Final Project Report: School District Assignment

1. Initial State Formulation and Assessment

Initial model formulation, m_0, yielded a total annual transportation cost of **$507,500** for student districts 1-6. Model formulation ensured every student was assigned to a feasible highschool, such that no student in district 2 was assigned to high school 1, no student from district 4 was assigned to high school 3, and so forth. Similarly, school population, both grade and overall, constraint capacities were enforced to avoid overcrowding, a phenomenon frequently associated with a decrease in educational quality. As a result, our model suggested the following initial state student-school distributions:

Table 1. m_0 High School 1 Suggested Distribution

	High School 1		
	10th Grade	**11th Grade**	**12th Grade**
Area 3	97.875	138.375	101.25
Area 4	142.5	131.25	101.25
Area 5	65.625	54.375	67.5
TOTAL	306	324	270

Table 2. m_0 High School 2 Suggested Distribution

	High School 2		
	10th Grade	**11th Grade**	**12th Grade**
Area 1	139.5	175.5	135
Area 2	180	135	185
Area 3	12	12.375	13.125
Area 4	32.625	46.125	33.75
TOTAL	364.125	369	366.875

Table 3. m_0 High School 3 Suggested Distribution

	High School 3		
	10th Grade	**11th Grade**	**12th Grade**
Area 3	164	169.125	179.375
Area 5	66.5	61.25	47.25
Area 6	74.27	61.625	76.5
TOTAL	304.77	292	303.125

2. Revised Costs Transportation Plan Analysis

Following our initial state assessment, we proceeded onto the proposed cost-reduction plans evaluation. The cost-reduction plans, detailed below, produced highly favorable monetary results.

- Only students living **over** 1.5 miles away from the school are to be provided access to transportation.
- Only students living **over** 2.0 miles away from the school are to be provided access to transportation.

In the first case, our revised model, m_1, determined the annual cost of transportation to be **$406, 641**, reducing yearly costs by **$100,859**. As for the second scenario, our model, m_2, determined the yearly transportation cost to be **$342, 500**, further reducing yearly costs by **$165,000**. However, these cost savings come at the expense of student convenience and safety, as in both scenarios a large percentage of the student body will be forced to either walk or ride a bicycle home. In scenario 1, 672.396 students will have to find other means of transportation, while in scenario 2, this number will increase to 950.

Considering this, we suggest discarding both of these cost-reduction plans and, instead, providing transportation for every student, as initially proposed in our initial-state model, m_0. Student safety should not be risked, furthermore, since we were not provided details regarding location specifics, i.e. *is there winter? How cold is it during the winter? Is every school area safe? Are there dangerous areas going from and to each of the high schools ?* We cannot risk having over 500 young adults walk or bicycle their way to school as a means of reducing costs. Hence, we restate that we **highly** suggest the school district board follow our initial-state student assignment suggestions regarding cost-saving plans.

3. District 6 Road Construction Analysis

It was brought to our attention that district 6 has been experiencing severe traffic delays due to ongoing road construction in the area. Seeing as this may increase the district's annual transportation costs for schools 1 & 2 by as much as 10%, we evaluated the potential impact this may have on annual transportation costs and student body distributions. Since we suggest the use of model, m_0, our analysis' scope is limited to this model's suggested distributions and costs.

Unfortunately, in the case of high school 1, the ongoing construction will definitely have an impact on ideal student-school distributions. This is due to the fact that our analysis indicates that the maximum yearly cost allowed, in order to retain current distributions and costs, is $300 per student, which is the current cost. As a result, a potential 10% increase in costs will yield entirely new school assignments.

Using this information, we evaluated a revised model, m_3, which utilized the updated annual costs ($330 rather than $300), assuming a 10% increase. Thus, taking into account potential district 6 traffic delays and increased costs, the model, m_3, suggests the following updated student-school distributions:

Table 4. m_3 Highschool 1 Suggested Distribution

	High School 1		
	10th Grade	**11th Grade**	**12th Grade**
Area 4	80	82.5	87.5
Area 5	67.67	95.67	70.0
Area 6	158.33	145.83	112.50
TOTAL	306	324	270

Table 5. m_3 High School 2 Suggested Distribution

	High School 2		
	10th Grade	**11th Grade**	**12th Grade**
Area 1	139.5	175.5	135
Area 2	156	117	160.33
Area 4	62.83	88.83	65
TOTAL	358.33	381.33	360.33

Table 6. m_3 High School 3 Suggested Distribution

	High School 3		
	10th Grade	**11th Grade**	**12th Grade**
Area 2	24	18	24.66
Area 3	96	99	105
Area 5	50.66	46.66	36
Area 6	140	116	144
TOTAL	310.66	279.66	309.66

Following the suggested student body assignments, the annual cost would remain the same as that of the initial state: **$507,500**. Hence, while the ongoing construction in district 6 will certainly affect the ideal distribution of students, if the model's suggested solution is followed, then there should be **no** change in annual costs.

As for the case of high school 2, notice that we do not assign any students from district 6 to high school 2 (Table 2), meaning that the traffic delays should have **no** effect on transportation costs and student distributions. In this scenario, the school board is advised to follow the initial state model (m_0) student-school distribution.

4. Portable Classroom Analysis

We have also analyzed the possibility of leasing portable classrooms for each of the high schools, as the use of these to accommodate an increased number of students per school may decrease annual transportation costs. Since the yearly lease for one portable classroom of capacity 20 is $800, i.e., $40 per student, it would only be favorable to make use of this option if one can obtain a cost-reduction **over** $40 per additional unit of capacity (student) for any of the given high schools.

Our initial state model analysis reveals that adding one unit of additional capacity for each of the high schools would yield a cost-reduction of:

- High School 1: $16.66
- High School 2: **$50.00**
- High School 3: $0.00

Consequently, the use of portable classrooms would only be favorable in high school 2, where their implementation would net savings of **$10** per additional unit of capacity, or **$200** per portable unit. In order to retain the ideal student population distribution, we advise to implement these units in a manner such that the total student body population does **not** exceed 1,363. Seeing as the initial school capacity is 1,100, the maximum number of suggested portable classrooms to be bought is 13.0. This number would ensure the total population remains under 1,363 and would save an additional **$2,600** per annum.

5. Conclusion

In this report we address the issue of student-school assignment and distribution for districts 1-6. We built initial-state and subsequent model (m_{0-3}), following the advised school capacity and school grade constraints to avoid overcrowding and guarantee quality of education for each individual. Furthermore, we also analyzed the possibility of implementing two different cost-reduction strategies, consisting of limiting transportation to students living a specified distance away from the high school. Unfortunately, we strongly advise against either of these potential cost-saving solutions as placing over 500 lives in potential risk does not justify monetary saving, even more so when we were not provided with any details, other than the distance, regarding the schools' locations. We also assessed the impact the ongoing construction in district 6 could have on annual costs and distributions. Fortunately, while the ideal distributions changed, the annual cost remained unchanged at **$507,500**, indicating that the traffic delays will not have a significant impact on student transportation. Finally, we evaluated the possibility of incorporating portable classrooms to increase capacity and reduce costs. Out of the three schools, only high school 2 would benefit from the leasing of portable units (suggested maximum number of leased units: 13) , yielding savings of **$200** per unit rented.

Technical Appendix

Parts A and B:

Area Supply: 450 (Area 1) , 500 (Area 2) , 550 (Area 3) , 450 (Area 4), 500 (Area 5) , and 400 (Area 6)
School Capacities: 900 (School 1 Capacity), 1100 (School 2 Capacity), 1000 (School 3 Capacity)
Each grade must be 30-36% of total school population.

Linear Program:

Objective function:

$$\begin{aligned} Minimize\ z = {} & 250S_{11} + 0S_{12} + 500S_{13}\ (area\ 1) \\ & + 10^{30}S_{21} + 250S_{22} + 300S_{23}\ (area\ 2) \\ & + 400S_{31} + 150S_{32} + 200S_{33}\ (area\ 3) \\ & + 250S_{41} + 300S_{42} + 10^{30}S_{43}\ (area\ 4) \\ & + 150S_{51} + 10^{30}S_{52} + 250S_{53}\ (area\ 5) \\ & + 300S_{61} + 200S_{62} + 0S_{63}\ (area\ 6) \end{aligned}$$

Note: Objective function coefficient values of 10^{30} denotes ∞ and the variables S_{21}, S_{43}, and S_{52} do not play a role in calculating the overall objective function value and each have values of 0.

subject to:

$$S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61} \leq 900 \quad \text{(School 1 Capacity)}$$
$$S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62} \leq 1100 \quad \text{(School 2 Capacity)}$$
$$S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63} \leq 1000 \quad \text{(School 3 Capacity)}$$
$$S_{11} + S_{12} + S_{13} = 450 \quad \text{(Area 1 Supply)}$$
$$S_{21} + S_{22} + S_{23} = 500 \quad \text{(Area 2 Supply)}$$
$$S_{31} + S_{32} + S_{33} = 550 \quad \text{(Area 3 Supply)}$$
$$S_{41} + S_{42} + S_{43} = 450 \quad \text{(Area 4 Supply)}$$
$$S_{51} + S_{52} + S_{53} = 500 \quad \text{(Area 5 Supply)}$$
$$S_{61} + S_{62} + S_{63} = 400 \quad \text{(Area 6 Supply)}$$

(School 1 10th grade lower and upper bounds)

$$0.31S_{11} + 0.36S_{21} + 0.32S_{31} + 0.29S_{41} + 0.38S_{51} + 0.35S_{61} \geq 0.30(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$
$$0.31S_{11} + 0.36S_{21} + 0.32S_{31} + 0.29S_{41} + 0.38S_{51} + 0.35S_{61} \leq 0.36(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$

(School 1 11th grade lower and upper bounds)

$$0.39S_{11} + 0.27S_{21} + 0.33S_{31} + 0.41S_{41} + 0.35S_{51} + 0.29S_{61} \geq 0.30(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$
$$0.39S_{11} + 0.27S_{21} + 0.33S_{31} + 0.41S_{41} + 0.35S_{51} + 0.29S_{61} \leq 0.36(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$

(School 1 12th grade lower and upper bounds)

$$0.30S_{11} + 0.37S_{21} + 0.35S_{31} + 0.30S_{41} + 0.27S_{51} + 0.36S_{61} \geq 0.30(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$
$$0.30S_{11} + 0.37S_{21} + 0.35S_{31} + 0.30S_{41} + 0.27S_{51} + 0.36S_{61} \leq 0.36(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$

(School 2 10th grade lower and upper bounds)

$$0.31S_{12} + 0.36S_{22} + 0.32S_{32} + 0.29S_{42} + 0.38S_{52} + 0.35S_{62} \geq 0.30(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$
$$0.31S_{12} + 0.36S_{22} + 0.32S_{32} + 0.29S_{42} + 0.38S_{52} + 0.35S_{62} \leq 0.36(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$

(School 2 11th grade lower and upper bounds)

$$0.39S_{12} + 0.27S_{22} + 0.33S_{32} + 0.41S_{42} + 0.35S_{52} + 0.29S_{62} \geq 0.30(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$
$$0.39S_{12} + 0.27S_{22} + 0.33S_{32} + 0.41S_{42} + 0.35S_{52} + 0.29S_{62} \leq 0.36(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$

(School 2 12th grade lower and upper bounds)

$$0.30S_{12} + 0.37S_{22} + 0.35S_{32} + 0.30S_{42} + 0.27S_{52} + 0.36S_{62} \geq 0.30(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$
$$0.30S_{12} + 0.37S_{22} + 0.35S_{32} + 0.30S_{42} + 0.27S_{52} + 0.36S_{62} \leq 0.36(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$

(School 3 10th grade lower and upper bounds)

$$0.31S_{13} + 0.36S_{23} + 0.32S_{33} + 0.29S_{43} + 0.38S_{53} + 0.35S_{63} \geq 0.30(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$
$$0.31S_{13} + 0.36S_{23} + 0.32S_{33} + 0.29S_{43} + 0.38S_{53} + 0.35S_{63} \leq 0.36(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$

(School 3 11th grade lower and upper bounds)

$$0.39S_{13} + 0.27S_{23} + 0.33S_{33} + 0.41S_{43} + 0.35S_{53} + 0.29S_{63} \geq 0.30(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$
$$0.39S_{13} + 0.27S_{23} + 0.33S_{33} + 0.41S_{43} + 0.35S_{53} + 0.29S_{63} \leq 0.36(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$

(School 3 12 th grade lower and upper bounds)

$$0.30S_{13} + 0.37S_{23} + 0.35S_{33} + 0.30S_{43} + 0.27S_{53} + 0.36S_{63} \geq 0.30(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$
$$0.30S_{13} + 0.37S_{23} + 0.35S_{33} + 0.30S_{43} + 0.27S_{53} + 0.36S_{63} \leq 0.36(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$

$S_{ij} \geq 0, where\ i = area\ and\ j = school$ (non-negativity constraint)

LP Solved Via Gurobi:

```
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (win64)
Thread count: 6 physical cores, 6 logical processors, using up to 6 threads
Optimize a model with 27 rows, 18 columns and 133 nonzeros
Model fingerprint: 0x5520a5c6
Coefficient statistics:
  Matrix range     [1e-02, 1e+00]
  Objective range  [2e+02, 1e+30]
  Bounds range     [0e+00, 0e+00]
  RHS range        [4e+02, 1e+03]
Warning: Model contains large objective coefficients
         Consider reformulating model or setting NumericFocus parameter
         to avoid numerical issues.
Presolve time: 0.01s
Presolved: 27 rows, 18 columns, 133 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.0250000e+05   1.185000e+03   0.000000e+00      0s
       7    5.0750000e+05   0.000000e+00   0.000000e+00      0s

Solved in 7 iterations and 0.01 seconds
Optimal objective  5.075000000e+05
```

```
for v in m.getVars():
    print('%s %g' % (v.varName, v.x))

print('Obj: %g' % m.objVal)
```

```
S11 0
S12 450
S13 0
S21 0
S22 433.333
S23 66.6667
S31 250
S32 0
S33 300
S41 233.333
S42 216.667
S43 0
S51 416.667
S52 0
S53 133.333
S61 0
S62 0
S63 400
Obj: 507500
```

Part C: Transportation only for > 1.5 miles otherwise zero cost

Objective function:

$$\begin{aligned} Minimize\ z = {} & 250S_{11} + 0S_{12} + 500S_{13}\ (area\ 1) \\ & +10^{30}S_{21} + 250S_{22} + 300S_{23}\ (area\ 2) \\ & + 400S_{31} + 0S_{32} + 200S_{33}\ (area\ 3) \\ & +250S_{41} + 300S_{42} + 10^{30}S_{43}\ (area\ 4) \\ & +0S_{51} + 10^{30}S_{52} + 250S_{53}\ (area\ 5) \\ & + 300S_{61} + 200S_{62} + 0S_{63}\ (area\ 6) \end{aligned}$$

subject to:

$$S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61} \leq 900 \quad \text{(School 1 Capacity)}$$
$$S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62} \leq 1100 \quad \text{(School 2 Capacity)}$$
$$S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63} \leq 1000 \quad \text{(School 3 Capacity)}$$
$$S_{11} + S_{12} + S_{13} = 450 \quad \text{(Area 1 Supply)}$$
$$S_{21} + S_{22} + S_{23} = 500 \quad \text{(Area 2 Supply)}$$
$$S_{31} + S_{32} + S_{33} = 550 \quad \text{(Area 3 Supply)}$$
$$S_{41} + S_{42} + S_{43} = 450 \quad \text{(Area 4 Supply)}$$
$$S_{51} + S_{52} + S_{53} = 500 \quad \text{(Area 5 Supply)}$$
$$S_{61} + S_{62} + S_{63} = 400 \quad \text{(Area 6 Supply)}$$

(School 1 10th grade lower and upper bounds)

$$0.31S_{11} + 0.36S_{21} + 0.32S_{31} + 0.29S_{41} + 0.38S_{51} + 0.35S_{61} \geq 0.30(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$
$$0.31S_{11} + 0.36S_{21} + 0.32S_{31} + 0.29S_{41} + 0.38S_{51} + 0.35S_{61} \leq 0.36(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$

(School 1 11th grade lower and upper bounds)

$$0.39S_{11} + 0.27S_{21} + 0.33S_{31} + 0.41S_{41} + 0.35S_{51} + 0.29S_{61} \geq 0.30(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$
$$0.39S_{11} + 0.27S_{21} + 0.33S_{31} + 0.41S_{41} + 0.35S_{51} + 0.29S_{61} \leq 0.36(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$

(School 1 12th grade lower and upper bounds)

$$0.30S_{11} + 0.37S_{21} + 0.35S_{31} + 0.30S_{41} + 0.27S_{51} + 0.36S_{61} \geq 0.30(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$
$$0.30S_{11} + 0.37S_{21} + 0.35S_{31} + 0.30S_{41} + 0.27S_{51} + 0.36S_{61} \leq 0.36(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$

(School 2 10th grade lower and upper bounds)

$$0.31S_{12} + 0.36S_{22} + 0.32S_{32} + 0.29S_{42} + 0.38S_{52} + 0.35S_{62} \geq 0.30(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$
$$0.31S_{12} + 0.36S_{22} + 0.32S_{32} + 0.29S_{42} + 0.38S_{52} + 0.35S_{62} \leq 0.36(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$

(School 2 11th grade lower and upper bounds)

$$0.39S_{12} + 0.27S_{22} + 0.33S_{32} + 0.41S_{42} + 0.35S_{52} + 0.29S_{62} \geq 0.30(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$
$$0.39S_{12} + 0.27S_{22} + 0.33S_{32} + 0.41S_{42} + 0.35S_{52} + 0.29S_{62} \leq 0.36(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$

(School 2 12th grade lower and upper bounds)

$$0.30S_{12} + 0.37S_{22} + 0.35S_{32} + 0.30S_{42} + 0.27S_{52} + 0.36S_{62} \geq 0.30(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$
$$0.30S_{12} + 0.37S_{22} + 0.35S_{32} + 0.30S_{42} + 0.27S_{52} + 0.36S_{62} \leq 0.36(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$

(School 3 10th grade lower and upper bounds)

$$0.31S_{13} + 0.36S_{23} + 0.32S_{33} + 0.29S_{43} + 0.38S_{53} + 0.35S_{63} \geq 0.30(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$
$$0.31S_{13} + 0.36S_{23} + 0.32S_{33} + 0.29S_{43} + 0.38S_{53} + 0.35S_{63} \leq 0.36(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$

(School 3 11th grade lower and upper bounds)

$$0.39S_{13} + 0.27S_{23} + 0.33S_{33} + 0.41S_{43} + 0.35S_{53} + 0.29S_{63} \geq 0.30(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$
$$0.39S_{13} + 0.27S_{23} + 0.33S_{33} + 0.41S_{43} + 0.35S_{53} + 0.29S_{63} \leq 0.36(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$

(School 3 12 th grade lower and upper bounds)

$$0.30S_{13} + 0.37S_{23} + 0.35S_{33} + 0.30S_{43} + 0.27S_{53} + 0.36S_{63} \geq 0.30(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$
$$0.30S_{13} + 0.37S_{23} + 0.35S_{33} + 0.30S_{43} + 0.27S_{53} + 0.36S_{63} \leq 0.36(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$

$$S_{ij} \geq 0, where\ i = area\ and\ j = school \quad \text{(non-negativity constraint)}$$

LP Solved Via Gurobi :

```
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (win64)
Thread count: 6 physical cores, 6 logical processors, using up to 6 threads
Optimize a model with 27 rows, 18 columns and 133 nonzeros
Model fingerprint: 0x740bde39
Coefficient statistics:
  Matrix range     [1e-02, 1e+00]
  Objective range  [2e+02, 1e+30]
  Bounds range     [0e+00, 0e+00]
  RHS range        [4e+02, 1e+03]
Warning: Model contains large objective coefficients
         Consider reformulating model or setting NumericFocus parameter
         to avoid numerical issues.
Presolve time: 0.00s
Presolved: 27 rows, 18 columns, 133 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    2.3750000e+05   1.185000e+03   0.000000e+00      0s
       5    4.0664062e+05   0.000000e+00   0.000000e+00      0s

Solved in 5 iterations and 0.01 seconds
Optimal objective  4.066406250e+05

                    S11 0
                    S12 450
                    S13 0
                    S21 0
                    S22 240.104
                    S23 259.896
                    S31 0
                    S32 297.396
                    S33 252.604
                    S41 337.5
                    S42 112.5
                    S43 0
                    S51 375
                    S52 0
                    S53 175
                    S61 187.5
                    S62 0
                    S63 212.5
                    Obj: 406641
```

Part D: Transportation only for > 2 miles otherwise zero cost

Objective function:

$$Minimize\ z = 250S_{11} + 0S_{12} + 500S_{13}\ (area\ 1)$$
$$+10^{30}S_{21} + 250S_{22} + 300S_{23}\ (area\ 2)$$
$$+ 400S_{31} + 0S_{32} + 0S_{33}\ (area\ 3)$$
$$+250S_{41} + 300S_{42} + 10^{30}S_{43}\ (area\ 4)$$
$$+0S_{51} + 10^{30}S_{52} + 250S_{53}\ (area\ 5)$$
$$+ 300S_{61} + 0S_{62} + 0S_{63}\ (area\ 6)$$

subject to:

$$S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61} \leq 900 \quad \text{(School 1 Capacity)}$$
$$S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62} \leq 1100 \quad \text{(School 2 Capacity)}$$
$$S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63} \leq 1000 \quad \text{(School 3 Capacity)}$$
$$S_{11} + S_{12} + S_{13} = 450 \quad \text{(Area 1 Supply)}$$
$$S_{21} + S_{22} + S_{23} = 500 \quad \text{(Area 2 Supply)}$$
$$S_{31} + S_{32} + S_{33} = 550 \quad \text{(Area 3 Supply)}$$
$$S_{41} + S_{42} + S_{43} = 450 \quad \text{(Area 4 Supply)}$$
$$S_{51} + S_{52} + S_{53} = 500 \quad \text{(Area 5 Supply)}$$
$$S_{61} + S_{62} + S_{63} = 400 \quad \text{(Area 6 Supply)}$$

(School 1 10th grade lower and upper bounds)

$$0.31S_{11} + 0.36S_{21} + 0.32S_{31} + 0.29S_{41} + 0.38S_{51} + 0.35S_{61} \geq 0.30(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$
$$0.31S_{11} + 0.36S_{21} + 0.32S_{31} + 0.29S_{41} + 0.38S_{51} + 0.35S_{61} \leq 0.36(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$

(School 1 11th grade lower and upper bounds)

$$0.39S_{11} + 0.27S_{21} + 0.33S_{31} + 0.41S_{41} + 0.35S_{51} + 0.29S_{61} \geq 0.30(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$
$$0.39S_{11} + 0.27S_{21} + 0.33S_{31} + 0.41S_{41} + 0.35S_{51} + 0.29S_{61} \leq 0.36(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$

(School 1 12th grade lower and upper bounds)

$$0.30S_{11} + 0.37S_{21} + 0.35S_{31} + 0.30S_{41} + 0.27S_{51} + 0.36S_{61} \geq 0.30(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$
$$0.30S_{11} + 0.37S_{21} + 0.35S_{31} + 0.30S_{41} + 0.27S_{51} + 0.36S_{61} \leq 0.36(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$

(School 2 10th grade lower and upper bounds)

$$0.31S_{12} + 0.36S_{22} + 0.32S_{32} + 0.29S_{42} + 0.38S_{52} + 0.35S_{62} \geq 0.30(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$
$$0.31S_{12} + 0.36S_{22} + 0.32S_{32} + 0.29S_{42} + 0.38S_{52} + 0.35S_{62} \leq 0.36(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$

(School 2 11th grade lower and upper bounds)

$$0.39S_{12} + 0.27S_{22} + 0.33S_{32} + 0.41S_{42} + 0.35S_{52} + 0.29S_{62} \geq 0.30(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$
$$0.39S_{12} + 0.27S_{22} + 0.33S_{32} + 0.41S_{42} + 0.35S_{52} + 0.29S_{62} \leq 0.36(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$

(School 2 12th grade lower and upper bounds)

$$0.30S_{12} + 0.37S_{22} + 0.35S_{32} + 0.30S_{42} + 0.27S_{52} + 0.36S_{62} \geq 0.30(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$
$$0.30S_{12} + 0.37S_{22} + 0.35S_{32} + 0.30S_{42} + 0.27S_{52} + 0.36S_{62} \leq 0.36(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$

(School 3 10th grade lower and upper bounds)

$$0.31S_{13} + 0.36S_{23} + 0.32S_{33} + 0.29S_{43} + 0.38S_{53} + 0.35S_{63} \geq 0.30(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$
$$0.31S_{13} + 0.36S_{23} + 0.32S_{33} + 0.29S_{43} + 0.38S_{53} + 0.35S_{63} \leq 0.36(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$

(School 3 11th grade lower and upper bounds)

$$0.39S_{13} + 0.27S_{23} + 0.33S_{33} + 0.41S_{43} + 0.35S_{53} + 0.29S_{63} \geq 0.30(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$
$$0.39S_{13} + 0.27S_{23} + 0.33S_{33} + 0.41S_{43} + 0.35S_{53} + 0.29S_{63} \leq 0.36(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$

(School 3 12 th grade lower and upper bounds)

$$0.30S_{13} + 0.37S_{23} + 0.35S_{33} + 0.30S_{43} + 0.27S_{53} + 0.36S_{63} \geq 0.30(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$
$$0.30S_{13} + 0.37S_{23} + 0.35S_{33} + 0.30S_{43} + 0.27S_{53} + 0.36S_{63} \leq 0.36(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$

$$S_{ij} \geq 0, where\ i = area\ and\ j = school \quad \text{(non-negativity constraint)}$$

LP Solved Via Gurobi:

```
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (win64)
Thread count: 6 physical cores, 6 logical processors, using up to 6 threads
Optimize a model with 27 rows, 18 columns and 133 nonzeros
Model fingerprint: 0x93c9e1fa
Coefficient statistics:
  Matrix range     [1e-02, 1e+00]
  Objective range  [3e+02, 1e+30]
  Bounds range     [0e+00, 0e+00]
  RHS range        [4e+02, 1e+03]
Warning: Model contains large objective coefficients
         Consider reformulating model or setting NumericFocus parameter
         to avoid numerical issues.
Presolve time: 0.01s
Presolved: 27 rows, 18 columns, 133 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    2.3750000e+05   6.180000e+02   0.000000e+00      0s
       5    3.4250000e+05   0.000000e+00   0.000000e+00      0s

Solved in 5 iterations and 0.01 seconds
Optimal objective  3.425000000e+05
```

```
for v in m.getVars():
    print('%s %g' % (v.varName, v.x))

print('Obj: %g' % m.objVal)
```

```
S11 0
S12 450
S13 0
S21 0
S22 500
S23 0
S31 0
S32 0
S33 550
S41 300
S42 150
S43 0
S51 400
S52 0
S53 150
S61 200
S62 0
S63 200
Obj: 342500
```

Part E:

Gurobi command: S61.SAObjUp

The maximum value cost from area 6 to school 1 is $300. This is identical to the current objective cost .Therefore, it is a sub 10% increase, and we need to reevaluate the model.

Revised LP Model

Objective function:

$$Minimize\ z = 250S_{11} + 0S_{12} + 500S_{13}\ (area\ 1)$$
$$+10^{30}S_{21} + 250S_{22} + 300S_{23}\ (area\ 2)$$
$$+ 400S_{31} + 150S_{32} + 200S_{33}\ (area\ 3)$$
$$+250S_{41} + 300S_{42} + 10^{30}S_{43}\ (area\ 4)$$
$$+0S_{51} + 10^{30}S_{52} + 250S_{53}\ (area\ 5)$$
$$+ 330S_{61} + 200S_{62} + 0S_{63}\ (area\ 6)$$

(Area 6 changed S61 from 300 to 330 for 10% increase)

subject to:

$$S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61} \leq 900 \quad \text{(School 1 Capacity)}$$
$$S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62} \leq 1100 \quad \text{(School 2 Capacity)}$$
$$S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63} \leq 1000 \quad \text{(School 3 Capacity)}$$
$$S_{11} + S_{12} + S_{13} = 450 \quad \text{(Area 1 Supply)}$$
$$S_{21} + S_{22} + S_{23} = 500 \quad \text{(Area 2 Supply)}$$
$$S_{31} + S_{32} + S_{33} = 550 \quad \text{(Area 3 Supply)}$$
$$S_{41} + S_{42} + S_{43} = 450 \quad \text{(Area 4 Supply)}$$
$$S_{51} + S_{52} + S_{53} = 500 \quad \text{(Area 5 Supply)}$$
$$S_{61} + S_{62} + S_{63} = 400 \quad \text{(Area 6 Supply)}$$

(School 1 10th grade lower and upper bounds)

$$0.31S_{11} + 0.36S_{21} + 0.32S_{31} + 0.29S_{41} + 0.38S_{51} + 0.35S_{61} \geq 0.30(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$
$$0.31S_{11} + 0.36S_{21} + 0.32S_{31} + 0.29S_{41} + 0.38S_{51} + 0.35S_{61} \leq 0.36(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$

(School 1 11th grade lower and upper bounds)

$$0.39S_{11} + 0.27S_{21} + 0.33S_{31} + 0.41S_{41} + 0.35S_{51} + 0.29S_{61} \geq 0.30(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$
$$0.39S_{11} + 0.27S_{21} + 0.33S_{31} + 0.41S_{41} + 0.35S_{51} + 0.29S_{61} \leq 0.36(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$

(School 1 12th grade lower and upper bounds)

$$0.30S_{11} + 0.37S_{21} + 0.35S_{31} + 0.30S_{41} + 0.27S_{51} + 0.36S_{61} \geq 0.30(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$
$$0.30S_{11} + 0.37S_{21} + 0.35S_{31} + 0.30S_{41} + 0.27S_{51} + 0.36S_{61} \leq 0.36(S_{11} + S_{21} + S_{31} + S_{41} + S_{51} + S_{61})$$

(School 2 10th grade lower and upper bounds)

$$0.31S_{12} + 0.36S_{22} + 0.32S_{32} + 0.29S_{42} + 0.38S_{52} + 0.35S_{62} \geq 0.30(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$
$$0.31S_{12} + 0.36S_{22} + 0.32S_{32} + 0.29S_{42} + 0.38S_{52} + 0.35S_{62} \leq 0.36(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$

(School 2 11th grade lower and upper bounds)

$$0.39S_{12} + 0.27S_{22} + 0.33S_{32} + 0.41S_{42} + 0.35S_{52} + 0.29S_{62} \geq 0.30(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$
$$0.39S_{12} + 0.27S_{22} + 0.33S_{32} + 0.41S_{42} + 0.35S_{52} + 0.29S_{62} \leq 0.36(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$

(School 2 12th grade lower and upper bounds)

$$0.30S_{12} + 0.37S_{22} + 0.35S_{32} + 0.30S_{42} + 0.27S_{52} + 0.36S_{62} \geq 0.30(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$
$$0.30S_{12} + 0.37S_{22} + 0.35S_{32} + 0.30S_{42} + 0.27S_{52} + 0.36S_{62} \leq 0.36(S_{12} + S_{22} + S_{32} + S_{42} + S_{52} + S_{62})$$

(School 3 10th grade lower and upper bounds)

$$0.31S_{13} + 0.36S_{23} + 0.32S_{33} + 0.29S_{43} + 0.38S_{53} + 0.35S_{63} \geq 0.30(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$
$$0.31S_{13} + 0.36S_{23} + 0.32S_{33} + 0.29S_{43} + 0.38S_{53} + 0.35S_{63} \leq 0.36(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$

(School 3 11th grade lower and upper bounds)

$$0.39S_{13} + 0.27S_{23} + 0.33S_{33} + 0.41S_{43} + 0.35S_{53} + 0.29S_{63} \geq 0.30(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$

$$0.39S_{13} + 0.27S_{23} + 0.33S_{33} + 0.41S_{43} + 0.35S_{53} + 0.29S_{63} \leq 0.36(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$

(School 3 12 th grade lower and upper bounds)

$$0.30S_{13} + 0.37S_{23} + 0.35S_{33} + 0.30S_{43} + 0.27S_{53} + 0.36S_{63} \geq 0.30(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$

$$0.30S_{13} + 0.37S_{23} + 0.35S_{33} + 0.30S_{43} + 0.27S_{53} + 0.36S_{63} \leq 0.36(S_{13} + S_{23} + S_{33} + S_{43} + S_{53} + S_{63})$$

$S_{ij} \geq 0, where\ i = area\ and\ j = school$ (non-negativity constraint)

LP Solved Via Gurobi:

```
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (win64)
Thread count: 6 physical cores, 6 logical processors, using up to 6 threads
Optimize a model with 27 rows, 18 columns and 133 nonzeros
Model fingerprint: 0x5520a5c6
Coefficient statistics:
  Matrix range     [1e-02, 1e+00]
  Objective range  [2e+02, 1e+30]
  Bounds range     [0e+00, 0e+00]
  RHS range        [4e+02, 1e+03]
Warning: Model contains large objective coefficients
         Consider reformulating model or setting NumericFocus parameter
         to avoid numerical issues.
Presolve time: 0.01s
Presolved: 27 rows, 18 columns, 133 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    4.0250000e+05   1.185000e+03   0.000000e+00      0s
       7    5.0750000e+05   0.000000e+00   0.000000e+00      0s

Solved in 7 iterations and 0.01 seconds
Optimal objective  5.075000000e+05
```

```
for v in m.getVars():
    print('%s %g' % (v.varName, v.x))

print('Obj: %g' % m.objVal)
```

```
S11 0
S12 450
S13 0
S21 0
S22 433.333
S23 66.6667
S31 250
S32 0
S33 300
S41 233.333
S42 216.667
S43 0
S51 416.667
S52 0
S53 133.333
S61 0
S62 0
S63 400
Obj: 507500
```

Parts F and G:

```
In [8]: #Parts f and g
        S62.SAObjUp #Maximum increase is infinite, this is greater than 10% so we do NOT need to reevaluate the model

Out[8]: inf
```

Part H:

The cost for leasing is $800 for 20 kids , so $40 per kid is the cost.

Gurobi commands to calculate shadow prices:

print(c1.pi)
print(c2.pi)
print(c3.pi)

Out of these options, ONLY school 2 is worth it, as adding 1 additional unit of capacity will reduce objective cost by $50.00. This nets a positive difference on cost vs benefit. School 1 is not worth it as the shadow price is sub $40.00, which is similar to school 3.

Part I:

We calculate the maximum allowable classroom capacity increase prior to objective function changing for each of the three high schools.

Gurobi commands:

print(c1.SARHSUp)
print(c2.SARHSUp)
print(c3.SARHSUp)

We then calculate the current right hand side values, which are the capacities of each of the three high schools.

Gurobi commands:

print(c1.RHS)
print(c2.RHS)
print(c3.RHS)

The current capacity according to model (b) for school 2 is 1100. Thus, we can add 13.15 (round to 13) portable classrooms for high school 2.