u\right) \tag{22}$$

From Lyapunov Stability theorem:

$$\dot{v} < 0$$

After solving, we get

$$u = 11.5\left(1 - \frac{2}{\sqrt{3+x_1^2}}\right)x_1 - x_2 - \sin(0.5t) \tag{23}$$

4.1.3 MATLAB simulation for the Duffing oscillator

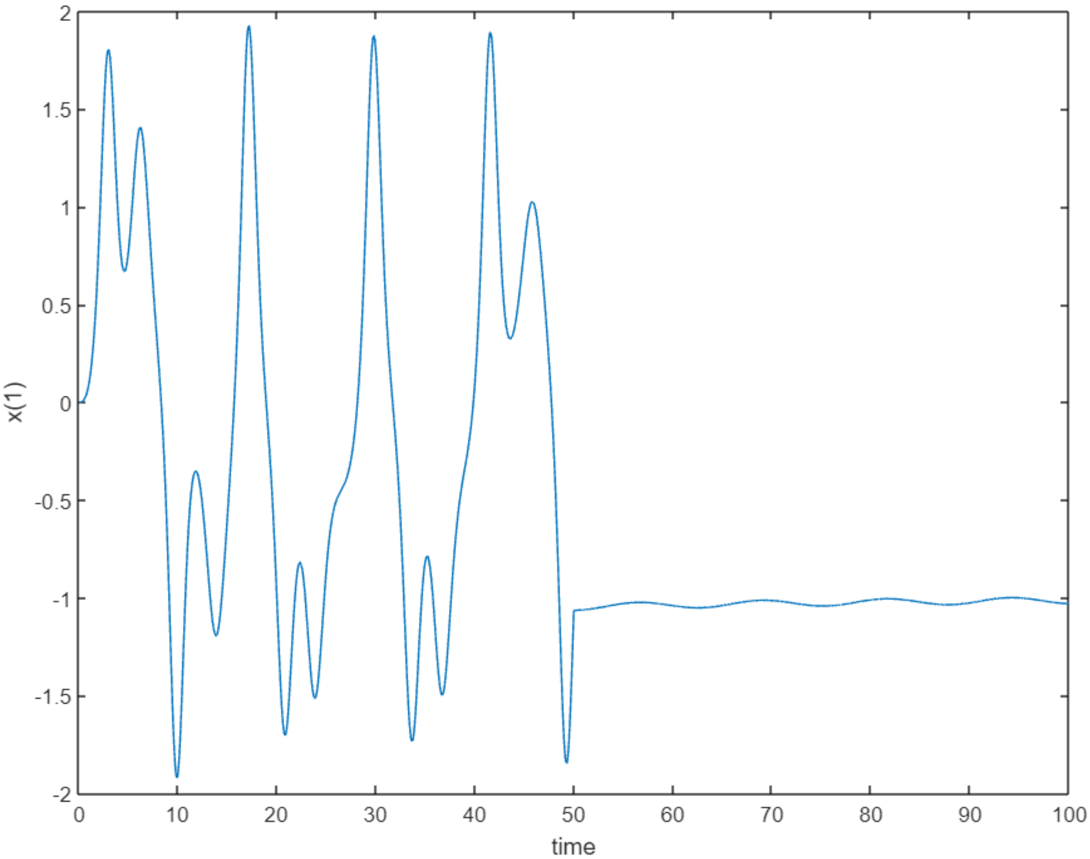

Figure 4.1.1 Time response of the Duffing's oscillator (for variable x_1)

As can be seen in figure 4.1.1, the chaotic behavior is illustrated by the state variable x_1. During this time, the robust controller is disabled by being switched off. After t >50 seconds, the robust controller is applied to the chaotic system, and the system's instability is suppressed, resulting in a relatively stable state.

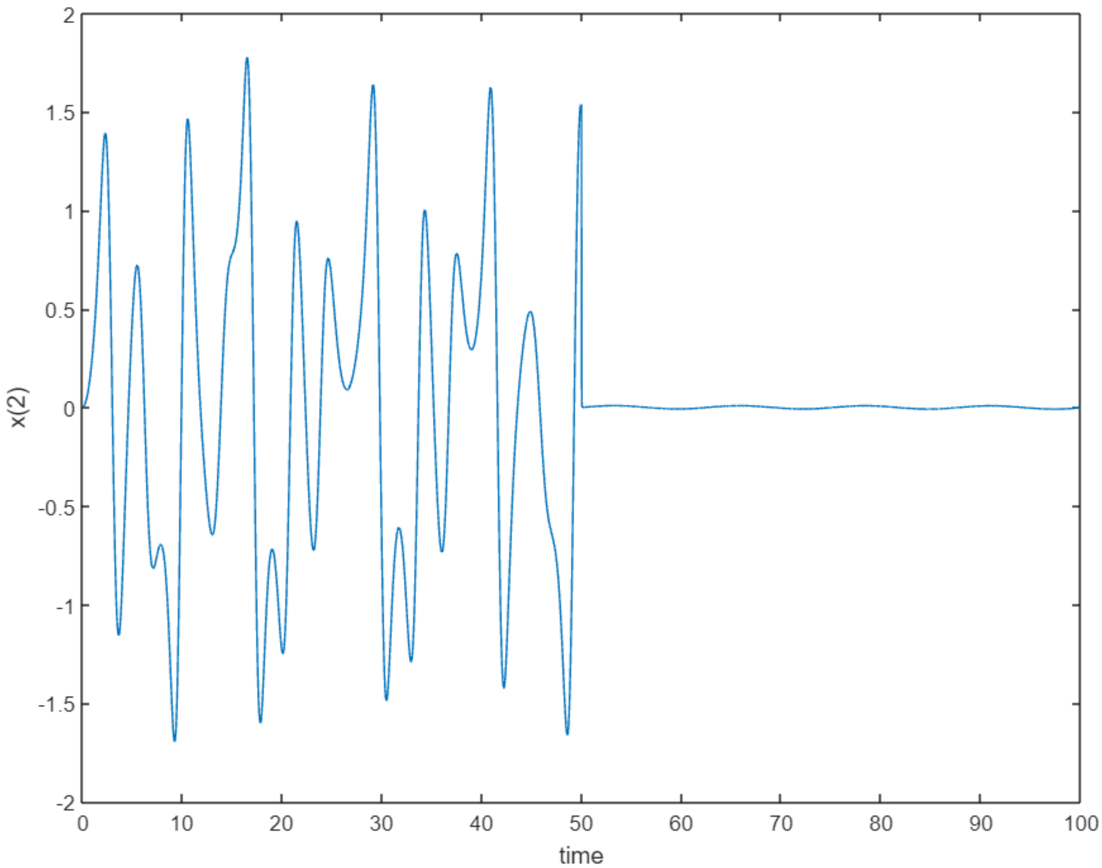

Figure 4.1.2 Time response of the Duffing's oscillator (for variable x_2)

Figure 4.1.2 demonstrates behavior that is strikingly similar to that depicted in figure 4.1.1. The chaotic behavior is illustrated by the state variable x_2. During this time, the robust controller is disabled by being switched off. After $t > 50$ seconds, the robust controller is applied to the chaotic system, and the system's instability is suppressed, resulting in a relatively stable state

4.2.1 Genesio System

The Genesio system captures many chaotic system characteristics. It consists of a simple square component and three simple ordinary differential equations whose solutions depend on three positive parameters. Following are the dynamic equations of the Genesio system:

$$\dot{x} = y \tag{24}$$
$$\dot{y} = z \tag{25}$$
$$\dot{z} = -cx - by - az + x^2 + u \tag{26}$$

where x, y and z are state variables; and a, b and c are positive constants satisfying $ab < c$. If the constants are specified as $a = 1.2$, $b = 4$ and $c = 6$, the uncontrolled system (i.e., $u = 0$) is chaotic and has two unstable equilibrium points, namely (0,0, 0) and (c, 0, 0).

Let the state (-0.2, 0.3, 1.2) be the set point

Using Lyapunov Stability Theorem, deriving the equation for control input:

$$\dot{v} = x\dot{x} + y\dot{y} + z\dot{z} \qquad \text{(For Genesio system)}$$
$$\dot{v} = xy + yz + z(-cx - by - az + x^2 + u) \tag{27}$$

From Lyapunov Stability theorem:

$$\dot{v} < 0$$

After solving we get,

$$u = cx + by - \frac{xy}{z} - y - x^2 \tag{28}$$

4.2.2 MATLAB simulation for the Genesio System

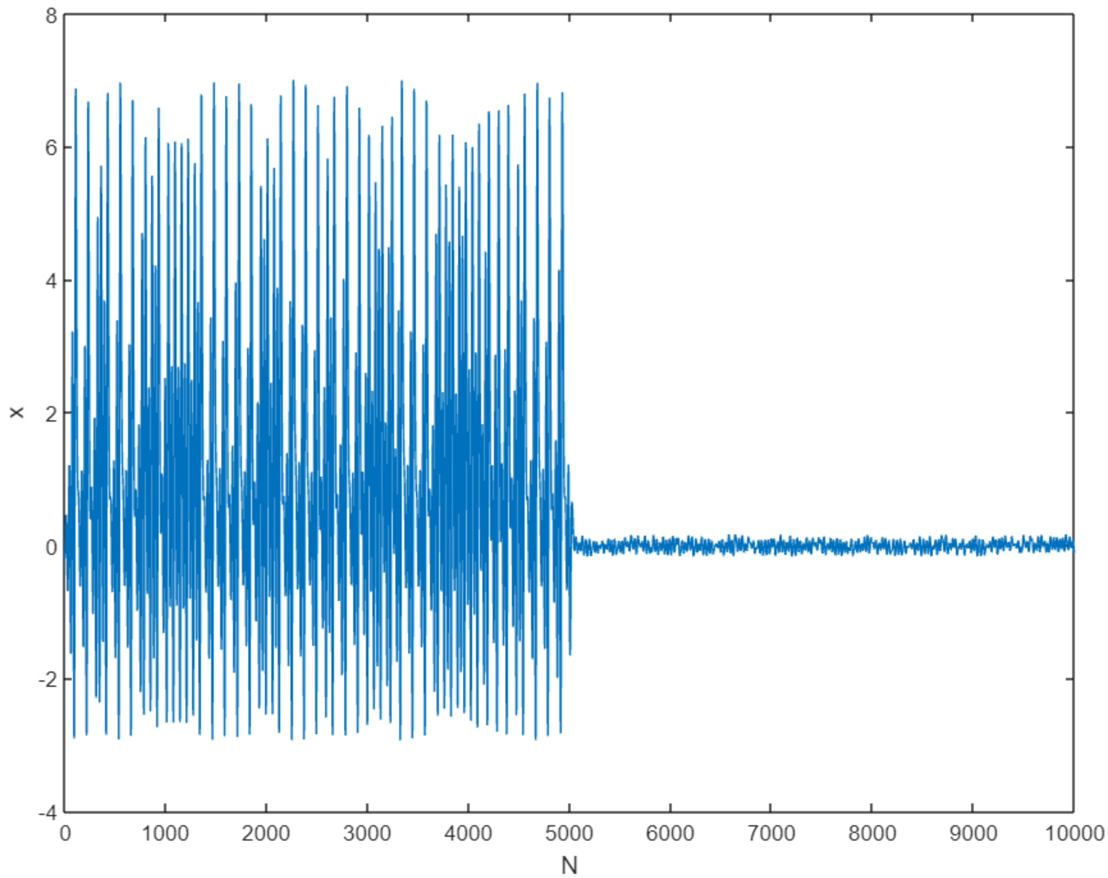

Figure 4.2.1 Time response of the Genesio System (for variable x)

As can be seen in figure 4.2.1, the chaotic behavior is illustrated by the state variable x. During this time, the robust controller is disabled by being switched off. After t >500 seconds (i.e. N > 5000), the robust controller is applied to the chaotic system, and the system's instability is suppressed, resulting in a relatively stable state.

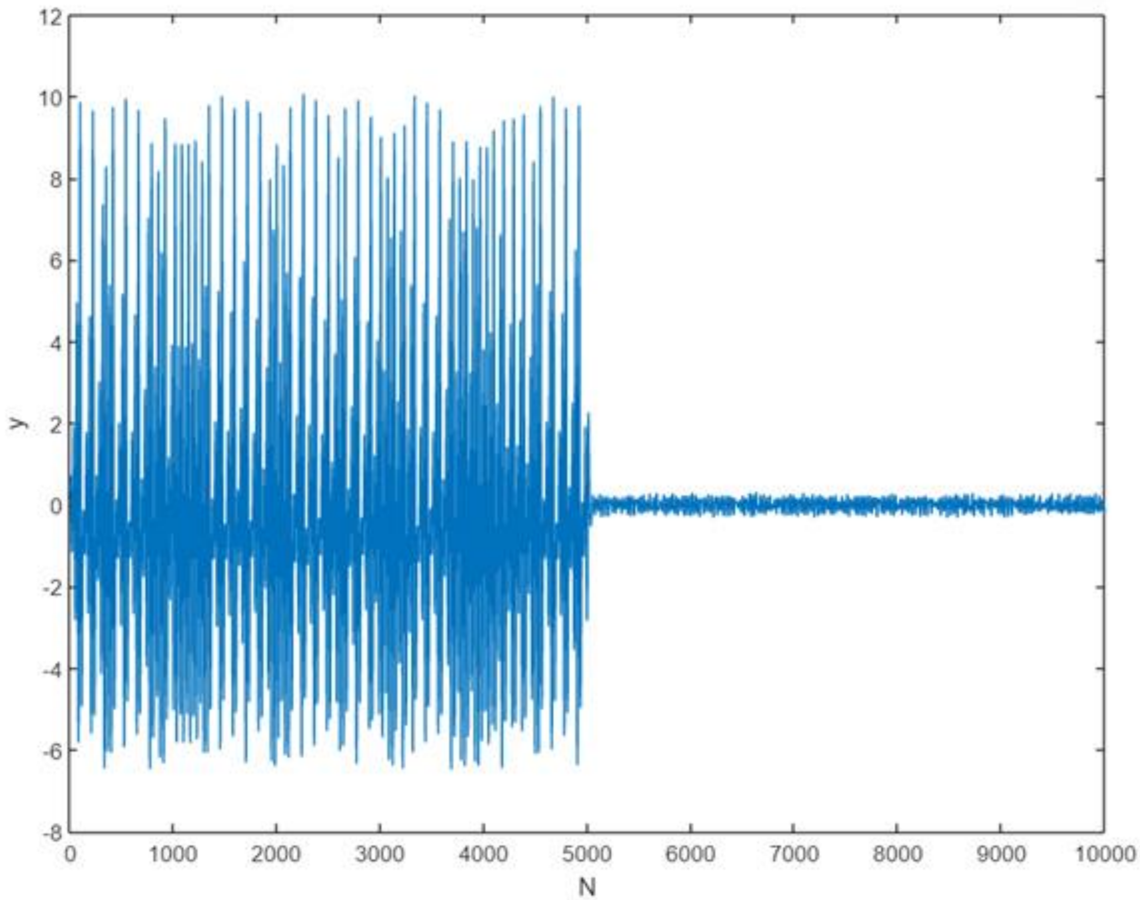

Figure 4.2.2 Time response of the Genesio System (for variable y)

Figure 4.2.2 demonstrates behavior that is strikingly similar to that depicted in figure 4.2.1. The state variable y provides a perfect example of the chaotic behavior, which can be observed in figure 4.2.2. During this time, the robust controller is disabled by being switched off. After t >500 seconds (i.e. N > 5000), the robust controller is applied to the chaotic system, and the system's instability is suppressed, resulting in a relatively stable state.

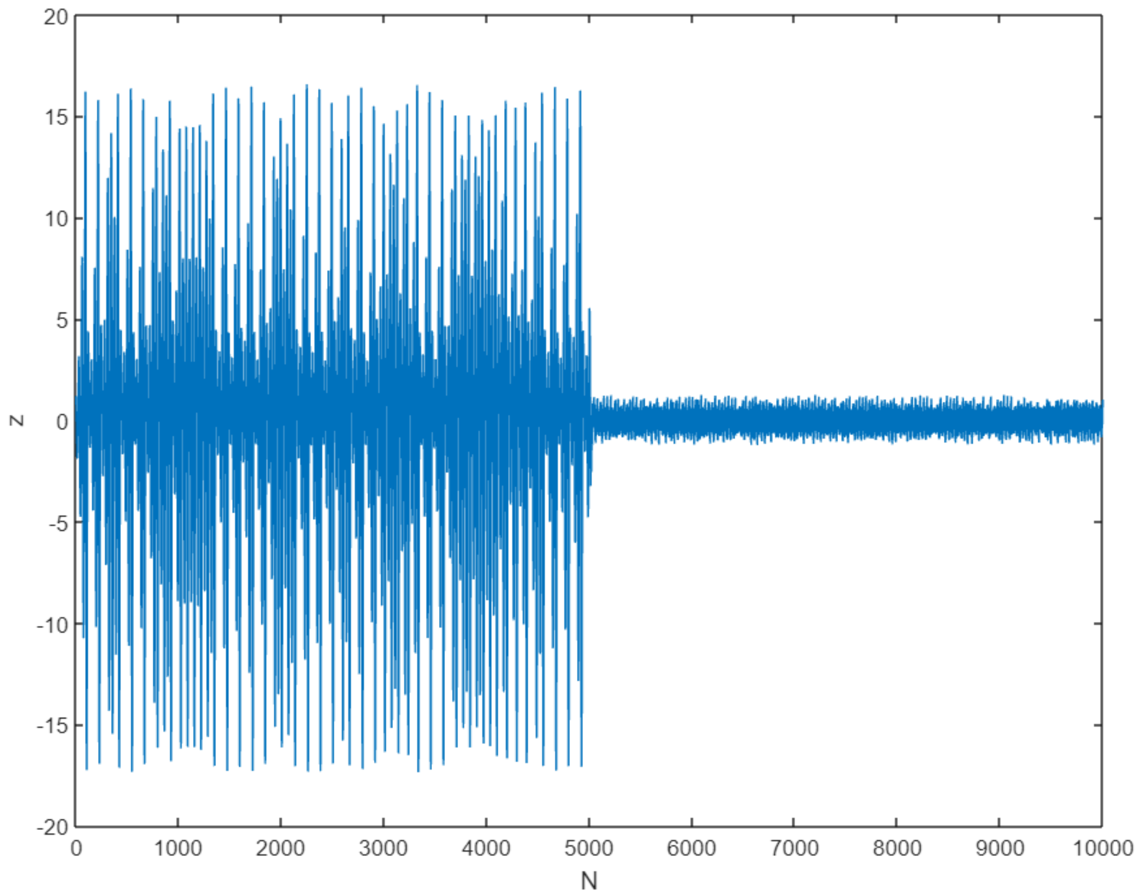

Figure 4.2.3 Time response of the Genesio System (for variable z)

Figure 4.2.3 also demonstrates similar behavior that depicted in figure 4.2.2 and Figure 4.2.2. The state variable z provides a perfect example of the chaotic behavior, which can be observed in figure 4.2.2. During this time, the robust controller is disabled by being switched off. After t > 500 seconds (i.e. N > 5000), the robust controller is applied to the chaotic system, and the system's instability is suppressed, resulting in a relatively stable state.

Chapter 5

MATLAB Simulation Code

The code used for the simulation of Duffing oscillator and Genesio System.

5.1 MATLAB code for the Duffing oscillator:

```
function func = Duffing(t,x)
        func = zeros(2,1);
        func(1) = x(2);

        T = 0.01;
        c = [100 20];
        u = zeros(10001,1);

        if(t<=50)
                func(2) = -11.5*(1-(2/sqrt(3+(x(1)*x(1)))))*x(1)-0.25*x(2)+sin(0.5*t);
                u = 11.5*(1-(2/sqrt(3+(x(1)*x(1)))))*x(1)-x(1)-sin(0.5*t);

        else
                for n = 4999:10001
                        u(n) = -c*x-(x(2)-x(1))/T-u(n-1);
                        func(2) = -11.5*(1-(2/sqrt(3+(x(1)*x(1)))))*x(1)-0.25*x(2)+sin(0.5*t)+u(n);
                end
        end
end
```

Simulation code:

```
clear all;
close all;
clc;
time = 0:0.01:100;
xint = [0,0];
[t,x] = ode23(@(t,x) Duffing(t,x),time,xint);
plot(t,x(:,1))
xlabel('time')
ylabel('x(1)')
figure;
plot(t,x(:,2))
xlabel('time')
ylabel('x(2)')
```

5.2 MATLAB code for Genesio System

```
clear all;
close all;
clc;

a = 1.2;
b = 4;
c = 6;
T = 0.1;
x = zeros(10000,1);
y = zeros(10000,1);
z = zeros(10000,1);


x(1) = -0.2;
y(1) = 0.3;
z(1) = 1.2;

u = zeros(10000,1);
N = 10000;

for i = 1:5000
   x(i+1,1) = x(i,1)+T*y(i,1);
   y(i+1,1) = y(i,1)+T*z(i,1);
   z(i+1,1) = z(i,1)+T*(-c*x(i,1)-b*y(i,1)-a*z(i,1)+(x(i,1)*x(i,1)));

   if(z(i,1) == 0)

                u(i,1) = c*x(i,1)+b*y(i,1)-((x(i,1)*y(i,1))/z(i,1)+0.01)-y(i,1)-(x(i,1)*x(i,1));

   else

                u(i,1) = c*x(i,1)+b*y(i,1)-((x(i,1)*y(i,1))/z(i,1))-y(i,1)-(x(i,1)*x(i,1));
   end

end

for i = 5001:N
       u(i,1) = -(12.5*x(i,1)+25*y(i,1)+1.5*z(i,1))-((z(i,1)-y(i,1))/T)-u(i-1,1);

   x(i+1,1) = x(i,1)+T*y(i,1);
   y(i+1,1) = y(i,1)+T*z(i,1);

        if(u(i,1)>1)
     u(i,1) = 1;
          z(i+1,1) = z(i,1)+T*(-c*x(i,1)-b*y(i,1)-   a*z(i,1)+(x(i,1)*x(i,1)))+u(i,1);

        elseif (u(i,1)<-1)
```

```
            u(i,1) = -1;
                        z(i+1,1) = z(i,1)+T*(-c*x(i,1)-b*y(i,1)-a*z(i,1)+(x(i,1)*x(i,1)))+u(i,1);
            else
                        z(i+1,1) = z(i,1)+T*(-c*x(i,1)-b*y(i,1)-a*z(i,1)+(x(i,1)*x(i,1)))+u(i,1);
    end

end


j = 1:1:N;
plot(j,x(j,1));
xlabel('N');
ylabel('x')
figure;

plot(j,y(j,1));
xlabel('N');
ylabel('y')
figure;

plot(j,z(j,1));
xlabel('N');
ylabel('z')
figure
```

Conclusion

An effective strategy for controlling a category of chaotic systems is analyzed here, wherein the system's precise dynamic equations are unknown. Here, the controller combines directly with a disturbance observer with a feedback linearization controller. This controller can prevent a nonlinear system with unknown dynamics from behaving chaotically in response to an unknown disturbance.

The Non-linear chaotic control scheme is then applied to two typical chaotic systems, namely Duffing's oscillator and the Genesio system, to demonstrate its feasibility and effectiveness. The simulation results show that the proposed controller effectively suppresses the chaotic behavior and achieves a satisfactory tracking performance for both systems.

Further, the controller can be used for hyperchaotic systems and higher order chaotic systems such as Lorenz system, Rössler system, and Chen system. It can also be extended to the control of uncertain chaotic systems with parameter uncertainties and external disturbances.

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