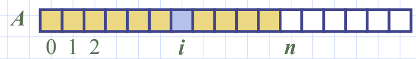
Array-Based Sequences

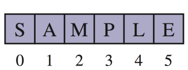
Python Sequence Classes

- Python has built-in types, **list**, **tuple**, and **str**.
- Each of these **sequence** types supports indexing to access an individual element of a sequence, using a syntax such as A[i]
- Each of these types uses an **array** to represent the sequence.
  - An array is a set of memory locations that can be addressed using consecutive indices, which, in Python, start with index 0.

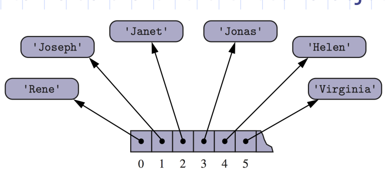


Arrays of Characters or Object References

- An array can store primitive elements, such as characters, giving us a **compact array.**



- An array can also store references to objects.



Compact Arrays

- Primary support for compact arrays is in a module named **array**.
  - That module defines a class, also named array, providing compact storage for arrays of primitive data types.
- The constructor for the array class requires a type code as a first parameter, which is a character that designates the type of data that will be stored in the array.

```
primes = array('i', [2, 3, 5, 7, 11, 13, 17, 19])
```

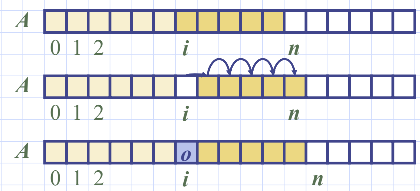
Type Codes in the array Class

- Python's array class has the following type codes:

Code	C Data Type	Typical Number of Bytes
'b'	signed char	1
'B'	unsigned char	1
'u'	Unicode char	2 or 4
'h'	signed short int	2
'H'	unsigned short int	2
'i'	signed int	2 or 4
'I'	unsigned int	2 or 4
'l'	signed long int	4
'L'	unsigned long int	4
'f'	float	4
'd'	float	8

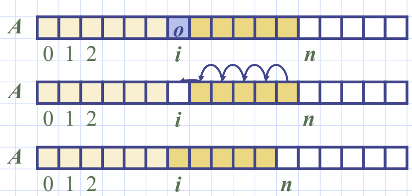
Insertion

- In an operation ***add***(i, o), we need to make room for the new element by shifting forward the $n - i$ elements $A[i], \ldots, A[n - 1]$
- In the worst case ($i = 0$), this takes $O(n)$ time



Element Removal

- In an operation ***remove***(i), we need to fill the hole left by the removed element by shifting backward the $n - i - 1$ elements $A[i + 1], \ldots, A[n - 1]$
- In the worst case ($i = 0$), this takes $O(n)$ time



Performance

- In an array based implementation of a dynamic list:
  - The space used by the data structure is $O(n)$
  - Indexing the element at I takes $O(1)$ time
  - ***add*** and ***remove*** run in $O(n)$ time in worst case
- In an ***add*** operation, when the array is full, instead of throwing an exception, we can replace the array with a larger one…

Growable Array-based Array List

- In an add(o) operation (without an index), we could always add at the end
- When the array is full, we replace the array with a larger one
- How large should the new array be?
  - Incremental strategy: increase the size by a constant c
  - Doubling strategy: double the size

```
Algorithm add(o)
  if t = S.length - 1 then
    A ← new array of
        size …
    for i ← 0 to n-1 do
      A[i] ← S[i]
    S ← A
  n ← n + 1
  S[n-1] ← o
```


Comparison of the Strategies

- We compare the incremental strategy and the doubling strategy by analyzing the total time $T(n)$ needed to perform a series of n add(o) operations
- We assume that we start with an empty stack represented by an array of size 1
- We call amortized time of an add operation the average time taken by an add over the series of operations, i.e., $T(n)/n$

Incremental Strategy Analysis

- We replace the array $k = n/c$ times
- The total time $T(n)$ of a series of n add operations is proportional to

$$n + c + 2c + 3c + 4c + \ldots + kc =$$
$$n + c(1 + 2 + 3 + \ldots + k) =$$
$$n + ck(k + 1)/2$$

- Since c is a constant, $T(n)$ is $O(n + k^2)$, i.e., $O(n^2)$
- The amortized time of an add operation is $O(n)$

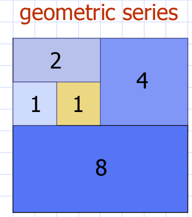
Doubling Strategy Analysis

- We replace the array $k = \log_2 n$ times
- The total time $T(n)$ of a series of n add operations is proportional to

$n + 1 + 2 + 4 + 8 + \ldots + 2^k =$
$n + 2^{k+1} - 1 =$
$3n - 1$

- $T(n)$ is $O(n)$
- The amortized time of an add operation is $O(1)$

geometric series



Python Implementation

```
import ctypes                                    # provides low-level arrays

class DynamicArray:
  """A dynamic array class akin to a simplified Python list."""

  def __init__(self):
    """Create an empty array."""
    self._n = 0                                  # count actual elements
    self._capacity = 1                           # default array capacity
    self._A = self._make_array(self._capacity)   # low-level array

  def __len__(self):
    """Return number of elements stored in the array."""
    return self._n

  def __getitem__(self, k):
    """Return element at index k."""
    if not 0 <= k < self._n:
      raise IndexError('invalid index')
    return self._A[k]                            # retrieve from array

  def append(self, obj):
    """Add object to end of the array."""
    if self._n == self._capacity:                # not enough room
      self._resize(2 * self._capacity)           # so double capacity
    self._A[self._n] = obj
    self._n += 1

  def _resize(self, c):                          # nonpublic utitity
    """Resize internal array to capacity c."""
    B = self._make_array(c)                      # new (bigger) array
    for k in range(self._n):                     # for each existing value
      B[k] = self._A[k]
    self._A = B                                  # use the bigger array
    self._capacity = c

  def _make_array(self, c):                      # nonpublic utitity
     """Return new array with capacity c."""
     return (c * ctypes.py_object)( )            # see ctypes documentation
```