

Homework 2

Vivek Gopalan

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R Markdown

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When you click the **Knit** button a document will be generated that includes both content as well as the output of any embedded R code chunks within the document. You can embed an R code chunk like this:

```
setwd("C:/ISE 5984")
library(data.table)

## Warning: package 'data.table' was built under R version 4.0.5

library(caret)

## Warning: package 'caret' was built under R version 4.0.5

## Loading required package: ggplot2

## Warning: package 'ggplot2' was built under R version 4.0.5

## Loading required package: lattice

library(pROC)

## Warning: package 'pROC' was built under R version 4.0.5

## Type 'citation("pROC")' for a citation.

##
## Attaching package: 'pROC'

## The following objects are masked from 'package:stats':
##
##   cov, smooth, var

library(mlbench)

## Warning: package 'mlbench' was built under R version 4.0.5

library(tm)

## Warning: package 'tm' was built under R version 4.0.5
```

```
## Loading required package: NLP

##
## Attaching package: 'NLP'

## The following object is masked from 'package:ggplot2':
##
##      annotate

library(class)
library(ISLR)

## Warning: package 'ISLR' was built under R version 4.0.5

library(MASS)
library(rpart)
library(rpart.plot)

## Warning: package 'rpart.plot' was built under R version 4.0.5

library(Rcpp)

## Warning: package 'Rcpp' was built under R version 4.0.5

library(InformationValue)

## Warning: package 'InformationValue' was built under R version 4.0.5

##
## Attaching package: 'InformationValue'

## The following objects are masked from 'package:caret':
##
##      confusionMatrix, precision, sensitivity, specificity

library(ISLR)
library("partykit")

## Warning: package 'partykit' was built under R version 4.0.5

## Loading required package: grid

## Loading required package: libcoin

## Warning: package 'libcoin' was built under R version 4.0.5

## Loading required package: mvtnorm

## Warning: package 'mvtnorm' was built under R version 4.0.5

library(dplyr)

## Warning: package 'dplyr' was built under R version 4.0.5
```

```

##
## Attaching package: 'dplyr'

## The following object is masked from 'package:MASS':
##
##      select

## The following objects are masked from 'package:data.table':
##
##      between, first, last

## The following objects are masked from 'package:stats':
##
##      filter, lag

## The following objects are masked from 'package:base':
##
##      intersect, setdiff, setequal, union

library(ROSE)

## Warning: package 'ROSE' was built under R version 4.0.5

## Loaded ROSE 0.0-4

library(car)

## Warning: package 'car' was built under R version 4.0.5

## Loading required package: carData

## Warning: package 'carData' was built under R version 4.0.5

##
## Attaching package: 'car'

## The following object is masked from 'package:dplyr':
##
##      recode

library(tidyverse)

## Warning: package 'tidyverse' was built under R version 4.0.5

## -- Attaching packages ----- tidyverse
1.3.1 --

## v tibble  3.1.6      v purrr   0.3.4
## v tidyr   1.2.0      v stringr 1.4.0
## v readr   2.1.2      v forcats 0.5.1

## Warning: package 'tibble' was built under R version 4.0.5

## Warning: package 'tidyr' was built under R version 4.0.5

```

```

## Warning: package 'readr' was built under R version 4.0.5
## Warning: package 'purrr' was built under R version 4.0.5
## Warning: package 'stringr' was built under R version 4.0.5
## Warning: package 'forcats' was built under R version 4.0.5

## -- Conflicts -----
tidyverse_conflicts() --
## x NLP::annotate()      masks ggplot2::annotate()
## x dplyr::between()     masks data.table::between()
## x dplyr::filter()      masks stats::filter()
## x dplyr::first()       masks data.table::first()
## x dplyr::lag()         masks stats::lag()
## x dplyr::last()        masks data.table::last()
## x purrr::lift()        masks caret::lift()
## x car::recode()        masks dplyr::recode()
## x dplyr::select()      masks MASS::select()
## x purrr::some()        masks car::some()
## x purrr::transpose()   masks data.table::transpose()

library(tinytex)

## Warning: package 'tinytex' was built under R version 4.0.5

library(lme4)

## Warning: package 'lme4' was built under R version 4.0.5

## Loading required package: Matrix

##
## Attaching package: 'Matrix'

## The following objects are masked from 'package:tidyr':
##
##     expand, pack, unpack

library(ggformula)

## Warning: package 'ggformula' was built under R version 4.0.5

## Loading required package: ggstance

## Warning: package 'ggstance' was built under R version 4.0.5

##
## Attaching package: 'ggstance'

## The following objects are masked from 'package:ggplot2':
##
##     geom_errorbarh, GeomErrorbarh

```

```
## Loading required package: scales
## Warning: package 'scales' was built under R version 4.0.5
##
## Attaching package: 'scales'
## The following object is masked from 'package:purrr':
##
##     discard
## The following object is masked from 'package:readr':
##
##     col_factor
## Loading required package: ggribes
## Warning: package 'ggribes' was built under R version 4.0.5
##
## New to ggformula? Try the tutorials:
##   learnr::run_tutorial("introduction", package = "ggformula")
##   learnr::run_tutorial("refining", package = "ggformula")
library(mosaic)
## Warning: package 'mosaic' was built under R version 4.0.5
## Registered S3 method overwritten by 'mosaic':
##   method                                from
##   fortify.SpatialPolygonsDataFrame ggplot2
##
## The 'mosaic' package masks several functions from core packages in order
to add
## additional features. The original behavior of these functions should not
be affected by this.
##
## Attaching package: 'mosaic'
## The following object is masked from 'package:scales':
##
##     rescale
## The following object is masked from 'package:lme4':
##
##     factorize
## The following object is masked from 'package:Matrix':
##
##     mean
```

```
## The following object is masked from 'package:purrr':  
##  
##   cross  
  
## The following objects are masked from 'package:car':  
##  
##   deltaMethod, logit  
  
## The following objects are masked from 'package:dplyr':  
##  
##   count, do, tally  
  
## The following object is masked from 'package:tm':  
##  
##   inspect  
  
## The following objects are masked from 'package:pROC':  
##  
##   cov, var  
  
## The following object is masked from 'package:caret':  
##  
##   dotPlot  
  
## The following object is masked from 'package:ggplot2':  
##  
##   stat  
  
## The following objects are masked from 'package:stats':  
##  
##   binom.test, cor, cor.test, cov, fivenum, IQR, median, prop.test,  
##   quantile, sd, t.test, var  
  
## The following objects are masked from 'package:base':  
##  
##   max, mean, min, prod, range, sample, sum  
  
library(Stat2Data)  
library(emmeans)  
  
## Warning: package 'emmeans' was built under R version 4.0.5  
  
library(psych)  
  
## Warning: package 'psych' was built under R version 4.0.5  
  
##  
## Attaching package: 'psych'  
  
## The following objects are masked from 'package:mosaic':  
##  
##   logit, rescale
```

```
## The following objects are masked from 'package:scales':
##
##   alpha, rescale

## The following object is masked from 'package:car':
##
##   logit

## The following objects are masked from 'package:ggplot2':
##
##   %+%, alpha

library(ICC)
library(naivebayes)

## Warning: package 'naivebayes' was built under R version 4.0.5

## naivebayes 0.9.7 loaded

##
## Attaching package: 'naivebayes'

## The following object is masked from 'package:data.table':
##
##   tables

library("FNN")

## Warning: package 'FNN' was built under R version 4.0.5

##
## Attaching package: 'FNN'

## The following objects are masked from 'package:class':
##
##   knn, knn.cv

library(pROC)
```

1.: Apply the Naïve Bayes, LDA, and Multiple Logistic Regression to the Zip code data using only digits 2, 3, and 5. Compare the testing error.

```
train <- read.table(file.path(getwd(), "zip.train.gz"))
test <- read.table(file.path(getwd(), "zip.test.gz"))
head(train)

##   V1 V2 V3 V4   V5   V6   V7   V8   V9   V10   V11   V12
## V13
## 1  6 -1 -1 -1 -1.000 -1.000 -1.000 -1.000 -0.631  0.862 -0.167 -1.000 -
## 1.000
```

```

## 2  5 -1 -1 -1 -0.813 -0.671 -0.809 -0.887 -0.671 -0.853 -1.000 -1.000 -
0.774
## 3  4 -1 -1 -1 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -0.996  0.147
1.000
## 4  7 -1 -1 -1 -1.000 -1.000 -0.273  0.684  0.960  0.450 -0.067 -0.679 -
1.000
## 5  3 -1 -1 -1 -1.000 -1.000 -0.928 -0.204  0.751  0.466  0.234 -0.809 -
1.000
## 6  6 -1 -1 -1 -1.000 -1.000 -0.397  0.983 -0.535 -1.000 -1.000 -1.000 -
1.000
##      V14      V15      V16 V17 V18 V19 V20      V21      V22      V23      V24      V25
## 1 -1.000 -1.000 -1.000  -1  -1  -1  -1 -1.000 -1.000 -1.000 -0.992  0.297
## 2 -0.180  0.052 -0.241  -1  -1  -1  -1  0.392  1.000  0.857  0.727  1.000
## 3 -0.189 -1.000 -1.000  -1  -1  -1  -1 -1.000 -1.000 -1.000 -1.000 -1.000
## 4 -1.000 -1.000 -1.000  -1  -1  -1  -1 -1.000 -0.114  0.974  0.917  0.734
## 5 -1.000 -1.000 -1.000  -1  -1  -1  -1 -1.000 -0.370  0.739  1.000  1.000
## 6 -1.000 -1.000 -1.000  -1  -1  -1  -1 -1.000 -1.000  0.692  0.536 -0.767
##      V26      V27      V28      V29      V30      V31      V32 V33 V34 V35 V36      V37
## 1  1.000  0.307 -1.000 -1.000 -1.000 -1.000 -1.000 -1  -1  -1  -1 -1.000
## 2  0.805  0.613  0.613  0.860  1.000  1.000  0.396 -1  -1  -1  -1 -0.548
## 3 -1.000 -0.882  1.000  0.390 -0.811 -1.000 -1.000 -1  -1  -1  -1 -1.000
## 4  0.994  1.000  0.973  0.391 -0.421 -0.976 -1.000 -1  -1  -1  -1 -0.323
## 5  1.000  1.000  0.644 -0.890 -1.000 -1.000 -1.000 -1  -1  -1  -1 -1.000
## 6 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1  -1  -1  -1 -1.000
##      V38      V39      V40      V41      V42      V43      V44      V45 V46      V47
V48
## 1 -1.000 -1.000 -0.410  1.000  0.986 -0.565 -1.000 -1.000 -1 -1.000 -
1.000
## 2  1.000  1.000  1.000  1.000  1.000  1.000  1.000  1.000  1  1.000
0.875
## 3 -1.000 -1.000 -1.000 -1.000 -1.000 -0.715  1.000  0.029 -1 -1.000 -
1.000
## 4  0.991  0.622 -0.738 -1.000 -0.639  0.023  0.871  1.000  1 -0.432 -
1.000
## 5  0.616  1.000  0.688 -0.455 -0.731  0.659  1.000 -0.287 -1 -1.000 -
1.000
## 6 -0.921  0.928 -0.118 -1.000 -1.000 -1.000 -1.000 -1.000 -1 -1.000 -
1.000
##      V49 V50 V51 V52      V53      V54      V55      V56 V57      V58      V59      V60
## 1 -1.000 -1  -1  -1 -1.000 -1.000 -0.683  0.825  1  0.562 -1.000 -1.000
## 2 -0.957 -1  -1  -1 -0.786  0.961  1.000  1.000  1  0.727  0.403  0.403
## 3 -1.000 -1  -1  -1 -1.000 -0.888 -0.912 -1.000 -1 -1.000 -0.549  1.000
## 4 -1.000 -1  -1  -1  0.409  1.000  0.000 -1.000 -1 -1.000 -1.000 -0.842
## 5 -1.000 -1  -1  -1 -1.000 -0.376 -0.186 -0.874 -1 -1.000 -0.014  1.000
## 6 -1.000 -1  -1  -1 -1.000 -0.394  1.000 -0.596 -1 -1.000 -1.000 -1.000
##      V61      V62      V63      V64 V65 V66 V67      V68      V69      V70      V71
V72
## 1 -1.000 -1.000 -1.000 -1.00  -1  -1  -1 -1.000 -1.000 -0.938  0.540
1.000
## 2  0.171 -0.314 -0.314 -0.94  -1  -1  -1 -1.000 -0.298  1.000  1.000

```



```

1.000
## 3  0.361 -1.000 -1.000 -1.00  -1  -1  -1 -1.000 -0.938  0.694  0.057 -
1.000
## 4  0.714  1.000 -0.534 -1.00  -1  -1  -1 -0.879  0.965  1.000 -0.713 -
1.000
## 5 -0.253 -1.000 -1.000 -1.00  -1  -1  -1 -1.000 -1.000 -1.000 -1.000 -
1.000
## 6 -1.000 -1.000 -1.000 -1.00  -1  -1  -1 -1.000 -1.000  0.060  0.900 -
0.951
##      V73      V74      V75      V76      V77      V78      V79 V80 V81 V82 V83      V84
## 1  0.778 -0.715 -1.000 -1.000 -1.000 -1.000 -1.000 -1 -1 -1 -1 -1.000
## 2  0.440  0.056 -0.755 -1.000 -1.000 -1.000 -1.000 -1 -1 -1 -1 -1.000
## 3 -1.000 -1.000 -0.382  1.000  0.511 -1.000 -1.000 -1 -1 -1 -1 -1.000
## 4 -1.000 -1.000 -1.000 -0.606  0.977  0.695 -0.906 -1 -1 -1 -1 -0.528
## 5 -1.000 -0.978  0.501  1.000 -0.540 -1.000 -1.000 -1 -1 -1 -1 -1.000
## 6 -1.000 -1.000 -1.000 -0.647  0.455 -0.333 -1.000 -1 -1 -1 -1 -1.000
##      V85      V86      V87      V88      V89      V90      V91      V92      V93      V94
V95 V96
## 1 -1.000  0.100  1.000  0.922 -0.439 -1.000 -1.000 -1.000 -1.000 -1.000 -
1 -1
## 2  0.366  1.000  1.000  1.000  1.000  1.000  0.889 -0.081 -0.920 -1.000 -
1 -1
## 3 -0.311  1.000 -0.043 -1.000 -1.000 -1.000 -0.648  1.000  0.644 -1.000 -
1 -1
## 4  1.000  0.931 -0.888 -1.000 -1.000 -1.000 -0.949  0.559  0.984 -0.363 -
1 -1
## 5 -1.000 -1.000 -0.998 -0.341  0.296  0.371  1.000  0.417 -0.989 -1.000 -
1 -1
## 6 -1.000  0.259  0.676 -1.000 -1.000 -1.000 -0.984  0.677  0.981  0.551 -
1 -1
##  V97 V98 V99  V100  V101  V102  V103  V104  V105  V106  V107  V108
## 1  -1  -1  -1 -1.00 -0.257  0.950  1.000 -0.162 -1.000 -1.00 -1.000 -0.987
## 2  -1  -1  -1 -1.00 -0.396  0.886  0.974  0.851  0.851  0.95  1.000  1.000
## 3  -1  -1  -1 -1.00  0.489  1.000 -0.493 -1.000 -1.000 -1.00 -0.564  1.000
## 4  -1  -1  -1 -0.97 -0.266 -0.555 -1.000 -1.000 -1.000 -1.00 -0.186  1.000
## 5  -1  -1  -1 -1.00 -1.000 -1.000 -0.008  1.000  1.000  1.00  1.000  0.761
## 6  -1  -1  -1 -1.00 -0.994  0.699  0.305 -1.000 -1.000 -1.00 -0.499  1.000
##      V109  V110 V111 V112 V113 V114 V115  V116  V117  V118  V119
V120
## 1 -0.714 -0.832  -1  -1  -1  -1  -1 -0.797  0.909  1.000  0.300 -
0.961
## 2  0.539 -0.754  -1  -1  -1  -1  -1 -1.000 -1.000 -0.886 -0.505 -
1.000
## 3  0.693 -1.000  -1  -1  -1  -1  -1 -0.966  0.988  1.000 -0.893 -
1.000
## 4  0.488 -1.000  -1  -1  -1  -1  -1 -1.000 -1.000 -1.000 -1.000 -
1.000
## 5 -0.731 -1.000  -1  -1  -1  -1  -1 -1.000 -1.000 -1.000  0.242
1.000
## 6 -0.092  0.751  -1  -1  -1  -1  -1 -1.000 -0.923  0.966 -0.107 -

```

```

1.000
##   V121   V122   V123   V124   V125   V126   V127 V128 V129 V130 V131   V132
## 1   -1 -1.000 -0.550 0.485  0.996  0.867  0.092  -1  -1  -1  -1  0.278
## 2   -1 -0.649  0.405 1.000  1.000  0.653 -0.838  -1  -1  -1  -1 -1.000
## 3   -1 -1.000 -0.397 1.000  0.903 -0.977 -1.000  -1  -1  -1  -1 -0.559
## 4   -1 -1.000  0.697 0.992 -0.458 -1.000 -1.000  -1  -1  -1  -1 -1.000
## 5    1  0.319  0.259 1.000  0.742 -0.757 -1.000  -1  -1  -1  -1 -1.000
## 6   -1 -1.000 -0.300 0.854 -0.382  0.617 -1.000  -1  -1  -1  -1 -1.000
##   V133   V134   V135   V136   V137   V138   V139   V140   V141   V142
V143
## 1  1.000  0.877 -0.824 -1.000 -0.905  0.145  0.977  1.000  1.000  1.000
0.990
## 2 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -0.550  0.993  1.000
0.618
## 3  1.000  1.000 -0.297 -1.000 -1.000 -1.000 -0.611  1.000  0.873 -0.698 -
0.552
## 4 -1.000 -1.000 -1.000 -1.000 -1.000 -0.341  1.000  0.608 -1.000 -1.000 -
1.000
## 5 -1.000 -1.000 -0.975 -0.467 -0.989 -1.000 -1.000 -0.171  0.998  0.669 -
0.945
## 6 -0.409  1.000 -0.529 -1.000 -1.000 -1.000  0.048  0.614 -0.268  0.544 -
1.000
##   V144 V145  V146  V147  V148  V149  V150  V151  V152  V153
V154
## 1 -0.745   -1 -1.00 -0.950  0.847  1.000  0.327 -1.000 -1.000  0.355
1.000
## 2 -0.869   -1 -0.96 -0.512  0.134 -0.343 -0.796 -1.000 -1.000 -1.000 -
1.000
## 3 -1.000   -1 -1.00 -1.000 -0.126  1.000  1.000  0.766 -0.764 -1.000 -
1.000
## 4 -1.000   -1 -1.00 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000
0.471
## 5 -1.000   -1 -1.00 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -
1.000
## 6 -1.000   -1 -1.00 -1.000 -1.000  0.050  0.971 -0.839 -1.000 -1.000 -
1.000
##   V155  V156  V157  V158  V159  V160 V161  V162  V163  V164
V165
## 1  0.655 -0.109 -0.185  1.000  0.988 -0.723   -1 -1.000 -0.63  1.000
1.000
## 2 -1.000 -1.000 -0.432  0.994  1.000  0.223   -1  0.426  1.00  1.000
1.000
## 3 -0.577  1.000  0.933  0.484 -0.197 -1.000   -1 -1.000 -1.00 -0.818 -
0.355
## 4  0.998 -0.416 -1.000 -1.000 -1.000 -1.000   -1 -1.000 -1.00 -1.000 -
1.000
## 5 -1.000 -1.000  0.228  1.000  0.038 -1.000   -1 -1.000 -1.00 -1.000 -
1.000
## 6  0.172  0.526 -0.003  0.307 -1.000 -1.000   -1 -1.000 -1.00 -1.000
0.398

```

[illegible]

```

## 4 -1.000 -1.000 0.773 0.958 -0.714 -1.000 -1.000 -1.000 -1.000 -1.000 -
1.000
## 5 -0.958 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -0.077 1.000 0.344 -
1.000
## 6 -1.000 -1.000 -1.000 -1.000 0.073 1.000 0.265 -1.000 -1.000 -1.000 -
1.000
##      V210      V211      V212      V213      V214      V215      V216      V217      V218      V219
V220
## 1 -1.000 -1.000 -0.452 0.828 1.000 1.000 1.000 1.000 1.000 1.000
1.000
## 2 -0.808 0.065 0.993 1.000 1.000 1.000 1.000 0.996 0.970 0.970
0.970
## 3 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -0.426
1.000
## 4 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -0.545 0.989 0.432 -1.000 -
1.000
## 5 -1.000 0.075 1.000 1.000 0.649 0.256 -0.200 -0.351 -0.733 -0.733 -
0.733
## 6 -1.000 -1.000 -1.000 0.563 0.210 -1.000 -1.000 -0.930 -0.127 0.890
0.935
##      V221      V222 V223      V224      V225 V226      V227      V228      V229      V230      V231
## 1 1.000 0.135 -1 -1.000 -1.000 -1 -1.000 -1.000 -0.483 0.813 1.000
## 2 0.998 1.000 1 1.000 0.109 -1 -1.000 -0.830 -0.242 0.350 0.800
## 3 0.555 -1.000 -1 -1.000 -1.000 -1 -1.000 -1.000 -1.000 -1.000 -1.000
## 4 -1.000 -1.000 -1 -1.000 -1.000 -1 -1.000 -1.000 -1.000 -1.000 -1.000
## 5 -0.433 0.649 1 0.093 -1.000 -1 -0.959 -0.062 0.821 1.000 1.000
## 6 -0.845 -1.000 -1 -1.000 -1.000 -1 -1.000 -1.000 0.093 0.793 -0.205
##      V232      V233      V234      V235      V236      V237      V238      V239      V240      V241
V242
## 1 1.000 1.000 1.000 1.000 1.000 0.219 -0.943 -1.000 -1.000 -1.00 -
1
## 2 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000 0.616 -0.93 -
1
## 3 -1.000 -1.000 -1.000 0.024 1.000 0.388 -1.000 -1.000 -1.000 -1.00 -
1
## 4 -0.348 1.000 0.798 -0.935 -1.000 -1.000 -1.000 -1.000 -1.000 -1.00 -
1
## 5 1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000 0.583 -0.843 -1.00 -
1
## 6 0.214 0.746 0.918 0.692 0.954 -0.882 -1.000 -1.000 -1.000 -1.00 -
1
##      V243 V244      V245      V246      V247      V248      V249      V250      V251      V252      V253
## 1 -1 -1 -1.000 -0.974 -0.429 0.304 0.823 1.000 0.482 -0.474 -0.991
## 2 -1 -1 -1.000 -1.000 -0.858 -0.671 -0.671 -0.033 0.761 0.762 0.126
## 3 -1 -1 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -0.109 1.000 -0.179
## 4 -1 -1 -1.000 -1.000 -1.000 -0.318 1.000 0.536 -0.987 -1.000 -1.000
## 5 -1 -1 -0.877 -0.326 0.174 0.466 0.639 1.000 1.000 0.791 0.439
## 6 -1 -1 -0.898 0.323 1.000 0.803 0.015 -0.862 -0.871 -0.437 -1.000
##      V254      V255      V256 V257
## 1 -1.000 -1.000 -1.000 -1

```

```
## 2 -0.095 -0.671 -0.828 -1
## 3 -1.000 -1.000 -1.000 -1
## 4 -1.000 -1.000 -1.000 -1
## 5 -0.199 -0.883 -1.000 -1
## 6 -1.000 -1.000 -1.000 -1
```

```
head(test)
```

```
##   V1 V2 V3 V4      V5   V6      V7      V8      V9      V10      V11      V12      V13
V14
## 1  9 -1 -1 -1 -1.000 -1.0 -0.948 -0.561  0.148  0.384  0.904  0.290 -0.782
-1
## 2  6 -1 -1 -1 -1.000 -1.0 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000
-1
## 3  3 -1 -1 -1 -0.593  0.7  1.000  1.000  1.000  1.000  0.853  0.075 -0.925
-1
## 4  6 -1 -1 -1 -1.000 -1.0 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000
-1
## 5  6 -1 -1 -1 -1.000 -1.0 -1.000 -1.000 -0.858 -0.106  0.802 -0.210 -1.000
-1
## 6  0 -1 -1 -1 -1.000 -1.0 -1.000  0.195  1.000  0.054 -1.000 -1.000 -1.000
-1
##   V15 V16 V17 V18 V19      V20      V21      V22      V23      V24      V25      V26
V27
## 1  -1  -1  -1  -1  -1 -1.000 -1.000 -0.748  0.588  1.000  1.000  0.991
0.915
## 2  -1  -1  -1  -1  -1 -1.000 -1.000 -1.000 -0.783 -0.973 -1.000 -1.000 -
1.000
## 3  -1  -1  -1  -1  -1 -0.553  0.998  1.000  1.000  1.000  1.000  1.000
1.000
## 4  -1  -1  -1  -1  -1 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -
1.000
## 5  -1  -1  -1  -1  -1 -1.000 -1.000 -1.000 -1.000 -0.854  0.597  1.000
0.798
## 6  -1  -1  -1  -1  -1 -1.000 -1.000 -1.000 -0.801  0.790  1.000  0.856 -
0.282
##      V28      V29      V30      V31 V32 V33 V34 V35      V36      V37      V38      V39
## 1  1.000  0.931 -0.476 -1.000  -1  -1  -1  -1 -1.000 -0.787  0.794  1.000
## 2 -1.000 -1.000 -1.000 -1.000  -1  -1  -1  -1 -1.000 -1.000 -0.364  0.789
## 3  1.000  0.961 -0.076 -0.999  -1  -1  -1  -1  0.228  1.000  0.849 -0.150
## 4 -1.000 -1.000 -1.000 -1.000  -1  -1  -1  -1 -1.000 -1.000 -1.000 -0.417
## 5 -0.388 -1.000 -1.000 -1.000  -1  -1  -1  -1 -1.000 -1.000 -1.000 -0.481
## 6 -0.831 -1.000 -1.000 -1.000  -1  -1  -1  -1 -1.000 -1.000 -0.937  0.498
##      V40      V41      V42      V43      V44      V45      V46      V47 V48 V49 V50 V51
## 1  0.727 -0.178 -0.693 -0.786 -0.624  0.834  0.756 -0.822 -1  -1  -1  -1
## 2 -0.371 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1  -1  -1  -1
## 3 -0.705 -1.000 -0.850 -0.333 -0.072  0.929  1.000 -0.451 -1  -1  -1  -1
## 4 -0.330 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1  -1  -1  -1
## 5  0.600  1.000  0.653 -0.780 -1.000 -1.000 -1.000 -1.000 -1  -1  -1  -1
## 6  1.000  0.975 -0.232  0.932  0.926 -0.069 -0.965 -1.000 -1  -1  -1  -1
```

##	V52	V53	V54	V55	V56	V57	V58	V59	V60	V61		
V62												
## 1	-0.922	0.810	1.000	0.010	-0.928	-1.000	-1.000	-1.000	-1	-0.390		
	1.000											
## 2	-1.000	-0.467	0.963	0.609	-0.986	-1.000	-1.000	-1.000	-1	-1.000		
	1.000											
## 3	-0.586	0.777	-0.524	-1.000	-1.000	-1.000	-1.000	-1.000	-1	0.344		
	1.000											
## 4	-1.000	-0.883	0.447	0.999	0.775	-1.000	-1.000	-1.000	-1	-1.000		
	1.000											
## 5	-1.000	-1.000	-0.386	0.913	1.000	0.658	-0.825	-1.000	-1	-1.000		
	1.000											
## 6	-1.000	-1.000	0.070	1.000	1.000	0.446	-0.953	-0.385	1	1.000		
	0.276											
##	V63	V64	V65	V66	V67	V68	V69	V70	V71	V72	V73	V74
## 1	0.271	-1.000	-1	-1	-1	0.012	1.000	0.248	-1.000	-1.000	-1.000	-1
## 2	-1.000	-1.000	-1	-1	-1	-0.875	0.605	0.960	-0.351	-1.000	-1.000	-1
## 3	0.544	-0.999	-1	-1	-1	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1
## 4	-1.000	-1.000	-1	-1	-1	-0.589	0.803	1.000	0.602	0.257	-1.000	-1
## 5	-1.000	-1.000	-1	-1	-1	-1.000	-0.373	0.939	0.999	0.129	-0.835	-1
## 6	-0.895	-1.000	-1	-1	-1	-1.000	-0.423	0.960	1.000	0.762	-0.799	-1
##	V75	V76	V77	V78	V79	V80	V81	V82	V83	V84	V85	
V86												
## 1	-1.000	-0.402	0.326	1	0.801	-0.998	-1	-1	-0.981	0.645	1.000	-
	0.687											
## 2	-1.000	-1.000	-1.000	-1	-1.000	-1.000	-1	-1	-1.000	0.050	1.000	
	0.096											
## 3	-1.000	-0.803	0.930	1	0.650	-0.999	-1	-1	-1.000	-1.000	-1.000	-
	1.000											
## 4	-1.000	-1.000	-1.000	-1	-1.000	-1.000	-1	-1	-0.588	0.862	0.981	
	0.150											
## 5	-1.000	-1.000	-1.000	-1	-1.000	-1.000	-1	-1	-1.000	-0.340	0.986	
	1.000											
## 6	-0.987	-0.238	0.941	1	-0.288	-1.000	-1	-1	-1.000	-0.989	0.321	
	1.000											
##	V87	V88	V89	V90	V91	V92	V93	V94	V95	V96	V97	
V98												
## 1	-1.000	-1.000	-1	-1	-0.792	0.976	1.000	1.000	0.413	-0.976	-1.000	-
	1.000											
## 2	-1.000	-1.000	-1	-1	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-
	1.000											
## 3	-1.000	-1.000	-1	-1	-0.579	0.821	1.000	1.000	-0.131	-1.000	-1.000	-
	1.000											
## 4	-0.980	-1.000	-1	-1	-1.000	-1.000	-0.969	-0.437	-0.295	-0.295	-0.939	-
	0.651											
## 5	0.007	-1.000	-1	-1	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-
	1.000											
## 6	0.924	-0.512	-1	-1	-1.000	-1.000	-0.015	1.000	0.906	-0.758	-1.000	-
	1.000											
##	V99	V100	V101	V102	V103	V104	V105	V106	V107	V108		

```

V109
## 1 -0.993  0.834  0.897 -0.951 -1.000  -1 -1.000 -0.831  0.140  1.000
1.000
## 2 -0.582  0.970  0.532 -0.922 -1.000  -1 -1.000 -1.000 -1.000 -1.000 -
0.602
## 3 -1.000 -1.000 -1.000 -1.000 -1.000  -1 -0.621  0.156  0.934  1.000
1.000
## 4  0.863  0.965 -0.219 -1.000 -1.000  -1 -1.000 -1.000 -0.946 -0.592
0.488
## 5 -0.741  0.763  1.000  0.119 -0.986  -1 -1.000 -1.000 -1.000 -1.000 -
1.000
## 6 -1.000 -0.530  0.992  1.000 -0.055  -1 -1.000 -1.000 -1.000 -1.000 -
0.651
##      V110   V111   V112   V113   V114   V115   V116   V117   V118   V119
V120
## 1  0.302 -0.889 -1.000 -1.000 -1.000 -1.000  0.356  0.794 -0.836 -1.000 -
0.445
## 2  0.307  0.718  0.718 -0.373 -0.998  0.723  1.000 -0.431 -1.000 -1.000 -
1.000
## 3  0.575 -0.933 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -0.952 -0.176
0.602
## 4  1.000  1.000  1.000  0.251  0.338  1.000  0.054 -1.000 -1.000 -1.000 -
1.000
## 5 -1.000 -1.000 -1.000 -1.000 -1.000  0.271  1.000  0.879 -0.871 -1.000 -
1.000
## 6  0.810  1.000  0.206 -1.000 -1.000 -1.000  0.424  1.000  0.868 -0.717 -
1.000
##      V121   V122   V123   V124   V125   V126   V127   V128   V129   V130
V131
## 1  0.074  0.833  1.000  1.000  0.696 -0.881 -1.000 -1.000 -1.000 -1.000 -
1.000
## 2 -1.000 -1.000 -0.817 -0.136  0.808  1.000  1.000  1.000  0.697 -0.670
0.965
## 3  1.000  1.000  1.000  0.952 -0.093 -1.000 -1.000 -1.000 -1.000 -1.000 -
1.000
## 4 -1.000 -0.698  0.628  1.000  1.000  0.673  0.059  0.890  0.995  0.995
0.878
## 5 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -0.620
0.933
## 6 -1.000 -1.000 -1.000 -1.000 -0.989  0.721  1.000 -0.125 -1.000 -1.000 -
0.944
##      V132   V133   V134 V135 V136 V137   V138   V139   V140   V141   V142
V143
## 1 -0.368  0.955  1.00    1    1    1  0.905  1.000  1.000 -0.262 -1.000 -
1.000
## 2  0.659 -1.000 -1.00    -1   -1   -1 -0.512  0.738  1.000  0.839 -0.336 -
0.977
## 3 -1.000 -1.000 -0.41    1    1    1  1.000  1.000  0.792 -0.715 -1.000 -
1.000
## 4 -0.814 -1.000 -1.00    -1   -1   -1  0.388  1.000  0.711 -0.501 -0.926 -

```

```

0.672
## 5  0.983 -0.349 -1.00  -1  -1  -1 -1.000 -1.000 -0.930 -0.356  0.552
0.647
## 6  0.867  1.000  0.00  -1  -1  -1 -1.000 -1.000 -1.000 -1.000  0.579
1.000
##      V144    V145    V146    V147    V148    V149    V150    V151    V152    V153
V154
## 1 -1.000 -1.000 -1.000 -1.000 -1.000 -0.507  0.451  0.692  0.692 -0.007 -
0.237
## 2  0.433  0.878  0.161  1.000 -0.102 -1.000 -1.000 -1.000 -1.000 -1.000 -0.643
0.870
## 3 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -0.345  0.333 -0.050 -
0.333
## 4  0.963  0.473  0.989  0.775 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000
0.677
## 5  0.618 -0.633  0.148  1.000  0.549 -1.000 -1.000 -1.000 -1.000 -1.000 -
0.853
## 6  0.344 -1.000 -1.000 -0.111  0.996  0.978 -0.604 -1.000 -1.000 -1.000 -
1.000
##      V155    V156    V157    V158    V159    V160    V161    V162    V163    V164
V165
## 1  1.000  0.882 -0.795 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -
1.000
## 2  0.970  0.264 -0.971 -1.000 -0.399  1.000  0.117  0.835  0.968 -0.701 -
1.000
## 3 -0.172  0.606  0.950 -0.101 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -
1.000
## 4  1.000 -0.654 -1.000 -1.000  0.122  1.000 -0.357  0.970  0.922 -0.506 -
0.994
## 5 -0.172  0.666  1.000  1.000  1.000  1.000  0.821  0.861  1.000  0.086 -
1.000
## 6 -1.000 -1.000 -1.000  0.111  1.000  0.013 -1.000 -1.000  0.406  1.000
0.988
##      V166 V167 V168    V169    V170    V171    V172    V173    V174    V175    V176
## 1 -1.000  -1  -1 -1.000  0.155  1.000  0.436 -1.000 -1.000 -1.000 -1.000
## 2 -1.000  -1  -1  0.198  1.000  0.052 -1.000 -1.000 -0.291  0.876  0.790
## 3 -1.000  -1  -1 -1.000 -1.000 -1.000 -0.838  0.548  1.000 -0.266 -1.000
## 4 -1.000  -1  -1 -1.000  0.440  0.262 -0.996 -0.985 -0.323  0.986  0.668
## 5 -1.000  -1  -1 -0.280  0.878  1.000  1.000  0.806 -0.097 -0.191  1.000
## 6 -0.861  -1  -1 -1.000 -1.000 -1.000 -1.000 -1.000  0.253  1.000  0.460
##      V177    V178    V179    V180    V181    V182    V183    V184    V185    V186
V187
## 1 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -0.991  0.703
1.000
## 2 -0.819  0.392  0.962 -0.461 -1.000 -1.000 -1.000 -0.948  0.820  1.000 -
0.168
## 3 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -
1.000
## 4 -0.954  0.053  1.000  1.000  0.499  0.038  0.179 -0.215 -0.215 -0.215 -
0.215

```



```

## 5  0.695  0.909  1.000 -0.666 -1.000 -1.000 -1.000 -0.316  0.999  1.000
0.712
## 6 -1.000 -1.000  0.148  1.000  0.811 -0.888 -1.000 -1.000 -1.000 -1.000 -
1.000
##      V188      V189      V190      V191      V192      V193      V194      V195      V196      V197
V198
## 1 -0.025 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -
1.000
## 2 -0.475  0.280  0.968  0.880 -0.613 -1.000 -0.551  0.854  0.980  0.498
0.324
## 3 -1.000 -0.285  1.000  0.729 -1.000 -1.000 -1.000 -1.000 -0.611 -0.944 -
1.000
## 4  0.184  0.637  1.000  0.443 -0.741 -1.000 -0.979  0.189  0.942  1.000
1.000
## 5 -0.530 -0.995 -0.859  0.569  1.000 -0.267  0.955  1.000 -0.552 -1.000 -
1.000
## 6 -1.000 -0.872  0.456  1.000  0.171 -1.000 -1.000 -0.465  0.913  1.000
0.281
##      V199      V200      V201      V202      V203      V204      V205      V206      V207      V208
V209
## 1 -1.000 -1.000 -0.833  0.959  1.000 -0.629 -1.000 -1.000 -1.000 -1.000 -
1.00
## 2  0.324  0.328  0.998  1.000  0.970  0.995  0.976  0.250 -0.642 -1.000 -
1.00
## 3 -1.000 -1.000 -1.000 -1.000 -1.000 -0.556  0.943  1.000  0.779 -0.943 -
1.00
## 4  1.000  1.000  1.000  1.000  1.000  1.000  0.896  0.177 -0.911 -1.000 -
1.00
## 5 -1.000  0.646  1.000  0.317 -0.926 -1.000 -0.849  0.598  1.000  0.169 -
0.97
## 6 -0.754 -1.000 -1.000 -1.000 -0.740 -0.436  0.657  1.000  1.000  0.008 -
1.00
##      V210      V211      V212      V213      V214      V215      V216      V217      V218      V219
V220
## 1 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -0.600  0.998  0.841 -
0.932
## 2 -1.000 -0.640  0.661  0.971  1.000  1.000  1.000  0.950  0.774  0.774
0.302
## 3 -1.000 -0.943  0.779  0.555 -0.333 -0.333 -0.333 -0.166  0.389  1.000
1.000
## 4 -1.000 -1.000 -0.723 -0.451 -0.081 -0.611 -0.021 -0.414 -0.021 -0.182 -
0.648
## 5  0.631  1.000  0.754  0.046 -0.244 -0.661  0.984  1.000  0.142 -0.584
0.075
## 6 -1.000 -1.000 -0.006  0.976  1.000  0.868  0.744  0.744  0.744  0.850
1.000
##      V221 V222  V223  V224 V225  V226  V227  V228  V229  V230  V231
## 1 -1.000  -1 -1.000 -1.000  -1 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000
## 2 -0.522  -1 -1.000 -1.000  -1 -1.000 -1.000 -1.000 -0.663 -0.606 -0.606
## 3  1.000   1  0.497 -1.000  -1 -1.000 -1.000  0.507  1.000  1.000  1.000

```

```

## 4 -0.780    -1 -1.000 -1.000    -1 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000
## 5  0.833     1  0.123 -0.963    -1 -0.537  0.896  1.000  1.000  1.000  1.000
## 6  1.000     1  0.782 -0.736    -1 -1.000 -1.000 -1.000 -0.310  0.686  1.000
##      V232    V233 V234    V235 V236  V237    V238    V239 V240 V241 V242    V243
## 1 -1.000 -0.424     1  0.732    -1 -1.00 -1.000 -1.000    -1  -1    -1 -1.000
## 2 -0.606 -0.688    -1 -1.000    -1 -1.00 -1.000 -1.000    -1  -1    -1 -1.000
## 3  1.000  1.000     1  1.000     1  0.83  0.053 -0.946    -1  -1    -1 -1.000
## 4 -1.000 -1.000    -1 -1.000    -1 -1.00 -1.000 -1.000    -1  -1    -1 -1.000
## 5  1.000  1.000     1  1.000     1  0.83 -0.387 -0.976    -1  -1    -1 -0.697
## 6  1.000  1.000     1  1.000     1  1.00  0.602 -0.906    -1  -1    -1 -1.000
##      V244    V245    V246    V247    V248    V249    V250    V251    V252    V253
V254
## 1 -1.000 -1.000 -1.000 -1.000 -1.000 -0.908  0.430  0.622 -0.973 -1.000 -
1.000
## 2 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -
1.000
## 3 -0.941  0.059  0.615  1.000  1.000  0.717  0.333  0.162 -0.393 -1.000 -
1.000
## 4 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -1.000 -
1.000
## 5 -0.108  0.312  0.901  0.901  0.901  0.901  0.901  0.290 -0.369 -0.867 -
1.000
## 6 -1.000 -1.000 -0.903  0.009  0.224  1.000  0.988  0.187  0.139 -0.641 -
0.812
##      V255 V256 V257
## 1    -1    -1    -1
## 2    -1    -1    -1
## 3    -1    -1    -1
## 4    -1    -1    -1
## 5    -1    -1    -1
## 6    -1    -1    -1

```

Filtering data set to 2's, 3's, and 5's

```

train <- train[train[,1] %in% c(2, 3,5),]
test  <- test[test[,1] %in% c(2, 3,5),]

```

Reducing models to key variables

```

pixel <- c("V1", "V3", "V5", "V7", "V15")
train <- train[,pixel]
test  <- test[,pixel]

```

```

train$V1 <- as.numeric(train$V1)
train$V3 <- as.numeric(train$V3)
train$V5 <- as.numeric(train$V5)
train$V7 <- as.numeric(train$V7)
train$V15<- as.numeric(train$V15)

```

```

cor(train[c("V1", "V3", "V5", "V7", "V15")])

```

```
##           V1           V3           V5           V7           V15
## V1  1.00000000 -0.08705182 -0.137484360 -0.27046609  0.431486903
## V3  -0.08705182  1.00000000  0.379312157  0.09936139 -0.017752337
## V5  -0.13748436  0.37931216  1.000000000  0.49781470  0.005154581
## V7  -0.27046609  0.09936139  0.497814705  1.00000000 -0.105463820
## V15  0.43148690 -0.01775234  0.005154581 -0.10546382  1.000000000
```

Building Naive Bayes Model

```
TrainY = train[,1]
TrainY = as.matrix(TrainY)
TrainY = factor(TrainY)
TrainX = train[2:5]
head(TrainX)
```

```
##      V3      V5      V7      V15
## 2  -1 -0.813 -0.809  0.052
## 5  -1 -1.000 -0.928 -1.000
## 7  -1 -0.830  1.000 -1.000
## 27 -1 -1.000 -0.104 -1.000
## 31 -1 -1.000 -1.000 -1.000
## 36 -1 -1.000  0.492 -1.000
```

```
bayes.model.train <- naive_bayes(TrainY~., data=train[,pixel])
bayes.model.train
```

```
##
## ===== Naive Bayes
## =====
##
## Call:
## naive_bayes.formula(formula = TrainY ~ ., data = train[, pixel])
##
## -----
##
## Laplace smoothing: 0
##
## -----
##
## A priori probabilities:
##
##      2      3      5
## 0.3758355 0.3383033 0.2858612
##
## -----
##
## Tables:
##
## -----
```

```
-----  
## ::: V1 (Gaussian)
```

```
## -----  
-----  
##
```

```
## V1      2 3 5  
## mean 2 3 5  
## sd    0 0 0  
##  
## -----  
-----
```

```
## ::: V3 (Gaussian)  
## -----  
-----  
##
```

```
## V3      2      3      5  
## mean -0.96039124 -0.98044225 -0.99509892  
## sd    0.21586006 0.13240860 0.05479478  
##  
## -----  
-----
```

```
## ::: V5 (Gaussian)  
## -----  
-----  
##
```

```
## V5      2      3      5  
## mean -0.7993461 -0.7305897 -0.9338058  
## sd    0.4652299 0.5079160 0.2659924  
##  
## -----  
-----
```

```
## ::: V7 (Gaussian)  
## -----  
-----  
##
```

```
## V7      2      3      5  
## mean -0.3736717 -0.1031353 -0.8024029  
## sd    0.7374306 0.7489323 0.4490188  
##  
## -----  
-----
```

```
## ::: V15 (Gaussian)  
## -----  
-----  
##
```

```
## V15      2      3      5  
## mean -0.99729275 -0.97465805 -0.59708993  
## sd    0.04194251 0.14583086 0.63029931  
##
```

```
## -----
summary(bayes.model.train)

##
## ===== Naive Bayes
##
## - Call: naive_bayes.formula(formula = TrainY ~ ., data = train[, pixel])
## - Laplace: 0
## - Classes: 3
## - Samples: 1945
## - Features: 5
## - Conditional distributions:
##   - Gaussian: 5
## - Prior probabilities:
##   - 2: 0.3758
##   - 3: 0.3383
##   - 5: 0.2859
##
## -----

predBayes <- predict(bayes.model.train, test)
summary(predBayes)

##    2    3    5
## 203 162 159

TestY = test[,1]
TestY = as.matrix(TestY)
TestY = factor(TestY)
TestX = test[2:5]
head(TestX)

##      V3      V5      V7 V15
## 3 -1.000 -0.593  1.000  -1
## 12  0.572  0.063 -0.847  -1
## 13 -1.000  0.413  1.000  -1
## 16 -1.000  0.264 -0.210  -1
## 21 -1.000 -1.000 -1.000  -1
## 22 -1.000 -0.831  0.140  -1

bayes.model.test <- naive_bayes(TestY~., data=test[,pixel])
bayes.model.test

##
## ===== Naive Bayes
##
## Call:
```

```
## naive_bayes.formula(formula = TestY ~ ., data = test[, pixel])
```

```
##
```

```
## -----
```

```
-----
```

```
##
```

```
## Laplace smoothing: 0
```

```
##
```

```
## -----
```

```
-----
```

```
##
```

```
## A priori probabilities:
```

```
##
```

```
##          2          3          5
```

```
## 0.3778626 0.3167939 0.3053435
```

```
##
```

```
## -----
```

```
-----
```

```
##
```

```
## Tables:
```

```
##
```

```
## -----
```

```
-----
```

```
## ::: V1 (Gaussian)
```

```
## -----
```

```
-----
```

```
##
```

```
## V1      2 3 5
```

```
## mean 2 3 5
```

```
## sd   0 0 0
```

```
##
```

```
## -----
```

```
-----
```

```
## ::: V3 (Gaussian)
```

```
## -----
```

```
-----
```

```
##
```

```
## V3          2          3          5
```

```
## mean -0.9503838 -0.9611627 -0.9868750
```

```
## sd   0.2558233 0.2393437 0.1008493
```

```
##
```

```
## -----
```

```
-----
```

```
## ::: V5 (Gaussian)
```

```
## -----
```

```
-----
```

```
##
```

```
## V5          2          3          5
```

```
## mean -0.8052374 -0.6965482 -0.9583687
```

```
## sd   0.4523138 0.5393461 0.1704971
```

```
##
```

```

## -----
## ::: V7 (Gaussian)
## -----
##
## V7          2          3          5
## mean -0.36917172 -0.03910241 -0.76425000
## sd    0.73638758  0.78476763  0.49284619
##
## -----
## ::: V15 (Gaussian)
## -----
##
## V15          2          3          5
## mean -0.9898788 -0.9705301 -0.5616562
## sd    0.1411382  0.1840315  0.6846652
##
## -----

summary(bayes.model.test)

##
## ===== Naive Bayes
## =====
##
## - Call: naive_bayes.formula(formula = TestY ~ ., data = test[, pixel])
## - Laplace: 0
## - Classes: 3
## - Samples: 524
## - Features: 5
## - Conditional distributions:
##   - Gaussian: 5
## - Prior probabilities:
##   - 2: 0.3779
##   - 3: 0.3168
##   - 5: 0.3053
##
## -----

# Conducting LDA analysis on train and test data
mylDA <- lda(formula = V1 ~.,data=train)
mylDA

## Call:
## lda(V1 ~ ., data = train)
##

```

```
## Prior probabilities of groups:
```

```
##           2           3           5
```

```
## 0.3758355 0.3383033 0.2858612
```

```
##
```

```
## Group means:
```

```
##           V3           V5           V7           V15
```

```
## 2 -0.9603912 -0.7993461 -0.3736717 -0.9972927
```

```
## 3 -0.9804422 -0.7305897 -0.1031353 -0.9746581
```

```
## 5 -0.9950989 -0.9338058 -0.8024029 -0.5970899
```

```
##
```

```
## Coefficients of linear discriminants:
```

```
##           LD1           LD2
```

```
## V3 -0.28206247 -3.0687501
```

```
## V5 -0.09066664  0.1476385
```

```
## V7 -0.92898907  1.0518466
```

```
## V15 2.29820633  1.4903142
```

```
##
```

```
## Proportion of trace:
```

```
##      LD1      LD2
```

```
## 0.9395 0.0605
```

```
summary(myLDA)
```

```
##           Length Class  Mode
## prior         3      -none- numeric
## counts        3      -none- numeric
## means        12      -none- numeric
## scaling        8      -none- numeric
## lev           3      -none- character
## svd            2      -none- numeric
## N              1      -none- numeric
## call           3      -none- call
## terms          3      terms  call
## xlevels        0      -none- list
```

```
# Predicting LDA training model on test data
```

```
predLDA <- predict(myLDA, test)
```

```
predLDA
```

```
## $class
```

```
## [1] 3 2 3 3 2 3 2 2 3 5 2 2 5 2 2 3 3 2 3 3 3 2 2 2 3 3 3 2 3 3 2 5 2 2
```

```
## [38] 2 2 3 3 2 3 2 3 2 3 2 3 3 2 2 2 3 2 3 3 5 2 2 2 2 2 2 3 2 2 2 2 3 2
```

```
## [75] 5 2 3 5 2 2 3 2 3 2 2 2 3 3 3 2 2 2 2 2 5 2 3 3 5 2 2 2 2 2 2 2 3 3
```

```
## [112] 3 2 3 3 3 2 3 2 3 2 3 3 2 2 2 5 2 2 3 2 2 2 3 2 2 3 3 3 2 3 3 2 2 5
```

```
## [149] 2 3 3 2 2 2 2 2 5 3 2 2 3 2 2 2 3 2 3 3 3 2 3 3 2 3 2 3 3 2 3 3 2 2
```

```
## [186] 3 5 2 2 2 2 3 2 2 5 2 2 2 2 2 2 2 2 3 2 2 2 2 2 2 2 3 2 3 2 2 3 3 2 3
```



```

2 2 2
## [223] 3 2 2 2 2 3 2 3 2 2 2 5 3 5 2 2 2 2 2 2 2 5 3 5 2 3 2 2 3 2 5 5
2 3 2
## [260] 2 5 3 2 2 2 3 3 2 2 3 2 2 2 3 2 2 5 3 2 3 2 3 3 2 2 2 2 3 3 2 5 5 2
2 5 2
## [297] 5 2 3 2 2 3 2 2 3 3 2 2 3 3 3 2 3 5 2 2 3 2 2 2 2 2 5 2 2 3 3 2 2 2
2 2 2
## [334] 3 2 2 2 2 2 5 5 3 5 3 2 5 5 2 3 5 3 3 5 3 2 5 2 2 2 2 3 3 5 2 5 2 2
3 5 5
## [371] 3 3 3 2 2 2 2 3 3 2 2 2 2 3 2 2 3 2 3 3 3 2 2 2 3 3 3 3 2 2 2 5 2 5
5 5 3
## [408] 2 3 3 2 2 3 5 2 3 2 2 2 3 3 5 2 2 2 5 5 2 2 3 2 2 2 3 3 2 3 2 3 2 2
2 2 2
## [445] 2 2 2 3 2 3 3 2 3 3 2 2 2 2 2 2 2 2 3 5 3 3 3 2 2 3 2 2 2 2 3 2 2 2
3 2 5
## [482] 5 3 2 2 5 5 2 2 5 3 3 2 2 2 2 5 2 2 2 5 5 2 3 2 2 2 5 2 2 2 2 2 5
5 2 2
## [519] 5 2 3 5 3 2
## Levels: 2 3 5
##
## $posterior
##           2           3           5
## 3      0.328187385 0.646369164 0.02544345
## 12     0.834447057 0.095674682 0.06987826
## 13     0.312854945 0.664927873 0.02221718
## 16     0.441825664 0.451488982 0.10668535
## 21     0.459908972 0.266728075 0.27336295
## 22     0.426280433 0.493934885 0.07978468
## 26     0.459908972 0.266728075 0.27336295
## 28     0.410250773 0.273733503 0.31601572
## 30     0.350289191 0.617442489 0.03226832
## 33     0.007521920 0.013293078 0.97918500
## 37     0.459908972 0.266728075 0.27336295
## 49     0.877380340 0.057629973 0.06498969
## 52     0.004589553 0.006049298 0.98936115
## 54     0.468661578 0.341079398 0.19025902
## 57     0.459908972 0.266728075 0.27336295
## 59     0.333286844 0.638306906 0.02840625
## 64     0.314718939 0.662690439 0.02259062
## 68     0.459908972 0.266728075 0.27336295
## 69     0.403598787 0.536029764 0.06037145
## 72     0.387638552 0.560490716 0.05187073
## 74     0.417735772 0.511472343 0.07079189
## 79     0.459908972 0.266728075 0.27336295
## 81     0.459908972 0.266728075 0.27336295
## 84     0.459908972 0.266728075 0.27336295
## 89     0.327896150 0.646725072 0.02537878
## 95     0.374279524 0.583435128 0.04228535
## 104    0.392128890 0.555919439 0.05195167
## 107    0.466744396 0.366388368 0.16686724

```

111 0.409860373 0.525646810 0.06449282
112 0.312976080 0.664782621 0.02224130
117 0.459908972 0.266728075 0.27336295
122 0.118739778 0.075094364 0.80616586
123 0.459908972 0.266728075 0.27336295
127 0.459908972 0.266728075 0.27336295
132 0.459908972 0.266728075 0.27336295
144 0.459908972 0.266728075 0.27336295
145 0.459908972 0.266728075 0.27336295
146 0.459908972 0.266728075 0.27336295
148 0.459908972 0.266728075 0.27336295
152 0.316115257 0.661011075 0.02287367
153 0.313530032 0.663838773 0.02263120
161 0.460471195 0.302415249 0.23711356
166 0.419678178 0.507763011 0.07255881
169 0.450342806 0.436793858 0.11286334
170 0.444348304 0.453371313 0.10228038
196 0.459908972 0.266728075 0.27336295
198 0.379599535 0.574987005 0.04541346
201 0.463419217 0.281047804 0.25553298
205 0.334438408 0.638696533 0.02686506
212 0.395830577 0.549912350 0.05425707
215 0.461937780 0.274530597 0.26353162
225 0.463171030 0.279894781 0.25693419
226 0.459908972 0.266728075 0.27336295
230 0.408505104 0.528356102 0.06313879
231 0.469951460 0.464363434 0.06568511
232 0.373125952 0.582404766 0.04446928
235 0.380895888 0.573491749 0.04561236
240 0.093344075 0.061051866 0.84560406
241 0.459908972 0.266728075 0.27336295
242 0.459908972 0.266728075 0.27336295
243 0.459908972 0.266728075 0.27336295
247 0.459908972 0.266728075 0.27336295
248 0.459908972 0.266728075 0.27336295
269 0.563463602 0.397353523 0.03918288
275 0.430342294 0.486180755 0.08347695
276 0.454851241 0.422639166 0.12250959
280 0.468781448 0.331546612 0.19967194
281 0.459908972 0.266728075 0.27336295
282 0.468761400 0.336508309 0.19473029
287 0.342358647 0.628232978 0.02940838
294 0.461657429 0.273384794 0.26495778
307 0.459908972 0.266728075 0.27336295
310 0.451996037 0.452028906 0.09597506
312 0.009110165 0.018116055 0.97277378
324 0.006307719 0.004930842 0.98876144
333 0.459908972 0.266728075 0.27336295
343 0.340941238 0.630121723 0.02893704
352 0.020159718 0.014511652 0.96532863

353 0.465682986 0.293224563 0.24109245
378 0.474631596 0.268022790 0.25734561
381 0.334438408 0.638696533 0.02686506
383 0.845214169 0.079963098 0.07482273
403 0.442915092 0.457054525 0.10003038
407 0.459908972 0.266728075 0.27336295
408 0.468384904 0.318490789 0.21312431
412 0.468738307 0.327979268 0.20328243
417 0.415175028 0.516274021 0.06855095
428 0.420137167 0.505480717 0.07438212
432 0.353011726 0.620369839 0.02661843
438 0.456509936 0.416913212 0.12657685
446 0.462437014 0.276634638 0.26092835
448 0.466369657 0.369381137 0.16424921
449 0.468359505 0.344249176 0.18739132
455 0.468629637 0.342073823 0.18929654
462 0.104637389 0.066539365 0.82882325
464 0.465835940 0.294196108 0.23996795
471 0.324925757 0.650347344 0.02472690
475 0.442762325 0.457441725 0.09979595
482 0.245003811 0.149478066 0.60551812
485 0.459908972 0.266728075 0.27336295
490 0.459908972 0.266728075 0.27336295
491 0.459908972 0.266728075 0.27336295
498 0.464611156 0.287024061 0.24836478
499 0.466821755 0.301211456 0.23196679
500 0.465494266 0.292059655 0.24244608
501 0.459908972 0.266728075 0.27336295
508 0.344231213 0.625221181 0.03054761
510 0.330074065 0.644060196 0.02586574
519 0.369029144 0.591147418 0.03982344
521 0.332470048 0.641119525 0.02641043
529 0.462392418 0.276443182 0.26116440
549 0.332226552 0.641366621 0.02640683
557 0.459908972 0.266728075 0.27336295
558 0.425631424 0.495991847 0.07837673
559 0.318115439 0.657658601 0.02422596
562 0.357848719 0.607077678 0.03507360
567 0.459908972 0.266728075 0.27336295
577 0.373448821 0.581944419 0.04460676
580 0.465851429 0.373171857 0.16097671
582 0.368493368 0.590023140 0.04148349
585 0.459908972 0.266728075 0.27336295
588 0.340799363 0.629649126 0.02955151
595 0.410532817 0.526740003 0.06272718
599 0.459908972 0.266728075 0.27336295
600 0.451420811 0.433434875 0.11514431
601 0.450804789 0.435421910 0.11377330
602 0.003031537 0.003090977 0.99387749
605 0.459908972 0.266728075 0.27336295

608 0.459908972 0.266728075 0.27336295
611 0.374061361 0.582986310 0.04295233
612 0.443477955 0.258329144 0.29819290
614 0.449268286 0.439926079 0.11080563
624 0.459908972 0.266728075 0.27336295
625 0.438932665 0.459243386 0.10182395
626 0.460325631 0.268245431 0.27142894
627 0.459908972 0.266728075 0.27336295
628 0.420393750 0.505419464 0.07418679
630 0.432469839 0.481351348 0.08617881
631 0.434685546 0.474214125 0.09110033
632 0.466989957 0.302579482 0.23043056
635 0.420185053 0.560151573 0.01966337
636 0.441061937 0.460753598 0.09818447
637 0.459908972 0.266728075 0.27336295
640 0.463387966 0.387229593 0.14938244
641 0.001774896 0.001313338 0.99691177
645 0.459908972 0.266728075 0.27336295
647 0.459908972 0.266728075 0.27336295
648 0.462568995 0.389125751 0.14830525
651 0.459908972 0.266728075 0.27336295
654 0.381234102 0.572975773 0.04579012
657 0.398307342 0.545626102 0.05606656
659 0.461323180 0.272049710 0.26662711
663 0.459908972 0.266728075 0.27336295
664 0.740171370 0.183083707 0.07674492
667 0.459908972 0.266728075 0.27336295
668 0.468468044 0.320069496 0.21146246
672 0.013443653 0.009827945 0.97672840
673 0.383482221 0.569526499 0.04699128
677 0.458316997 0.409703996 0.13197901
678 0.459908972 0.266728075 0.27336295
683 0.332147317 0.641516177 0.02633651
693 0.459908972 0.266728075 0.27336295
701 0.459908972 0.266728075 0.27336295
705 0.529101655 0.437299402 0.03359894
715 0.441527341 0.460535773 0.09793689
717 0.467300694 0.361400894 0.17129841
719 0.395906812 0.548927126 0.05516606
721 0.316464612 0.660590456 0.02294493
731 0.377613734 0.578460360 0.04392591
733 0.462874307 0.278551148 0.25857455
736 0.417918553 0.508546343 0.07353510
751 0.360423247 0.504988927 0.13458783
752 0.459908972 0.266728075 0.27336295
755 0.413647801 0.516127896 0.07022430
770 0.459908972 0.266728075 0.27336295
775 0.422261079 0.502734136 0.07500479
776 0.333592372 0.639738776 0.02666885
779 0.465031945 0.289345644 0.24562241

789 0.443072638 0.449152622 0.10777474
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## 1874 0.361285739 0.601364538 0.03734972
## 1877 0.464704677 0.380551777 0.15474355
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## 1881 0.010959557 0.007596968 0.98144348
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## 1999 0.036437410 0.035354592 0.92820800
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##          LD1          LD2
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## 26	0.289558127	-0.769197689
## 28	0.410058518	-0.347965362
## 30	-1.441852904	1.193844022
## 33	3.528717757	3.748178641
## 37	0.289558127	-0.769197689
## 49	-0.249181188	-6.630510434
## 52	3.994063307	3.234696211
## 54	-0.064386710	-0.368444123
## 57	0.289558127	-0.769197689
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## 69	-0.994110749	0.697504170
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## 89	-1.607044009	1.397389587
## 95	-1.251816080	0.976111147
## 104	-1.103925484	0.808572255
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## 215	0.251469575	-0.726071977

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1880 -0.090398404 -0.338992417
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1925 -0.231604744 -0.179111730
1928 -0.316142749 -0.083393686
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1933 0.289558127 -0.769197689
1934 3.365639656 1.354576805
1938 0.289558127 -0.769197689
1939 0.289558127 -0.769197689
1942 0.289558127 -0.769197689
1943 3.470384094 3.829340588

```

## 1944  1.689630587  2.739649433
## 1947  0.289558127 -0.769197689
## 1951 -1.514206566  1.283548134
## 1952 -0.216740918 -0.195941276
## 1959  0.289558127 -0.769197689
## 1964  0.289558127 -0.769197689
## 1965  4.289670969 -3.946933941
## 1968  0.280268236 -0.758679223
## 1969 -0.300659086 -0.050995738
## 1973  0.289558127 -0.769197689
## 1974  0.289558127 -0.769197689
## 1978  0.106547279 -0.561983903
## 1979  0.289558127 -0.769197689
## 1983  2.811982771  2.447993964
## 1984  2.764726348  0.835870706
## 1987  0.289558127 -0.769197689
## 1988  0.317136603 -0.751313919
## 1989  3.888549245  1.564634351
## 1993 -0.066289787 -0.280347807
## 1995 -1.573950686  1.343501521
## 1999  2.655275355  1.762998073
## 2000 -0.975680346  0.664266498
## 2003 -0.252042503 -0.155971104

mylDA2 <- lda(formula = V1 ~.,data=test)
mylDA2

## Call:
## lda(V1 ~ ., data = test)
##
## Prior probabilities of groups:
##      2      3      5
## 0.3778626 0.3167939 0.3053435
##
## Group means:
##      V3      V5      V7      V15
## 2 -0.9503838 -0.8052374 -0.36917172 -0.9898788
## 3 -0.9611627 -0.6965482 -0.03910241 -0.9705301
## 5 -0.9868750 -0.9583687 -0.76425000 -0.5616562
##
## Coefficients of linear discriminants:
##      LD1      LD2
## V3  1.1541120  1.6637425
## V5  0.1541224 -0.3413991
## V7  0.9146936 -0.9803365
## V15 -2.0420320 -1.4628063
##
## Proportion of trace:
##      LD1      LD2
## 0.9377 0.0623

```

```
summary(my1DA2)
```

```
##           Length Class  Mode
## prior      3      -none- numeric
## counts     3      -none- numeric
## means     12      -none- numeric
## scaling    8      -none- numeric
## lev        3      -none- character
## svd         2      -none- numeric
## N           1      -none- numeric
## call       3      -none- call
## terms      3      terms  call
## xlevels    0      -none- list
```

2.:Apply the kNN method to the entire data set of the Home Equity Loan data. Find the best value of k using the cross validation error.

```
data = read.csv("hmeq.csv",stringsAsFactors = FALSE)
```

```
# Convert the variable 'BAD' from integer to factor
```

```
data$BAD <- as.factor(data$BAD)
```

```
# Convert integer variables to numeric variables
```

```
data$LOAN <- as.numeric(data$LOAN)
```

```
data$NINQ <- as.numeric(data$NINQ)
```

```
data$CLNO <- as.numeric(data$CLNO)
```

```
data$DEROG <- as.numeric(data$DEROG)
```

```
data$DELINQ <- as.numeric(data$DELINQ)
```

```
str(data)
```

```
## 'data.frame':   5960 obs. of  13 variables:
## $ BAD      : Factor w/ 2 levels "0","1": 2 2 2 2 1 2 2 2 2 2 ...
## $ LOAN      : num  1100 1300 1500 1500 1700 1700 1800 1800 2000 2000 ...
## $ MORTDUE   : num  25860 70053 13500 NA 97800 ...
## $ VALUE     : num  39025 68400 16700 NA 112000 ...
## $ REASON    : chr   "HomeImp" "HomeImp" "HomeImp" "" ...
## $ JOB       : chr   "Other" "Other" "Other" "" ...
## $ YOJ       : num  10.5 7 4 NA 3 9 5 11 3 16 ...
## $ DEROG     : num  0 0 0 NA 0 0 3 0 0 0 ...
## $ DELINQ    : num  0 2 0 NA 0 0 2 0 2 0 ...
## $ CLAGE     : num  94.4 121.8 149.5 NA 93.3 ...
## $ NINQ      : num  1 0 1 NA 0 1 1 0 1 0 ...
## $ CLNO      : num  9 14 10 NA 14 8 17 8 12 13 ...
## $ DEBTINC   : num  NA NA NA NA NA ...
```

```
summary(data)
```

```
##  BAD           LOAN           MORTDUE           VALUE           REASON
##  0:4771   Min.    : 1100     Min.    : 2063   Min.    : 8000   Length:5960
##  1:1189   1st Qu.:11100   1st Qu.: 46276   1st Qu.: 66076   Class
:character
##                Median :16300   Median : 65019   Median : 89236   Mode
```

```

:character
##           Mean    :18608    Mean    : 73761    Mean    :101776
##           3rd Qu.:23300    3rd Qu.: 91488    3rd Qu.:119824
##           Max.    :89900    Max.    :399550    Max.    :855909
##           NA's    :518      NA's    :112
##           JOB           YOJ           DEROG           DELINQ
## Length:5960           Min.    : 0.000    Min.    : 0.0000    Min.    : 0.0000
## Class :character      1st Qu.: 3.000    1st Qu.: 0.0000    1st Qu.: 0.0000
## Mode  :character      Median : 7.000    Median : 0.0000    Median : 0.0000
##                               Mean    : 8.922    Mean    : 0.2546    Mean    : 0.4494
##                               3rd Qu.:13.000    3rd Qu.: 0.0000    3rd Qu.: 0.0000
##                               Max.    :41.000    Max.    :10.0000    Max.    :15.0000
##                               NA's    :515      NA's    :708      NA's    :580
##           CLAGE           NINQ           CLNO           DEBTINC
## Min.    : 0.0    Min.    : 0.000    Min.    : 0.0    Min.    : 0.5245
## 1st Qu.:115.1    1st Qu.: 0.000    1st Qu.:15.0    1st Qu.:29.1400
## Median :173.5    Median : 1.000    Median :20.0    Median :34.8183
## Mean    :179.8    Mean    : 1.186    Mean    :21.3    Mean    :33.7799
## 3rd Qu.:231.6    3rd Qu.: 2.000    3rd Qu.:26.0    3rd Qu.:39.0031
## Max.    :1168.2    Max.    :17.000    Max.    :71.0    Max.    :203.3121
## NA's    :308      NA's    :510      NA's    :222      NA's    :1267

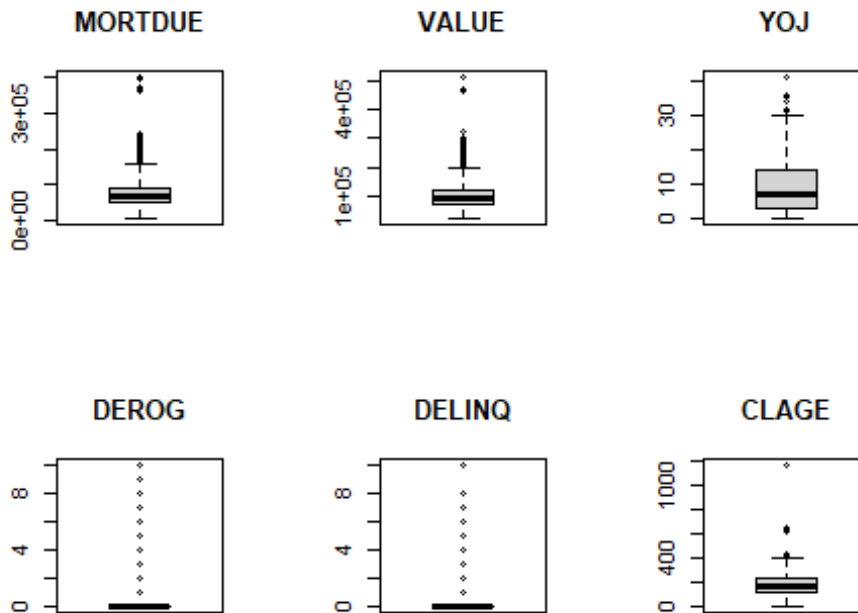
data<-na.omit(data)
dim(data)

## [1] 3515    13

par(mfrow = c(2,3))

boxplot(data$MORTDUE, main = "MORTDUE")
boxplot(data$VALUE, main = "VALUE")
boxplot(data$YOJ, main = "YOJ")
boxplot(data$DEROG, main = "DEROG")
boxplot(data$DELINQ, main = "DELINQ")
boxplot(data$CLAGE, main = "CLAGE")

```

```
boxplot(data$NINQ, main = "NINQ")
boxplot(data$CLNO, main = "CLNO")
boxplot(data$DEBTINC, main = "DEBTINC")
```

#remove the outliers

```
outliers_removal <- function(a){
  df <- a
  aa<-c()
  count<-1
  for(i in 1:ncol(df)){
    if(is.numeric(df[,i])){
      Q3 <- quantile(df[,i], 0.75, na.rm = TRUE)
      Q1 <- quantile(df[,i], 0.25, na.rm = TRUE)
      IQR <- Q3 - Q1
      upper <- Q3 + 1.5 * IQR
      lower <- Q1 - 1.5 * IQR
      for(j in 1:nrow(df)){
        if(is.na(df[j,i]) == TRUE){
          next
        }
        else if(df[j,i] > upper | df[j,i] < lower){
          aa[count]<-j
          count<-count+1
        }
      }
    }
  }
}
```

```

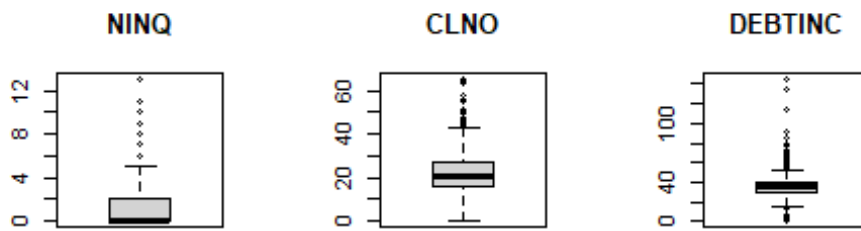
    }
    df<-df[-aa,]
  }

data <- outliers_removal(data)
dim(data)

## [1] 2302   13

par(mfrow = c(2,2))

```



```

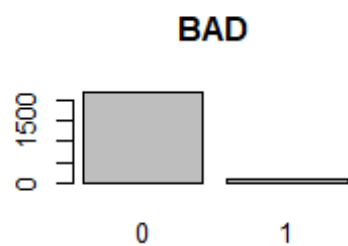
barplot(table(data$REASON), main = "REASON")
barplot(table(data$JOB), main = "JOB")
barplot(table(data$BAD), main = "BAD")

```

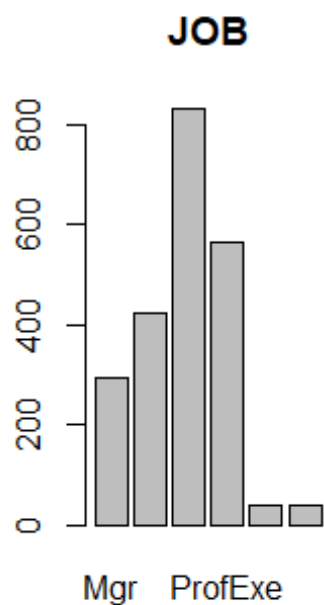
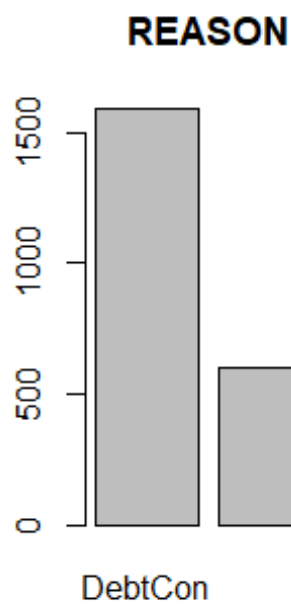
```

#correct the errors
data <- data[!(data$REASON == ""),]
data <- data[!(data$JOB == ""),]
par(mfrow = c(1,2))

```



```
barplot(table(data$REASON), main = "REASON")
barplot(table(data$JOB), main = "JOB")
```



```

dim(data)

## [1] 2184 13

cor(data[c("LOAN", "MORTDUE", "VALUE", "YOJ", "DEROG", "DELINQ", "CLAGE",
"NING", "CLNO", "DEBTINC")])

## Warning in stats::cor(x, y, ...): the standard deviation is zero

##           LOAN      MORTDUE      VALUE      YOJ  DEROG  DELINQ
## LOAN      1.000000000  0.17852824  0.34809670  0.077232083    NA    NA
## MORTDUE    0.178528245  1.00000000  0.84001285 -0.070994141    NA    NA
## VALUE      0.348096704  0.84001285  1.00000000  0.024763530    NA    NA
## YOJ         0.077232083 -0.07099414  0.02476353  1.000000000    NA    NA
## DEROG              NA              NA              NA          1    NA
## DELINQ              NA              NA              NA          NA    1
## CLAGE      0.067864250  0.07690745  0.20400150  0.223298794    NA    NA
## NING      0.005072811  0.06095620  0.03552415 -0.023927232    NA    NA
## CLNO      0.097396871  0.30790517  0.29373600  0.004381988    NA    NA
## DEBTINC    0.156149422  0.23864832  0.18322318 -0.035695438    NA    NA
##           CLAGE      NING      CLNO      DEBTINC
## LOAN      0.06786425  0.00507281  0.09739687  0.15614942
## MORTDUE    0.07690745  0.06095620  0.30790517  0.23864832
## VALUE      0.20400150  0.03552415  0.29373599  0.18322318
## YOJ         0.22329879 -0.02392723  0.00438198 -0.03569544
## DEROG              NA              NA              NA          NA
## DELINQ              NA              NA              NA          NA
## CLAGE      1.00000000 -0.03480325  0.19747514  0.04023405
## NING      -0.03480326  1.00000000  0.14720475  0.24175019
## CLNO      0.19747514  0.14720475  1.00000000  0.16301041
## DEBTINC    0.04023405  0.24175019  0.16301041  1.00000000

data$DEROG <- NULL
data$DELINQ <- NULL
# multicollinearity is present between variables MORTDUE and VALUE
data$MORTDUE <- NULL # delete MORTDUE
dim(data)

## [1] 2184 10

# The training data is created
input_ones <- data[which(data$BAD == 1), ] #all 1's
input_zeros <- data[which(data$BAD == 0), ] # all 0's
set.seed(100) # for repeatability of sample
input_ones_training_rows <- sample(1:nrow(input_ones), 0.7 *
nrow(input_ones)) #1's for training
input_zeros_training_rows <- sample(1:nrow(input_zeros), 0.7 *
nrow(input_zeros)) #0's for training
#pick as many as 0's and 1's
training_ones <- input_ones[input_ones_training_rows, ]
training_zeros <- input_zeros[input_zeros_training_rows, ]

```

```

# We row bind the 0's and 1's for training data
trainingData <- rbind(training_ones, training_zeros)

# The test data is created
test_ones <- input_ones[-input_ones_training_rows, ]
test_zeros <- input_zeros[-input_zeros_training_rows, ]
# We row bind the 0's and 1's for test data
testData <- rbind(test_ones, test_zeros)

table(trainingData$BAD)

##
##      0      1
## 1456    72

prop.table(table(trainingData$BAD))

##
##              0              1
## 0.95287958 0.04712042

dim(trainingData)

## [1] 1528    10

set.seed(111)
trControl <- trainControl(method = "cv",
                           number = 5)
s = function(seeds_list,k){
  seeds_list = lapply(seeds_list,"[,1:k)
  seeds_list[[length(seeds_list)+1]] = 999
  seeds_list
}

k = 1
model0 <- train(BAD~.,data=data,method="knn",
                 trControl = trainControl(method = 'LOOCV'),tuneGrid =
expand.grid(k = 1:k))
model0

## k-Nearest Neighbors
##
## 2184 samples
##      9 predictor
##      2 classes: '0', '1'
##
## No pre-processing
## Resampling: Leave-One-Out Cross-Validation
## Summary of sample sizes: 2183, 2183, 2183, 2183, 2183, 2183, ...
## Resampling results:
##

```

```

## Accuracy Kappa
## 0.9285714 0.1826347
##
## Tuning parameter 'k' was held constant at a value of 1

k = 5

model <- train(BAD~.,data=data,method="knn",
               trControl = trainControl(method = 'LOOCV'),tuneGrid =
expand.grid(k = 1:k))
model

## k-Nearest Neighbors
##
## 2184 samples
## 9 predictor
## 2 classes: '0', '1'
##
## No pre-processing
## Resampling: Leave-One-Out Cross-Validation
## Summary of sample sizes: 2183, 2183, 2183, 2183, 2183, 2183, ...
## Resampling results across tuning parameters:
##
## k Accuracy Kappa
## 1 0.9285714 0.18263473
## 2 0.9299451 0.15425112
## 3 0.9491758 0.17075774
## 4 0.9523810 0.12218650
## 5 0.9510073 0.06101128
##
## Accuracy was used to select the optimal model using the largest value.
## The final value used for the model was k = 4.

k = 10

model1 <- train(BAD~.,data=data,method="knn",
                trControl = trainControl(method = 'LOOCV'),tuneGrid =
expand.grid(k = 1:k))
model1

## k-Nearest Neighbors
##
## 2184 samples
## 9 predictor
## 2 classes: '0', '1'
##
## No pre-processing
## Resampling: Leave-One-Out Cross-Validation
## Summary of sample sizes: 2183, 2183, 2183, 2183, 2183, 2183, ...
## Resampling results across tuning parameters:
##

```

```
## k Accuracy Kappa
## 1 0.9285714 0.18263473
## 2 0.9299451 0.16262705
## 3 0.9491758 0.17075774
## 4 0.9514652 0.13182527
## 5 0.9510073 0.06101128
## 6 0.9523810 0.06506849
## 7 0.9519231 0.06369427
## 8 0.9523810 0.04960836
## 9 0.9528388 0.03480589
## 10 0.9528388 0.03480589
##
## Accuracy was used to select the optimal model using the largest value.
## The final value used for the model was k = 10.

k = 20

model2 <- train(BAD~.,data=data,method="knn",
                trControl = trainControl(method = 'LOOCV'),tuneGrid =
expand.grid(k = 1:k))
model2

## k-Nearest Neighbors
##
## 2184 samples
## 9 predictor
## 2 classes: '0', '1'
##
## No pre-processing
## Resampling: Leave-One-Out Cross-Validation
## Summary of sample sizes: 2183, 2183, 2183, 2183, 2183, 2183, ...
## Resampling results across tuning parameters:
##
## k Accuracy Kappa
## 1 0.9285714 0.18263473
## 2 0.9331502 0.18065206
## 3 0.9491758 0.17075774
## 4 0.9482601 0.10755923
## 5 0.9510073 0.06101128
## 6 0.9505495 0.05970149
## 7 0.9519231 0.06369427
## 8 0.9523810 0.04960836
## 9 0.9528388 0.03480589
## 10 0.9523810 0.03362832
## 11 0.9532967 0.03600360
## 12 0.9528388 0.01815706
## 13 0.9532967 0.03600360
## 14 0.9528388 0.01815706
## 15 0.9523810 0.00000000
## 16 0.9528388 0.01815706
```

```
## 17 0.9523810 0.00000000
## 18 0.9523810 0.00000000
## 19 0.9523810 0.00000000
## 20 0.9523810 0.00000000
##
## Accuracy was used to select the optimal model using the largest value.
## The final value used for the model was k = 13.

# The optimal value for k is 13, which gives the highest accuracy in
predicting the defaulters
```

3. : Split randomly the Home Equity Loan data into 3 parts (50% training, 25% validating, and 25% testing). Repeat this 3 times and compare the training, validating, and testing error of (i) the logistic regression model, (ii) LDA, and (iii) the tree model. Compare the results.

```
data1 = read.csv("hmeq.csv")
# The variable 'BAD' is converted from integer to factor
data1$BAD <- as.factor(data1$BAD)
# Integer variables are converted to numeric variables
data1$LOAN <- as.numeric(data1$LOAN)
data1$NINQ <- as.numeric(data1$NINQ)
data1$CLNO <- as.numeric(data1$CLNO)
data1$DEROG <- as.numeric(data1$DEROG)
data1$DELINQ <- as.numeric(data1$DELINQ)
str(data1)

## 'data.frame': 5960 obs. of 13 variables:
## $ BAD : Factor w/ 2 levels "0","1": 2 2 2 2 1 2 2 2 2 2 ...
## $ LOAN : num 1100 1300 1500 1500 1700 1700 1800 1800 2000 2000 ...
## $ MORTDUE: num 25860 70053 13500 NA 97800 ...
## $ VALUE : num 39025 68400 16700 NA 112000 ...
## $ REASON : chr "HomeImp" "HomeImp" "HomeImp" "" ...
## $ JOB : chr "Other" "Other" "Other" "" ...
## $ YOJ : num 10.5 7 4 NA 3 9 5 11 3 16 ...
## $ DEROG : num 0 0 0 NA 0 0 3 0 0 0 ...
## $ DELINQ : num 0 2 0 NA 0 0 2 0 2 0 ...
## $ CLAGE : num 94.4 121.8 149.5 NA 93.3 ...
## $ NINQ : num 1 0 1 NA 0 1 1 0 1 0 ...
## $ CLNO : num 9 14 10 NA 14 8 17 8 12 13 ...
## $ DEBTINC: num NA NA NA NA NA ...

summary(data1)

## BAD LOAN MORTDUE VALUE REASON
## 0:4771 Min. : 1100 Min. : 2063 Min. : 8000 Length:5960
## 1:1189 1st Qu.:11100 1st Qu.: 46276 1st Qu.: 66076 Class
```



```

:character
##           Median :16300   Median : 65019   Median : 89236   Mode
:character
##           Mean    :18608   Mean     : 73761   Mean     :101776
##           3rd Qu.:23300   3rd Qu.: 91488   3rd Qu.:119824
##           Max.    :89900   Max.     :399550   Max.     :855909
##           NA's    :518     NA's      :112
##           JOB      YOJ      DEROG      DELINQ
## Length:5960      Min.    : 0.000   Min.    : 0.0000   Min.    : 0.0000
## Class :character 1st Qu.: 3.000   1st Qu.: 0.0000   1st Qu.: 0.0000
## Mode  :character Median : 7.000   Median : 0.0000   Median : 0.0000
##           Mean    : 8.922   Mean     : 0.2546   Mean     : 0.4494
##           3rd Qu.:13.000   3rd Qu.: 0.0000   3rd Qu.: 0.0000
##           Max.    :41.000   Max.     :10.0000   Max.     :15.0000
##           NA's    :515     NA's      :708     NA's      :580
##           CLAGE     NINQ      CLNO      DEBTINC
## Min.    : 0.0   Min.    : 0.000   Min.    : 0.0   Min.    : 0.5245
## 1st Qu.:115.1   1st Qu.: 0.000   1st Qu.:15.0   1st Qu.: 29.1400
## Median :173.5   Median : 1.000   Median :20.0   Median : 34.8183
## Mean    :179.8   Mean     : 1.186   Mean     :21.3   Mean     : 33.7799
## 3rd Qu.:231.6   3rd Qu.: 2.000   3rd Qu.:26.0   3rd Qu.: 39.0031
## Max.    :1168.2   Max.     :17.000   Max.     :71.0   Max.     :203.3121
## NA's    :308     NA's      :510     NA's      :222     NA's      :1267

```

```

data1<-na.omit(data1)
dim(data1)

```

```

## [1] 3515   13

```

#The outliers are removed

```

outliers_removal <- function(a){
  df <- a
  aa<-c()
  count<-1
  for(i in 1:ncol(df)){
    if(is.numeric(df[,i])){
      Q3 <- quantile(df[,i], 0.75, na.rm = TRUE)
      Q1 <- quantile(df[,i], 0.25, na.rm = TRUE)
      IQR <- Q3 - Q1
      upper <- Q3 + 1.5 * IQR
      lower <- Q1 - 1.5 * IQR
      for(j in 1:nrow(df)){
        if(is.na(df[j,i]) == TRUE){
          next
        }
        else if(df[j,i] > upper | df[j,i] < lower){
          aa[count]<-j
          count<-count+1
        }
      }
    }
  }
}

```

```

    }
  }
  df<-df[-aa,]
}

data1 <- outliers_remover(data1)
dim(data1)

## [1] 2302    13

par(mfrow = c(2,2))
barplot(table(data1$REASON), main = "REASON")
barplot(table(data1$JOB), main = "JOB")
barplot(table(data1$BAD), main = "BAD")

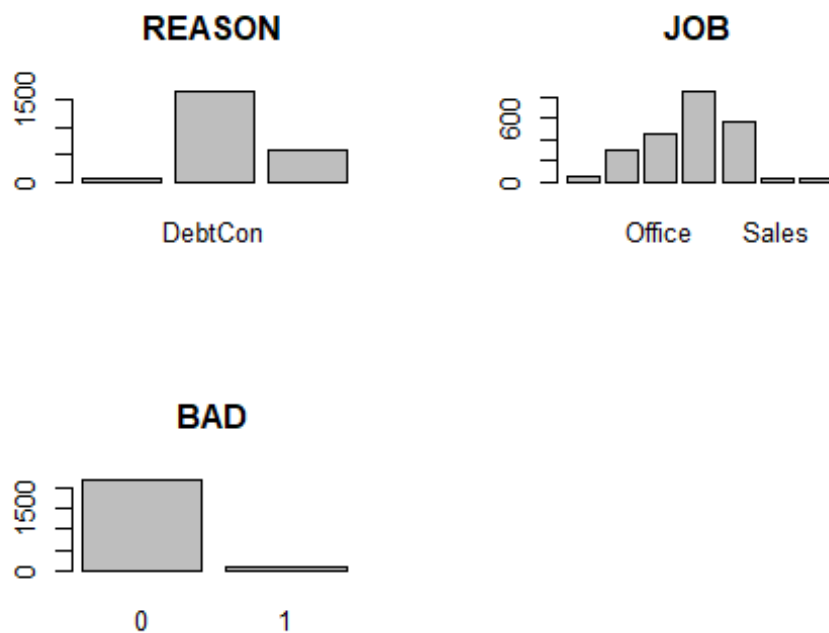
```

#The errors are corrected

```

data1 <- data1[!(data1$REASON == ""),]
data1 <- data1[!(data1$JOB == ""),]
par(mfrow = c(1,2))

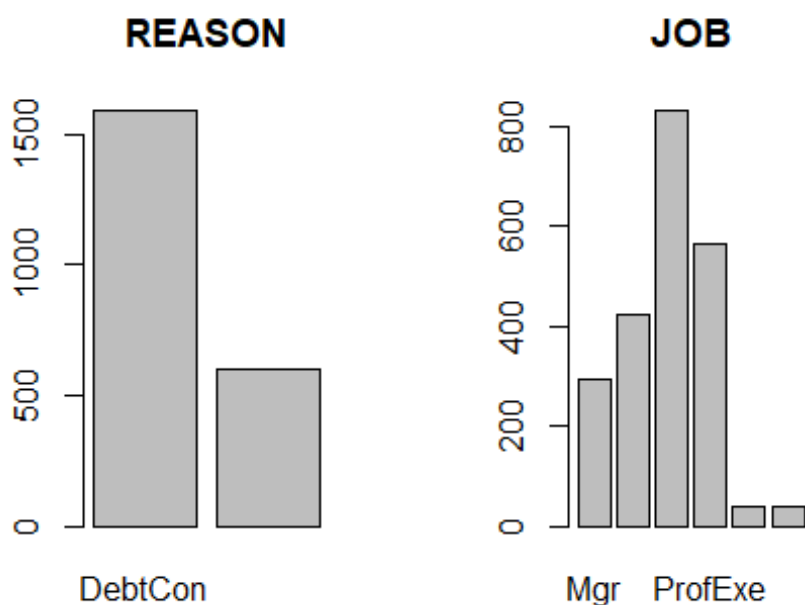
```



```

barplot(table(data1$REASON), main = "REASON")
barplot(table(data1$JOB), main = "JOB")

```



```
dim(data1)
## [1] 2184 13

#Correlation Matrix is created
cor(data1[c("LOAN", "MORTDUE", "VALUE", "YOJ", "DEROG", "DELINQ", "CLAGE",
"NING", "CLNO", "DEBTINC")])

## Warning in stats::cor(x, y, ...): the standard deviation is zero

##          LOAN      MORTDUE      VALUE      Yoj  DEROG  DELINQ
## LOAN      1.000000000  0.17852824  0.34809670  0.077232083    NA     NA
## MORTDUE    0.178528245  1.00000000  0.84001285 -0.070994141    NA     NA
## VALUE      0.348096704  0.84001285  1.00000000  0.024763530    NA     NA
## Yoj        0.077232083 -0.07099414  0.02476353  1.000000000    NA     NA
## DEROG       NA         NA         NA         NA         1     NA
## DELINQ      NA         NA         NA         NA         NA     1
## CLAGE      0.067864250  0.07690745  0.20400150  0.223298794    NA     NA
## NING        0.005072811  0.06095620  0.03552415 -0.023927232    NA     NA
## CLNO        0.097396871  0.30790517  0.29373600  0.004381988    NA     NA
## DEBTINC     0.156149422  0.23864832  0.18322318 -0.035695438    NA     NA
##          CLAGE      NING      CLNO      DEBTINC
## LOAN      0.06786425  0.005072811  0.097396871  0.15614942
## MORTDUE    0.07690745  0.060956203  0.307905172  0.23864832
## VALUE      0.20400150  0.035524149  0.293735998  0.18322318
## Yoj        0.22329879 -0.023927232  0.004381988 -0.03569544
## DEROG       NA         NA         NA         NA
```

```

## DELINQ          NA          NA          NA          NA
## CLAGE    1.00000000 -0.034803256 0.197475140 0.04023405
## NINQ     -0.03480326 1.000000000 0.147204757 0.24175019
## CLNO      0.19747514 0.147204757 1.000000000 0.16301041
## DEBTINC    0.04023405 0.241750193 0.163010411 1.00000000

data1$DEROG <- NULL
data1$DELINQ <- NULL
# there is multicollinearity between MORTDUE and VALUE
data1$MORTDUE <- NULL
dim(data1)

## [1] 2184    10

# The training set and validation set data is created
input_ones <- data1[which(data1$BAD == 1), ] #all 1's
input_zeros <- data1[which(data1$BAD == 0), ] # all 0's
set.seed(100) #This is for the repeatability of sample
input_ones_training_rows <- sample(1:nrow(input_ones), 0.5 *
nrow(input_ones)) #1's for training
input_zeros_training_rows <- sample(1:nrow(input_zeros), 0.5 *
nrow(input_zeros)) #0's for training
input_ones_validation_rows <- sample(1:nrow(input_ones), 0.25 *
nrow(input_ones)) #1's for validation
input_zeros_validation_rows <- sample(1:nrow(input_zeros), 0.25 *
nrow(input_zeros)) #0's for validation
#pick as many as 0's and 1's
training_ones <- input_ones[input_ones_training_rows, ]
training_zeros <- input_zeros[input_zeros_training_rows, ]
validation_ones <- input_ones[input_ones_validation_rows, ]
validation_zeros <- input_zeros[input_zeros_validation_rows, ]
# We row bind the 0's and 1's
trainingData <- rbind(training_ones, training_zeros)
validationData <- rbind(validation_ones, validation_zeros)

#We create the test set data
test_ones <- input_ones[-input_ones_training_rows, ]
test_zeros <- input_zeros[-input_zeros_training_rows, ]
#1's and 0's and row binded
testData <- rbind(test_ones, test_zeros)

table(trainingData$BAD)

##
##      0      1
## 1040    52

prop.table(table(trainingData$BAD))

```

```
##
##           0           1
## 0.95238095 0.04761905

treeMod <- rpart(BAD ~., data = trainingData)
pred_treeMod <- predict(treeMod, newdata = testData)

accuracy.meas(testData$BAD, pred_treeMod[,2])

##
## Call:
## accuracy.meas(response = testData$BAD, predicted = pred_treeMod[,
##      2])
##
## Examples are labelled as positive when predicted is greater than 0.5
##
## precision: 0.281
## recall: 0.173
## F: 0.107

#We re-sample the data set

data_balanced_over <- ovun.sample(BAD ~., data = trainingData, method =
"over", N = 2912)$data
table(data_balanced_over$BAD)

##
##      0      1
## 1040 1872

data_balanced_both <- ovun.sample(BAD ~., data = trainingData, method =
"both", p = 0.5, N = 1528)$data
table(data_balanced_both$BAD)

##
##      0      1
## 770 758

#Decision Tree Models are built
tree.over <- rpart(BAD ~., data = data_balanced_over)
tree.both <- rpart(BAD ~., data = data_balanced_both)

# We make predictions on test data
pred_tree.over <- predict(tree.over, newdata = testData)
pred_tree.both <- predict(tree.both, newdata = testData)

#ROC Curves are created to show error rates for Decision Tree Algorithm
par(mfrow = c(2,2))
```

```

roc.curve(testData$BAD, pred_tree.over[,2], col = "BLACK", main = "ROC curve
of oversampling")

## Area under the curve (AUC): 0.686

roc.curve(testData$BAD, pred_tree.both[,2], col = "RED", main = "ROC curve of
balanced sampling")

## Area under the curve (AUC): 0.686

# We conduct a logistic regression on the training data
logisticModel <- glm(BAD ~., data = trainingData, family = binomial(link =
"logit"))
pred_logit <- predict(logisticModel, testData)
summary(logisticModel)

##
## Call:
## glm(formula = BAD ~ ., family = binomial(link = "logit"), data =
trainingData)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -1.1989  -0.2982  -0.1812  -0.1182   3.3995
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  -3.715e+00  1.293e+00  -2.872  0.00407 **
## LOAN          -5.057e-05  2.467e-05  -2.050  0.04040 *
## VALUE         -3.340e-06  5.577e-06  -0.599  0.54928
## REASONHomeImp -3.206e-01  3.795e-01  -0.845  0.39819
## JOBOffice     -1.465e+00  7.252e-01  -2.021  0.04331 *
## JOBOther      -2.109e-01  4.754e-01  -0.444  0.65734
## JOBProfExe    -5.771e-01  5.759e-01  -1.002  0.31632
## JOBSales       1.804e+00  7.913e-01  2.280  0.02262 *
## JOBSelf        1.403e+00  9.687e-01  1.448  0.14748
## YOJ           -2.701e-02  2.812e-02  -0.961  0.33678
## CLAGE         -1.155e-02  2.780e-03  -4.154  3.27e-05 ***
## NINQ           1.347e-01  1.391e-01  0.968  0.33304
## CLNO          -2.303e-02  2.046e-02  -1.125  0.26050
## DEBTINC        1.261e-01  3.218e-02  3.918  8.92e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 418.11  on 1091  degrees of freedom
## Residual deviance: 334.96  on 1078  degrees of freedom
## AIC: 362.96

```

```
##
## Number of Fisher Scoring iterations: 7

summary(pred_logit)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -7.210  -4.650  -3.801  -3.754  -2.722   1.403

#Doing a Logistic Regression on the validation set to fine tune the model
mylogit1 <- glm(BAD ~., data=validationData , family =binomial(link =
"logit"))
pred_logit1 <- predict(mylogit1, testData)
summary(mylogit1)

##
## Call:
## glm(formula = BAD ~ ., family = binomial(link = "logit"), data =
validationData)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -1.3731  -0.2875  -0.1570  -0.0769   3.5393
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  -2.351e-01  1.714e+00  -0.137  0.89091
## LOAN         -1.196e-04  4.182e-05  -2.859  0.00425 **
## VALUE        -1.738e-05  9.448e-06  -1.839  0.06589 .
## REASONHomeImp -6.243e-01  5.359e-01  -1.165  0.24398
## JOBOffice     6.312e-01  9.198e-01   0.686  0.49253
## JOBOther      6.130e-01  8.263e-01   0.742  0.45820
## JOBProfExe    6.756e-01  9.557e-01   0.707  0.47961
## JOBSales      3.582e+00  1.503e+00   2.382  0.01720 *
## JOBSelf       3.295e+00  1.506e+00   2.188  0.02865 *
## YOJ          -1.850e-02  4.046e-02  -0.457  0.64754
## CLAGE        -1.375e-02  4.233e-03  -3.248  0.00116 **
## NINQ         3.947e-01  1.740e-01   2.269  0.02326 *
## CLNO        -4.557e-02  3.112e-02  -1.465  0.14304
## DEBTINC      7.374e-02  4.038e-02   1.826  0.06784 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 209.06  on 545  degrees of freedom
## Residual deviance: 157.79  on 532  degrees of freedom
## AIC: 185.79
##
## Number of Fisher Scoring iterations: 7

summary(pred_logit1)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -10.197  -5.348  -4.016  -4.124  -2.840   2.466

# We Look at the VIF to check for multicollinearity in the regression model
vif(mylogit1)

##              GVIF Df GVIF^(1/(2*Df))
## LOAN      1.141095  1      1.068221
## VALUE     1.300624  1      1.140449
## REASON    1.207600  1      1.098909
## JOB       1.766971  5      1.058578
## YOJ       1.123303  1      1.059860
## CLAGE     1.383429  1      1.176193
## NINQ      1.206555  1      1.098433
## CLNO      1.224231  1      1.106450
## DEBTINC   1.230260  1      1.109171

#Doing a Logistic Regression on the test set after fine tuning the regression
model
mylogit2 <- glm(BAD ~ NINQ +VALUE +DEBTINC +CLAGE, data=testData, family =
binomial(link = "logit"))
pred_logit2 <- predict(mylogit2, testData)
summary(pred_logit2)

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -6.8241 -4.1406 -3.3808 -3.4603 -2.6921 -0.9078

vif(mylogit2)

##      NINQ      VALUE  DEBTINC      CLAGE
## 1.054234 1.072927 1.102163 1.025191

summary(mylogit2)

##
## Call:
## glm(formula = BAD ~ NINQ + VALUE + DEBTINC + CLAGE, family = binomial(link
= "logit"),
##      data = testData)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -0.7443  -0.3432  -0.2486  -0.1564   3.1113
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -3.442e+00  1.047e+00  -3.286 0.001016 **
## NINQ         6.422e-02  1.222e-01   0.526 0.599079
## VALUE       -2.071e-05  5.669e-06  -3.653 0.000259 ***
## DEBTINC      9.674e-02  2.872e-02   3.368 0.000757 ***
## CLAGE       -7.705e-03  2.322e-03  -3.318 0.000905 ***
## ---
```



```

## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 418.11  on 1091  degrees of freedom
## Residual deviance: 375.18  on 1087  degrees of freedom
## AIC: 385.18
##
## Number of Fisher Scoring iterations: 7

# Returns the cutoff that gives minimum misclassification error.
cutoff <- optimalCutoff(testData$BAD, pred_logit)[1]
cutoff

## [1] 0.1628139

# Calculating Error Rate for Logistic Regression
misClassError(testData$BAD, pred_logit2, threshold = cutoff)

## [1] 0.0476

misClassError(testData$BAD, pred_logit2, threshold = 0.5)

## [1] 0.0476

confusionMatrix(testData$BAD, pred_logit, cutoff)

##      0  1
## 0 1040 50
## 1    0  2

specificity(testData$BAD, pred_logit2, cutoff)

## [1] 1

sensitivity(testData$BAD, pred_logit2, cutoff)

## [1] 0

accuracy.meas(testData$BAD, pred_logit2, cutoff)

##
## Call:
## accuracy.meas(response = testData$BAD, predicted = pred_logit2,
##      threshold = cutoff)
##
## Examples are labelled as positive when predicted is greater than 0.1628139
##
## precision: NaN
## recall: 0.000
## F: NaN

```

```

plotROC(testData$BAD, pred_logit)
# We conduct an LDA analysis on training and validation datasets
Equity.lda <- lda(BAD ~., data = trainingData)
Equity.lda

## Call:
## lda(BAD ~ ., data = trainingData)
##
## Prior probabilities of groups:
##          0          1
## 0.95238095 0.04761905
##
## Group means:
##      LOAN      VALUE REASONHomeImp  JOBOffice  JOBOther  JOBProfExe
JOBSales
## 0 17427.12 99054.77      0.2903846 0.21346154 0.3759615 0.2548077
0.01346154
## 1 14590.38 87962.25      0.2500000 0.05769231 0.5576923 0.1346154
0.07692308
##      JOBSelf      YOJ      CLAGE      NINQ      CLNO  DEBTINC
## 0 0.01442308 9.212500 177.3966 0.837500 21.05192 33.53054
## 1 0.03846154 6.365385 123.6583 1.115385 18.94231 37.09308
##
## Coefficients of linear discriminants:
##                      LD1
## LOAN          -5.266976e-05
## VALUE          -2.189875e-06
## REASONHomeImp  -4.149341e-01
## JOBOffice      -8.040156e-01
## JOBOther       -2.561738e-02
## JOBProfExe     -3.256789e-01
## JOBSales       2.836376e+00
## JOBSelf        1.716459e+00
## YOJ            -1.200019e-02
## CLAGE          -7.648944e-03
## NINQ           9.624250e-02
## CLNO           -1.672174e-02
## DEBTINC        7.581878e-02

fit.hat <- predict(Equity.lda,testData)
summary(fit.hat)

##          Length Class  Mode
## class      1092  factor numeric
## posterior   2184   -none- numeric
## x           1092   -none- numeric

Equity.lda2 <- lda(BAD ~., data = validationData)
Equity.lda2

```

```

## Call:
## lda(BAD ~ ., data = validationData)
##
## Prior probabilities of groups:
##           0           1
## 0.95238095 0.04761905
##
## Group means:
##      LOAN      VALUE REASONHomeImp JOBOffice  JOBOther JOBProfExe
JOBSales
## 0 17832.50 100894.4      0.2923077 0.2000000 0.3692308 0.2596154
0.009615385
## 1 12780.77 80641.5      0.2692308 0.1923077 0.5000000 0.1538462
0.038461538
##      JOBSelf      YOJ      CLAGE      NINQ      CLNO  DEBTINC
## 0 0.02115385 9.303846 177.4786 0.8865385 20.88077 33.91158
## 1 0.03846154 6.538462 121.3760 1.3846154 16.96154 35.74296
##
## Coefficients of linear discriminants:
##                      LD1
## LOAN          -7.369884e-05
## VALUE          -3.871734e-06
## REASONHomeImp -5.007599e-01
## JOBOffice      1.416515e-01
## JOBOther       3.654024e-01
## JOBProfExe     3.388812e-01
## JOBSales       2.784490e+00
## JOBSelf        1.718807e+00
## YOJ            -6.706146e-03
## CLAGE          -7.750737e-03
## NINQ           2.295962e-01
## CLNO           -4.034104e-02
## DEBTINC        5.219889e-02

fit.hat2 <- predict(Equity.lda2,testData)
# There are vast differences in the means for the variable JOB
# We remove the variable JOB from the LDA analysis on the test to see if it
improves accuracy of model
Equity.lda3 <- lda(BAD ~ LOAN+VALUE+REASON+YOJ+CLAGE+NINQ+CLNO+DEBTINC, data
= testData)
Equity.lda3

## Call:
## lda(BAD ~ LOAN + VALUE + REASON + YOJ + CLAGE + NINQ + CLNO +
##      DEBTINC, data = testData)
##
## Prior probabilities of groups:
##           0           1
## 0.95238095 0.04761905
##

```

```

## Group means:
##      LOAN      VALUE REASONHomeImp      YOJ      CLAGE      NINQ      CLNO
DEBTINC
## 0 17446.06 98186.70      0.2548077 8.928846 175.2786 0.8788462 20.36442
33.73609
## 1 13753.85 79327.69      0.3076923 6.269231 134.3807 1.0961538 18.28846
36.10330
##
## Coefficients of linear discriminants:
##              LD1
## LOAN          -5.174706e-05
## VALUE         -1.038321e-05
## REASONHomeImp  1.315388e-01
## YOJ           -2.734217e-02
## CLAGE         -6.293539e-03
## NINQ          6.419849e-02
## CLNO         -2.055394e-02
## DEBTINC       9.110237e-02

fit.hat3 <- predict(Equity.lda3,testData)
# It turns out that re-sampling the data improves the efficiency of the LDA model
Equity.lda4 <- lda(BAD ~., data =data_balanced_both )
Equity.lda4

## Call:
## lda(BAD ~ ., data = data_balanced_both)
##
## Prior probabilities of groups:
##      0      1
## 0.5039267 0.4960733
##
## Group means:
##      LOAN      VALUE REASONHomeImp  JOBOoffice  JOBOther  JOBProfExe
JOBSales
## 0 17284.42 100964.58      0.2987013 0.21818182 0.3805195 0.2532468
0.01038961
## 1 15058.05 88857.97      0.2546174 0.05277045 0.5501319 0.1477573
0.08179420
##      JOBSelf      YOJ      CLAGE      NINQ      CLNO      DEBTINC
## 0 0.01948052 8.990909 174.463 0.8402597 21.07792 33.59819
## 1 0.04089710 6.362797 124.857 1.1015831 19.34433 37.06649
##
## Coefficients of linear discriminants:
##              LD1
## LOAN          -2.900461e-05
## VALUE         -7.297490e-07
## REASONHomeImp -2.406279e-01
## JOBOoffice    -1.308094e+00
## JOBOther      1.186391e-01

```

```
## JOBProfExe    -2.287448e-01
## JOBSales      1.475184e+00
## JOBSelf       8.979253e-01
## YOJ           -3.475800e-02
## CLAGE         -8.837612e-03
## NINQ          2.644739e-03
## CLNO          -4.082716e-03
## DEBTINC       8.761021e-02
```

Out of the three models, the decision tree had the lowest error rate, followed by the logistic regression model.

