# Performance Prediction of Asphalt Mixtures using Heterogeneous Properties

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# Performance Prediction of Asphalt Mixtures using Heterogeneous Properties

*Dissertation submitted in partial fulfilment*

*Of the requirements for the degree of*

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*in*

***Civil Engineering***

*by*



***Vivek Kumar Singh***

(Roll Number: 220CE3462)



*Based on research*

*carried out under the*

*supervision of*

***Prof. Gourab Saha***

May 2022



Department of Civil Engineering

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# Certificate of Examination

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Title of Research: Performance Prediction of Asphalt Mixtures using Heterogeneous Properties. We hereby state that we have given our approval to submit this dissertation for the partial fulfilment of the degree of Master of Technology in Transportation Engineering in the Department of Civil Engineering at National Institute of Technology, Rourkela. We have checked this dissertation and found that it fulfill all the requirements which is needed to award degree of Master of Technology in Civil Engineering.

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May 2022

# Supervisors' Certificate

This is to certify that the work presented in the thesis entitled "Performance Prediction of Asphalt Mixtures using Heterogeneous Properties" submitted by *Vivek Kumar Singh*, Roll Number 220CE3462, is a dissertation of genuine work carried by him for getting degree of Master of Technology in Transportation Engineering in the Department of Civil Engineering at National Institute of Technology, Rourkela. I have personally guided him throughout this study and I assure that this work is genuine and it has never been submitted before for getting any degree anywhere.

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# Dedication

***"I dedicate this dissertation to my beloved parents who has always kept me motivated and supported to achieve my goals."***

# Declaration of Originality

I, Vivek Kumar Singh whose Roll Number is *220CE3462* hereby state that this dissertation whose title is *Performance Prediction of Asphalt Mixtures using Heterogeneous Properties* is original work of mine as per my knowledge which is carried out for completing degree of Master of Technology in Transpiration Engineering in Department of Civil Engineering at National Institute of Technology, Rourkela. This dissertation do not include any content which is done, written or published by any other author before as per my knowledge. The persons who have helped me during this research is mentioned in the Acknowledgement clearly and all the research papers which has been studied during this study has been clearly mentioned in the reference section

I know that the senate of National Institute of Technology, Rourkela has right to cancel my awarded degree if they found any discrepancy in dissertation in future.

May, 2022 — Vivek Kumar Singh

NIT Rourkela

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# Abstract

According to IRC 37-2018 each layer of flexible pavement is assumed to be homogenous and static loading is considered in the design of flexible pavement. But in reality dynamic loads are applied on the pavement and all layers are not perfectly homogenous. Thus, it is important to consider the effect of non- homogenous asphalt layer and dynamic loading in the design of flexible pavement. With this background, this study investigated the effect of dynamic loading and non-homogenous property of asphalt layer on the thickness of different layers of flexible pavement while designing of flexible pavement and quantified the impact on the overall design. To calculate the effect a suitable model is created in ABAQUS software whose results are validated with the results of IITPAVE software. A pavement section was modelled in the ABAQUS program and after analysing the pavement model the critical strain values are obtained. The values of critical strains obtained from ABAQUS when dynamic loading is applied is compared with control one. The critical strains obtained from dynamic loading is found to be higher than the strains which are obtained when static loading is applied. To make the asphalt layer as non-homogenous layer, asphalt layer is divided in different layers with different values of resilient modulus. The effect of heterogeneous nature of asphalt layer is checked with different values of coefficient of variation on the thickness of different layers of flexible pavement. Various types of variation in resilient moduli were considered in the model and model output was considered in the pavement design. It was found that the thickness of different layers of flexible pavement is highly influenced by considering dynamic loading and non-homogenous asphalt layer. If the strength of the asphalt layer was underestimated, then the thickness required for asphalt layer was high which led to an uneconomical design. Thus, for the realistic and economical design of the flexible pavement, the behaviour of asphalt layer should be taken as heterogeneous material and dynamic loading should be considered instead of static loading.

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# CHAPTER 1

# INTRODUCTION

## 1.1 Background

In general, flexible pavement is composed of a bituminous surface which is laid over base and sub-base layer. The bituminous surface course may consist of one or more bituminous layers. Whenever load acts on this pavement, it bends because this type of pavement has very low flexural strength. The structural performance of this pavement is calculated by the combined action of all the layers. The loads applied on the pavement gets distributed over the layers in the form of truncated cone. The top surface is directly exposed to traffic so it has the maximum value of resilient modulus. The impact of load gets reduced throughout the depth of the pavement, so lesser superior material is used from top to bottom of the pavement. The layers of the flexible pavement are designed in such a way that the load which reaches the subgrade should be less than the bearing capacity of the subgrade soil.

When loads are applied on the flexible pavement then it gets deflected or flexes. The structure of flexible pavement is composed of different layers of different materials. As the loads are applied to the flexible pavement it gets distributed to all the layers of flexible pavement. The top layer transfers the loads to its bottom layer and it goes on transferring until the load reaches subgrade. This transferring of loads from top to bottom layers of flexible pavement has an effect on stress generation on different layers. Since the maximum amount of load is acting on the top layer so the maximum stress gets generated on top layer. Since the lower layer carries minimum load so it experiences minimum stress. To take the advantage of stress variation in different layers of flexible pavement it is advised to use material of high load-bearing capacity at the top and lower bearing capacity material at the bottom.

The design of the flexible pavement is performed with the help of the Linear-Elastic Layered theory in which the properties of the material used in each layer are considered in the design. For designing the flexible pavement, the resilient modulus and Poisson's ratio of each layer are considered as inputs. Typically, the resilient moduli of the soil subgrade, granular material, and base material are found by various laboratory tests. These tests are conducted on the materials used in different layers, and then the required properties are determined and used in the pavement design. Since the flexible pavement is designed as per the Linear-Elastic Layered theory, each layer plays an important role in transferring the load to the bottom of the soil subgrade. Thus, it is important to account for the material properties of each layer

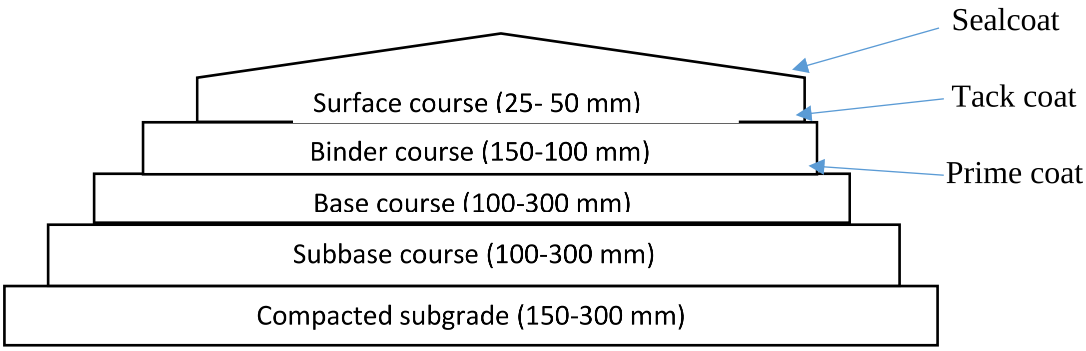



Fig. 1. 1 – Various layers of flexible pavement.

## 1.2 Problem statement

Since the asphalt layer carries the maximum amount of stress when the pavement is subjected to loading. So it is important to calculate accurate response of asphalt layer for effective design. To make the design simple, IRC considered the asphalt layer as homogenous but in reality, it is not the case. For designing of asphalt layer, it is assumed as homogenous and a value of resilient modulus is taken as input. While doing so the heterogeneous nature of the asphalt layer is neglected which makes this design insufficient. The heterogeneous nature of asphalt layer is considered by taking spatially varied values of instantaneous resilient modulus as input. This pavement is modelled in ABAQUS and the response is measured.

For extensive use of flexible pavement, it is important to incorporate heterogeneous property of asphalt layer. Since the asphalt layer carries maximum percentage of load coming to it so it is important to access the property of asphalt layer accurately to predict the performance of flexible pavement effectively. Designers need to find a way to incorporate the heterogeneous nature of asphalt layer and give an alternate effective design for flexible pavement.

## 1.3 Research objective

The prime objective to carry out this research is to find the effect of different level of heterogeneity in bituminous layer and dynamic loading in the design of flexible pavement in terms of different layer thickness.

## 1.4 Scope

There are following scope of the study included:

- Comprehensive literature review.
- Design a suitable pavement section using IRC 37.
- Design a simple model using finite element method.
- Compare the results of IITPAVE and ABAQUS.
- Design a real model using finite element method.
- Evaluate the effects on layer thickness.

## 1.5 Report organisation

Chapter 1 of this dissertation contains the problem statement, research objective, and scope of this study. Chapter 2 contains a brief discussion of the literature pertaining to the characterization and design approach of asphalt layer. This chapter also includes a brief discussion on the design of flexible pavement. Chapter 3 discusses the various methods adopted to achieve the objective of this study briefly. This chapter discussed the steps to be followed to incorporate heterogeneous properties in asphalt layer and also discusses the analysis of impact of heterogeneous material in design of flexible pavement. Chapter 4 discusses about results obtained from this study. This chapter also contains various observations from the results with the help of tables and figures. Chapter 5 discusses about summary, research significance, findings and references of this study.

# CHAPTER 2

# LITERATURE REVIEW

## 2.1 General information

Flexible pavement generally consists of several layers. Superior quality of the material is placed at the top layer and inferior quality material is placed at lower layers. The different layers of pavement have crucial role in transferring of load for above layer to below layer. The pavements were examined using a technique known as "linear elastic layer theory," which simulates the flexible pavement as a combination of different layers. The uppermost layers are assumed to have an infinite horizontal length and thickness, whereas the bottom layer (soil subgrade) is considered semi-infinite. The critical stress, strain, and deflection produced by the imposed standard load of 80 kN at the pavement surface are calculated using Poisson's ratio, elastic modulus, and the thickness of individual layer as input parameters. The pavement portion is analysed using IITPAVE software in accordance with IRC 37 (2018). The IITPAVE is an improved version of an FPAVE that was created for the MORTH Scheme R-56 for the analysis of flexible pavement and its design. The strain which acts as vertical and compressive in nature on top of subgrade is considered critical in the given guidelines by the IRC 37 (2018) code of practice for controlling subgrade rutting, and the strain which is horizontal and tensile in nature at the bottom of the bituminous layer is considered critical for controlling bottom-up cracking in the bituminous layer according to the IRC 37 (2018) code of practice for controlling bottom-up cracking in the bituminous layer. Similarly, the tensile stress and strain at the bottom of the cement-treated foundation are regarded as crucial factors that must be monitored in order to assure safety and avoid coming off. Rutting inside the bituminous layer is caused by the accumulation of persistent (plastic) distortion in the bituminous layer caused by the repetitive application of traffic loads; this is the greatest distress that develops in the flexible pavement. Because of the intense traffic load and high pavement temperature, excessive rutting in the bituminous layer may develop earlier, reducing the fraction of plastic (irrecoverable) deformation in the total deformation. The moisture degradation of mixes and brittle cracking create excessive age hardening of bitumen in the higher layers, which are the key concerns to be considered in the pavement analysis. This type of distress is addressed by integrating mix design with structural design by including mix volumetric parameters into performance parameters based models and making appropriate suggestions for the type of binder and type of mix to be used in the various layers of the flexible pavement. The IRC 37 (2018) recommends that the pavement sections be selected to satisfy the limiting strains and stresses recommended by the pavement performance models used in the IRC guidelines for bottom-up cracking to ensure that the magnitude of the distresses is within allowable limits and that the bituminous pavements perform satisfactorily during the service life period (Cement Treated Bases). In India, flexible pavement is developed in accordance with the IRC 37 (2018) regulation. According to the code, there are two key design characteristics known as performance criteria. Subgrade rutting and fatigue cracking are two of the performance requirements. Many elements influence these two criteria, including traffic volume, reliability, Modulus of Resilience value, percentage volume of air voids, and percentage volume of bitumen in the mix design. The linear elastic model is used for calculation of critical strains and stress generated on a three-layered flexible pavement model. For determining the adequate performance of pavement when it is subjected to repeated vehicular load the following three distresses are considered:

1. The strain which is vertical and compressive in nature which is acting on the top surface of subgrade.
2. The strain which is Horizontal and tensile in nature and which is acting on the lower surface of bituminous layer.
3. The deformation of pavement inside the bituminous layer.

By conducting a proper mix design the deformation of the pavement in the bituminous layer ca easily be controlled. The analytical design approach is conducted to find the suitable thickness of bituminous and granular layer so that the value of critical strains generated should be less than the value of allowable strains. The resilient modulus value of dense bituminous macadam layer with 60/70 bitumen is being used while carrying out this analysis for determining tensile strains at the bottom surface of bituminous layer,

## Performance Criteria

- Fatigue Criteria:

In this criteria, if the percentage of fatigue cracking in the section which is under observation is 20 or more then this type of pavement will be considered as failed pavement due to fatigue. The number of equivalent standard axle load repetitions that can be allowed on the pavement, before it reaches the critical condition of crack whose value is 20% or more of the total pavement area under consideration, is determined by given equations with reliability values of 80% and 90% respectively.

$N_f = 1.6064*C*10^{-4}*(1/_{\varepsilon t})^{3.89}*(1/_{Mrm})^{0.854}$ (for 80 % reliability)

$N_f = 0.5161*C*10^{-4}*(1/_{\varepsilon t})^{3.89}*(1/_{Mrm})^{0.854}$ (for 90 % reliability)

Where

$C = 10^M$ and $M = 4.84*(\frac{Vbe}{Va+Vbe} - 0.69)$

$V_a$ = amount of air void in mix in percentage

$V_{be}$ = effective bitumen percentage in the mix

$N_f$ = fatigue life of bituminous layer.

$\varepsilon_t$= critical fatigue strain.

$M_{rm}$ = value of resilient modulus of bituminous layer.

The value of resilient modulus of subgrade is calculated based on the value of California Bearing Ratio of subgrade soil by following equations

$M = 10*CBR$ (for CBR ≤ 5 %)

$M_{RS} = 17.6*(CBR)^{0.64}$ (for CBR > 5 %)

Where,

$M_{RS}$ = modulus of resilience of subgrade soil in MPa.

CBR = California bearing ratio of subgrade soil in %.

The modulus value of the GSB layer is determined based on the modulus value of the soil subgrade and the estimated thickness of the GSB layer. The modulus value of the soil subgrade and the thickness of the GSB layer are related by

$M_{RGRAN} = 0.2*(h)^{0.45} * M_{RSUPPORT}$

Where,

h = granular layer thickness in mm

$M_{RGRAN}$ = modulus of resilience of the granular layer in MPa.

$M_{RSUPPORT}$ = modulus of resilience of the supporting layer in MPa.

- Rutting Criteria:

According to IRC 37-2018 a pavement is said to be undergone rutting failure or critical rutting failure when the value of average rut depth measured along the path of wheel is 20 mm or more. The value of rutting life i.e. the number of equivalent standard axle load which can act on the pavement before it experienced a rut depth of 20 mm or more is given by following equations with 80 percent and 90 recent reliability respectively.

$N_R = 4.1656 \text{ X } 10^{-8} *(1/_{\varepsilon v})^{4.5337}$ (for 80 % reliability)

$N_R = 1.4100 \text{ X } 10^{-8} *(1/_{\varepsilon v})^{4.5337}$ (for 90 % reliability)

Where,

$N_R$ = Rutting life

$\varepsilon_V$ = critical rutting strain

## 2.2 Area of works

The flexible pavement which is designed using IRC 37 (2018) assumed asphalt layer as homogenous layer. This method of design of flexible pavement ignore the heterogeneous nature of asphalt layer. A single input value of resilient modulus is used in IRC method of design which enables it to predict the exact behaviour of asphalt layer and a large void is created for researchers to exactly analyse the performance of asphalt layer by considering a different number of values of instantaneous modulus which varied along the space of asphalt layer. This research tried to find a computational way to incorporated heterogeneous nature of asphalt layer and to provide an effective method of design of flexible pavement. If asphalt layer is considered homogenous during the design then it is very difficult to predict accurate performance of asphalt layer even if strict quality control measures has been taken during construction. This uncertainty leads the researchers to think in the direction of heterogeneous nature of asphalt layer.

A large number of researchers has worked in this area. They have used the concept of finite element modelling to predict the heterogeneous nature of asphalt layer. In finite element modelling the asphalt layer is divided in a large numbers of discrete elements and each element is analysed to find bulk response of the asphalt layer. This paper also utilises the concept of finite element modelling with the help of ABAQUS software to introduce heterogeneity properties in the asphalt layer. A different level of heterogeneity can be introduced in asphalt layer. A different number of models has been created in this research with ABAQUS to assess the effect of different level of heterogeneity in asphalt layer to the thickness of remaining layers of flexible pavement.

There are various method to introduce the heterogeneous property in asphalt layer

- Microstructural model:

Image acquisition and X-ray computed tomography are used to create the microstructure model in 2D or 3D. Accurate modelling of particular microstructures can yield extremely detailed findings; but, as with any trade-off, this approach has a larger computing cost than its homogenous counterparts.

Sectional model:

In this model the material is divided into a large number of delimited areas or "sections" (enclosed planar zones if in 2D; enclosed volumes if in 3D). A different material properties is assigned to each of these sections. The assumptions of sectional model did not used the microstructure property of asphalt layer because the sections considered in this model do not includes explicit aggregates or air voids of the mix.

Numerical image processing technique:

In this method, a large number of two-dimensional images are captured for asphalt mixtures using a high-resolution camera or a computed tomography scanner. These 2D images are fed inside geometrical models which contains numerical aggregates with the shape, gradation and distribution near to nature with the help of image boundary recognition technique. This method has been mainly used to show the fracture mechanism in the mixture of asphalt by You and Buttlar and Dai and You and with the finite element method by Kim *et al*.. However, it is found to be lengthy and very costly to fabricate and cut experimental specimens and then to deal with the scanned images.

- Parameterization modelling technique:

In this technique, uneven distribution of aggregates are introduced with the help of various numerical algorithms which depends on gradation of aggregate and quantity of aggregate. Yang *et al*. discovered a more highly efficient algorithm, in which the gradation of aggregates were modelled as regular convex polyhedrons with different sizes and aggregate content could be controlled well.

## 2.3 Literature review

Whenever load is applied on asphalt material, it shows the variable responses. Therefore there is an urgent need to take care of heterogeneous nature of asphalt material in the design of flexible pavement. Now a days the researchers are taking more interest to incorporate heterogeneity in asphalt layer while dealing with computational modelling. Various researchers utilizes different methods to include heterogeneous property in asphalt layer in 2D or 3D to predict pavement performance when load acts on it.

Castillo *et al* (2018) proposed method to section to incorporate heterogeneous properties in asphalt layer. In this method different models were created in which asphalt material is divided in delimited areas or sections. The delimited areas is enclosed section which can be in 2D or 3D. If it is in 2D then it is planar zones and if it is in 3D then it is enclosed volumes. This method do not include the microstructural level of asphalt layer. Each sections of asphalt layer is provided with different values of resilient modulus. Different values of resilient modulus is found with the help of random variable. A field of random variable is represented in the form of 3D matrix in which each element of the matrix represents each sections of asphalt layer. Castillo *et al* found that if we take higher variability in the mechanical properties while assigning to asphalt layer, it produces higher variation in asphalt layer strain. He also concluded that the value of coefficient of variability of maximum longitudinal strain under the asphalt layer is overestimated with microstructural method. Sectional method is found to be cheaper and convenience method than microstructural method of analysis of heterogeneous material.

Castillo *et al* (2019) proposed an effective method called Domain analysis to quantify bulk response of pavement. It is the most recent method in which bulk performance of the material is calculated. This method breaks the traditional approach of predicting performance in which localised response is calculated. It helps to find comprehensive evaluation of pavement in multiaxial direction. It also helps the researchers to find the damage potential of critical areas which is below the loaded area in the pavement. The key components considered in domain analysis are

- The value of stress and strain generated on all axis generated during simulation of loading pavement.
- The value of generated stresses and strains are compared with fatigue criteria.
- A cumulative single scalar value is calculated which represents the volumetric response of the structure of the pavement.

Various researchers found that if the variability in the value resilient modulus is taken close to 10% or 6.3% then shear becomes the main factor which shows more variation in its magnitude. Several researchers has also calculated the value of cumulative stress ratio (CRS) and cumulative strain ratio (CRE) for the first time using domain analysis.

Coleri and Harvey (2013) used x-ray computed tomography to incorporate heterogeneous nature in asphalt layer. In this method a resolution camera or computed tomography scanner is used to capture 2D digital images of asphalt mixture. These digital images are transferred to geometrical models with the help of boundary recognition technique. The geometrical models shows the simulation of numerical aggregates in terms of shape, gradation and distribution in the asphalt layer. You and Buttlar (2006) used this method along with discrete element method to simulated two dimensional fracture in asphalt layer. Dai and You (2008) used this method along with finite element method. They found this method to be more time-consuming and costly to use (Kim *et al*. 2008).

For assessing the importance of non-homogeneity in asphalt layer up to microstructural level it is important to show the accurate variation of aggregate in asphalt material (You *et al*. 2008; Ying *et al*. 2014). Some of the research papers have used cohesive zone model to predict two dimensional fracture in bituminous layer by considering it as linear viscoelastic material (Aragao *et al*. 2011). Many authors have tried to validate the computational models of Asphalt layer in which the model shows the response of Asphalt layer with cyclic loading with finite element modelling (Ying *et al*. 2014) and with discrete elements (You *et al*. 2008). Through this paper it is clear that the final response of the material depends on its specific geometry, properties of different phases in asphalt material and the interfaces between them. This study shows the importance of heterogeneity in asphalt layer for the design of flexible pavement with the help of computational approach using finite element modelling.

Several studies had used finite element modelling to simulate pavements structures (Elseifi *et al*. 2006; Mulungye *et al*. 2007; Kim *et al.* 2009; Castillo *et al*. 2019 etc). The main focus of these studies is to find the performance of pavement with different combination of layers geometry, material properties and loading methods. Generally the authors are trying to create models with varying complexity in materials or by changing the properties of interaction between the layers. The results obtained from these studies are found to be very effective in solving real-life problem. Those studies which considered the asphalt layer to be homogeneous they neglect the heterogeneous properties of asphalt layer.

It is very much possible to generate multiphase model of a multiphase material such as asphalt material. This model can be in two dimensional or three dimensional which depends on our analysis. The multiphase model consists of more than one homogenous phases. If asphalt material is modelled in multiphase model then multiphase model may consists of different homogeneous phases such as a mixed phase composed of fine aggregate and binder, aggregates and air voids. In practice this multiphase model has been prepared by microstructural model with the help of image acquisition (Dai and You 2008; Sefidmazgi *et al*. 2012), X-ray tomography (You *et al*. 2013) and parametrization technique (Kim *et al*. 2005). This type of microstructural models gives the accurate result by modelling each microstructural elements. This approach of modelling has a higher computational coast than its homogeneous one.

An alternate method of modelling asphalt pavement to incorporate heterogeneous nature is proposed by Castillo *et al*. 2018. This method is found to be less time consuming and less expensive. The Asphalt layer is divide into different sections and each sections is provided with different values of mechanical properties. A different level of heterogeneity can also be introduced with this method of simulation.

Aragao *et al*. (2011) conducted their study in the simulation of fracture in heterogeneous Asphalt layer using finite element method and he concluded that the model gives a more accurate value which is close to exact one when heterogeneous nature is considered. Han *et al* (2020) studied the effect of material properties in the design of asphalt pavement to predict thermal conductivity behaviour of Asphalt layer using FEM on microstructure level. One of the researchers studied about the creep behaviour in Asphalt layer when heterogeneous nature of Asphalt layer is considered and he found that heterogeneous nature of Asphalt layer helps to improve creep resistance in asphalt pavement (Ma *et al*. 2017). Caro *et al*. (2010) found in his studies that the moisture present in the material cause a negative impact on the interfacial bond strength between matrix and aggregate.

The researchers are mainly using FEM and DEM (discrete element method) for microstructural analysis of composite material such as asphalt mixture. Both methods are found to be more effective in predicting the behaviour of asphalt pavement by various authors. These are the two methods which is generally used by various researchers to prepare two or three dimensional heterogeneous models. X-ray CT are used to take accurate pictures of three dimensional state of asphalt mixture which depicts the three phases of asphalt mixture clearly at microstructural level. Xing *et al*. (2019) used the concept of X-ray CT images and scan the microstructural model of asphalt mixture to find the accurate aggregate gradation using digital image processing technology. Elseifi *et al*. (2011) studied the range of damage in dynamic modulus test using X-ray CT. Liu *et al*. (2019) created the three dimensional model asphalt mixture and found that there is a nonlinear relationship between compactness and tensile strength in asphalt mixture. The application of X-ray CT is still limited due to many reasons. For example X-ray CT can only create the microstructure of already available asphalt mixture specimen. So if the formed asphalt is not constructed properly then this method will fail to predict exact microstructural model. To make the constructed specimen to be accurate, several specimen were casted which is time consuming. Same experiments have to be repeated for more than one time for different samples which makes this method tedious. It may be possible that the different components in image may get intersect due to equipment inaccuracy and climatic condition, which leads to a difficult situation to differentiate different materials.

A different number of mechanical properties is found with the help of random field. A large number of correlated random values of mechanical properties is generated with the help of random field. Random field is a vector in which correlated random values are spatially varied. This makes the random field as a sufficient method to model heterogeneous behaviour of material (Vanmarcke 1983). Random field is not a new concept. Many researchers have used this method for finding different things such as Griffiths and Fenton (2004) used to find slope and foundation stability, Glimm and Sharp (1991) uses this to study about water permeability conduction in aquifers, Robin *et al.* (1993) uses this found effect of extreme conditions on the sensitivity of system, forecasting of uncertainties and estimation of effective properties of macroscopically equivalent structures. This method is found to be more suitable with finite element method when load is applied because an individual element or group of elements can be assigned to every section in the random field.

Now-a-days researchers have taken more interest in random field to predict heterogeneous nature of pavement. Caro *et al*. (2011) used this method to predict the heterogeneous nature of asphalt material. He created a two dimensional model to simulate the Asphalt layer which consists of air void phase and asphalt material. A more recent study was conducted by Lea and Harvey (2015) who presented a statistical analysis about the spatially varying properties of pavement layers. Lua and Sues (1996) used the concept of random field in predicting performance of pavement performance for the first time. Lea (2010) found that if we model a three-dimensional full stress-strain simulation of heterogeneous Asphalt layer, then it would be more expensive. So the author proposed a new method instead of random field named Fourier technique, which is less expensive than former one. Castillo and Caro (2014) also conducted a study about heterogeneous nature of pavement layers which contrast the traditional way of designing of pavement.

Dondi *et al*. (2007) and Pei and Chang (2009) used dual wheel load to find the strain and displacement distribution within heterogeneous pavement using discrete element method. The author used circular and spherical shapes of aggregate to decrease the model runtime. But author did not compared the response of the pavement obtained with the in-situ response. Another author Kim *et al*. (2011) proposed a quasi-static model which is a multiscale computational model used to predict mechanical behaviour of Asphalt layer of asphalt pavement.

Random reconstruction technique is another method of modelling heterogeneous nature of Asphalt layer in asphalt pavement. In this method there is no need to construct a specimen to create digital image of asphalt mixture. Digital image of asphalt mixtures are created directly which makes it less time consuming and less expensive than X-ray CT method of producing digital image of asphalt mixture. The real aggregate in asphalt mixture is modelled with predefined shape. The digital aggregates are randomly placed in the predefined digital boundary of model in descending order of their sizes. Every time when the new particle is being generated is tested for interaction with other pre-existing particles. If we found that there is no interaction between newly generated aggregate and pre-existing aggregate then aggregate generation is allowed otherwise the aggregates are again randomly distributed until we found no interaction between newly and pre-existed aggregate. This will create an endless loop because as more and more particles get generated the newly generated particle will always be in a contact with existing particle. As the particles are randomly distributed so it is not possible that the gap between the particles remain small. It leads to sparse distribution of aggregates. One of the major drawback of this method is that the heterogeneous property which is generated by random distribution of digital aggregated is also not stable. But if we see aggregate packing in the real asphalt mix which is either manufactured in laboratory or taken from field has interlocked grain o grain contact between them. The aggregates are stable in this case.

The main purpose of this study is to present a computational method to study heterogeneous behaviour of asphalt layer and to study the effect of heterogeneity of asphalt layer on the depth of different layers of flexible pavement. To carry out this study a three dimensional simple section of flexible pavement is modelled in ABAQUS initially by following all the assumptions of IRC 37-2018 guidelines. Then a three dimensional model of asphalt pavement is developed with fully heterogeneous asphalt layer. This model is loaded with a dynamic loading. The dynamic loading is represented by a moving circular tyre imprint. Random field method is used to find different values of mechanical properties. Then the different values obtained are correlated, this means that the average values of all the values gives a unique value which will be equal to the mechanical property of homogeneous asphalt layer. These different values of asphalt layers are assigned to different sections of asphalt layer. Finite element method is used to predict the behaviour of asphalt pavement using ABAQUS. The obtained performance of pavement is calibrated with tradition approach of modelling of pavement. A detailed analysis is being done to analyse the impact of heterogeneity of Asphalt layer in the design of flexible pavement.

Xu and Chang (2014) studied about the structural responses of asphalt pavement when geospatial heterogeneity property of asphalt material is considered. This studied was conducted with intelligent compaction technology. He found that even if we keep all the lower values of elastic moduli with respect to mean value it is not necessary that always it will give lower performance when structural analysis is done. He also found the high performance of pavement when higher geospatial heterogeneity is considered for material elastic moduli.

## 2.4 Summary

Findings of various researchers has been summarised below

Table 2. 1 – Key findings of various literatures

| Author | Year | Key Findings |
|---|---|---|
| Castillo *et. al.* | 2018 | Higher variation of resilient modulus of asphalt layer generates higher variation in strains.<br>Sectional method is cheaper and less time consuming than microstructural model. |
| Castillo *et. al.* | 2019 | Variability in asphalt layer affect the overall response of the pavement structure. |
| Coleri and Harvey | 2013 | For thick bituminous layer deformation due to shear strain do not affect the bottom part of the layer. |
| Yin *et. al.* | 2014 | Distribution of coarse aggregate change the direction of crack propagation locally but has a slight influence in the direction of overall crack. |
| Xu and Chang | 2014 | The higher values of heterogeneity of material using different values of elastic moduli results in higher response values of deflection, tensile stress and strain. |
| Gamez *et. al.* | 2018 | Introduced domain analysis method to find bulk volume response of pavement. |
| Wang *et. al*. | 2020 | The amount and frequency of dynamic loads created by cars, which are heavily influenced by axle arrangement, pavement surface roughness, and vehicle speed, have an impact on pavement performance. |
| Cho *et. al.* | 2018 | Dynamic response of flexible pavement depends upon contact area and size of mesh. |
| Han *et. al.* | 2021 | For reconstructing the heterogeneous structure of asphalt mixtures with relatively low air void content, the method in which heterogeneous asphalt mixtures are considered to be composed of disperse phase |

| | | |
|---|---|---|
| | | including coarse aggregate, air voids, and continuous phase including fine asphalt matrix is reliable. |
| Lea | 2010 | Random field method was found to be more expensive than Fourier technique. |
| Aragao *et al.* | 2011 | Cohesive zone can be used to predict fracture in linear viscoelastic material and gives accurate model for heterogeneous nature of Asphalt layer. |
| You *et al.* | 2008 | Validated the computational model for performance of Asphalt layer under cyclic loading using discrete element method. |
| Gungor *et al.* | 2006 | FEM is a versatile method of modelling which can accommodate many geometries and material behaviour of pavement layers. |
| Caro *et al.* | 2014 | The modulus of asphalt material gets changed due to change in air voids with depth of Asphalt layer. |
| Kassem *et al.* | 2018 | The coefficient of variation of modulus of random Asphalt layer was proposed to be 10%. |
| Ma *et al* | 2017 | The heterogeneous nature of Asphalt layer can improves the creep resistance of asphalt pavement. |
| Lie *et al.* | 2019 | There is a nonlinear relationship exists between compactness and tensile strength of asphalt mixture. |
| Dondi *et al.* | 2007 | The circular and spherical shapes of aggregate in Asphalt layer can be used to decrease model runtime. |
| Ziari *et al.* | 2019 | When the crack lies between the two wheels of vehicle then the asphalt pavement will have lowest fatigue life. The stiffness of HMA layer plays an important role on the fatigue life of the pavement. |
| Shaker *et al.* | 2019 | The fracture parameters are not found to be sensitive to the shape of the aggregate in aggregate generation and packing algorithm method. |
| Chen *et al.* | 2015 | The thermal conductivity of asphalt concrete gets reduced when percentage of air voids is increased in the heterogeneous Asphalt layer of the asphalt pavement. |

## 2.5 Research gap

The study has been conducted in the field of heterogeneity of asphalt layer by various researchers. When analysing pavement responses, they have mainly focused on studying the appearance and development of damage in the pavement materials. They have mainly focused on the performance of Asphalt layer with the heterogeneous property. Some of the researchers have predicted the crack propagation in Asphalt layer and other researchers have worked regarding thermal conductivity of Asphalt layer by considering Asphalt layer as heterogeneous material. Since the asphalt layer is one of the expansive material used in the construction of flexible pavement. So the property of asphalt layer should be utilised properly during design of asphalt pavement. Therefore it is necessary to find the impact of heterogeneous property of Asphalt layer in the design of flexible pavement. There is a very little work found regarding the impact on the thickness of different layers of flexible pavement due to different level of heterogeneity in Asphalt layer which plays an important role for effective design of flexible pavement. The main aim of this study is to assess the effect of different level of heterogeneity in Asphalt layer in the design of flexible pavement.

# CHAPTER 3

# RESEARCH METHODOLOGY

## 3.1 Introduction

The traditional design of flexible pavement assumed the different layers of pavement as homogeneous. Different studies have stated that if we assume asphalt layer as heterogeneous then the performance of flexible pavement can easily predicted with high precision. The different constituents of asphalt layer make it heterogeneous so it is important to take heterogeneous property of asphalt layer while designing for effective use of properties of asphalt layer. This chapter will discuss about the methods to introduce heterogeneous property in Asphalt layer of asphalt pavement. A step-by-step procedure has been provided in this chapter to take care of heterogeneous property of Asphalt layer.

## 3.2 Methodology

The design of flexible pavement is done with IRC 37-2018. IRC 37-2018 uses elastic layer theory for designing. In this method of design all the layers of flexible pavement is assumed to be homogeneous and a single resilient value is used for each layer while designing. But in actual case it is different. If asphalt layer of flexible pavement is considered then it shows different phases such as air void, aggregate and asphalt material. So it not the effective design of flexible pavement if we assume Asphalt layer as homogenous. To make the design effective Asphalt layer should be taken as heterogeneous material and spatially varied value of mechanical properties should be used while designing of Asphalt layer. This study has been conducted to find the effect on design of flexible pavement if we consider Asphalt layer as heterogeneous material. A series of steps which were followed during completing this project is presented below. The explanation of each step is also provided.

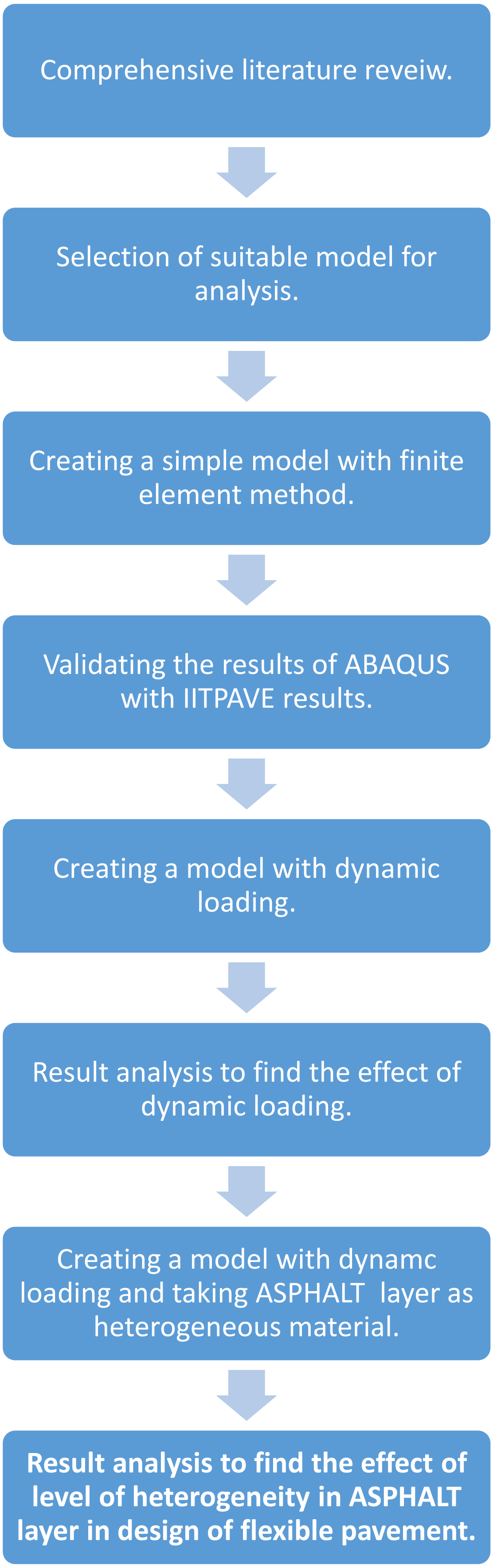



Fig. 3. 1 - Methodology

### 3.2.1 Comprehensive literature review

A comprehensive literature review is conducted about the heterogeneous properties of asphalt layer. Various literature has been read about design of flexible pavement. IRC 37-2018 is found to me mostly adopted in India for design of flexible pavement. This method of design considers Asphalt layer as homogeneous one to make the design simple. As the time passes and when the computational work becomes easier various researchers have shifted towards heterogeneous properties of Asphalt layer which found to be more accurate than its counterparts. A number of research paper has been read regarding incorporating of heterogeneous properties in Asphalt layer and predicting different performance of asphalt pavement.

### 3.2.2 Selection of suitable model for analysis

A suitable model of asphalt pavement is selected using IRC 37-2018 guidelines. In this method, the thickness of different layers has been found using IITPAVE software by conducting trial and error method. The thickness of different layers obtained should be such that it is the minimum thickness of the layers which can be adopted for safe design. In the initial trial a random trial depth of different layers has been adopted to conduct strain analysis using IITPAVE. Since the thickness of asphalt layer control the fatigue criteria of the pavement so an economical depth of asphalt layer is found using fatigue criteria. After finding economical depth of the asphalt layer, thickness of base layer is reduced and at each reduced depth the design is checked for safe condition. The least depth of base layer for which design is safe is taken for analysis. The depth of subgrade layer is assumed as 500 mm as per IRC 37-2018 guidelines. The cross-section of the model is assumed as square with side length as 3750 mm.

The guidelines given in IRC 37-2018 is used while designing of as section of pavement using a particular value of design traffic and CBR value of subgrade soil as input parameters.. The value of strains generated at critical sections when load is applied is computed using IITPAVE software. These value of computed strains were compared with the allowable limit values of rutting and fatigue performance criteria and it is also checked for the suitability of the pavement section for the design traffic capability. These critical strain values were also used in the further analysis of the pavement section when the physical properties of the granular materials were considered. Typical interfaces of IITPAVE software and its result window were shown below.

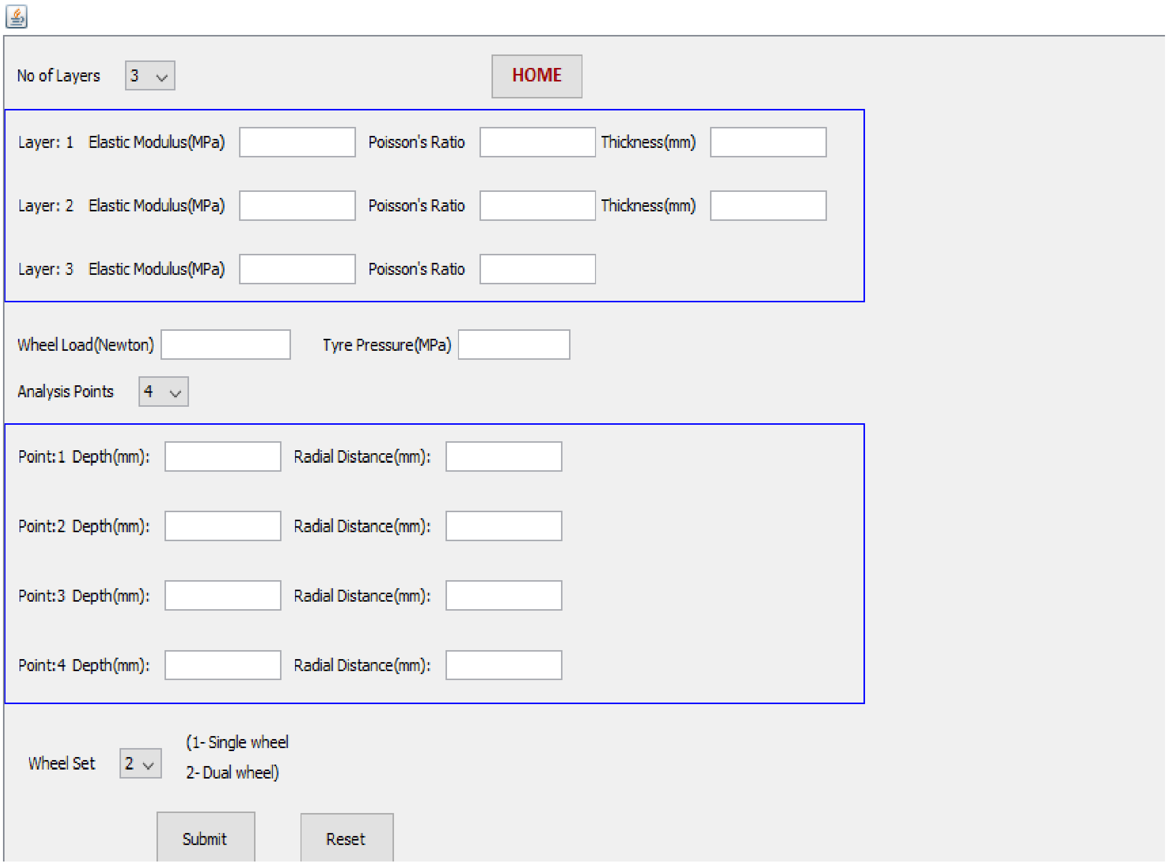



Fig. 3. 2 – Input parameters for IITPAVE software

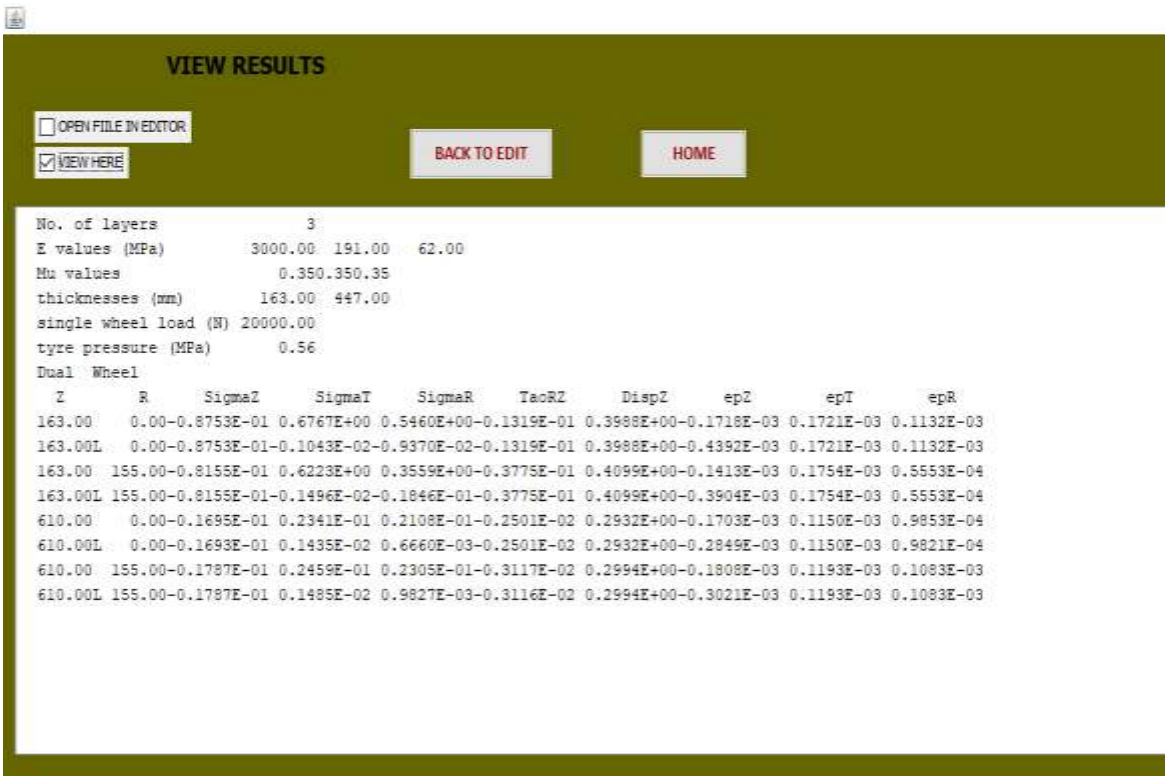



Fig. 3. 3 – Output interface of IITPAVE software

### 3.2.3 Creating a simple model with finite element method

A simple model of flexible pavement with suitable thickness of different layers and cross section which has been found in above step is created using finite element method. The flexible pavement is modeled using ABAQUS software. The guidelines of IRC 37-2018 is followed while modeling of asphalt pavement in ABAQUS. A suitable interaction properties within different layers and boundary condition is applied. As per IRC guidelines all the layers are taken as homogeneous and static loading is applied for analysis. The values of input mechanical properties of different layers of flexible pavement is kept same which is used during analysis with IITPAVE software.

For the preparation of this 3D isotropic pavement model the following steps if followed:

• Modeled each layer of the pavement section with particular dimensions

• Assign the properties of each layer in the pavement section

• Each layer of the modeled pavement section is combined together

• Assign the type of loading condition which is used in the analysis of the pavement section

• Assign the interaction properties between the two contact surfaces

• Define the loading position and assign the pressure intensity to that defined position

• Each pavement layer has meshed with a predefined mess size

• Input data is checked for the analysis of the pavement model

• Effect of the loading on the modeled pavement section analyzed

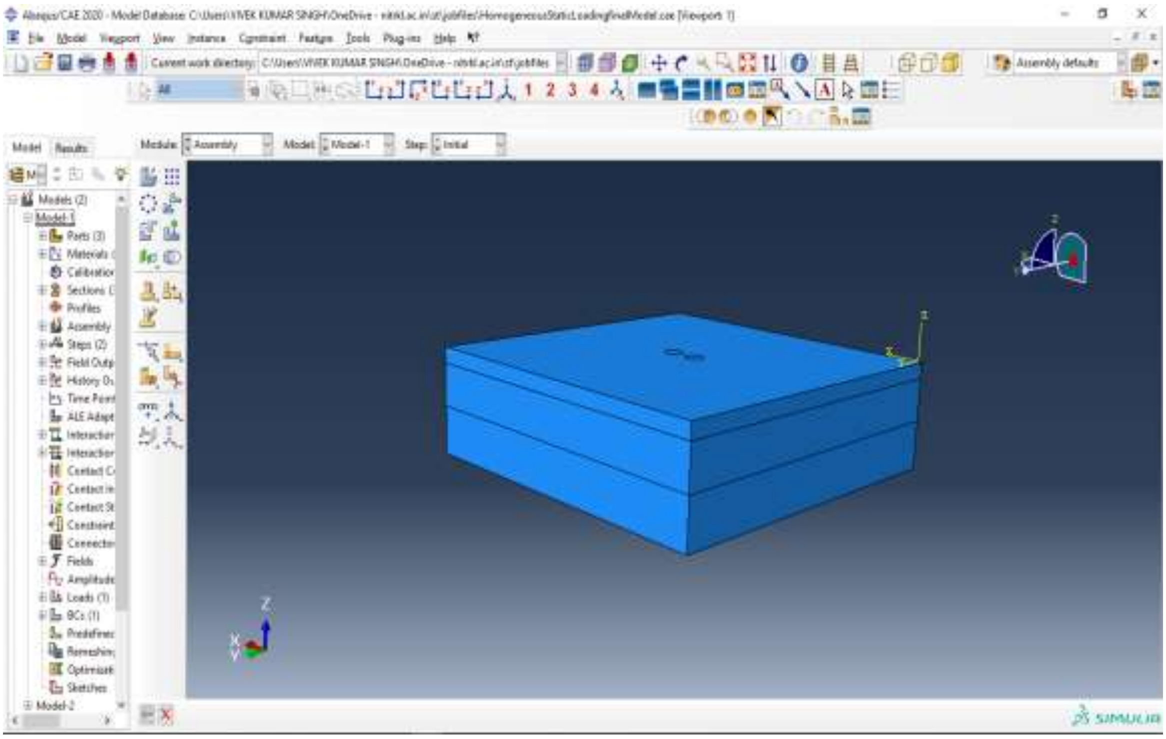


Fig. 3. 4 – ABAQUS interface

### 3.2.4 Validating the results of ABAQUS with IITPAVE results

The results obtained from ABAQUS and IITPAVE are analyzed. Since the model created in ABAQUS follows the guidelines of IRC 37-2018 so the results obtained from ABAQUS should match with IITPAVE results. Initially the results obtained from ABAQUS do not match with IITPAVE results. Since ABAQUS is a user defined software so we need to adjust the modeling condition of ABAQUS to match it with IITPAVE results. The adjustment is done in the mesh step of the ABAQUS model. A series of mesh analysis is carried out with trial and error method. Each time a different set of mesh is given to the different layers of flexible pavement for analysis and result obtained is checked with IITAPVE results. A set of mesh that gives the nearly exact result as IITPAVE result is being adopted for further study.

### 3.2.5 Creating a model with dynamic loading

In this step a section of flexible pavement is being modeled using ABAQUS. This time the static loading is replaced by dynamic loading for analysis of pavement. All the conditions were kept same as previous step except the loading conditions. The dynamic loading is applied using step option in ABAQUS. In step option time of loading is given as input. The magnitude of the load is varied as sine function. The results obtained is noted in terms of critical strain analysis. The value of fatigue and rutting strain were noted.

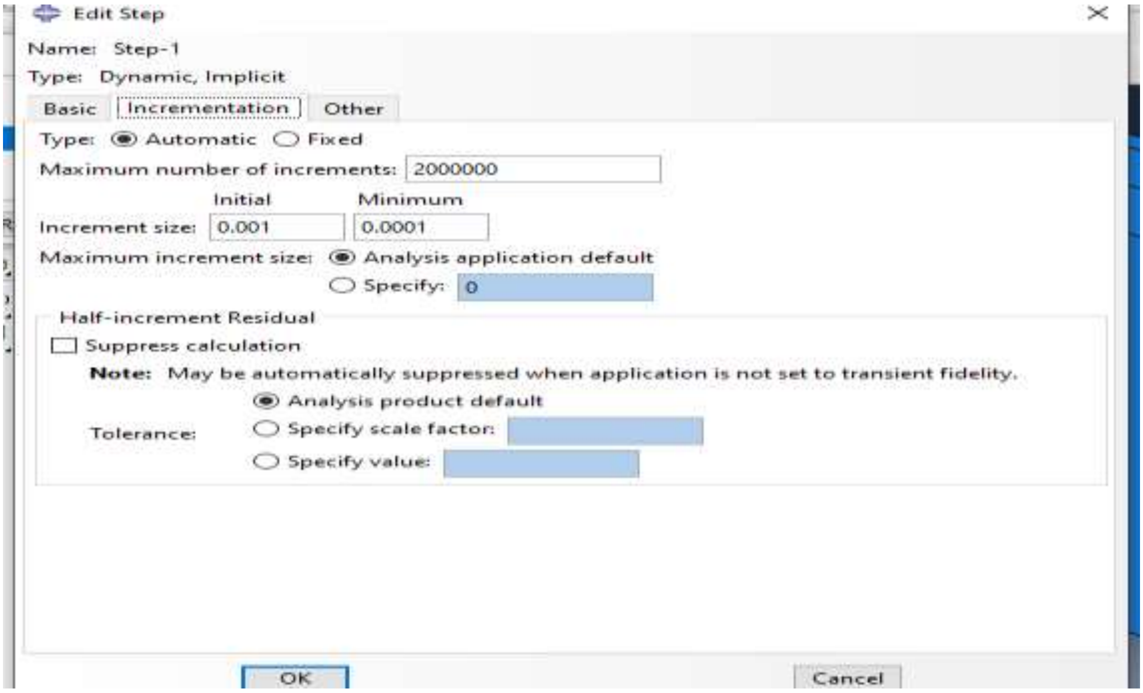



Fig. 3. 5 – Input parameters for step module in ABAQUS

### 3.2.6 Result analysis to find the effect of dynamic loading

The result obtained from modeling a section of flexible pavement with dynamic loading is compared with the results of simple model which is created in third step. The obtained critical strain values is compared with allowable critical strain values. After comparing both values of critical strains the depth of different layers are adjusted and modeled again in ABAQUS by trial and error method until the values of critical strain becomes equal to allowable values of critical strain. Then the new depth of different layers obtained is compared with old depth of different layers of flexible pavement.

### 3.2.7 Creating a model with dynamic loading and taking Asphalt layer as heterogeneous material

The model which is obtained after fifth step is again modelled in ABAQUS. The all condition of fifth step is kept same except the behaviour of Asphalt layer. Asphalt layer is considered a heterogeneous material in this step. Asphalt layer is divided into a large number of sections and for every section a unique value of mechanical properties is assigned. The different values of mechanical properties is obtained with the help of random field. After running the model in ABAQUS the critical values of strain is noted. The different level of heterogeneity is introduced in the Asphalt layer and each time the value of critical strain is noted.

### 3.2.8 Result analysis to find the effect of level of heterogeneity in Asphalt layer

The value of critical strain obtained in seventh step is compared with third step. The value of critical strain obtained at each level of heterogeneity in Asphalt layer is compared with the value of strain obtained by modeling a simple model in third step. The depth of different layers of flexible pavement is adjusted for each level of heterogeneity in Asphalt layer such that the critical values of strain obtained after modeling becomes equal to allowable values of critical strain. The effect of different level of heterogeneity in asphalt layer is computed.

## 3.3 Summary

This chapter briefly describes the overall methods used in conducting this research. Every stage in this process was designed with the goal of achieving the study's goal. The main goal was to investigate the impact of varying levels of heterogeneous behaviour in the Asphalt layer on flexible design. To achieve the study goals, a mix of mathematics and finite element modelling was used. The study technique was created in such a way that the IRC 37-2018 may be used to scale the quantifications of diverse behaviour.

# CHAPTER 4

# RESULTS AND DISCUSSION

## 4.1 Introduction

In this chapter, the objective of this study is presented in a systematic manner. A 3D pavement model is prepared for this purpose. A finite element-based software ABAQUS is used for the preparation of the model of the pavement section. Analysis of the pavement model is done and critical strain values for this model are obtained. These critical strain values are compared with the design critical strain values of the real pavement section which is obtained by IITPAVE software. The different values of critical strains obtained during this study is tabulated in this chapter.

## 4.2 Finding a suitable size of pavement for analysis

A suitable cross section of the flexible pavement is found out by running different trials on IITPAVE software. Each time a different values of thickness of different layers is entered in IITPAVE and critical strains are calculated at each step. The computed values of critical strains are compared with the allowable values of critical strains. The above step is repeated until we found an economical depths of different layers of flexible pavement which is safe.

Design of flexible pavement according to guidelines of IRC 37-2018

| | | |
|---|---|---|
| Input parameters | | |
| CBR | 7 | % |
| Initial traffic | 5000 | cvpd |
| Traffic growth rate | 6 | % |
| Design period | 20 | years |
| VDF | 5.2 | |
| $V_a$ | 3 | % |
| $V_{be}$ | 11.5 | % |
| LDF | 0.75 | |

| | | | |
|---|---|---|---|
| Initial directional traffic | 2500 | cvpd | (assuming 50% in each direction) |
| No of standard axles | 130.91 | msa | (from eq. 4.5 ) |
| Resilient modulus of subgrade | 61.1470 | Mpa | (from eq. 6.2 ) |

Trial thickness of different layers

| | | |
|---|---|---|
| Bituminous | 190 | mm |
| Granular layer | 450 | mm |

| | | | |
|---|---|---|---|
| Resilient modulus of granular layer | 191.1398 | Mpa | (from eq. 7.1) |
| Allowable vertical strain on subgrade (Rutting) | 0.0003006 | | (from eq. 3.2) |
| Allowable tensile strain on bituminous (Fatigue) | 0.0003527 | | (from eq. 3.4) |
| C | 3.155155 | | (from eq. 3.4) |
| M | 0.499021 | | (from eq. 3.4) |

The following trials were conducted to find the economic depth of different layers of flexible pavement using IITPAVE software.

Table 4. 1 – Finding suitable thickness for pavement layers

| | **Trial thickness** | | **From IRC 37-2018** | | **IITPAVE** | | | |
|---|---|---|---|---|---|---|---|---|
| **s.no.** | **Bituminous layer (mm)** | **Granular layer (mm)** | **Allowable rutting** | **Allowable fatigue** | **Rutting** | **Fatigue** | **Rutting** | **Fatigue** |
| 1 | 190 | 450 | 3.90E-04 | 1.76E-04 | 2.61E-04 | 1.49E-04 | Safe | Safe |
| 2 | 180 | 450 | 3.90E-04 | 1.76E-04 | 2.75E-04 | 1.58E-04 | Safe | Safe |
| 3 | 170 | 450 | 3.90E-04 | 1.76E-04 | 2.89E-04 | 1.68E-04 | Safe | Safe |
| 4 | 168 | 450 | 3.90E-04 | 1.76E-04 | 2.92E-04 | 1.70E-04 | Safe | Safe |
| 5 | 165 | 450 | 3.90E-04 | 1.76E-04 | 2.97E-04 | 1.73E-04 | Safe | Safe |
| 6 | 163 | 450 | 3.90E-04 | 1.76E-04 | 3.00E-04 | 1.75E-04 | Safe | Safe |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | 162 | 450 | 3.90E-04 | 1.76E-04 | 3.02E-04 | 1.76E-04 | Safe | Unsafe |
| 8 | 161 | 450 | 3.90E-04 | 1.76E-04 | 3.03E-04 | 1.77E-04 | Safe | Unsafe |
| 9 | 160 | 450 | 3.90E-04 | 1.76E-04 | 3.05E-04 | 1.78E-04 | Safe | Unsafe |
| 10 | 150 | 450 | 3.90E-04 | 1.76E-04 | 3.22E-04 | 1.90E-04 | Safe | Unsafe |
| 11 | 140 | 450 | 3.90E-04 | 1.76E-04 | 3.40E-04 | 2.02E-04 | Safe | Unsafe |
| 12 | 130 | 450 | 3.90E-04 | 1.76E-04 | 3.59E-04 | 2.15E-04 | Safe | Unsafe |
| 13 | 120 | 450 | 3.90E-04 | 1.76E-04 | 3.80E-04 | 2.29E-04 | Safe | Unsafe |
| 14 | 110 | 450 | 3.90E-04 | 1.76E-04 | 4.02E-04 | 2.43E-04 | Unsafe | Unsafe |
| 15 | 163 | 440 | 3.90E-04 | 1.76E-04 | 3.10E-04 | 1.77E-04 | Safe | Unsafe |
| 16 | 163 | 445 | 3.90E-04 | 1.76E-04 | 3.06E-04 | 1.76E-04 | Safe | Unsafe |
| 17 | 163 | 446 | 3.90E-04 | 1.76E-04 | 3.05E-04 | 1.76E-04 | Safe | Unsafe |
| 18 | 163 | 447 | 3.90E-04 | 1.76E-04 | 3.05E-04 | 1.76E-04 | Safe | Safe |
| 19 | 163 | 430 | 3.90E-04 | 1.76E-04 | 3.17E-04 | 1.78E-04 | Safe | Unsafe |
| 20 | 163 | 420 | 3.90E-04 | 1.76E-04 | 3.24E-04 | 1.79E-04 | Safe | Unsafe |
| 21 | 163 | 410 | 3.90E-04 | 1.76E-04 | 3.32E-04 | 1.80E-04 | Safe | Unsafe |
| 22 | 163 | 350 | 3.90E-04 | 1.76E-04 | 3.82E-04 | 1.88E-04 | Safe | Unsafe |
| 23 | 163 | 340 | 3.90E-04 | 1.76E-04 | 3.91E-04 | 1.89E-04 | Unsafe | Unsafe |
| 24 | 163 | 345 | 3.90E-04 | 1.76E-04 | 3.87E-04 | 1.89E-04 | Safe | Unsafe |
| 25 | 163 | 342 | 3.90E-04 | 1.76E-04 | 3.90E-04 | 1.89E-04 | Safe | Unsafe |
| 26 | 163 | 341 | 3.90E-04 | 1.76E-04 | 3.90E-04 | 1.89E-04 | Safe | Unsafe |

The size of final pavement model selected for analysis is

Table 4. 2 – Pavement Section details

| **Layers** | **Depth (mm)** | **Resilient modulus (Mpa)** | **Poisons ratio** |
|---|---|---|---|
| Layer 1 | 163 | 3000 | 0.35 |
| Layer 2 | 447 | 191 | 0.35 |
| Layer 3 | 500 | 62 | 0.35 |

## 4.3 Creating a ABAQUS model with static loading

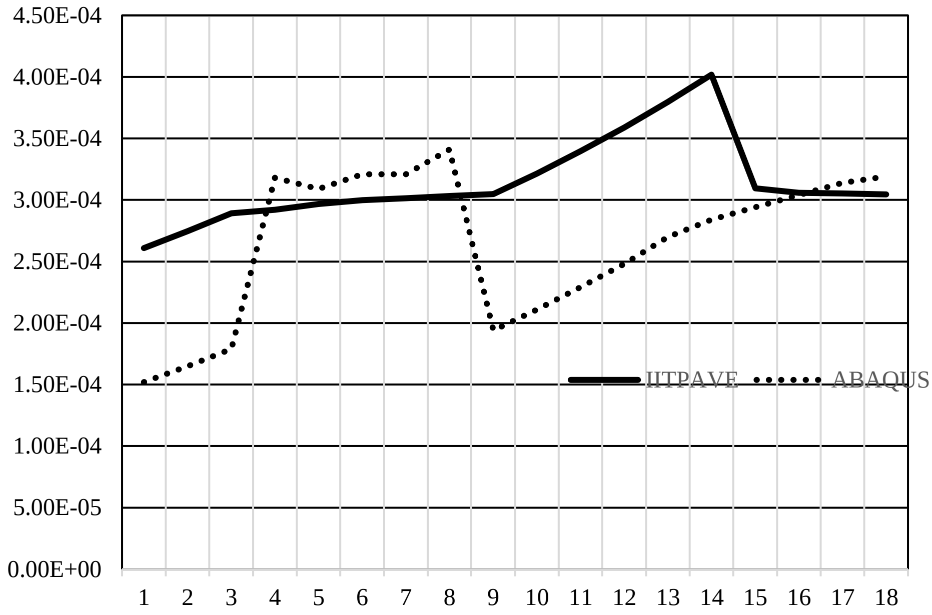
A pavement section model is created using ABAQUS software. The loading conditions is kept same as per IRC 37-2018 guidelines. So the value of computed critical strains obtained from ABAQUS software should come same or nearly same to IITPAVE software. Initially these values will not match. In ABAQUS software we need to give every condition of field as input values to make it effective model.

After creating model in ABAQUS software, the values of critical strains were found. The values of critical strains is presented in table 4.3.

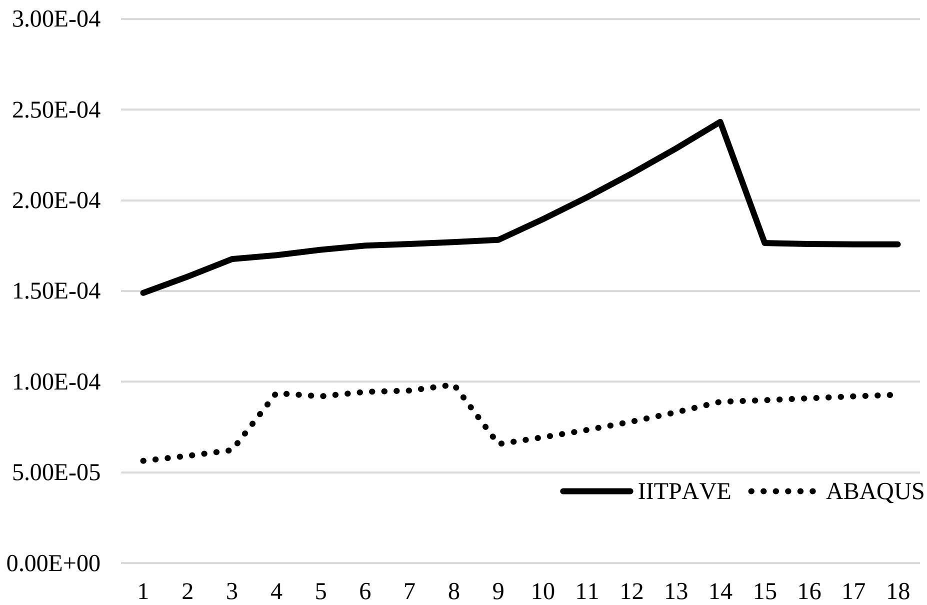
Table 4. 3 – Comparing IITPAVE results with ABAQUS results

| | TRIAL THICKNESS | | FROM IRC37-2018 | | IITPAVE | | | |
|---|---|---|---|---|---|---|---|---|
| S.No. | BITUMINOUS LAYER (mm) | GRANULAR LAYER (mm) | ALLOWABLE RUTTING | ALLOWABLE FATIGUE | RUTTING | FATIGUE | Rutting | Fatigue |
| 1 | 190 | 450 | 3.90E-04 | 1.76E-04 | 2.61E-04 | 1.49E-04 | Safe | Safe |
| 2 | 180 | 450 | 3.90E-04 | 1.76E-04 | 2.75E-04 | 1.58E-04 | Safe | Safe |
| 3 | 170 | 450 | 3.90E-04 | 1.76E-04 | 2.89E-04 | 1.68E-04 | Safe | Safe |
| 4 | 168 | 450 | 3.90E-04 | 1.76E-04 | 2.92E-04 | 1.70E-04 | Safe | Safe |
| 5 | 165 | 450 | 3.90E-04 | 1.76E-04 | 2.97E-04 | 1.73E-04 | Safe | Safe |
| 6 | 163 | 450 | 3.90E-04 | 1.76E-04 | 3.00E-04 | 1.75E-04 | Safe | Safe |
| 7 | 162 | 450 | 3.90E-04 | 1.76E-04 | 3.02E-04 | 1.76E-04 | Safe | Unsafe |
| 8 | 161 | 450 | 3.90E-04 | 1.76E-04 | 3.03E-04 | 1.77E-04 | Safe | Unsafe |
| 9 | 160 | 450 | 3.90E-04 | 1.76E-04 | 3.05E-04 | 1.78E-04 | Safe | Unsafe |
| 10 | 150 | 450 | 3.90E-04 | 1.76E-04 | 3.22E-04 | 1.90E-04 | Safe | Unsafe |
| 11 | 140 | 450 | 3.90E-04 | 1.76E-04 | 3.40E-04 | 2.02E-04 | Safe | Unsafe |
| 12 | 130 | 450 | 3.90E-04 | 1.76E-04 | 3.59E-04 | 2.15E-04 | Safe | Unsafe |
| 13 | 120 | 450 | 3.90E-04 | 1.76E-04 | 3.80E-04 | 2.29E-04 | Safe | Unsafe |
| 14 | 110 | 450 | 3.90E-04 | 1.76E-04 | 4.02E-04 | 2.43E-04 | Unsafe | Unsafe |
| 15 | 163 | 440 | 3.90E-04 | 1.76E-04 | 3.10E-04 | 1.77E-04 | Safe | Unsafe |
| 16 | 163 | 445 | 3.90E-04 | 1.76E-04 | 3.06E-04 | 1.76E-04 | Safe | Unsafe |
| 17 | 163 | 446 | 3.90E-04 | 1.76E-04 | 3.05E-04 | 1.76E-04 | Safe | Unsafe |
| 18 | 163 | 447 | 3.90E-04 | 1.76E-04 | 3.05E-04 | 1.76E-04 | Safe | Safe |

The graph 4.1 and 4.2 shows the variation of critical strain obtained from IITPAVE and ABAQUS model.




(a)




(b)

Fig. 4. 1 – Variation of critical strains for static loading (a) Rutting strain (b) Fatigue strain.

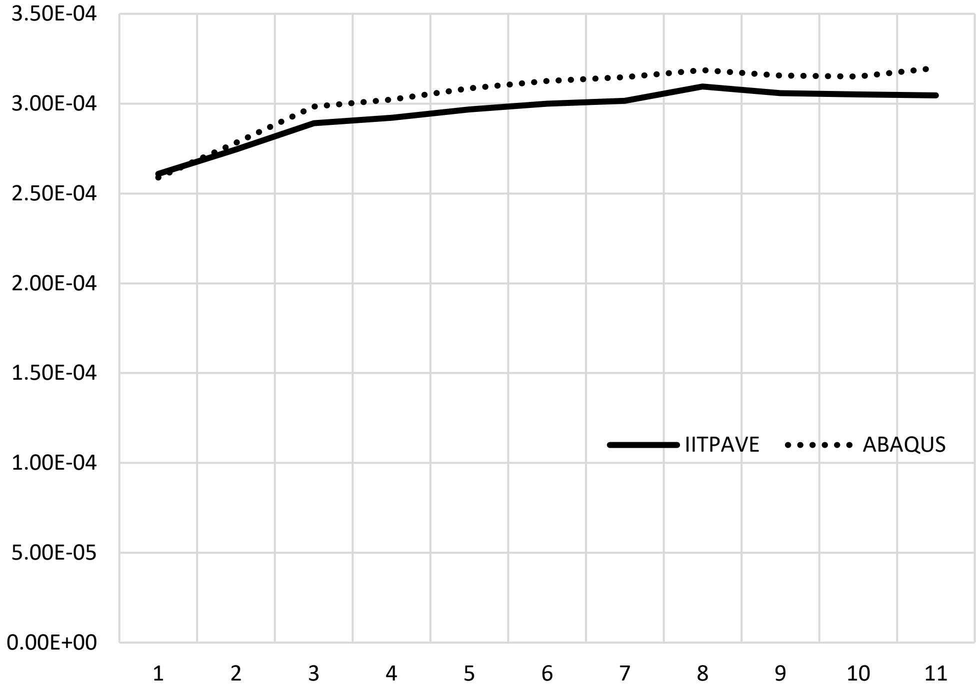
## 4.4 Validation of ABAQUS model with static loading

We can easily observed from the above table 4.3 and graph 4.1 that the values of critical strains obtained from ABAQUS software do not match with IITPAVE software. So a calibration is needed in the model to make it equal. To calibrate the model a different values of mesh sizes are adopted and the values of critical strains are checked at each time. After conducting a number of trails a required value of mesh size is obtained for the model in which both the values of critical strains gets nearly matched. The matched values of critical strains are given in table 4.4.

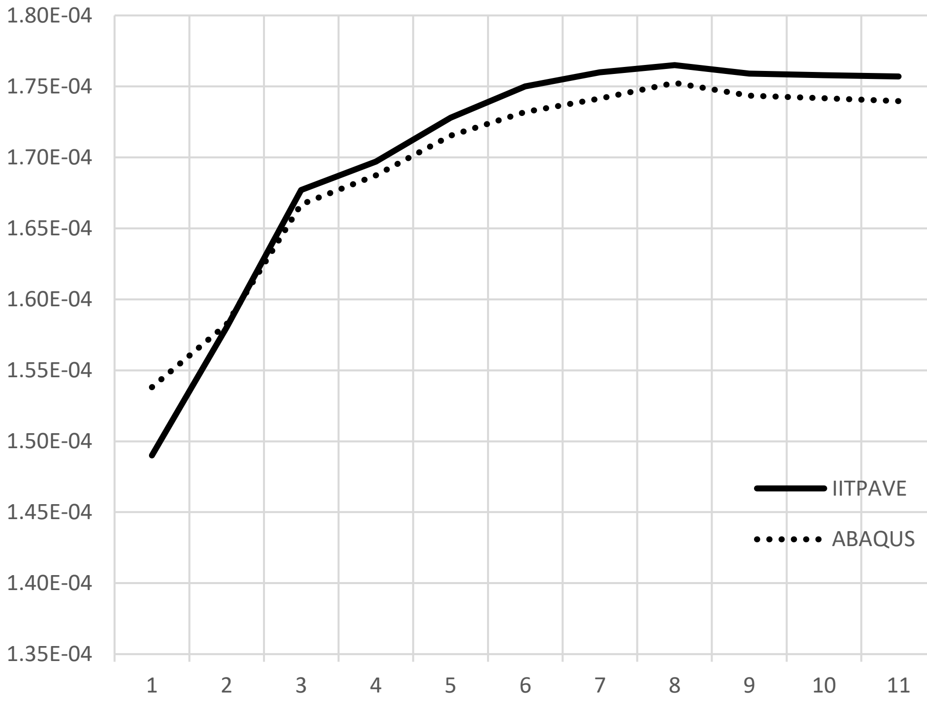
Table 4. 4 – Validation of ABAQUS model

| | Trial thickness | | IRC 37-2018 | | IITPAVE | | ABAQUS | | ABAQUS/IITPAVE | |
|---|---|---|---|---|---|---|---|---|---|---|
| s.no. | Bituminous layer (mm) | Granular layer (mm) | Allowable rutting | Allowable fatigue | Rutting | Fatigue | Rutting | Fatigue | Rutting | Fatigue |
| 1 | 190 | 450 | 3.90E-04 | 1.76E-04 | 2.61E-04 | 1.49E-04 | 2.59E-04 | 1.54E-04 | 0.99 | 1.03 |
| 2 | 180 | 450 | 3.90E-04 | 1.76E-04 | 2.75E-04 | 1.58E-04 | 2.78E-04 | 1.58E-04 | 1.01 | 1.00 |
| 3 | 170 | 450 | 3.90E-04 | 1.76E-04 | 2.89E-04 | 1.68E-04 | 2.98E-04 | 1.67E-04 | 1.03 | 0.99 |
| 4 | 168 | 450 | 3.90E-04 | 1.76E-04 | 2.92E-04 | 1.70E-04 | 3.02E-04 | 1.69E-04 | 1.04 | 0.99 |
| 5 | 165 | 450 | 3.90E-04 | 1.76E-04 | 2.97E-04 | 1.73E-04 | 3.08E-04 | 1.72E-04 | 1.04 | 0.99 |
| 6 | 163 | 450 | 3.90E-04 | 1.76E-04 | 3.00E-04 | 1.75E-04 | 3.13E-04 | 1.73E-04 | 1.04 | 0.99 |
| 7 | 162 | 450 | 3.90E-04 | 1.76E-04 | 3.02E-04 | 1.76E-04 | 3.15E-04 | 1.74E-04 | 1.04 | 0.99 |
| 8 | 163 | 440 | 3.90E-04 | 1.76E-04 | 3.10E-04 | 1.77E-04 | 3.19E-04 | 1.75E-04 | 1.03 | 0.99 |
| 9 | 163 | 445 | 3.90E-04 | 1.76E-04 | 3.06E-04 | 1.76E-04 | 3.16E-04 | 1.74E-04 | 1.03 | 0.99 |
| 10 | 163 | 446 | 3.90E-04 | 1.76E-04 | 3.05E-04 | 1.76E-04 | 3.15E-04 | 1.74E-04 | 1.03 | 0.99 |
| 11 | 163 | 447 | 3.90E-04 | 1.76E-04 | 3.05E-04 | 1.76E-04 | 3.20E-04 | 1.74E-04 | 1.05 | 0.99 |

The graphical representation of calibrated strain values of ABAQUS and IITPAVE are as follows




(a)




(b)

Fig. 4. 2 – Variation of validated critical strains (a) Rutting strain (b) Fatigue strain

From Fig. 4.2 it’s clear that our model is properly calibrated for conducting further study. The final model of pavement section is shown below with static loading.

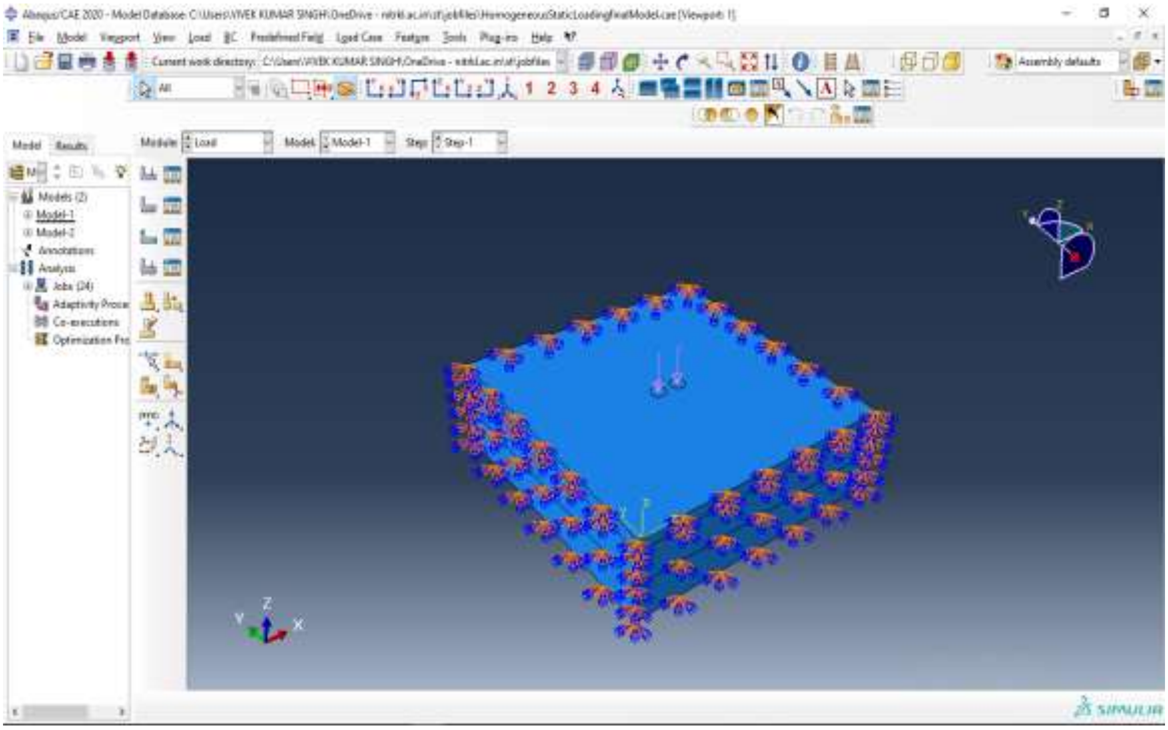


Fig. 4. 3 – Validated model

## 4.5 Creation of pavement model with dynamic loading

Dynamic loading is applied on the flexible pavement using ABAQUS. Dynamic load is applied using following two steps

- Varying the magnitude of the load with time
- Varying the position of loading with time

To find apply the dynamic loading on the flexible pavement following input parameters has been taken

- Speed of the vehicle = 60 kmph.
- Total time period = 0.225 seconds.
- Peak load = 0.56 Mpa.

The variation of load with time given in Fig 4.4. The results obtained after running the ABAQUS model with dynamic loading is given in Table 4.5.

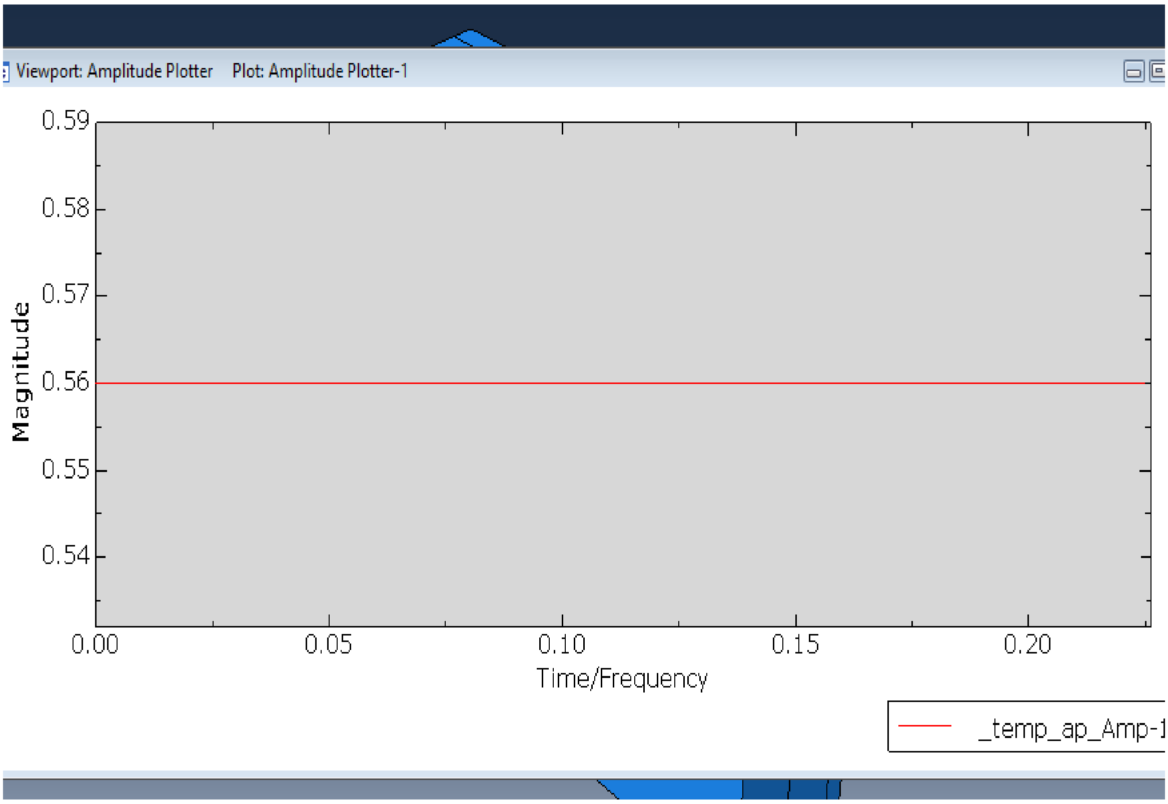



Fig. 4. 4 – Variation of load with time.

Table 4. 5 – Dynamic strain

| Thickness of different layers | | | Static strains | | Dynamic strains | | % variation | |
|---|---|---|---|---|---|---|---|---|
| Bituminous layer (mm) | Granular layer (mm) | Subgrade (mm) | Rutting | Fatigue | Rutting | Fatigue | Ruttin g | Fatigue |
| 190 | 447 | 500 | 3.05e-04 | 1.76e-04 | 1.04e-05 | 3.23e-06 | 97% | 98% |

From Table 4.5 we can easily observe that the value of critical strains produced when static load is applied on the pavement is nearly 97% higher than the values of critical strains produced when the dynamic load is applied on the pavement. Since one of the researchers Bayat and Knight (2012) has mentioned in their study that the value of strains obtained from the dynamic loading should be 10-15 % higher than strains produced from static loading. So our dynamic model needs a calibration to carry out further study. The calibration of this model is done in next section.

## 4.6 Calibration of ABAQUS model with dynamic loading

The ABAQUS model with dynamic loading is calibrated such that the strains produced should be greater than static loading. It is done by trial and error method. In each trial the magnitude of the load is increased and strains values are checked. A large numbers of trails has been performed to obtain the calibrated model which is given in Table 4.6. During this calibration the speed of the vehicle is kept constant (60 kmph). The total time period is also kept constant.

Table 4. 6 – Calibration of dynamic model

| Load (Mpa) | Dynamic fatigue strain | Static fatigue strain | % variation in strains |
|---|---|---|---|
| 0.56 | 3.23E-06 | 1.76E-04 | 98% |
| 1 | 6.08E-05 | 1.76E-04 | 65% |
| 2 | 1.22E-04 | 1.76E-04 | 31% |
| 2.5 | 1.52E-04 | 1.76E-04 | 13% |
| 3 | 1.83E-04 | 1.76E-04 | 4% |
| 2.9 | 1.77E-04 | 1.76E-04 | 0% |
| 2.8 | 1.70E-04 | 1.76E-04 | 3% |

From Table 4.6 it can be seen that the percentage variation between the values of static and dynamic strains goes on decreasing as the value of peak load increases. This trend is followed till the value of load reaches to 2.9 Mpa. When the value of load increase from 2.9 then again the percentage variation of starts increasing. This trends will continue till the value of load keeps on increasing. We observe that when the value of load is equal to 2.9 Mpa then the values of both the strains becomes nearly equal. So for further study of dynamic loading we will be taking 2.9 Mpa as the magnitude of load.

## 4.7 Effect of speed of vehicle on the performance of flexible pavement

In this section the effect of speed of vehicle on the performance of flexible pavement is calculated by keeping the magnitude of the load constant. Different speeds of the vehicle is applied on the dynamic ABAQUS model to check the effect of the speeds. The values of strain variation is observed at different speed to quantify the effect. The values of strains calculated at different speeds are tabulated in Table 4.7.

Table 4. 7 – Effect of speed of vehicle

| Vehicular speeds (kmph) | Static fatigue strain | Total time period (seconds) | Dynamic fatigue strain | % variations in strains |
|---|---|---|---|---|
| 40 | 1.76E-04 | 0.3375 | 1.96E-04 | 12% |
| 60 | 1.76E-04 | 0.225 | 1.77E-04 | 0% |
| 80 | 1.76E-04 | 0.16875 | 1.82E-04 | 4% |
| 100 | 1.76E-04 | 0.135 | 1.86E-04 | 6% |

From Table 4.7 it can be observed that the value of strain obtained when dynamic load is applied is higher than the value strain obtained when static load is applied. Whether the speed of vehicle is higher or lower, the value of dynamic strain is always higher than static strain. This shows that the value of dynamic strain will always be higher than static strain irrespective of the speed of the vehicle. The variation of the strains with the speed of the vehicle is given in Fig 4.5.

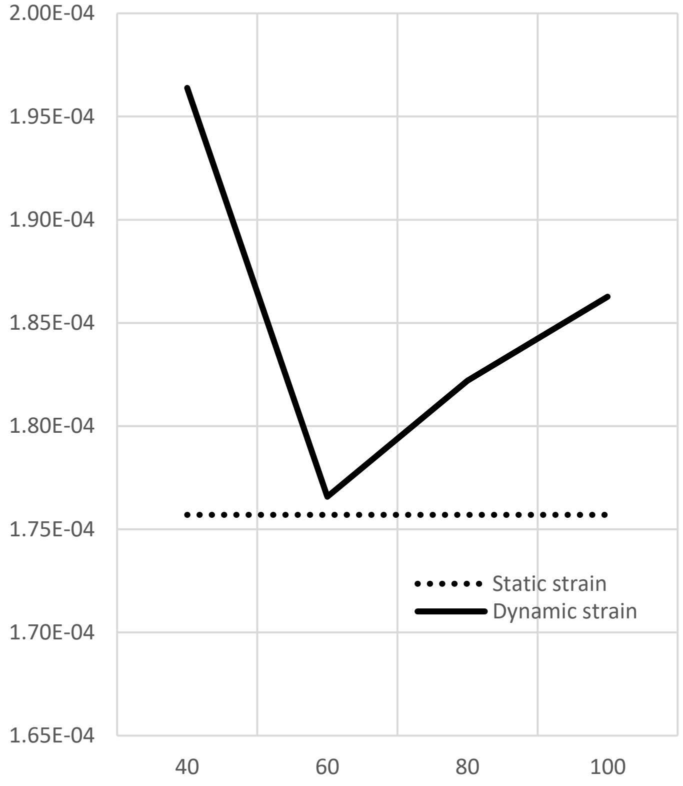



Fig. 4. 5 – Variation of strain with speed of vehicle

## 4.8 Variation of strains with different mix

The variation of static and dynamic strains with different mix of asphalt layer is quantified in this section. The value of only resilient modulus of asphalt layer is changed with different mix during the analysis of static and dynamic loading of pavement. Since the value of resilient modulus depends on frequency of loading and temperature, so the value of resilient modulus of different mix is taken at 25 $^0$C temperature and 10 Hz frequency. The variation of strains with different mix is given in Table 4.8. The total time period is taken as 0.1 second and peak load is taken as 2.9 Mpa.

Table 4. 8 – Variation of strains with different mix

| Mix type | Temperature ($^0$C) | Frequency (Hz) | Resilient modulus (Mpa) | Dynamic strains | Static strains | % Variations |
|---|---|---|---|---|---|---|
| Dense graded mix | 25 | 10 | 2982 | 6.91E-04 | 1.76E-04 | 159% |
| Polymer modified gap graded mix | | | 4103 | 4.55E-04 | 1.47E-04 | 371% |
| Asphalt rubber gap graded mix | | | 2616 | 8.52E-04 | 1.89E-04 | 351% |

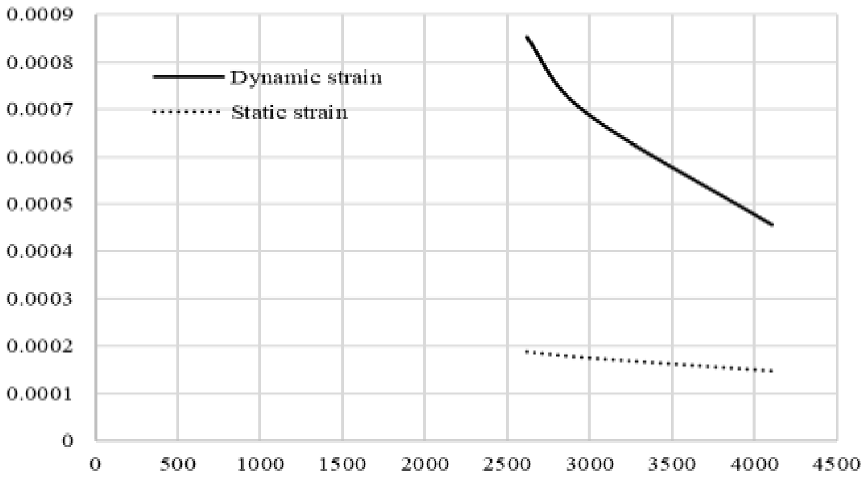



Fig. 4. 6 – Variation of strains with different resilient modulus

From Fig. 4.6 it can be seen that the value of dynamic and static strain decreases with increase of resilient modulus. The rate of decrease of strain with increase in resilient modulus is more in dynamic loading than static loading. It can also be observed that the value of dynamic strain produced due to different mix is three times than strain produced due to static loading.

# 4.9 Effect of non-homogenous Asphalt layer

In this section the effect of non-homogenous asphalt layer in the design of flexible pavement is evaluated. To the asphalt layer as non-homogenous material, it is divided into different layers with different values of resilient modulus. Now this pavement model is analysed using ABAQUS and strain produced are noted. Each time a different level of coefficient of variability (COV) is taken between resilient modulus values of different asphalt layers. To carry out this analysis a different section of pavement model is created whose detail is given in Table 4.9.

Table 4. 9 – Section details

| COV (%) | Resilient modulus values of different layers | | | | | Poisson's ratio |
|---|---|---|---|---|---|---|
| | AC1 | AC2 | AC3 | Base | Subgrade | |
| 10 | 2728 | 3000 | 3273 | 192 | 62 | 0.35 |
| 20 | 2500 | 3000 | 3500 | 192 | 62 | 0.35 |
| 30 | 2308 | 3000 | 3693 | 192 | 62 | 0.35 |
| 40 | 2143 | 3000 | 3858 | 192 | 62 | 0.35 |
| 50 | 2000 | 3000 | 4000 | 192 | 62 | 0.35 |

The effect of heterogeneity in asphalt layer is analysed in two ways. First way of analysis is by applying static loading on the pavement and calculating its effect. The second way of analysis is by applying dynamic loading on the pavement and calculating its effect. Both way of analysis is presented in subsequent section. In both way of analysis the same section is taken whose details is given in Table 4.9.

## 4.9.1 Analysis of non-homogenous asphalt layer with static loading

In this analysis the section details of Table 4.9 is used. For applying static loading the magnitude of load is taken as 0.56 Mpa. The values of strains obtained at different values of coefficient of variation of different asphalt layers is given in Table 4.10.

Table 4. 10 – Variation of static strain with different COV of asphalt layers

| COV(%) | Computed fatigue strain | Homogenous fatigue strain | % Variations in strains |
|---|---|---|---|
| 10 | 9.33E-05 | 1.76E-04 | 47% |
| 20 | 8.90E-05 | 1.76E-04 | 49% |
| 30 | 8.56E-05 | 1.76E-04 | 51% |
| 40 | 8.28E-05 | 1.76E-04 | 53% |
| 50 | 8.05E-05 | 1.76E-04 | 54% |

From Table 4.10 it can be easily observed that the value of strain produced due to static loading is inversely proportional to coefficient of variation of resilient modulus of different asphalt layers. The value of strain obtained due to static loading on pavement section in which asphalt layer is considered as non-homogeneous layer is nearly 50% less than the strain produced in the pavement section in which asphalt layer is assumed as homogenous layer.

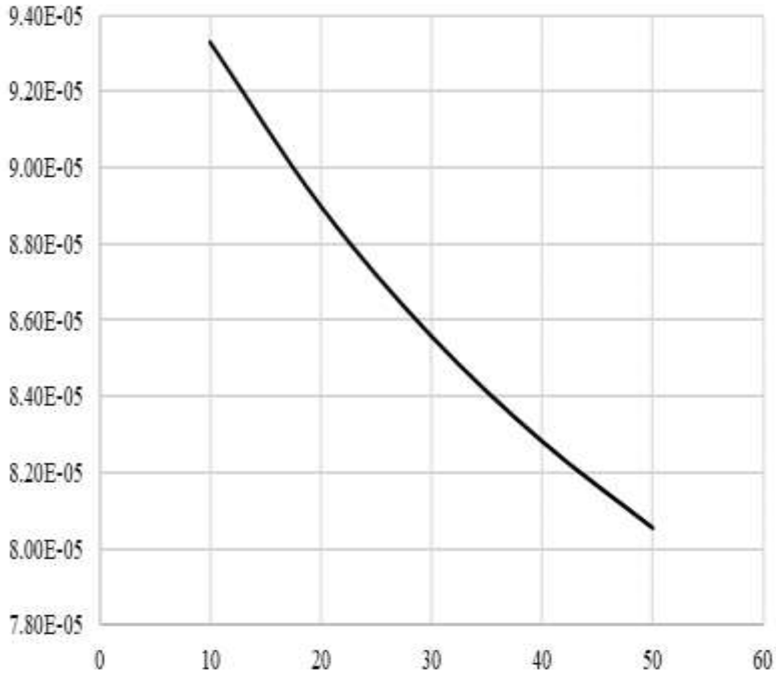



Fig. 4. 7 – Variation of static strain with COV of asphalt layer

The variation of strain produced due to pavement section with non-homogenous asphalt layer with the coefficient of variation of different asphalt layer is given in Fig. 4.7.

### 4.9.2 Analysis of non-homogenous asphalt layer with dynamic loading

For analysis of a pavement section with non-homogenous asphalt layer, the details of the pavement section given in Table 4.9 is used. To apply the dynamic loading on this pavement section the speed of the vehicle is taken as 60 kmph and total time period of analysis is taken as 0.225 seconds. The value of strains generated at different values of coefficient of variation of asphalt layers when dynamic loading is applied is given in Table 4.11.

Table 4. 11 – Variation of dynamic strain with different values of COV of asphalt layers

| COV(%) | Computed fatigue strain | Homogenous fatigue strain | % Variations in strains |
|---|---|---|---|
| 10 | 6.81E-04 | 1.76E-04 | 288% |
| 20 | 6.54E-04 | 1.76E-04 | 272% |
| 30 | 6.33E-04 | 1.76E-04 | 260% |
| 40 | 6.12E-04 | 1.76E-04 | 248% |
| 50 | 5.98E-04 | 1.76E-04 | 240% |

From Table 4.11 it can be observed that the value of strains produced when dynamic load is applied is inversely proportional to the coefficient of variation of different asphalt layers. The value of strains produced when dynamic load is applied on the section in which the asphalt layer is assumed to be non-homogenous material is nearly 3 times of the strain produced on the pavement section of homogenous asphalt layer when static load is applied for lower values of coefficient of variation of asphalt layer. When the coefficient of variation increases than the strain produced is nearly 2.5 times.

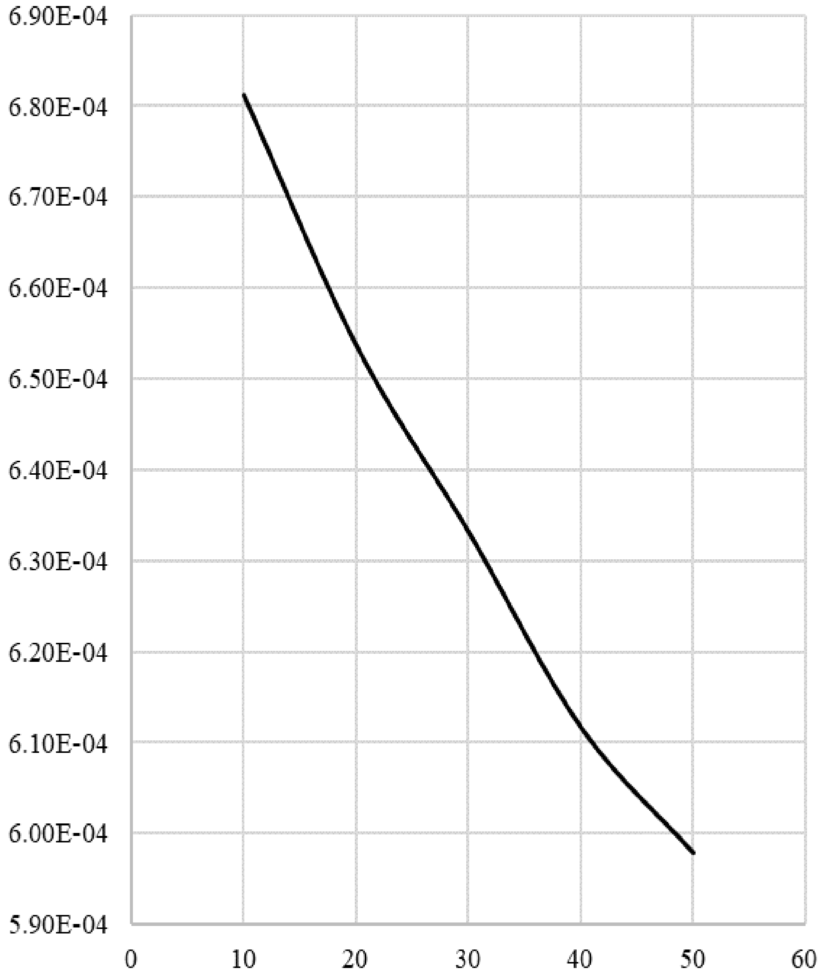



Fig. 4. 8 - Variation of dynamic strain with COV of asphalt layer

The variation of computed fatigue strain on the pavement section with non-homogenous asphalt layer with coefficient of variation of different asphalt layer is given in Fig 4.8.

CHAPTER 5

# SUMMARY AND CONCLUSIONS

## 5.1 Introduction

This chapter gives an overview of the study conducted during this project. It contains a brief details of the study conducted. It also contains various findings during this study. At the end of this chapter it will suggest the future scope of the project. In this chapter we will be able to know the limitations of this project. It will also discuss about the importance of this study. This chapter also contains various references which has been referred for carryout this study.

## 5.2 Summary

- A most economical section of the flexible pavement is found with the help of IITPAVE software and IRC 37-2018.
- A pavement section by applying same guidelines of IRC 37-2018 is created in ABAQUS software and strains are calculated.
- The values of strains obtained from ABAQUS model did not match with IITPAVE results. So the ABAQUS model is calibrated in such a way that the value of strain should get nearly equal to IITPAVE results.
- A dynamic loading is applied on the pavement model using ABAQUS to quantify the effect on design of flexible pavement.
- A dynamic load is applied on flexible pavement with non-homogeneous asphalt layer to quantify the effects of heterogeneity of asphalt layer on the design of flexible pavement.
- The results are presented with the help of different tables, graphs, figures etc.

## 5.3 Findings

The following points has been found during this study

- The strain produced from dynamic loading is always greater than the strain produced from static loading, whether the speed of the vehicle is more or less.
- The magnitude of dynamic strain decreases with the increase in speed of the vehicle.
- The magnitude of dynamic strain is directly proportional to the total time of analysis.
- The rate of variation of magnitude of dynamic strain is higher than static strain with various type of mixes of different resilient modulus value.
- Higher the resilient modulus of the asphalt mix lower the value of dynamic strain produced.
- As the COV of asphalt layer increases, the magnitude of strain gets reduced which is produced due to application of dynamic loading on the flexible pavement with non-homogenous asphalt layer.
- The magnitude of strain produced due to dynamic loading will be higher in case of flexible pavement with heterogeneous asphalt layer than the flexible pavement with homogenous asphalt layer.

## 5.4 Research significance

- Since the magnitude of strain produced due to dynamic loading is always higher than static loading so the thickness of pavement designed using IRC 37-2018 will not be always suitable in dynamic case.
- The required thickness of different layers of flexible pavement which is safe against dynamic loading also is slightly higher than static loading case.
- The thickness of the flexible pavement constructed for high speed vehicles is lower than that of pavement constructed for slower vehicles.
- The thickness of the pavement gets reduced if asphalt mix of higher resilient modulus value is used.
- The COV of resilient modulus of asphalt layer affects the thickness of flexible pavement.

## 5.5 Limitations and future scope

This study has been conducted virtually using ABAQUS as a software for analysing the model. It has been tried to create a suitable model of flexible pavement with same environmental conditions of field. This accuracy of model creation depends on one's experience. The tyre imprint area is taken as circular for this study. Any type of experiments has not been conducted in the field for validation of obtained results. The strain values obtained from IRC 37-2018 has been used as a base for safe performance of flexible pavement during this study. In this study only the effect of heterogeneity of one layer of flexible pavement is taken. We can extend this study further by taking effect of heterogeneity of others remaining layers of flexible pavement by applying dynamic loading.

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