

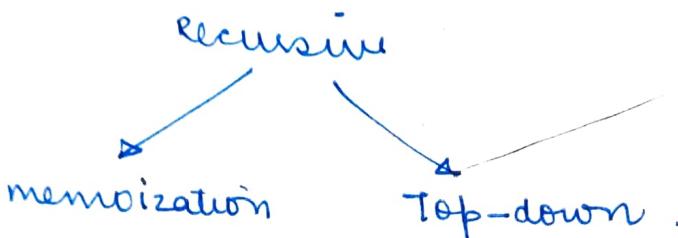
Dynamic Programming

DP → enhanced Recursion

Identification

(to check if a problem
is of DP or not)

choice of including the
current element
optimization is asked



- 1) ~~1) 0-1 knapsack (6)~~
- 2) ~~2) unbounded knapsack (5)~~
- 3) Fibonacci (7)
- 4) ~~4) LCS (15)~~
- 5) LIS (10)
- 6) Kandane's Algorithm (6)
- 7) Matrix chain multiplication (7) \star
- 8) DP on trees (4)
- 9) BP on grids (14)
- 10) Others (5)

0-1 knapsack

- subset sum
- equal sum
- count of subset sum
- minimum subset sum
- Target sum
- # of subset with given difference

Knapsack

Fractional knapsack

0/1

unbounded knapsack

greedy

add items
as a
whole.

can add
multiple
instances of
an element

I/P → weight [] → []
value [] → []
W → +

O/P → max profit

choice diagram
for item $\downarrow \rightarrow$

item \downarrow

$w_i \cdot \text{weight} > w$

$w_i \cdot \text{weight} \leq w$

choice \checkmark \times

take
item
not take

// Base diagram
// three of smallest valid input
if ($n == 0 \text{ or } w == 0$) return 0;

if ($wt[n-1] \leq w$) {

return max(knapsack(wt, val, cap, n-1),

val[n-1] + knapsack(wt[0:n-1],

3. else return knapsack (wt, val, cap, n-1);

Memoization \Rightarrow Recur code + 2 lines

this nature will be have
params which are changing in recursive call

dp \rightarrow

\Downarrow

$dp[n+1][w+1]$

size of knapsack
of knapsack

returning the max value with -1

max(dp, -1, sizeOf(dp))

Before calling the recursive function if
the max value is already filled on
the mat

int knapsack (int wt[], int val[], int w, int n);

if ($n == 0 \text{ or } w == 0$) return 0;

if ($t[n][w] != -1$) return t[n][w];

if ($wt[n-1] \leq w$) {

$t[n][w] = \max(\text{val}[n-1] + \text{knapsack}(wt[0:n-1], val[0:n-1]),$
 $\text{knapsack}(wt, val, w, n-1))$

return t[n][w];

} else {

$t[n][w] = \text{knapsack}(wt, val, w, n-1);$

return t[n][w];

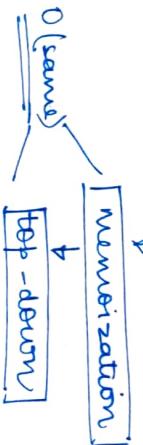
Top-down approach

recursion

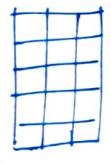
memoization



Recursion +



Top down →



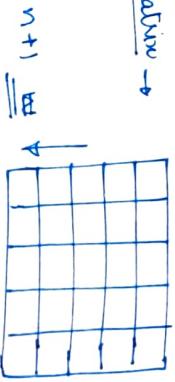
~~no recursion~~

ans →

stack overflow avoided

as not recursion calls are there.

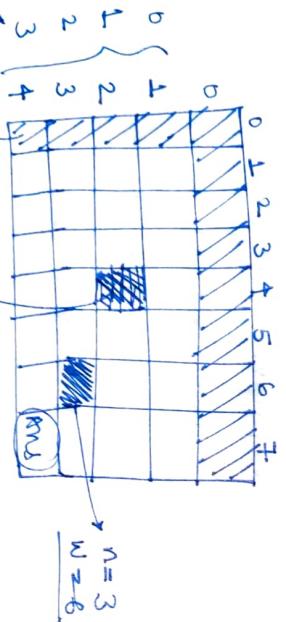
matrix →



w+1

step-1 → initialize
step-2 → recursion calls → iterative version

$n = 4$
 $N = 4$



initialization

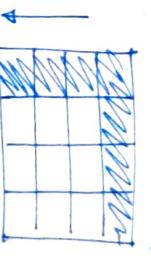
two elements only.

index-

max profit
when $n = 2 \neq N = 4$

Base condition → initialization
(row)

(top-down)



return 0 → $n == 0 \text{ || } w == 0$



n

$y(n == 0 \text{ || } w == 0)$

return 0;

$t[i][w]$

$t[i][w] = 0$

$y(w > n \leq w)$

$t[n][w] = \max(t[n-w][w], t[n][w])$

$y(w < n \leq w)$

$t[n][w] = \max(t[n][w-w], t[n][w])$

$t[n][w] = \max(v[n][w] + t[n-1][w-v[n][w]], t[n-1][w])$

$v[n][w] = \max(v[n-1][w], v[n-1][w-v[n][w]])$

$y(w < n \leq w)$

$t[n][w] = \max(v[n][w] + t[n-1][w-v[n][w]], t[n-1][w])$

$v[n][w] = \max(v[n-1][w], v[n-1][w-v[n][w]])$

use

$t[n][w] = t[n-1][w]$

now → $(n, w) \rightarrow (i, j)$
and run a loop

```

for (int i = 1; i < n+1; i++) {
    for (int j = 1; j < n+1; j++) {
        if (arr[i-1] <= j) {
            t[i][j] = max ( val[i-1] + t[i-1][j-1],
                            t[i-1][j] );
        }
    }
}

```

$$t[i][j] = \max (\underbrace{val[i-1] + t[i-1][j-1]}_{t[i-1][j-1]} , \underbrace{t[i-1][j]}_{t[i-1][j]})$$

```

else {
    t[i][j] = t[i-1][j];
}

```

Return $t[n][n]$;

4) Subset sum Problem

I/P \rightarrow 2 3 7 8 10 \rightarrow T/F \Rightarrow given an array
check if there is a subset sum is

sum



base case (int i, arr) {
if (i == arr.size())
return false;
if (sum == 0) return T;
if (arr[i] ≤ sum) {
return dfs(i+1, sum - arr[i],
arr);
} else {
return dfs(i+1, sum, arr);
}

dfs (int i, sum, arr) {
if (i == arr.size())
return true;
else {
if (arr[i] ≤ sum) {
t[i][sum] = true;
} else {
t[i][sum] = false;
} } } }

3.

dfs (int i, sum, arr);

~~base~~ \rightarrow sum
n = 0 \rightarrow T
sum = 0 \rightarrow F
n = 0 \rightarrow F
sum = 0 \rightarrow T

		sum			
		T	F	F	T
		T			
n	\rightarrow				
1					
2					
3					
4					
5					

Initialization of matrix

sum = 0 \rightarrow T
sum = 0 \rightarrow F
n = 0 \rightarrow F
sum = 0 \rightarrow T

for (int i = 0 \rightarrow n)
for (int j = 0 \rightarrow sum) {
if (i == 0) \rightarrow t[i][j] = False;
if (j == 0) \rightarrow t[i][j] = True.

= 3.

for (int i = 0 \rightarrow n)
for (int j = 0 \rightarrow sum) {
if (arr[i] ≤ j) {
t[i][j] = t[i+1][j - arr[i]]

||
else
t[i][j] = t[i+1][j];
} } }

Return $t[n][sum]$

Equal sum partition

arr[] → 3 1 5 1 1 5 3 → divide array in

O/P → T/F

two
subsets

such that both
have equal sum

sum → sum(arr) = 22

at any element

s₁ /
s₂

initially all
items are
there

sum + I

sum

book subsetsum (int arr[], int sum, int n){

→

s₁ + s₂ → sum

as s₁ = s₂

as = sum

hence sum must be even

we just have to find a subset with sum of sum/2

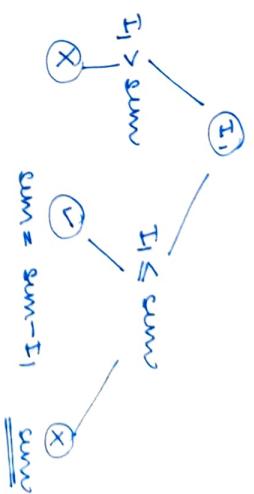
return subsetsum (arr, 0, sum/2);

Count of subsets of given sum :-

I/P → arr[] → 2 3 5 6 8 10
sum → 10

dp[n+1][sum+1]

n = 0 → F
sum = 0 → T



int dfa (int i, arr, sum) {

'i == arr.size ()' return 0;

if (dfa[i][sum] != -1) return dfa[i][sum];

if (sum == 0) return 1;

if (arr[i] <= sum).

dfa[i][sum] = dfa (i+1, arr, sum - arr[i]) +

dfa (i+1, arr, sum);

else dfa[i][sum] = dfa (i+1, arr, sum);

return dfa[i][sum];

→ sum

0	0	0	0	0	0	0
1						
-1						
1						
-1						

```

for (int i = 1 → n) {
    for (int j = 1 → sum) {
        if (arr[i] ≤ sum) {

```

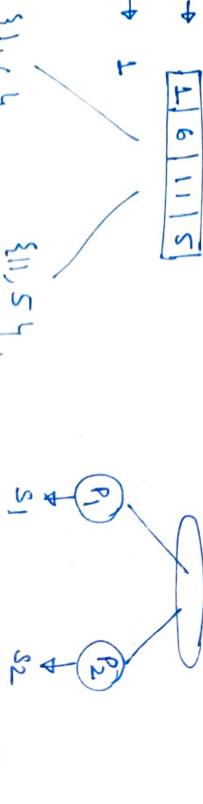
$$dp[i][j] = dp[i+1][j - arr[i]] + dp[i+1][j];$$

↓

else
 $dp[i][j] = dp[i+1][j];$

minimum subset sum difference

$arr[] \rightarrow [1 | 6 | 1 | 1 | 5]$



$\text{abs}(16 - 7) \rightarrow 9.$

\downarrow

$\{1, 6, 5\}$

(1)

$0/p \rightarrow \text{abs}(12 + 1) \underline{\underline{= 1}}$

we opt for subset arr, int s1);

~~s2 = sum - s1;~~

~~if (s2 == arr.size()) return INT_MAX;~~

~~return~~

~~max (abs(i+1, arr, s1+arr[i]),~~

~~min (abs(i+1, arr, s1))~~

y.

$s_2 \rightarrow s_2 - s_1 \rightarrow (\min)$
 $(\text{range} - s_1 - s_1) \rightarrow \min$
 $\text{Range} - 2s_1 \rightarrow \min$

as we have to return abs then $s_2 - s_1$ always smallest.

we have to calculate maximum value
 $s_1 < \text{range}/2$

$$\begin{array}{c} 0 \\ \times \quad \times \quad \times \quad \times \\ \text{range}/2 \\ \text{range} \end{array}$$

	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										

for range/2

the maximum val

of s for which

the set is T^o is

the value of s_1 .

we'll see min now as it will give

the value of possible subset

sums :: min

num of a = n .

←

we can do some new stuff about sum, problem

\downarrow

comp count the # of subsets with given difference:

$arr[] \rightarrow [1 | 1 | 2 | 3]$

$diff \rightarrow 1$

$0/p \underline{\underline{= 3}}$

$\{1, 2\} \{1, 3\}$

$\{1, 3\} \{1, 2\}$

$\{1, 1, 2\} \{3\}$

\downarrow

$s_1 - s_2 = d$

$s_1 + s_2 = arr$

$s_1 = \frac{d + arr}{2}$

$$\text{sum}(s_1) = \frac{a + \text{sum}(\text{arr})}{2}$$

problem reduced to count of subset
 $\text{sum}(\text{arr}) = \frac{a + \text{sum}(\text{arr})}{2}$

$$S = \frac{a + \text{sum}(\text{arr})}{2}$$

$$\boxed{\frac{a + \text{sum}(\text{arr})}{2}}$$

Target sum

$$\begin{array}{|c|c|c|c|} \hline \text{arr} & [1 & 1 & 2 & 3] \\ \hline \text{sum} & 1 \\ \hline \text{o/p} & 3 \\ \hline \end{array}$$

assign $\text{arr}_i +/-$
 in front of every
 element \Rightarrow then
 clean the result

$$\text{Ex:- } + + + + + 1 - 1 - 2 + 3$$

$$= 1$$

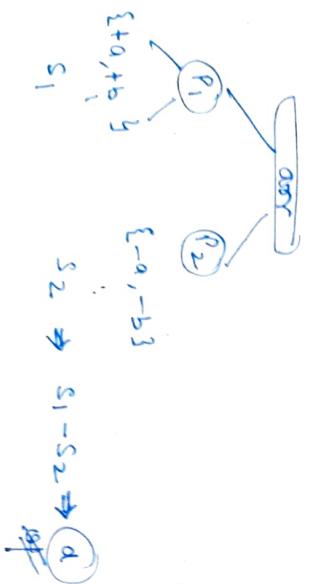
$$+ 1 - 1 - 2 + 3$$

$$- 1$$

$$\vdots$$

for approach \rightarrow
 $\text{dfs}(\text{i}+1, \text{nums}, \text{sum} + \text{arr}[\text{i}]);$
 $\text{dfs}(\text{i}+1, \text{nums}, \text{sum} - \text{arr}[\text{i}]);$

no approach \Rightarrow same as previous one



$$s_1$$

$$s_2 \Rightarrow s_1 - s_2 \Rightarrow \underline{a}$$

Unbounded Knapsack

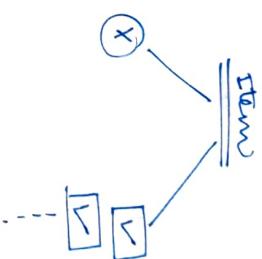
variations

\Rightarrow multiple occurrences
 of same item is
 allowed.

- road cutting
- coin change
- knapsack
- Maximum knapsack

previously

\vdash either item
 is taken or
 not it is taken
 as considered
 (processed)



return $\max \left(\text{val}[\text{i}] + \text{dfs}(\text{i}-1, \text{cap} - \text{wt}[\text{i}]), \text{dfs}(\text{i}-1, \text{cap}) \right)$

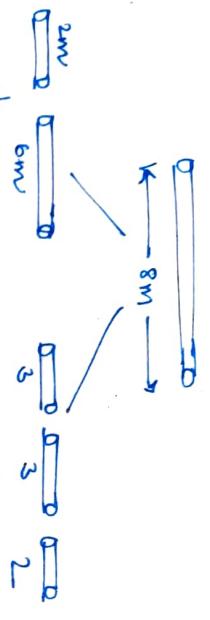
\Downarrow unbounded.

$$\max \left(\text{val}[\text{i}] + \text{dfs}(\text{i}, \text{cap} - \text{wt}[\text{i}]), \text{dfs}(\text{i}, \text{cap}) \right) \Rightarrow t[\text{i}][\text{cap}] = \max \left(t[\text{i}][\text{cap} - \text{wt}[\text{i}]] + \text{val}[\text{i}], t[\text{i}][\text{cap}] \right)$$

$$s_1$$

Rod-cutting Problem

$\text{lengths} \rightarrow 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8$
 $\text{price } \rightarrow 1 \ 5 \ 8 \ 9 \ 10 \ 17 \ 17 \ 20$
 $L \rightarrow 8$



$$\begin{aligned}
 & S+17 \rightarrow 22 \\
 & 8+8+5 \\
 & 16+5 \rightarrow 21
 \end{aligned}$$

↑
maximum profit

$\text{length} > \text{available}$
 $\text{length} \leq L$

$\text{X} \rightarrow \text{we can include any piece - two or more times}$

$$\begin{aligned}
 \text{length} &\rightarrow \boxed{1 \dots n} \\
 \text{price} &\rightarrow \underline{\underline{p_1 \dots p_n}}
 \end{aligned}$$

global $\rightarrow \text{arr}[], \text{price}[]$

$\text{dfs}(i, \text{arr}) \quad \text{int } \text{dfs}(i, \text{arr}) \in$
 $\text{y}(i) \geq \text{arr.size()} \text{return } 0$

$\text{y}(i) \leq \text{arr.size() - 1} \quad \text{int } \text{dfs}(i, \text{arr}, s) \in$
 $\text{y}(i) \geq \text{arr.size()} \text{return } 0; \quad \text{if } (s == 0) \text{return } 1$

else return

$\text{max}(\text{dfs}(i, \text{arr}, s) + \text{price}[i])$
 $\text{dfs}(i+1, \text{arr})$

else

$\Rightarrow \text{return } \text{dfs}(i+1, \text{arr})$

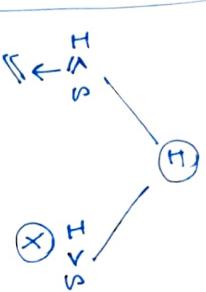
Coin Change - I # of ways

$\text{coins} \rightarrow \boxed{1 \ 2 \ 3}$
 $\text{sum} \rightarrow 5$

$$\begin{aligned}
 & 1+2+2 \\
 & 2+1+1+1 \\
 & 3+2 \\
 & 1+1+1+1+1 \\
 & 3+2+1
 \end{aligned}
 \left\{ \text{sum} = 5 \right.$$

$\text{O/P} \rightarrow 5$

5 numbers of ways to produce 5 as output



$\text{int } \text{dfs}(\text{int } i, \text{int } \text{arr}[], \text{int } s) \{$
 $\quad \text{if } (\text{arr}[i] > s) \text{return } 0;$
 $\quad \text{return } \text{dfs}(i, \text{arr}, s - \text{arr}[i]) +$

$\text{dfs}(i+1, \text{arr}, s)$
 $\}$

$\text{int } \text{dfs}(\text{int } i, \text{int } \text{arr}, \text{int } s) \{$
 $\quad \text{if } (i \geq \text{arr.size()}) \text{return } 0; \quad \text{if } (s == 0) \text{return } 1$
 $\quad \text{if } (\text{arr}[i] \leq s) \{$

$\quad \text{return } \text{dfs}(i, \text{arr}, s - \text{arr}[i]) +$
 $\quad \text{dfs}(i+1, \text{arr}, s)$

else
 $\text{return } \text{dfs}(i+1, \text{arr}, s);$

unbounded
 \rightarrow because we can have multiple instances of single coin

using DP → sum.

1	0	0	1	0	...
↓					

size ↓

$y(w[i:i]) \leq j$

$$t[i][j] = t[i][j-1] + t[i-1][j]$$

use

$$t[i][j] = t[i-1][j];$$

Column Change - II

coin[] → [1 1 2 1 3] // Y

sum → 5

$$O/P \rightarrow \min \# \text{ of coins}$$

$$\begin{matrix} & 2+3 \\ 1+1+1+2. & \underbrace{\quad\quad\quad}_{\min \# = 2} \\ 1+1+3 \\ 1+1+1+1+1 \end{matrix}$$

$$\xrightarrow{\text{sum}} \rightarrow \text{sum}$$



Longest common subsequence

I/P → x: ① ② ③ ④ ⑤
y: ② ③ ④ ⑤ ⑥ ⑦

lcs → abab
→ O/P → 4

subsequence

we have to choose continuous

subsequence

we have to skip element

int lcs (string x, string y, i, j) {

~~if (i == 0 || j == 0)~~

return 0;

$y(x[i]) = y[j]) \{$

return 1 + lcs (x, y, i+1, j+1);

use {

}

return max (lcs (x, y, i+1, j),
lcs (x, y, i, j+1));

}

use {

memoized

built an array / vector of longest imp + 1

vector<vector<int>> memo (n+1, vector<int> (m+1, -1))

n → x.length()
m → y.length()

Top down DP

$x : \underline{a} \underline{b} \underline{c}$ $\underline{d} \underline{e} \underline{f}$	$\rightarrow abc$
$y : \underline{a} \underline{c} \underline{b} \underline{c} \underline{f}$	$\rightarrow abc$

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$\rightarrow dp[i+1][m+1]$

$$y(x[i-1]) == y(i-1) \}$$

$$t[i][j] = L + t[i-1][j-1];$$

use ?

$$t[i][j] = \max(t[i-1][j], t[i][j-1]);$$

longest common substring

$x \rightarrow \underline{a} \underline{b} \underline{c} \underline{d} \underline{e}$
 $y \rightarrow \underline{a} \underline{b} \underline{c} \underline{d} \underline{e}$
 $O/P \rightarrow \textcircled{2}$

int afs (string x, string y, int i, int j, int count)

{ if (i >= x.length() || j >= y.length()) return 0;

if (x[i] == y[j]) {

int temp = afs (x, y, i+1, j+1);

res = max (res, temp);

else afs (i+1, j+1)

return count;

3

Print longest common subsequence

$x : \textcircled{a} \textcircled{b} \textcircled{c} \textcircled{d} \textcircled{e}$
 $y : \textcircled{a} \textcircled{b} \textcircled{c} \textcircled{d} \textcircled{e}$

O/P $\rightarrow abc$

		0	1	2	3	4	5	6
0	0	0	0	0	0	0	0	0
1	0	1	1	1	1	1	1	1
2	0	1	1	2	2	2	2	2
3	0	1	2	2	2	2	2	2
4	0	1	2	3	3	3	3	3
5	0	1	2	3	3	3	3	4

start with $i = n-2, j = m-1$
 what do we
 reverse
 engineering
 $\{ \text{else } \}$
 $\{ \text{if } \}$

$y(x[i-1]) == y(j-1) \}$

$y(i-1) == y(j-1) \}$
 $y(i-1) == y(j-1) \}$
 $y(i-1) == y(j-1) \}$

if equal $i, j \rightarrow i--, j--$
 else $i, j \rightarrow \max \left(\begin{matrix} i-1, j \\ i, j-1 \end{matrix} \right)$

$\text{int } i = m; j = n;$
 while $(i > 0 \text{ and } j > 0)$ {

$b(a[i-1]) == b(t[j-1]) \{$

$\text{s.push_back}(a[i-1]);$

$i--; j--;$

else {

$t[i] > t[j-1])$

$i--;$

else $i--;$

$\}$
 $\}$
 $\}$
 $\}$
 $\}$
 $\}$
 $\}$
 $\}$

shortest common supersequence

a: "geek"
 b: "eek"
 +
 merge \rightarrow geek eek \rightarrow geekeee
geeee

shortest length $\rightarrow m+n - \underline{\text{LCS}}$

$\xrightarrow{\text{to calculate the length we can just}}$
 $n+m - \underline{\text{LCS}}$

$\xrightarrow{\text{in other terms, it's called as LCS}}$
 $\xrightarrow{\text{we have used this only once as it's present in both.}}$

$\xrightarrow{\text{A(G)G(T)X A(Y)B}}$
 $\xrightarrow{\text{length} \rightarrow 9}$
 $\xrightarrow{\text{shortest supersequence}}$

$\xrightarrow{\text{both sequences are present in this string}}$
 $\xrightarrow{\text{not necessarily contiguous}}$
 $\xrightarrow{\text{continuous.}}$

$\xrightarrow{\text{a: AGGTAB}}$
 $\xrightarrow{\text{b: GXTXAYB}}$
 $\xrightarrow{\text{AGGTGXABTXAYB}}$

Minimum # of insertion and deletion

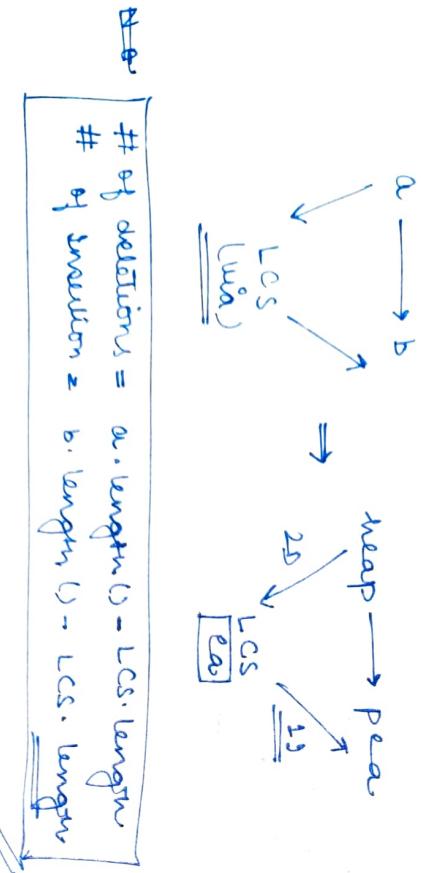
$a \rightarrow \text{pear}$

$b \rightarrow \text{pear}$
 perform insert
and delete on
string a to
remove .
remove it to
connect it to
'b'

~~p~~~~pear~~ → pear

pear

insert p
remove to
remove p .
ins: 1
del: 2



- ① if we note two special point

- ② ea remains intact as this is new

LCS

Longest Palindromic Subsequence

I/P → S: abcba

we can have
discontinuous / → that is
continuous sequence
longest

I/O → S

- ④ we can find the longest common subsequence of S and reverse (S)

$$\text{LPS}(a) \equiv \text{LCS}(a, \text{reverse}(a))$$

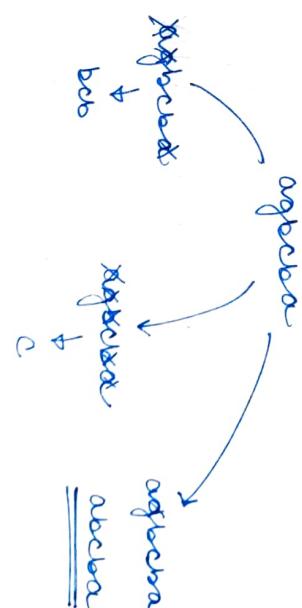
Minimum # of deletions in a string to make it Palindrome

minimum #
of deletion

=

S.length -

LPS(S)



Print SCS → shortest common supersequence

worst case → m+n

better
 $\hookrightarrow m+n-LCS$

↓
 unoptimized

abcba

abcba

Longest Repeating Subsequence

$\text{str} = "AABEBCDD"$

We have to find a subsequence in this string which is repeating.

	a	b	c	d	e	f	g
a	0	0	0	0	0	0	0
b	0	1	1	2	2	2	2
c	0	1	2	2	2	2	2
d	0	1	2	3	3	3	3
e	0	1	2	3	3	3	4

LCS $\rightarrow \boxed{abcd}$

$\text{while } i > 0 \quad \& \quad j > 0 \quad \{$

~~if $a[i-1] == b[j-1]$~~

~~$i--$; $j--$~~

~~res.push_back(a[i-1])~~
~~$i--$; $j--$~~
~~res.push_back(b[j-1])~~

$\text{while } (j-1) > 0 \quad \{$

~~if $a[i-1] == b[j-1]$~~
~~$j--$~~
~~res.push_back(b[j-1])~~

$\text{reverse}(res)$

~~if $(+t[i-1][j-1] > +t[i-1][j])$~~
~~$t[i-1][j] = +t[i-1][j-1] + 1$~~
~~else {~~
~~res.push_back(b[j-1])~~

~~if $(a[i-1] == b[j-1] \& \& i_1 == j)$~~
~~$i_1 = j \Rightarrow \underline{\text{include}}$~~

$+t[i][j] = \max (+t[i][i-1], +t[i-1][j])$

$+t[i][j] = \max (+t[i][i-1], +t[i-1][j])$

We will find LCS among ~~str~~ and ~~dup(str)~~

~~A A B E B C D
A B E B C D D
A B E B C D D
A B E B C D D~~

while including in result

~~if $a[i-1] == b[j-1] \& \& i == j$~~

~~\rightarrow we will not include it~~

$a[i-1] == b[j-1] \& \& i_1 == j \Rightarrow \underline{\text{include}}$

$+t[i][j] = \max (+t[i][i-1], +t[i-1][j])$

Sequence Pattern Matching

Matrix Chain Multiplication

$S/P \rightarrow a: "AXY"$

$b: "ADXCOPY"$

"A" is a subsequence of B

we can sum two loops and check if it is equal or we can just try LCS

$LCS("AXY", "ADXCOPY") == "AXY"$

→ true
use false

Minimum Number of insertions to make it Palindrome

$s: a e b e b d a$

~~a@e b e b d a~~

$s \rightarrow a e b c b d a$

we'll find the LCS and the remaining elements (which are not included in LCS) are more which do not have a pair

string - (LCS.length)

we'll find the LCS and the remaining elements (which are not included in LCS)

for ($i = 0; i < n; i++$) {
 temp = solve(...);
 ans = temp + solve(...);
}

ans = fun(temp, ans);

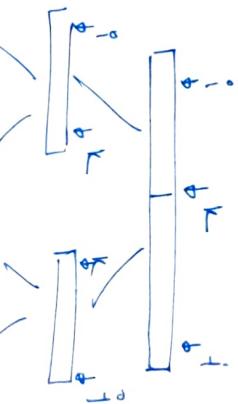
- 1) MCM
- 2) printing MCM
- 3) Evaluate Exp to True / Boolean Parenthesis
- 4) Max min value.
- 5) Palindrome partitioning
- 6) Scramble string
- 7) Egg dropping.

Identification + formal

breaking the string at any point

$fun(i, j)$

$fun(i, k) \quad fun(k, j)$



int solve (int arr[], int i, int j) {

if ($i > j$) return 0;

for (int k = i; k < j; k++) {
 temp = solve(...);
 ans = temp + solve(...);
 }

Matrix chain multiplication

cost of multiplying
two matrices

$$\begin{array}{c} A_1 \\ \downarrow \\ a \times b \end{array}$$

$$\begin{array}{c} A_2 \\ \downarrow \\ b \times c \end{array}$$

$$arr[i] \rightarrow \boxed{\quad \quad \quad}$$

$$a_1 \ a_2 \ a_3 \ a_4$$

$$\begin{array}{c} ((a_1 a_2) \ a_3 \ a_4) \\ ((a_1 (a_2 \ a_3)) a_4) \\ \hline \end{array}$$

$$\begin{array}{c} A \rightarrow 5 * 30 \\ B \rightarrow 30 * 7 \\ C \rightarrow 7 * 10 \\ \hline \end{array}$$

$$ABC \rightarrow \boxed{5 \ 30 \ 7 \ 10}$$

$$\begin{array}{c} AB \rightarrow (5 \times 30 \times 7) \\ = 5 \times 1050 \\ \hline \end{array}$$

return minimum cost after multiplications

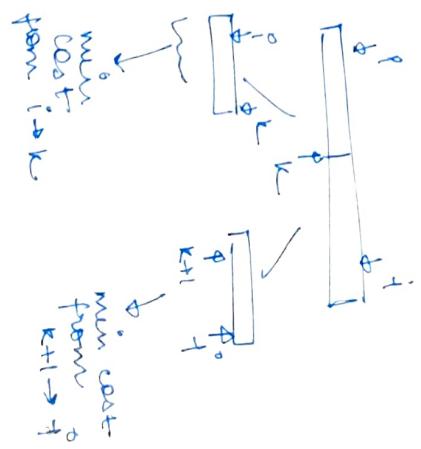
$$arr[i] \rightarrow \begin{matrix} 0 & 20 & 20 & 30 & 40 & 30 & 0 \end{matrix}$$

no of matrices
 $= \text{size}(C) - 1$

$$\begin{array}{c} a_1 \rightarrow 40 \times 20 \\ a_2 \rightarrow 20 \times 30 \\ a_3 \rightarrow 30 \times 10 \\ a_4 \rightarrow 10 \times 30 \end{array}$$

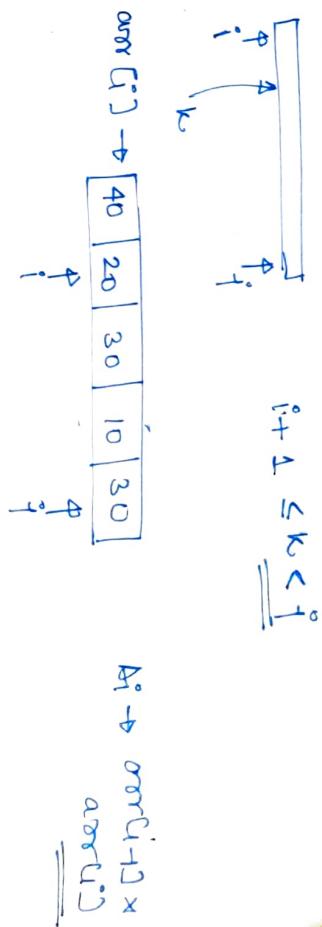
$$A_i \rightarrow arr[i-1] * arr[i]$$

we have ensured
that a group
contains atleast
1 matrix



$$\begin{array}{c} (AB)(CD) \\ \downarrow \\ \min \text{ cost of } m.(CD) \end{array}$$

$$\begin{array}{c} \text{new cost} \\ \downarrow \\ (A_1)(A_2 \ A_3 \ A_4) \\ \hline \end{array}$$



int solve (int i, int j, int arr[]) {

if ($i \geq j$) return 0;

// move k
 $i \rightarrow j$

for (int k = i; k < j; k++) {

solve (arr, i, k);

solve (arr, k+1, j);

}

$$i+1 \leq k < j$$

40	20	30	10	45	60
----	----	----	----	----	----

min cost
of multiplication
of $i \rightarrow k$

(40×20)
 $\underline{(20 \times 30)}$

min cost of
multiplication of

$\underline{30 \times 10}$
 $\underline{10 \times 45}$
 $\underline{45 \times 60}$

temp ans 1 = value (i^o, k^o)
temp ans 2 = value ($k+1^o, j^o$)

sum → temp ans 1 + temp ans 2 +

dimensions
of res mat
 $\underline{40 \times 30}$
 $\underline{30 \times 45}$
 $\underline{45 \times 60}$

\Rightarrow $y(\text{dp}[i][j] * 10) = -1$ return $\text{dp}[i][j]$
 $\Rightarrow \text{dp}[i][j] = \text{temp_res}$
 \Rightarrow return $\text{dp}[i][j]$

\Rightarrow size of array
size of array
 \Rightarrow int memo [n+1][n+1];

return memo [i+1][j+1];

return memo;

$y(\text{temp} < \text{mn})$ mn = temp;

Palindrome Partitioning

worst case

partition after every
character as
worst case is a
single character

\Rightarrow temp_res = temp 1 + temp 2 + arr[i-1] *
arr[i] * arr[i] * arr[i].

int solve (int arr[], int i, int j) {

if ($i \geq j$) return 0; int mn = INT_MAX;

for (int k = i; k <= j-1; k++) {

int temp_res = value (arr, i, k) +

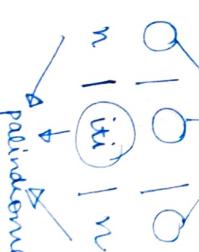
value (arr, k+1, j) +

arr[i-1] * arr[k] * arr[j];

min

no of partitions \Rightarrow 2

if $\min \rightarrow 4$

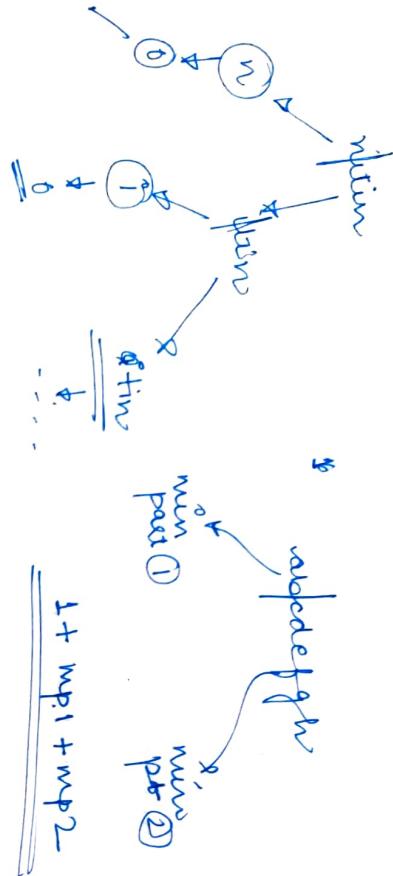


partition

optimization → ~~Efficient memorization~~

There is a probability that one recursive problem, i.e. same (i, k) or same $(k+1, j)$ might get solved previously.

$$\text{int temp} = \text{sum}(\text{arr}, i, k) + \text{sum}(\text{arr}, k+1, j);$$



whenever we find
a palindrome
string we
need 0 partitions

`int sume (int arr[], int i, int j){`

`if ($i > j$) return 0; int res = INT_MAX;`
 `if (isPalindrome(arr, i, j)) return 0;`
 `for (int k = i; k < j; k++) {`

`int temp1 = sume (arr, i, k);`
 `int temp2 = sume (arr, k+1, j);`
 `int temp_res = 1 + temp1 + temp2;`
 `res = min (res, temp_res);`

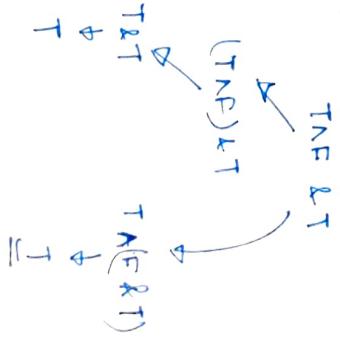
Evaluate Expressions to True | Boolean Parenthesis

string → T | F & F

put pos brackets in a way so that the evaluated string becomes true.

symbols can be of T, F, 1, k, n

```
    }  
    return res;  
}
```



$T \mid F \& T \wedge F$

$$\downarrow \\ T \mid (F \wedge (T \wedge F))$$

$$T \wedge T \\ \uparrow \\ T$$

$$T \wedge T \\ \uparrow \\ T$$

- * we'll put the position of k in such a way that $k+1$ we know brackets

$$T \mid (F \wedge T \wedge F) \\ \uparrow \\ T \wedge F \text{ and } T \wedge F \\ \uparrow \\ k \\ \exp^x \text{ XOR } \exp^x \\ (\text{base}) \\ k \\ \exp^x \text{ XOR } \exp^x \\ (k+1 \text{ to } i)$$

- * we'll always put a bracket on operation to jump on operators always $[k = k + 2]$

1) find $i \neq j$

$$i = 1, j = n - 2$$

$$n \rightarrow s.length()$$

2) find base condition

$$if (i > j) return false$$

$$if (i == j) \{$$

$$if (list[i] == true)$$

$$return s[i] == 'T';$$

$$else return s[i] == 'F';$$

if we wanted
true over else f
then \neg

\boxed{T}

- we have to find both
- when the expression is evaluated true
 - when the expression is evaluated false

$$exp_1 \wedge exp_2 \\ \uparrow \quad \uparrow \\ T \wedge P \rightarrow T \rightarrow T$$

$$exp_1 \quad exp_2 \\ \uparrow \quad \uparrow \\ T \rightarrow 2 \quad T \rightarrow 3 \\ P \rightarrow 4 \quad P \rightarrow 5 \\ 2 \times 5 + 4 \times 3 \\ \boxed{10 + 12 = 22}$$

so for doing function call, we'll pass parameters

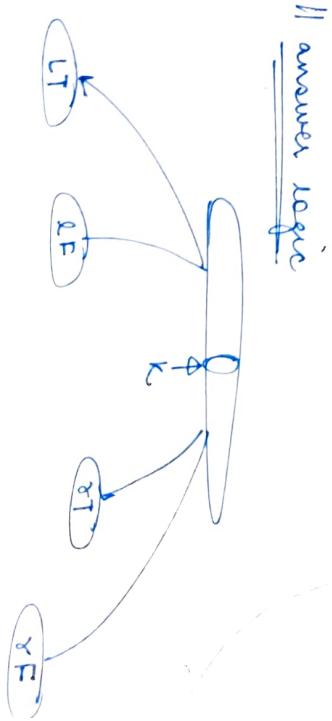
$$(T/F)$$

some (i, i, T)
some (i, i, F)

for $(int k = i+1; k <= j-1; k = k+2) \{$

```
int LT = some (i, k-1, T);
int RT = some (k+1, j, T);
int LF = some (i, k-1, F);
int RF = some (k+1, j, F);
```

|| answer logic



$y(s[i] == 'R')$

$y(i \text{ true} == \text{true}) \{$

$\text{ans} = \text{ans} + (\text{LT} * \text{NT});$

else
 $\text{ans} += (\text{LF} * \text{RT} + \text{LF} * \text{RF} + \text{LF} * \text{RF});$

$y(s[k] == 'I') \{$

$y(\text{isTrue}) \{$

$\text{ans} +=$

$$\begin{pmatrix} \text{LT} * \text{RT} + \\ \text{LT} * \text{RF} + \\ \text{LF} * \text{RT} \end{pmatrix}$$

$y(\text{use})$

$\text{ans} += \text{LF} * \text{RF};$

$y(s[i] == 'N') \{$

$y(\text{isTrue}) \{$

$\text{ans} += \begin{pmatrix} \text{LF} * \text{RF} + \\ \text{LT} * \text{RF} \end{pmatrix};$

$y(\text{use})$

$\text{ans} += \begin{pmatrix} \text{LF} * \text{RF} + \\ \text{LT} * \text{RF} \end{pmatrix};$

y
return ans;

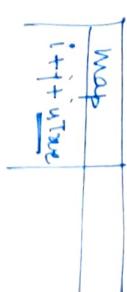
memorization

Dimension depends on
number of variables changing

Here ~~variables~~ changing \rightarrow i, j, isTrue
dimensions $\rightarrow [n+1][n+1][2]$

Better way
we can use map
 $\text{map} \xrightarrow{i+j+\text{isTrue}}$

dimensions $\rightarrow [n+1][n+1][2]$



$\text{key} = \text{to_string}(i) + \text{to_string}(j) + \text{to_string}(\text{isTrue})$

$$= "5+9+\text{F}"$$

$\xrightarrow{\text{key of the map}}$

Scrambled string

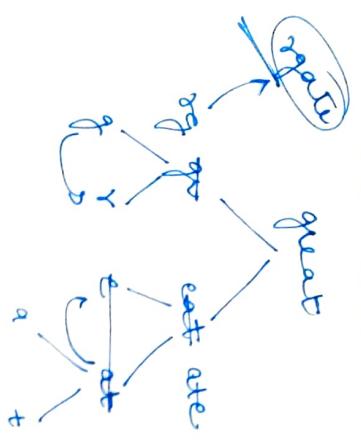
I/P \rightarrow a: "great"
b: "repeat"
O/P \rightarrow True

We represent two string as a binary tree by partitioning it into two non-empty string recursively.

Scrambled string \rightarrow to generate a scrambled string we can choose any two non-leaf node and swap them.

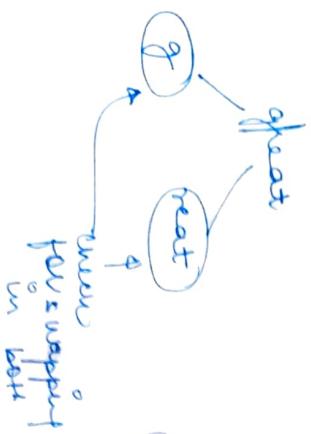
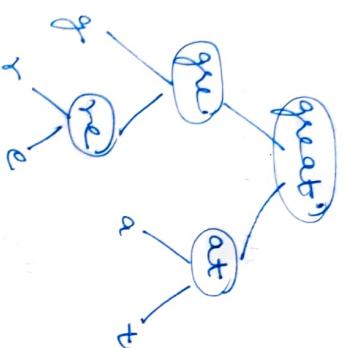
1D | 2D | 3D

D) binary tree only
d) No empty string
child



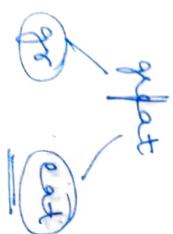
Non-leaf
node can be
swapped by
children

great > scrambled
great stings T



$$[K = i^0 + 1 \rightarrow n-1]$$

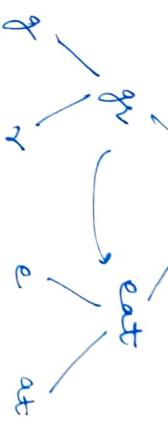
great



even
far = swapped
in last

swapping → zero or more

$i=2$
 $great \Rightarrow catgr$



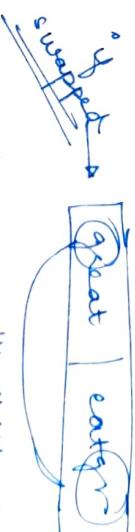
great → great
great → catgr

swap X
swap V

return True
in both
cases.

not swap

we can have
2 case at
any i
1. fine swap
2. not swap



we'll check the
first K letters of
string with last K
if it's scrambled.

not
swapped →



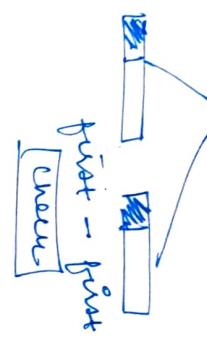
even in order only

scramble

swap ✓
swap ✗.

base
lcase
rcase

first → last
check



y (scramble (a.substr (0, k), b.substr (n-k, r)))

kk

return true;

scramble (a.substr (k, n-k), b.substr (0, n-k))

return true;

scramble

y (scramble (a.substr ('0', k),

b.substr ('0', k))

kk

scramble (a.substr ('k', n-k),

b.substr ('k', n-k))

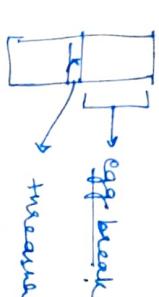
return true;

y (scramble (a.substr (0, k), b.substr (n-k, k)))

kk

scramble (a.substr (0, k), b.substr (0, n-k))

return true;



minimum number
of attempts to
find critical
floor.

measurable/
critical floor.

Egg Dropping Problem

$$\begin{cases} e = 3 \\ f = 5 \end{cases}$$

memoization → use map + key can be
 $a + '!' + b$
 $a + '!' + b$
 $a + '#' + b$

base case (string a, string b) &
y (a.compare (b) == 0)
return true;

y (a.length () <= 1)
return false;

int n = a.length ();
bool flag = false;
for (int i = 1; i < n; i++) {
 if (cond1 || cond2) {
 flag = true;
 break;
 }
}

return flag;

-

measurable/
critical floor.

wall wall
✓ ✓

- we drop egg from every floor and check if it breaks.

we have to use 'e' numbers of eggs ^{so that} we can find the critical floor.

OR

suppose no of floors >> number of eggs.

in this case, if we don't use the eggs wisely we won't be able to find the critical floor after using all the eggs also.

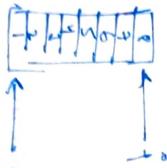
Also, suppose we'll start from bottom we can find the critical floor in 1 egg only but the attempts \rightarrow huge!!

• use egg wisely & minimize # of attempts.

one



\downarrow
floors



basically we don't know from which floor to start my first attempt.

return mn;

4

```
int temp = 1 + max ( save ( e-1, k-1 ),
                      save ( e, f-k ) );
```

```
mn = min ( mn, temp );
```

```
int mn = INT_MAX;
```

```
for ( int k = 1; k <= f; k++ ) {
```

```
    if ( b == 0 || t == 1 ) return t;
    if ( e == 1 ) return t;
```

```
    int save ( int e, int f ) {
```

```
        if ( b == 0 || t == 1 ) return t;
```

```
        if ( e == 1 ) return t;
```

```
        int mn = INT_MAX;
```

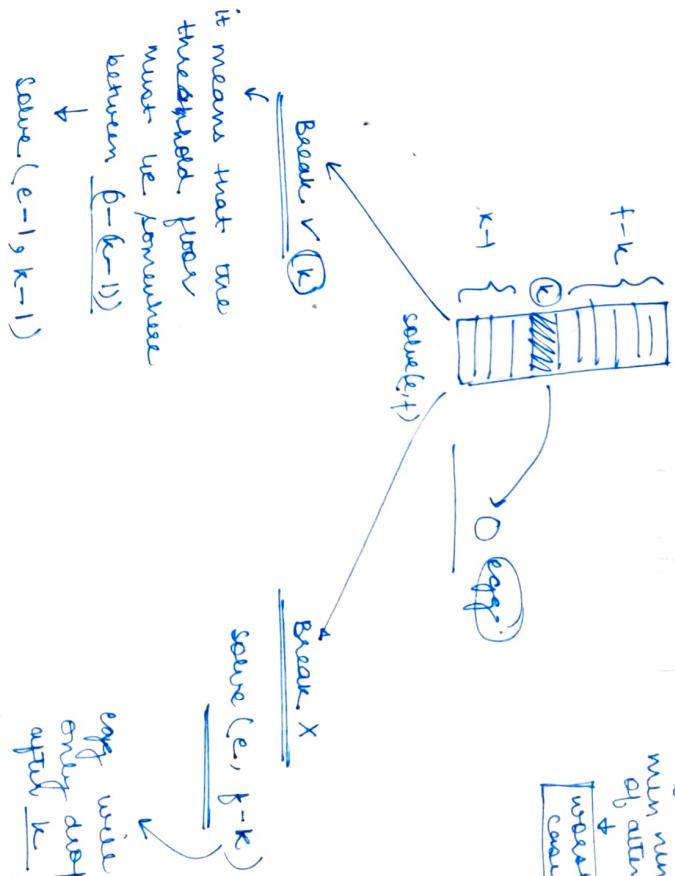
```
        for ( int k = 1; k <= f; k++ ) {
```

```
            int temp = 1 + max ( save ( e-1, k-1 ),
                                  save ( e, f-k ) );
```

```
            mn = min ( mn, temp );
```

```
        }
```

min number of attempts
+ worst case



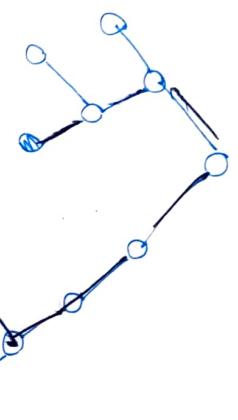
DP ON TREES

diameter of tree

- maximum path between two leaf nodes

identification

- ↳ we're traversing a tree and on each node we again need to call recursive functions.



General syntax

```
int score ( Node *root, int *res ) {
    if (root == NULL) return 0;
    int l = score (root->left, res);
    int r = score (root->right, res);
    int ans // Suppose
    int temp = calculate temp ans
    *res = max (temp, l+r);
```

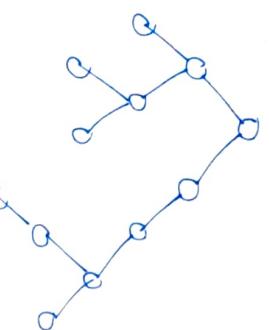
(\downarrow) question dependent.

```
int res = INT-MAXMIN;
int score ( Node *root, int &res ) {
    if (root == NULL) return 0;
    int ans = max (max (root->left->val, root->right->val),
                  max (root->left->val + root->right->val));
    res = max (temp, ans);
    return ans;
```

diameter of binary tree

int res = INT-MAXMIN;

```
int score ( Node *root ) {
    if (root == NULL)
        return 0;
    int l = score (root->left);
    int r = score (root->right);
    int temp-ans = l+r;
    res = max (res, temp-ans);
    return l+max (l,r);
```



Maximum Path sum

maximum path sum from any node to any node

↳ question becomes different when value of node becomes negative

↳ if the value of 'l' or 'r' comes out to be negative then we won't include that

```
int max ( max (l,r)+root->val, )
        (root->val);
```

```
int score ( Node *root, int &res ) {
    if (root == NULL) return 0;
```

```
    int ans = max (temp,
                  max (l+r, l+r+root->val));
    res = max (temp, ans);
    return ans;
```

Maximum Path sum from leaf to leaf

```
int solve ( Node *root) {  
    if (root == NULL) return 0;  
    int l = solve (root->left);  
    int r = solve (root->right);  
    int tempAns = max (l,r) + root->val;  
    res = max (res, root->val + l+r);  
    int ans = max (tempAns, l+r+root->val);  
    res = max (ans+res);  
    return temp;  
}
```

