MATH 8650

Advanced Data Structures

Term Project

Optimization of Bellman Ford Algorithm

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November - 30, 2018

Abstract

Dijkstra's and Bellman Ford algorithms are two of the popular algorithms used for path finding. Though Dijkstra's outperforms Bellman Ford, it does not work in the presence of negative edge weight cycle. To get the best of both algorithms, Shortest Path Faster Algorithm(SPFA) is used. This report presents the comparison study of the above mentioned algorithms.

1 Introduction

This report considers the problem of shortest path finding. Shortest path is the minimum cost between two vertices in a given graph. The common algorithms used to find the shortest paths are Dijkstra, A*, Floyd-Warshall and Bellman Ford. Bellman Ford algorithm works with negative edge weights and it detects negative cycles in the given graph. Detecting negative cycles are important as they can cause the algorithm to run into an infinite loop. Edge weights are not only restricted to represent length, but can also represent time, money and negative costs like loss.

Consider the example of traffic congestion on a map. Here the weights represent traffic conditions in the map. More unfavorable conditions results is more negative weights. In such cases, Bellman Ford algorithm is considered. In order to optimize the performance of Bellman Ford algorithm, SPFA is considered.

Shortest Path Faster Algorithm (SPFA) is an optimization of Bellman Ford Algorithm which computes single-source shortest path in a weighted directed graph. Though the worst-case complexity of SPFA is same as Bellman Ford, it performs better on practical random graphs.

In this report, we do a comparison study between Bellman Ford, Dijkstra's and several versions of SPFA which are SPFA - FIFO, SPFA - LIFO and SPFA - PAPE.

2 Implementation

This section describes the implementation of Bellman Ford and the three versions of SPFA algorithms.

The input to the Bellman Ford Algorithm is (G, src), where $G = (V, E)$ is a directed graph with V vertices and E Edges. src $\in$ V is the source node and the algorithm finds the shortest path from src to all other nodes.

The following steps are considered while implementing the Bellman Ford Algorithm:

1. Create a distance array, dist[] and initialize it to infinity for all nodes and zero for source.
2. Perform relaxation of nodes for V - 1 times. This finds the shortest distances.
3. Perform negative cycle check by doing one more relaxation.
4. Display the results.

2.1 Negative Cycle Check

For a given source node, Bellman Ford algorithm first calculates the shortest distance which has at-most one edge in the path. Then, it calculates shortest path with at-most two edges, and so on. After V - 1 iterations, shortest distance to all the vertices in the graph will be calculated. Another shorter distance found after the $V - 1^{th}$ iteration indicates the presence of a Negative Cycle. The algorithm can be stopped here otherwise it will fall into an infinite loop.

2.2 Node Relaxation

Relaxation is the most important step in Bellman-Ford. It is what increases the accuracy of the distance to any given vertex. Relaxation works by continuously shortening the calculated distance between vertices comparing that distance with other known distances.

The distance of the current node is compared with the sum of the weight(w) of the edge and the distance of the node connected to the edge. If the sum is lesser than then distance of the current node, then the current node distance is updated. i.e if dist[v] $<$ w $+$ dist[u] then update dist[v]. Here v is the current node and u is the node connected to the edge.

2.3 Shortest Path Faster Algorithm

The Bellman Ford Algorithm scans every node in the graph for each iteration. This is computationally expensive. The complexity of the Bellman Ford Algorithm is O($|V| \cdot |E|$), where $|V|$ and $|E|$ represent the number of vertices and edges respectively. This process can be optimized by scanning only the nodes that were relaxed/updated in the previous iteration.

In the SPFA, a double-ended-queue is maintained, which holds the vertices that are recently updated. A vertex is popped from the queue, and all its adjacent vertices are scanned for shorter paths. If any vertex is updated, it is added to the queue unless it is already in the queue.The algorithm ends when there are no elements left in the queue.

The performance of the SPFA is strongly determined by the order in which candidate vertices are used to relax other vertices. Three different orders were selected for this report.

1. SPFA-FIFO: Here a First In First Out Queue is chosen, so that the initially updated vertex will be popped and processed first. Here the vertices are appended to the tail and popped from the head of the queue.

2. SPFA-LIFO: In this method, the most recently updated vertex is processed first. To achieve this order, vertices are appended to the tail of the queue and popped from the tail of the queue.

3. SPFA-PAPE: In this method, pop vertices from the head. Append first-time vertices to the tail, others are appended to the head.

2.4 Data Structures Used

- Priority Queues were used in Dijkstra's algorithm.
- set() was used to store edges and vertices in Bellman Ford and SPFA algorithms to prevent duplication of nodes.
- lists were used to store distance values.
- Dictionary was used to map the edges to its corresponding weights.
- Deque was used in SPFA to maintain the order of vertex update.
- networkx and random was used to generate random graphs.

3 Results

This section describes the results of the python3.0 implementation that was discussed in section 2.

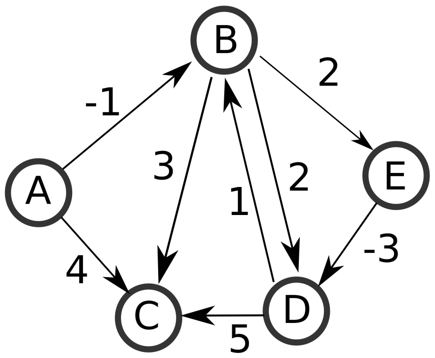


Figure 1: Directed graph

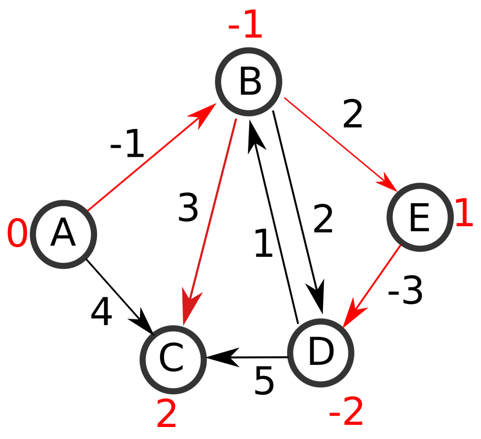
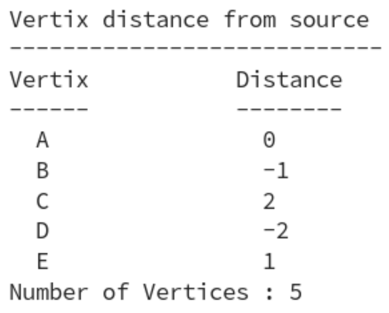


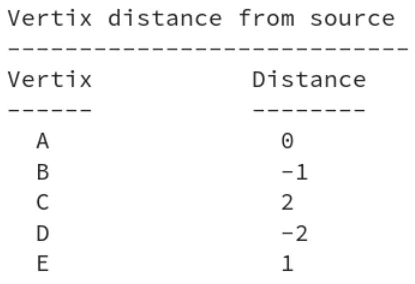
Figure 2: Graph with shortest paths highlighted

Figure 1 represents a five vertex graph that is used to test the Bellman Ford algorithm.
Figure 2 shows the shortest paths calculated by the Bellman Ford Algorithm.
Figure 3 represents the output of the implemented python code, that shows the shortest distance from Source node 'A' to other nodes for the graph shown in Figure 1.



```
Vertix distance from source
---------------------------
Vertix          Distance
------          --------
  A               0
  B               -1
  C               2
  D               -2
  E               1
Number of Vertices : 5
```

Figure 3: The output of Bellman Ford algorithm



```
Vertix distance from source
---------------------------
Vertix          Distance
------          --------
  A               0
  B               -1
  C               2
  D               -2
  E               1
```

Figure 4: The output of SPFA

Figure 4 represents the output of SPFA algorithm for the same graph that was used for Bellman Ford verification.

It can be observed that the output for SPFA and Bellman Ford are the same. Since all versions of SPFA give the same shortest distance, only one output was included above.

The Figures 1 , 2, 3 and 4 are just a sample dataset, chosen to verify the Bellman Ford and SPFA algorithms.

Algorithm	nodes = 10	nodes = 50	nodes = 100	nodes = 250	nodes = 500
Dijkstra	51.6 μs	832 μs	2.35 ms	10.4 ms	38.2 ms
Bellman Ford	428 μs	57.4 ms	477 ms	8.43 s	1 m 20 s
SPFA FIFO	15.1 μs	393 μs	1.49 ms	11 ms	43.9 ms
SPFA LIFO	19.3 μs	472 μs	1.7 ms	9.62 ms	60.9 ms
SPFA PAPE	16.5 μs	366 μs	1.56 ms	12.3 ms	50.9 ms

Table 1: Non-negative weight runtime comparison

Table 1 displays the average runtime for implemented algorithms for graphs of various node count with positive edge weights.

Algorithm	nodes = 10	nodes = 50	nodes = 100	nodes = 250	nodes = 500
Bellman Ford	453 μs	60.1 ms	496 ms	9.26 s	1 m 25 s
SPFA FIFO	14.4 μs	424 μs	2.73 ms	42.6 ms	380 ms
SPFA LIFO	15.8 μs	377 μs	2.51 ms	68.4 ms	720 ms
SPFA PAPE	14.9 μs	473 μs	3.14 ms	57.6 ms	710 ms

Table 2: Non-negative and negative edge weight runtime comparison

Table 2 represents the runtime comparisons of implemented algorithms for a combination of negative and non-negative weights.

4 Conclusion

Bellman Ford and three versions of SPFA was implemented using python3.0. Table 1 and Table 2 shows the average runtime for these algorithms. The averages were taken over 7 runs of each algorithm.

For non-negative weight graphs, Dijkstra runs 80 times faster than Bellman Ford and SPFA's perform 1.4 times faster than Dijkstra's for 250 node random graphs. Among the variations of SPFA on average FIFO performs better than the other two.

For combination of negative and non-weight graphs, SPFA-FIFO performs better than other SPFA variations.

References

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