**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans = Work start after 10 minutes so average(μ) will grow to 55 Min. Sample(X) = 60 min

Z Score = (X-μ)/σ, (60-55)/8 = 0.625 & P(x>60=1-P(x<60)=**0.266(**the probability that the service manager cannot meet his commitment)

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans = mean=38,X=400, *σ = 6. Z score=(x-μ)/σ* = 60.33, P= (38<x<400) = P(x<400) - 0.5= 50%

mean=44,X=38, *σ = 6. Z score=(x-μ)/σ* = 1, P= (x<44) =- 0.84= 84%

Employees between age 38 to 44 = 84-50 = 34 % which is 136 out of 400

Hence the statement of employees are older than 44 than 38 - 44 is **True**.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans = mean=30,X=38, *σ = 6. Z score=(x-μ)/σ* = 1.33, P= (x<30) = 0.0912

Under age of 30 = 0.0912\* 400=36.48(36 employees), So as the statement of employees under the age of 30 at the center would be expected to attract about 36 employees is **TRUE.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans = If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are two independent variables then,

1. X+Y ~ N(mu1+mu2,s1^2+s2^2), (b) X-Y~N(mu1-mu2,s1^2-s2^2)

Z= (a)X+(b)Y , as X and Y mentioned above, Z~N(a mu1+bmu2,s1^2+s2^2)

So now , 2X1~N( 2mu,4,s^2) , X1+X2~N(mu+mu,s^2+s^2)~N(2mu,2s^2)

2X1-(X1+X2) = N(4mu,6 S^2).

The Mean of 2 X1 and X1+X2 is equal.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans = Z =( X-mu)/s = (0.99-100)/20 = -4.9505,Prpbability of 0.99<x<100=0.5

Z Value is -2.575.

(x-100)/20=>x=20+100

* - (-2.575)\*20+100 = 151.5
* (- 2.575)\*20+100 = 48.5

Thus, Option **D** is a Correct One.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans = Mean (5+7)\*45=540 Million

Std. deviation(9+16)\*45 =225 Million [np.sqrt((9)+(16))]

Range= stats.norm.interval(0.95,540,225) in Millions

(99.00810347848784, 980.9918965215122)

Specify the 5th percentile of profit (in Rupees) for the company

5th Percentile Z value is -1.645, Thus, X=mu+Z\*s = 540+(-1.645)\*225 = 169.87(Profit of 5th percentile.)

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans = Div 1 ( stats.norm.cdf(0,5,3) = 0.04779 >> Has a larger probability of making loss

Div 2 ( stats.norm.cdf(0,7,4) = 0.04005