**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676**
4. 0.5
5. 0.6987

Solution: We need first find the Z value for normal distribution.

Value of Z is obtained from formula Z=(X-*μ)/σ*

X value is 50 as each customers car work begins after 10 mins and customers are not supposed to wait for more than an hour. Then the probability of time taking less than 50 mins is obtained from the python code stats.norm.cdf(Z) which gives p(X<=50) whose value is 0.73401

We have to find P(X>50) which is 1-P(X<=50) whose value is 0.26598

Therefore, the solution is option B

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Solution:

1. Z=(X-*μ)/σ*

X=44

Mean=38

Standard deviation=6

P(X>=44) = P(Z>=1) = 1-P(Z<1)

1-0.841344

0.1586 is the probability that more employees are older than 44.

Between 38 and 44

P(X<44)=P(Z<1)=0.5-P(Z>=1)

0.5-0.841

0.341 Is the probability that more employees are between age 38 and 44

P(Employees older than 44) < P(Employees between 38 and 44)

Therefore, the statement is **FALSE**

1. X=30

Mean=38

Standard deviation=6

Z=(X-*μ)/σ*

Z=(30-38)/6

Z=-1.333

P(X<30)=P(Z<-1.3333)=0.0912=9.12%

9.12% of 400 is 36.

Therefore, the above statement is **TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Solution: 2X1:

Mean of 2X1 is 2μ

Variance becomes (2σ)2 which becomes 4σ2

Therefore, it becomes 2*X1* ~ *N*(2μ,4σ2)

*X*1 + *X*2:

The mean is μ+ μ =2μ

Whereas the variance is σ2+ σ2 =2σ2

X1+X2 ~ N (2μ,2σ2)

The mean and variance of 2X1 is 2μ and 4σ2 respectively

The mean and variance of X1+X2 is 2μ and 2σ2 respectively

Both random variables have same mean whereas the variances differ. The variance of 2X1 will be higher that of X1+X2

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9

Solution:

stats.norm.interval(0.99,100,20)

On executing this code in python we get output as (48.48341392902199, 151.516586070978)

Therefore, **D** is the right answer

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Solution: Overall mean = mean1+mean2

= $12 =12X45=540

Mean = 540

Stdev2 =Stdev12+stdev22

=9+16

Standard deviation=$5 =5X45

Standard deviation=225

The range of Rs (99.00810347848784, 980.9918965215122)

1. Specify the 5th percentile of profit (in Rupees) for the company

Solution: X = μ + Z\*σ

Z value is -1.644 at 5% significance level

= 540 + (-1.644)\*225

=170.1

1. Which of the two divisions has a larger probability of making a loss in a given year?

Solution: Let X be the amount of profit a division makes.

Then the probability of making a loss P(X<0) can be calculated by stats.norm.cdf function in python

Therefore, for the first division we get a probability of 0.0477903522728147

Which is 4.77%

For second divison:

P(X<0) is 0.040059156863817086

Which is 4% approximately.