Indian Institute of Information Technology Ranchi
Jharkhand-834010

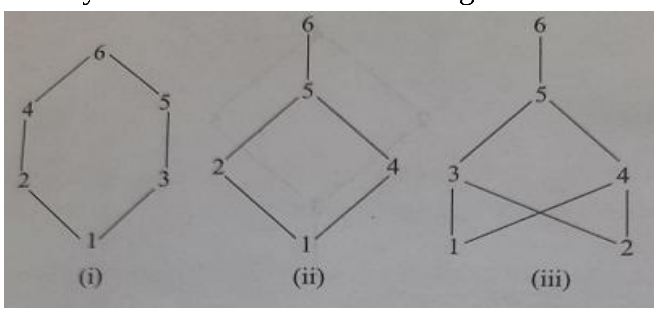
Department of Mathematics

Assignment-III

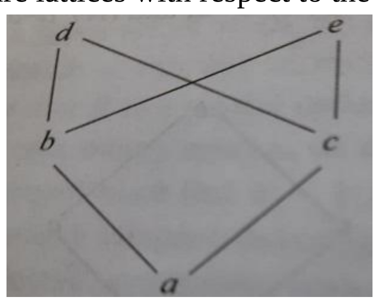
Subject Name: Discrete Mathematics, **Subject Code:** CS1004

Instruction: Solve all the questions systematically and submit it **by 5:30PM** on **27th July**, 2023 to the respective faculty.

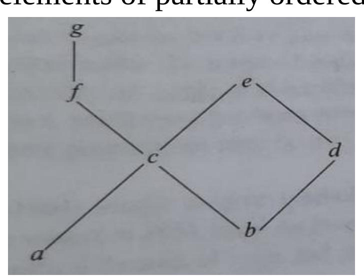
1. Which of the partially ordered sets shown in the figures below are lattices?



2. Consider the partially ordered set $S = \{a, b, c, d, e\}$ shown in the figure given below. Find the two subsets of S that are lattices with respect to the operation on *S*.



3. Find the minimal and maximal elements of partially ordered set A given as



4. Let L be a bounded distributive lattice. Then prove that their complements are unique if they exist.
5. If $(L, \leq)$ is a lattice, then show that $(L, \geq)$ is also a lattice.

6. Show that a finite partial ordered set has (i) at most one greatest element and (ii) at most one least element.
7. In a group of six people, at least three must be mutual friends or at least three must be mutual strangers.
8. Show that a monoid is a group if and only if cancellation laws hold in it.
9. If in a semi group S, $x^2y = y = y^2x$, for all x and y, then show that S is an abelian group.
10. Let G be the group of all 2×2 non singular matrices over the real numbers, find the centre of G.
11. Union of two subgroups is a subgroup iff one of them is contained in the other.
12. Coset is not essentially a subgroup, justify your answer.
13. If $G = S_3$ and $H = \{I, (13)\}$, then write all the left cosets of H in G.
14. Show that a cyclic group is abelian. However, converse need not be true.
15. Show that the group of rational numbers under addition is not cyclic.
16. If in a ring R with unity, $(xy)^2 = x^2y^2$ for all x and y in R, then show that R is commutative.
17. If in a ring R, the equation $ax = b$ for all a, b ($a \neq 0$) has a solution then show that R is a division ring.
18. Let R be the ring of 3×3 matrices over real numbers. Show that $S = \left\{ \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} \mid x \text{ is real} \right\}$ is a subring of R, and has unity different from the unity of R.
19. A ring of order p^2, (p a prime) may not be commutative.
20. A field is a ring but converse need not be true, justify your answer.
21. Z_7 is a field but Z_8 is not, why? Justify your answer.
22. Draw the diagram of each of the following graphs $G = (V, E)$:
    (a) $V = \{1, 2, 3, 4, 5, 6\}$ and $E = \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 5\}, \{2, 6\}, \{3, 5\}, \{3, 6\}, \{4, 5\}, \{4, 6\}\}$.
    (b) $V = \{1, 2, 3, 4, 5\}$ and $E = \{\{1, 2\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 5\}\}$
23. Prove that the sum of the degrees of all vertices in a graph G is equal to twice the number of edges in G.
24. The number of odd vertices in a graph is always even. Justify it
25. Find the number of edges in the complete graph with n vertices.
26. Show that a graph is a tree if and only if there is a unique path between every pair of vertices in the graph.
27. Show that a graph is a tree if and only if it is connected and every edge in it is a bridge.

28. A connected graph G is an Euler graph if and only if the degree of every vertex in G is even.
29. Describe Prim's algorithm.
30. Show that a graph G with n-vertices, n-1 edges and no circuits is tree.

*****************************DO SMILE*******************************